Generating Ultralow-Emittance Ion Beams in a Storage Ring

Y. Yuri and H. Okamoto

Department of Quantum Matter, Graduate School of Advanced Sciences of Matter, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima 739-8530, Japan

(Received 16 June 2004; published 11 November 2004)

Laser cooling of heavy-ion beams in a storage ring is systematically studied with a multiparticle simulation code where not only exact lattice characteristics and space-charge forces but also realistic laser-ion interactions can be incorporated. The *resonant coupling method* is applied in order to extend the powerful longitudinal photon pressure to the transverse degrees of freedom. It is shown that, in spite of a space-charge-induced tune shift, the synchrobetatron resonance mechanism required for fast damping of transverse oscillations operates throughout the cooling process. Extremely efficient three-dimensional cooling of stored ion beams is thus feasible. It is demonstrated that, at low line density, normalized root-mean-squared emittances of the order of 10^{-12} m · rad can be reached in all three directions by employing only existing technologies.

DOI: 10.1103/PhysRevLett.93.204801 PACS numbers: 29.20.Dh, 29.27.Fh, 41.75.–i

Generating "zero-emittance" beams is an ultimate goal in accelerator physics. The emittance, the μ-space volume occupied by a beam, is the measure of beam quality that we wish to control freely. This fundamental quantity of great practical importance is, however, known to be invariant in conservative dynamical systems. Some dissipative mechanism must, therefore, be developed to compress a beam in phase space. The procedure of improving the beam quality is called *cooling* because the beam temperature in the rest frame actually becomes lower in that process. The purpose of this Letter is to show what level of beam quality we can obtain by employing only existing, well established accelerator technologies. As demonstrated below, a projected normalized root-mean-squared (rms) emittance of the order of 10^{-12} m · rad will be attainable in all 3 degrees of freedom.

To the best of our knowledge, the most powerful dissipative force is brought by laser photons [1]. *Laser cooling,* although it is applicable only to ion beams, enables us to reach a temperature range far below those achievable with other cooling techniques [2,3]. In fact, the Doppler limit of laser cooling is in a milli-Kelvin range or even lower, which means that we can come very close to a zero-emittance state, at least in theory. Note also that laser cooling works, in principle, much faster than other conventional methods. It is quite natural to state that laser cooling is currently the most promising means for us to realize an ultracold beam.

There is, however, an essential problem in the Doppler cooling process, unfortunately; since the photon pressure operates only in the direction of laser propagation, no efficient transverse cooling is available as long as the beam is circulating at great speed [2,3]. Although we may expect some "sympathetic" transverse cooling through particle collisions, this effect is generally negligible unless the beam is well conditioned before laser cooling [4]. In order to overcome this practical difficulty, Okamoto, Sessler, and Möhl have proposed a simple solution that is referred to as the *resonant coupling method* [5,6]. The basic idea is not limited to laser cooling but can be applied to any general situation where we have a one (or two) dimensional dissipation that ought to be extended to the other degree(s) of freedom. All we have to do is just to introduce a linear coupling source and then excite a difference resonance. For three-dimensional (3D) laser cooling, the following two conditions must be satisfied initially

$$\nu_x - \nu_y \approx m, \qquad \nu_x - \nu_s \approx n \qquad (m, n = \text{integer}), \tag{1}$$

where ν_x and ν_y are the betatron tunes of the storage ring, and ν_s is the synchrotron tune. Since ν_s must be finite to avoid transverse integer resonances under the condition (1), this 3D cooling scheme is relevant only to bunched beams [7]. As a source of synchrobetatron coupling, we can use either a special radio-frequency (rf) cavity operating in a deflective mode [5] or a regular rf cavity sitting at a dispersive position [6]. Coupling between the horizontal and vertical motions can readily be provided, e.g., by a solenoid or a skew quadrupole magnet. It is probably informative to mention that a stronger coupling does not necessarily result in more efficient indirect cooling. As first proven in Ref. [5], the indirect cooling rate cannot be enhanced without resonance even if we drastically increase the coupling constant.

It order to perform a systematic study of the present subject, we have developed a multiparticle simulation code CRYSTAL in which an almost exact laser-cooling process can be taken into account. As is well known, the dissipative force generated by a laser light can be expressed as

$$\mathbf{F} = \frac{1}{2}\hbar\mathbf{k}\Gamma\frac{S}{1 + S + (2\Delta/\Gamma)^2}, \tag{2}$$

where $\hbar\mathbf{k}$ is the momentum vector of a laser photon, Γ is the natural line width of the cooling transition, S is the saturation parameter, and Δ is the frequency detuning dependent on the amount of the Doppler shift. Denoting the angular frequency of the laser in the laboratory frame and the transition frequency in the beam frame to be ω and ω_0, respectively, we have the detuning $\Delta = \omega\gamma(1 \pm v_{\parallel}/c) - \omega_0$, where γ is the Lorentz factor, c is the speed of light, $v_{\parallel}$ is the longitudinal velocity of an ion, and the sign depends on which direction we inject a laser. In what follows, we consider a Gaussian distribution of photons, assuming a spatially nonuniform saturation parameter; namely, $S = S_0 \exp[-2(x^2 + y^2)/\sigma^2]$ where S_0 corresponds to the on-axis saturation power, (x, y) are the transverse coordinates, and σ is the laser spot size that depends on the Rayleigh length and longitudinal coordinate. Owing to the random nature of spontaneous photon emission and absorption, diffusive heating takes place, which determines the Doppler cooling limit. We have incorporated this heating mechanism into the code by using the Monte Carlo algorithm. It has been confirmed that our algorithm actually gives the correct Doppler limit.

CRYSTAL is based on a Hamiltonian defined in the beam frame [8]. It is possible to consider the exact lattice structure of a storage ring, the effects of regular and coupling rf cavities, uniform transverse focusing if necessary, adiabatic beam capturing, several different cooling models, etc. Apart from cooling interactions, the equations of motion are integrated in a symplectic manner. When the beam is continuous, the molecular dynamics (MD) method is employed to evaluate not only short-range but also long-range Coulomb interactions among particles. For a bunched beam, the longitudinal length of a unit numerical cell for space-charge evaluation is automatically set equal to that of an rf bucket. Similarly to the standard MD approach, real particles (not macroparticles) are used to calculate the short-range Coulomb interactions within the reference bucket. The Coulomb potential originating from particles in the other buckets is approximately analyzed under the assumption that particle distributions in all buckets are identical.

We have so far explored cooling properties in four different lattice structures including the Small Laser-equipped Storage Ring (S-LSR) now under construction at Kyoto University [9]. All the test rings have relatively high lattice symmetry, so that the betatron phase advance μ_0 per single superperiod can be small. Ideally, μ_0 should be below 90° to avoid crossing linear resonance stopbands through the whole cooling process [10]. The present simulations have suggested, however, that it is possible to go beyond even a linear stopband if the beam intensity is very low. The restriction imposed upon the phase advance is then understood to be $\mu_0 < \pi/\sqrt{2}$ ($\approx 127°$) [11]. We have found that, with this condition satisfied, the reachable beam emittance is insensitive to the details of lattice structures at low line density. In the following, therefore, we only show results on the S-LSR lattice.

Among a wide range of choices, $^{24}Mg^+$ will be adopted for future experiments at S-LSR because it is coolable with a single laser of the wavelength of ~280 nm. The saturation intensity is 254 mW/cm^2. The main simulation parameters are summarized in Table I. The lattice of S-LSR, whose circumference is 22.6 m, has sixfold symmetry. The bare betatron tunes can be chosen below 2.12 in both transverse planes, so that the condition outlined above is fulfilled. A regular rf cavity is located in the middle of a straight section where the size of momentum dispersion is 0.80 m. Longitudinal dissipation can thus be transferred into the horizontal direction through the dispersive coupling [6]. In order to excite a synchrobetatron resonance, the synchrotron tune ν_s must be close to 0.07 in the present parameter setup. The rf amplitude required for this high tune is less than 30 V due to the low beam energy and high harmonic number. For horizontal-vertical coupling, we turn on a solenoid magnet installed in a straight section for electron cooling. The necessary field strength is weak enough to maintain the approximate lattice symmetry.

We assume the initial temperature of a beam to be rather high (typically, $\gtrsim 10^4$ K) in all directions. Two longitudinal lasers, one copropagating and the other counterpropagating with the beam, are introduced in separate straight sections, where each ion interacts with laser photons many times every turn. In order to try capturing all tail particles, we scan the laser frequency over a wide range in 2.6 sec. Figure 1(a) shows the time evolution of normalized rms emittances when the operating point of the ring is far away from synchrobetatron resonance; namely, $\nu_s = 0.01$. The total number of $^{24}Mg^+$ ions in the ring is 800, i.e., eight particles per bunch, in this example. After a frequency scan (completed at 60 000th turn), the longitudinal normalized emittance has reached on the order of 10^{-11} m · rad, while no cooling effect is observed in both transverse directions. We now increase the synchrotron tune to 0.07 such that the resonance conditions in Eq. (1) are approximately satisfied. The result is given in Fig. 1(b) where an ex-

TABLE I. Simulation parameters.

Lattice	S-LSR (Kyoto Univ.)
Ion species	$^{24}Mg^+$
Kinetic beam energy	35 keV
Superperiodicity (without solenoid)	6
Bare betatron tunes (ν_x, ν_y)	(2.067, 1.073)
Rf harmonic number	100
Length of the solenoid magnet	0.8 m
Saturation parameter (on-axis) S_0	1.0
Minimum laser spot size σ	5 mm

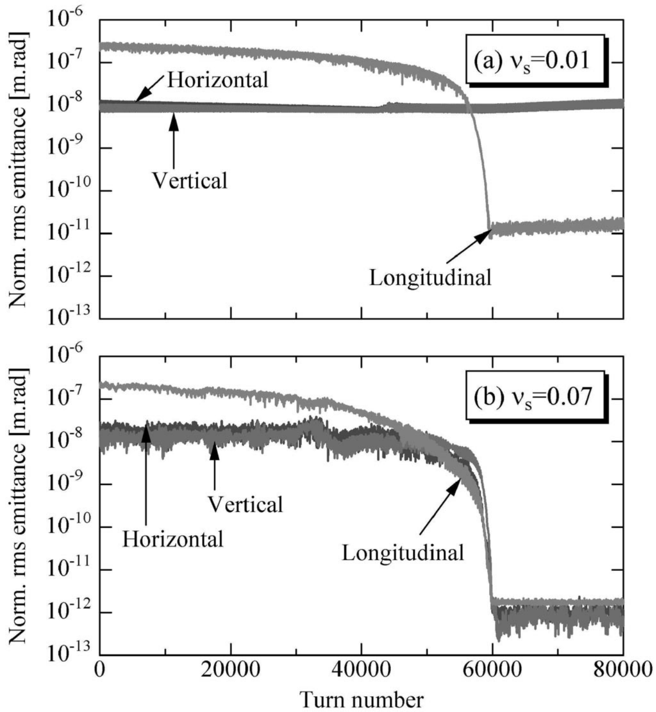

FIG. 1. Time evolution of normalized rms emittances during laser cooling. The synchrotron tune has been adjusted to (a) $\nu_s = 0.01$ and to (b) $\nu_s = 0.07$. Other parameters are listed in Table I. Although the scan of the laser frequency has been initiated at the first turn, many ions are out of resonance with photons until after 50 000 turns.

tremely fast reduction of the emittances has occurred in all three directions. In the final state, the normalized emittances have been equalized on the order of 10^{-12} m · rad. Note that the transverse beam size was roughly twice larger than the laser spot size at the beginning because of the high initial temperature. Nevertheless, efficient 3D cooling has been carried out successfully. As seen from the figure, the actual transverse cooling time is about 200 msec that can be even shortened by speeding the laser-frequency scanning. The ν_s dependence of beam emittances at the end of a laser scan is shown in Fig. 2.

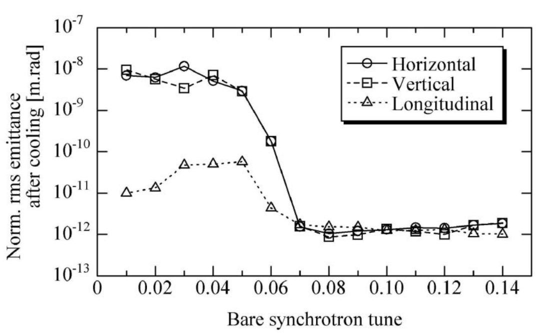

FIG. 2. Dependence of achievable normalized emittances on the bare synchrotron tune ν_s.

We find that 3D cooling can be done whenever the synchrotron tune is above the optimum resonant value. The explanation to this result is given later.

The real-space configuration of a cooled beam is depicted in Fig. 3. Clearly, a *stringlike* order has been established. The synchrotron motion of each individual ion has been completely frozen out in this state. We have also confirmed that this curious beam profile can be transformed into a *zigzag* configuration either by increasing the synchrotron tune or by adding two more ions in the bucket. The threshold synchrotron tune at which a string-to-zigzag transition occurs is $\nu_s \approx 0.09$ with the parameters considered here.

The resonant coupling method was originally intended for achieving ultrafast 3D cooling of an ordinary *hot* beam into a *cold* state where intrabeam scattering is dominant. Once the beam becomes sufficiently dense, we can then rely on the sympathetic effect to proceed further [12]. The efficiency of transverse cooling will, however, be lowered if the expected breakdown of the synchrobetatron resonance really takes place due to a space-charge-induced tune-shift. In fact, the sympathetic cooling is much less effective than the indirect cooling through artificial resonant coupling. Furthermore, the possible reduction of the transverse cooling rate may limit the attainable beam emittance, because intrabeam scattering not only builds up coupling but also produces heating [11]. If the peak heating rate is higher than the indirect cooling rate, the beam will settle into a sort of equilibrium at relatively high temperature. Contrary to this natural perspective, the resonant coupling appears to be quite effective throughout the cooling process, as indicated by the result in Fig. 1(b). The reason why the coupling mechanism keeps working even in the ultralow-emittance regime is, in one word, that any storage ring has momentum dispersion due to the existence of bending magnets. If an rf cavity is there, each particle receives some energy from the rf field every turn. The amount of the energy gain (or loss) depends on the timing when the particle passes through the cavity. As a 3D cooling pro-

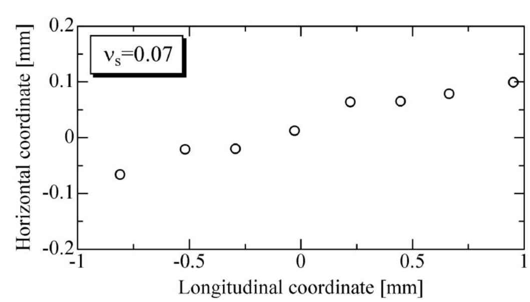

FIG. 3. Real-space profile of the laser-cooled beam in Fig. 1(b). Each circle represents a single $^{24}Mg^+$ ion circulating in the cooler storage ring S-LSR at the kinetic energy of 35 keV. The linear configuration survives for many turns even without the cooling force.

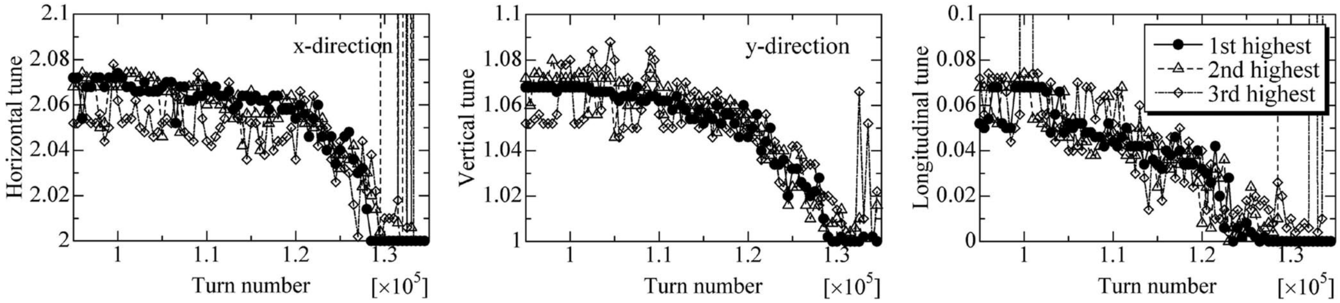

FIG. 4. Effective tunes of transverse and longitudinal particle oscillations evaluated from simulation data. The incoherent tunes corresponding to the three highest peaks identified in the Fourier spectrum of a typical single-particle trajectory have been plotted as a function of turn number. The scan of the laser frequency is ended at the 1.3×10^5 turn in this picture. The same lattice parameters as in Fig. 1 have been assumed, while the number of $^{24}Mg^+$ ions in a bunch is now 200. The initial value of ν_s is 0.07. The final normalized rms emittance is somewhat higher than the case in Fig. 1(b) due to the dispersive heating [12], but still around 10^{-11} m · rad in all three directions. The final configuration of the bunch is a long ellipsoid whose axis transversely oscillates in the same way as the string beam in Fig. 3.

cess advances, this timing for a particular particle gradually becomes nonrandom because the synchrotron motion is more and more suppressed. Finally, the ring dispersion, which forces off-momentum particles to deviate from the design orbit, causes the whole beam to oscillate horizontally [13]. We actually notice that the string beam in Fig. 3 is not along the beam line but slightly tilted. It has been verified that the beam executes a periodic "head-tail" oscillation, keeping its linear profile.

In an ultralow-emittance state, the transverse tune of this dispersive oscillation is equal to the round number nearest to the bare betatron tune (provided that the strict lattice superperiodicity including cavities is unity) [13]. Since $(\nu_x, \nu_y) = (2.067, 1.073)$ in the present case, we expect that the effective incoherent tunes eventually converge not at zero but at 2.0 in the horizontal direction and 1.0 in the vertical direction [14]. The validity of this expectation has been demonstrated in Fig. 4 where the effective tunes obtained from Fourier analysis are plotted as a function of turn number. Recalling that the tune-shift problem is more severe at higher intensity, we have increased the total number of stored ions to 2×10^4 in this simulation. It is evident that the space-charge repulsion gradually depresses the three tunes but their fractions are always close enough for resonance. The pictures also provide an explanation to the observed ν_s dependence of achievable emittances in Fig. 2. In an off-resonance situation, the laser-cooling force only affects the longitudinal motion, leading to a shift of ν_s (while no tune shifts occur in the transverse directions). Consequently, the decrease of the effective longitudinal tune automatically brings the beam onto a synchrobetatron resonance whenever ν_s is initially above the optimum value. Once the 3D coupling mechanism begins to work, all 3 degrees of freedom are simultaneously cooled as shown in Fig. 1(b).

To conclude, it is possible to generate ultralow-emittance ion beams in a properly-designed storage ring by means of the 3D laser-cooling scheme utilizing artificial resonant coupling. The results of this Letter strongly suggest that a heavy-ion beam with the highest quality ever reached, or even a phase transition to a Coulomb crystallized state, can be realized at the cooler storage ring S-LSR.

[1] D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. **20**, 637 (1975); T. Hänsch and A. Schawlow, Opt. Commun. **13**, 68 (1975).
[2] S. Schröder *et al.*, Phys. Rev. Lett. **64**, 2901 (1990).
[3] J. S. Hangst *et al.*, Phys. Rev. Lett. **67**, 1238 (1991).
[4] For instance, it is necessary to precool the beam with another cooling device, such that the beam density becomes high enough for the sympathetic coupling.
[5] H. Okamoto, A. M. Sessler, and D. Möhl, Phys. Rev. Lett. **72**, 3977 (1994).
[6] H. Okamoto, Phys. Rev. E **50**, 4982 (1994).
[7] Needless to say, the beam is not necessarily bunched for 2D cooling in the transverse phase space. In order to extend a horizontal (vertical) cooling force to the vertical (horizontal) direction, only the first condition in Eq. (1) is required.
[8] J. Wei, X.-P. Li, and A. M. Sessler, Brookhaven National Laboratory Report No. BNL-52381, 1993.
[9] A. Noda, Nucl. Instrum. Methods Phys. Res., Sect. A **532**, 150 (2004).
[10] K. Okabe and H. Okamoto, Jpn. J. Appl. Phys. **42**, 4584 (2003).
[11] J. Wei, H. Okamoto, and A. M. Sessler, Phys. Rev. Lett. **80**, 2606 (1998).
[12] If we wish to make a stable 3D *crystalline beam*, we eventually need the so-called "tapered" force; see, e.g., H. Okamoto and J. Wei, Phys. Rev. E **58**, 3817 (1998).
[13] H. Okamoto, Y. Yuri, and K. Okabe, Phys. Rev. E **67**, 046501 (2003).
[14] The vertical oscillation is driven by the vertical dispersion induced through the solenoid coupling.