Charmonium spectrum from quenched QCD on anisotropic lattices *

CP-PACS Collaboration : S. Aoki[a], R. Burkhalter[a,b], S. Ejiri[b] †, M. Fukugita[c], S. Hashimoto[d], N. Ishizuka[a,b], Y. Iwasaki[a,b], K. Kanaya[a], T. Kaneko[d], Y. Kuramashi[d], V. Lesk[b], K. Nagai[b‡], M. Okamoto[a], M. Okawa[d], Y. Taniguchi[a], A. Ukawa[a,b] and T. Yoshié[a,b]

[a]Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan

[b]Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan

[c]Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan

[d]High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

We present our final results of the charmonium spectrum in quenched QCD on anisotropic lattices. Simulations are made with the plaquette gauge action and a tadpole improved clover quark action employing $\xi = a_s/a_t = 3$. We calculate the spectrum of S- and P-states and their excitation, and study the scaling behavior of mass splittings. Comparison is made with the experiment and previous lattice results. The issue of hyperfine splitting for different choices of the clover coefficients obtained by Klassen is discussed.

1. Introduction

Standard lattice QCD actions on space-time isotropic lattices encounter serious obstacles for heavy quarks with currently accessible lattice spacings because mass-dependent $O(ma)$ discretization errors are very large. Aiming to reduce such errors, Klassen[1,2] has proposed to employ anisoropic lattices with $ma_t \ll 1$ for heavy quark simulations. In this paper, we summarize our final results of the quenched charmonium spectrum using the anisotropic method[3,4]. We also address the problem with hyperfine splitting[2] that different choices of clover coefficients lead to disagreeing results in the continuum limit.

2. Simulations

We use the standard anisotropic gauge action given by $S_g = \beta \sum(1/\xi_0 P_{ss'} + \xi_0 P_{st})$. The bare anisotropy ξ_0 is tuned to obtain a desired value of the renormalized anisotropy $\xi \equiv a_s/a_t$, adopting Klassen's parametrization[5].

Table 1
Simulation parameters. a_s is fixed by $r_0 = 0.5$ fm.

β	$a_s^{r_0}$[fm]	$L^3 \times T$	La_s[fm]	#conf
5.7	0.204	$8^3 \times 48$	1.63	1000
5.9	0.137	$12^3 \times 72$	1.65	1000
6.1	0.099	$16^3 \times 96$	1.59	600
6.35	0.070	$24^3 \times 144$	1.67	400

For quark we use an anisotropic clover quark action:

$$\begin{aligned} S_f &= \sum\{\bar{\psi}_x\psi_x \\ &-K_t[\bar{\psi}_x(1-\gamma_0)U_{0,x}\psi_{x+\hat{0}} + \bar{\psi}_{x+\hat{0}}(1+\gamma_0)U_{0,x}^\dagger\psi_x] \\ &-K_s[\bar{\psi}_x(1-\gamma_i)U_{i,x}\psi_{x+\hat{i}} + \bar{\psi}_{x+\hat{i}}(1+\gamma_i)U_{i,x}^\dagger\psi_x] \\ &+iK_s[c_s\bar{\psi}_x\sigma_{ij}F_{ij}(x)\psi_x + c_t\bar{\psi}_x\sigma_{0i}F_{0i}(x)\psi_x]\}. \quad (1) \end{aligned}$$

The bare quark mass is given by $m_0 = 1/2K_t - 3/\zeta - 1$ with $\zeta \equiv K_t/K_s$. For ζ we adopt the tree level tadpole improved value for massive quarks. For clover coefficients c_s and c_t, we employ the values in the massless limit. We note that our choice of c_s is still correct for massive quarks because it has no mass dependence at the tree level[4]. The tadpole factors are determined as $\langle U_s\rangle = \langle P_{ss'}\rangle^{1/4}$ with $P_{ss'}$ the spatial plaquette and $\langle U_t\rangle = 1$.

Simulation parameters are summarized in Ta-

*Talk presented by M. Okamoto
†present address: Department of Physics, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, UK
‡present address: Theory Division, CERN, CH-1211 Geneva 23, Switzerland

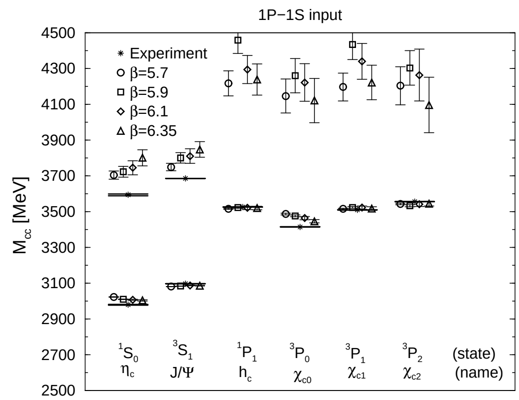

Figure 1. $c\bar{c}$ spectrum with $1\bar{P}-1\bar{S}$ input.

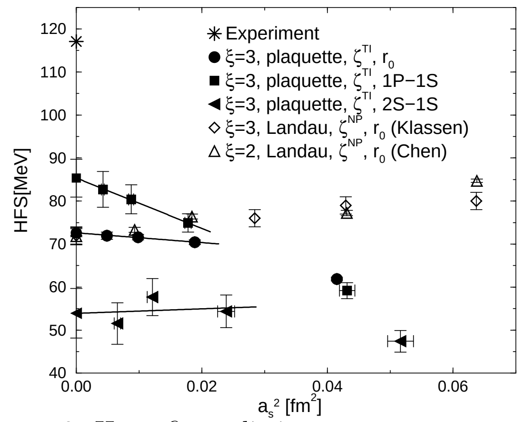

Figure 2. Hyperfine splitting.

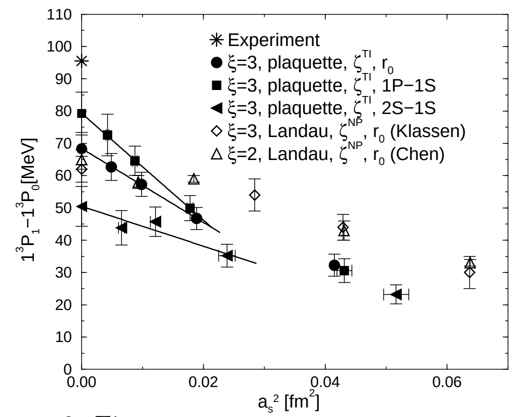

Figure 3. Fine structure.

ble 1. We adopt lattices with $\xi = 3$ and $La_s \sim$ 1.6 fm. Runs are made at four values of β which correspond to $a_s = 0.07$-0.20 fm. For each β, we measure S- and P-state meson correlation functions at two values of bare quark mass. Results are then inter(extra)polated to the charm quark mass where $1\bar{S}$ mass has its experimental value. The lattice scale is set by either the Sommer scale $r_0 = 0.5$ fm, $1\bar{P}-1\bar{S}$ splitting or $2\bar{S}-1\bar{S}$ splitting.

3. Results

In Fig.1, we show results of the charmonium spectrum with the scale from the $1\bar{P}-1\bar{S}$ splitting. Gross features of the spectrum are consistent with the experiment, *e.g.* splittings between χ_c states are well resolved with correct ordering. The deviation of 2S masses from the experiment is in part ascribed to the quenching effect and in part to contaminations from higher excited states.

3.1. Hyperfine splitting

In Fig.2, we plot by filled symbols the lattice spacing dependence of the hyperfine splitting $\Delta M(1^3S_1 - 1^1S_0)$ for three inputs for the scale. Data at finite a_s are extrapolated to the continuum limit adopting an a_s^2-linear ansatz. The results largely depend on scale inputs, and are much smaller than the experimental value (*e.g.,* by about 30% with $1\bar{P}-1\bar{S}$ input). Thus quenching effects are very large for the hyperfine splitting.

In the same figure, we also plot results by Klassen (open diamonds; $\xi = 3$)[2] and Chen (open triangles; $\xi = 2$)[6] with the same action. Their simulations differ from ours in that we determine the tadpole factor u_0 from the plaquette average and adopt for the parameter ζ the tree-level tadpole improved value $\zeta^{\rm TI}$, while they use the mean link in the Landau gauge for u_0 and a non-perturbative estimate $\zeta^{\rm NP}$ determined from the meson dispersion relation. Nonetheless, their results and ours, using the same scale r_0, all converge to a consistent value of about 70 MeV in the continuum limit.

3.2. Fine structure

Figure 3 shows results of the fine structure $\Delta M(1^3P_1 - 1^3P_0)$. The deviation from the experimental value is smaller than that for the hyperfine splitting (about 20% with $1\bar{P}-1\bar{S}$ input). Our result with r_0 input is again consistent with those of Refs. [2,6].

4. Effect of c_s for hyperfine splitting

The results described so far all use the tadpole improved value $\tilde{c_s} = 1$ for the spatial clover coefficient. In Refs.[1,2], Klassen employed a different choice $\tilde{c_s} = 1/\nu$ ($\nu \equiv \xi_0/\zeta$). He obtained HFS$(a_s = 0, r_0) \approx 90$ MeV for the continuum limit of the hyperfine splitting, which is much larger than the result above HFS$(a_s = 0, r_0) \approx 70$ MeV with $\tilde{c_s} = 1$. We note that $\tilde{c_s} = 1/\nu$ is correct *only* in the massless limit, while $\tilde{c_s} = 1$ is valid for *any* quark mass, at the tree level.

To resolve this problem, we attempt an effective analysis. The potential model predicts that the hyperfine splitting is due to the spin-spin interaction of quarks, which originates from the $\mathbf{\Sigma} \cdot \mathbf{B}$ term in the nonrelativistic Hamiltonian $H^{\rm NR}$. We therefore define a "tree-level effective hyperfine splitting"

$$\text{HFS}^{\rm eff} \equiv (a_t \tilde{M}_1 / a_t \tilde{M}_B)^2 \ , \qquad (2)$$

where

$$\frac{1}{a_t M_B} = \frac{2\xi^2/\zeta^2}{m_0(2+m_0)} + \frac{\xi^2 c_s/\zeta}{1+m_0} \qquad (3)$$

is the tree level coefficient of the $\mathbf{\Sigma} \cdot \mathbf{B}$ term in $H^{\rm NR}$. The pole mass $a_t M_1 = \log(1 + m_0)$ is inserted to normalize to unity in the continuum limit, and tildes denote the tadpole improvement.

In Fig.4 we compare the scaling behavior of HFS$^{\rm eff}$ (left panel) and the actual data HFS (right panel) for $\tilde{c_s} = 1/\nu$. A similar comparison for $\tilde{c_s} = 1$ is made in Fig. 5. We find that results of HFS are qualitatively well reproduced by those of HFS$^{\rm eff}$. For $\tilde{c_s} = 1/\nu$, HFS$^{\rm eff}$ remains large even at $(a_s \tilde{M}_1)^2 \sim 1$, which suggests that the actual HFS should rapidly decrease as $a_s \to 0$, and hence a naive estimation ≈ 90 MeV[1,2] from an a_s^2-linear continuum fit is misleading for this case. On the other hand, HFS$^{\rm eff}$ is already close to unity for $(a_s \tilde{M}_1)^2 \lesssim 1$ for $\tilde{c_s} = 1$. Thus an a_s^2-linear continuum estimation (≈ 70 MeV) for this case appears much more reliable than that for $\tilde{c_s} = 1/\nu$.

5. Conclusions

We have computed the charmonium spectrum accurately using quenched anisotropic lattices

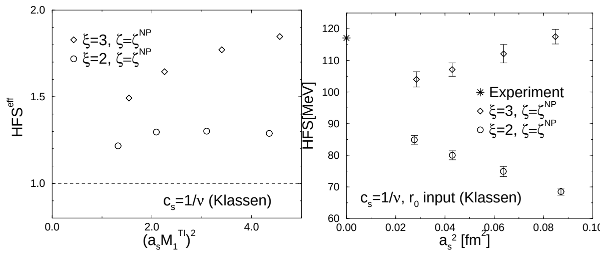

Figure 4. HFS$^{\rm eff}$ and HFS for $\tilde{c_s} = 1/\nu$.

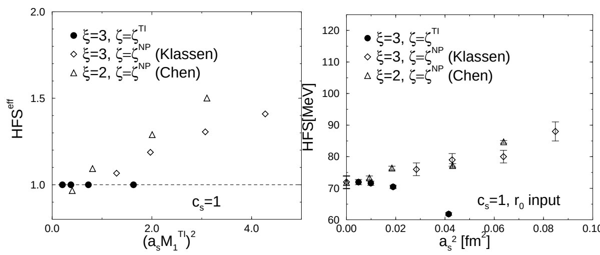

Figure 5. HFS$^{\rm eff}$ and HFS for $\tilde{c_s} = 1$.

with $\xi = 3$. We find that the spin splittings largely depend on the scale input and are smaller than the experimental values. Our results are consistent with previous results [2,6] when the same clover coefficients are used. We have also shown that a large hyperfine splitting reported in Ref. [1,2] with a different choice of the clover coefficients is likely an overestimate arising from the continuum extrapolation.

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