Monte Carlo Study of SU(4) Gauge Theory at Finite Temperature

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The pure SU(4) Yang-Mills theory is studied at finite temperature. We observe a first-order transition at $8/g_c^2 = 10.50 \pm 0.02$.

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In this Letter we present the results obtained from a Monte Carlo study of the finite-temperature SU(4) gauge theory. Before we do so, however, let us review some of the physics necessary to understand why SU(4) is interesting. [For the real world the relevant gauge group is of course SU(3) —not SU(4).]

The pure SU(N) Yang-Mills theory is believed to have the property of confinement, i.e., the physical states of the theory are singlets under SU(N) transformations. However, this property does not persist indefinitely when the theory is heated up. At a finite physical temperature the theory undergoes a transition to a new phase—an essentially ideal gas of gluons.[1] From a purely theoretical point of view the study of this transition is important in that it adds to our understanding of the structure of gauge theories and the confinement mechanism. But there are also experimental implications: The temperatures and densities necessary to drive such a transition might be available in relativistic heavy-ion colliders in the near future. Two important questions about the nature of the deconfining phase transition have to be answered. What is its order and what happens to it when quarks are added? Both of these questions have been addressed by many authors. Monte Carlo studies of the SU(2) and SU(3) gauge theories[2] show that the transitions are second- and first-order, respectively. The numerical results of Hasenfratz, Karsch, and Stamatescu[3] indicate that the first-order SU(3) transition is wiped out when quarks of a sufficiently small mass are added. Parallel to these numerical investigations the nature of the deconfining phase transition has also been analyzed analytically. It was argued[4] that the $N \geqslant 4$ transitions might fall into the same universality class as those of some $Z(N)$-invariant spin systems which are known to undergo second-order transitions. Mean-field calculations seemed to support this view.[5] Gocksch and Neri[6] argued that the $N=\infty$ transition should be first order. Recently[7] it was pointed out that although the transition might indeed be that of an effective $Z(N)$ spin system, the "spins" are still SU(N) valued and the fluctuations in the group manifold cannot be neglected. Mean-field analysis showed that it is precisely these fluctuations which drive the $N \geqslant 3$ transitions first order. From the point of view of Ref. 7 the case $N=2$ is special: There the transition is second order in agreement with the Monte Carlo data. Our motivation for studying SU(4) is therefore that it is the easiest way to check the theoretical ideas described above.

We have studied the model defined by

$$S = (2N/g^2) \sum_{\square} (1 - \mathrm{Re}\,\mathrm{tr}\, U_{\square}) \tag{1}$$

at $N=4$. The sum in (1) is over all plaquettes of the lattice. The thermodynamics of the model follows from the partition function

$$Z = \int \prod_{\{n,\mu\}} [dU_\mu(n)] e^{-S(U)}. \tag{2}$$

To implement the condition of finite physical temperature the links $U_\mu(n)$ satisfy

$$U_\mu(n + n_\tau \hat{e}_0) = U_\mu(n), \tag{3}$$

where n is a site, $\hat{e}_0$ a unit vector in the "time" direction, and n_τ the extent of the lattice in this direction. The inverse temperature and the lattice spacing a are related by

$$\beta = n_\tau a(g^2). \tag{4}$$

The action (1) has a by now well-known symmetry under multiplication by an element of Z_N of all

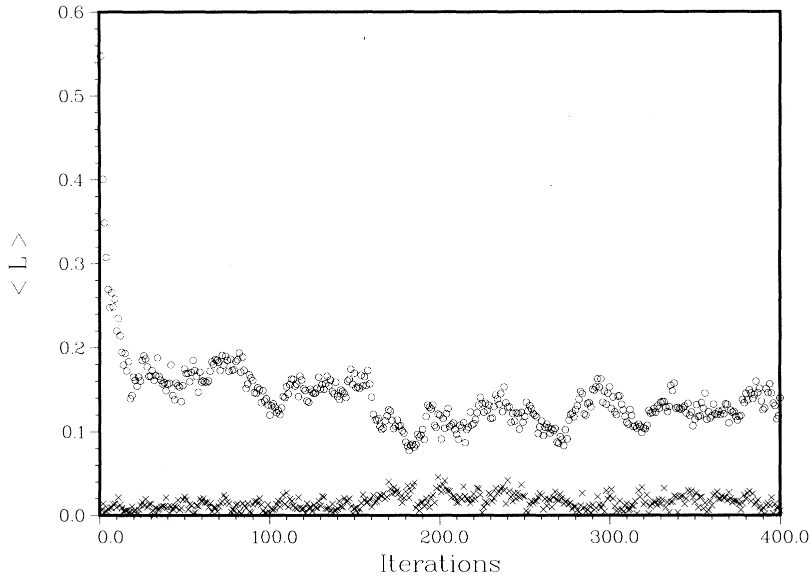

FIG. 1. The Wilson line at $1/g^2 = 1.3125$. The upper curve is obtained from an ordered start and the lower one from a disordered start.

links in a fixed timelike plane. The Wilson line

$$L(\vec{n}) = \frac{1}{N} \operatorname{tr} \prod_{m=0}^{n_\tau - 1} U_0(\vec{n} + m\hat{e}_0) \tag{5}$$

transforms nontrivially under this operation and must therefore vanish in the confining phase:

$$\langle L \rangle \sim e^{-\beta F}. \tag{6}$$

Here the expectation value is with respect to (2) and F is the free energy of an isolated heavy quark. In the deconfining phase F is finite and L can take on a nonvanishing expectation value. Hence $\langle L \rangle$ serves as an order parameter for the phase transition under consideration.

We monitored $\langle L \rangle$ on both an $8^3 \times 3$ and $8^3 \times 4$ lattice. Our results indicate that on the $8^3 \times 3$ lattice the transition takes place very close to the well-known SU(4) crossover transition which happens at $1/g^2 = 1.28$.[8] Results from such a lattice are there-

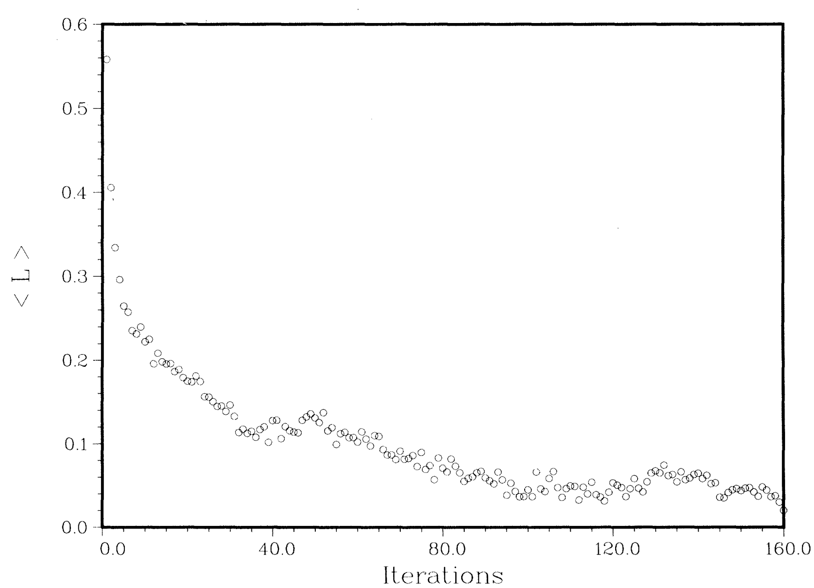

FIG. 2. Ordered start, $1/g^2 = 1.31$.

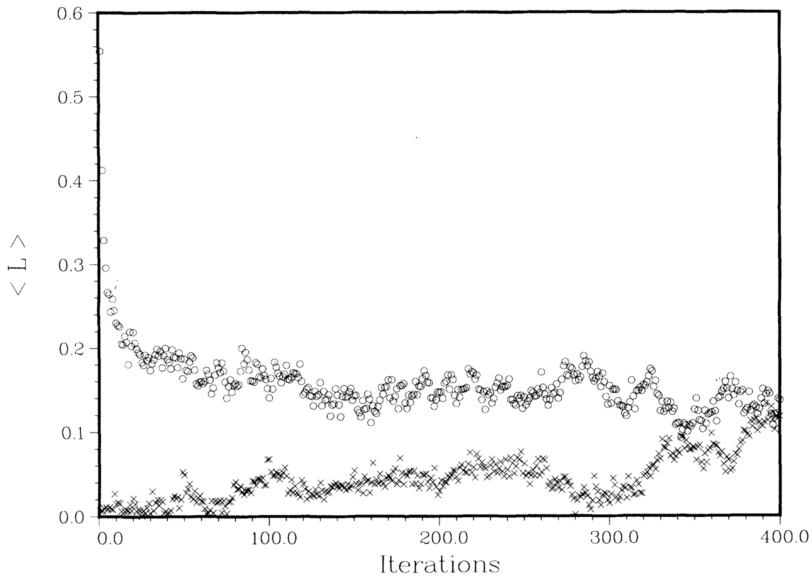

FIG. 3. Ordered and disordered starts, $1/g^2 = 1.315$.

fore not to be trusted. For $n_\tau = 4$ the transition occurs at a critical $1/g^2$ of about $1/g_c^2 = 1.3125$. In Fig. 1 we show the result of 400 iterations starting from both ordered and disordered configurations. The quantity $\langle L \rangle$ displayed there is measured by the procedure outlined in Refs. 2 and 3 . We observe a clear two-state signal characteristic of a first-order phase transition. In Figs. 2 and 3 we show our results with slightly detuned values of $1/g^2$ [which corresponds to raising and lowering the temperature—see Ref. 4]. Note the rapid drift from the unstable to the stable phase. This is also consistent with the first-order nature of the transition. The change in $1/g^2$ of about 0.19% whch destroys the two-state signal is approximately half of what it is in the case of SU(3).[2,3] Unfortunately we are not able to check the scaling prediction

$$T_c = \frac{\Lambda_L}{n_\tau}\left(\frac{11}{12\pi^2} g_c^2(n_\tau)\right)^{51/121} \exp\left(\frac{6\pi^2}{11 g_c^2(n_\tau)}\right) \tag{7}$$

for the critical temperature. (One should go to a $10^3 \times 5$ lattice for that.) However, (7) is expected to hold well because of the sharp crossover from weak to strong coupling in the case of SU(4). For $1/g_c^2 = 1.3125$ and $n_\tau = 4$, Eq. (7) implies $T_c \sim 95.9\Lambda_L$ which maps into $1/g_c^2 \sim 1.26$ at $n_\tau = 3$. This is in the strong coupling region which implies that results from an $8^3 \times 3$ lattice do not scale.

To summarize, we have presented our Monte Carlo data for the deconfining phase transition in SU(4) gauge theory. The results indicate that recent theoretical ideas are indeed correct: The $N \geqslant 3$ deconfining phase transitions are first order.[7] The mean-field predictions for both critical coupling and the jump in the order parameter are roughly a factor of 2 too large. They are, however, derived from what is essentially a strong coupling approximation and are expected to be reasonably accurate only for smaller values of n_τ.

The calculations reported here were performed on the Brookhaven National Laboratory CDC 7600. We employed the SU(2) subgroup heat bath algorithm.[9] It took approximately 3.5 ms to update one link when Wilson lines and plaquettes were measured.

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Note added.—After submitting this paper we received preprints by G. Batrouni and B. Svetitsky and by J. F. Wheater and M. Gross in which they also examine the SU(4) deconfining phase transition.

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