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Abstract

Ignoring distortions, the social benefit from an urban transportation improvement is typically measured as the reduction in transportation costs at a particular location, holding travel fixed at the pre-improvement level, summed over all locations. The result has been seen in specific urban transportation models, however, not formally derived as a general result. Through a transformation of variables first employed in Arnott and Stiglitz (1981), this paper adopts a different perspective to generalize the result. From this perspective, the benefits of a transportation improvement derive from the increase in the residential land area of better accessibility that the improvement brings about. The generalized result tells that a transportation improvement changes the land area at varying levels of accessibility, as well as land rents throughout the city, however, the social benefit of the transportation improvement is then measured as the increase in aggregate differential land rents due to a change in land area at each level of accessibility, rather than that due to a change in land rent, by holding the function relating land rent to accessibility at its pre-improvement levels. This paper presents this result by using a basic monocentric model as a vehicle of demonstration and then shows that the result generalizes beyond the geographic features of the city, to a broad class of first-best urban economies with multiple transportation modes and employment centers and multiple household groups.

Keywords: Optimal transportation network, Land use models

JEL codes: R14, R42, R52

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1 Introduction

Ignoring distortions, the social benefit from an urban transportation improvement is typically measured as the reduction in transportation costs at a particular location, holding travel fixed at the pre-improvement level, summed over all locations. The result has been seen in a range of urban transportation models, each of which assumes its specific geographic features. However, the result is not formally derived as a general result. Through a transformation of variables first employed in Arnott and Stiglitz (1981), this paper adopts a different perspective to generalize the result. It measures the accessibility of each location, and then regards a transportation improvement as raising that location's accessibility. The social benefit of the transportation improvement derive from an increase in the residential land area of better accessibility that the improvement brings about. The generalized result tells that a transportation improvement changes the quantities of land at different accessibility levels as well as land rents throughout the city, however, the social benefit of the transportation improvement is then measured as the increase in aggregate differential land rents due to a change in land area –rather than that due to a change in land rent– summed over all levels of accessibility, holding the function relating land rent to accessibility at its pre-improvement levels. This paper start out by presenting this result in the context of a basic monocentric city model precisely for expositional simplicity, and then shows that this result is robust to the generalization not only to the geographic features of the city and its transportation network, but also to a broad class of first-best urban economies such as those with endogenous time allocation, multiple household groups, and multiple transportation modes and employment subcenters, as well as the use of land in transportation and traffic congestion.

More formally: In the basic monocentric model, where η denotes transportation capacity, define the transportation cost shape of the city $\Xi(C;\eta)$ to be the residential land area for which transportation costs to the city center are less than or equal to C with $\xi(C;\eta) = d\Xi(C;\eta)/dC$, $R(C;\eta)$ be the equilibrium land rent at C, and R_A be the agricultural rent. Letting subscript 0 denote the pre-improvement equilibrium, the marginal social benefit from a transportation improvement equals $\int_0^{\bar{C}} (R(C;\eta_0) - R_A)\,\partial\xi(C;\eta_0)/\partial\eta dC$ where $\bar{C}$ is transportation costs to the city center at the city boundary. Also, where $K(\eta)$ denotes the construction cost of capacity η, with optimal transportation capacity, η^*, $K'(\eta^*) = \int_0^{\bar{C}} (R(C;\eta^*) - R_A)\,\partial\xi(C;\eta^*)/\partial\eta dC$. It can be show that the change in the aggregate differential land rent has two parts: $\int_0^{\bar{C}} (R(C) - R_A)\,\partial\xi(C)/\partial\eta dC$ and $\int_0^{\bar{C}} \xi(C)\,\partial(R(C) - R_A)/\partial\eta dC$. However, optimality calls for only the former to be equal to the marginal cost. In words, the marginal social benefit of a transportation improvement equals the increase in aggregate differential land rents it induces due to a change in land area, not in the land rent, evaluated at pre-improvement rents, and the optimal level of capacity is such that the marginal social benefit of a transportation improvement equals the marginal construction cost.

Section 2 derives the result for the basic monocentric model. Section 3 examines how the result generalizes when the model is extended in a variety of ways in the direction of realism. Section 4 relates the result to previous results obtained in the urban land use literature.

2 Basic Model and Main Result

2.1 Standard monocentric land use models

We first set up and review a standard land use model in which a fixed number of identical households reside, commute and work. Production occurs within the central business district (CBD). We assume full employment; one member per household commutes to the CBD and works for fixed hours. This labor is the sole production factor, with which generic good is produced under constant returns to scale. Each household receives an equal share of the generic good as a daily wage, say W. The size of and, therefore, the transportation cost within the CBD are assumed to be negligible.

Let us denote by α the fraction of land ownership by city dwellers, with $\alpha \in [0, 1]$. Thus, $\alpha = 0$ means that all land is owned by absentee landlords, and $\alpha = 1$ means that all land is owned by residents. There is an alternative use of the land for agriculture with a uniform opportunity cost R_A. The land allocated to the transport network is not considered.

Households' problem

Let (x_1, x_2) represent a location in the city, say in polar coordinate where x_1 is a radial distance from the CBD and x_2 is an angler displacement, and $C(x_1, x_2)$ be the cost of a round-trip commute from (x_1, x_2) to the CBD. Given the transportation capacity η and resulting commuting cost $C(x_1, x_2)$ for all (x_1, x_2), each household chooses a location (x_1, x_2) and the amount of consumption of residential land Q and all other non-residential goods Z as a numeraire composite good so as to maximize utility subject to its budget constraint. In addition to the wage income it earns in the CBD, each household receives, according to its ownership of land, an equal share of the aggregated differential land rent denoted by Φ. Households also pay a lump-sum tax T to finance transport infrastructure.

We here describe the dual of the above. When we denote the land rent at location (x_1, x_2) by $\widetilde{R}(x_1, x_2)$, each household's expenditure minimization problem becomes

$$\begin{aligned} E\left(\widetilde{R}(x_1, x_2), U; \eta\right) &= \min_{Z,Q} Z + \widetilde{R}(x_1, x_2) Q \\ & \quad s.t.\ U = U(Z, Q) \end{aligned}$$

where E is the expenditure function. Solving this minimization problem yields the compensated demand functions

$$\begin{aligned} Q^c &= Q^c\left(\widetilde{R}(x_1, x_2), U\right) \\ Z^c &= Z^c\left(\widetilde{R}(x_1, x_2), U\right). \end{aligned}$$

The bid rent function $\widetilde{R}(x)$ is the maximum amount that a household is willing to pay for a land lot of unit area provided that it achieves a given utility level:

$$\widetilde{R}(x_1, x_2) = \frac{1}{Q^c}\left[W - T + \alpha \frac{\Phi}{N} - C(x_1, x_2) - Z^c\right].$$

Spatial equilibrium

The homogeneity of households implies that they achieve the same level of utility regardless of their location. We denote the spatial equilibrium level of utility by $\overline{U}$. The land rent adjusts across locations to yield the same utility level.

Let N be the number of households and $\mathfrak{R}$ be the set of all locations in the city such that $\mathfrak{R} \equiv \left\{ r, \theta \,\middle|\, \widetilde{R}(x_1, x_2) \geq R_A \right\}$, where R_A is an agricultural rent that is the opportunity cost of land. The fact that population N resides within the city area $\mathfrak{R}$ yields that

$$N = \iint_{x_1, x_2 \in \mathfrak{R}} \frac{x_1 dx_1 dx_2}{Q^c\left(\widetilde{R}(x_1, x_2), \overline{U}\right)}. \tag{1}$$

At the city boundary, the residential bid rent must be equal to the agricultural land rent. This gives

$$\widetilde{R}(\bar{x}_1, \bar{x}_2) = R_A \tag{2}$$

where $(\bar{x}_1, \bar{x}_2)$ represent the location at the city boundary. The aggregate differential land rent Φ is then finally obtained as

$$\Phi = \iint_{x_1, x_2 \in \mathfrak{R}} \left(\widetilde{R}(x_1, x_2) - R_A\right) x_1 dx_1 dx_2 \tag{3}$$

which is redistributed to the city dwellers and thus constitutes a part of their budget, according to the share of land ownership α. Solving Equations (1), (2) and (3) together yields $\overline{U}$, $(\bar{x}_1, \bar{x}_2)$ and Φ in terms of all of the exogenous variables and the lump-sum tax T.

2.2 A New Way of Evaluating the Benefits of a Transportation Improvement

Calibrating the above set of spatial equilibrium conditions for the optimal transport network requires specifying the geographic features of the city. This is the point of departure among studies in the literature, where each model goes a separate way according to their model specifications, to find the above mentioned property that optimum is such that social benefit is equal to transport time or cost reduction multiplied by travel volume at pre-improvement level at each location, summed over all locations. This result in the literature is robust in the sense that it has always been holding in these specific models so far, but not in the sense that it is established to hold in general. We attain this by using isomorphism; isomorphism abstracts away the geographic features of the city such as that of a monocentric city, which we used so far just to describe the spatial equilibrium. We will see how it is generalized in this section.

Rewriting the above spatial equilibrium

Using isomorphism, we can express the above set of spatial equilibrium equations in terms of the commuting cost at each location.[1] Let us redefine the land rent as a function of the transportation

[1] The idea of isomorphism was first introduced by Arnott and Stiglitz (1981), who described the spatial equilibrium of a monocentric city in terms of transport cost as a measure of distance.

cost C and denote it by R. That is,

$$R(C(x_1,x_2)) = \widetilde{R}(x_1,x_2).$$

Using this, we rewrite the optimal consumption of lots and composite goods in terms of C:

$$Q^*(C) = Q^c\left(R(C), \overline{U}\right) \tag{4}$$

$$Z^*(C) = Z^c\left(R(C), \overline{U}\right) \tag{5}$$

where $\overline{U}$ is the spatial equilibrium level of utility as obtained above. With these items, land rent $R(C)$ satisfies the following:

$$R(C) = \frac{1}{Q^*(C)}\left[W - T + \alpha\frac{\Phi}{N} - C - Z^*(C)\right]. \tag{6}$$

We define $\xi(C;\eta)\,dC$ as the land area where the transportation cost is equal to C given transport capacity η. That is, by letting $\Xi(C;\eta)$ be the area of the city where the transportation cost is less than or equal to C, we have $\xi(C;\eta) = d\Xi(C;\eta)/dC$ for $C \geq 0$. The spatial equilibrium can now be rewritten in terms of C:

$$N = \int_0^{\bar{C}} \frac{\xi(C;\eta)}{Q^*(C)}dC \tag{7}$$

$$R\left(\bar{C}\right) = \frac{W - T + \alpha\Phi/N - \bar{C} - Z^*\left(\bar{C}\right)}{Q^*\left(\bar{C}\right)} = R_A \tag{8}$$

$$\Phi = \int_0^{\bar{C}} \left(R(C) - R_A\right)\xi(C;\eta)\,dC \tag{9}$$

where $\bar{C}$ is the transportation cost at the city boundary, i.e., $\bar{C} = C(\bar{x})$. The equilibrium utility $\overline{U}$, the transportation cost at the city boundary $\bar{C}$, and the aggregated differential land rent Φ satisfy the above three equations.[2]

The generalized optimality condition

The construction cost of the transport network depends on the characteristics of the network improvement, such as the width and/or length of highways and the frequency and capacity of a railway. We denote the construction cost and the characteristics of the transport network by K and a vector η, respectively, that is,

$$K = K(\eta). \tag{10}$$

Transport authority is assumed to maintain a balanced budget:

$$K = TN. \tag{11}$$

[2] It can be shown that the ratio of the aggregate transport cost to the aggregate differential land rent becomes

$$\frac{\int_0^{\bar{C}} \frac{C\xi(C)}{Q^*(C)}dC}{\int_0^{\bar{C}} \frac{\Xi(C)}{Q^*(C)}dC}$$

as implied by Arnott and Stiglitz (1981). See Appendix A.3.

With a slight abuse of notation, let K_η be $\partial K/\partial \eta_i$ where η_i is an arbitral element of the transport network characteristics vector η. For notational simplicity, we abbreviate its subscript i in what follows. The transport authority's problem is then to maximize the equilibrium utility $\overline{U}$ by optimizing characteristics of the transport network η. Total differentiation of spatial equilibrium conditions (7) through (9) as well as the construction technology and budget constraints faced by the transport authority given in (10) and (11), together with $d\overline{U}/d\eta = 0$ yields the necessary conditions for the optimal transport network.[3]

Optimality then implies that

$$K_\eta + (1-\alpha)\frac{d\Phi}{d\eta} = \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC \tag{12}$$

and that

$$\frac{d\Phi}{d\eta} - N\frac{d\bar{C}}{d\eta} = \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC \tag{13}$$

where in the second line we use the fact that from (9), the change in the aggregate differential land rent Φ from a marginal improvement in transport network characteristics η consists of two parts, namely, one due to a change in land *area* and another due to a change in land *rent* at each level of commuting cost:

$$\frac{d\Phi}{d\eta} = \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC + \int_0^{\bar{C}} \xi(C)\frac{\partial (R(C) - R_A)}{\partial \eta} dC. \tag{14}$$

The right-hand side of the optimality condition (12) is the same as the first term of the marginal aggregate differential land rent given in (14), i.e., the change in the aggregate differential land rent Φ due to a change in land *area* at each level of commuting cost. We summarize this result in the following proposition.

Proposition 1. *In the optimal transport network, the marginal cost of transport network improvement plus the payment of land rent to the absentee landlord, if any, is equal to the change in the aggregate differential land rent due to a change in the land area at each level of commuting cost while holding the land rent constant at the pre-improvement level.*

Sufficiency in the case of resident land ownership

The optimality condition for the case of resident land ownership immediately obtains by letting $\alpha = 1$ in (12):

$$K_\eta = \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C;\eta^*)}{\partial \eta} dC, \tag{15}$$

for which the sufficiency condition is straightforwardly obtained as

$$K_{\eta\eta} > \frac{d}{d\eta}\left\{\int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C;\eta^*)}{\partial \eta} dC\right\}, \tag{16}$$

[3] Appendix A.1 provides the derivation.

i.e., the gradient of the marginal construction cost should be strictly greater than the gradient of the marginal benefit that is the aggregate differential land rent due to the change in land area with respect to the transportation capacity η.[4]

The optimality condition (15) and sufficiency condition (16) contain neither the change in the aggregate differential land rent $d\Phi/d\eta$ nor the change in the land rent $\partial R/\partial\eta$ at any location. This result carries policy relevance since in determining the optimal transport improvement, the transport authority only needs to know the resulting change in commuting costs in the city and not the anticipated change in the land rent, which is considerably more difficult to estimate.

An example

Our optimality condition can be indeed considered as a generalization of the results of more specific models in the literature. One clear example is that by Kanemoto (1984). Kanemoto (1984) investigated the optimal railway network in a monocentric city with resident land ownership.[5] In this city, commuters walk circumferentially to the nearest radial railway that is accessible anywhere along it and then take the railway to travel radially to the CBD. By letting x_1 be the radial distance from the CBD and x_2 be the circumferential distance from the nearest railway, he found that the optimal railway length is such that

$$2\int_0^{\bar{x}_2(\bar{x}_1)} \left[\widetilde{R}\left(\bar{x}_1, x_2\right) - R_A\right] dx_2 \quad = \quad K_\eta \tag{17}$$

where bar $(-)$ indicates the city boundary, and K_η is the marginal cost of extending the railway a unit distance.

This is indeed a special case of our general optimality condition. Extending the railway does not change the commuting cost anywhere in the city except at the margin. That is, $\partial\xi\left(C\left(\bar{x}_1, x_2\right)\right)/\partial\eta = 2\left(\partial C\left(\bar{x}_1, x_2\right)/\partial x_2\right)^{-1}$ for any $C \in \left[C\left(\bar{x}_1, 0\right), \bar{C}\right]$ and zero otherwise.[6] Hence, our optimality condition in (15) transforms into the following:

$$K_\eta \quad = \quad 2\int_{C(\bar{x}_1,0)}^{\bar{C}} \left(R\left(C\right) - R_A\right)\left(\frac{\partial C\left(\bar{x}_1, x_2\right)}{\partial x_2}\right)^{-1} dC.$$

Substituting $dC = \left[\partial C\left(\bar{x}_1, x_2\right)/\partial x_2\right] dx_2$ in the above integration together with the fact that $\bar{C} = C\left(\bar{x}_1, \bar{x}_2\left(\bar{x}_1\right)\right)$ yields Kanemoto's result in (17).

3 Extensions

Here in this section we show that the optimality condition is indeed extends to a wide class of urban land use models, namely those with endogenous time allocation, multiple transportation modes and employment centers, and multiple household groups.

[4]See Appendix A.2 for its derivation.

[5]Another example is the result by Anas and Moses (1979). Anas and Moses (1979) discussed the optimization of the width of radial highways in a two-dimensional monocentric city with absentee landlords. Their optimization condition can also be derived as a special case of our results shown in equations (12) and (13).

[6]It is multiplied by two for both sides of the railway.

Time costs of travel

The results are unchanged when we alternatively consider households optimizing their time allocation among work, commuting and leisure. Let L be the leisure time consumption, and replace W with wH, where w is the hourly wage and H is the household's time endowment. By defining the land rent at location (r, θ) as $\widetilde{R}(x_1, x_2)$, the household's budget constraint becomes

$$Z + \widetilde{R}(x_1, x_2) Q + wL = wH - T + \alpha \frac{\Phi}{N} - C_h(x_1, x_2)$$

where $C_h(x_1, x_2)$ is now treated as the generalized transportation cost that includes both the monetary and time costs of commuting. That is,

$$C_h = C_h^m(x_1, x_2) + wC_h^t(x_1, x_2)$$

where C_h^m and C_h^t are the monetary and time costs, respectively, for a round-trip commute from (x_1, x_2) to the CBD via route h.

Multiple routes and multiple CBDs

A city can contain multiple travel modes, and their combinations create different travel routes from a given location to the CBD. For example, modes of travel can include automobile, rail, biking, and walking. Commuters can choose a route say on highways, city streets, or both to travel to the CBD, denoted by h. Solving the household's problem yields the indirect utility function conditional on the location and the travel mode h.

In other words, each household chooses a travel route h to maximize this conditional indirect utility. This problem is identical to the route-choice problem of minimizing the transportation cost. Let $C_h(x_1, x_2)$ be the cost of a round-trip commute from (x_1, x_2) to the CBD via route h. The bid rent function $\widetilde{R}_h(x_1, x_2)$ of a household commuting via route h is

$$\widetilde{R}_h(x_1, x_2) = \frac{1}{Q^c}\left[W - T + \alpha \frac{\Phi}{N} - C_h(x_1, x_2) - Z^c\right].$$

Because the land is given to the highest bidder, the land rent is the maximum of the bid rents:

$$\widetilde{R}(x_1, x_2) = \max_h \widetilde{R}_h(x_1, x_2).$$

The argument of the maximum here coincides with that of the minimum of the transportation cost over routes h. That is, residents in each location chooses a route to the CBD that incurs the lowest commuting cost. Then we define $C(x_1, x_2)$ as the resulting minimum commuting cost at location (x_1, x_2) such that

$$C(x_1, x_2) = \min_h C_h(x_1, x_2) \tag{18}$$

to obtain the above results.

A case with multiple CBDs with different productivities can be interpreted within the above setting. Let $C_{h,i}(x_1, x_2)$ be the transportation cost from (x_1, x_2) to the ith CBD via route h. Define i^* such that $i^* = \arg\max_i W_i - C_i(x_1, x_2)$, where $C_i(x_1, x_2) = \min_h C_{h,i}(x_1, x_2)$, and W_i is the daily wage in the ith CBD. Then, results are unchanged by defining the transportation cost $C(x_1, x_2)$ alternatively as

$$C(x_1, x_2) = C_{i^*}(x_1, x_2) + \left(\max_i W_i - W_{i^*}\right)$$

i.e., adding the wage difference $(\max_i W_i - W_{i^*})$ to the commuting cost $C_{i^*}(x_1, x_2)$, while viewing $\max_i W_i$ as W. By defining the commuting cost in this way, everything then follows as in the previous section above.[7]

Heterogeneous residents

Let us consider two groups of households that differ in income.[8] We denote by W_r and W_p the income of rich and poor households, respectively.[9] We denote the equilibrium level of utility by $\overline{U}_j$ for household class $j = r, p$. Using this, we rewrite the optimal consumption of lots and composite goods in terms of C:

$$Q_j^*(C) = Q_j^c\left(R_j(C), \overline{U}_j\right) \tag{19}$$

$$Z_j^*(C) = Z_j^c\left(R_j(C), \overline{U_j}\right). \tag{20}$$

With these items, bid rent $R_j(C)$ satisfies the following:

$$R_j(C) = \frac{1}{Q_j^*(C)}\left[W_j - T + \alpha\frac{\Phi}{N} - C - Z_j^*(C)\right]. \tag{21}$$

Differentiating the above with respect to the commuting cost C and noting that $d\overline{U}_j/dC = 0$ in a spatial equilibrium yields

$$\frac{dR_j}{dC} = -\frac{1}{Q_j^*(C)}.$$

Assuming that land is normal, this means that for any given land rent R and commuting cost C, the bid rent curve of the poor is steeper than that of the rich. This further implies that their bid rents intersect only once, say at $\hat{C}$, and that poor households live "inside" of $\hat{C}$ and the rich live "outside" of $\hat{C}$.[10]

The spatial equilibrium conditions are then expressed as

$$\begin{aligned} N_p &= \int_0^{\widehat{C}} \frac{\xi(C)}{Q_p^*(C)} dC \\ N_r &= \int_{\widehat{C}}^{\bar{C}} \frac{\xi(C)}{Q_r^*(C)} dC \\ R_p\left(\widehat{C}\right) &= R_r\left(\widehat{C}\right) \\ R_r(\bar{C}) &= \frac{W_r - T + \alpha\Phi/N - \bar{C} - Z^*\left(\bar{C}\right)}{Q_r^*\left(\bar{C}\right)} = R_A \end{aligned}$$

[7]Viewing this wage difference $(\max_i W_i - W_{i^*})$ as a cost of "teleportation" simply tells that residents can now choose to take either the "teleportation" or "transportation" route from any CBDs to the CBD with the highest wage, whichever the cost is lower. Interpreting multiple CBDs in this way widens the scope of our "city" to a much greater geographical area, such as an entire country, where internal migration is free but immigration is not.

[8]The result readily extends to the case of more than two household classes.

[9]Here and in what follows, subscripts r and p denote rich and poor, respectively.

[10]That is, the poor live in the area where the commuting cost is in $\left[0, \hat{C}\right]$ and rich live where it is in $\left[\hat{C}, \bar{C}\right]$.

$$\Phi = \int_0^{\widehat{C}} \left(R_p\left(C\right) - R_A\right) \xi\left(C\right) dC + \int_{\widehat{C}}^{\bar{C}} \left(R_r\left(C\right) - R_A\right) \xi\left(C;\eta\right) dC.$$

The first two lines show that the population of rich and poor, namely N_r and N_p, must fit in the area with commuting costs above and below $\hat{C}$, respectively, where $N_r + N_p = N$. The third line indicates equality of bid rents of rich and poor at $\hat{C}$. The fourth line indicates that at the city boundary, the bid rent of the rich is equal to the agricultural rent. The last line gives the aggregate differential land rent Φ.

The transport authority again faces the budget constraint that the construction cost of the transport network K is financed by the lump-sum tax T:

$$K = TN.$$

Subject to the spatial equilibrium conditions and the budget constraint above, the transport authority maximizes social welfare Ω, which is a function of equilibrium utility for rich and poor:

$$\max_{\eta} \Omega = \Omega\left(\overline{U}_r, \overline{U}_p\right).$$

At the optimum, we thus have

$$\frac{d\Omega}{d\eta} = \frac{\partial\Omega}{\partial\overline{U}_r}\frac{d\overline{U}_r}{d\eta} + \frac{\partial\Omega}{\partial\overline{U}_p}\frac{d\overline{U}_p}{d\eta} = 0. \tag{22}$$

Note here that the opportunity cost of marginally increasing the spatial-equilibrium level of utility for rich and poor is

$$\int_{\widehat{C}}^{\bar{C}} \lambda_r\left(C\right) \frac{\xi\left(C;\eta\right)}{Q_r^*\left(C\right)} dC$$

$$\int_0^{\widehat{C}} \lambda_p\left(C\right) \frac{\xi\left(C;\eta\right)}{Q_p^*\left(C\right)} dC$$

respectively, where $\lambda_j = \partial E_j / \partial \overline{U}_j$ is the marginal expenditure necessary to increase the utility of type j, and $\xi\left(C\right)/Q_j^*\left(C\right)$ is the population at a location where the commuting cost is C for $j = r, p$. We therefore define the social welfare function such that at the optimum, the marginal rate of substitution is equal to the relative price given as above, or equivalently that

$$\frac{\partial\Omega/\partial\overline{U}_r}{\partial\Omega/\partial\overline{U}_p} = \frac{\int_{\widehat{C}}^{\bar{C}} \lambda_r\left(C\right) \frac{\xi\left(C;\eta^*\right)}{Q_r^*\left(C\right)} dC}{\int_0^{\widehat{C}} \lambda_p\left(C\right) \frac{\xi\left(C;\eta^*\right)}{Q_p^*\left(C\right)} dC}. \tag{23}$$

Total differentiation of the spatial equilibrium conditions implies

$$\begin{aligned} K_\eta + \left(1-\alpha\right)\frac{d\Phi}{d\eta} &= \int_0^{\widehat{C}} \left(R_p\left(C\right) - R_A\right) \frac{\partial\xi\left(C;\eta^*\right)}{\partial\eta} dC + \int_{\widehat{C}}^{\bar{C}} \left(R_r\left(C\right) - R_A\right) \frac{\partial\xi\left(C;\eta^*\right)}{\partial\eta} dC \\ &\quad - \left[\int_0^{\widehat{C}} \lambda_p\left(C\right) \frac{\xi\left(C;\eta^*\right)}{Q_p^*\left(C\right)} dC\right] \frac{d\overline{U}_p}{d\eta} - \left[\int_{\widehat{C}}^{\bar{C}} \lambda_r\left(C\right) \frac{\xi\left(C;\eta^*\right)}{Q_r^*\left(C\right)} dC\right] \frac{d\overline{U}_r}{d\eta} \end{aligned} \tag{24}$$

where the second line goes to zero by using equations (22) and (23).[11] By noting that $R(C) = \max\{R_r(C), R_p(C)\}$ and hence that

$$R(C) = \begin{cases} R_p(C), & \forall C \in \left[0, \hat{C}\right] \\ R_r(C), & \forall C \in \left[\hat{C}, \bar{C}\right], \end{cases}$$

this finally yields the same optimality condition as described in Proposition 1.

4 Relationships to corollaries in the land-use models

In this section, we contrast our result with other well-known corollaries in land-use models.

Samuelson condition in land-use models

The Samuelson condition in land-use models states that the marginal project benefit is equal to the marginal project cost. In our case, there is no direct project benefit; benefits arise indirectly through the reduction in commuting costs and savings in the payment to absentee landlords, if any. This is expressed in our notation as

$$K_\eta = -\frac{d}{d\eta}\left[\left(N\bar{C} - \Phi\right) + (1-\alpha)\Phi\right] \tag{25}$$

which is confirmed by equating the left hand sides of our optimality conditions in (12) and (13).[12]

For the case of resident land ownership, this becomes

$$K_\eta = -\frac{d}{d\eta}\left[N\bar{C} - \Phi\right].$$

Noting that $N\bar{C} - \Phi$ is the aggregate transportation cost, it simply confirms that the marginal cost of a transport network improvement is equal to the resulting savings in the aggregate transportation cost. For the case of absentee landlords, it becomes

$$K_\eta = -\frac{d}{d\eta}\left[\Phi + \left(N\bar{C} - \Phi\right)\right],$$

which then verifies that optimality implies that the the marginal cost of a transport network improvement is equal to the resulting savings in the land rent payment and aggregate transportation cost in the case of absentee landlords.

Henry George theorem

The Henry George Theorem states that at the optimal population, the shadow profit from decreasing-returns-to-scale production and the shadow loss from increasing-returns-to-scale production are equal.[13] The production of residential land area exhibits decreasing returns to scale in terms of aggregate commuting costs, and transport infrastructure and the land rent payment to the absentee landlord exhibit

[11] See Appendix A.4 for the derivation.

[12] Recall that our primary result as summarized in Proposition 1 is given by (12), which is therefore clearly different from the Samuelson condition.

[13] See Arnott (2004).

increasing returns to scale, while the production of the other good exhibits constant returns to scale. In our context, the aggregate transportation cost is $N\bar{C}-\Phi$ and the payment to the absentee landlord is $(1-\alpha)\Phi$. Therefore, for the Henry George Theorem to hold, it must be the case that

$$N\frac{d\left(N\bar{C}-\Phi\right)}{dN}-\left(N\bar{C}-\Phi\right) \quad = \quad N\frac{d\left[K+(1-\alpha)\Phi\right]}{dN}-\left[K+(1-\alpha)\Phi\right], \tag{26}$$

which is indeed implied by $d\overline{U}/dN=0$.

Heuristically, this may be easiest to see by focusing on the utility of the households living at the city boundary. Since at the city boundary the relative price between Z and Q is fixed at $1/R_A$, maximizing utility there reduces to maximizing Z. Note that when Z is maximized with respect to N, we have $Z=d(NZ)/dN$. From the local constant returns at the optimal population, the resource constraint gives Z as follows:

$$Z \quad = \quad W-\frac{1}{N}\left[K+\left(N\bar{C}-\Phi\right)+(1-\alpha)\Phi\right].$$

Solving $Z=d(NZ)/dN$ and rearranging terms gives (26).[14]

Capitalization hypothesis

According to the capitalization hypothesis, the total marginal project benefit must be equal to the marginal aggregate differential land rent. Therefore, for the capitalization hypothesis to hold, we need

$$-\frac{d}{d\eta}\left[(1-\alpha)\Phi+\left(N\bar{C}-\Phi\right)\right] \quad = \quad \frac{d\Phi}{d\eta}.$$

This will hold, for example, when (i) $\alpha=1$ and $d\bar{C}/d\eta=0$, i.e., all land is owned by residents and the transport network improvement changes the commuting cost everywhere except at the city boundary; alternatively, (ii) $\alpha=0$, and $-Nd\bar{C}/d\eta=d\Phi/d\eta$, i.e., the change in the commuting cost at the city boundary is equal to the change in the differential land rent per household. However, in our model settings, the capitalization hypothesis in general does not hold, even given the optimal transport network.

5 Conclusion

This paper presented a new way of evaluating the benefits of a transportation improvement for the basic monocentric model, and extended it to a broad class of first-best urban economies, i.e., multiple household groups, multiple transportation modes, and multiple employment subcenters. Ignoring distortions, the social benefit from an urban transportation improvement is typically measured as the reduction in transportation costs at a particular location, holding travel fixed at the pre-improvement level, summed over all locations. Through a transformation of variables first employed in Arnott and

[14] Alternatively, comparative statics analysis yields the same result, as we have

$$d\overline{U} \quad = \quad -U_Z d\bar{C}-U_Z dT+\alpha\frac{U_Z}{N}d\Phi-\alpha\frac{U_Z\Phi}{N^2}dN$$

where letting $d\overline{U}/dN=0$ and noting $K=TN$ yield (26).

Stiglitz (1981), we derived that the optimal transportation network requires the marginal cost of the network improvement – plus the payment to absentee landlord, if any – to be equal to the resulting change in the aggregate differential land rent due to a change in land area at each level of commuting cost, evaluated at the current levels of land rent. This optimality condition for the transportation network improvement has been found in more specific city models in the literature that address the shape and size of a city and its transportation network. The condition for an optimal railway network by Kanemoto (1984) and that for an optimal highway width by Anas and Moses (1979) are relevant examples.

The result carries policy implications in the case of resident land ownership. Information on the change in the land rent resulting from the transport improvement is not required, which is usually much more difficult to predict than the resulting change in commuting costs. We also examined the relationship of the results to the corollaries in land use models. While we confirmed the Samuelson condition to hold and interpreted the Henry George theorem within our model setup, the capitalization hypothesis does not hold in general. Future studies could include a congestion externality, housing rent rather than land rent, transportation capacity using up space, and endogenous trip frequency. Population growth can also be introduced, as Miyao (1977) did for a monocentric city, in general settings.

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A Derivations of Relevant Conditions

A.1 Derivation of optimality conditions

Total differentiation of equations (7) through (11) as well as the network characteristics η yields

$$
\begin{aligned}
0 &= \int_0^{\bar{C}} \frac{1}{Q^*(C)} \left[-\frac{\xi(C)}{Q^*(C)} dQ^* + \frac{\partial \xi(C)}{\partial \eta} d\eta \right] dG + \frac{\xi(\bar{C})}{Q^*(\bar{C})} d\bar{C} \\
0 &= -dT + \alpha \frac{d\Phi}{N} - d\bar{C} - dZ^*|_{\bar{C}} - R_A\, dQ^*|_{\bar{C}} \\
d\Phi &= \int_0^{\bar{C}} \left[\left(\frac{-dT + \alpha \frac{d\Phi}{N} - \lambda(C)\, d\overline{U}}{Q^*(C)} \right) \xi(C) + (R(C) - R_A) \frac{\partial \xi(C)}{\partial \eta} d\eta \right] dC \\
dK &= K_\eta d\eta \\
dT &= \frac{dK}{N}
\end{aligned}
$$

where $\lambda(C) \equiv \partial E / \partial U = Z_U^c + R(C) Q_U^c$.[15] From the compensated demand functions (4) and (5) we get

$$
\begin{aligned}
dQ^* &= Q_R^c dR + Q_U^c d\overline{U} \\
dZ^* &= Z_R^c dR + Z_U^c d\overline{U}
\end{aligned}
$$

and differentiation of the bid rent function (6) yields

$$
Q^* dR + R dQ^* = -dT + \alpha \frac{d\Phi}{N} - dZ^*.
$$

By noting the $Z_R^c + RQ_R^c = 0$ as implied by Shephard's lemma, we have

$$
dR = \frac{1}{Q^*(C)} \left[-dT + \alpha \frac{d\Phi}{N} - \lambda(C)\, d\overline{U} \right]
$$

and

$$
\begin{aligned}
dQ^* &= \frac{Q_R^c}{Q^*(C)} \left(-dT + \alpha \frac{d\Phi}{N} \right) + \left[Q_U^c - \frac{\lambda(C)\, Q_R^c}{Q^*(C)} \right] d\overline{U} \\
dZ^* &= \frac{Z_R^c}{Q^*(C)} \left(-dT + \alpha \frac{d\Phi}{N} \right) + \left[Z_U^c - \frac{\lambda(C)\, Z_R^c}{Q^*(C)} \right] d\overline{U}.
\end{aligned}
$$

Using these, we can rewrite the total differentiation equations above as follows:

$$
\begin{aligned}
0 = - & \left[\int_0^{\bar{C}} \frac{\xi(C)}{(Q^*(C))^3} Q_R^c dC \right] \left(-dT + \alpha \frac{d\Phi}{N} \right) - \left[\int_0^{\bar{C}} \frac{\xi(C)}{(Q^*(C))^2} \left(Q_U^c - \frac{\lambda(C)\, Q_R^c}{Q^*(C)} \right) dC \right] d\overline{U} \\
& + \left[\int_0^{\bar{C}} \frac{1}{Q^*(C)} \frac{\partial \xi(C)}{\partial \eta} dC \right] d\eta + \frac{\xi(\bar{C})}{Q^*(\bar{C})} d\bar{C} \qquad (27)
\end{aligned}
$$

$$
d\bar{C} = -dT + \alpha \frac{d\Phi}{N} - \lambda(\bar{C})\, d\overline{U} \qquad (28)
$$

$$
\begin{aligned}
d\Phi &= N \left(-dT + \alpha \frac{d\Phi}{N} \right) - \left[\int_0^{\bar{C}} \lambda(C) \frac{\xi(C)}{Q^*(C)} dC \right] d\overline{U} \\
& \quad + \left[\int_0^{\bar{C}} (R(C) - R_A) \frac{\partial \xi(C)}{\partial \eta} dC \right] d\eta \qquad (29)
\end{aligned}
$$

$$
dT = \frac{1}{N} K_\eta d\eta, \qquad (30)
$$

[15]Subscripts indicate partial derivatives.

where the third equation uses $\int_0^{\bar{C}} \xi(C)/Q^*(C)\, dC = N$.

Now, by letting $d\overline{U}/d\eta = 0$, we finally obtain the following optimality conditions:

$$K_\eta + (1-\alpha)\frac{d\Phi}{d\eta} = \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC \tag{31}$$

$$\frac{d\bar{C}}{d\eta} = \frac{1}{N}\left[\int_0^{\bar{C}} \xi(C)\frac{\partial (R(C) - R_A)}{\partial \eta} dC\right] \tag{32}$$

where deriving $d\bar{C}/d\eta$ in the second line uses (14).

A.2 Sufficiency condition under resident land ownership

Solving (27) through (30) gives

$$\frac{d\overline{U}}{d\eta} = \frac{-K_\eta + \int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC}{\int_0^{\bar{C}} \lambda(C)\frac{\xi(C)}{Q^*(C)} dC}.$$

Using the first-order optimality condition $d\overline{U}/d\eta = 0$, the second-order derivative at the optimum is expressed as

$$\frac{d^2\overline{U}}{d\eta^2} = \frac{-K_{\eta\eta} + \frac{d}{d\eta}\left\{\int_0^{\bar{C}} (R(C) - R_A)\frac{\partial \xi(C)}{\partial \eta} dC\right\}}{\int_0^{\bar{C}} \lambda(C)\frac{\xi(C)}{Q^*(C)} dG}.$$

Because $\lambda(C)$ and $\xi(C)$ are positive, sufficiency condition $d^2\overline{U}/d\eta^2 < 0$ eventually implies (16).

A.3 Deriving the result of Arnott and Stiglitz (1981) in our model

The aggregate transport cost (ATC) is the sum of the total transport cost within the city's boundaries, which is

$$ATC = \int_0^{\bar{C}} \frac{C\xi(C)dC}{Q^*(C)}.$$

Integrating by parts rewrites our aggregate differential land rent ($ADRL$) as

$$\begin{aligned} ADLR &= \int_0^{\bar{C}} (R(C) - R_A)\xi(C)dC \\ &= \int_0^{\bar{C}} R(C)\frac{d\Xi(C)}{dC} dC - R_A\Xi(\bar{C}) \\ &= -\int_0^{\bar{C}} \frac{dR(C)}{dC}\Xi(C)\, dC. \end{aligned}$$

Differentiating $R(C)$ with respect to C and noting that $d\overline{U}/dC = 0$ in a spatial equilibrium yields

$$\frac{dR(C)}{dC} = -\frac{1}{Q^*(C)}.$$

Substituting this into the above gives

$$\frac{ATC}{ADLR} = \frac{\int_0^{\bar{C}} \frac{1}{Q^*(C)} C\xi(C)dC}{\int_0^{\bar{C}} \frac{1}{Q^*(C)} \Xi(C)dC}$$

which is the same as Equation (11) of Arnott and Stiglitz (1981).

A.4 Total differentiation of spatial equilibrium conditions in heterogeneous resident case

Total differentiation of the spatial equilibrium equation for the aggregate differential land rent Φ, as well as the construction cost function and transport authority's budget constraint, gives

$$\begin{aligned} d\Phi &= \int_0^{\widehat{C}} \left[\left(\frac{-dT + \alpha \frac{d\Phi}{N} - \lambda_p(C)\, d\overline{U}_p}{Q_p^*(C)} \right) \xi(C) + (R_p(C) - R_A) \frac{\partial \xi(C)}{\partial \eta} d\eta \right] dC \\ &+ \int_{\widehat{C}}^{\bar{C}} \left[\left(\frac{-dT + \alpha \frac{d\Phi}{N} - \lambda_r(C)\, d\overline{U}_r}{Q_r^*(C)} \right) \xi(C) + (R_r(C) - R_A) \frac{\partial \xi(C)}{\partial \eta} d\eta \right] dC \\ dK &= K_\eta d\eta \\ dT &= \frac{dK}{N} \end{aligned}$$

where the first equation exploits that

$$dR_j = \frac{1}{Q_j^*(C)} \left[-dT + \alpha \frac{d\Phi}{N} - dC - \lambda_j(C)\, d\overline{U}_p \right].$$

By noting that $\int_0^{\widehat{C}} \xi(C)/Q_p^*(C)\, dC + \int_{\widehat{C}}^{\bar{C}} \xi(C)/Q_r^*(C)\, dC = N$ the above reduces to (24).