

Doctoral Dissertation

**Conceptualizing Pre-service Mathematics Teachers' Knowledge for
Teaching Probability in Egypt from the Perspective of Probabilistic
Reasoning**

(Summary)

SAMAH GAMAL AHMED ELBEHARY

Graduate School for International Development and Cooperation
Hiroshima University

March 2021

Probability knowledge is needed for all citizens to reason on every day uncertain situations; besides, it is also required to train many professionals at the university level. Furthermore, an essential value for learning probability contributes to the formation of a specific type of reasoning: probabilistic reasoning, which helps learners formally structure their vague thinking about random phenomena. For such reasons, the probability was recognized by educational authorities and included in the curricula of many countries at different levels, from the primary stage to teacher education. However, several concerns have been discussed in the literature regarding the deficiency of probability education, which may negatively impact learners' acquisition of probability knowledge and the development of their probabilistic reasoning.

Among other issues, the following ones have been raised: (a) the probability curriculum was criticized in terms of acquiring a too narrow view of probability; it strengthens the statistical side that is relevant to the objective mathematical rules rather than the epistemic side that interprets probability as a personal degree of belief. And (b) there is an inadequate preparation of Pre-Service Mathematics Teachers (PSMTs) to teach probability efficiently. Both issues were found in the Egyptian context where, on one hand, ideas of independence and conditional probability, probability distribution and expectation, and convergence and the law of large numbers were disregarded from the intended curriculum; and, on the other hand, the implemented curriculum emphasized merely theoretical and axiomatic approaches to probability. Moreover, for the PSMTs, only about 9% of all subjects, which they were studying throughout the whole duration of their preparation program, were assigned to statistics and probability. Besides, there was no particular discussion concerning probability instruction. These issues cause various learning difficulties, and they also create further challenges for teachers, notably because of the distinct characteristics of probability that are not often found in other mathematics areas.

From this aspect, and acknowledging the influence of teachers' knowledge on students' learning and achievement, this study highlighted PSMTs' knowledge for teaching probability; as argued by Dollard (2011), "One way to improve this situation is to ensure that new teachers graduating from teacher education programs have a good understanding of the fundamental concepts of probability" (p. 27). Moreover, this study embraced the perspective of probabilistic reasoning (PoPR) to approach such knowledge. This was decided in the light of reviewing the historical development of probability education research wherein the PoPR was assumed to construct "a more unified development of the classical, frequentist, and subjective approaches to probability" (Jones et al., 2007, p. 949), and to clarify issues of subjective probability that did not receive much attention in the field of mathematics education.

Based on the previous discussion, this study was intended to *conceptualize PSMTs' knowledge for teaching probability in Egypt from the PoPR* through answering these questions:

RQ1. What is the current status of "statistics and probability" education in Egypt?

RQ2. What is the definition of mathematics teachers' professional knowledge for teaching probability from the PoPR?

RQ3. What are the characteristics of PSMTs' knowledge for teaching probability in Egypt from the PoPR?

To answer the first research question, issues of statistics and probability education in Egypt were outlined locally and, then, internationally. From a local perspective, several documents

were reviewed. As a result, it was evident that the Egyptian government advocated the need to enhance PSMTs' professional competence to meet pupils' needs, especially to teach contents of statistics and probability. This content constituted a less emphasized area of study during PSMTs' preparation; furthermore, Egyptian pupils' achievement in the content area of Data and chance stayed the lowest among other mathematics areas, as reported by TIMSS 2003 and 2007. More specifically, **probability** denoted a core concept for which most of the textbooks' activities aimed, which is promoting pupils' probabilistic understanding.

Additionally, both the intended and implemented Egyptian school curricula of probability were analyzed from an international viewpoint. The intended curriculum was compared with the New Zealand curriculum, where ideas of independence and conditional probability, probability distribution and expectation, and convergence and the law of large numbers appeared to be ignored within the Egyptian curriculum. Moreover, analyzing the implemented curriculum exhibited a lack of addressing experimental probability interpretation; also, the subjective probability approach was revealed to be neglected until grade 9.

To answer the second research question, two essential steps were performed. While the first step outlined mathematics teachers' professional knowledge for teaching probability as defined in the literature (knowledge for practice), the second step manifested the psychological facet of teachers' knowledge; it was exemplified through their conceptions (knowledge in practice) and reasoning processes.

For the first step, aspects of **KoP**, **KoTP**, **KoSPK**, and **KoPL** were explored. Moreover, the **KoP** was sharpened in this study since it determined the heart of mathematics teachers' professional knowledge for teaching probability. It also indicated teachers' epistemological reflection on the meaning of probability, which requires an understanding of its (a) objective facet that emphasizes the mathematical rules that govern random processes, and (b) subjective facet that sharpens the information available to the person assigning that probability. Despite this, these components disregarded the dynamic aspect of teachers' knowledge; they neither considered teachers' reasoning processes nor the cognitive biases that shape their pedagogical practices. Thus, to exhibit these aspects, the second step was recognized through introducing the study premises. These premises were: (a) conceptions represent knowledge in evolution, (b) reasoning indicates an individual cognitive process to interpret the acquired knowledge, and (c) there is a reciprocal relationship between conceptions and reasoning.

In light of both steps, the study framework was represented. Accordingly, mathematics teachers' professional knowledge for teaching probability was redefined from the PoPR to include these aspects: **R(in)P**, **R(in)PL**, **R(in)TP**, and **R(in)SPK**, which determine their reasoning in a situation that involves knowledge of probability, probability language, teaching probability, and students probability knowledge, respectively.

Finally, answering the third research question incorporated characterizing **PSMTs' R(in)P** that is related to (a) *simple unconditional* and (b) *conditional* probabilistic situations. Both issues of (a) and (b) were handled through a field study, in which a sample of sixty-eight PSMTs, who studied the mathematics teachers' preparation program during the academic year 2018-2019 at the Faculty of Education, Tanta University, Egypt, was engaged in this study.

The data were collected using a questionnaire; it included six items that were developed in terms of acknowledging (a) the value of adopting a social problem, (b) the school curriculum viewpoint, (c) the pupils' perspective, and (d) issues of previous research. As a result, PSMTs'

reasoning in a *simple unconditional probabilistic situation* was characterized within four major categories: mathematical [M], subjective [S], outcome [O], and intuitively [I] oriented thinkers. Besides, several cognitive biases emerged (e.g., equiprobable bias, insensitivity to the prior probability of outcomes, representativeness heuristic, overgeneralization heuristic, prediction bias, dependence conception, Allah's will, prediction conception, and causal conception). Furthermore, and essentially, the three factors of variability, randomness, and contextual recognition emerged in all manners of reasoning.

Additionally, PSMTs' reasoning in a *conditional probabilistic situation* was also characterized within two broad categories of intrasubjective and intersubjective thinkers. The intrasubjective included those who shared the overgeneralization heuristic, confusion between joint and conditional probability, the combination of the confusion between conditioning and conditioned event and independence conception, and the illusion of validity. Besides, the intersubjective thinkers incorporated those who practiced the availability heuristic; the reluctance to believe that the condition restricts not only the sample space but also the favorable outcome; the combination of the confusion between the conditioned event and another event, and the reluctance to believe that the condition restricts not only the sample space but also the favorable outcome; the combination of the confusion between conditioning and conditioned event, and the reluctance to believe that the condition restricts not only the sample space but also the favorable outcome; unawareness of basic probability axioms; the gambler fallacy; the causal conception; the fallacy of transposed conditional; and the confusion between the conditioned event and another event in the experiment.

Based on such findings, some directions for future research were proposed. For example, similar studies might be conducted over different groups (e.g., PSMTs in other universities, in-service teachers) to get a broad and profound understanding of the current state of mathematics teachers' knowledge for teaching probability in Egypt. Besides, to validate this study's results, different investigations might be carried out using more specific questions.

Furthermore, while the study findings exposed, on one hand, some misconceptions that mismatch with the probability theory, and on the other hand, concepts of variability, randomness, and contextual recognition as crucial factors to reason probabilistically, more areas for future research can be adopted. For example, how to improve PSMTs' conceptual knowledge of probability through a pedagogical treatment; also, how to change the traditional way of teaching probability and instead focus on concepts of variability, randomness, and contextual recognition.