Simulation analysis using multi-agent systems for social norms

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Abstract: With the existence of the social customs or norms, Naylor demonstrates a possibility of stable long-run equilibria of support for a strike in a labor market, and this implies that at least some individuals will behave cooperatively and hence the prisoners' dilemma could be escaped. In this paper, using an agent-based simulation model in which artificial adaptive agents have mechanisms of decision making and learning based on neural networks and genetic algorithms, we compare the results of our simulation analysis with that of the mathematical model by Naylor. In particular, while Naylor's model is based on rationality as it relates to individual utility maximization, agents behave adaptively in our agent-based simulation model; agents make decisions by trial and error, and they learn from experiences to make better decisions.

Key words: social norms, reputation, simulation, adaptive agents.
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1 Introduction

In most mathematical models for analyzing social behaviors of people, utility functions of individuals are defined so as to meet social situations to be analyzed, and it is assumed that individuals make choices maximizing such utility functions. For examining behavior of individuals with respect to social norms, Akerlof (1980) develops a model including a social reputation on the norm as well as motivation of monetary rewards, and explains involuntary unemployment by considering the social custom or norm of a fair wage. In the model, the personal tastes or attitudes toward the social norm are heterogeneous, and the utility function is designed so that individuals obtain a better reputation if they obey the norm, otherwise they must pay the penalty for disobeying it. He shows that once such a social norm is established, it may continue to be followed with a stable fraction of the population believing in it and also following it.

Various models for social norms related to the research by Akerlof have been reported (Naylor, 1989; Holländer, 1990; Kandel and Lazear, 1992; Barron and Gjerde, 1997; Kübler, 2001; Huck et al., 2001). Naylor (1989) proposes a model similar to that of Akerlof in order to explain the logic of collective strike action, and then shows the effectiveness of an approach considering the reputation arising from obedience of social norms. Kübler (2001) attempts to endogenize norms by investigating how individuals, or groups of individuals, can be influenced by the two types of norms: bandwagon norms which are characterized by the property that once a critical number of norm followers is reached, the reputation value of norms increases sharply; and snob norms, which yield the most reputation to the followers when only a small number of people follow them. Huck *et al.* (2001) deal with the interplay between economic incentives and social norms in firms, and model the team production in a linear incentive scheme with a social norm.

Recently, approaches based on laboratory experiments analyzing social norms have been attempted. Gachter and Fehr (1999) investigate the impact and the limitations of social rewards on people's behavior in the provision of a public good, and find that if subjects have some social familiarity with each other, approval incentives generate a rise in cooperation. Rege and Telle (2004) also conduct laboratory experiments on public goods, and argue that revealing each player's identity and his contribution to the public good may increase voluntary contributions. Nyborg and Rege (2003) study the formation of social norms for considerate smoking behavior, and report the results with empirical evidence based on interviews with many people.

Simulation is a growing field in the social sciences, and analysis through the use of simulation on the formation of social norms and cultures has been developed. Based on the n-person prisoner's dilemma game, Axelrod (1986) considers a norm game with a mechanism punishing players who disobey the norm, and investigates the dynamics of the norm by using a technique of evolutionary theory. Using a simulation model with many autonomous and adaptive agents, Axelrod (1997a) examines how cultural regions emerge, develop, and settle down from viewpoints of differences of cultures and interaction among them. Bowles and Gintis (2004) classify agents into three types by judging whether agents obey the norm or not and whether they punish other agents who violate it or not, and study cooperation in the population by agent-based simulations.

Although the mathematical models by Akerlof (1980) and Naylor (1989) reach interesting results, it is assumed that players are rational and maximize their payoffs, and they can discriminate between two payoffs with a minute difference. Such optimization approaches are not always appropriate for analyzing human behaviors and social phenomena, and models based on adaptive behavior can be alternatives to such optimization models. In these models, there exist two groups of agents—those who believe the social norm and those who do not believe it—, and the penalty to believers of the social norm for disobedience is not the same as that of nonbelievers. An individual who obeys the social norm in the prior period of the game is defined as a believer, and an individual who disobeys it in the prior period is defined as a nonbeliever. As Akerlof points out, however, it is natural to suppose that the degree of belief of an individual who continues to obey the social norm in the long run is not the same as that of an individual who obeys it only in the prior period.

To incorporate adaptive behaviors and the degree of belief based on an agent's history of actions, we employ a multi-agent simulation model. Agents in a simulation model evaluate results of their decisions and revise policies to choose one of several alternatives as actual decision makers do. As concerns approaches based on adaptive behavior models, Holland and Miller (1991) interpret most economic systems as complex adaptive systems, and point out that simulations using artificial societies with adaptive agents is effective for analysis of such economic systems. Axelrod (1997b) insists on the need for simulation analysis in social sciences, and states that the purposes of simulation analysis include prediction, performance, training, entertainment, education, proof and discovery. Brenner (1998) examines the use of evolutionary algorithms in social science studies, and suggests that it is important to incorporate histories of agents in simulation systems.

For the iterated prisoner's dilemma game, Axelrod (1987) examines the effectiveness of strategies generated in an artificial social system, in which agents endowed with strategies are adaptively evolved by using a genetic algorithm. Dorsey et al. (1994) employ an artificial decision making mechanism using neural networks to imitate the decision making of auctioneers, and compare artificial agents' behavior with that of real auctioneers which often deviate from the Nash equilibria. To estimate bid functions of bidders, i.e., to establish the appropriate weights of a neural network, they employ the genetic adaptive neural network algorithm based on genetic algorithms instead of the error backpropagation algorithm which is the most commonly used method.

Andreoni and Miller (1995) use genetic algorithms to model decision making in auctions. In a way similar to the approach of Dorsey et al. (1994), they compare decisions of artificial adaptive agents with decisions observed in the experiments with human subjects, and find that the two types of decisions by the artificial agents and the human subjects resemble each other. Erev and Rapoport (1998) investigate a market entry game by using an adaptive learning model based on reinforcement learning proposed by Roth and Erev (1995). Rapoport et al. (2002) also deal with market entry games. They compare decisions observed in experiments with human subjects with decisions of artificial adaptive agents with a learning mechanism using reinforcement learning, and analyze behavioral patterns on the aggregate level.

Leshno et al. (2002) consider equilibrium problems in market entry games through agent-based simulations with a decision making mechanism of agents based on neural networks, and the neural networks are trained not by some teacher signals but by the outcomes of games. They compare the results of the simulations with the results of the experiments with human subjects conducted by Sundali et al. (1995), and find some similarities between phenomena of the simulations and the experiments. Nishizaki et al. (2005) investigate the effectiveness of a socio-economic system for preserving the global commons by simulation analysis. A number of attempts have been made for performing multi-agent based simulations and developing the related techniques underlying the ideas from distributed artificial intelligence and multi-agent systems (Epstein and Axtell, 1996; Conte et al., 1997; Chellapilla and Fogel, 1999; Downing et al., 2001; Moss and Davidsson, 2001; BanerjeSen, 2002; Niv et al., 2002; Parsons et al., 2002; Sichman et al., 2003).

In the model by Akerlof, a labor market is considered and then two types of individuals, laborers and capitalists, are required, while there is only one type of individual in the model by Naylor. In this paper, we focus on the model by Naylor because of relative simplicity and develop a multi-agent system for simulation analysis, in which agents behave adaptively and their belief with respect to the social norm is created by a series of actions in the long run. Adaptive behavior of agents is implemented by incorporating mechanisms of decision making and learning based on neural networks and genetic algorithms. Using the multi-agent system, we compare the results of the model by Naylor with those of the simulations, and examine behavior of agents with respect to the social norm by varying values of several parameters. In section 2, we briefly review the mathematical model by Naylor and summarize the results of the study. After we describe our simulation system with its many adaptive artificial agents in section 3, we examine the results of the simulations in section 4. Finally, in section 5 we summarize the results and findings from the simulations.

2 The mathematical model by Naylor

In the model by Naylor (1989), the utility involves the effect of reputation arising from obedience of the social norm in a way similar to that of Akerlof (1980). An individual obeys the social norm if the utility obtained from doing so is at least as great as the utility derived from not doing so. Otherwise, he or she disobeys the social norm. The utility function of an individual has five arguments: money income M; reputation R; a decision variable s with respect to obedience of the social norm; a state variable b with respect to the individual's belief of the social norm; and the individual's personal taste ε. Then, the utility function of individual i is represented by

$$U_i = U(M, R, s, b, \varepsilon_i). \tag{1}$$

The personal taste ε_i of individual i represents sensitivity concerning reputation in a society, and it is assumed to be distributed uniformly in the interval $[\varepsilon^L, \varepsilon^H]$. From the distribution of ε it follows that heterogeneous individuals are dealt with in this model. Suppose that there exist two groups of agents: one of which believes the social norm, and one which does not believe it. Then, the personal tastes of agents in the group of believers are larger than those in the group of non-believers.

The money income M is defined as d if the individual obeys the social norm and w if the individual disobeys it; it is assumed that $d \leq w$. In the context of Naylor, the social norm is invoking workers to support a strike. If the individual disobeys the social norm, the individual suffers disutility consequent upon the act of disobedience. The disutility is a constant $\bar{c}$ if the individual is a believer of the social norm, and it is $\bar{g}$ if a non-believer; it is assumed that $\bar{g} < \bar{c}$. Then, the money income M is represented by

$$G = ds + w(1-s) - b(1-s)\bar{c} - (1-b)(1-s)\bar{g}, \tag{2}$$

where s is 1 if the individual obeys the social norm, and 0 if not; and b is 1 if the individual is a believer, and 0 if not.

The utility from reputation accrues to individuals only when the individuals obey the social norm, i.e., $s = 1$. It depends on the rate of obedience μ in the population and the personal taste ε_i of individual i, and it is represented by

$$R = \alpha s \mu \varepsilon_i, \tag{3}$$

where α is a coefficient of the reputation. The utility R from reputation increases with the rate of obedience μ and the personal taste ε_i.

Then, the utility function is represented by

$$U_i = ds + w(1-s) + \alpha s \mu \varepsilon_i - b(1-s)\bar{c} - (1-b)(1-s)\bar{g}. \tag{4}$$

The behavior of an individual is determined by the utility function (4), and it depends on the belief of the social norm. If a believer i obeys the social norm, i.e., $b = 1$ and $s = 1$, then the believer i obtains $U_i^{bs} = d + \alpha\varepsilon_i\mu$; otherwise, $b = 1$ and $s = 0$, then the believer i obtains $U_i^{b\bar{s}} = w - \bar{c}$. These values are compared and then if $U_i^{bs} \geq U_i^{b\bar{s}}$, the believer i obeys the social norm; otherwise, the believer i disobeys it. Similarly, if a nonbeliever $\bar{i}$ obeys the social norm, i.e., $b = 0$ and $s = 1$, then the nonbeliever $\bar{i}$ obtains $U_{\bar{i}}^{\bar{b}s} = d + \alpha\varepsilon_{\bar{i}}\mu$; otherwise, $b = 0$ and $s = 0$, then the nonbeliever $\bar{i}$ obtains $U_{\bar{i}}^{\bar{b}\bar{s}} = w - \bar{g}$. These values are compared and then if $U_{\bar{i}}^{\bar{b}s} \geq U_{\bar{i}}^{\bar{b}\bar{s}}$, the nonbeliever $\bar{i}$ obeys the social norm; otherwise the nonbeliever $\bar{i}$ disobeys it. The behavior of an individual i also depends on the personal taste ε_i of self, and it can be represented graphically by Figure 1.

An individual's personal taste ε is uniformly distributed over the interval $[\varepsilon^L, \varepsilon^H]$, and the behavior of an individual is governed by the two hyperbolic curves, as seen in Figure 1. For the configuration of parameters given in Figure 1, there are multiple equilibria: the point o, the intervals k-m and p-q. The rate of obedience

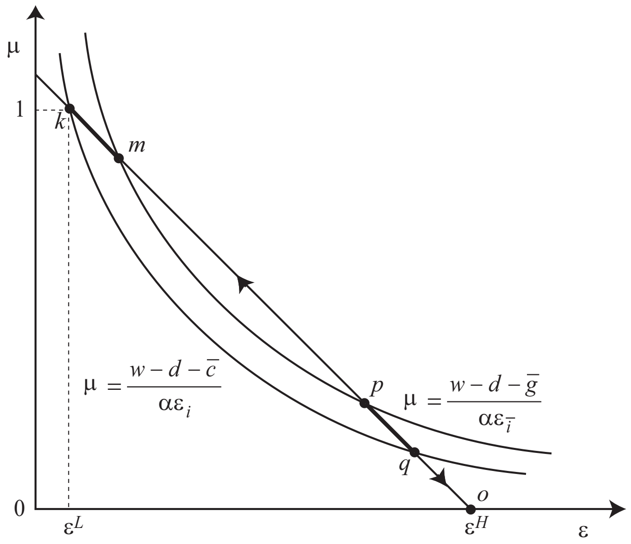


Figure 1: The model by Naylor

μ in the population increases in the interval *m*-*p*, and it decreases in the interval *q*-*o* as time goes on. The three main results of the study by Naylor (1989) are summarized as follows.

(i) For particular configurations of parameters, three types of equilibria can be found: the maintenance state of the social norm with higher levels of the obedience rate (the interval *k*-*m*), the maintenance state of the social norm with lower levels of the obedience rate (the interval *p*-*q*), and the extinction state of the social norm (the point *o*).

(ii) Regardless of an individual's belief with respect to the social norm, the obedience rate of the social norm increases with the penalty, $\bar{c}$ or $\bar{g}$, for disobedience of the social norm.

(iii) The obedience rate of the social norm increases with the payoff *d* by obeying the social norm, and it decreases with the payoff *w* by disobeying the social norm.

Naylor suggests that the existence of the social norm induces some social-regarding individuals to behave cooperatively and hence to possibly escape the prisoners' dilemma.

3 An agent-based simulation system

Although the mathematical model by Naylor (1989) reaches interesting results, it is assumed that individuals are rational and maximize their payoffs, and consequently they can discriminate between two payoffs with a minute difference. Such modeling of human behaviors might be not always appropriate for analyzing social phenomena. In contrast, models based on adaptive behavior can be alternatives to such mathematical approaches. Furthermore, while the penalty for disobeying the social norm depends on the belief with respect to it, the belief is defined only by an action in the prior period. Namely, an individual who obeys the social norm in the prior period is a believer of the social norm and an individual who disobeys it in the prior period is a nonbeliever. It is natural to consider that the belief of individuals who obey the social norm in the long term is at variance with that of individuals who obey it only in the prior period. Akerlof (1980) himself points out that the alternative expression of the belief should be extended.

In this paper, we examine the variances between those individuals and the alternative expression of belief. Specifically, we employ a simulation model with adaptive artificial agents. In our simulation model, each agent has a decision making and learning mechanism based on neural networks and a genetic algorithm. We also assume that the personal tastes ε of all the agents are uniformly distributed over the interval $[\varepsilon^L, \varepsilon^H]$, and we restrict interaction between agents within a group of agents with similar values of personal tastes ε_i. Namely, all of the agents are divided into multiple groups in terms of the personal tastes; agents in a group are evolved independently of agents in the other groups.

As we mentioned in the previous section, reinforcement learning, learning classifier systems, neural networks, genetic algorithms, etc. are applicable as mechanisms for the learning of artificial agents. Because we deal with changes and the development of social norms in this paper, we need to employ a model in which decision making of an agent depends not only on the outcome of an action selected by the agent, but also on a social state such as the obedience rate of the social norm. In reinforcement learning, learning of an agent is effected by revising the selection probabilities of strategies according to the outcomes of actions selected by the agent. The learning classifier systems which consist of multiple 'if-then' rules receive information from the environment and classify states of the environment; and the action given by the if-then rule matching a state of the environment is performed. According to rewards from the environment, a set of the rules is evolved. In the learning system with decision making and learning mechanisms based on neural networks and genetic algorithms developed by Nishizaki (2007), an action of an agent is determined by an output of the neural network which is a nonlinear function of multiple inputs, and the parameters of the neural network are revised according to the outcomes of actions of the agent in the framework of genetic algorithms.

Applicabilities of these methods of analyzing the social norm can be evaluated as follows. For reinforcement learning, it is difficult to relate the state of the society to decisions of agents. It is natural to think that, for environments varying continuously, such as changes in and the development of social norms, it is appropriate to employ the learning system based on neural networks and genetic algorithms which outputs continuous real numbers rather than the learning classifier systems with discrete outputs based on if-then rules. In view of the characteristics of these methods, we employ the learning system based on neural networks and genetic algorithms in this paper.

To examine changes in and development of the social norm, an action and a payoff of an agent in the prior period, the belief of the agent with respect to the social norm, actions and payoffs of the other agents, and the obedience rates of the social norm in the population are used as inputs of the neural network, and an action of the agent is determined as an output of the nonlinear function embedded in the neural network. Moreover, the history of the agent's actions is held together with a chromosome of the agent for the genetic algorithm, and the degree of belief with respect to the social norm is determined by using the history of actions.

A neural network which is the decision making mechanism of an agent is characterized by the synaptic weights between two nodes and the thresholds for the output functions of nodes, and these parameters are treated as a chromosome in the genetic algorithm. This evolutionary system with the neural networks and the genetic algorithm reproduces dominant agents which take appropriate actions for the changing environment. Agents receive individual information and external information such as societal states and actions of the other agents, and they adapt to the environment through a trial and error process. Agents which can obtain higher payoffs survive, and the society is constituted of such dominant agents. Because the knowledge of agents is accumulated and embedded in the parameters of the neural networks, the agents can learn collectively in the changing environment.

Brenner (1998) considers the use of evolutionary algorithms in social science studies, and points out that a history of actions of agents should be taken into consideration in systems implemented for analyzing social phenomena. In our system for analyzing changes in and development of the social norm, a history of actions of an agent is held, and it determines the degree of belief with respect to the social norm. Therefore, it can be said that our system is consistent with his claim.

Because each agent has different personal tastes, the population of agents is considered to be heterogeneous. In the context of the social norm analysis, we think that there is hardly any opportunity for interaction between agents with exceedingly different values of personal tastes. From this viewpoint, we divide the population of agents into multiple subpopulations, and an agent is allowed to interact only within the same subpopulation. In contrast, Flache and Mäs (2008a,b) investigate the performance of teams where agents with different attributes interact with each other. This aspect is also interesting in analyzing social norms, and we investigate the influence of the interaction between heterogeneous agents on changes in and development of the social norm. To do so, we conduct simulations on the scope of interactions among agents, and examine how the scope of interactions influences changes in and development of social norms.

3.1 Decision making by a neural network

An agent corresponds to a neural network which is characterized by synaptic weights between two nodes in the neural network and thresholds which are parameters for the output functions of nodes. Because a structure of neural networks is specified by the number of layers and the number of nodes in each layer, an agent is prescribed by the fixed number of parameters when the structure is determined. In our model, we provide strings composed of these parameters identifying agents and they are used as chromosomes of the agents in an artificial genetic system.

Each agent chooses one of obedience or disobedience for the social norm in each period, and each of the groups with similar agents evolves into that of agents with larger payoffs. The neural network structure and chromosome string are depicted in Figure 2.

An output of a neural network is determined by a vector of inputs, the synaptic weights, and the thresholds. The inputs of the neural network consist of the following six values:

(i) The choice of agent i in the prior period: s_i^{prior}

(ii) The obedience rate of the population in the prior period: μ^{prior}

(iii) The personal taste of agent i: ε_i

(iv) The utility obtained by agent i in the prior period: U_i^{prior}

(v) The sum of utilities of all the agents in the prior period: U^{total}

(vi) The degree of belief of agent i for the social norm: b_i.

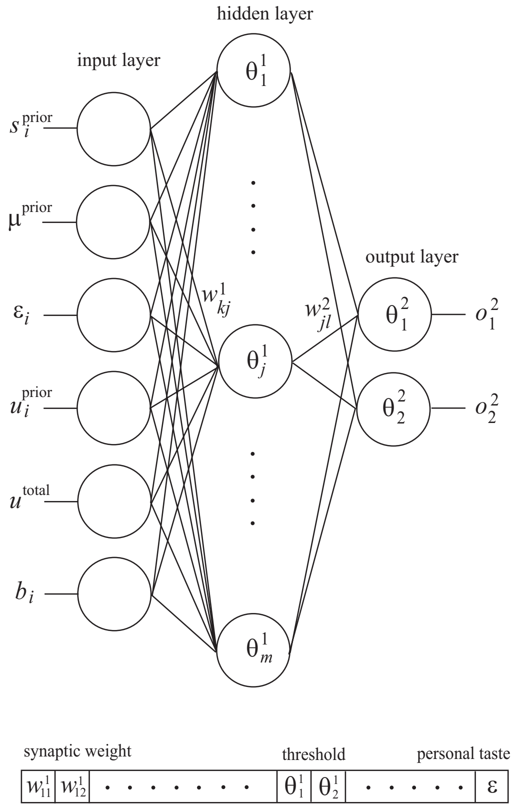


Figure 2: A neural network and the corresponding chromosome

It is natural to consider that decision making of an individual depends on actions of self and others in the previous periods. Therefore, the input data of agent i include (i) the prior choice of self, and (ii) the rate of obedience in the population in the prior period. The prior choice $s_i^{\text{prior}} \in \{0,1\}$ of agent i is 1 if agent i obeys the social norm, and 0 if agent i disobeys it, and the obedience rate $\mu^{\text{prior}} \in [0,1]$ is considered an index of the social norm in the society, which is the aggregated information of actions of all the agents. Because the utility in the case of obedience depends on (iii) the personal taste ε_i of agent i concerning the reputation, it is included in the set of input data. An agent behaves adaptively, and the decision depends on the utility obtained in the previous periods. Because we suppose that the decision of the agent also depends on the situation of a society as well as the utility, (iv) the utility of agent i and (v) the sum of utilities of all the agents, which can be interpreted as a certain societal situation, are included in the set of input data. In our simulation model, the penalty for disobeying the social norm depends on the belief with respect to it, and then (vi) the belief of agent i is also one of the input data.

In the model by Naylor (1989), the belief $b \in \{0,1\}$ is defined only by an action in the prior period. Namely, b is 1 if an individual obeys the social norm in the prior period, and 0 if the individual disobeys it. In contrast, we assume that the belief with respect to the social norm is created by a series of actions in the long run, and define the degree of belief of agent i as follows.

$$b_i = \Big(\sum_{t=1}^{n} s_t \delta^{t-1}\Big) \Big/ \Big(\sum_{t=1}^{n} \delta^{t-1}\Big), \tag{5}$$

where δ is a discount factor, and s_t represents an action of an agent t periods before: s_t is 1 if the agent obeys the social norm, and 0 if the agent disobeys it. Because $0^0 = 1$, we have $b = s_1$ if $\delta = 0$, which means that the definition of belief (5) with $\delta = 0$ is reduced to that of the model by Naylor. As δ is made larger, the influence of actions in the past on the belief increases in seriousness. To connect the belief depending not only on the prior action but also on actions in wide-ranging past periods with the utility of an agent, we define the following utility function and employ it in our agent-based simulation system.

$$U_i = ds + w(1-s) + \alpha s \mu \varepsilon_i - (1-s)\{b(\bar{c} - \bar{g}) + \bar{g}\}. \tag{6}$$

It follows that the penalty for disobeying the social norm varies in proportion to the degree of belief b in this utility function.

Let z_k^p and w_{kj}^p, $k = 1, \ldots, m^p$ denote an input value and a synaptic weight of node j in the hidden layer $(p = 1)$ or the output layer $(p = 2)$, and let θ_j^p denote a threshold of node j. Then an output o_j^p of node j is represented by

$$o_j^p = f\left(\sum_{k=1}^{m^p} w_{kj}^p z_k^p - \theta_j^p\right), \tag{7}$$

where f is an output function which is a logistic function $f(z) = 1/(1+\exp(-z))$. As shown in Figure 2, there are two nodes in the output layer, and an action of an agent is determined by the two output values. Namely, if the value o_1^2 of output node 1 is larger than or equal to the value o_2^2 of output node 2, the agent obeys the social norm, and otherwise the agent disobeys.

As we mentioned above, there are six units in the input layer and two units in the output layer. Let m be the number of units in the hidden layer. Then because the number of synaptic weights is $8m$ and the number of units in the hidden and the output layers is $m+2$, the neural network corresponding to an agent can be determined by the synaptic weights w_l, $l = 1,\ldots,8m$ and the thresholds θ_l, $l = 1,\ldots,m+2$. These parameters and the input values determine an action of the agent, and the synaptic weights and the thresholds are adjusted through the genetic algorithm so that the initial population evolves into the population of agents obtaining larger payoffs.

3.2 Evolutionary learning through the genetic algorithm

Each agent chooses between obedience and disobedience every period, and the agent obtains the utility defined by (6). By evaluating utilities arising from a series of decisions by way of the fitness, the population of agents evolves. The structure of the simulation model is shown in Figure 3.

3.2.1 Initial population

A chromosome of an agent consists of the synaptic weights w_l, $l = 1,\ldots,8m$, the thresholds θ_l, $l = 1,\ldots,m+2$ and the personal taste ε_i; it is represented by a string like that of Figure 2. A population is composed of N agents, and the personal tastes ε_i, $i = 1,\ldots,N$ are uniformly distributed over the interval $[0.2, 1.2]$. The synaptic weights w_l and the thresholds θ_l are initialized to be adjusted to a given parameter μ_0 of the initial rate of obedience.

In an initial population, for any given initial rate of obedience μ_0, we set up the

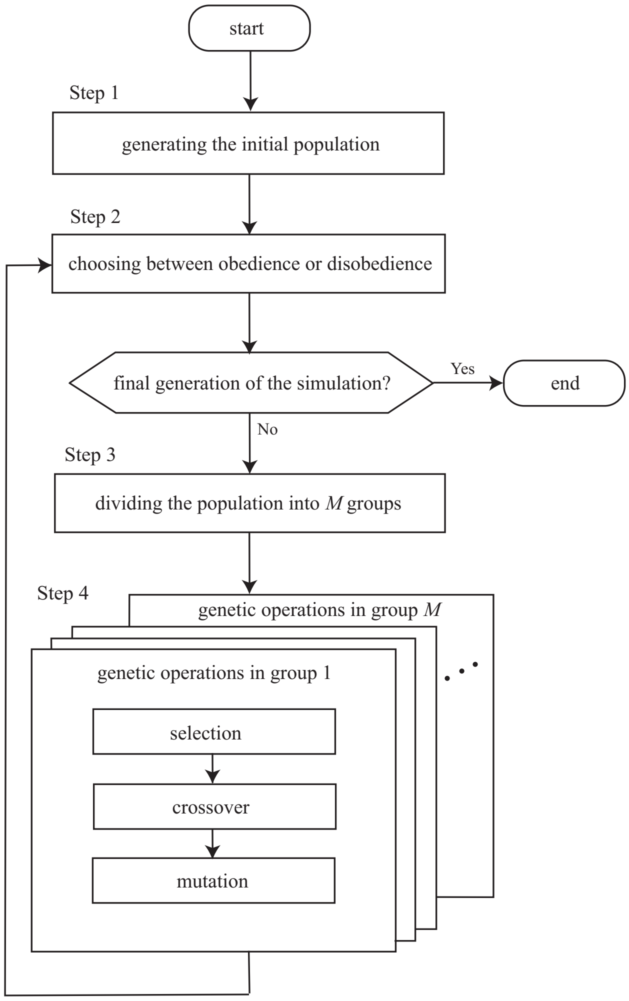


Figure 3: Flowchart of the simulation model

synaptic weights and the thresholds such that $N\mu_0$ agents who are believers, $b = 0$, obey the social norm, and $N(1-\mu_0)$ agents who are non-believers, $b = 1$, disobey it. To do so, we first divide all N agents into $N\mu_0$ agents with relatively larger personal tastes and $N(1-\mu_0)$ agents with relatively narrower personal tastes. Then, the synaptic weights and the thresholds are adjusted by using the error back propagation algorithm (e.g. Hassoun (1995)) with the teacher signals shown in Table 1. Through this procedure, we can obtain an initial population with the specified initial rate of obedience μ_0.

Table 1: Teacher signals for the error back propagation algorithm

	believers		non-believers
s_i^{prior}	1.0		0.0
μ^{prior}		μ_0*	
ε_i		ε_i*	
U_i^{prior}	$d+\alpha\mu_0\varepsilon_i$		$w-\bar{g}$
U^{total}		$\sum(d+\alpha\mu_0\varepsilon_i)+\sum(w-\bar{g})$*	
b_i	1.0		0.0
o_1	1.0		0.0
o_2	0.0		1.0

* The teacher signals of μ^{prior}, ε_i and U^{total} are the same both for believers and for non-believers.

3.2.2 Genetic operations

Because the parameter ε_i of agent i represents personal taste with respect to the social norm, it is natural to suppose that there is hardly any opportunity for agents with exceedingly different values of the parameter to interact with one another. To implement such situations, we divide the population of agents into multiple subpopulations, and an agent is allowed to interact only within the same subpopulation. Namely, N agents are divided into M groups, and the following genetic operations (e.g. Goldberg (1989)) are executed for each group.

Reproduction As a reproduction operator, the roulette wheel selection is adopted. Let $N' = N/M$ denote the number of agents in each subpopulation. A chromosome of an agent is selected into the next generation by a roulette wheel with slots sized by the probability $p_i^s = U_i/(\sum_{i=1}^{N'} U_i)$, where U_i is a utility of agent i at this period and it is also interpreted as the fitness in the artificial genetic system.

Crossover A single-point crossover operator is applied to any pair of chromosomes with the probability of crossover p^c. A point of crossover on the chromosomes is randomly selected and then two new chromosomes are created by swapping subchromosomes which are part of the right side of the original chromosomes from the selected point of crossover. A new population is formed by exchanging a specified rate of portions of the current population for that of the modified population in which the crossover operation is executed; the rate is called the generation gap g. The utility of a newly created offspring by the crossover operation is determined by inheriting those of its parents in the proportion of sizes of the swapped subchromosomes. An agent keeps the history of actions from past periods. To create the history of the offspring, those of the parents are also utilized. It is determined by choosing from two series of actions of the parents with probabilities corresponding to the sizes of the swapped subchromosomes.

Mutation With a given small probability of mutation p^m, each gene which represents a synaptic weight w_l, a threshold θ_l or the personal taste ε_i in a chromosome is randomly changed. If the selected gene is w_l or θ_l, it is replaced with a random number in $[-1,1]$, and if it is ε_i, it is replaced with a random number in $[0.2,1.2]$.

4 Results of the simulations

4.1 The details of the simulations

We conduct the basic simulation comparing the results of the Naylor model with those of our simulation model, and we also provide four supplementary simulations for the degree of belief with respect to the social norm, the penalties for disobeying the social norm, the payoffs by obeying and disobeying the social norm, and the scope of interaction among agents. Then, we arrange the following five simulations.

(i) Simulation *Basis*: Comparison between the model by Naylor and our simulation model.

(ii) Simulation *Belief*: Degree of belief with respect to the social norm.

(iii) Simulation *Penalty*: Penalties for disobeying the social norm.

(iv) Simulation *Payoff*: Payoffs by obeying and disobeying the social norm.

(v) Simulation *Interaction*: Scope of interaction among agents.

In Simulation *Basis*, the same setting as that of the model by Naylor is used in our agent-based simulation system, and we examine whether or not the three types of equilibria obtained in the model by Naylor can be observed. We introduce a parameter of the belief with respect to the social norm which is created by a series of actions in the long run, and in Simulation *Belief* we examine the influence of the belief and the penalty defined by the belief on the decision making of agents. We focus on the penalty for disobeying the social norm in Simulation *Penalty*. In the model by Naylor, it is concluded that reduction of the penalty leads to a fall of the rate of obedience and increase of the penalty leads to a rise of it. We examine whether or not the results of Simulation *Penalty* support this conclusion. The pecuniary payoffs depend on choices of agents between obedience and disobedience, and in Simulation *Payoff* we focus on these payoffs. The mathematical consideration by Naylor forms the conclusion that the obedience rate of the social norm increases with the payoff by obeying the social norm and it decreases with the payoff by disobeying the social norm. We verify this claim through Simulation *Payoff*. Belief and personal tastes of individuals are formed by interacting with other people, and it is natural to suppose that such interactions frequently occur among people with similar beliefs or personal tastes. From this viewpoint, it is appropriate to restrict the scope of interaction among agents in a simulation model with adaptive artificial agents. Because in our simulation model, interaction among agents is implemented as the genetic operations in the evolutionary process, the genetic operations are performed within a subpopulation of agents with similar personal tastes. In Simulation *Interaction*, we examine the influence of the scope of interaction among agents on maintenance or extinction of the social norm by varying the size of subpopulations.

The standard setting of the parameters of the utility function (6) with the degree of belief (5) used in the simulations is shown as follows:

pecuniary payoffs: $d = 0.1$ for obedience, $w = 1.0$ for disobedience;

penalties: $\bar{c} = 0.7$ for believers, $\bar{g} = 0.6$ for nonbelievers;
coefficient of reputation: $\alpha = 1.0$;
discount factor: $\delta = 0.0$;
personal tastes: a random number ε_i in the interval $[0.2, 1.2]$.

Artificial adaptive agents have a mechanism of decision making and learning based on a neural network and a genetic algorithm, and the standard setting of the parameters of the neural network and the genetic algorithm is also given as follows:

the number of nodes in the neural network:
6 in the input layer, 8 in the hidden layer, 2 in the output layer;
the number of individuals (agents): $N = 10000$;
the number of subpopulations: $M = 100$;
the parameters of genetic operations:
crossover $p^c = 0.6$, mutation $p^m = 0.01$, generation gap $g = 0.5$.

4.2 Simulation *Basis*

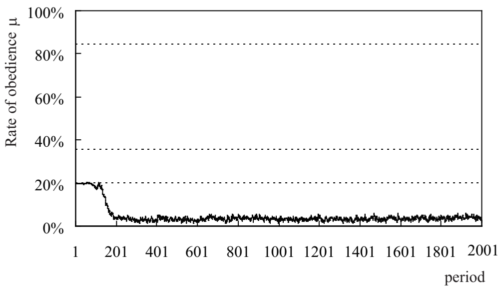
In Simulation *Basis*, the standard setting of the parameters is employed, and we perform 11 treatments of the simulation with the initial rate of obedience in the society from $\mu_0 = 0.0$ to $\mu_0 = 1.0$ at intervals of 0.1. In particular, for the treatments where there are multiple states of convergence, we execute further trials of the simulation with different values of the initial rate of obedience. Each treatment is performed 100 runs, and we observe transitions of the rate of obedience in the whole population and measure the time needed for convergence. Because of the fact that until 1500 periods have been performed, all preparatory runs converge at certain levels of the obedience rate which mean the extinction and the maintenance of the social norm. Accordingly, we set the maximal periods of the simulation at 2000 periods.

In Figure 4, we show the results of the four treatments of the simulation with $\mu_0 = 0.2, 0.3, 0.4, 1.0$ which characterize the steady states of the simulation. Each of graphs on the left hand side depicts a transition of the obedience rate of the social norm. For the treatments of $\mu_0 = 0.2, 0.4, 1.0$, the transition shown in the figure is one of 100 runs, and for the treatment of $\mu_0 = 0.3$, two transitions are shown because there are two levels at which the obedience rate converges in the long run. In graphs on the right hand side, we give the distributions of obedience rates over the interval of the personal tastes $[0.2, 1.2]$; the black curve is the average of 100 runs after period 100, and the gray curve is the average after period 2000.

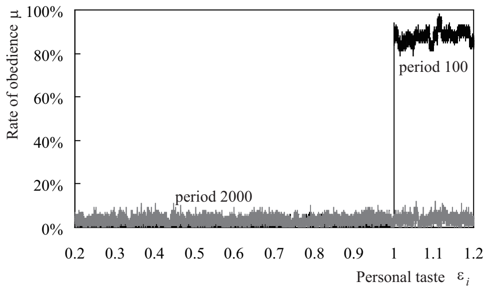
In the treatment of $\mu_0 = 0.2$, at the beginning of the simulation, the synaptic weights and the thresholds of the neural network are adjusted by using the error back propagation algorithm such that agents with personal tastes $0.2 \leq \varepsilon_i \leq 1.0$ which account for 80% of the whole population disobey the social norm and the rest of the agents with personal tastes $1.0 \leq \varepsilon_i \leq 1.2$ obey it. As an example shown in Figure 4(a), in each of the 100 runs of the treatment of $\mu_0 = 0.2$, the rate of obedience converges at almost 0% and the social norm becomes extinct in the long run. As seen in Figure 4(b), it is observed that 20% of the group of the agents with personal tastes $1.0 \leq \varepsilon_i \leq 1.2$ which obey the social norm at the beginning of the simulation disobey it already after only period 100, and almost all the agents of the group eventually disobey it by the last period 2000.

In the treatment of $\mu_0 = 0.3$, as seen in Figure 4(c), there are two levels at which the obedience rate converges in the long run: the maintenance state of the social norm in which 80% of agents of the population obey the social norm, and the extinction state of the social norm in which all of the agents disobey the social norm. However, the maintaining state of the social norm is observed only one time among the 100 runs. In fact, as seen in Figure 4(d), on average it follows that a part of the group of agents with personal tastes $0.9 \leq \varepsilon_i \leq 1.0$ come to disobey it after period 100, and finally at period 2000, almost all the agents disobey it.

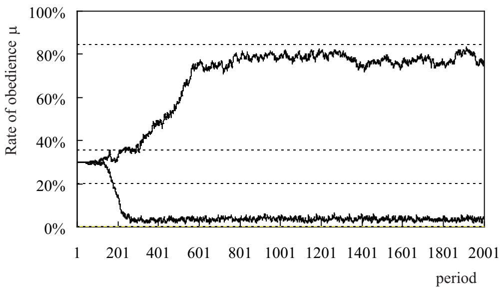
In all the 100 runs of the treatment of $\mu_0 = 0.4$, as seen in Figure 4(e), the rate of obedience μ increases from $\mu_0 = 0.4$ at the beginning to approximately $\mu = 0.78$, and the social norm is maintained with a high level of the obedience rate.



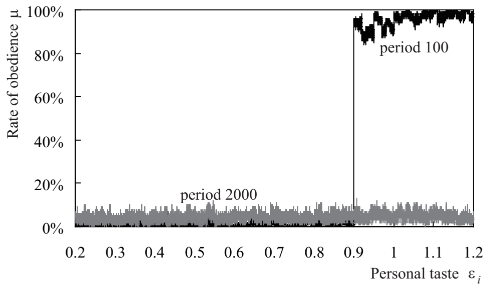
(a) Transition of obedience: $\mu_0 = 0.2$



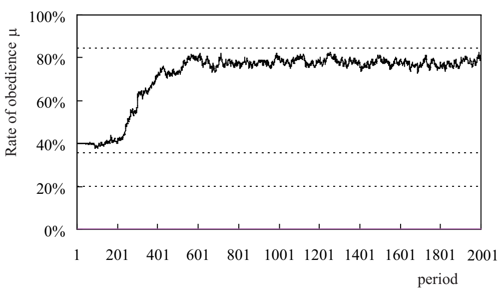
(b) Distribution of obedience: $\mu_0 = 0.2$



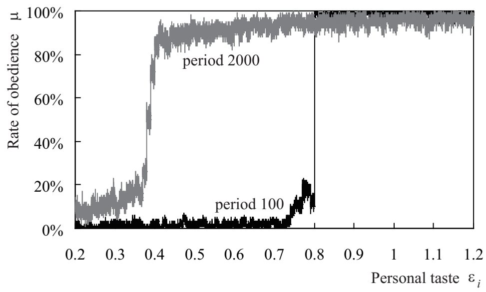
(c) Transition of obedience: $\mu_0 = 0.3$



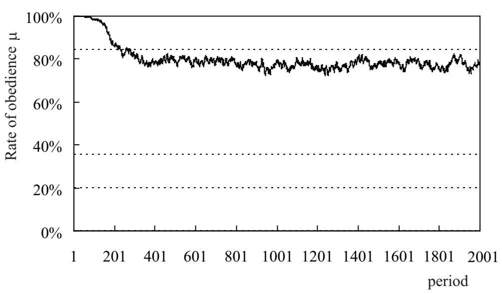
(d) Distribution of obedience: $\mu_0 = 0.3$



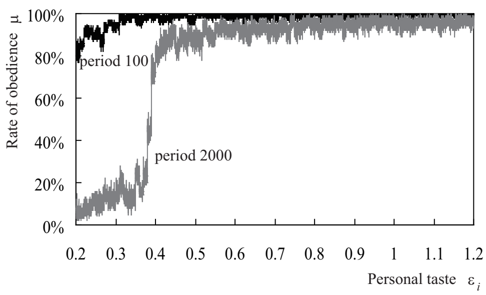
(e) Transition of obedience: $\mu_0 = 0.4$



(f) Distribution of obedience: $\mu_0 = 0.4$



(g) Transition of obedience: $\mu_0 = 1.0$



(h) Distribution of obedience: $\mu_0 = 1.0$

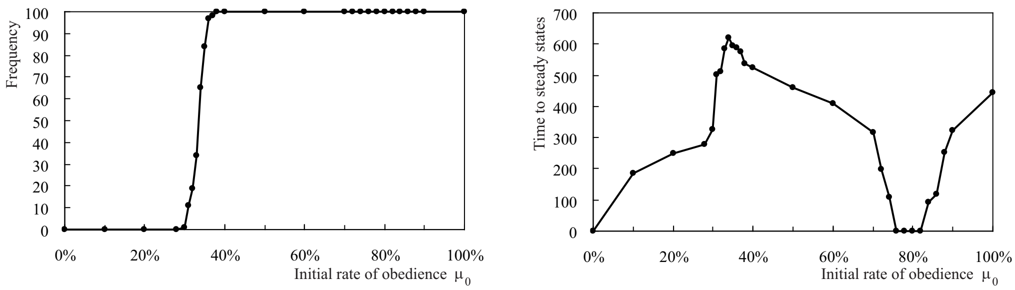
Figure 4: Equilibria in the long run

As seen in Figure 4(f), although agents with personal tastes $0.2 \leq \varepsilon_i \leq 0.8$ which account for 60% of the population disobey the social norm at the beginning of the simulation, a part of them, that is, agents with personal tastes $0.7 \leq \varepsilon_i \leq 0.8$ begin to obey it after period 100, and finally at period 2000, the population converges at a state that agents with personal tastes $\varepsilon_i \geq 0.385$ obey it with a probability of 0.8 and over.

In each of the 100 runs of the treatment of $\mu_0 = 1.0$, the rate of obedience decreases from $\mu_0 = 1.0$ at the beginning to approximately $\mu = 0.78$. An example is shown in Figure 4(g). The rate does not decrease any further and the social norm is maintained with a high level of the obedience rate. As seen in Figure 4(h), although all the agents obey the social norm at the beginning of the simulation, a part of the agents with personal tastes $0.2 \leq \varepsilon_i \leq 0.3$ begin to disobey it after period 100, and finally at period 2000, the population converges at a state that agents with personal tastes $0.2 \leq \varepsilon_i \leq 0.385$ obey it with only a probability of 0.2 and below. This state is the same as the steady state of the treatment of $\mu_0 = 0.4$.

From the above observation, two types of the steady states are observed in the results of the simulation. It follows that if the initial rate of obedience is smaller than 0.3, the social norm becomes extinct in the long run, and otherwise the social norm is maintained at the obedience rate of about 0.78. Let τ^0 and τ^* denote the steady states which mean the extinction and the maintenance of the social norm, respectively. To explore a diverging point of the initial rate of obedience to the two steady states and to measure the time needed for converging to the steady states, we perform further trials of the simulation starting from a large variety of the initial rates of obedience. In Figure 5(a), we show the number of runs such that the social norm is maintained for all treatments with different initial rates of obedience μ_0, and in Figure 5(b), the time needed for converging to the steady states is shown for each of all the treatments. The time needed to reach the steady states is measured as follows. We compute the mean θ and the standard deviation σ of the obedience rate after period 2000 for the 100 runs, and define the time needed to reach the steady states as a minimal period that the rate of obediences μ

of more than 90 runs are in the interval $[\theta - 2\sigma, \theta + 2\sigma]$.



(a) The maintenance of the social norm

(b) Time to converge to the steady states

Figure 5: The maintenance of the social norm and time to steady states

As seen in Figure 5(a), all of the treatments of $\mu_0 \in [0.0, 0.28]$ converge only to the extinction state of the social norm τ^0; the treatments of $\mu_0 \in [0.30, 0.37]$ converge to either of the two steady states τ^0 and τ^*; and all of the treatments of $\mu_0 \in [0.38, 1.0]$ converge to the maintenance state of the social norm τ^*. Concerning the time needed for converging to the steady states, as seen in Figure 5(b), in the treatments of $\mu_0 \in [0.31, 0.40]$, it takes more than 500 periods to converge to either of the two steady states, and it takes a long time until the consequence of the social norm—which is either extinction or maintenance—becomes clear. Viewing this situation from a different angle, we can interpret this to be that the social norm is maintained at a lower level of the obedience rate in the short term.

To compare the results of the simulation with those of the mathematical model by Naylor, first we summarize the equilibria of the mathematical model. (M1) If the initial rate μ_0 of obedience of a society is in the interval $[0.0, 0.20)$, i.e., $\mu_0 \in [0.0, 0.20)$, the rate of obedience μ decreases and finally reaches the equilibrium which means the extinction of the social norm, $\mu = 0$, and it corresponds to the point o in Figure 1. (M2) If $\mu_0 \in [0.20, 0.355]$, the rate of obedience does not change and it is in equilibrium, which corresponds to any point in the interval p-q with the lower level of the obedience rate in Figure 1. (M3) If $\mu_0 \in (0.355, 0.845)$, the rate of obedience increases and finally reaches the equilibrium which means the maintenance of the social norm, $\mu = 0.845$, and it corresponds to the point *m* in Figure 1. (M4) If $\mu_0 \in [0.845, 1.0]$, the rate of obedience does not change and it is in equilibrium, which corresponds to any point in the interval *k*-*m* with the higher level of the obedience rate in Figure 1.

Next, we give a summary of the results of the simulation as follows. (S1) If the simulation starts from any initial rate of obedience in the interval $[0.0, 0.20)$, i.e., $\mu_0 \in [0.0, 0.20)$, the rate of obedience converges to the extinction state of the social norm τ^0. Thus, the result of the simulation supports that of the mathematical model. (S2) If $\mu_0 \in [0.20, 0.355]$, all of the treatments of $\mu_0 \in [0.20, 0.30)$ converge to the extinction state τ^0, and some runs of the treatments of $\mu_0 \in [0.30, 0.355]$ reach τ^0 and the others eventually arrive at the maintenance state of the social norm τ^*. Thus, the result of the simulation does not support the mathematical model's result that the social norm is maintained in the lower level of the obedience rate. However, in the treatments of $\mu_0 \in [0.31, 0.355]$, it takes more than 500 periods to converge to the steady states, and we can interpret this situation with the lower level of the obedience rates as a short-term stable state. (S3) If $\mu_0 \in (0.355, 0.845]$, in a small number of runs in the treatments of $\mu_0 \in (0.355, 0.37]$, the social norm becomes extinct, but in most of the runs, the social norm is maintained at the higher level of the obedience rate. Thus, it follows that the results of the simulation support that of the mathematical model. (S4) If $\mu_0 \in [0.845, 1.0]$, the rate of obedience μ decreases in all runs but the social norm is maintained at the higher level of obedience rate, that is, the rate of obedience converges to τ^*. Although in one sense, because the social norm is maintained in the mathematical model, the results of the simulation support those of the mathematical model. However, in the sense that the rate of obedience does not change from the initial rate, the results of the simulation do not support those of the mathematical model.

In the mathematical model, because believers and non-believers with the same personal tastes differ in their optimal actions when the obedience rate μ of the society is in the interval *p*-*q* or the interval *k*-*m* in Figure 1 on the assumption that individuals can discriminate between two utilities with a minute difference, any point in the intervals becomes a state of equilibrium. On the other hand, because agents in the simulation model adaptively behave and agents obtaining larger utilities are likely to survive, if an agent which has obtained smaller utilities in the short term comes to obtain higher utilities later, there is some possibility that the agent survives. Although if an agent which is a believer disobeys the social norm, the agent suffers disutility consequent upon the act of disobedience, the agent becomes a non-believer in the next period and the penalty for disobeying the social norm becomes small as long as the agent disobeys it. Therefore, because actions of agents depend on the difference between the utility for agents continuing to obey the social norm and the utility for agents continuing to disobey it over the long run, it is thought that, in the simulation, there does not exist equilibria of intervals such as *p*-*q* or *k*-*m* in Figure 1.

Moreover, the equilibria of the interval *p*-*q* in the mathematical model are considered to be less stable than the interval *k*-*m* because if the rate of obedience deviates from the interval *p*-*q* even slightly, it goes to the points *m* or *o*. In the simulation, there exists a steady state τ^* corresponding to the interval *k*-*m* but a steady state corresponding to the interval *p*-*q* cannot be observed.

To compare behavior of agents in our simulation with that of individuals in the mathematical model by Naylor, consider an agent with relatively small personal taste, $\varepsilon_i = 0.21$, in the treatment of the initial rate of obedience $\mu_0 = 1.0$. The utility for obedience is $U_i = d + \mu\varepsilon_i = 0.1 + 1.0 \cdot 0.21 = 0.31$; the utility just after changing from obedience to disobedience is $U_i = w + \bar{c} = 1.0 - 0.7 = 0.3$; and the utility for continuing disobedience is $U_i = w + \bar{g} = 1.0 - 0.6 = 0.4$. In the mathematical model, an individual with the same personal taste obeys the social norm because the utility of 0.31 for obedience is larger than that of 0.30 for disobedience. On the other hand, because in the simulation, behavior of agents is characterized by trial and error, some agents that obeyed the social norm may disobey it in the next period. Such agents obtain 0.4 of utility by continuing disobedience and consequently they may obtain larger utilities than utilities obtained by agents continuing to obey the social norm. Thus, it is disadvantageous in an evolutionary viewpoint that agents with small values of personal tastes such as $\varepsilon_i = 0.21$ continue obeying the social norm, and there is little chance that such agents can survive in the long run.

The point m of an equilibrium in the mathematical model is $(\mu^m, \varepsilon^m) = (0.845, 0.355)$. At this point a non-believer obtains the same utility irrespective of his or her choice between the actions. Thus, from $\mu^m = 1.2 - \varepsilon^m$, we have $d + \mu^m \varepsilon^m = d + (1.2 - \varepsilon^m)\varepsilon^m = w + \bar{g} = 0.4$ and therefore $(\mu^m, \varepsilon^m) = (0.845, 0.355)$. In the simulation model with adaptive artificial agents, it is thought that it is an even chance that agents with personal tastes around a certain border value ε^* obey the social norm. There is a larger chance of obedience for an agent with a larger value of personal tastes than ε^*, and conversely an agent with a smaller value of personal tastes than ε^* is likely to disobey it. The individual obedience rate of agents with the personal taste ε in the long run can be estimated as a certain nonlinear function $f(\varepsilon; \varepsilon^*)$ like the gray curve depicted in Figure 4(f) or 4(h). Provided that such a function $f(\varepsilon; \varepsilon^*)$ is given, the rate of obedience in the society can be obtained by computing $\mu(\varepsilon^*) = \int_{0.2}^{1.2} f(\varepsilon; \varepsilon^*) d\varepsilon$. Particularly in the setting of this simulation, we have a pair of the obedience rate and the border value of the personal taste $(\mu^* = \mu(\varepsilon^*), \varepsilon^*) = (0.78, 0.385)$ satisfying $d + \mu(\varepsilon^*)\varepsilon^* = w + \bar{g} = 0.4$, as seen in the steady state represented by the gray curve of Figure 4(f) or 4(h). The obedience rate $\mu^* = 0.78$ of the steady state τ^* is slightly smaller than the obedience rate $\mu^m = 0.845$ of the equilibrium m, and the border value $\varepsilon^* = 0.385$ of the personal taste is larger than the border value $\varepsilon^m = 0.355$ of the equilibrium m.

The result of the simulation suggests that when a social norm is maintained, a section of individuals who do not care about their reputation disobey it. That is, there does not exist a society where all the individuals obey the social norm. Moreover, the results also suggest that even if in the short term there exists a social norm supported by a small number of people like a situation corresponding to the interval p-q in the mathematical model, there is little chance that such a social norm would be maintained in the long run.

4.3 Simulation *Belief*

In the simulation model, the degree of belief (5) is determined by a series of an agent's actions in the long run, and the agent obtains utility (6) depending on the degree of belief. In Simulation *Belief*, we examine the influence of the belief and the penalty based on the belief on decision making of agents by varying a value of the discount factor for past actions.

To make results of the simulation clearer, we expand the difference between the penalties $\bar{c}$ and $\bar{g}$ for believers and nonbelievers by changing them from $\bar{c} = 0.7$ and $\bar{g} = 0.6$ of the standard setting to $\bar{c} = 0.7$ and $\bar{g} = 0.55$. For each of the three treatments of the discount factor $\delta = 0.0, 0.5, 0.9$, we conduct trials with the initial rate of obedience from $\mu_0 = 0.0$ to $\mu_0 = 1.0$ at intervals of 0.1, and each trial is performed for 100 runs. The results of the simulation are shown in Figure 6. In Figure 6, the horizontal axis is the initial rate of obedience and the vertical axis is the number of runs such that the social norm is maintained among the 100 runs. In Figure 7, we give transitions of the obedience rate for the treatments of $\delta = 0.0$ and $\delta = 0.9$ which start from the initial rate of obedience $\mu_0 = 1.0$. For the treatments of $\delta = 0.9$, the transition shown in the figure is one of 100 runs, and for the treatment of $\delta = 0.0$, two transitions are shown because there are two levels at which the obedience rate converges in the long run. The broken lines in the graphs show the rate of obedience $\mu = 0.7$, which means the lower limit of the equilibria with the higher level of the obedience rate in the mathematical model for the parameters of this simulation; the rate of $\mu = 0.7$ corresponds to a point m in Figure 1.

As can be seen in Figure 6, the frequency of maintenance of the social norm increases as the discount factor rises. For the treatment of $\delta = 0.9$, the social norm is maintained in all of the 100 runs when the initial rate of obedience is larger than 0.6, $\mu_0 \geq 0.6$, and for the treatment of $\delta = 0.0$, even if the initial rate of obedience is larger than 0.7, $\mu_0 \geq 0.7$, the social norm becomes extinct in 10 runs among the 100 runs. In the two runs depicted in Figure 7(a), both of them temporarily

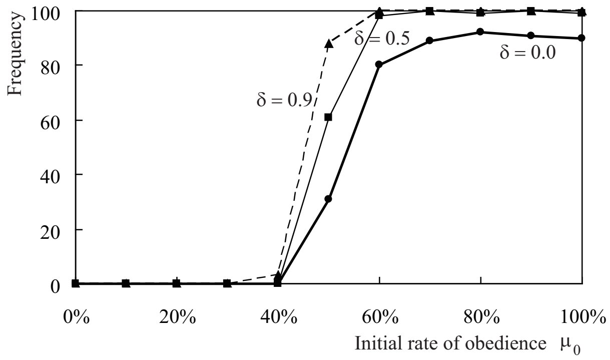
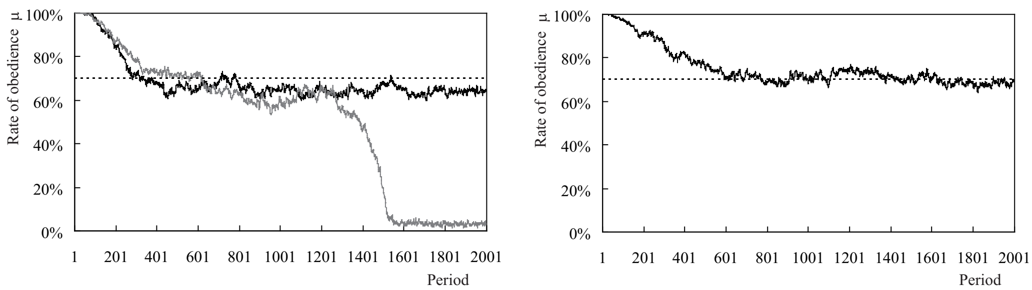


Figure 6: Frequency of maintenance of the social norm with respect to the discount factors



(a) $\delta = 0.0$ (b) $\delta = 0.9$

Figure 7: The transition of the rate of obedience

converge at around $\mu = 0.7$ until period 400, and thereafter in one of them the social norm is maintained to the end of the simulation, but in the other one, it suddenly decreases after period 1300 and finally becomes extinct. The transition shown in Figure 7(b) converges to the steady state until around period 600. From the transitions of (a) and (b) in Figure 7, it is observed that the rate of obedience in the treatment of $\delta = 0.0$ converges quickly but it is unstable, compared with the treatment of $\delta = 0.9$.

To examine stability of maintenance of the social norm in detail, we conduct treatments of the discount factor from $\delta = 0.0$ to $\delta = 0.9$ at intervals of 0.1, fixing the initial rate of obedience at $\mu_0 = 1.0$; each treatment is performed 100 runs. The frequency of maintenance of the social norm is shown in Figure 8, and the mean and the standard deviation of runs where the social norm is maintained is shown in Figure 9.

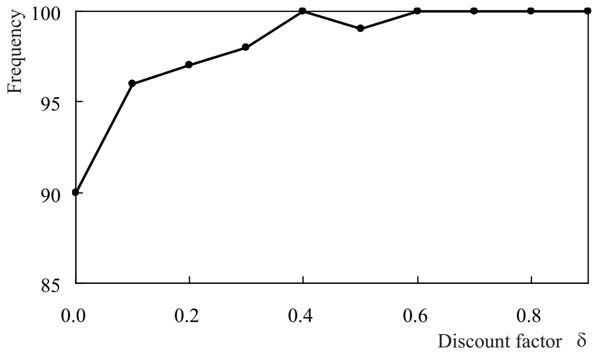
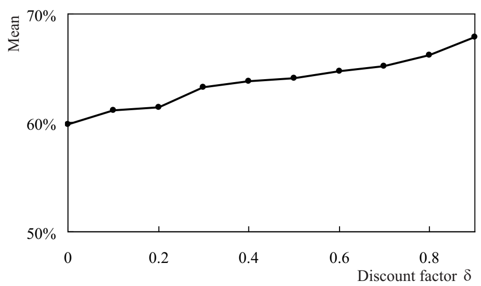
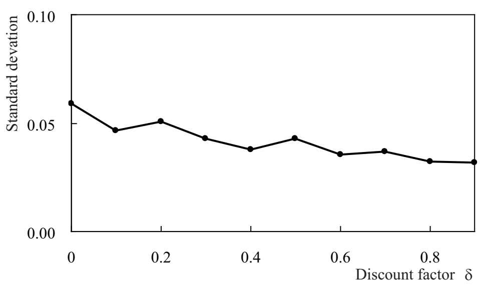


Figure 8: Stability of maintenance of the social norm



(a) Mean



(b) Standard deviation

Figure 9: Mean and standard deviation of the maintenance rate of the social norm

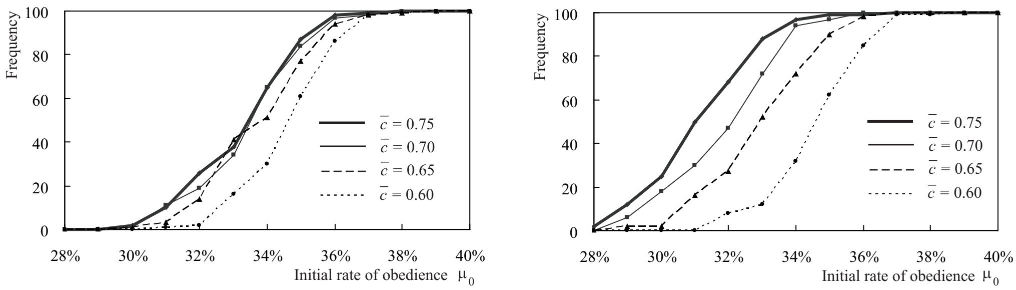
As seen in Figures 8 and 9, as the discount factor δ is made larger, which means that the influence of actions from the past on the belief becomes serious, the frequency of maintenance and the mean value of the obedience rate increases and its standard deviation decreases. The reason for this fact is that the fluctuation in utility by changing actions is small when the discount factor is large. For the treatment of $\delta = 0.0$, it follows that the degree of belief b is defined only by one action in the prior period, and because b is 0 or 1, the penalty for a shift of actions from obedience to disobedience is larger than that of the treatment of $\delta = 0.5$ or $\delta = 0.9$. Thus, behavior of agents is likely to settle either to obey or to disobey, and as seen in Figure 7, the time needed to reach the steady states increases with the value of δ. In the treatment of $\delta = 0.0$, because of the above mentioned reason, actions chosen by agents are likely to be one-sided, if the rate of obedience becomes less than a certain level, the social norm can become extinct even with a high initial rate of obedience as seen Figure 7(a) because of the stochastic reproduction based on the roulette wheel selection in the artificial genetic system.

We summarize the results of Simulation *Belief* as follows. When the degree of belief is determined by a series of an agent's actions in the long run, it is likely that the rate of obedience rises and the state of the artificial society remains stable. On the other hand, for the case where the degree of belief is determined only by one action in the prior period, there is some chance that the social norm becomes extinct even if the initial rate of obedience is sufficiently large, and it is observed that the variance of the rate of obedience is large even in the case of maintenance of the social norm. Namely, if agents act myopically, that is, the discount factor is small, uncertainty or diversity of existence of the social norm increases.

4.4 Simulation *Penalty*

In Simulation *Penalty*, we focus on the penalty for disobeying the social norm. There are two types of penalties for disobeying the social norm in the model by Naylor: the penalty $\bar{c}$ to believers and the penalty $\bar{g}$ to non-believers. In our simulation model, we employ a similar setting, that is, $\bar{c}$ is the penalty to agents with the degree of belief $b = 1$ and $\bar{g}$ is the penalty to agents with the degree of belief $b = 0$. In the model by Naylor, it is concluded that the decrease of the penalty leads to a fall of the rate of obedience and the increase of the penalty leads to a rise of it. We examine whether or not the results of Simulation *Penalty* support this conclusion.

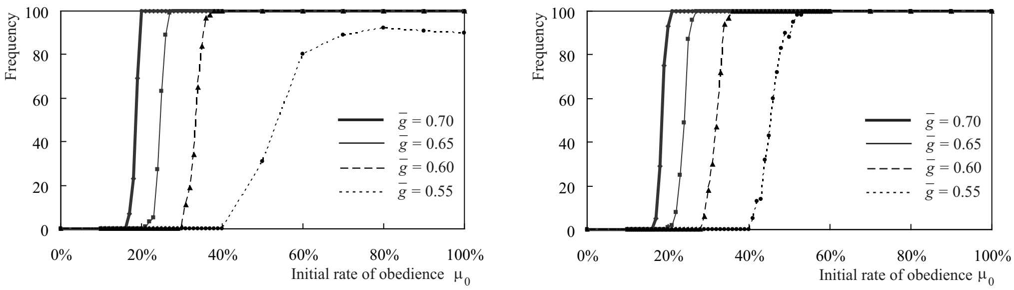
We conduct two treatments of $\bar{c}$ and $\bar{g}$. For each of them, the discount factor is set at $\delta = 0.0$ and $\delta = 0.9$: $\delta = 0.0$ where the degree of belief is determined only by one action in the prior period; $\delta = 0.9$ where the degree of belief is determined by a series of an agent's actions in the long run. In the treatment $\bar{c}$, four trials $\bar{c} = 0.60, 0.65, 0.70, 0.75$ are performed, fixing at $\bar{g} = 0.6$. In the treatment $\bar{g}$, four trials $\bar{g} = 0.55, 0.60, 0.65, 0.70$ are performed, fixing at $\bar{c} = 0.7$. Each set of trails is performed 100 runs. The results are shown in Figures 10 and 11. In both of the figures, the horizontal axis is the initial rate of obedience and the vertical axis is the number of runs such that the social norm is maintained among the 100 runs. Moreover, the rate of obedience only in runs such that the social norm is maintained is shown in Figures 12 and 13.



(a) $\delta = 0.0$ (b) $\delta = 0.9$

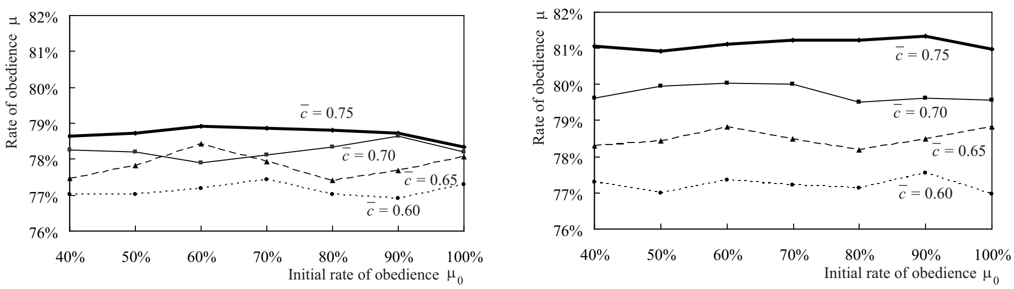
Figure 10: The maintenance number of the norm with respect to the penalties $\bar{c}$

As seen in (b) of Figure 10, for the treatment of $\bar{c}$ with $\delta = 0.9$, an increase of the penalty $\bar{c}$ for agents with the degree of belief $b = 1$ evidently leads to a rise of the maintenance number of the social norm. For the treatment of $\bar{c}$ with $\delta = 0.0$, a similar phenomenon can be only just observed but there is not much difference between the trials $\bar{c} = 0.65$, 0.70, and 0.75. We can observe a similar result in Figure 12; in $\delta = 0.9$, an increase of the penalty $\bar{c}$ causes the rate of obedience to rise, and in $\delta = 0.0$, the difference of the rate of obedience between $\bar{c} = 0.65$, 0.70, and 0.75 is not clear. The reason for these facts is supposed as follows. Consider an agent which continues obeying the social norm. If the agent disobeys in the



(a) $\delta = 0.0$ (b) $\delta = 0.9$

Figure 11: The maintenance number of the norm with respect to the penalties $\overline{g}$

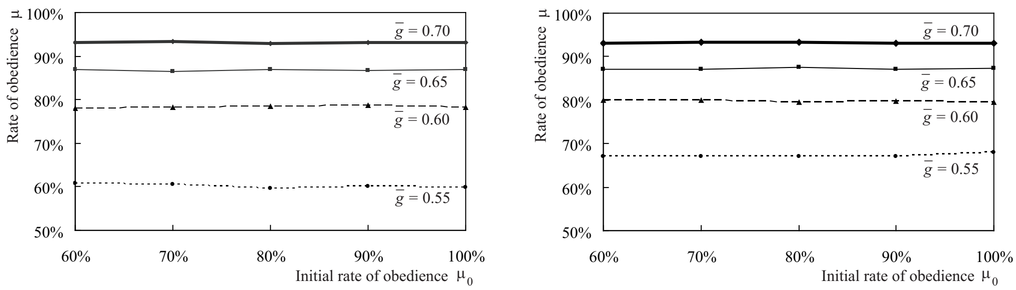


(a) $\delta = 0.0$ (b) $\delta = 0.9$

Figure 12: The rate of obedience with respect to the penalties $\overline{c}$

next period, although the agent is affected by the penalty $\overline{c}$ only one period in the case of $\delta = 0.0$, it continues to suffer the penalties close to $\overline{c}$ over many periods in the case of $\delta = 0.9$. Thus, in the treatment of $\overline{c}$ with $\delta = 0.0$, it is supposed that variation in the value of $\overline{c}$ does not have a large influence on the maintenance number of the social norm and the rate of obedience.

As seen in Figure 11, for the treatment of $\overline{g}$ both with $\delta = 0.0$ and with $\delta = 0.9$, an increase of the penalty $\overline{g}$ for agents with the degree of belief $b = 0$ leads to a rise of the maintenance number of the social norm. There is hardly any difference between the cases of $\delta = 0.0$ and $\delta = 0.9$ in the trials of $\overline{g} = 0.70, 0.65, 0.60$, and the variation of the discount factor δ does not have much effect on the maintenance number of the social norm when the gap $\overline{c} - \overline{g}$ is relatively small. In the trial of



(a) $\delta = 0.0$ (b) $\delta = 0.9$

Figure 13: The rate of obedience with respect to the penalties $\bar{g}$

$\bar{g} = 0.55$ with $\delta = 0.0$, the maintenance number of the social norm does not reach 100 even if the initial rate of obedience is larger than 60%, i.e, $\mu_0 \geq 0.6$. As seen in Figure 13, an increase of $\bar{g}$ leads to a rise of the obedience rate in runs where the social norm is maintained, and only in the trial of $\bar{g} = 0.55$, there exists some difference in the rate of obedience between $\delta = 0.0$ and $\delta = 0.9$.

From the above analysis, it is found that variation in the value of $\bar{g}$ has a larger impact on the maintenance number of the social norm and the rate of obedience than that of $\bar{c}$. To examine this fact in detail, we give the maintenance number of the social norm among the total 1100 runs for each of the treatments with a rate of obedience from $\mu_0 = 0.0$ to $\mu_0 = 1.0$ at intervals of 0.1 in Table 2, where the results of the treatments of $\bar{c} = 0.70$ and $\bar{g} = 0.60$ are arranged in the same column because these values of $\bar{c}$ and $\bar{g}$ are the standard setting.

Table 2: Fluctuations in the maintenance number of the social norm respect to the penalties $\bar{g}$ and $\bar{c}$

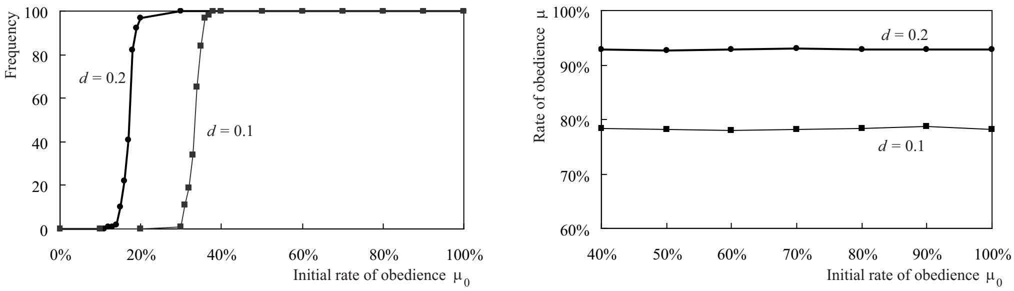
$\bar{c}$		0.75	**0.70**	0.65	0.6
$\delta = 0.0$		702	701	701	700
$\delta = 0.9$		725	718	702	700
$\bar{g}$	0.70	0.65	**0.60**	0.55	
$\delta = 0.0$	900	800	701	473	
$\delta = 0.9$	893	801	718	588	

As seen in Table 2, for the cases of $\delta = 0.0$ and $\delta = 0.9$, the differences between $\bar{c} = 0.75$ and $\bar{c} = 0.65$, which shift 0.05 from the standard setting $\bar{c} = 0.70$, are 1 and 23, respectively. On the other hand, the differences between $\bar{g} = 0.65$ and $\bar{g} = 0.55$ are 327 and 213, respectively. From this fact, it is found that an increase of the penalty $\bar{g}$ leads to a rise of the maintenance number of the social norm effectively, compared with the increase of the penalty $\bar{c}$. The penalty $\bar{c}$ to agents with $b = 1$ motivates them to continue obeying, and it has an influence on the utility only when agents continuing to obey switch to disobeying. In contrast, the penalty $\bar{g}$ to agents with $b = 0$ motivates them to merely obey the social norm. In the simulation, behavior of agents is characterized by trial and error, and therefore they are likely to change their actions. Consequently, it is supposed that variation in the value of $\bar{c}$ does not have a large influence on the maintenance number of the social norm and the rate of obedience, compared with $\bar{g}$.

4.5 Simulation *Payoff*

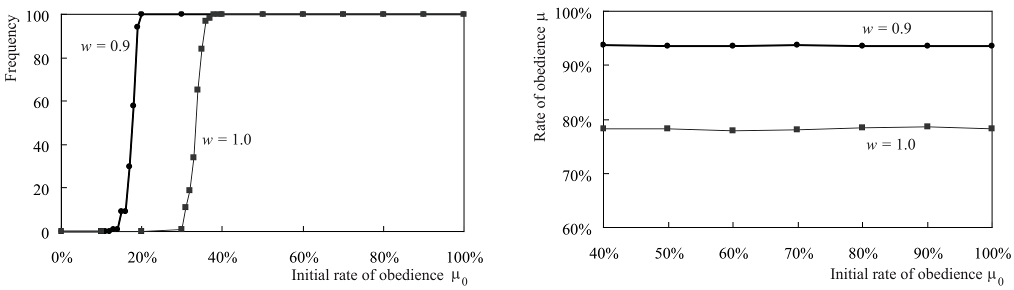
Simulation *Payoff* deals with the payoff d obtained by obeying the social norm and the payoff w obtained by disobeying the social norm. In the model by Naylor, it is concluded that the obedience rate of the social norm increases with the payoff d and it decreases with the payoff w. We examine whether or not the results of the simulation support this claim.

Because the discount factor δ has no connection with the payoffs d and w in the definition (6) of the utility U_i of an agent, in Simulation *Payoff*, we perform only cases of $\delta = 0.0$. In the treatment of obedience payoff d, three trials $d = 0.20$, 0.10, 0.00 are performed, fixing w at 1.0. In the treatment of the disobedience payoff w, three trials $w = 0.9$, 1.0, 1.1 are also performed, fixing d at 0.1. Each of the trials with an initial rate of obedience from $\mu_0 = 0.0$ to $\mu_0 = 1.0$ at intervals of 0.1 is performed 100 runs; altogether it comes to 1100 runs. The results of the simulation are shown in Figures 14 and 15. The graphs (a) of the figures show the number of runs such that the social norm is maintained among the 100 runs for each of the different initial rates μ_0; the graphs (b) of the figures show the rate of obedience in runs where the social norm is maintained. The results of the treatments of $d = 0.0$ and $w = 1.1$ are not shown in the figures because the social norm is not maintained for any of the 1100 runs. Furthermore, to compare the result of the treatment of d with that of the treatment of w, we give the maintenance number of the social norm among the total 1100 runs in Table 3.



(a) Frequency (b) Rate of obedience

Figure 14: Frequency of maintenance with respect to and the rate of obedience the payoffs *d*



(a) Frequency (b) Rate of obedience

Figure 15: Frequency of maintenance and the rate of obedience with respect to the payoffs *w*

As seen in Figure 14, for the treatment of d, an increase of the obedience payoff d leads to a rise of the maintenance number of the social norm, and it also leads to a rise of the obedience rate. In contrast, from Figure 15, for the

Table 3: Fluctuations in frequency of maintenance of the social norm with respect to the payoffs *d* and *w*

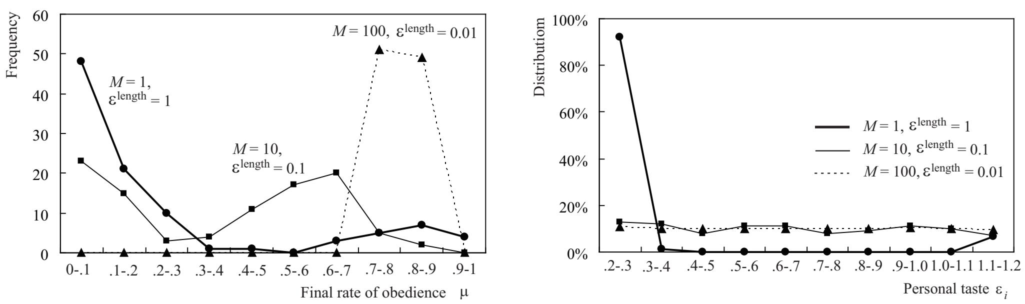
d	0.2	**0.1**	0.0
frequency	897	701	0
w	0.9	**1.0**	1.1
frequency	900	701	0

treatment of w, it is apparent that an increase of the disobedience payoff w results in a fall of the maintenance number of the social norm, and it also leads to a fall of the obedience rate. Thus, it follows that these results of the simulation support the conclusion of the mathematical model by Naylor. From Table 3, it is also observed that the results of the treatment of d are similar to that of the treatment of w, and therefore it is found that a gap between d and w which means a pecuniary incentive to disobey the social norm has an effect on maintenance of the social norm. Thus, as the value of $w - d$ grows large, the social norm is likely to become extinct because the pecuniary incentive to disobey it is strong, and vice versa.

4.6 Simulation *Interaction*

Because interaction among agents in our artificial agent system is implemented as the genetic operations in the evolutionary process, the interaction is limited within a subpopulation of agents with similar personal tastes. In Simulation *Interaction*, we verify whether or not the maintenance of the social norm depends on the scope of interaction among agents, and if so, we examine how the scope of interaction influences maintenance or extinction of the social norm.

In the simulations described in the previous sections, N agents in the whole population are divided into M groups, and each group is a subpopulation within which the genetic operations are executed. In Simulation *Interaction*, we vary the number of groups M, and conduct three treatments $M = 1$, 10, 100, fixing the number of agents at $N = 10000$. Each of the treatments starts at the initial rate of obedience of $\mu_0 = 0.5$, and it is performed 100 runs. In the treatment $M = 1$, there is only one subpopulation, and the genetic operations are executed in the whole population of agents with personal tastes in the interval $[0.2, 1.2]$. For the treatments $M = 10$ and $M = 100$, after the personal tastes of all the agents are uniformly distributed in the interval $[0.2, 1.2]$, in the treatment $M = 10$, subpopulations are formed by dividing the interval $[0.2, 1.2]$ into 10 subintervals, and in the treatment $M = 100$, subpopulations are formed by dividing the interval $[0.2, 1.2]$ into 100 subintervals. Therefore, for the treatments $M = 1$, 10, 100, the lengths of the subintervals are $\varepsilon^{\text{length}} = 1.0, 0.1, 0.01$, respectively. In Figure 16, we show the final frequency distribution of the obedience rate and the final distribution of the personal tastes after period 2000.



(a) The rate of obedience (b) The personal tastes

Figure 16: The results of Simulation *Interaction*

For the treatment $M = 100$ with $\varepsilon^{\text{length}} = 0.01$, as mentioned in Simulation *Basis*, because the rate of obedience starts from $\mu_0 = 0.5$ at the beginning of the simulation, the final rate of obedience after period 2000 converges in rates of obedience from $\mu = 0.7$ to $\mu = 0.9$. As seen in Figure 16(b), the personal taste is almost uniformly distributed even at the end of the simulation because the whole population is divided into a large number of subpopulations.

For the treatment $M = 10$ with $\varepsilon^{\text{length}} = 0.1$, as seen in Figure 16(a), the distribution of the rate of obedience at the end of the simulation has two peaks; there are high frequencies in the intervals $[0.0, 0.2]$ and $[0.4, 0.7]$. It is supposed that the former peak indicates the extinction of the social norm and the latter indicates the maintenance of it. Compared with the treatment $M = 100$, as seen in Figure 16(b), the distribution of the personal tastes of the treatment $M = 10$ is slightly biased toward lower values. In two thirds of the 100 runs, the personal taste is almost uniformly distributed to the end of the simulation as well as the treatment $M = 100$ and the social norm is maintained. In the rest of the runs, the distribution of the personal tastes is slightly biased toward lower values, and the social norm becomes extinct in the long run.

For the trial $M = 1$ with $\varepsilon^{\text{length}} = 1$, as seen in Figure 16(a), the distribution of the rate of obedience at the end of the simulation has two peaks; there are high frequencies in the intervals $[0.0, 0.2]$ and $[0.8, 1.0]$. The former peak is higher than the latter one, and the latter peak shifts to the right hand side, compared with the treatment $M = 10$. This means that the rate of obedience is higher than the other treatments when the social norm is maintained. The distribution of the personal tastes concentrates in the interval $[0.2, 0.3]$, except for a small portion from $\varepsilon_i = 1.1$ to $\varepsilon_i = 1.2$. Because in the treatment $M = 1$, interaction among agents is not restricted and the scope of interaction is equivalent to the whole population, most of the agents with higher values of personal tastes which pay attention to the reputation arising from obedience of the social norm do not survive in the long run. Consequently in most part of the runs the social norm becomes extinct. However, it is interesting that inversely in a small part of the runs, agents with higher values of personal tastes hold a majority and the social norm is maintained with high rates of obedience in the long run.

The results of Simulation *Interaction* are summarized as follows. When the scope of interaction is extremely restricted, maintenance or extinction of the social norm depends on the initial rate of obedience and uncertainty about outcomes of the simulation is not very high. When the scope of interaction is broad, although the social norm is likely to become extinct generally, the rate of obedience is high if the social norm is maintained; on the whole the uncertainty of outcomes increases.

5 Conclusions

In this paper, we have developed a simulation system where artificial agents act adaptively and their belief depends on a series of actions in wide-ranging past periods. Using the system, we have compared the results of the mathematical model by Naylor with those of the simulations. Furthermore, we have examined the influence of the scope of interaction among agents. The results of the simulations roughly support the claim from the mathematical model by Naylor, except for the existence of the equilibria with lower rates of obedience and maintenance at high obedience rates such as 100%. When the degree of belief is determined by a series of an agent's actions in the long run, the obedience rate of the social norm in the artificial society rises and uncertainty or diversity within the existence of the social norm decreases. From the simulation for the scope of interaction, we have found that if the scope of interaction is broad, the social norm is likely to become extinct generally and uncertainty of states of the artificial society increases.

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