

Anisotropic lattice with nonperturbative accuracy*

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We determine the nonperturbative anisotropic parameter of the gauge action in the quenched approximation with less than 1% accuracy using the Sommer scale measured by the Lüscher-Weisz algorithm or smearing technique. We also study the nonperturbative $O(a)$ -improvement of the quark action. The bare quark anisotropy is determined using the masses from the temporal and spatial directions. For the determination of the $O(a)$ improvement coefficients, we apply the Schrödinger functional method.

1. Introduction

Anisotropic lattices whose temporal lattice spacing a_τ is finer than the spatial one a_σ have become a powerful tool in various subjects of lattice QCD simulations. Among these applications, computations of heavy-light matrix elements [1,2,3,4] require the most accurate parameter tuning, which should be performed nonperturbatively. In this paper, we report the status of our project to develop the anisotropic lattice framework for such precision computations with accuracy of a few percent level [5].

Here we briefly summarize our strategy. For precise computations of heavy-light matrix elements, we need a framework of the heavy quark in which one should be able to (i) take the continuum limit, (ii) compute the parameters in the action and the operators nonperturbatively, (iii) and compute the matrix elements with a modest computational cost. The anisotropic lattice is a candidate of such framework, if a method which fulfills the above condition (ii) is provided. Our expectation is that on anisotropic lattices the mass dependence of the parameters becomes so mild that one can adopt coefficients determined nonperturbatively at massless limit. We also need to control all the systematic errors in the continuum extrapolations. Feasibility studies performed so far for the level of $O(10\%)$ computations are encouraging for further development

[3,4]. We therefore investigate calibration procedures at the accuracy less than one percent, both for the gauge and quark actions in the quenched approximation at $\xi = 4$ [5].

2. Calibration of gauge field

To achieve a few percent accuracy in the final results, the tuning of parameters must be performed at much less than this accuracy. As the goal of present work, we intend to determine the anisotropy parameters at $O(0.2\%)$ level. The elaborated work by Klassen [6], the $O(1\%)$ level calibration for the Wilson action, is therefore no longer meets the present condition. For more precise calibration of the gauge field, we need to measure the static quark potential very accurately. For this purpose, we adopt the Lüscher-Weisz noise reduction technique [9] as well as the standard smearing technique while applied in the anisotropic plane. The former method can drastically reduce the statistical errors while requires larger memory resources than the latter.

We define the renormalized anisotropy ξ_G through the hadronic radii r_0 measured in the coarse and fine directions. Since we carry out the continuum extrapolation in terms of the lattice scale set by r_0 , the renormalized anisotropy is kept fixed during the extrapolation. This avoids the systematic uncertainties due to the anisotropy which may remain in the continuum limit.

Figure 1 shows a result of calibration at $\beta = 5.75$. The top panel shows the result for the

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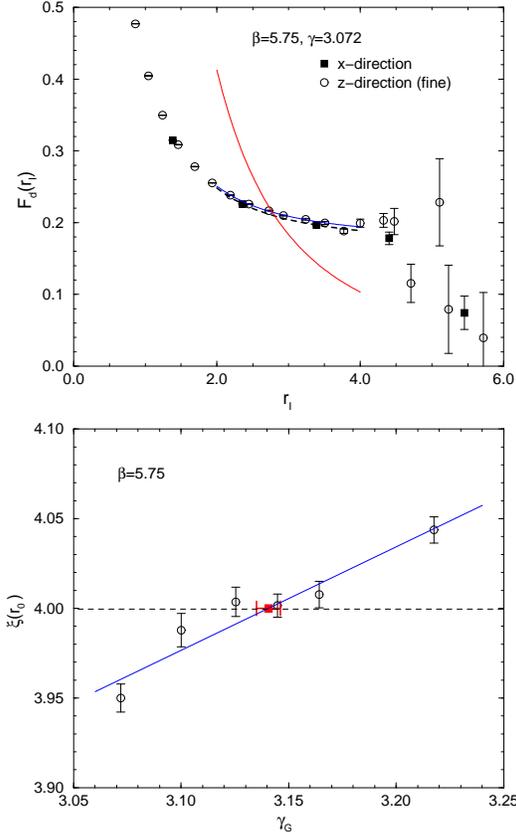


Figure 1. Determination of the gauge field anisotropy at $\beta = 5.75$.

static potential determined with the Lüscher-Weisz technique at $\gamma_G = 3.072$. The renormalized anisotropy ξ_G is determined with 0.2% accuracy. A linear fit of the results at several values of γ_G determines γ_G^* for which $\xi_G = 4$ holds with 0.2% accuracy as displayed in the bottom panel.

In Figure 2, the results at several values of β are collected. Although the precisions of the result with potential determined with the standard smearing technique are still not enough, sufficient precision is achieved at $\beta \leq 6$ where we used the L-W method. Improvement of calculation at $\beta > 6$ and global fit analysis are in progress.

3. Calibration of quark field

Our heavy quark formulation basically follows the Fermilab approach [7] but is formulated on

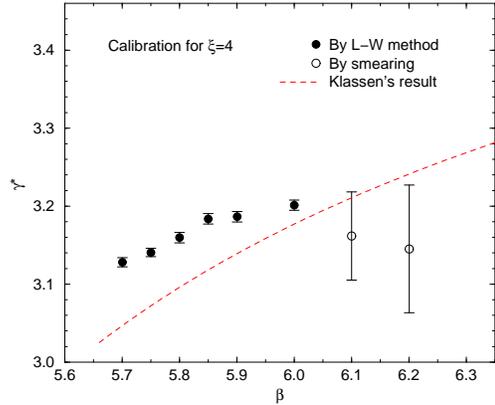


Figure 2. Result of calibration of gauge field.

the anisotropic lattices [1,8]. The quark action is represented as

$$\begin{aligned}
 S_F &= \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y), \quad (1) \\
 K(x,y) &= \delta_{x,y} - \kappa_\tau \left[(1 - \gamma_4) U_4(x) \delta_{x+\hat{4},y} \right. \\
 &\quad \left. + (1 + \gamma_4) U_4^\dagger(x - \hat{4}) \delta_{x-\hat{4},y} \right] \\
 &\quad - \kappa_\sigma \sum_i \left[(r - \gamma_i) U_i(x) \delta_{x+\hat{i},y} \right. \\
 &\quad \left. + (r + \gamma_i) U_i^\dagger(x - \hat{i}) \delta_{x-\hat{i},y} \right] \\
 &\quad - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} \\
 &\quad - r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y}, \quad (2)
 \end{aligned}$$

where κ_σ and κ_τ are the spatial and temporal hopping parameters, r the spatial Wilson parameter and c_E and c_B the clover coefficients. For a given κ_σ , in principle, the four parameters $\gamma_F \equiv \kappa_\tau/\kappa_\sigma$, r , c_E and c_B should be tuned. We can set $r = 1/\xi$ without loss of generality [1,7].

We must calibrate γ_F , c_E , and c_B to the level which enables computations of matrix elements within a few percent accuracy. We also need to perform the nonperturbative renormalization of the operators such as the heavy-light axial current. The nonperturbative renormalization technique [10] is one of the most powerful methods to perform such a program. Following our strategy, this technique can also be applied with a little modification for the anisotropic lattice.

We perform the calibration of γ_F , c_E , c_B , and

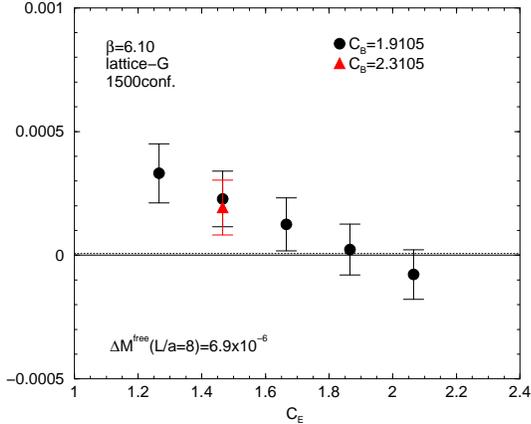


Figure 3. Tuning of c_E at $\beta = 6.1$.

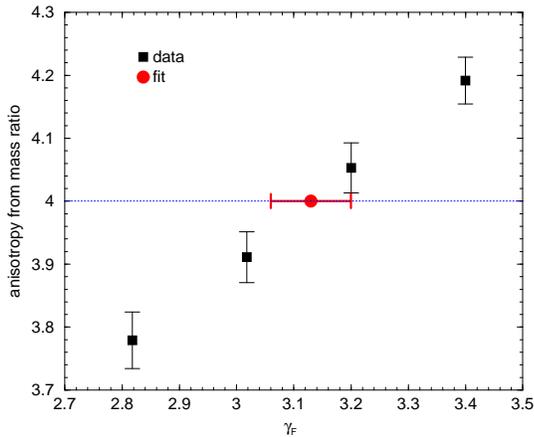


Figure 4. Determination of γ_F^* at $\beta = 5.75$.

the renormalization coefficients of the axial current along the following steps. (1) Tuning of c_E by Schrödinger functional method. (2) Calibration of γ_F (and c_B if possible) by requiring the physical isotropy conditions for m_{PS} and m_V in the coarse and fine directions on lattices with $T, L \gtrsim 2$ fm. (3) Determination of c_B , κ_c and the renormalization coefficients of the axial current by Schrödinger functional method. We also need to verify that the systematic errors are under control by calculating the hadron spectra and the dispersion relations and by taking the continuum limit. It is also necessary to verify that the tuned parameters in the massless limit is also available in the heavy quark mass region.

To verify the feasibility of the step (1), we de-

termine c_E with fixed values of γ_F and c_B . At $\beta = 9.5$, the method is successfully applicable, and the result for c_E is close to the 1-loop mean-field value. Figure 3 shows the result at $\beta = 6.1$ ($a_\sigma \simeq 2$ GeV). The tuned value of c_E is obtained as the value at which ΔM , the difference of the quark mass defined through the axial Ward identity under different kinematical conditions, vanishes up to $O(a^2)$ effects. The result of c_E is larger than the tadpole improved tree level value. It is also found that ΔM is not sensitive to the change of c_B .

Figure 4 shows the result for the step (2) at $\beta = 5.75$ on a $12^2 \times 24 \times 96$ lattice. The values of ξ_F is determined from the meson masses in the fine and coarse directions. We note that the result for γ_F^* is consistent with that from the dispersion relation [2]. At this stage, the precision of γ_F^* is still not sufficient, while several techniques to reduce the statistical noise are yet to be tested. For the determination of c_B , the ratio of the hyperfine splittings in the fine and coarse directions is not feasible because of large statistical noise. Other procedures, such as Schrödinger functional method with boundaries in the coarse direction, are under investigation.

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