

Calculation of $K \rightarrow \pi$ matrix elements in quenched domain-wall QCD *

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We explore the possibility for an evaluation of non-leptonic $\Delta S = 1$ K decay amplitudes through the calculation of $K \rightarrow \pi$ matrix elements using domain-wall QCD. The relation between the physical $K \rightarrow \pi\pi$ matrix elements and $K \rightarrow \pi$ matrix elements deduced from chiral perturbation theory is recapitulated. Quenched numerical simulations are performed on an $16^3 \times 32 \times 16$ lattice at a lattice spacing $a^{-1} \approx 2\text{GeV}$ for both the standard plaquette gauge action and a renormalization-group improved gauge action, and reasonable signals for $K \rightarrow \pi$ matrix elements are obtained. Preliminary results are reported on the $K \rightarrow \pi\pi$ matrix elements, and results from two actions are compared.

1. INTRODUCTION

Despite extensive efforts over the years, lattice QCD calculation of the matrix elements relevant for understanding the $K \rightarrow \pi\pi$ decays, including the $\Delta I = 1/2$ rule and the direct CP violation parameter ε'/ε , have achieved only limited success to the present[1]. One reason behind the slow progress is lack of full chiral symmetry in the Kogut-Susskind[2] and Wilson[3] formulation for lattice fermions employed in the past attempts. The domain wall fermion formalism [4,5] offers a possibility of resolving this problem. In this article we report on our study of the $K \rightarrow \pi\pi$ amplitudes in quenched QCD using this formalism[6].

The framework of our study is the reduction formulae[7] derived from chiral perturbation theory(χ PT), which relate $K \rightarrow \pi$ ampli-

tudes to the physical $K \rightarrow \pi\pi$ amplitudes. We briefly review these formulae to expose some limitations[8,9].

Chiral property of domain wall QCD has been examined in detail recently [10,11]. We have shown in particular [10] that the residual chiral symmetry breaking due to finite fifth-dimensional size is significantly reduced for a renormalization-group improved gluon action as compared to the standard plaquette action. We therefore calculate the $K \rightarrow \pi$ matrix elements for the two gluon actions in parallel.

2. χ PT REDUCTION FORMULAE

The effective Hamiltonian H_W for $\Delta S = 1$ K meson decays is written as

$$H_W^{(\text{eff})} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} W(\mu)_i Q_i(\mu), \quad (1)$$

where $W(\mu)_i$ are Wilson coefficients. The basis four-quark operators Q_i are decomposed as $Q_i^{(0)} + Q_i^{(2)}$ corresponding to iso-spin in the final state

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$I = 0$ and 2.

Under $SU(3)_L \otimes SU(3)_R$ chiral transformation, the basis operators Q_i transform as $(27, 1) \oplus (8, 1)$ for $i = 1, \dots, 4, 9, 10$, $(8, 1)$ for $i = 5, 6$ and $(8, 8)$ for $i = 7, 8$. To leading order in χ PT the operators that transform according to these flavor representations are given by[7-9]

$$(8, 1) : \quad \mathbf{A} = (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)_{23}, \quad (2)$$

$$\mathbf{B} = (\Sigma^\dagger M + M^\dagger \Sigma)_{23} \quad (3)$$

$$(27, 1) : \quad \mathbf{C} = 3(\Sigma^\dagger \partial_\mu \Sigma)_{23} (\Sigma^\dagger \partial_\mu \Sigma)_{11} \\ + 2(\Sigma^\dagger \partial_\mu \Sigma)_{13} (\Sigma^\dagger \partial_\mu \Sigma)_{21} \quad (4)$$

$$(8, 8) : \quad \mathbf{D} = \Sigma_{21}^\dagger \Sigma_{13}, \quad (5)$$

where we consider $(8, 8)$ operator only at $O(p^0)$ order due to the reason explained later.

Using unknown parameters, a, b, c, d, \dots , one may write the Q_i 's in terms of the χ PT operators according to

$$Q_i = a_i \mathbf{A} + b_i \mathbf{B} + c_i \mathbf{C} \quad (i = 1, \dots, 6, 9, 10) \quad (6)$$

$$Q_i = d_i \mathbf{D} \quad (i = 7, 8) \quad (7)$$

for the case of $I = 0$ and 2 separately.

For Q_i in (6), $K^+ \rightarrow \pi^+$ matrix elements are proportional to $a_i + b_i - 2c_i$, whereas $K^0 \rightarrow \pi^+ \pi^-$ matrix elements to $a_i - 2c_i$. To eliminate b_i one has to introduce an operator

$$Q_{\text{sub}} = (m_s + m_d) \bar{s} d - (m_s - m_d) \bar{s} \gamma_5 d = b_{\text{sub}} \mathbf{B} \quad (8)$$

and construct $Q_i - \alpha_i Q_{\text{sub}}$, where $\alpha_i = b_i / b_{\text{sub}}$ can be determined by $K^0 \rightarrow 0$ matrix elements (See (11)). One then obtains the reduction formula, (9) below, relating the $K \rightarrow \pi\pi$ and $K \rightarrow \pi$ amplitudes originally given by Bernard *et.al*[7].

In (7) there would appear too many unknown parameters to invalidate the reduction relation, if all $(8, 8)$ operators up to $\mathcal{O}(p^2)$ order were included as in the case of other representation. To exclude contributions from these operators at $O(p^2)$, we take the chiral limit where the reduced relation is justified. In this limit, the matrix elements for $K \rightarrow \pi\pi$ are obtained in proportion to those for $K \rightarrow \pi$ [8,9].

It should be stressed that it is only in the chiral limit that one can obtain a complete set of reduction formulae at the lowest order of χ PT.

Table 1

Simulation parameters. Scale a^{-1} is fixed from string tension $\sqrt{\sigma} = 440$ MeV.

gauge action	plaquette	RG-improved
β	6.0	2.6
a^{-1}	2.0 GeV	1.94 GeV
lattice size	$16^3 \times 32 \times 16$	
DW height M	1.8	
m_f	0.01 – 0.06 in steps of 0.01	
#config.	111	110
m_{5q} [10]	2.96(34) MeV	0.283(42) MeV

3. NUMERICAL SIMULATION

Parameters of our numerical simulation are summarized in Table. 1. Calculations are carried out, assuming degenerate bare quark mass $m_f = m_u = m_d = m_s$, on a $16^3 \times 32 \times 16$ lattice at a coupling constant corresponding to a lattice spacing of $a^{-1} \approx 2$ GeV. With these parameters the anomalous quark mass m_{5q} representing residual chiral symmetry breaking is similar to u-d quark mass for the plaquette action and quite small for the RG-improved action. We set the scale from measurement of the string tension assuming $\sqrt{\sigma} = 440$ MeV[10].

To evaluate quark loops in some types of contractions, we employ the random noise method with the number of noises taken to be 2. This number is decided from a numerical test.

We calculate the $K \rightarrow \pi$ matrix elements using the wall source and dividing by the normalization factor $\langle \pi^+ | A_4 | 0 \rangle \langle 0 | A_4 | K^+ \rangle$. In this convention, χ PT reduction formulae for $i = 1, \dots, 6, 9, 10$ take the form,

$$\langle \pi^+ \pi^- | Q_i^{(I)} | K^0 \rangle = \sqrt{2} f_\pi (m_K^2 - m_\pi^2) \\ \times \frac{\langle \pi^+ | Q_i^{(I)} - \alpha_i Q_{\text{sub}} | K^+ \rangle}{\langle \pi^+ | A_4 | 0 \rangle \langle 0 | A_4 | K^+ \rangle}, \quad (9)$$

and for $i = 7, 8$,

$$\langle \pi^+ \pi^- | Q_i^{(I)} | K^0 \rangle = \\ -\sqrt{2} f_\pi \times m_M^2 \frac{\langle \pi^+ | Q_i^{(I)} | K^+ \rangle}{\langle \pi^+ | A_4 | 0 \rangle \langle 0 | A_4 | K^+ \rangle} \quad (10)$$

where m_M is the meson mass on the lattice, while the experimental values are to be substituted in

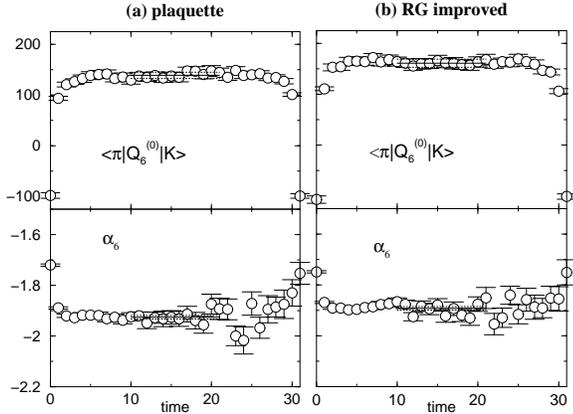


Figure 1. Propagators relevant to $Q_6^{(0)}$ with $m_f = 0.03$ for (a) plaquette and (b) RG-improved gauge actions. Fit ranges and jack knife errors are represented by horizontal lines.

f_π, m_π, m_K . In (9) the subtraction term $\alpha_i Q_{\text{sub}}$ appears only for $I = 0$. The parameter α_i is written as

$$\alpha_i = -\frac{\frac{d}{dm_s} \langle 0 | Q_i^{(0)} | K^0 \rangle |_{m_s=m_d}}{\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle}, \quad (11)$$

where the differentiation by m_s is implemented by a double inversion of the Dirac operator.

4. RESULTS

For limitation of space we concentrate on the matrix elements of $Q_6^{(0)}$ and $Q_8^{(2)}$, which represent the main contributions to ε'/ε .

Examples of propagator ratios for $Q_6^{(0)}$ which appears on the right hand-side of (9) are shown in Fig. 1 for (a) plaquette and (b) RG-improved gauge actions. In each figure, the upper panel is for the $K \rightarrow \pi$ matrix element and the lower for α_6 defined in (11). Although both propagators contain quark loops, which have been known to make signals worse, reasonable signals are obtained at the level of propagator ratios. This also justifies our choice of 2 random noises to evaluate the quark loops.

A constant fitting, as shown in Fig. 1, yields the matrix element, which we plot as a function of m_f in Fig. 2. Circles and diamonds show the contribution of the $K \rightarrow \pi$ matrix element and the sub-

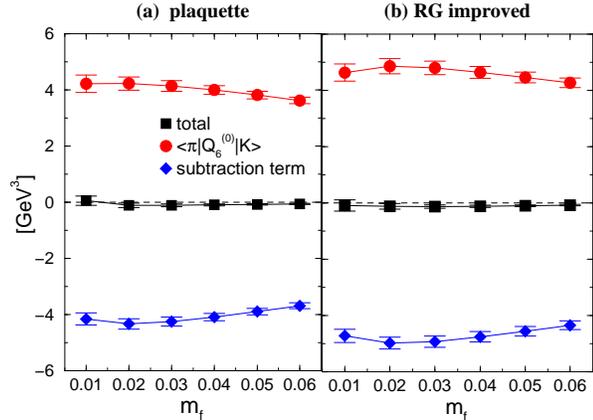


Figure 2. $Q_6^{(0)}$ calculated by the χ PT reduction formula (9). Original matrix element (circles), subtraction term (diamonds) and the total value (squares) are separately plotted. Lines are guide to eyes.

traction term to the $K \rightarrow \pi\pi$ matrix element of $Q_6^{(0)}$ as given in the reduction formula (9). There is a severe cancellation between the two contributions, leading to the total value of the $K \rightarrow \pi\pi$ matrix element (squares) which is more than an order of magnitude smaller than the individual contributions. Nonetheless, the enlarged plots in Fig. 3(a) and (b) show that the total values have a reasonable signal.

There is no subtraction term for $Q_8^{(2)}$. We obtain clear signals for this operator using the reduction formula (10) as shown in Fig. 4.

The remaining procedure is to make a linear fitting, as shown in Figs. 3 and 4, to take the chiral limit $m_f \rightarrow 0$ where both of the reduction formulae (9) and (10) are valid to estimate the physical matrix element, and to convert the values to continuum theory renormalized in the $\overline{\text{MS}}$ scheme with NDR. For the latter, we employ the renormalization factors calculated in perturbation theory at one-loop level[12] with mean field improvement.

Prior to this final step, we are currently increasing the statistics to reduce the errors further. We are also performing a new simulation with a larger lattice volume to investigate finite size effect in the matrix elements.

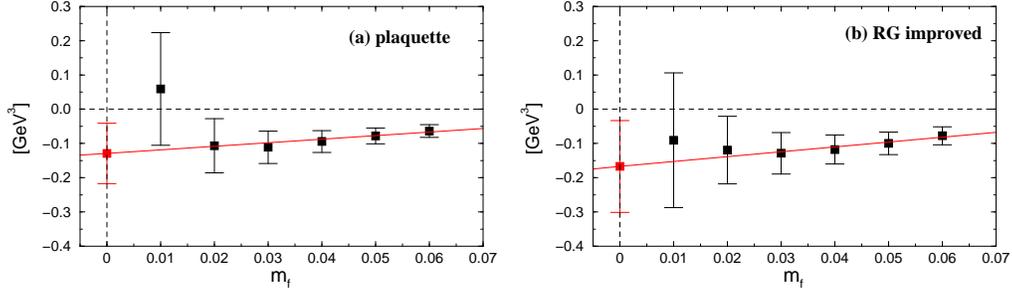


Figure 3. $K \rightarrow \pi\pi$ matrix element of $Q_6^{(0)}$ and linear extrapolation to the limit $m_f \rightarrow 0$ for the plaquette and RG-improved gauge action.

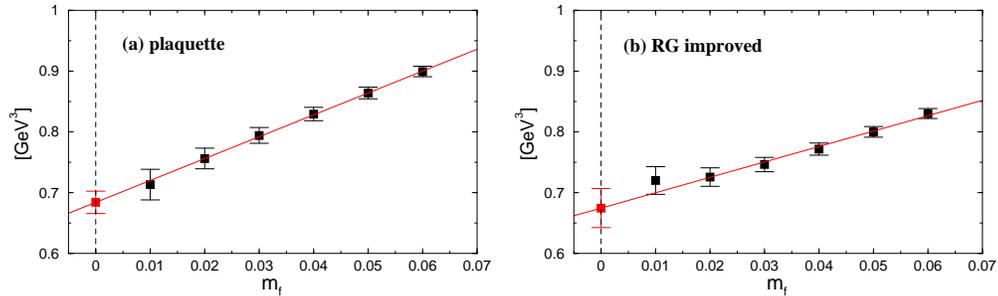


Figure 4. Counterpart of Fig. 3 for the operator $Q_8^{(2)}$.

5. OUTLOOK

We have presented our preliminary results for the $K \rightarrow \pi\pi$ decay amplitudes based on the χ PT reduction formulae and domain wall QCD. Matrix elements of reasonable statistical quality are obtained from about a hundred gauge configurations in our quenched numerical simulation. Values for the matrix elements from plaquette and RG-improved gauge actions seems consistent within the errors. These results make us hopeful that the present approach yields precise information about the $\Delta I = 1/2$ rule and direct CP violation in the standard model with more statistics and detailed analysis.

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