

Chiral extrapolation of light-light and heavy-light decay constants in unquenched QCD*

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We test the one-loop chiral perturbation theory formula on unquenched lattice data of pseudoscalar meson decay constants. The chiral extrapolation including the effect of the chiral logarithm is attempted and its uncertainty is discussed.

1. Introduction

In the unquenched QCD simulations using the Hybrid Monte Carlo algorithm, the computational cost rapidly grows as the chiral limit of sea quark is approached. In practical simulations with Wilson-type fermions the sea quark mass is limited to be heavier than $\sim m_s/2$. To obtain the physical results for u and d quarks, therefore, the chiral extrapolation is indispensable.

In the chiral extrapolation the chiral perturbation theory (ChPT) may be used to decide the functional form, as it is an effective theory valid for low energy QCD. Once the available lattice data are confirmed to be consistent with ChPT, the extrapolation to the u and d quark masses using the ChPT formula is justified. The practical question is, then, whether the lattice results could reproduce the sea quark mass dependence, especially the chiral logarithm, predicted by ChPT.

In this talk, we present the sea quark mass dependence of the pseudoscalar meson decay constant obtained in unquenched QCD, and compare them with the one-loop ChPT prediction. The unquenched simulations are done using the stan-

dard gauge and nonperturbatively $O(a)$ -improved fermion action at $\beta = 5.2$ with sea quark masses corresponding to the pion mass in the range 550–1000 MeV. Further details of the simulation are discussed in a separate talk [1].

We also discuss the uncertainty associated with the chiral extrapolation taking light-light and heavy-light decay constants as examples.

2. Test of the ChPT formula

In full QCD the ChPT predicts a specific functional dependence of physical quantities on the quark mass, *i.e.* the chiral logarithm, at the one-loop order. For N_f flavors of degenerate quarks with a mass m_S , the pseudoscalar meson decay constant f_{SS} is given by [2]

$$\frac{f_{SS}}{f} = 1 - \frac{N_f}{2} y_{SS} \ln y_{SS} + \frac{y_{SS}}{2} [\alpha_5 + N_f \alpha_4], \quad (1)$$

with $y_{SS} = 2B_0 m_S / (4\pi f)^2$. While the low energy constants α_i are unknown parameters, the chiral log term $y_{SS} \ln y_{SS}$ appears with a definite coefficient depending only on the number of flavors.

Figure 1 shows the lattice results together with fitting curves. If we leave the coefficient of the

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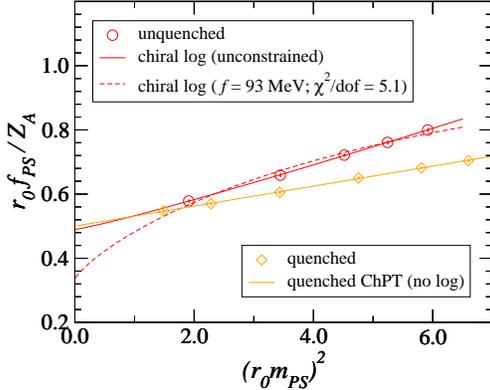


Figure 1. Fit of pseudoscalar meson decay constant with the one-loop ChPT formula.

chiral log term as a free parameter, the fit result is consistent with zero (solid line), while the fit with the fixed coefficient $N_f/2$ gives a bad χ^2/dof (dashed line). A similar observation is obtained for the PCAC relation $M_{SS}^2/2m_S$ [3].

Partially quenched ChPT [4,5] may be used to explicitly test the presence of the chiral logarithm. For non-degenerate pions composed of quarks with mass m_S and m_V (V denotes a valence quark), the low energy constants cancel out in the double ratios

$$\left(\frac{M_{VS}^2}{m_V + m_S}\right)^2 = 1 + \frac{y_{SS}}{N_f} t, \quad (2)$$

$$\frac{f_{VS}}{\sqrt{f_{SS} f_{VV}}} = 1 - \frac{y_{SS}}{4N_f} t, \quad (3)$$

with $t \equiv \ln(y_{VV}/y_{SS}) + 1 - (y_{VV}/y_{SS})$, and only the chiral log terms remain. A parameter y_{SS}/N_f obtained as a coefficient of t in (2) and in (3) are plotted as a function of M_{SS}^2 in Figure 2. We find that the results are much smaller than the prediction of the partially quenched ChPT shown by a steep dashed line.

3. Uncertainty in the chiral extrapolation

Since the lattice data do not support the presence of the chiral logarithm for the sea quark masses used in the simulation, the chiral extrapolation using the one-loop ChPT formula is not

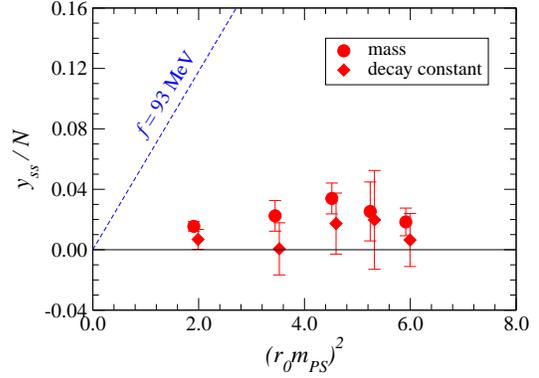


Figure 2. Ratio test of the partially quenched ChPT formula.

fully justified. Instead, we consider several possible functional forms to approach the chiral limit and discuss their associated uncertainty.

Our observation suggests that the mass region where the chiral logarithm becomes important is around or below 500 MeV, and the ChPT ceases to converge above that scale. Then, a possible way to extrapolate the data including the effect of the chiral logarithm is to use a polynomial fit (quadratic fit, for example) above some energy scale μ , and then switch to the one-loop ChPT formula below μ . An example is shown in Figure 3 for $\mu = 300$ and 500 MeV. The limit of $\mu = 0$ MeV corresponds to the usual polynomial fit. Since the scale μ is unknown, the variation of several fit curves, about $\pm 10\%$ in the chiral limit, should be taken as systematic uncertainty.

Another possible functional form suggested by the Adelaide-MIT group [6] is the one-loop ChPT with a hard momentum cutoff μ . It amounts to replace the chiral log term $m_\pi^2 \ln(m_\pi^2/\mu^2)$ by $m_\pi^2 \ln(m_\pi^2/(m_\pi^2 + \mu^2))$. Changing the unknown ‘‘cutoff’’ scale μ from 0 to 1 GeV, we obtain the similar size of uncertainty in the chiral limit.

The SU(3) breaking ratio in the decay constant f_K/f_π may be obtained with a fit to partially quenched lattice data. Our result from the above fit functions changes from 1.167(3) (quadratic fit, corresponding to $\mu = 0$ MeV) or 1.190(3) (Adelaide-MIT fit, $\mu = 500$ MeV) to 1.276(7) (chiral log plus quadratic, corresponding to $\mu = \infty$ MeV). Although it is clear that our

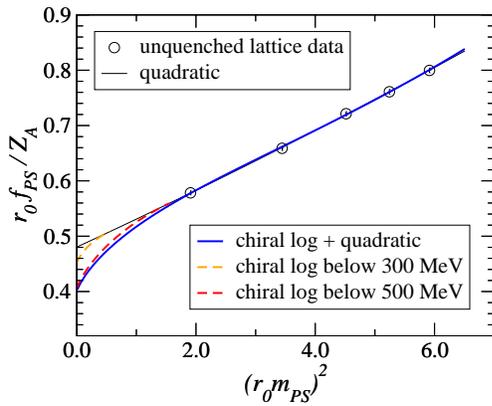


Figure 3. Uncertainty in the chiral extrapolation.

two-flavor QCD result is significantly higher than the quenched result $1.081(5)(17)$ [7] and closer to the physical value 1.22, the uncertainty is still sizable.

For the heavy-light decay constant the prediction of ChPT is available in the heavy quark limit for quenched, partially quenched and full QCD [8,9]. The chiral logarithm appears with a definite coefficient but including an additional coupling constant g describing the $B^*B\pi$ interaction. The chiral extrapolation of f_B and f_{B_s} including the chiral logarithm is shown in Figure 4 with two representative values of g . As in the pion decay constant, the uncertainty in the chiral limit is enhanced by the chiral logarithm.

The ratio f_{B_s}/f_B is needed in the extraction of the CKM matrix element $|V_{td}/V_{ts}|$. Since the bulk of systematic errors cancels in the ratio, one may expect better accuracy than the determination of $|V_{td}|$ solely from ΔM_d . Our preliminary result varies from 1.24 (quadratic fit) to 1.38 (chiral log, $g = 0.59$). It suggests that the ratio and its error can be significantly larger than the previous world average 1.16(4). Similar discussion, but using quenched data, has been made in [10].

4. Conclusions

The chiral logarithm expected from ChPT is not observed in the unquenched lattice data with m_{PS} greater than about 500 MeV, which suggests that ChPT may only be applied in smaller mass

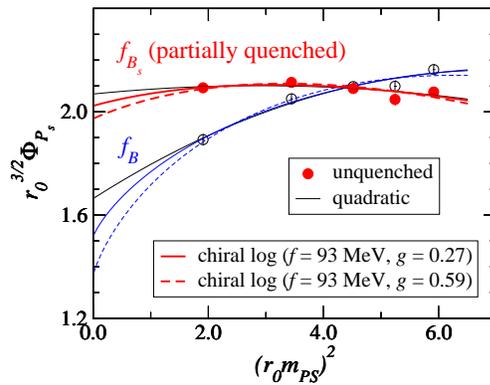


Figure 4. Chiral extrapolation of heavy-light decay constants.

regions. The estimate using model functions for the chiral extrapolation leads to the uncertainty as large as $\pm 10\%$ for the decay constants.

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