

B_K with the Wilson Quark Action: A Non-Perturbative Resolution of Operator Mixing using Chiral Ward Identities*

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We propose a non-perturbative method to determine the mixing coefficients of $\Delta s = 2$ four-quark operators for the Wilson quark action using chiral Ward identities. The method is applied to calculate B_K in quenched QCD.

1. Introduction

An essential step in the calculation of B_K with the Wilson quark action is the resolution of the mixing problem of the $\Delta s = 2$ four-quark operators, which is made difficult by the chiral symmetry breaking effects of the Wilson term. An apparent deficiency of perturbation theory for this problem has been well known[1], and most calculations have tried to resolve the mixing non-perturbatively with the aid of chiral perturbation theory[2]. This method, however, has not been successful, since it contains large systematic uncertainties from higher order effects which survive even in the continuum limit. Recently the method of non-perturbative renormalization[3] has yielded a $\Delta s = 2$ operator with a good chiral behavior[4]. However, the underlying mechanism of improvement in this approach is not quite apparent.

Our aim is to calculate B_K with a method which explicitly incorporates the chiral properties of the Wilson action, and to examine whether the result is consistent with that using the Kogut-Susskind action. In this report we propose a non-

perturbative method to resolve the operator mixing problem based on chiral Ward identities(WI), and report first results of a calculation of B_K carried out on VPP500/80 at KEK.

2. Formulation of the method

Let us consider a set of weak operators in the continuum $\{\hat{O}_i\}$ which closes under chiral rotation $\delta^a \hat{O}_i = ic_{ij}^a \hat{O}_j$. The continuum operators are given by a linear combination of a set of lattice operators $\{O_\alpha\}$, $\hat{O}_i = \sum_\alpha Z_{i\alpha} O_\alpha$. We choose the mixing coefficients $Z_{i\alpha}$ such that the Green's functions of $\{\hat{O}_i\}$ with quarks in the external states satisfy the relevant chiral Ward identities to $O(a)$. The identities can be derived in a standard manner[5] and take the form given by

$$\begin{aligned} & -2\rho Z_A \langle \sum_x P^a(x) \hat{O}_i(0) \prod_k \tilde{\psi}(p_k) \rangle \\ & + c_{ij}^a \langle \hat{O}_j(0) \prod_k \tilde{\psi}(p_k) \rangle \\ & - i \sum_l \langle \hat{O}_i(0) \prod_{k \neq l} \tilde{\psi}(p_k) \delta^a \tilde{\psi}(p_l) \rangle + O(a) = 0 \end{aligned} \quad (1)$$

with p_k the momentum of external quark.

The four-quark operator relevant for B_K may be schematically written as $\hat{O}_{VV+AA} = VV + AA$ with $V = \bar{s}\gamma_\mu d$ and $A = \bar{s}\gamma_\mu \gamma_5 d$. Together with $\hat{O}_{VA} = VA$, it forms a minimal set which closes under λ^3 chiral rotation. The

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mixing pattern of these operators takes the form $\hat{O}_{VV+AA}/2 = Z_{VV+AA} (O_0 + z_1 O_1 + \dots + z_4 O_4)$ and $O_{VA} = Z_{VV+AA} \cdot z_5 O_5$ where the lattice operators in the Fierz eigenbasis are given by $O_0 = (VV + AA)/2$, $O_1 = (SS + TT + PP)/2$, $O_2 = (SS - TT/3 + PP)/2$, $O_3 = (VV - AA)/2 + (SS - PP)$, $O_4 = (VV - AA)/2 - (SS - PP)$ and $O_5 = VA$.

Let us take four external quarks with an equal momentum $p^2 = \mu^2$. Let Γ_{VV+AA} and Γ_{VA} be the sum of Green's function on the left hand side of (1) with external quark legs amputated. Using the projection operator P_i corresponding to O_i , we can write $\Gamma_{VV+AA} = \Gamma_5 P_5$ and $\Gamma_{VA} = \Gamma_0 P_0 + \Gamma_1 P_1 + \dots + \Gamma_4 P_4$. We then have six equations for the five coefficients z_1, \dots, z_5 ,

$$\Gamma_i / Z_{VV+AA} = c_0^i + c_1^i z_1 + \dots + c_5^i z_5 = O(a) \quad (2)$$

for $i = 0, \dots, 5$. We may choose five equations to exactly vanish on the right hand side. In the present analysis our choice is $i = 1, \dots, 5$. The remaining overall factor Z_{VV+AA} is determined by the non-perturbative renormalization method of ref. [3]. We convert final results for matrix elements into those of the \overline{MS} scheme with naive dimensional regularization (NDR) in the continuum at the renormalization scale $\mu = 2\text{GeV}$.

3. Parameters of numerical simulation

In Table 1 we summarize parameters of our simulations. The lattice spacing is estimated from m_ρ . At each β we employ four values of the hopping parameter such that the physical point for the K meson can be interpolated from data. We take degenerate s and d quark masses, and estimate $m_s a/2$ from $m_K/m_\rho = 0.648$.

For calculating Green's functions in Ward identities quark propagators are solved in the Landau gauge for point source at the origin with the periodic boundary condition. We extract B_K from a fit of plateau of the ratio of K^0 - \bar{K}^0 Green's function of \hat{O}_{VV+AA} divided by the vacuum saturation of the AA operator. For this calculation quark propagators are solved without gauge fixing for wall source at the edges of lattice using the Dirichlet boundary condition in the time direction. Errors are estimated by the single elimi-

Table 1
Run parameters.

β	5.9	6.1	6.3
$L^3 \times T$	$24^3 \times 64$	$32^3 \times 64$	$40^3 \times 96$
#conf.	300	100	50
a^{-1}	1.95(5)	2.65(11)	3.41(20)
$m_s a/2$	0.0294(14)	0.0198(16)	0.0144(17)

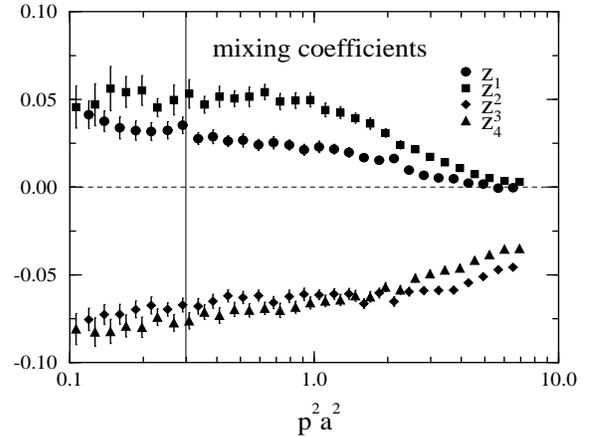


Figure 1. Mixing coefficients z_1, \dots, z_4 for $K = 0.15034$ at $\beta = 6.3$. Vertical line corresponds to $p \approx 2\text{GeV}$.

nation jackknife procedure for all measured quantities.

4. Results for B_K

In Fig. 1 a representative result for the mixing coefficients is plotted as a function of external quark momenta. Data show only a weak scale dependence in the range $0.2 \lesssim p^2 a^2 \lesssim 1.0$. We take values of coefficients at $p \approx 2\text{GeV}$, which falls within this range for our runs at $\beta = 5.9 - 6.3$, in the following analysis.

We note that non-zero values for z_2 contrasts with the one-loop perturbative result $z_2 = 0$. Other coefficients agree in sign, albeit larger in magnitude, and approach perturbative values as β increases.

We show in Fig. 2 the a dependence of the ratio $\langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle / |\langle 0 | \hat{P} | K^0 \rangle|^2$ extrapolated to $m_q = 0$, which measures the contribution of chiral symmetry breaking terms. Results are plotted both for our method (WI) and with tadpole-improved one-loop perturbation theory

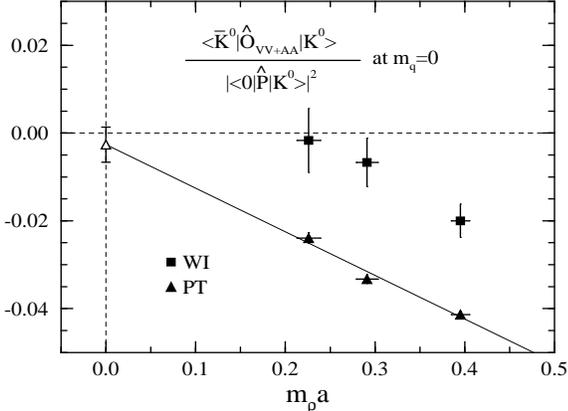


Figure 2. Effect of chiral symmetry breaking for Ward identity (WI) and perturbative (PT) methods.

(PT). The scalar density \hat{P} in the denominator is perturbatively renormalized for both cases. A significant improvement achieved with the use of Ward identities is clearly seen, the ratio becoming consistent with zero even at lattice spacing as large as $m_\rho a \approx 0.2 - 0.3$.

Within the one-loop resolution of operator mixing chiral breaking effects are expected to appear as terms of $O(g^4)$ and $O(a)$. A roughly linear behavior of the PT values is consistent with the presence of the $O(a)$ term. Also they linearly extrapolate to zero within errors at $a=0$. This suggests that $O(g^4)$ terms left out in the one-loop treatment are actually small.

Our results for $B_K(\text{NDR}, 2\text{GeV})$ are summarized in Fig. 3. The WI method gives reasonable values even at a finite lattice spacing. Errors, however, are large, and a continuum extrapolation is difficult at this stage.

In order to reduce statistical errors at each β , we employ an alternative method(WI[VS]) in which the denominator of the ratio for extracting B_K is estimated from the vacuum saturation of \hat{O}_{VV+AA} constructed by the WI method. While this method gives results different from those of WI at $a \neq 0$, the discrepancy is expected to vanish as $a \rightarrow 0$. A linear extrapolation in a yields $B_K(\text{NDR}, 2\text{GeV}) = 0.59(8)$. This is consistent with a recent JLQCD result for the Kogut-Susskind action, $B_K(\text{NDR}, 2\text{GeV}) = 0.587(7)(17)[6]$.

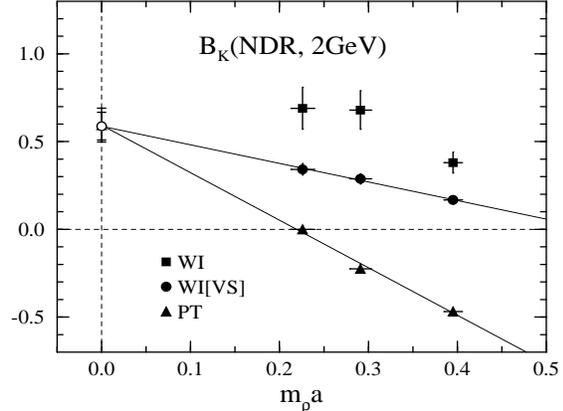


Figure 3. B_K as a function of a .

The tadpole-improved one-loop results (PT), if extrapolated linearly in a , give $B_K(\text{NDR}, 2\text{GeV})=0.59(10)$, which agrees well those obtained with the WI or WI[VS] method. Uncertainties associated with a large extrapolation has to be resolved for assessing the reliability of this approach, however.

In conclusion our results for B_K demonstrates the effectiveness of the method of chiral Ward identities for constructing the $\Delta s = 2$ operator with the correct chiral property. This makes us hopeful to achieve the goal of a precision determination of B_K with the Wilson quark action with further improvement of our simulations.

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