DOCTORAL THESIS

A Study of the Third Family Quark Mass Hierarchy and Flavor-Changing Neutral Current in the Universal Seesaw Model

(ユニバーサルシーソー模型における

第三世代のクォーク質量階層性と

フレーバーを変える中性カレントの研究)

2024

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Abstract

The Universal Seesaw Model is an extension of the Standard Model (SM) that aims to explain the mass hierarchy problem between fermions by introducing heavy vector-like fermions (VLFs). These VLFs mix with the SM fermions, providing a seesaw-like mechanism that naturally explains the small masses of the light quarks and leptons while accommodating the heavy masses of the third family quarks. In addition, flavor-changing neutral currents (FCNC) are present at the tree level.

In this thesis, we present the study of the quark sector of the universal seesaw model with $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ gauge symmetry in the massless case of the two lightest quark families. This model aims to explain the mass hierarchy of the third family quark by introducing a vector-like quark (VLQ) partner for each quark. In this model, we introduce $\mathrm{SU(2)_L}$ and $\mathrm{SU(2)_R}$ Higgs doublets.

We derive the Lagrangian of the model explicitly for the quark sector, Higgs sector, and kinetic terms of the gauge fields. Starting from a Lagrangian invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$, we systematically present the Lagrangian at each stage of symmetry breaking. After the $\mathrm{SU(2)_R}$ Higgs doublet acquires a non-zero vacuum expectation value (vev), the Lagrangian becomes invariant under the SM gauge symmetry, and further breaking to $\mathrm{U(1)_{em}}$ occurs when the $\mathrm{SU(2)_L}$ Higgs doublet acquires its vev. At each stage of the symmetry breaking, we present the Lagrangian with the remaining gauge symmetry. Additionally, we investigate the flavor-changing neutral currents (FCNC) of Higgs (h) and Z-boson in the interaction with the top, heavy top, bottom, and heavy bottom quark.

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Chapter 1

Introduction

1.1 Background

The Standard Model (SM) of particle physics is the most successful framework for describing the fundamental elementary particles and their interactions. The precision test measurements conducted at the Large Hadron Collider (LHC), Large Electron-Positron Collider (LEP), and other facilities have validated the model's predictions to an extraordinary degree of accuracy. The discovery of the Higgs boson by the ATLAS [1] and CMS [2] experiments in 2012 confirmed the existence of all elementary particles predicted by the SM.

Despite its successes, the Standard Model (SM) has limitations. Several phenomena remain unexplained, such as the origin of neutrino mass. In the SM, neutrinos are massless. However, experimental results of neutrino oscillation [3–6] indicate that neutrinos have non-vanishing mass. Another mystery is the observational evidence from phenomena such as galaxy rotation curves and gravitational lensing, which supports the existence of dark matter [7].

Moreover, the SM does not fully explain particle-antiparticle asymmetry [10]. Experiments such as Belle [8] and BaBar [9] have studied CP violation in B meson decay to uncover the imbalance of particles and antiparticles. Another issue is the fermion mass hierarchy, which leads to unnatural fine-tuning of the Yukawa couplings. Consequently, many physicists attempt to address these issues by exploring theories beyond the Standard Model.

One of the intriguing aspects is the quark mass hierarchy. The Particle Data Group (PDG) provides recent data on quark masses [11]. Using the following tree-

Table 1.1: Quark masses and their corresponding Yukawa couplings. The values of m_u, m_d, and m_s are from $\overline{\text{MS}}$ at $\mu = 2$ GeV, m_c and m_b are from $\overline{\text{MS}}$ at $\mu = \overline{m}$, and m_t is from direct measurement. $v = 246.22$ GeV is used. Data from Ref.[11].

Quark mass	**Yukawa coupling**
$m_u = 2.16$ MeV	1.24×10^{-5}
$m_d = 4.70$ MeV	2.7×10^{-5}
$m_s = 93.5$ MeV	5.37×10^{-4}
$m_c = 1.273$ GeV	7.31×10^{-3}
$m_b = 4.183$ GeV	2.4×10^{-2}
$m_t = 172.57$ GeV	0.99

level mass of quark (m_q),

$$m_q = \frac{y_q^{\text{SM}}}{\sqrt{2}} v, \tag{1.1}$$

where $q \in \{u, d, c, s, b, t\}$ and v is the vacuum expectation value of SM Higgs, one can obtain the SM Yukawa coupling of quark q, denoted as y_q^{SM}. The list of Yukawa couplings for the corresponding quark masses is given in Table 1.1. One can see that the range of Yukawa couplings for each quark is very large.

The seesaw mechanism is a well-known approach to explain the smallness of neutrino masses [12–19]. It introduces heavy right-handed neutrinos that mix with left-handed neutrinos, giving them a small mass. This inspired the construction of a similar model, which can be applied to other cases. The universal seesaw model (USM) [20–35], is an extension of the SM that applies a seesaw-like mechanism to the quark sector to solve the mass hierarchy problem. For example, the small mass of the up quark can be explained with a tiny ratio of $\text{SU}(2)_\text{R}$ breaking scale and a vector-like quark (VLQ) with mass parameter M_U [36]. The corresponding Yukawa coupling for up quark is given by a seesaw-like formula,

$$y_u^{\text{SM}} = \frac{y_{u_L} v_R y_{u_R}}{\sqrt{2} M_U} \simeq \frac{v_R}{\sqrt{2} M_U} \simeq 10^{-5} \tag{1.2}$$

where y_{u_L} and y_{u_R} are the Yukawa coupling between SM quark and the VLQ partner. These Yukawa couplings are taken $y_{u_L} \simeq y_{u_R} \simeq \mathcal{O}(1)$. The top quark mass in the seesaw model of quark has been studied in Ref [27–29]. From Eq.(1.2), introducing vector-like quarks (VLQs) into this model is essential.

VLQs have left- and right-handed components that transform identically under some gauge group. Using this property, they can mix with SM quarks, resulting in modified mass matrices that can be diagonalized and generate a tiny seesaw-like mass. Various studies about the addition of VLQ to SM have been explored, for example, introducing one down-type VLQ isosinglet [37], one up-type VLQ isosinglet [38, 39], and both one up-type and down type VLQ isosinglet [40]. The presence of VLQs also has implications for flavor physics, as they can introduce flavor-changing neutral currents (FCNCs) [41] and weak-basis invariants have been analyzed to understand the flavor structures [42, 43]. Effective field theory approaches to VLQs have been studied to understand their contributions to low-energy observables [36, 44]. A review of the theory and phenomenology of isosinglet VLQs can be found in Ref.[45].

From the background that has been pointed out above, we aim to study the quark sector of the universal seesaw model with $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ gauge symmetry, focusing on the massless case of the two lightest quark families. This model aims to explain the mass hierarchy of the third family quark by introducing a vector-like quark (VLQ) partner for each quark. In our model, we introduce $\mathrm{SU(2)_L}$ and $\mathrm{SU(2)_R}$ Higgs doublets.

We derive explicitly the Lagrangian for the quark sector, Higgs sector, and kinetic terms of the gauge fields, starting from the Lagrangian, which is invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ gauge symmetry. At each stage of the symmetry breaking, we present the Lagrangian with the remaining gauge symmetry. Additionally, we investigate the flavor-changing neutral currents (FCNC) of Higgs (h) and Z-boson in the interaction with the top, heavy top, bottom, and heavy bottom quark.

1.2 Outline of the Thesis

The outline of this thesis is as follows. In chapter 2, some parts of the Standard Model (SM) is reviewed. In chapter 3, we reviewed the universal seesaw model. We introduce the particle contents and the Lagrangian of our model in section 3.3 based on Ref.[46].

After these chapters, we present our results based on Ref.[46]. Chapter 4 focuses on the quark sector and Yukawa interactions. We explain the derivation of the Lagrangian of the kinetic terms and Yukawa interactions. Starting with the Lagrangian which is invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$, in each stage of the symmetry breaking we present the Lagrangian with the remaining gauge symmtery. The quark mass eigenvalues and the identification of FCNC within the massive third family quarks and their VLQ partners are discussed.

Chapter 5 discusses the Higgs sector of this model. The kinetic terms and Higgs potential are also derived step by step. In the end, we classify the terms based on the number of the fields in the term as linear, quadratic, cubic, and quartic, ensuring a clear understanding of the interactions of the gauge sector. In addition, we also provide the exact diagonal mass of $Z - Z'$ bosons and $h - H$ bosons.

The kinetic terms of gauge fields are discussed in Chapter 6. In the final derivation, we show the difference between our model and SM. In Chapter 7, we presented our result about the hierarchy of VLQ's mass parameters, the non-zero vacuum expectation value of $\mathrm{SU(2)_L}$ Higgs doublet (v_L), and the non-zero vacuum expectation value of $\mathrm{SU(2)_R}$ Higgs doublet (v_R). In chapter 8 we analyze the interaction between Higgs and Z-boson with the quarks. This leads to a discussion about flavor-changing neutral currents in this model.

Chapter 2

Standard Model

In this chapter, the Standard Model (SM) is reviewed. The main part of the review is the quark sector. For a more comprehensive review, see, e.g., Ref [47, 48]

2.1 Introduction

The SM is based on the gauge group $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$. The $\mathrm{SU(3)_C}$ describes the strong interaction with gluon as the corresponding gauge bosons. This gauge group remains unbroken so that gluons are massless. This interaction binds quarks together to form protons, neutrons, and other hadrons. $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ describes the electroweak interaction, which unifies the electromagnetic and weak interactions. After this electroweak symmetry breaking, the gauge bosons mediating the weak interactions, $W^{\pm}$ and Z bosons, become massive. On the other hand, the gauge boson of the electromagnetic interaction, the photon, remains massless. This spontaneous symmetry breaking is explained by the Higgs mechanism, which introduces the Higgs field. This electroweak symmetry breaking also leads to a conserved quantity: electromagnetic charge. The relation between electromagnetic charge (Q), third component weak isospin (I^3), and hypercharge Y is,

$$Q = I^3 + Y. \tag{2.1}$$

2.2 Particle Contents

The particle contents of the SM according to their transformation properties under SM gauge groups are shown in Table (2.1). SM categorizes fermions into three generations, where each successive generation is heavier than the previous one. These

Table 2.1: The particle content with their quantum numbers under the SM gauge groups. The index $i \in \{1, 2, 3\}$ denotes the generation of quarks. The index $\alpha \in \{e, \mu, \tau\}$ denotes the flavor of charged leptons. The symbols G^a_μ, W^I_μ, and B_μ with $a \in \{1, \ldots, 8\}, I \in \{1, 2, 3\}$ represent the $\mathrm{SU(3)_C}$, $\mathrm{SU(2)_L}$, and $\mathrm{U(1)_Y}$ corresponding gauge bosons, respectively. The symbol Φ represents the $\mathrm{SU(2)_L}$ Higgs doublet.

Fields	$\mathrm{SU(3)_C}$	$\mathrm{SU(2)_L}$	$U(1)_Y$
$q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u^i_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$
d^i_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
$L^\alpha_L = \begin{pmatrix} \nu^\alpha_L \\ \ell^\alpha_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
ℓ^α_R	$\mathbf{1}$	$\mathbf{1}$	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$
G^a_μ	$\mathbf{8}$	$\mathbf{1}$	0
W^I_μ	$\mathbf{1}$	$\mathbf{3}$	0
B_μ	$\mathbf{1}$	$\mathbf{1}$	0

fermions are called leptons and quarks. Leptons and quarks are both spin 1/2 particles, but they are distinguished by their interactions. Leptons do not interact with gluons, so they do not have strong interactions. Quarks carry color charge (red, blue, green) and interact via the strong interactions. The L and R subscripts denote the left-handed and right-handed chirality components, respectively.

The left-handed components of both leptons and quarks transform as doublets under $\mathrm{SU(2)_L}$ gauge group, whereas their right-handed components are singlet under this group. The three generations of leptons consist of the electron, muon, and tau, which have an electromagnetic charge $Q_\ell = -1$, along with their corresponding neutrinos which is neutral. In the Standard Model, only left-handed neutrinos exist. Both component of left-handed quarks carry non-zero electromagnetic charge. The up-type quarks have $Q_u = 2/3$, while the down-type quarks have $Q_d = -1/3$.

Moreover, SM includes spin-1 vector bosons that mediated the fundamental interactions. Each local gauge symmetry has corresponding gauge bosons whose number matches the number of the symmetry's generators, and they transform according to the adjoint representations of the corresponding gauge groups. The gauge bosons G^a_μ correspond to the $\mathrm{SU}(3)_\mathrm{C}$ gauge group, where $a \in \{1, \ldots, 8\}$, and are called gluons. The gauge bosons W^I_μ correspond to the $\mathrm{SU}(2)_\mathrm{L}$ gauge group, where $I \in \{1, 2, 3\}$. Lastly, B_μ is the gauge boson corresponding to the $\mathrm{U}(1)_\mathrm{Y}$ gauge group. The SM also includes a spin-0 particle known as the Higgs boson which is a part of the $\mathrm{SU}(2)_\mathrm{L}$ Higgs doublet Φ. When the neutral component of the Higgs field acquires non-zero vacuum expectation value (vev), it breaks the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ symmetry down to $\mathrm{U}(1)_\mathrm{em}$. Through this spontaneous symmetry mechanism, the W^I_μ and B_μ are mixed and become the massive $W^\pm$ and Z bosons, and the massless spin-1 photon (γ).

2.3 Quark Sector of the Standard Model

The Lagrangian of the SM quark sector which is invariant under SM gauge groups is as follows,

$$\mathcal{L}^q_\mathrm{SM} = \mathcal{L}^q_\mathrm{kin} + \mathcal{L}^q_\mathrm{Yuk}, \tag{2.2}$$

where,

$$\mathcal{L}_{q,\mathrm{kin}} = \overline{q}^i_L i\gamma^\mu D_\mu q^i_L + \overline{u}^i_R i\gamma^\mu D_\mu u^i_R + \overline{d}^i_R i\gamma^\mu D_\mu d^i_R \tag{2.3}$$

$$\mathcal{L}^q_\mathrm{Yuk} = -\overline{q^i_L}(y^\mathrm{SM}_u)^{ij}\tilde{\Phi}u^j_R - \overline{q^i_L}(y^\mathrm{SM}_d)^{ij}\Phi d^j_R - h.c.. \tag{2.4}$$

The covariant derivatives in Eq.(2.3) are defined as (excluding the $\mathrm{SU}(3)_\mathrm{C}$ part),

$$D_\mu q^i_L = \left(\partial_\mu + igW^I_\mu \frac{\tau^I_L}{2} + ig'Y_q B_\mu\right) q^i_L \tag{2.5}$$

$$D_\mu u^i_R = \left(\partial_\mu + ig'Y_{u_R} B_\mu\right) u^i_R \tag{2.6}$$

$$D_\mu d^i_R = \left(\partial_\mu + ig'Y_{d_R} B_\mu\right) d^i_R, \tag{2.7}$$

where g is $\mathrm{SU}(2)_\mathrm{L}$ gauge coupling, τ^I is the Pauli matrix, g' is $\mathrm{U}(1)_\mathrm{Y}$ gauge coupling, $Y \in \{Y_q, Y_{u_R}, Y_{d_R}\}$ are the corresponding hypercharge for each quark fields. The charge conjugation of the Higgs field is defined as $\tilde{\Phi} = i\tau^2\Phi^*$. In Eq.(2.4), y^SM_u and y^SM_d are 3×3 matrices for up-type and down-type Yukawa matrices, respectively. The electroweak gauge bosons (W^I_μ and B_μ) are not the physical eigenstates we observed. Through the spontaneously symmetry breaking, W^1_μ and W^2_μ mix to form the charged

$W^\pm_\mu$ bosons which are defined as,

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp W^2_\mu). \tag{2.8}$$

W^3_μ and B_μ mix together to form the neutral Z boson and the photon A_μ with the following transformation,

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{2.9}$$

where θ_W denotes as weak mixing angle and is called as Weinberg angle. It has the following expression with the weak gauge coupling g and g',

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}. \tag{2.10}$$

Furthermore, the electromagnetic coupling e is related to the gauge couplings g and g' as,

$$e = g\sin\theta_W = g'\cos\theta_W. \tag{2.11}$$

Therefore, using Eqs.(2.8),(2.9),(2.11), and (2.1), the covariant derivatives in Eqs.(2.5), (2.6), and (2.7) are expressed as follows,

$$\begin{aligned} D_\mu q^i_L = \Bigg(& \partial_\mu + ieQ_{q_L}A_\mu + i\frac{g}{\sqrt{2}}\left(W^+_\mu\tau^+ + W^-_\mu\tau^-\right) \\ & + i\frac{g}{2\cos\theta_W}\left(\tau^3 - 2\sin^2\theta_W Q_{q_L}\right)Z_\mu\Bigg) q^i_L \end{aligned} \tag{2.12}$$

$$D_\mu u^i_R = \left(\partial_\mu + ieQ_uA_\mu - i\frac{g}{\cos\theta_W}\sin^2\theta_W Q_u Z_\mu\right) u^i_R \tag{2.13}$$

$$D_\mu d^i_R = \left(\partial_\mu + ieQ_dA_\mu - i\frac{g}{\cos\theta_W}\sin^2\theta_W Q_d Z_\mu\right) d^i_R, \tag{2.14}$$

where,

$$Q_{q_L} = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}, \quad Q_u = \frac{2}{3}, \quad Q_d = -\frac{1}{3}, \quad \tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2). \tag{2.15}$$

Finally, using Eqs.(2.12),(2.13), and (2.14), the Lagrangian in Eq.(2.3) becomes,

$$\begin{aligned}\mathcal{L}_{q,\text{kin}} &= \overline{u^i} i\gamma^\mu \partial_\mu u^i + \overline{d^i} i\gamma^\mu \partial_\mu d^i \\ &- e\left(\frac{2}{3}\overline{u^i}\gamma^\mu u^i - \frac{1}{3}\overline{d^i}\gamma^\mu d^i\right) A_\mu \\ &- \frac{g}{\sqrt{2}}\left(\overline{u^i_L}\gamma^\mu d^i_L W^+_\mu + \overline{d^i_L}\gamma^\mu u^i_L W^-_\mu\right) \\ &- \frac{g}{2\cos\theta_W}\left\{\left(\overline{u^i_L}\gamma^\mu u^i_L - \overline{d^i_L}\gamma^\mu d^i_L\right) - 2\sin^2\theta_W\left(\frac{2}{3}\overline{u^i}\gamma^\mu u^i - \frac{1}{3}\overline{d^i}\gamma^\mu d^i\right)\right\} Z_\mu,\end{aligned} \tag{2.16}$$

where we define $u^i = u^i_L + u^i_R$ and $d^i = d^i_L + d^i_R$. The first line of Eq.(2.16) is the kinetic terms of up-type and down-type quarks. The second line details the electromagnetic interaction between quarks mediated by photon. The third line describes the charged weak currents, where the left-handed up-type and down-type quarks interact with the $W^\pm$ bosons. Finally, the fourth line explains the neutral weak currents, mediated by Z boson.

We define electromagnetic and weak isospin current in the quark sector of SM as $j^\mu_{\text{em,q}}$ and $j^\mu_{3,\text{q}}$ with following expressions,

$$j^\mu_{\text{em,q}} = \frac{2}{3}\overline{u^i}\gamma^\mu u^i - \frac{1}{3}\overline{d^i}\gamma^\mu d^i \tag{2.17}$$

$$j^\mu_{3,\text{q}} = \overline{u^i_L}\gamma^\mu u^i_L - \overline{d^i_L}\gamma^\mu d^i_L. \tag{2.18}$$

By using Eqs.(2.17) and (2.18), the Lagrangian in Eq.(2.16) can be written as,

$$\begin{aligned}\mathcal{L}_{q,\text{kin}} &= \overline{u^i} i\gamma^\mu \partial_\mu u^i + \overline{d^i} i\gamma^\mu \partial_\mu d^i - e j^\mu_{\text{em,q}} A_\mu \\ &- \frac{g}{\sqrt{2}}\left(\overline{u^i_L}\gamma^\mu d^i_L W^+_\mu + \overline{d^i_L}\gamma^\mu u^i_L W^-_\mu\right) \\ &- \frac{g}{2\cos\theta_W}\left(j^\mu_{3,\text{q}} - 2\sin^2\theta_W j^\mu_{\text{em,q}}\right) Z_\mu.\end{aligned} \tag{2.19}$$

2.3.1 Generating Quark Masses

The symmetry breaking $\text{SU(2)}_\text{L} \times \text{U(1)}_\text{Y}$ into U(1)_em occurs when the neutral component of the Higgs field acquires non-zero vev with following form,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.20}$$

The Yukawa interaction in Eq.(2.4) becomes,

$$\mathcal{L}^q_{\text{Yuk}} \to \mathcal{L}^{\text{SM}}_{\text{mass}} = -\overline{u^i_L}(m_u)^{ij} u^j_R - \overline{d^i_L}(m_d)^{ij} d^j_R - h.c., \tag{2.21}$$

where,

$$(m_u)^{ij} = \frac{v}{\sqrt{2}}(y^{\text{SM}}_u)^{ij} \tag{2.22}$$

$$(m_d)^{ij} = \frac{v}{\sqrt{2}}(y^{\text{SM}}_d)^{ij}. \tag{2.23}$$

The mass matrices in Eqs.(2.22) and (2.23) is diagonalized by changing from the flavor eigenstate to physical mass eigenstate of quark fields using the following transformations,

$$u^i_L = \sum_{j=1}^{3}(K_{u_L})^{ij}(u^m_L)^j, \tag{2.24}$$

$$u^i_R = \sum_{j=1}^{3}(K_{u_R})^{ij}(u^m_R)^j, \tag{2.25}$$

$$d^i_L = \sum_{j=1}^{3}(K_{d_L})^{ij}(d^m_L)^j, \tag{2.26}$$

$$d^i_R = \sum_{j=1}^{3}(K_{d_R})^{ij}(d^m_R)^j, \tag{2.27}$$

where $K_{u_L}, K_{u_R}, K_{d_L}$, and K_{d_R} are 3×3 unitary matrices. The diagonalization of quark mass matrices in Eqs.(2.22) and (2.23) reads,

$$(K^\dagger_{u_L})^{ji}(m_u)^{ij}(K_{u_R})^{jk} = (m^{\text{diag}}_u)^{jj}\delta^{jk}, \tag{2.28}$$

$$(K^\dagger_{d_L})^{ji}(m_d)^{ij}(K_{d_R})^{jk} = (m^{\text{diag}}_d)^{jj}\delta^{jk}, \tag{2.29}$$

where,

$$m^{\text{diag}}_u = \text{diag}(m_u, m_c, m_t), \tag{2.30}$$

$$m^{\text{diag}}_d = \text{diag}(m_d, m_s, m_b), \tag{2.31}$$

are the physical up-type and down-type quark masses, respectively. Finally, after changing from the flavor eigenstate to the mass eigenstate using Eqs.(2.24),(2.25), (2.26), and (2.27), respectively, the Lagrangian in Eq.(2.21) becomes,

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm mass} &= -\overline{(u^m_L)^j}(m_u^{\rm diag})^{jj}(u^m_R)^j - \overline{(d^m_L)^j}(m_d^{\rm diag})^{jj}(d^m_R)^j - h.c.\\ &= -\overline{(u^m)^j}(m_u^{\rm diag})^{jj}(u^m)^j - \overline{(d^m)^j}(m_d^{\rm diag})^{jj}(d^m)^j, \end{aligned}\tag{2.32}$$

where by including the Hermitian conjugate terms, we define $u^m = u^m_L + u^m_R$ and $d^m = d^m_L + d^m_R$.

2.3.2 Charged Currents

Extracting the charged current Lagrangian from Eq.(2.19) as follows,

$$\mathcal{L}_{q,\rm kin} \supset \mathcal{L}^{\rm SM,q}_{\rm CC} = -\frac{g}{\sqrt{2}}\left(\overline{u^i_L}\gamma^\mu d^i_L W^+_\mu + \overline{d^i_L}\gamma^\mu u^i_L W^-_\mu\right). \tag{2.33}$$

Changing from the quark flavor eigenstate to the mass eigenstate using Eqs.(2.24) and (2.26), the charged current Lagrangian in Eq.(2.33) becomes,

$$\begin{aligned}\mathcal{L}^{\rm SM,q}_{\rm CC} &= -\frac{g}{\sqrt{2}}\left((\overline{u^m_L})^j\gamma^\mu(K^\dagger_u K_d)^{jk}(d^m_L)^k W^+_\mu + (\overline{d^m_L})^j\gamma^\mu(K^\dagger_d K_u)^{jk}(u^m_L)^k W^-_\mu\right)\\ &= -\frac{g}{\sqrt{2}}\left((\overline{u^m_L})^j\gamma^\mu(V^{\rm SM}_{\rm CKM})^{jk}(d^m_L)^k W^+_\mu + (\overline{d^m_L})^j\gamma^\mu(V^{\rm SM\dagger}_{\rm CKM})^{jk}(u^m_L)^k W^-_\mu\right), \end{aligned}\tag{2.34}$$

where $j, k \in \{1, 2, 3\}$. The mixing matrix in Eq.(2.34) is called Cabibbo-Kobayashi-Maskawa (CKM) matrix which is defined as,

$$V^{\rm SM}_{\rm CKM} = K^\dagger_u K_d. \tag{2.35}$$

In Eq.(2.34), the CKM matrix is a general 3×3 unitary matrix parameterized by three mixing angles and six phases. However, we can remove the unphysical phases of the CKM matrix.

One has the freedom to rephase the quark field in the mass basis, which leaves the mass terms of quarks in Eq.(2.32) unchanged. We define the following phase transformations,

$$(u^m_L)^j = e^{i\phi^j_u}(\hat{u}^m_L)^j, \tag{2.36}$$

$$(u^m_R)^j = e^{i\phi^j_u}(\hat{u}^m_R)^j, \tag{2.37}$$

$$(d^m_L)^j = e^{i\phi^j_d}(\hat{d}^m_L)^j, \tag{2.38}$$

$$(d^m_R)^j = e^{i\phi^j_d}(\hat{d}^m_R)^j. \tag{2.39}$$

By applying Eqs.(2.36) and (2.38), the charged current in Eq.(2.34) becomes

$$\mathcal{L}_{\rm CC}^{\rm SM,q} = -\frac{g}{\sqrt{2}}\left(\overline{(\hat{u}_L^m)}^j\gamma^\mu(\hat{V}_{\rm CKM}^{\rm SM})^{jk}(\hat{d}_L^m)^k W_\mu^+ + \overline{(\hat{d}_L^m)}^j\gamma^\mu(\hat{V}_{\rm CKM}^{\rm SM\dagger})^{jk}(\hat{u}_L^m)^k W_\mu^-\right), \quad (2.40)$$

where $\hat{V}_{\rm CKM}^{\rm SM}$ is the rephased CKM matrix. Considering the rephasing and the unitarity of CKM Matrix, the number of parameters of the $N_g \times N_g$ CKM matrix is,

$$\text{Number of mixing angle} = \frac{1}{2}N_g(N_g-1) \quad (2.41)$$

$$\text{Number of physical phase} = \frac{1}{2}(N_g-1)(N_g-2) \quad (2.42)$$

where N_g is the number of generations. In SM with $N_g = 3$, the CKM matrix has three mixing angles and one physical phase [49, 50].

2.3.3 Weak Neutral Currents

The electromagnetic and weak isospin current in Eqs.(2.17) and (2.18) transform into the expression in mass basis using Eqs.(2.24)-(2.27) and are rephased using Eqs.(2.36)-(2.39). They become,

$$j_{\rm em,q}^\mu = \frac{2}{3}\overline{(\hat{u}^m)^i}\gamma^\mu(\hat{u}^m)^i - \frac{1}{3}\overline{(\hat{d}^m)^i}\gamma^\mu(\hat{d}^m)^i \quad (2.43)$$

$$j_{3,\rm q}^\mu = \overline{(\hat{u}_L^m)^i}\gamma^\mu(\hat{u}_L^m)^i - \overline{(\hat{d}_L^m)^i}\gamma^\mu(\hat{d}_L^m)^i. \quad (2.44)$$

The weak neutral current Lagrangian extracted from Eq.(2.19) is:

$$\mathcal{L}_{q,\rm kin} \supset \mathcal{L}_{\rm NC}^{\rm SM,q} = -\frac{g}{2\cos\theta_W}\left(j_{3,\rm q}^\mu - 2\sin^2\theta_W j_{\rm em,q}^\mu\right)Z_\mu. \quad (2.45)$$

By substituting Eqs.(2.43) and (2.44) into Eq.(2.45), we see that the weak neutral current interaction does not involve any mixing of quark flavors. In the SM, flavor-changing neutral currents (FCNCs) are absent at the tree level.

Moreover, the weak neutral current Lagrangian in Eq.(2.45) can be expressed in the following forms,

$$\mathcal{L}_{\rm NC}^{\rm SM,q} = -\frac{g}{2\cos\theta_W}\left[\sum_\alpha \overline{q^\alpha}\gamma^\mu\left(g_V^\alpha - g_A^\alpha\gamma^5\right)q^\alpha\right]Z_\mu, \quad (2.46)$$

where $\alpha \in \{(\hat{u}^m)^i, (\hat{d}^m)^i\}$; g_V and g_A are the vector and axial-vector couplings, respectively. The definition of these couplings are as follows,

$$g_V^\alpha = (\tau_L^3)^\alpha - 2Q^\alpha \sin^2\theta_W, \tag{2.47}$$

$$g_A^\alpha = (\tau_L^3)^\alpha, \tag{2.48}$$

where $\tau_L^3 = 1/2$ for $\alpha = (\hat{u}^m)^i$ and $\tau_L^3 = -1/2$ for $\alpha = (\hat{d}^m)^i$. Q^α is the electromagnetic charge which is written in Eq.(2.15).

Chapter 3

Universal Seesaw Model (USM)

In this chapter, we explain the Universal Seesaw Model (USM). We begin by introducing the general framework of the USM. After that, we introduce our model.

3.1 Neutrino and the Seesaw Mechanism

Historically, the Standard Model (SM) includes only left-handed neutrinos and right-handed anti-neutrinos [51–53]. This was consistent with early experimental observations, which did not indicate the presence of right-handed neutrinos [54–56]. However, the discovery of neutrino oscillations, where neutrinos change flavor as they propagate, provided clear evidence that neutrinos have a small but non-zero mass [3–6]. This observation required an extension of the SM, as the original framework could not accommodate massive neutrinos.

One of the well-known explanations for the tiny masses of neutrinos is the seesaw mechanism [12–19]. The type I Seesaw mechanism introduces heavy right-handed neutrinos that is singlet under the SM gauge group, but can mix with the left-handed neutrinos [19]. The mass matrix for neutrinos is then modified, and the smallness of the observed neutrino masses is achieved through the large Majorana mass term for the right-handed neutrinos. The large mass scale of these right-handed neutrinos leads to a suppression of the neutrino masses, making them small.

3.2 USM: General Framework

The Universal Seesaw Model (USM) extends the seesaw mechanism to explain the mass hierarchy of all fermions, including quarks and charged leptons. In this model, each fermion acquires its mass through interactions with heavy singlet fermions, analogous to the right-handed neutrinos in the Type I Seesaw mechanism. These heavy singlet fermions are often referred to as Vector Like Fermions (VLFs) because they have both left-handed and right-handed components that transform identically under the gauge group. These VLFs are singlet under SM gauge group. The mass terms for the SM fermions arise from mixing with these heavy VLFs, leading to a seesaw-like suppression of their masses.

Additionally, the gauge symmetry in the USM is usually extended to include an additional gauge group, such as $\mathrm{SU(2)_R}$ and $\mathrm{U(1)_{B-L}}$, alongside the $\mathrm{SU(2)_L}$ gauge group [24]. An additional Higgs field is required to break the new gauge symmetries. This additional Higgs field is a doublet under the new gauge group $\mathrm{SU(2)_R}$, which is different from the traditional left-right symmetric model that introduces a Higgs bi-doublet [57, 58]. The spontaneous breaking of this extended symmetry leads to the mass generation for both the SM fermions and the heavy VLFs.

Our model follows this general framework but focuses on the third family of quarks to address their mass hierarchy. We also investigate the flavor-changing neutral currents (FCNCs) in our model.

3.3 USM: The Third Family Quark Framework

We consider an extension of SM with $\mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ gauge symmetry in the massless case of the two lightest quark families. Alongside of the $\mathrm{SU(2)_L}$ SM Higgs doublet (ϕ_L), we introduce a $\mathrm{SU(2)_R}$ Higgs doublet (ϕ_R). Additionally, the model incorporates one up-type and one down-type isosinglet vector-like quark (VLQ), labeled as T and B, respectively. The charge convention adopted in this model is as follows,

$$Q = I_L^3 + I_R^3 + Y', \tag{3.1}$$

where $Q, I^3_{L(R)}$ and Y' represent the electromagnetic charge, left (right) weak isospin, and $\mathrm{U(1)_{Y'}}$ hypercharge, respectively. The particle contents and their corresponding charge assignments under the model's gauge group are detailed in Table 3.1.

Table 3.1: Quark and Higgs fields with their quantum numbers under the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{Y}'}$ gauge groups, where $i \in \{1, 2, 3\}$ is the family index.

Quark and Higgs Fields	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{SU}(2)_\mathrm{R}$	$\mathrm{U}(1)_{\mathrm{Y}'}$
$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	**3**	**2**	**1**	1/6
$q_R^i = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix}$	**3**	**1**	**2**	1/6
$T_{L,R}$	**3**	**1**	**1**	2/3
$B_{L,R}$	**3**	**1**	**1**	−1/3
$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}$	**1**	**2**	**1**	1/2
$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	**1**	**1**	**2**	1/2

The Lagrangian of this model (excluding the QCD part) is as follows,

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_H + \mathcal{L}_{\text{gauge}}, \tag{3.2}$$

$$\begin{aligned}\mathcal{L}_q = &\overline{q_L^i} i\gamma^\mu D_{L\mu} q_L^i + \overline{q_R^i} i\gamma^\mu D_{R\mu} q_R^i + \overline{T} i\gamma^\mu D_{T\mu} T + \overline{B} i\gamma^\mu D_{B\mu} B \\ &- \left(Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R + Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 + \overline{q_L^i} y_{d_L}^i \phi_L B_R + \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i + h.c. \right) \\ &- \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - h.c.,\end{aligned} \tag{3.3}$$

$$\mathcal{L}_H = (D_L^\mu \phi_L)^\dagger (D_{L\mu} \phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu} \phi_R) - V(\phi_L, \phi_R), \tag{3.4}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{L\mu\nu}^a F_L^{a\mu\nu} - \frac{1}{4} F_{R\mu\nu}^a F_R^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}, \tag{3.5}$$

where,

$$V(\phi_L, \phi_R) = \mu_L^2 \phi_L^\dagger \phi_L + \mu_R^2 \phi_R^\dagger \phi_R + \lambda_L (\phi_L^\dagger \phi_L)^2 + \lambda_R (\phi_R^\dagger \phi_R)^2 + 2\lambda_{LR} (\phi_L^\dagger \phi_L)(\phi_R^\dagger \phi_R), \tag{3.6}$$

$$D_{L(R)\mu} q_{L(R)}^i = \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_q B'_\mu \right) q_{L(R)}^i, \tag{3.7}$$

$$D_{L(R)\mu} \phi_{L(R)} = \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_\phi B'_\mu \right) \phi_{L(R)}, \tag{3.8}$$

$$D_{T\mu} T = \left(\partial_\mu + i g'_1 Y'_T B'_\mu \right) T, \tag{3.9}$$

$$D_{B\mu} B = \left(\partial_\mu + i g'_1 Y'_B B'_\mu \right) B, \tag{3.10}$$

$$F^a_{L\mu\nu} = \partial_\mu W^a_{L\nu} - \partial_\nu W^a_{L\mu} - g_L \epsilon^{abc} W^b_{L\mu} W^c_{L\nu}, \tag{3.11}$$

$$F^a_{R\mu\nu} = \partial_\mu W^a_{R\nu} - \partial_\nu W^a_{R\mu} - g_R \epsilon^{abc} W^b_{R\mu} W^c_{R\nu}, \tag{3.12}$$

$$B'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu. \tag{3.13}$$

The Lagrangian in Eq.(3.2) is divided into three parts. The first part is the kinetic terms of quark doublet and isosinglet VLQs, Yukawa interactions, and mass terms of isosinglet VLQs, which are contained in Eq.(3.3). The second part is the kinetic terms and potential of Higgs doublet which are contained in Eq.(3.4). The third part is the kinetic terms of the gauge fields, which are written in Eq.(3.5).

The first line of Eq.(3.3) is the kinetic terms of quark doublet and isosinglet VLQs where the definition of the covariant derivatives are written in Eqs.(3.7), (3.9) and (3.10) respectively, where $g_{L(R)}$ is $\mathrm{SU(2)_{L(R)}}$ gauge coupling, τ^a is the Pauli matrix, g'_1 is $\mathrm{U(1)_{Y'}}$ gauge coupling and Y' is the corresponding $\mathrm{U(1)_{Y'}}$ hypercharge. For the Yukawa interaction part, one can choose in a weak-basis where the Yukawa couplings of up-type quark doublet ($Y^3_{u_L}$ and $Y^3_{u_R}$) are real positive numbers. In contrast, the Yukawa couplings of down-type quark are general complex vectors as shown in the second line of Eq.(3.3). The derivation of this weak-basis is briefly explained in Appendix A. The family index for SM quarks is denoted as $i = 1, 2, 3$, the charge conjugation of Higgs fields is defined as $\tilde{\phi}_{L(R)} = i\tau^2 \phi^*_{L(R)}$. In the third line of Eq.(3.3), M_T and M_B are isosinglet VLQs mass parameters that we take as real numbers.

The first two terms of Eq.(3.4) are the kinetic terms of Higgs doublet where the definition of the covariant derivatives are written in Eq.(3.8). The third term is the Higgs potential which is shown in Eq.(3.6), containing the mass terms and quartic interactions of Higgs doublet. The interaction between ϕ_L and ϕ_R is also included in this term. Later ϕ_R and ϕ_L acquire non-zero vacuum expectation values (vevs) denoted as v_R and v_L that break $\mathrm{SU(2)_R}$ and $\mathrm{SU(2)_L}$ respectively. They satisfy the hierarchy, $v_R \gg v_L$.

Chapter 4

Quark Sector and Yukawa Interactions

In this chapter, we derived the kinetic terms of quark doublet and isosinglet VLQs, Yukawa interactions, and mass terms of isosinglet VLQs, as written in Eq. (3.3). Once the $\mathrm{SU(2)_R}$ Higgs doublet acquires non-zero vev, we obtain the Lagrangian which is invariant under SM gauge symmetry. Furthermore, the SM gauge group is subsequently broken down to $\mathrm{U(1)_{em}}$ after $\mathrm{SU(2)_L}$ Higgs doublet acquires non-zero vev. Finally, we obtain the masses of top and bottom quarks, their heavy partners, Z, Z', h, and H. Additionally, the derivation also accounts the flavor-changing neutral currents (FCNCs) and the CKM matrix to be generated.

4.1 $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

In this stage, the neutral scalar component of $\mathrm{SU(2)_R}$ Higgs doublet acquires non-zero vev and is expanded around the vev as follows,

$$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_R^+ \\ v_R + h_R + i\chi_R^3 \end{pmatrix}, \tag{4.1}$$

where v_R is the non-zero vev. h_R is the neutral CP-even state and χ_R^3 is the neutral CP-odd state. The charged component is denoted as, $\chi_R^+ = \frac{1}{\sqrt{2}}(\chi_R^1 + i\chi_R^2)$. In addition, we rotate the gauge fields with the following transformation,

$$\begin{pmatrix} B'_\mu \\ W^3_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} B_\mu \\ Z_{R\mu} \end{pmatrix}, \tag{4.2}$$

where the mixing angle,

$$\sin\theta_R = \frac{g_1'}{\sqrt{g_R^2 + g_1'^2}}, \qquad \cos\theta_R = \frac{g_R}{\sqrt{g_R^2 + g_1'^2}}. \tag{4.3}$$

We also define the SM $\mathrm{U(1)_Y}$ gauge coupling as,

$$g' = g_1' \cos\theta_R = g_R \sin\theta_R. \tag{4.4}$$

After this spontaneously symmtery breaking, the Lagrangian in Eq.(3.3) become,

$$\begin{aligned}
\mathcal{L}_q = {} & \overline{q_L^i} i\gamma^\mu D_{\mathrm{SM}\mu} q_L^i + \overline{T_L} i\gamma^\mu D_{\mathrm{SM}\mu} T_L + \overline{B_L} i\gamma^\mu D_{\mathrm{SM}\mu} B_L \\
& + \overline{u_R^i} i\gamma^\mu D_{\mathrm{SM}\mu} u_R^i + \overline{d_R^i} i\gamma^\mu D_{\mathrm{SM}\mu} d_R^i + \overline{T_R} i\gamma^\mu D_{\mathrm{SM}\mu} T_R + \overline{B_R} i\gamma^\mu D_{\mathrm{SM}\mu} B_R \\
& - \frac{g_R}{\sqrt{2}} \overline{u_R^i} \gamma^\mu d_R^i W_{R\mu}^+ - h.c. \\
& + g' \tan\theta_R \left(\overline{q_L^i} \gamma^\mu Y_q q_L^i + \frac{2}{3} \overline{T_L} \gamma^\mu T_L - \frac{1}{3} \overline{B_L} \gamma^\mu B_L \right) Z_{R\mu} \\
& - \left\{ \frac{g_R}{2\cos\theta_R} (\overline{u_R^i} \gamma^\mu u_R^i - \overline{d_R^i} \gamma^\mu d_R^i) - g' \tan\theta_R \left(\frac{2}{3} (\overline{u_R^i} \gamma^\mu u_R^i + \overline{T_R} \gamma^\mu T_R) \right.\right. \\
& \left.\left. - \frac{1}{3} (\overline{d_R^i} \gamma^\mu d_R^i + \overline{B_R} \gamma^\mu B_R) \right) \right\} Z_{R\mu} \\
& - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \frac{v_R}{\sqrt{2}} \overline{T_L} u_R^3 - \overline{T_L} M_T T_R - h.c. \\
& - Y_{u_R}^3 \overline{T_L} \left(\frac{1}{\sqrt{2}} u_R^3 (h_R + i\chi_R^3) - d_R^3 \chi_R^+ \right) - h.c. \\
& - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \frac{v_R}{\sqrt{2}} d_R^i - \overline{B_L} M_B B_R - h.c. \\
& - \overline{B_L} y_{d_R}^{i*} \left(\frac{1}{\sqrt{2}} d_R^i (h_R - i\chi_R^3) + u_R^i \chi_R^- \right) - h.c.,
\end{aligned} \tag{4.5}$$

where $i \in \{1, 2, 3\}$ is the family index and the SM covariant derivatives have following expressions,

$$D_{\mathrm{SM}\mu} q_L^i = \left(\partial_\mu + i g_L \frac{\tau^a}{2} W_{L\mu}^a + i g' Y_{q_L} B_\mu \right) q_L^i, \tag{4.6}$$

$$D_{\mathrm{SM}\mu} f_u = \left(\partial_\mu + \frac{2}{3} i g' B_\mu \right) f_u, \tag{4.7}$$

$$D_{\mathrm{SM}\mu} f_d = \left(\partial_\mu - \frac{1}{3} i g' B_\mu \right) f_d, \tag{4.8}$$

where $f_u \in \{u_R^i, T_{L,R}\}$ and $f_d \in \{d_R^i, B_{L,R}\}$. At this stage, $\mathrm{U(1)_Y}$ hypercharge can be obtained as following Eq.(3.1), $Y = I_R^3 + Y'$. In Eqs.(4.7) and (4.8), we write the $\mathrm{U(1)_Y}$ hypercharge of the corresponding fields explicitly. Next, we follow several steps to reach the Lagrangian invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge symmetry.

- **Step 1**: Rotate d_R^i by the following transformation,

$$d_R^i = (V_{d_R})^{ij}(d'_R)^j, \tag{4.9}$$

where V_{d_R} is 3×3 unitary matrix, which related to Yukawa coupling parameterization as shown in Eq.(A.3),

$$y_{d_R} = \begin{pmatrix} \sin\theta_R^d \sin\phi_R^d e^{i\alpha_{d_R}^1} \\ \sin\theta_R^d \cos\phi_R^d e^{i\alpha_{d_R}^2} \\ \cos\theta_R^d e^{i\alpha_{d_R}^3} \end{pmatrix} Y_{d_R}^3 = \mathbf{e}_{R_3}^d Y_{d_R}^3, \tag{4.10}$$

$$V_{d_R} = \begin{pmatrix} \mathbf{e}_{R_1}^d & \mathbf{e}_{R_2}^d & \mathbf{e}_{R_3}^d \end{pmatrix}. \tag{4.11}$$

If we multiply Eq.(4.11) by the Hermitian conjugate of Eq.(4.10) from the left, it can be shown that the terms in Eq.(4.5) which proportional to complex vector $y_{d_R}^*$ are replaced by a real positive number $Y_{d_R}^3$ multiply with δ^{3j}. Then we can extract the mass terms from the Lagrangian as follows,

$$\begin{aligned} \mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = &-\overline{T_L}\begin{pmatrix} Y_{u_R}^3 \frac{v_R}{\sqrt{2}} & M_T \end{pmatrix}\begin{pmatrix} u_R^3 \\ T_R \end{pmatrix} - h.c. \\ &- \overline{B_L}\begin{pmatrix} Y_{d_R}^3 \frac{v_R}{\sqrt{2}} & M_B \end{pmatrix}\begin{pmatrix} (d'_R)^3 \\ B_R \end{pmatrix} - h.c. \end{aligned} \tag{4.12}$$

After doing transformation in Eq.(4.9), V_{d_R} appears as CKM-like matrix in the right-handed charged current term,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{RCC}} = -\frac{g_R}{\sqrt{2}} \sum_{i,j=1}^{3} \overline{u_R^i}\gamma^\mu (V_{d_R})^{ij}(d'_R)^j W_{R\mu}^+ - h.c. \tag{4.13}$$

From Eq.(4.12), we can see that the first and second families are decoupled from the Yukawa coupling. This lead to the fact that we have freedom to do another U(2) transformation for the right-handed quark fields. This rotation should keep the third family unchanged.

- **Step 2:** Rotate u_R^i and $(d'_R)^i$ by the following transformations,

$$u_R^i = \sum_{j=1}^{3} (\widetilde{U}_{u_R})^{ij} (\tilde{u}_R)^j, \tag{4.14}$$

$$(d'_R)^i = \sum_{j=1}^{3} (\widetilde{W}_{d_R})^{ij} (\tilde{d}'_R)^j, \tag{4.15}$$

where $\widetilde{U}_{u_R}$ and $\widetilde{W}_{d_R}$ are 3×3 unitary matrix and written in matrix form as follows,

$$\widetilde{U}_{u_R} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{U_{u_R}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.16}$$

$$\widetilde{W}_{d_R} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{W_{d_R}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.17}$$

with U_{u_R} and W_{d_R} are $2{\times}2$ unitary matrices that rotate (u_R^1, u_R^2) and $((d'_R)^1, (d'_R)^2)$, respectively. By applying the transformations in Eqs.(4.14) and (4.15) to the charged current in Eq.(4.13), we further define

$$\widetilde{V}_{d_R} = \widetilde{U}_{u_R}^{\dagger} V_{d_R} \widetilde{W}_{d_R}. \tag{4.18}$$

As shown in Eq.(B.6), by choosing $\widetilde{U}_{u_R}$ and $\widetilde{W}_{d_R}$ properly, the unphysical phases and angles in V_{d_R} are removed and $\widetilde{V}_{d_R}$ has the following matrix form,

$$\widetilde{V}_{d_R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_R^d & \sin\theta_R^d e^{i\frac{\alpha_{d_R}^3}{2}} \\ 0 & -\sin\theta_R^d e^{i\frac{\alpha_{d_R}^3}{2}} & \cos\theta_R^d e^{i\alpha_{d_R}^3} \end{pmatrix}. \tag{4.19}$$

The details of the parameterization and the procedure for the removal of unphysical phases and angles of V_{d_R} are shown in Appendix B.

- **Step 3**: Rotate $(\tilde{u}_R)^\alpha$ and $(\tilde{d}'_R)^\alpha$ by the following transformations

$$(\tilde{u}_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{W}_{T_R})^{\alpha\beta} (\tilde{u}'_R)^\beta, \tag{4.20}$$

$$(\tilde{d}'_R)^\alpha = \sum_{\beta=1}^{4} (\widetilde{W}_{B_R})^{\alpha\beta} (\tilde{d}''_R)^\beta, \tag{4.21}$$

where $\alpha = \{1, 2, 3, 4\}$, $(\tilde{u}_R)^4 = T_R$, and $(\tilde{d}'_R)^4 = B_R$. The 4×4 unitary matrices $\widetilde{W}_{T_R}$ and $\widetilde{W}_{B_R}$ are expressed as follows,

$$\widetilde{W}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & W_{T_R} \end{pmatrix}, \tag{4.22}$$

$$\widetilde{W}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & W_{B_R} \end{pmatrix}, \tag{4.23}$$

where I_2 and 0_2 are the 2×2 identity matrix and zero matrix, respectively. The 2×2 submatrices W_{T_R} and W_{B_R} rotate $((\tilde{u}_R)^3, (\tilde{u}_R)^4)$ and $((\tilde{d}'_R)^3, (\tilde{d}'_R)^4)$, respectively by following expressions,

$$(\tilde{u}_R)^i = \sum_{j=3}^{4} (W_{T_R})^{ij} (\tilde{u}'_R)^j, \tag{4.24}$$

$$(\tilde{d}'_R)^i = \sum_{j=3}^{4} (W_{B_R})^{ij} (\tilde{d}''_R)^j, \tag{4.25}$$

where $i \in \{3, 4\}$. The explicit matrix form of W_{T_R} and W_{B_R} are as follows,

$$W_{T_R} = \begin{pmatrix} \cos\theta_{T_R} & \sin\theta_{T_R} \\ -\sin\theta_{T_R} & \cos\theta_{T_R} \end{pmatrix}, \tag{4.26}$$

$$W_{B_R} = \begin{pmatrix} \cos\theta_{B_R} & \sin\theta_{B_R} \\ -\sin\theta_{B_R} & \cos\theta_{B_R} \end{pmatrix}, \tag{4.27}$$

where the mixing angles have the following expressions,

$$\cos\theta_{T_R} = \frac{M_T}{m_{u_4}}, \quad \sin\theta_{T_R} = \frac{Y^3_{u_R}}{m_{u_4}} \frac{v_R}{\sqrt{2}}, \quad \cos\theta_{B_R} = \frac{M_B}{m_{d_4}}, \quad \sin\theta_{B_R} = \frac{Y^3_{d_R}}{m_{d_4}} \frac{v_R}{\sqrt{2}},$$
$$m_{u_4} = \sqrt{\frac{(Y^3_{u_R})^2 v_R^2}{2} + M_T^2}, \qquad m_{d_4} = \sqrt{\frac{(Y^3_{d_R})^2 v_R^2}{2} + M_B^2}. \tag{4.28}$$

By using Eqs.(4.24) and (4.25), the mass terms in Eq.(4.12) transform into,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = -m_{u_4} \overline{T_L} (\tilde{u}'_R)^4 - m_{d_4} \overline{B_L} (\tilde{d}''_R)^4 - h.c. \tag{4.29}$$

The right-handed charged current in Eq.(4.13) becomes,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{RCC}} = -\frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(\tilde{u}'_R)^\alpha} \gamma^\mu (V_R^{\text{CKM}})^{\alpha\beta} (\tilde{d}''_R)^\beta W^+_{R\mu} - h.c., \tag{4.30}$$

where

$$(V_R^{\text{CKM}})^{\alpha\beta} = \sum_{i,j=1}^{3} (\widetilde{W}^\dagger_{T_R})^{\alpha i} (\tilde{V}_{d_R})^{ij} (\widetilde{W}_{B_R})^{j\beta}; \quad \alpha, \beta \in \{1, 2, 3, 4\} \tag{4.31}$$

is 4×4 "intermediate" right-handed CKM-like matrix. We call this matrix intermediate because it is not the final expression of the right-handed CKM-like matrix. The explicit matrix form of V_R^{CKM} is shown in Eq.(D.1).

In addition, we define the right-handed weak isospin current in Eq.(4.5) as

$$j^\mu_{3R} \equiv \overline{u^i_R} \gamma^\mu u^i_R - \overline{d^i_R} \gamma^\mu d^i_R. \tag{4.32}$$

Then, by following steps 1 to 3, Eq.(4.32) is transformed into,

$$\begin{aligned} j^\mu_{3R} = & \sum_{i=1}^{2} \overline{(\tilde{u}'_R)^i} \gamma^\mu (\tilde{u}'_R)^i + \sum_{j,k=3}^{4} \overline{(\tilde{u}'_R)^j} \gamma^\mu (Z_{T_R})^{jk} (\tilde{u}'_R)^k \\ & - \sum_{i=1}^{2} \overline{(\tilde{d}''_R)^i} \gamma^\mu (\tilde{d}''_R)^i - \sum_{j,k=3}^{4} \overline{(\tilde{d}''_R)^j} \gamma^\mu (Z_{B_R})^{jk} (\tilde{d}''_R)^k \end{aligned} \tag{4.33}$$

where the tree-level FCNC couplings are generated with the following definitions,

$$(Z_{T_R})^{jk} \equiv (W^\dagger_{T_R})^{j3} (W_{T_R})^{3k}, \tag{4.34}$$

$$(Z_{B_R})^{jk} \equiv (W^\dagger_{B_R})^{j3} (W_{B_R})^{3k}, \tag{4.35}$$

where $j, k \in \{3, 4\}$. Furthermore, Eqs.(4.34) and (4.35) can be expressed explicitly in 2×2 matrix form as follows,

$$Z_{T_R} = \begin{pmatrix} \cos^2\theta_{T_R} & \sin\theta_{T_R}\cos\theta_{T_R} \\ \sin\theta_{T_R}\cos\theta_{T_R} & \sin^2\theta_{T_R} \end{pmatrix}, \tag{4.36}$$

$$Z_{B_R} = \begin{pmatrix} \cos^2\theta_{B_R} & \sin\theta_{B_R}\cos\theta_{B_R} \\ \sin\theta_{B_R}\cos\theta_{B_R} & \sin^2\theta_{B_R} \end{pmatrix}. \tag{4.37}$$

These tree-level FCNC couplings are generated due to mixing between the third flavor of up and down quark with their corresponding isosinglet right-handed VLQ.

After following steps 1 to 3, the Lagrangian in Eq.(4.5) becomes,

$$
\begin{aligned}
\mathcal{L}_q =\ & \overline{q_L^i} i\gamma^\mu D_{\mathrm{SM}\mu} q_L^i + \overline{T_L} i\gamma^\mu D_{\mathrm{SM}\mu} T_L + \overline{B_L} i\gamma^\mu D_{\mathrm{SM}\mu} B_L \\
& + \overline{(\tilde{u}'_R)^\alpha} i\gamma^\mu D_{\mathrm{SM}\mu} (\tilde{u}'_R)^\alpha + \overline{(\tilde{d}''_R)^\alpha} i\gamma^\mu D_{\mathrm{SM}\mu} (\tilde{d}''_R)^\alpha \\
& - \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(\tilde{u}'_R)^\alpha} \gamma^\mu (V_R^{\mathrm{CKM}})^{\alpha\beta} (\tilde{d}''_R)^\beta W^+_{R\mu} - h.c \\
& + g' \tan\theta_R \left(\overline{q_L^i} \gamma^\mu Y_q q_L^i + \frac{2}{3} \overline{T_L} \gamma^\mu T_L - \frac{1}{3} \overline{B_L} \gamma^\mu B_L \right) Z_{R\mu} \\
& - \left\{ \frac{g_R}{2\cos\theta_R} (j^\mu_{3_R}) - g' \tan\theta_R \left(\frac{2}{3} \overline{(\tilde{u}'_R)^\alpha} \gamma^\mu (\tilde{u}'_R)^\alpha - \frac{1}{3} \overline{(\tilde{d}''_R)^\alpha} \gamma^\mu (\tilde{d}''_R)^\alpha \right) \right\} Z_{R\mu} \\
& - Y^3_{u_L} \overline{q_L^3} \tilde{\phi}_L \left(\sum_{j=3}^{4} (W_{T_R})^{4j} (\tilde{u}'_R)^j \right) - m_{u_4} \overline{T_L} (\tilde{u}'_R)^4 - h.c. \\
& - \frac{m_{u_4}}{v_R} \overline{T_L} \left[\left(\sum_{j=3}^{4} (Z_{T_R})^{4j} (\tilde{u}'_R)^j \right) (h_R + i\chi^3_R) - \sqrt{2} \left(\sum_{\beta=2}^{4} (V_R^{\mathrm{CKM}})^{4\beta} (\tilde{d}''_R)^\beta \right) \chi^+_R \right] - h.c. \\
& - \overline{q_L^i} y^i_{d_L} \phi_L \left(\sum_{j=3}^{4} (W_{B_R})^{4j} (\tilde{d}''_R)^j \right) - m_{d_4} \overline{B_L} (\tilde{d}''_R)^4 - h.c. \\
& - \frac{m_{d_4}}{v_R} \overline{B_L} \left[\left(\sum_{j=3}^{4} (Z_{B_R})^{4j} (\tilde{d}''_R)^j \right) (h_R - i\chi^3_R) + \sqrt{2} \left(\sum_{\beta=2}^{4} (V_R^{\mathrm{CKM}\dagger})^{4\beta} (\tilde{u}'_R)^\beta \right) \chi^-_R \right] - h.c.
\end{aligned}
\tag{4.38}
$$

where $i = \{1,2,3\}$, $\alpha = \{1,2,3,4\}$ and the definition of $W_{T_R}, W_{B_R}, m_{u_4}, m_{d_4}, V_R^{\mathrm{CKM}}$, Z_{T_R}, and Z_{B_R} are written in Eqs.(4.26),(4.27),(4.28),(D.1),(4.36), and (4.37) respectively. One can show that Lagrangian in Eq.(4.38) is invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge symmetry.

4.2 $\mathbf{SU(2)_L \times U(1)_Y \to U(1)_{em}}$

In this stage, the neutral scalar component of $\mathrm{SU(2)_L}$ Higgs doublet acquires non-zero vev and is expanded around vev's as follows,

$$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_L^+ \\ v_L + h_L + i\chi_L^3 \end{pmatrix}, \tag{4.39}$$

where v_L is the non-zero vev, h_L is the neutral CP-even state and χ_L^3 is the neutral CP-odd state. The charged component is defined as, $\chi_L^+ = \frac{1}{\sqrt{2}}(\chi_L^1 + i\chi_L^2)$. In addition, we rotate the gauge fields with the following transformation,

$$\begin{pmatrix} B_\mu \\ W_{L\mu}^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{L\mu} \end{pmatrix}, \tag{4.40}$$

where the mixing angles are defined as,

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L^2 + g'^2}}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g_L^2 + g'^2}}. \tag{4.41}$$

We also define the electromagnetic $\mathrm{U(1)_{em}}$ gauge coupling as,

$$e = g'\cos\theta_W = g_L \sin\theta_W. \tag{4.42}$$

After this breaking, the Lagrangian in Eq.(4.38) becomes

$$\begin{aligned}
\mathcal{L}_q = {} & \overline{u_L^i} i\gamma^\mu D_{\mathrm{em}\mu} u_L^i + \overline{T_L} i\gamma^\mu D_{\mathrm{em}\mu} T_L + \overline{d_L^i} i\gamma^\mu D_{\mathrm{em}\mu} d_L^i + \overline{B_L} i\gamma^\mu D_{\mathrm{em}\mu} B_L \\
& + \overline{(\tilde{u}_R')^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\tilde{u}_R')^\alpha + \overline{(\tilde{d}_R'')^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\tilde{d}_R'')^\alpha \\
& - \frac{g_L}{\sqrt{2}} \overline{u_L^i} \gamma^\mu d_L^i W_{L\mu}^+ - h.c. \\
& - \left(\frac{g_L}{2\cos\theta_W} (j_{3L}^\mu) - e\tan\theta_W (j_{\mathrm{em}}^\mu) \right) Z_{L\mu} \\
& - \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(\tilde{u}_R')^\alpha} \gamma^\mu (V_R^{\mathrm{CKM}})^{\alpha\beta} (\tilde{d}_R'')^\beta W_{R\mu}^+ - h.c \\
& - \left\{ \frac{g_R}{2\cos\theta_R} (j_{3_R}^\mu) - g'\tan\theta_R \left((j_{\mathrm{em}}^\mu) - \frac{1}{2}(j_{3_L}^\mu) \right) \right\} Z_{R\mu} \\
& - Y_{u_L}^3 \frac{v_L}{\sqrt{2}} \overline{u_L^3} \left(\sum_{j=3}^{4} (W_{T_R})^{4j} (\tilde{u}_R')^j \right) - m_{u_4} \overline{T_L} (\tilde{u}_R')^4 - h.c.
\end{aligned}$$

$$
\begin{aligned}
&- Y^3_{u_L}\left(\frac{1}{\sqrt{2}}\overline{u^3_L}\left(\sum_{j=3}^{4}(W_{T_R})^{4j}(\tilde{u}'_R)^j\right)(h_L - i\chi^3_L) - \overline{d^3_L}\left(\sum_{j=3}^{4}(W_{T_R})^{4j}(\tilde{u}'_R)^j\right)\chi^-_L\right) - h.c.\\
&- \frac{m_{u_4}}{v_R}\overline{T_L}\left[\left(\sum_{j=3}^{4}(Z_{T_R})^{4j}(\tilde{u}'_R)^j\right)(h_R + i\chi^3_R) - \sqrt{2}\left(\sum_{\beta=2}^{4}(V_R^{\text{CKM}})^{4\beta}(\tilde{d}''_R)^\beta\right)\chi^+_R\right] - h.c.\\
&- y^i_{d_L}\frac{v_L}{\sqrt{2}}\overline{d^i_L}\left(\sum_{j=3}^{4}(W_{B_R})^{4j}(\tilde{d}''_R)^j\right) - m_{d_4}\overline{B_L}(\tilde{d}''_R)^4 - h.c.\\
&- y^i_{d_L}\left(\frac{1}{\sqrt{2}}\overline{d^i_L}\left(\sum_{j=3}^{4}(W_{B_R})^{4j}(\tilde{d}''_R)^j\right)(h_L + i\chi^3_L) + \overline{u^i_L}\left(\sum_{j=3}^{4}(W_{B_R})^{4j}(\tilde{d}''_R)^j\right)\chi^+_L\right) - h.c.\\
&- \frac{m_{d_4}}{v_R}\overline{B_L}\left[\left(\sum_{j=3}^{4}(Z_{B_R})^{4j}(\tilde{d}''_R)^j\right)(h_R - i\chi^3_R) + \sqrt{2}\left(\sum_{\beta=2}^{4}(V_R^{\text{CKM}\dagger})^{4\beta}(\tilde{u}'_R)^\beta\right)\chi^-_R\right] - h.c.,
\end{aligned}
\tag{4.43}
$$

where the covariant derivatives are,

$$D_{\text{em}\mu}f'_u = \left(\partial_\mu + \frac{2}{3}ieA_\mu\right)f'_u, \tag{4.44}$$

$$D_{\text{em}\mu}f'_d = \left(\partial_\mu - \frac{1}{3}ieA_\mu\right)f'_d. \tag{4.45}$$

The left-handed weak isospin current and electromagnetic current are

$$j^\mu_{3L} = \overline{u^i_L}\gamma^\mu u^i_L - \overline{d^i_L}\gamma^\mu d^i_L, \tag{4.46}$$

$$
\begin{aligned}
j^\mu_{\text{em}} &= \frac{2}{3}\left(\overline{u^i_L}\gamma^\mu u^i_L + \overline{T_L}\gamma^\mu T_L + \overline{(\tilde{u}'_R)^\alpha}\gamma^\mu(\tilde{u}'_R)^\alpha\right)\\
&\quad - \frac{1}{3}\left(\overline{d^i_L}\gamma^\mu d^i_L + \overline{B_L}\gamma^\mu B_L + (\tilde{d}''_R)^\alpha\gamma^\mu(\tilde{d}''_R)^\alpha\right),
\end{aligned}
\tag{4.47}
$$

where $f'_u \in \{u^i_L, (\tilde{u}'_R)^\alpha, T_L\}$, $f'_d \in \left\{d^i_L, (\tilde{d}''_R)^\alpha, B_L\right\}$, $i \in \{1,2,3\}$, $\alpha \in \{1,2,3,4\}$ and the right-handed weak isospin current j^μ_{3R} is written in Eq.(4.33). Our main goal is to obtain the mass eigenvalues of the top and bottom quarks and their heavy partners. The following steps outline our approach: (the number of counting steps continues from the previous section)

- **Step 4:** Rotate d^i_L by following transformation,

$$d^i_L = (V_{d_L})^{ij}(d'_L)^j, \tag{4.48}$$

where V_{d_L} is 3×3 unitary matrix, associated with the parameterization of

Yukawa couplings as demonstrated in Eq.(A.3),

$$y_{d_L} = \begin{pmatrix} \sin\theta_L^d \sin\phi_L^d e^{i\alpha^1_{d_L}} \\ \sin\theta_L^d \cos\phi_L^d e^{i\alpha^2_{d_L}} \\ \cos\theta_L^d e^{i\alpha^3_{d_L}} \end{pmatrix} Y^3_{d_L} = \mathbf{e}^d_{L_3} Y^3_{d_L}, \tag{4.49}$$

$$V_{d_L} = \begin{pmatrix} \mathbf{e}^d_{L_1} & \mathbf{e}^d_{L_2} & \mathbf{e}^d_{L_3} \end{pmatrix}. \tag{4.50}$$

If we multiply Eq.(4.49) by the hermitian conjugate of Eq.(4.50) from the left, it can be shown that the terms in Eq.(4.43) that proportional to the complex vector y_{d_L} are replaced by the product of a real positive number $Y^3_{d_L}$ and δ^{j3}. The mass terms can be extracted from the Lagrangian and written as follows,

$$\begin{aligned} \mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = &- \begin{pmatrix} \overline{u^3_L} & \overline{T_L} \end{pmatrix} \begin{pmatrix} Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{43} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix} \begin{pmatrix} (\tilde{u}'_R)^3 \\ (\tilde{u}'_R)^4 \end{pmatrix} - h.c. \\ &- \begin{pmatrix} \overline{(d'_L)^3} & \overline{B_L} \end{pmatrix} \begin{pmatrix} Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{43} & Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix} \begin{pmatrix} (\tilde{d}''_R)^3 \\ (\tilde{d}''_R)^4 \end{pmatrix} - h.c. \end{aligned} \tag{4.51}$$

Additionally, an important outcome of the transformation in Eq.(4.48) is that V_{d_L} appears as CKM-like matrix in the left-handed charged current term,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{LCC}} = -\frac{g_L}{\sqrt{2}} \sum_{i,j=1}^{3} \overline{u^i_L} \gamma^\mu (V_{d_L})^{ij} (d'_L)^j W^+_{L\mu} - h.c. \tag{4.52}$$

From Eq.(4.51), we have freedom to do another U(2) transformation to the left-handed quark fields with keep the third family unchanged.

- **Step 5:** Rotate u^i_L and $(d'_L)^i$ by the following transformations

$$u^i_L = \sum_{j=1}^{3} (\widetilde{U}_{u_L})^{ij} (\tilde{u}_L)^j, \tag{4.53}$$

$$(d'_L)^i = \sum_{j=1}^{3} (\widetilde{W}_{d_L})^{ij} (\tilde{d}'_L)^j, \tag{4.54}$$

where $\widetilde{U}_{u_L}$ and $\widetilde{W}_{d_L}$ are 3×3 unitary matrices and written in the matrix form as follows,

$$\widetilde{U}_{u_L} = \begin{pmatrix} \begin{matrix} U_{u_L} \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix}, \tag{4.55}$$

$$\widetilde{W}_{d_L} = \begin{pmatrix} \begin{matrix} W_{d_L} \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix}, \tag{4.56}$$

with U_{u_L} and W_{d_L} are 2×2 unitary matrices which rotate (u_L^1, u_L^2) and $((d_L')^1, (d_L')^2)$, respectively. By applying the transformations in Eqs.(4.53) and (4.54) to the charged current in Eq.(4.52), we further define

$$\widetilde{V}_{d_L} = \widetilde{U}_{u_L}^{\dagger} V_{d_L} \widetilde{W}_{d_L}. \tag{4.57}$$

By properly choosing $\widetilde{U}_{u_L}$ and $\widetilde{W}_{d_L}$, the unphysical phases and angles in V_{d_L} are eliminated, resulting in $\widetilde{V}_{d_L}$, which has the same matrix form as Eq.(4.19), with the R index replaced by L.

- **Step 6:** Rotate $(\tilde{u}_L)^\alpha$, $(\tilde{u}_R')^\alpha$, $(\tilde{d}_L')^\alpha$, and $(\tilde{d}_R'')^\alpha$ into the mass basis by the following transformations,

$$(\tilde{u}_L)^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{T_L})^{\alpha\beta} (u_L^m)^\beta, \tag{4.58}$$

$$(\tilde{u}_R')^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{T_R})^{\alpha\beta} (u_R^m)^\beta, \tag{4.59}$$

$$(\tilde{d}_L')^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{B_L})^{\alpha\beta} (d_L^m)^\beta, \tag{4.60}$$

$$(\tilde{d}_R'')^\alpha = \sum_{\beta=1}^{4} (\widetilde{K}_{B_R})^{\alpha\beta} (d_R^m)^\beta, \tag{4.61}$$

where $\alpha \in \{1, 2, 3, 4\}$, $(\tilde{u}_L)^4 = T_L$ and $(\tilde{d}_L')^4 = B_L$. The 4×4 unitary matrices $\widetilde{K}_{T_L}$, $\widetilde{K}_{T_R}$, $\widetilde{K}_{B_L}$, and $\widetilde{K}_{B_R}$ are expressed as follows,

$$\widetilde{K}_{T_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_L} \end{pmatrix}, \tag{4.62}$$

$$\widetilde{K}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_R} \end{pmatrix}, \tag{4.63}$$

$$\widetilde{K}_{B_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_L} \end{pmatrix}, \tag{4.64}$$

$$\widetilde{K}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_R} \end{pmatrix}, \tag{4.65}$$

where I_2 and 0_2 are the 2×2 identity matrix and zero matrix, respectively. The 2×2 unitary submatrices $K_{T_L}, K_{T_R}, K_{B_L}$, and K_{B_R} rotate $((\tilde{u}_L)^3, (\tilde{u}_L)^4)$, $((\tilde{u}'_R)^3, (\tilde{u}'_R)^4)$, $((\tilde{d}'_L)^3, (\tilde{d}'_L)^4)$ and $((\tilde{d}''_R)^3, (\tilde{d}''_R)^4)$ pairs, respectively where the explicit forms are written in Eqs.(C.19), (C.20), (C.24), and (C.25).

We denote the top and bottom quarks as the third component of the fields in the mass basis, while the heavy top and bottom quarks are the fourth component. We can diagonalize the mass matrices in Eq.(4.51), which are defined as

$$\mathbb{M}_t \equiv \begin{pmatrix} Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{43} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix}, \tag{4.66}$$

$$\mathbb{M}_b \equiv \begin{pmatrix} Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{43} & Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix}, \tag{4.67}$$

by using the appropriate submatrices in Eqs.(4.58) - (4.61) resulting in:

$$K^\dagger_{T_L} \mathbb{M}_t K_{T_R} = (m_t^{\text{diag}}) = \text{diag}(m_t, m_{t'}), \tag{4.68}$$

$$K^\dagger_{B_L} \mathbb{M}_b K_{B_R} = (m_b^{\text{diag}}) = \text{diag}(m_b, m_{b'}). \tag{4.69}$$

From this diagonalization process, we obtain,

$$m_{t(b)} = -\frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} - m_{u(d)_L})^2}}{2} + \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} + m_{u(d)_L})^2}}{2}, \tag{4.70}$$

$$m_{t'(b')} = \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} - m_{u(d)_L})^2}}{2} + \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} + m_{u(d)_L})^2}}{2}, \tag{4.71}$$

where $m_{t(b)}$ and $m_{t'(b')}$ are mass of top(bottom) and heavy-top(bottom) exact mass eigenvalues, respectively. The definitions of $m_{u_L}, m_{u_R}, m_{d_L}$, and m_{d_R} are shown in Eqs.(C.16) and (C.23). The diagonalization procedure is explained in Appendix C. The mass eigenvalues for t and t' in Eqs.(4.70) and (4.71) agree with Eq.(10) of Ref.[36].

Moreover, the left-handed and right-handed charged currents in Eqs.(4.52) and (4.30), now become

$$\begin{aligned}\mathcal{L}_q \supset \mathcal{L}_{\mathrm{CC}} &= \mathcal{L}_{\mathrm{LCC}} + \mathcal{L}_{\mathrm{RCC}} \\ &= -\frac{g_L}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(u_L^m)^\alpha} \gamma^\mu (\mathcal{V}_L^{\mathrm{CKM}})^{\alpha\beta} (d_L^m)^\beta W_{L\mu}^+ - h.c. \\ &\quad - \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(u_R^m)^\alpha} \gamma^\mu (\mathcal{V}_R^{\mathrm{CKM}})^{\alpha\beta} (d_R^m)^\beta W_{R\mu}^+ - h.c., \end{aligned} \tag{4.72}$$

where

$$(\mathcal{V}_L^{\mathrm{CKM}})^{\alpha\beta} = \sum_{i,j=1}^{3} (\widetilde{K}_{T_L}^\dagger)^{\alpha i} (\widetilde{V}_{d_L})^{ij} (\widetilde{K}_{B_L})^{j\beta}, \tag{4.73}$$

$$(\mathcal{V}_R^{\mathrm{CKM}})^{\alpha\beta} = \sum_{\rho,\eta=1}^{4} (\widetilde{K}_{T_R}^\dagger)^{\alpha\rho} (V_R^{\mathrm{CKM}})^{\rho\eta} (\widetilde{K}_{B_R})^{\eta\beta} \tag{4.74}$$

are the left-handed and right-handed CKM-like matrices. The matrix forms are shown in Eqs.(D.3) and (D.5), respectively. However, there are some unphysical phases which can be eliminated from the left-handed and right-handed CKM-like matrices. We have the freedom to rephase the quark fields with the following transformations,

$$(u_{L(R)}^m)^\alpha = (\theta_{u_{L(R)}})^\alpha \delta^{\alpha\beta} (\hat{u}_{L(R)}^m)^\beta, \tag{4.75}$$

$$(d_{L(R)}^m)^\alpha = (\theta_{d_{L(R)}})^\alpha \delta^{\alpha\beta} (\hat{d}_{L(R)}^m)^\beta, \tag{4.76}$$

where,

$$\theta_{u_{L(R)}} = \mathrm{diag}(e^{i\theta_{u_{L(R)1}}}, e^{i\theta_{u_{L(R)2}}}, e^{i\theta_{u_3}}, e^{i\theta_{u_4}}), \tag{4.77}$$

$$\theta_{d_{L(R)}} = \mathrm{diag}(e^{i\theta_{d_{L(R)1}}}, e^{i\theta_{d_{L(R)2}}}, e^{i\theta_{d_3}}, e^{i\theta_{d_4}}). \tag{4.78}$$

After rephasing the quark fields, the left-handed and right-handed CKM-like matrices become the final versions denoted as $\hat{\mathcal{V}}_L^{\mathrm{CKM}}$ and $\hat{\mathcal{V}}_R^{\mathrm{CKM}}$, whose matrix forms are as follows,

$$\hat{\mathcal{V}}_L^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_L^d} & s_{\theta_L^d} c_{\phi_{B_L}} & -s_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & -c_{\phi_{T_L}} s_{\theta_L^d} & c_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & -c_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta_L^d} & -s_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & s_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \end{pmatrix}, \tag{4.79}$$

$$\hat{\mathcal{V}}_R^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & -s_{\theta_R^d} c_{\beta_{B_R}} e^{i\frac{\delta}{2}} & s_{\theta_R^d} s_{\beta_{B_R}} e^{i\frac{\delta}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & c_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & -c_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \\ 0 & -s_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & -s_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & s_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \end{pmatrix}, \tag{4.80}$$

where

$$\begin{aligned} c_{\theta_L^d} &= \cos\theta_L^d, \quad s_{\theta_L^d} = \sin\theta_L^d, \quad c_{\phi_{T_L}} = \cos\phi_{T_L}, \\ s_{\phi_{T_L}} &= \sin\phi_{T_L}, \quad c_{\phi_{B_L}} = \cos\phi_{B_L}, \quad s_{\phi_{B_L}} = \sin\phi_{B_L}, \\ c_{\theta_R^d} &= \cos\theta_R^d, \quad s_{\theta_R^d} = \sin\theta_R^d, \quad c_{\beta_{T_R}} = \cos\beta_{T_R}, \\ s_{\beta_{T_R}} &= \sin\beta_{T_R}, \quad c_{\beta_{B_R}} = \cos\beta_{B_R}, \quad s_{\beta_{B_R}} = \sin\beta_{B_R}, \\ \beta_{T_R} &= \theta_{T_R} - \phi_{T_R}, \quad \beta_{B_R} = \theta_{B_R} - \phi_{B_R}, \quad \delta = \alpha_{d_R}^3 - \alpha_{d_L}^3. \end{aligned} \tag{4.81}$$

The number of CP violating phase in this model is one. This agrees with the result in Ref.[30] for the $N = 1$ case. The details of the rephasing process is explained in Appendix D.

In addition, the final expression of the left-handed FCNC couplings, which appears in the left-handed weak isospin current in Eq.(4.46), are defined as follows,

$$(\mathcal{Z}_{T_L})^{ij} \equiv (K_{T_L}^\dagger)^{i3}(K_{T_L})^{3j}, \tag{4.82}$$
$$(\mathcal{Z}_{B_L})^{ij} \equiv (K_{B_L}^\dagger)^{i3}(K_{B_L})^{3j}, \tag{4.83}$$

where $i, j \in \{3, 4\}$. These have explicit matrix form as follows,

$$\mathcal{Z}_{T_L} = \begin{pmatrix} \cos^2\phi_{T_L} & -\sin\phi_{T_L}\cos\phi_{T_L} \\ -\sin\phi_{T_L}\cos\phi_{T_L} & \sin^2\phi_{T_L} \end{pmatrix}, \tag{4.84}$$
$$\mathcal{Z}_{B_L} = \begin{pmatrix} \cos^2\phi_{B_L} & -\sin\phi_{B_L}\cos\phi_{B_L} \\ -\sin\phi_{B_L}\cos\phi_{B_L} & \sin^2\phi_{B_L} \end{pmatrix}. \tag{4.85}$$

Similarly, for the right-handed weak isospin current from Eq.(4.33), the intermediate right-handed FCNC couplings transforms into the final expressions as,

$$(\mathcal{Z}_{T_R})^{ij} \equiv \sum_{k,l=3}^{4} (K_{T_R}^{\dagger})^{ik} (Z_{T_R})^{kl} (K_{T_R})^{lj}, \tag{4.86}$$

$$(\mathcal{Z}_{B_R})^{ij} \equiv \sum_{k,l=3}^{4} (K_{B_R}^{\dagger})^{ik} (Z_{B_R})^{kl} (K_{B_R})^{lj}, \tag{4.87}$$

where $i, j \in \{3, 4\}$. These can be expressed in matrix form as follows,

$$\mathcal{Z}_{T_R} = \begin{pmatrix} \cos^2\beta_{T_R} & -\sin\beta_{T_R}\cos\beta_{T_R} \\ -\sin\beta_{T_R}\cos\beta_{T_R} & \sin^2\beta_{T_R} \end{pmatrix}, \tag{4.88}$$

$$\mathcal{Z}_{B_R} = \begin{pmatrix} \cos^2\beta_{B_R} & -\sin\beta_{B_R}\cos\beta_{B_R} \\ -\sin\beta_{B_R}\cos\beta_{B_R} & \sin^2\beta_{B_R} \end{pmatrix}. \tag{4.89}$$

with $\beta_{T_R} = \theta_{T_R} - \phi_{T_R}$ and $\beta_{B_R} = \theta_{B_R} - \phi_{B_R}$.

Finally, we obtain the expression of the Lagrangian for the quark and Yukawa interaction after following all steps as follows,

$$\begin{aligned}
\mathcal{L}_q = & \sum_{\alpha=1}^{4} \overline{(\hat{u}^m)^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\hat{u}^m)^\alpha + \sum_{\alpha=1}^{4} \overline{(\hat{d}^m)^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\hat{d}^m)^\alpha \\
& - \frac{g_L}{\sqrt{2}} \left(\sum_{\alpha,\beta=1}^{4} \overline{(\hat{u}_L^m)^\alpha} \gamma^\mu (\hat{\mathcal{V}}_L^{\mathrm{CKM}})^{\alpha\beta} (\hat{d}_L^m)^\beta W_{L\mu}^+ + h.c. \right) \\
& - \left(\frac{g_L}{2\cos\theta_W} (j_{3L}^\mu) - e\tan\theta_W (j_{\mathrm{em}}^\mu) \right) Z_{L\mu} \\
& - \frac{g_R}{\sqrt{2}} \left(\sum_{\alpha,\beta=1}^{4} \overline{(\hat{u}_R^m)^\alpha} \gamma^\mu (\hat{\mathcal{V}}_R^{\mathrm{CKM}})^{\alpha\beta} (\hat{d}_R^m)^\beta W_{R\mu}^+ + h.c \right) \\
& - \left\{ \frac{g_R}{2\cos\theta_R} (j_{3_R}^\mu) - g' \tan\theta_R \left((j_{\mathrm{em}}^\mu) - \frac{1}{2} (j_{3_L}^\mu) \right) \right\} Z_{R\mu} \\
& - \sum_{j=3}^{4} (m_t^{\mathrm{diag}})^{jj} \overline{(\hat{u}^m)^j} (\hat{u}^m)^j - \sum_{j=3}^{4} (m_b^{\mathrm{diag}})^{jj} \overline{(\hat{d}^m)^j} (\hat{d}^m)^j \\
& - \frac{1}{v_L} \sum_{k,i=3}^{4} \Big((\mathcal{Z}_{T_L} m_t^{\mathrm{diag}})^{ki} \overline{(\hat{u}_L^m)^k} (\hat{u}_R^m)^i + (m_t^{\mathrm{diag}} \mathcal{Z}_{T_L})^{ki} \overline{(\hat{u}_R^m)^k} (\hat{u}_L^m)^i \\
& \qquad + (\mathcal{Z}_{B_L} m_b^{\mathrm{diag}})^{ki} \overline{(\hat{d}_L^m)^k} (\hat{d}_R^m)^i + (m_b^{\mathrm{diag}} \mathcal{Z}_{B_L})^{ki} \overline{(\hat{d}_R^m)^k} (\hat{d}_L^m)^i \Big) h_L
\end{aligned}$$

$$
\begin{aligned}
&-\frac{\sqrt{2}}{v_L}\left[\sum_{k=3}^{4}\sum_{\alpha=2}^{4}\left(\overline{(\hat{u}_L^m)^\alpha}(\hat{\mathcal{V}}_L^{\rm CKM}m_b^{\rm diag})^{\alpha k}(\hat{d}_R^m)^k-\overline{(\hat{u}_R^m)^k}(m_t^{\rm diag}\hat{\mathcal{V}}_L^{\rm CKM})^{k\alpha}(\hat{d}_L^m)^\alpha\right)\chi_L^+ + h.c.\right]\\
&+\frac{1}{v_L}\sum_{k,i=3}^{4}\left((\mathcal{Z}_{T_L}m_t^{\rm diag})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i-(m_t^{\rm diag}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\right.\\
&\qquad\qquad\left.-(\mathcal{Z}_{B_L}m_b^{\rm diag})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i+(m_b^{\rm diag}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\right)i\chi_L^3\\
&-\frac{1}{v_R}\sum_{k,i=3}^{4}\left(((1-\mathcal{Z}_{T_L})m_t^{\rm diag}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i+(\mathcal{Z}_{T_R}m_t^{\rm diag}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\right.\\
&\quad\left.+((1-\mathcal{Z}_{B_L})m_b^{\rm diag}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i+(\mathcal{Z}_{B_R}m_b^{\rm diag}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\right)h_R\\
&-\frac{\sqrt{2}}{v_R}\left[\sum_{k=3}^{4}\sum_{\alpha=2}^{4}\left(\overline{(\hat{u}_R^m)^\alpha}(\hat{\mathcal{V}}_R^{\rm CKM}m_b^{\rm diag}(1-\mathcal{Z}_{B_L}))^{\alpha k}(\hat{d}_L^m)^k\right.\right.\\
&\qquad\qquad\left.\left.-\overline{(\hat{u}_L^m)^k}((1-\mathcal{Z}_{T_L})m_t^{\rm diag}\hat{\mathcal{V}}_R^{\rm CKM})^{k\alpha}(\hat{d}_R^m)^\alpha\right)\chi_R^+ + h.c.\right]\\
&+\frac{1}{v_R}\sum_{k,i=3}^{4}\left(((1-\mathcal{Z}_{B_L})m_b^{\rm diag}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i-(\mathcal{Z}_{B_R}m_b^{\rm diag}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\right.\\
&\left.-((1-\mathcal{Z}_{T_L})m_t^{\rm diag}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i+(\mathcal{Z}_{T_R}m_t^{\rm diag}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\right)i\chi_R^3,
\end{aligned}
\tag{4.90}
$$

where we define $\hat{u}^m = \hat{u}_L^m + \hat{u}_R^m$ and $\hat{d}^m = \hat{d}_L^m + \hat{d}_R^m$. As mentioned before, the top and bottom quarks are the third component of the fields in the mass basis, while the heavy partners are the fourth component,

$$
(\hat{u}^m_{L(R)})^3 = t_{L(R)},\quad (\hat{u}^m_{L(R)})^4 = t'_{L(R)},\quad (\hat{d}^m_{L(R)})^3 = b_{L(R)},\quad (\hat{d}^m_{L(R)})^4 = b'_{L(R)}. \tag{4.91}
$$

The left-handed, right-handed weak-isospin, and electromagnetic current in Eq.(4.90) now have following final expressions,

$$
\begin{aligned}
j^\mu_{3L} &= \sum_{i=1}^{2}\overline{(\hat{u}_L^m)^i}\gamma^\mu(\hat{u}_L^m)^i+\sum_{l,j=3}^{4}\overline{(\hat{u}_L^m)^l}\gamma^\mu(\mathcal{Z}_{T_L})^{lj}(\hat{u}_L^m)^j\\
&-\sum_{i=1}^{2}\overline{(\hat{d}_L^m)^i}\gamma^\mu(\hat{d}_L^m)^i-\sum_{l,j=3}^{4}\overline{(\hat{d}_L^m)^l}\gamma^\mu(\mathcal{Z}_{B_L})^{lj}(\hat{d}_L^m)^j,
\end{aligned}
\tag{4.92}
$$

$$
\begin{aligned}
j^\mu_{3R} &= \sum_{i=1}^{2}\overline{(\hat{u}_R^m)^i}\gamma^\mu(\hat{u}_R^m)^i+\sum_{l,j=3}^{4}\overline{(\hat{u}_R^m)^l}\gamma^\mu(\mathcal{Z}_{T_R})^{lj}(\hat{u}_R^m)^j\\
&-\sum_{i=1}^{2}\overline{(\hat{d}_R^m)^i}\gamma^\mu(\hat{d}_R^m)^i+\sum_{l,j=3}^{4}\overline{(\hat{d}_R^m)^l}\gamma^\mu(\mathcal{Z}_{B_R})^{lj}(\hat{d}_R^m)^j,
\end{aligned}
\tag{4.93}
$$

$$j^{\mu}_{\rm em} = \frac{2}{3}\sum_{\alpha=1}^{4}\overline{(\hat{u}^m)^{\alpha}}\gamma^{\mu}(\hat{u}^m)^{\alpha} - \frac{1}{3}\sum_{\alpha=1}^{4}\overline{(\hat{d}^m)^{\alpha}}\gamma^{\mu}(\hat{d}^m)^{\alpha}, \tag{4.94}$$

where the definitions and matrix forms of FCNC couplings are shown in Eqs.(4.82)-(4.89). It should be noted that the Lagrangian written in Eq.(4.90) can be expressed in the mass eigenstates of the Higgs and Z bosons. We will discuss this further in chapter 5.

Chapter 5

Higgs Sector

In this chapter, we derived the kinetic terms and potential of Higgs, which are contained in Eq.(3.4). In the same way as in chapter 4, we derive it step by step from the $\mathrm{SU(2)_R \times U(1)_{Y'}}$ breaking into $\mathrm{U(1)_Y}$ and finally $\mathrm{SU(2)_L \times U(1)_Y}$ breaking into $\mathrm{U(1)_{em}}$.

5.1 $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

This stage occurs after the $\mathrm{SU(2)_R}$ Higgs doublet acquires non-zero vev and is parameterized as written in Eq.(4.1). Additionally, there is mixing between B'_μ and $W^3_{R\mu}$ into B_μ and $Z_{R\mu}$, following the transformation shown in Eq.(4.2). We will analyze the kinetic terms and potential separately. Furthermore, we classify the terms based on the number of the fields in the term as linear, quadratic, cubic, and quartic. The gauge fields inside the covariant derivatives are not counted as fields in this classification.

5.1.1 Kinetic Terms

The kinetic terms in Eq.(3.4) become,

$$\begin{aligned}\mathcal{L}_H \supset \mathcal{L}_{\text{kin}} = {} & (D^\mu_{\text{SM}}\phi_L)^\dagger(D_{\text{SM}\mu}\phi_L) \\ & - ig'Y_\phi \tan\theta_R Z_{R\mu}\{(D_{\text{SM}\mu}\phi_L)^\dagger\phi_L - \phi_L^\dagger(D^\mu_{\text{SM}}\phi_L)\} \\ & + g'^2Y_\phi^2\tan^2\theta_R Z^\mu_R Z_{R\mu}\phi_L^\dagger\phi_L \\ & + (D^\mu_{\text{SM}}\chi_R^-)(D_{\text{SM}\mu}\chi_R^+) + i\frac{g_Rv_R}{2}\{W_R^{+\mu}(D_{\text{SM}\mu}\chi_R^-) - W_R^{-\mu}(D_{\text{SM}\mu}\chi_R^+)\} \\ & + \frac{g_R^2v_R^2}{4}W_R^{-\mu}W^+_{R\mu}\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}(\partial_\mu h_R)^2+\frac{1}{2}\left(\partial_\mu\chi_R^3-\frac{g_R v_R}{2\cos\theta_R}Z_{R\mu}\right)^2\\
&-\frac{g_R}{2}\chi_R^3\{(W_R^{+\mu}D_{\mathrm{SM}\mu}\chi_R^-)+W_R^{-\mu}(D_{\mathrm{SM}\mu}\chi_R^+)\}\\
&+i\frac{g_R}{2}\{W_R^{+\mu}(D_{\mathrm{SM}\mu}\chi_R^-)-W_R^{-\mu}(D_{\mathrm{SM}\mu}\chi_R^+)\}h_R+\frac{g_R^2 v_R}{2}h_R W_R^{-\mu}W_{R\mu}^+\\
&+i\frac{g_R}{2}\frac{\cos 2\theta_R}{\cos\theta_R}Z_R^\mu\{\chi_R^+(D_{\mathrm{SM}\mu}\chi_R^-)-\chi_R^-(D_{\mathrm{SM}\mu}\chi_R^+)\}\\
&+\frac{g_R^2 v_R}{4}\left(\frac{\cos 2\theta_R-1}{\cos\theta_R}\right)(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+)Z_R^\mu\\
&+\frac{g_R}{2}(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+)\partial^\mu\chi_R^3-i\frac{g_R}{2}(W_{R\mu}^+\chi_R^--W_{R\mu}^-\chi_R^+)\partial^\mu h_R\\
&+\frac{g_R}{2\cos\theta_R}\{\chi_R^3(\partial^\mu h_R)-(\partial^\mu\chi_R^3)h_R\}Z_{R\mu}+\left(\frac{g_R}{2\cos\theta_R}\right)^2 v_R h_R Z_R^\mu Z_{R\mu}\\
&+\frac{g_R^2(\cos 2\theta_R)-1}{4\cos\theta_R}\{(W_{R\mu}^+\chi_R^--W_{R\mu}^-\chi_R^+)i\chi_R^3+(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+)h_R\}Z_R^\mu\\
&+\frac{g_R^2}{4}\left(\frac{1}{2\cos^2\theta_R}Z_{R\mu}Z_R^\mu+W_R^{+\mu}W_{R\mu}^-\right)((\chi_R^3)^2+h_R^2)\\
&+\frac{g_R^2}{2}\left(W_R^{+\mu}W_{R\mu}^-+\frac{\cos^2 2\theta_R}{2\cos^2\theta_R}Z_R^\mu Z_{R\mu}\right)(\chi_R^-\chi_R^+),
\end{aligned}
\tag{5.1}
$$

where

$$
D_{\mathrm{SM}\mu}\phi_L=\left(\partial_\mu+ig_L W_{L\mu}^a\frac{\tau_L^a}{2}+ig'Y_\phi B_\mu\right)\phi_L, \tag{5.2}
$$

$$
D_{\mathrm{SM}\mu}\chi_R^+=(\partial_\mu+ig'B_\mu)\chi_R^+ \tag{5.3}
$$

are the definition of SM covariant derivatives for ϕ_L and χ_R^+ respectively.

5.1.2 Higgs Potential

The Higgs potential which is written in Eq.(3.6) now becomes,

$$
\begin{aligned}
V(\phi_L,\phi_R)=&(\mu_L^2+\lambda_{LR}v_R^2)\phi_L^\dagger\phi_L+\lambda_L(\phi_L^\dagger\phi_L)^2\\
&+2\lambda_{LR}v_R(\phi_L^\dagger\phi_L)h_R+2\lambda_{LR}(\phi_L^\dagger\phi_L)\left(\chi_R^-\chi_R^++\frac{1}{2}(h_R^2+(\chi_R^3)^2)\right)\\
&+\frac{\mu_R^2}{2}v_R^2+\frac{\lambda_R}{4}v_R^4\\
&+h_R(\mu_R^2 v_R+\lambda_R v_R^3)\\
&+\frac{h_R^2}{2}(\mu_R^2+3\lambda_R v_R^2)+(\mu_R^2+\lambda_R v_R^2)\left(\chi_R^-\chi_R^++\frac{1}{2}(\chi_R^3)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ 2v_R h_R \lambda_R \left(\chi_R^- \chi_R^+ + \frac{1}{2}(h_R^2 + (\chi_R^3)^2) \right) \\
&+ \lambda_R \left(\chi_R^- \chi_R^+ + \frac{1}{2}(h_R^2 + (\chi_R^3)^2) \right)^2 .
\end{aligned} \tag{5.4}
$$

5.2 $\mathbf{SU(2)_L \times U(1)_Y \to U(1)_{em}}$

This stage occurs after $\mathrm{SU(2)_L}$ Higgs doublet acquires non-zero vev as parameterized in Eq.(4.39). Similar to what happens in SM, there is a mixing between B_μ and $W^3_{L\mu}$ into A_μ and $Z_{L\mu}$, following the transformation shown in Eq.(4.40).

5.2.1 Kinetic Terms

At this stage, it can be shown that the first line of Eq.(5.1) yields similar results to the breaking of $\mathrm{SU(2)_R \times U(1)_{Y'}}$ when substituting $R \to L$, $\theta_R \to \theta_W$, and $D_{\mathrm{SM}} \to D_{\mathrm{em}}$. After computing all terms, the kinetic terms of the Higgs in Eq.(5.1) become,

$$
\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\mathrm{kin}} = {} & (D^\mu_{\mathrm{em}} \chi_L^-)(D_{\mathrm{em}\mu} \chi_L^+) + (D^\mu_{\mathrm{em}} \chi_R^-)(D_{\mathrm{em}\mu} \chi_R^+) \\
& + i \frac{g_L v_L}{2} \{ W_L^{+\mu} (D_{\mathrm{em}\mu} \chi_L^-) - W_L^{-\mu} (D_{\mathrm{em}\mu} \chi_L^+) \} + \frac{g_L^2 v_L^2}{4} W_L^{-\mu} W_{L\mu}^+ \\
& + i \frac{g_R v_R}{2} \{ W_R^{+\mu} (D_{\mathrm{em}\mu} \chi_R^-) - W_R^{-\mu} (D_{\mathrm{em}\mu} \chi_R^+) \} + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W_{R\mu}^+ \\
& + \frac{1}{2}(\partial_\mu h_L)^2 + \frac{1}{2} \left(\partial_\mu \chi_L^3 - \frac{g_L v_L}{2\cos\theta_W} Z_{L\mu} \right)^2 \\
& + \frac{1}{2}(\partial_\mu h_R)^2 + \frac{1}{2} \left(\partial_\mu \chi_R^3 - \frac{g_R v_R}{2\cos\theta_R} Z_{R\mu} \right)^2 \\
& + \frac{1}{2} g' \tan\theta_R Z_{R\mu} \left\{ -v_L (\partial^\mu \chi_L^3) + \frac{g_L v_L^2}{2\cos\theta_W} Z_L^\mu \right\} + \frac{1}{8} v_L^2 g'^2 \tan^2\theta_R Z_R^\mu Z_{R\mu} \\
& - \frac{g_L}{2} \chi_L^3 \{ W_L^{+\mu} (D_{\mathrm{em}\mu} \chi_L^-) + W_L^{-\mu} (D_{\mathrm{em}\mu} \chi_L^+) \} \\
& - \frac{g_R}{2} \chi_R^3 \{ W_R^{+\mu} (D_{\mathrm{em}\mu} \chi_R^-) + W_R^{-\mu} (D_{\mathrm{em}\mu} \chi_R^+) \} \\
& + i \frac{g_L}{2} \{ W_L^{+\mu} (D_{\mathrm{em}\mu} \chi_L^-) - W_L^{-\mu} (D_{\mathrm{em}\mu} \chi_L^+) \} h_L + \frac{g_L^2 v_L}{2} h_L W_L^{-\mu} W_{L\mu}^+ \\
& + i \frac{g_R}{2} \left\{ W_R^{+\mu} (D_{\mathrm{em}\mu} \chi_R^-) - W_R^{-\mu} (D_{\mathrm{em}\mu} \chi_R^+) \right\} h_R + \frac{g_R^2 v_R}{2} h_R W_R^{-\mu} W_{R\mu}^+ \\
& + i \frac{g_L}{2} \frac{\cos 2\theta_W}{\cos\theta_W} \{ \chi_L^+ (D_{\mathrm{em}\mu} \chi_L^-) - \chi_L^- (D_{\mathrm{em}\mu} \chi_L^+) \} Z_L^\mu \\
& + i \frac{g_R}{2} \frac{\cos 2\theta_R}{\cos\theta_R} \{ \chi_R^+ (D_{\mathrm{em}\mu} \chi_R^-) - \chi_R^- (D_{\mathrm{em}\mu} \chi_R^+ \}) Z_R^\mu
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{g_L^2 v_L}{4}\left(\frac{\cos 2\theta_W - 1}{\cos\theta_W}\right)(W^+_{L\mu}\chi^-_L + W^-_{L\mu}\chi^+_L)Z^\mu_L \\
&+\frac{g_R^2 v_R}{4}\left(\frac{\cos 2\theta_R - 1}{\cos\theta_R}\right)(W^+_{R\mu}\chi^-_R + W^-_{R\mu}\chi^+_R)Z^\mu_R \\
&+\frac{g_L}{2}(W^+_{L\mu}\chi^-_L + W^-_{L\mu}\chi^+_L)\partial^\mu\chi^3_L - i\frac{g_L}{2}(W^+_{L\mu}\chi^-_L - W^-_{L\mu}\chi^+_L)\partial^\mu h_L \\
&+\frac{g_R}{2}(W^+_{R\mu}\chi^-_R + W^-_{R\mu}\chi^+_R)\partial^\mu\chi^3_R - i\frac{g_R}{2}(W^+_{R\mu}\chi^-_R - W^-_{R\mu}\chi^+_R)\partial^\mu h_R \\
&+\frac{g_L}{2\cos\theta_W}\{\chi^3_L(\partial_\mu h_L) - (\partial_\mu\chi^3_L)h_L\}Z^\mu_L + \left(\frac{g_L}{2\cos\theta_W}\right)^2 v_L h_L Z^\mu_L Z_{L\mu} \\
&+\frac{g_R}{2\cos\theta_R}\{\chi^3_R(\partial_\mu h_R) - (\partial_\mu\chi^3_R)h_R\}Z^\mu_R + \left(\frac{g_R}{2\cos\theta_R}\right)^2 v_R h_R Z^\mu_R Z_{R\mu} \\
&- ie\tan\theta_W\{\chi^+_R(D_{\mathrm{em}\mu}\chi^-_R) - \chi^-_R(D_{\mathrm{em}\mu}\chi^+_R)\}Z^\mu_L \\
&- i\frac{1}{2}g'\tan\theta_R\{\chi^+_L(D_{\mathrm{em}\mu}\chi^-_L) - \chi^-_L(D_{\mathrm{em}\mu}\chi^+_L)\}Z^\mu_R \\
&- \frac{g_R}{2}v_R e\tan\theta_W\left(W^+_{R\mu}\chi^-_R + W^-_{R\mu}\chi^+_R\right)Z^\mu_L \\
&- \frac{g_L}{2}v_L g'\tan\theta_R(W^+_{L\mu}\chi^-_L + W^-_{L\mu}\chi^+_L)Z^\mu_R \\
&+ g'\frac{1}{2}\tan\theta_R\left\{(\partial_\mu h_L)\chi^3_L - (\partial_\mu\chi^3_L)h_L\right\}Z^\mu_R \\
&+ g'\frac{1}{2}\tan\theta_R\frac{g_L}{\cos\theta_W}v_L h_L Z_{R\mu}Z^\mu_L + v_L g'^2\frac{1}{4}\tan^2\theta_R h_L Z_{R\mu}Z^\mu_R \\
&+ \frac{g_L^2(\cos 2\theta_W - 1)}{4\cos\theta_W}\{(W^+_{L\mu}\chi^-_L - W^-_{L\mu}\chi^+_L)i\chi^3_L + (W^+_{L\mu}\chi^-_L + W^-_{L\mu}\chi^+_L)h_L\}Z^\mu_L \\
&+ \frac{g_R^2(\cos 2\theta_R - 1)}{4\cos\theta_R}\{(W^+_{R\mu}\chi^-_R - W^-_{R\mu}\chi^+_R)i\chi^3_R + (W^+_{R\mu}\chi^-_R + W^-_{R\mu}\chi^+_R)h_R\}Z^\mu_R \\
&- i\frac{g_R}{2}e\tan\theta_W\chi^3_R\left(W^+_{R\mu}\chi^-_R - W^-_{R\mu}\chi^+_R\right)Z^\mu_L \\
&- i\frac{g_L}{2}g'\tan\theta_R\chi^3_L\left(W^+_{L\mu}\chi^-_L - W^-_{L\mu}\chi^+_L\right)Z^\mu_R \\
&- \frac{g_R}{2}e\tan\theta_W h_R\left(W^+_{R\mu}\chi^-_R + W^-_{R\mu}\chi^+_R\right)Z^\mu_L \\
&- \frac{g_L}{2}g'\tan\theta_R h_L\left(W^+_{L\mu}\chi^-_L + W^-_{L\mu}\chi^+_L\right)Z^\mu_R \\
&+ \frac{g_L^2}{4}\left(W^{+\mu}_L W^-_{L\mu}\right)((\chi^3_L)^2 + h_L^2) + \frac{g_R^2}{4}\left(W^{+\mu}_R W^-_{R\mu}\right)((\chi^3_R)^2 + h_R^2) \\
&+ \frac{g_L^2}{2}W^{+\mu}_L W^-_{L\mu}\chi^+_L\chi^-_L + \frac{g_R^2}{2}W^{+\mu}_R W^-_{R\mu}\chi^+_R\chi^-_R \\
&+ \frac{g_L^2}{2}\frac{\cos^2 2\theta_W}{2\cos^2\theta_W}\chi^+_L\chi^-_L Z_{L\mu}Z^\mu_L + \frac{g_R^2}{2}\frac{\cos^2 2\theta_R}{2\cos^2\theta_R}\chi^+_R\chi^-_R Z_{R\mu}Z^\mu_R \\
&+ e^2\tan^2\theta_W\chi^+_R\chi^-_R Z_{L\mu}Z^\mu_L + \frac{g'^2}{4}\tan^2\theta_R\chi^-_L\chi^+_L Z_{R\mu}Z^\mu_R
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{g_R}{2}e\tan\theta_W\frac{\cos 2\theta_R}{\cos\theta_R}(2\chi_R^+\chi_R^-)Z_{L\mu}Z_R^\mu-\frac{g_L}{2}g'\tan\theta_R\frac{\cos 2\theta_W}{\cos\theta_W}\chi_L^-\chi_L^+Z_{L\mu}Z_R^\mu\\
&+\frac{g_L^2}{4}\frac{1}{2\cos^2\theta_W}Z_{L\mu}Z_L^\mu((\chi_L^3)^2+h_L^2)+\frac{g_R^2}{4}\frac{1}{2\cos^2\theta_R}Z_{R\mu}Z_R^\mu((\chi_R^3)^2+h_R^2)\\
&+\frac{g'^2}{4}\frac{1}{2}\tan^2\theta_R Z_{R\mu}Z_R^\mu\left((\chi_L^3)^2+h_L^2\right), \qquad (5.5)
\end{aligned}
$$

where,

$$
D_{\mathrm{em}\mu}\chi_{L(R)}^+=(\partial_\mu+ieA_\mu)\chi_{L(R)}^+. \qquad (5.6)
$$

5.2.2 Higgs Potential

At this stage, the Higgs potential in Eq.(5.4) becomes,

$$
\begin{aligned}
V(\phi_L,\phi_R)=&\frac{\mu_L^2}{2}v_L^2+\frac{\mu_R^2}{2}v_R^2+\frac{\lambda_L}{4}v_L^4+\frac{\lambda_R}{4}v_R^4+\frac{\lambda_{LR}}{2}v_R^2v_L^2\\
&+h_L(\mu_L^2v_L+\lambda_Lv_L^3+\lambda_{LR}v_R^2v_L)+h_R(\mu_R^2v_R+\lambda_Rv_R^3+\lambda_{LR}v_Rv_L^2)\\
&+h_L(2\lambda_{LR}v_Rv_L)h_R+\frac{h_L^2}{2}(\mu_L^2+3\lambda_Lv_L^2+\lambda_{LR}v_R^2)+\frac{h_R^2}{2}(\mu_R^2+3\lambda_Rv_R^2+\lambda_{LR}v_L^2)\\
&+(\mu_L^2+\lambda_Lv_L^2+\lambda_{LR}v_R^2)\left(\chi_L^-\chi_L^++\frac{1}{2}(\chi_L^3)^2\right)\\
&+(\mu_R^2+\lambda_Rv_R^2+\lambda_{LR}v_L^2)\left(\chi_R^-\chi_R^++\frac{1}{2}(\chi_R^3)^2\right)\\
&+2v_L\left\{\lambda_L\left(\chi_L^-\chi_L^++\frac{1}{2}(h_L^2+(\chi_L^3)^2)\right)+\lambda_{LR}\left(\chi_R^-\chi_R^++\frac{1}{2}(h_R^2+(\chi_R^3)^2)\right)\right\}h_L\\
&+2v_R\left\{\lambda_R\left(\chi_R^-\chi_R^++\frac{1}{2}(h_R^2+(\chi_R^3)^2)\right)+\lambda_{LR}\left(\chi_L^-\chi_L^++\frac{1}{2}(h_L^2+(\chi_L^3)^2)\right)\right\}h_R\\
&+\lambda_L\left(\chi_L^-\chi_L^++\frac{1}{2}(h_L^2+(\chi_L^3)^2)\right)^2+\lambda_R\left(\chi_R^-\chi_R^++\frac{1}{2}(h_R^2+(\chi_R^3)^2)\right)^2\\
&+2\lambda_{LR}\left(\chi_L^-\chi_L^++\frac{1}{2}(h_L^2+(\chi_L^3)^2)\right)\left(\chi_R^-\chi_R^++\frac{1}{2}(h_R^2+(\chi_R^3)^2)\right), \qquad (5.7)
\end{aligned}
$$

where μ_L^2 and μ_R^2 are negative. The minimization conditions of the potential are,

$$
v_L(\mu_L^2+\lambda_Lv_L^2+\lambda_{LR}v_R^2)=0, \qquad (5.8)
$$

$$
v_R(\mu_R^2+\lambda_Rv_R^2+\lambda_{LR}v_L^2)=0. \qquad (5.9)
$$

The expressions for the non-zero vevs can be obtained as follows,

$$v_L = \sqrt{\frac{\lambda_{LR}\mu_R^2 - \lambda_R\mu_L^2}{\lambda_R\lambda_L - \lambda_{LR}^2}} \quad \text{and} \quad v_R = \sqrt{\frac{\lambda_{LR}\mu_L^2 - \lambda_L\mu_R^2}{\lambda_R\lambda_L - \lambda_{LR}^2}}, \tag{5.10}$$

where the vev's are taken to be positive. It can be shown that the linear terms of the Higgs fields and the quadratic terms of $\chi^{\pm}_{L(R)}, \chi^3_{L(R)}$ will vanish by using Eqs.(5.8) and (5.9).

5.3 Boson Mass

We collect the quadratic terms from kinetic terms Eq.(5.5) and Higgs potential Eq.(5.7) below,

$$\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} = &\, (D^\mu_{\text{em}}\chi^-_L)(D_{\text{em}\mu}\chi^+_L) + (D^\mu_{\text{em}}\chi^-_R)(D_{\text{em}\mu}\chi^+_R) \\
&+ i\frac{g_L v_L}{2}\{W_L^{+\mu}(D_{\text{em}\mu}\chi^-_L) - W_L^{-\mu}(D_{\text{em}\mu}\chi^+_L)\} + \frac{g_L^2 v_L^2}{4} W_L^{-\mu} W^+_{L\mu} \\
&+ i\frac{g_R v_R}{2}\{W_R^{+\mu}(D_{\text{em}\mu}\chi^-_R) - W_R^{-\mu}(D_{\text{em}\mu}\chi^+_R)\} + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W^+_{R\mu} \\
&+ \frac{1}{2}\left(\frac{g_L}{2}\frac{v_L}{\cos\theta_W}\right)^2 Z^\mu_L Z_{L\mu} + \frac{1}{2}\left\{\left(\frac{g_R}{2}\frac{v_R}{\cos\theta_R}\right)^2 + \left(\frac{g'}{2}v_L\tan\theta_R\right)^2\right\} Z^\mu_R Z_{R\mu} \\
&+ \frac{g' v_L}{2}\tan\theta_R \frac{g_L}{2}\frac{v_L}{\cos\theta_W} Z^\mu_L Z_{R\mu} \\
&+ \frac{1}{2}(\partial_\mu\chi^3_L)^2 + \frac{1}{2}(\partial_\mu\chi^3_R)^2 \\
&- \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}(\partial^\mu\chi^3_L) - \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}(\partial^\mu\chi^3_R) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}(\partial^\mu\chi^3_L) \\
&+ \frac{1}{2}(\partial_\mu h_L)^2 + \frac{1}{2}(\partial_\mu h_R)^2 \\
&- h_L(2\lambda_{LR}v_R v_L)h_R - \frac{h_L^2}{2}(2\lambda_L v_L^2) - \frac{h_R^2}{2}(2\lambda_R v_R^2).
\end{aligned} \tag{5.11}$$

From Eq.(5.11), we obtain the masses for W_L and W_R as follows,

$$M_{W_L} = \frac{g_L}{2}v_L, \tag{5.12}$$

$$M_{W_R} = \frac{g_R}{2}v_R. \tag{5.13}$$

Since there is mixing between Z_L and Z_R as well as h_L and h_R, we need to diagonalize the mass matrices to obtain the mass eigenstates for the Z bosons and the Higgs bosons. In line with that, the Nambu-Goldstone bosons χ_L^3 and χ_R^3 also mix.

5.3.1 Z and Z' Boson Mass

We define the following transformation from the Z_L and Z_R basis into the mass eigenstates,

$$\begin{pmatrix} Z_{L\mu} \\ Z_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}. \tag{5.14}$$

From Eq.(5.11), the mass matrix in the Z_L and Z_R basis is given by,

$$\mathbb{M}_Z^2 = \begin{pmatrix} (\frac{g_L v_L}{2\cos\theta_W})^2 & \frac{1}{2}g' v_L \tan\theta_R \frac{g_L v_L}{2\cos\theta_W} \\ \frac{1}{2}g' v_L \tan\theta_R \frac{g_L v_L}{2\cos\theta_W} & (\frac{g_R v_R}{2\cos\theta_R})^2 + \left(\frac{1}{2}g' v_L \tan\theta_R\right)^2 \end{pmatrix}. \tag{5.15}$$

The matrix $\mathbb{M}_Z^2$ can be diagonalized as,

$$\mathcal{O}_Z^T \mathbb{M}_Z^2 \mathcal{O}_Z = \mathrm{diag}(M_Z^2, M_{Z'}^2), \tag{5.16}$$

where $\mathcal{O}_Z$ is the mixing matrix defined in Eq.(5.14). The exact mass eigenvalues and mixing angles are as follows,

$$M_Z^2 = \frac{M_{W_R}^2}{2c_R^2}\left\{1 + (c_R^2 + t_W^2)\frac{M_{W_L}^2}{M_{W_R}^2} - \sqrt{1 - \frac{2M_{W_L}^2}{M_{W_R}^2}\left(\frac{c_R^2 - s_W^2 s_R^2}{c_W^2}\right) + (c_R^2 + t_W^2)^2 \left(\frac{M_{W_L}^2}{M_{W_R}^2}\right)^2}\right\}, \tag{5.17}$$

$$M_{Z'}^2 = \frac{M_{W_R}^2}{2c_R^2}\left\{1 + (c_R^2 + t_W^2)\frac{M_{W_L}^2}{M_{W_R}^2} + \sqrt{1 - \frac{2M_{W_L}^2}{M_{W_R}^2}\left(\frac{c_R^2 - s_W^2 s_R^2}{c_W^2}\right) + (c_R^2 + t_W^2)^2 \left(\frac{M_{W_L}^2}{M_{W_R}^2}\right)^2}\right\}, \tag{5.18}$$

$$\tan 2\theta = \frac{2c_R s_R^3 s_W \frac{v_L^2}{v_R^2}}{s_W^2 - s_R^2\left(s_W^2 \cos 2\theta_R + c_W^2 c_R^2\right)\frac{v_L^2}{v_R^2}}, \quad 0 \le \theta \le \frac{\pi}{4}, \tag{5.19}$$

where,

$$c_R = \cos\theta_R, \;\; s_R = \sin\theta_R, \;\; c_W = \cos\theta_W, \;\; s_W = \sin\theta_W, \;\; t_W = \tan\theta_W. \tag{5.20}$$

When $M_{W_R} \gg M_{W_L}$, the masses of the Z and Z' bosons are approximately given by,

$$M_Z^2 \simeq \frac{M_{W_L}^2}{c_W^2}\left(1 - \frac{M_{W_L}^2}{M_{W_R}^2} s_R^2 t_W^2\right), \tag{5.21}$$

$$M_{Z'}^2 \simeq \frac{M_{W_R}^2}{c_R^2}\left(1+\frac{M_{W_L}^2}{M_{W_R}^2}s_R^2 t_W^2\right). \tag{5.22}$$

5.3.2 Higgs Boson Mass

We define the transformation from the h_L and h_R basis into the mass eigenstate as follows,

$$\begin{pmatrix} h_L \\ h_R \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} h \\ H \end{pmatrix}. \tag{5.23}$$

The mass matrix of the Higgs in the h_L and h_R basis are is given by,

$$\mathbb{M}_h^2 = \begin{pmatrix} 2\lambda_L v_L^2 & 2\lambda_{LR} v_R v_L \\ 2\lambda_{LR} v_R v_L & 2\lambda_R v_R^2 \end{pmatrix}. \tag{5.24}$$

By defining the mixing matrix in Eq.(5.23) as $\mathcal{O}_h$, we can diagonalize $\mathbb{M}_h$ as,

$$\mathcal{O}_h^T \mathbb{M}_h^2 \mathcal{O}_h = \mathrm{diag}(m_h^2, m_H^2), \tag{5.25}$$

which yieds the exact mass eigenvalues,

$$m_h^2 = \lambda_L v_L^2 + \lambda_R v_R^2 - \sqrt{(\lambda_L v_L^2 - \lambda_R v_R^2)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}, \tag{5.26}$$

$$m_H^2 = \lambda_L v_L^2 + \lambda_R v_R^2 + \sqrt{(\lambda_L v_L^2 - \lambda_R v_R^2)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}. \tag{5.27}$$

Additionally, the mixing angle in Eq.(5.23) is given by,

$$\tan 2\phi = \frac{2\lambda_{LR} v_R v_L}{\lambda_R v_R^2 - \lambda_L v_L^2}, \quad 0 \le |\phi| \le \frac{\pi}{4}. \tag{5.28}$$

Furthermore, the mass eigenvalues and mixing angle can be approximated as follows,

$$m_h^2 \simeq 2\lambda_L\left(1 - \frac{\lambda_{LR}^2}{\lambda_L \lambda_R}\right) v_L^2, \tag{5.29}$$

$$m_H^2 \simeq 2\lambda_R v_R^2, \tag{5.30}$$

$$\tan 2\phi \simeq \frac{2\lambda_{LR}}{\lambda_R}\frac{v_L}{v_R} \tag{5.31}$$

if we ignore the correction of $\mathcal{O}\left(v_L^2/v_R^2\right)$.

5.4 χ_L^3 and χ_R^3 Mixing

From Eq.(5.11), we extract the following form,

$$\begin{aligned}\mathcal{L}_{\text{quad}} \supset \mathcal{L}_\chi = &\frac{1}{2}(\partial_\mu \chi_L^3)^2 + \frac{1}{2}(\partial_\mu \chi_R^3)^2 \\ &- \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}(\partial^\mu \chi_L^3) - \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}(\partial^\mu \chi_R^3) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}(\partial^\mu \chi_L^3).\end{aligned} \tag{5.32}$$

By changing into the mass eigenstate using Eq.(5.14) and writing in terms of the diagonal mass eigenvalues $(M_Z, M_{Z'})$, Eq.(5.32) can be rewritten as,

$$\begin{aligned}\mathcal{L}_{\text{quad}} \supset \mathcal{L}_\chi = &\frac{1}{2}(\partial_\mu \chi_Z)^2 + \frac{1}{2}(\partial_\mu \chi_{Z'})^2 \\ &- M_Z\ (\partial^\mu \chi_Z) Z_\mu - M_{Z'}(\partial^\mu \chi_{Z'}) Z'_\mu,\end{aligned} \tag{5.33}$$

where,

$$\begin{pmatrix} \chi_L^3 \\ \chi_R^3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \chi_Z \\ \chi_{Z'} \end{pmatrix}, \tag{5.34}$$

$$\cos\alpha = \frac{M_Z \cos\theta}{\sqrt{M_Z^2\cos^2\theta + M_{Z'}^2 \sin^2\theta}}, \tag{5.35}$$

$$\sin\alpha = \frac{M_{Z'} \sin\theta}{\sqrt{M_Z^2\cos^2\theta + M_{Z'}^2 \sin^2\theta}}. \tag{5.36}$$

Therefore, the quadratic terms in Eq.(5.11) can be written in terms of the mass basis of the Z bosons, Higgs bosons, and Nambu-Goldstone bosons,

$$\begin{aligned}\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} = &\left(D_{\text{em}}^\mu \chi_L^- - iM_{W_L} W_L^{\mu -}\right)\left(D_{\text{em}\mu}\chi_L^+ + iM_{W_L} W_{L\mu}^+\right) \\ &+ \left(D_{\text{em}}^\mu \chi_R^- - iM_{W_R} W_R^{\mu -}\right)\left(D_{\text{em}\mu}\chi_R^+ + iM_{W_R} W_{R\mu}^+\right) \\ &+ \frac{1}{2}\left(\partial_\mu \chi_Z - M_Z Z_\mu\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_{Z'} - M_{Z'} Z'_\mu\right)^2 \\ &+ \frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{1}{2} m_h^2 h^2 + \frac{1}{2}\left(\partial_\mu H\right)^2 - \frac{1}{2} m_H^2 H^2,\end{aligned} \tag{5.37}$$

where the covariant derivatives of χ_L and χ_R are given in Eq.(5.6). We have shown explicitly that χ_L^3 and χ_R^3 are mixed in this model. From Eq.(5.37), it is shown clearly that the degrees of freedom χ_Z and $\chi_{Z'}$ become the longitudinal components of the massive Z and Z' bosons, respectively.

Chapter 6

Kinetic Terms of the Gauge Fields

In this chapter we derive the kinetic terms of the gauge fields starting from Lagrangian in Eq.(3.5).

6.1 $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

At this stage, the kinetic terms of the gauge fields change from the B'_μ and $W_{R\mu}$ basis into B_μ and $Z_{R\mu}$ basis. Following the transformation in Eq.(4.2), the Lagrangian in Eq.(3.5) becomes,

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} =& -\frac{1}{4}F^a_{L\mu\nu}F^{a\mu\nu}_L - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& -\frac{1}{2}(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu})(\partial^\mu W_R^{-\nu} - \partial^\nu W_R^{-\mu}) \\
& - i(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu})(g_R\cos\theta_R Z^\nu_R + g'B^\nu)W_R^{-\mu} \\
& + i(\partial^\mu W_R^{-\nu} - \partial^\nu W_R^{-\mu})(g_R\cos\theta_R Z_{R\nu} + g'B_\nu)W^+_{R\mu} \\
& - \big\{(g_R\cos\theta_R Z_{R\nu} + g'B_\nu)W^+_{R\mu}(g_R\cos\theta_R Z^\nu_R + g'B^\nu)W_R^{-\mu} \\
& \quad -(g_R\cos\theta_R Z_{R\mu} + g'B_\mu)W^+_{R\nu}(g_R\cos\theta_R Z^\nu_R + g'B^\nu)W_R^{-\mu}\big\} \\
& -\frac{1}{4}F^0_{Z_R\mu\nu}F^{0\mu\nu}_{Z_R} + iW^-_{R\mu}W^+_{R\nu}(g_R\cos\theta_R F^{0\mu\nu}_{Z_R} + g'B^{\mu\nu}) \\
& +\frac{1}{2}g_R^2(W^-_{R\mu}W^+_{R\nu} - W^+_{R\mu}W^-_{R\nu})(W_R^{-\mu}W_R^{+\nu}),
\end{aligned} \tag{6.1}$$

where,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{6.2}$$

$$F^a_{L\mu\nu} = \partial_\mu W^a_{L\nu} - \partial_\nu W^a_{L\mu} - g_L\epsilon^{abc}W^b_{L\mu}W^c_{L\nu}, \tag{6.3}$$

$$F^0_{Z_R\mu\nu} = \partial_\mu Z_{R\nu} - \partial_\nu Z_{R\mu}. \tag{6.4}$$

6.2 $\mathbf{SU(2)_L \times U(1)_Y \to U(1)_{em}}$

At this stage, there is a mixing between B_μ and $W^3_{L\mu}$ into A_μ and $Z_{L\mu}$ following the transformation shown in Eq.(4.40). Additionally, we express the fields in the diagonal basis of Z and Z' where the transformation is shown in Eq.(5.14). Thus, the Lagrangian in Eq.(6.1) becomes,

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} = & -\frac{1}{4}F^0_{Z\mu\nu}F^{0\mu\nu}_Z - \frac{1}{4}F^0_{Z'\mu\nu}F^{0\mu\nu}_{Z'} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& - \frac{1}{2}\left(\mathcal{D}_\mu W^+_{L\nu} - \mathcal{D}_\nu W^+_{L\mu}\right)\left(\mathcal{D}^\mu W^{-\nu}_L - \mathcal{D}^\nu W^{-\mu}_L\right) \\
& - \frac{1}{2}\left(\mathcal{D}_\mu W^+_{R\nu} - \mathcal{D}_\nu W^+_{R\mu}\right)\left(\mathcal{D}^\mu W^{-\nu}_R - \mathcal{D}^\nu W^{-\mu}_R\right) \\
& + \frac{g_L^2}{2}\left((W^-_L \cdot W^-_L)(W^+_L \cdot W^+_L) - \left(W^-_L \cdot W^+_L\right)^2\right) \\
& + \frac{g_R^2}{2}\left((W^-_R \cdot W^-_R)(W^+_R \cdot W^+_R) - \left(W^-_R \cdot W^+_R\right)^2\right) \\
& + i\left\{g_L \cos\theta_W \cos\theta F^{0\mu\nu}_Z + g_L \cos\theta_W \sin\theta F^{0\mu\nu}_{Z'} + eF^{\mu\nu}\right\}\left(W^-_{L\mu}W^+_{L\nu}\right) \\
& + i\left\{-(g_R \cos\theta_R \sin\theta + e\tan\theta_W \cos\theta)F^{0\mu\nu}_Z\right. \\
& \qquad \left. +(g_R \cos\theta_R \cos\theta - e\tan\theta_W \sin\theta)F^{0\mu\nu}_{Z'} + eF^{\mu\nu}\right\}\left(W^-_{R\mu}W^+_{R\nu}\right),
\end{aligned} \tag{6.5}$$

where,

$$\begin{aligned}
F^0_{Z\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, \\
F^0_{Z'\mu\nu} &= \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
\mathcal{D}_\mu W^+_{R\nu} &= (D_{\text{em}\mu} W^+_{R\nu}) - i(e\tan\theta_W Z_{L\mu} - g_R \cos\theta_R Z_{R\mu})W^+_{R\nu}, \\
\mathcal{D}_\mu W^+_{L\nu} &= (D_{\text{em}\mu} W^+_{L\nu}) + ig_L \cos\theta_W Z_{L\mu} W^+_{L\nu}, \\
D_{\text{em}\mu} G_\nu &= (\partial_\mu + ieA_\mu)G_\nu,
\end{aligned} \tag{6.6}$$

with $G_\nu \in \{W^+_{R\nu}, W^+_{L\nu}\}$.

Chapter 7

Hierarchy of VLQ's Mass Parameters, v_L, and v_R

In this chapter, we discuss about the hierarchy of VLQ's mass parameters, v_L, and v_R. From Eqs.(4.70) and (4.71), we have the exact mass eigenvalues of top and bottom quarks, as well as the heavy top and bottom quarks, respectively. One of the motivations for the universal seesaw model in the quark sector is to explain the mass hierarchy of quarks. The hierarchy of VLQ's mass parameters (M_T and M_B), v_L, and v_R is important in our model. We give the analytical and numerical analysis.

7.1 Analytical analysis

The exact mass eigenvalue of the top quark in Eq.(4.70) can be expressed as follows,

$$\begin{aligned} m_t &= \frac{\sqrt{M_T^2 + m_{u_R}^2 + m_{u_L}^2 + 2m_{u_L}m_{u_R}}}{2} - \frac{\sqrt{M_T^2 + m_{u_R}^2 + m_{u_L}^2 - 2m_{u_L}m_{u_R}}}{2} \\ &\simeq \left(\frac{m_{u_R}}{\sqrt{M_T^2 + m_{u_R}^2}} \right) m_{u_L}. \end{aligned} \tag{7.1}$$

From the first line to the second line of Eq.(7.1), we apply the condition $m_{u_L} < m_{u_R}$. The second line of Eq.(7.1) can then be rewritten in terms of Yukawa couplings, using Eq.(C.16), as follows,

$$m_t \simeq \left(\frac{\frac{Y_{u_R}^3 v_R}{\sqrt{2}}}{\sqrt{M_T^2 + \frac{(Y_{u_R}^3)^2 v_R^2}{2}}} \right) \frac{Y_{u_L}^3 v_L}{\sqrt{2}}. \tag{7.2}$$

Assuming $Y^3_{u_L} = Y^3_{u_R} \simeq \mathcal{O}(1)$ and that the factor inside the parentheses is $\mathcal{O}(1)$, we can approximate the top quark mass as $m_t \simeq v_L$. This implies that $M_T < v_R$. To determine the hierarchy between M_T and v_R for the large top quark mass, the ratio M_T/v_R can be derived from Eq.(7.2) as follows,

$$\frac{M_T}{v_R} = \frac{Y^3_{u_L} Y^3_{u_R}}{\sqrt{2}} \sqrt{\frac{1}{(y_t^{\mathrm{SM}})^2} - \frac{1}{(Y^3_{u_L})^2}}, \tag{7.3}$$

where y_t^{SM} is the SM Yukawa coupling of top quark and $Y^3_{u_L} \geq y_t^{\mathrm{SM}}$. If we further require that the Yukawa couplings are in the perturbative region, $y_t^{\mathrm{SM}} \leq Y^3_{u_L}, Y^3_{u_R} \leq 1$, the upper and the lower limit of the ratio M_T/v_R is given by

$$0 \leq \frac{M_T}{v_R} \leq \frac{1}{\sqrt{2}} \sqrt{\frac{1}{(y_t^{\mathrm{SM}})^2} - 1}. \tag{7.4}$$

If we take $y_t^{\mathrm{SM}} = 0.9912$, we find that the upper limit of the ratio M_T/v_R is ≤ 0.0944. This demonstrates how the seesaw mechanism accounts for both the top quark mass and the hierarchy between M_T and v_R.

Similarly, in the bottom sector, by applying the condition $m_{d_L} < m_{d_R}$, the bottom quark mass can be expressed as follows,

$$m_b \simeq \left(\frac{\frac{Y^3_{d_R} v_R}{\sqrt{2}}}{\sqrt{M_B^2 + \frac{(Y^3_{d_R})^2 v_R^2}{2}}} \right) \frac{Y^3_{d_L} v_L}{\sqrt{2}}. \tag{7.5}$$

Assuming $Y^3_{d_L} = Y^3_{d_R} \simeq \mathcal{O}(1)$ and that the factor inside the parentheses is much smaller than $\mathcal{O}(1)$, we can derive the light bottom quark mass. This implies $M_B \gg v_R$, allowing us to express Eq.(7.5) as follows,

$$m_b \simeq \frac{v_R Y^3_{d_R} Y^3_{d_L} v_L}{2 M_B}. \tag{7.6}$$

To determine the hierarchy between M_B and v_R for the light bottom quark mass, the ratio M_B/v_R can be obtained from Eq.(7.6) as follows,

$$\frac{M_B}{v_R} = \frac{Y^3_{d_L} Y^3_{d_R}}{\sqrt{2}} \frac{1}{y_b^{\mathrm{SM}}}, \tag{7.7}$$

where y_b^{SM} is the SM Yukawa coupling of bottom quark. If we further require that the Yukawa couplings are in the perturbative region, $Y^3_{d_L}, Y^3_{d_R} \leq 1$, the upper limit of the ratio M_B/v_R is given by

$$\frac{M_B}{v_R} \leq \frac{1}{\sqrt{2}}\frac{1}{y_b^{\rm SM}}. \tag{7.8}$$

If we take $y_b^{\rm SM} = 2.4 \times 10^{-2}$, the upper limit of the ratio M_B/v_R is ≤ 29.46. Equality holds when the Yukawa couplings $Y^3_{d_L} = Y^3_{d_R} = 1$. This demonstrates how the seesaw mechanism accommodates the bottom quark mass and the hierarchy between M_B and v_R. Therefore, when all Yukawa couplings $Y^3_{d_L}$, $Y^3_{d_R}$, $Y^3_{u_L}$, and $Y^3_{u_R}$ are $\mathcal{O}(1)$, the hierarchy among the three scales is $M_T < v_R \ll M_B$. If we include the v_L, the hierarchy has two possibilities depending on the numerical inputs: either $v_L < M_T < v_R \ll M_B$ or $M_T < v_L < v_R \ll M_B$.

To summarize, by using the hierarchy that we discussed before, from the exact mass eigenvalues in Eqs.(4.70) and (4.71) we can obtain the approximate form as follows,

$$m_t^{\rm approx} \simeq \frac{v_R Y^3_{u_R} Y^3_{u_L} v_L}{2\sqrt{\frac{v_R^2}{2}(Y^3_{u_R})^2 + M_T^2}}, \tag{7.9}$$

$$m_{t'}^{\rm approx} \simeq \sqrt{\frac{v_R^2}{2}(Y^3_{u_R})^2 + M_T^2}, \tag{7.10}$$

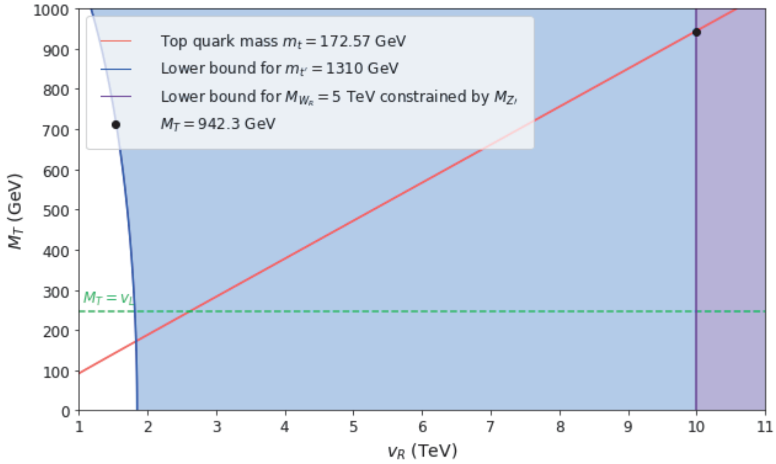
$$m_b^{\rm approx} \simeq \frac{v_R Y^3_{d_R} Y^3_{d_L} v_L}{2M_B}, \tag{7.11}$$

$$m_{b'}^{\rm approx} \simeq M_B. \tag{7.12}$$

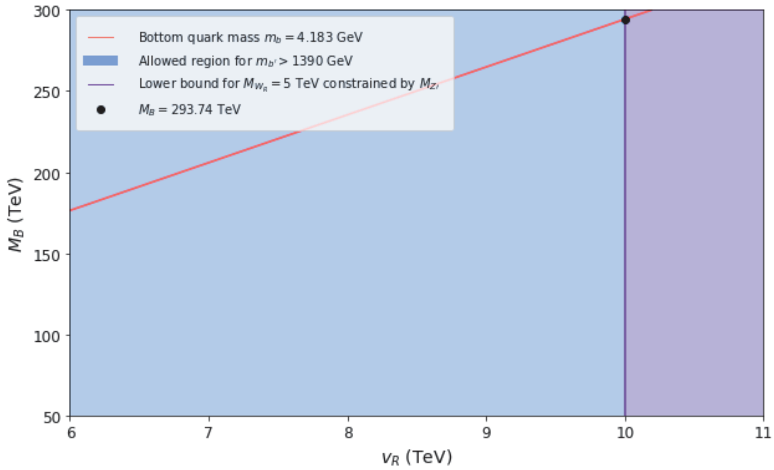
Our results in Eqs.(7.9) and (7.10) agree with Eqs.(7) and (8) in Ref.[28], as well as Eqs.(3.19) and (3.17) in Ref.[29], respectively. While our results in Eqs.(7.11) and (7.12) agree with Eqs.(14) and (15) in Ref.[28], as well as Eq.(3.9) in Ref.[29], respectively.

7.2 Numerical analysis

We start by analyzing the constraints in the top sector, as shown in Fig (7.1a). We consider an asymmetric left-right model with $g_L \neq g_R$. By assuming $g_R \simeq 1$ and using the value of $g' \simeq 0.357$, we obtain θ_R with Eq.(4.4). Additionally, we assume $Y^3_{u_R} \simeq Y^3_{u_L} \simeq 1$. The following constraints are used [11]: (1) the top quark mass obtained by the direct measurement is $m_t = 172.57$ GeV; (2) the lower bound for the heavy top quark mass is set to be $m_{t'} > 1310$ GeV; (3) the lower bound for the Z' boson mass is set to be $M_{Z'} > 5150$ GeV. Using the exact mass eigenvalue for



(a)



(b)

Figure 7.1: Constraints on v_R and VLQ's mass parameters of different sectors. (a) Top sector. (b) Bottom sector. These figures are taken from Figure 1 in Ref.[46].

the Z' boson mass in Eq.(5.18), we compute the lower bound for W_R boson mass as $M_{W_R} \gtrsim 5$ TeV. Consequently, we find the constraint for v_R using Eq. (5.13), yielding $v_R \gtrsim 10$ TeV. At $v_R = 10$ TeV, M_T is 942.3 GeV as shown by the black dot in Fig (7.1a). Using these v_R and M_T values, we further calculate the heavy top quark mass with Eq.(4.71) and obtain $m_{t'} = 7.13$ TeV.

Next, we analyze the constraints in the bottom sector, as depicted in Fig (7.1b). Here, we also assume $Y^3_{d_R} \simeq Y^3_{d_L} \simeq 1$. The constraints are [11]: (1) the SM bottom quark mass we use is the running mass at bottom mass $m_b = 4.183$ GeV; (2) the lower bound for the heavy bottom quark mass is set to be $m_{b'} > 1390$ GeV; (3) the constraint for $v_R \gtrsim 10$ is derived from the lower bound for the Z' boson mass. For the bottom sector, at $v_R = 10$ TeV, M_B is 293.74 TeV as indicated by the black dot in Fig (7.1b). Using these v_R and M_B values, we further calculate the heavy bottom quark mass with Eq.(4.71) and obtain $m_{b'} = 293.82$ TeV. This result indicates that $m_{b'} \simeq M_B$.

From the above facts, the mass parameter of the top partner VLQ (M_T) is smaller than v_R but could be larger or smaller than v_L depending on other parameters. On the other hand, in the bottom sector, the mass parameter of the bottom partner VLQ (M_B) is significantly larger compared to v_R. This explains the mass hierarchy problem, where the smallness of the bottom quark mass is suppressed by the large mass of the bottom VLQ through a seesaw mechanism. Mathematically, our choice of numerical input satisfies the following hierarchy: (1) for the top sector: $v_L < M_T < v_R$; (2) for the bottom sector: $v_L < v_R \ll M_B$.

Using our chosen numerical inputs, one can compute the masses in the approximation form given in Eqs.(7.9), (7.10), (7.11) and (7.12) and obtain $m_t^{\text{approx}} = 172.58$ GeV, $m_{t'}^{\text{approx}} = 7.13$ TeV, $m_b^{\text{approx}} = 4.19$ GeV, and $m_{b'}^{\text{approx}} = 293.74$ TeV. These values are very close to the exact mass eigenvalues formula. For the rest of our numerical analysis, we will use $v_R = 10$ TeV. This $v_R = 10$ TeV is also used in Ref.[34], although unlike this paper, they considered the model with left-right symmetry where $g_L = g_R$.

Chapter 8

Flavor-changing Neutral Current

In this chapter, we discuss flavor-changing neutral currents (FCNCs) in this model.

8.1 Higgs FCNC

In this section, we discuss the interaction between Higgs and quarks in our model. From Eq.(4.90), we extract the interactions between h_L and h_R with quarks, given by

$$\begin{aligned}\mathcal{L}_q \supset \mathcal{L}_{hH} = &-\frac{1}{v_L}\sum_{k,i=3}^{4}\Big[(\mathcal{Z}_{T_L}m_t^{\text{diag}})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i + (m_t^{\text{diag}}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i \\ &+(\mathcal{Z}_{B_L}m_b^{\text{diag}})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i + (m_b^{\text{diag}}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\,h_L \\ &-\frac{1}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\text{diag}}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i \\ &+(\mathcal{Z}_{T_R}m_t^{\text{diag}}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i + ((1-\mathcal{Z}_{B_L})m_b^{\text{diag}}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i \\ &+(\mathcal{Z}_{B_R}m_b^{\text{diag}}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\,h_R, \end{aligned} \tag{8.1}$$

where $\mathcal{Z}_{T_L}, \mathcal{Z}_{B_L}, \mathcal{Z}_{T_R}, \mathcal{Z}_{B_R}, m_t^{\text{diag}}$, and m_b^{diag} are given in Eqs.(4.84),(4.85),(4.88), (4.89), (4.68), and (4.69), respectively. By transforming $h_L - h_R$ basis into $h - H$ mass eigenstate with Eq.(5.23), the Lagrangian in Eq.(8.1) transforms into,

$$\begin{aligned}\mathcal{L}_{hH} = -\Bigg\{\frac{\cos\phi}{v_L}\sum_{k,i=3}^{4}\Big[&(\mathcal{Z}_{T_L}m_t^{\text{diag}})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i + (m_t^{\text{diag}}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i \\ &+(\mathcal{Z}_{B_L}m_b^{\text{diag}})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i + (m_b^{\text{diag}}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\end{aligned}$$

$$
\begin{aligned}
&-\frac{\sin\phi}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\text{diag}}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i+(\mathcal{Z}_{T_R}m_t^{\text{diag}}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\\
&\quad+((1-\mathcal{Z}_{B_L})m_b^{\text{diag}}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i+(\mathcal{Z}_{B_R}m_b^{\text{diag}}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\Big\}\,h\\
&-\Big\{\frac{\sin\phi}{v_L}\sum_{k,i=3}^{4}\Big[(\mathcal{Z}_{T_L}m_t^{\text{diag}})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i+(m_t^{\text{diag}}\mathcal{Z}_{T_L})^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\\
&\qquad+(\mathcal{Z}_{B_L}m_b^{\text{diag}})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i+(m_b^{\text{diag}}\mathcal{Z}_{B_L})^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\\
&+\frac{\cos\phi}{v_R}\sum_{k,i=3}^{4}\Big[((1-\mathcal{Z}_{T_L})m_t^{\text{diag}}\mathcal{Z}_{T_R})^{ki}\overline{(\hat{u}_L^m)^k}(\hat{u}_R^m)^i+(\mathcal{Z}_{T_R}m_t^{\text{diag}}(1-\mathcal{Z}_{T_L}))^{ki}\overline{(\hat{u}_R^m)^k}(\hat{u}_L^m)^i\\
&+((1-\mathcal{Z}_{B_L})m_b^{\text{diag}}\mathcal{Z}_{B_R})^{ki}\overline{(\hat{d}_L^m)^k}(\hat{d}_R^m)^i+(\mathcal{Z}_{B_R}m_b^{\text{diag}}(1-\mathcal{Z}_{B_L}))^{ki}\overline{(\hat{d}_R^m)^k}(\hat{d}_L^m)^i\Big]\Big\}\,H,
\end{aligned}
\tag{8.2}
$$

where h and H denote the Higgs and the heavy Higgs, respectively. In this discussion, we will focus on the interaction of the Higgs with the quarks in our model.

8.1.1 Top Sector

We collect the interaction terms between Higgs with top quark (t) and heavy top quark (t') from Eq. (8.2)

$$
\begin{aligned}
\mathcal{L}_{hH}\supset\mathcal{L}_{ht}=&-\Big[\frac{\cos\phi}{v_L}\cos^2\phi_{T_L}m_t-\frac{\sin\phi}{v_R}\big(\sin^2\phi_{T_L}\cos^2\beta_{T_R}m_t\\
&\qquad-\sin\phi_{T_L}\cos\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R}m_{t'})\Big]\,\bar{t}th\\
&+\Big[\frac{\cos\phi}{v_L}\sin\phi_{T_L}\cos\phi_{T_L}m_{t'}+\frac{\sin\phi}{v_R}\big(\sin\phi_{T_L}\cos\phi_{T_L}\sin^2\beta_{T_R}m_{t'}\\
&\qquad-\sin^2\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R}m_t\big)\Big]\,(\bar{t}_Lt'_R+\bar{t}'_Rt_L)h\\
&+\Big[\frac{\cos\phi}{v_L}\sin\phi_{T_L}\cos\phi_{T_L}m_t+\frac{\sin\phi}{v_R}\big(\sin\phi_{T_L}\cos\phi_{T_L}\cos^2\beta_{T_R}m_t\\
&\qquad-\cos^2\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R}m_{t'}\big)\Big]\,(\bar{t}'_Lt_R+\bar{t}_Rt'_L)h\\
&-\Big[\frac{\cos\phi}{v_L}\sin^2\phi_{T_L}m_{t'}-\frac{\sin\phi}{v_R}\big(\cos^2\phi_{T_L}\sin^2\beta_{T_R}m_{t'}\\
&\qquad-\sin\phi_{T_L}\cos\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R}m_t)\Big]\,\bar{t}'t'h,
\end{aligned}
\tag{8.3}
$$

where we substitute the elements of $\mathcal{Z}_{T_L}$ and $\mathcal{Z}_{T_R}$ in Eqs.(4.84) and (4.88), respectively. Then, we approximate the mixing angles using Eqs.(C.26) and (D.13). Additionally, by using the hierarchy in the top sector that is $v_L < M_T < v_R$, and the approximation for the mixing angle ϕ in Eq.(5.31), we derive the interaction between the Higgs and the top-sector quarks as follows,

$$\mathcal{L}_{ht} \simeq -\cos\phi\frac{m_t}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi\frac{M_T}{m_{u_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)(\bar{t}_L t'_R + \bar{t}'_R t_L)h$$
$$- \cos\phi\frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1+\frac{\lambda_{LR}}{\lambda_R}\right)(\bar{t}'_L t_R + \bar{t}_R t'_L)h - \cos\phi\frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2}-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h. \quad (8.4)$$

In this expression, we also assume that $Y_{u_L}^3 \simeq Y_{u_R}^3 \simeq 1$. From Eq.(8.4) we extract useful informations regarding our model. Higgs-top quark coupling receives a small correction, while Higgs-heavy top quark coupling receives an overall suppression of $\mathcal{O}(v_L/v_R)$. Another significant point is that the tree-level FCNC interaction is suppressed. The Higgs FCNC of $\bar{t}'_L t_R$ and $\bar{t}_R t'_L$ types are more suppressed by a factor of $\mathcal{O}(v_L/v_R)$ compared to the $\bar{t}_L t'_R$ and $\bar{t}'_R t_L$ type.

8.1.2 Bottom Sector

In the same way as in the top quark sector, from Eq. (8.2), we collect the interaction between Higgs with the bottom quark sector. By expressing $\mathcal{Z}_{B_L}$ and $\mathcal{Z}_{B_R}$ in terms of their elements, we obtain,

$$\begin{aligned}\mathcal{L}_{hH} \supset \mathcal{L}_{hb} = &-\Big[\frac{\cos\phi}{v_L}\cos^2\phi_{B_L} m_b - \frac{\sin\phi}{v_R}\big(\sin^2\phi_{B_L}\cos^2\beta_{B_R} m_b \\ &\qquad - \sin\phi_{B_L}\cos\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_{b'}\big)\Big]\bar{b}bh \\ &+\Big[\frac{\cos\phi}{v_L}\sin\phi_{B_L}\cos\phi_{B_L} m_{b'} + \frac{\sin\phi}{v_R}\big(\sin\phi_{B_L}\cos\phi_{B_L}\sin^2\beta_{B_R} m_{b'} \\ &\qquad - \sin^2\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_b\big)\Big](\bar{b}_L b'_R + h.c.)h \\ &+\Big[\frac{\cos\phi}{v_L}\sin\phi_{B_L}\cos\phi_{B_L} m_b + \frac{\sin\phi}{v_R}\big(\sin\phi_{B_L}\cos\phi_{B_L}\cos^2\beta_{B_R} m_b \\ &\qquad - \cos^2\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_{b'}\big)\Big](\bar{b}'_L b_R + h.c.)h \\ &-\Big[\frac{\cos\phi}{v_L}\sin^2\phi_{B_L} m_{b'} - \frac{\sin\phi}{v_R}\big(\cos^2\phi_{B_L}\sin^2\beta_{B_R} m_{b'} \\ &\qquad - \sin\phi_{B_L}\cos\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_b\big)\Big]\bar{b}'b'h. \quad (8.5)\end{aligned}$$

By using the approximations for the mixing angles in Eqs.(C.26), (D.13), and (5.31), and considering the hierarchy in the bottom sector $v_L < v_R \ll M_B$, we obtain the interaction between the Higgs and bottom-sector quarks as follows

$$\begin{aligned}\mathcal{L}_{hb} \simeq & -\cos\phi\frac{m_b}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi\frac{m_b m_{b'}}{m_{d_L}m_{d_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)(\bar{b}_L b'_R + \bar{b}'_R b_L)h \\ & -\frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R}+\frac{v_R^2}{M_B^2}\right)(\bar{b}'_L b_R + \bar{b}_R b'_L)h - \cos\phi\frac{m_{d_L}}{m_{b'}}\left(1-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h. \qquad (8.6)\end{aligned}$$

Similar to the top sector, the interaction between the Higgs and the bottom quark pairs receives a small correction compared to the SM. The interaction between the Higgs and the heavy bottom quark pairs is suppressed by a factor $\mathcal{O}(v_L/M_B)$. The Higgs FCNC of $\bar{b}'_L b_R$ and $\bar{b}_R b'_L$ types are suppressed by a factor $\mathcal{O}(v_L/v_R)$. On the other hand, the Higgs FCNC of $\bar{b}_L b'_R$ and $\bar{b}'_R b_L$ type is not suppressed. This is because we assume $Y^3_{d_L} \simeq 1$.

8.2 Z FCNC

In this section we discuss the interaction between the Z boson and quarks. We begin by extracting the interaction terms between $Z_L - Z_R$ and quarks from Eq.(4.90), which reads as follows,

$$\begin{aligned}\mathcal{L}_q \supset \mathcal{L}_{ZZ'} = & -\left[\frac{g_L}{2\cos\theta_W}(j^\mu_{3L}) - e\tan\theta_W(j^\mu_{\text{em}})\right]Z_{L\mu} \\ & -\left[\frac{g_R}{2\cos\theta_R}(j^\mu_{3R}) - g'\tan\theta_R\left(j^\mu_{\text{em}} - \frac{1}{2}(j^\mu_{3L})\right)\right]Z_{R\mu}. \qquad (8.7)\end{aligned}$$

Here j^μ_{3L}, j^μ_{3R}, and j^μ_{em} are defined in Eqs.(4.92)-(4.94), respectively. Next, we change the basis from $Z_L - Z_R$ basis to the $Z - Z'$ basis using Eq.(5.14), which yields

$$\begin{aligned}\mathcal{L}_{ZZ'} = & -\left[\frac{1}{2\cos\theta_W}(g_L\cos\theta - e\tan\theta_R\sin\theta)j^\mu_{3L} - \frac{g_R\sin\theta}{2\cos\theta_R}j^\mu_{3R}\right. \\ & \left. -\frac{e}{\cos\theta_W}(\sin\theta_W\cos\theta - \tan\theta_R\sin\theta)j^\mu_{\text{em}}\right]Z_\mu \\ & -\left[\frac{1}{2\cos\theta_W}(g_L\sin\theta + e\tan\theta_R\cos\theta)j^\mu_{3L} + \frac{g_R\cos\theta}{2\cos\theta_R}j^\mu_{3R}\right. \\ & \left. -\frac{e}{\cos\theta_W}(\sin\theta_W\sin\theta + \tan\theta_R\cos\theta)j^\mu_{\text{em}}\right]Z'_\mu. \qquad (8.8)\end{aligned}$$

In this discussion, we will focus on the interaction between SM Z-boson with quarks. We expressed the Z-boson interaction in terms of vector and axial-vector couplings as follows,

$$\mathcal{L}_{ZZ'} \supset \mathcal{L}^{Z}_{\bar{q}q} = -\frac{g_L}{2\cos\theta_W}\sum_{\alpha,\beta=1}^{4}\overline{(\hat{u}^m)^\alpha}\gamma^\mu\left[(g_V)_u^{\alpha\beta} - (g_A)_u^{\alpha\beta}\gamma^5\right](\hat{u}^m)^\beta Z_\mu$$
$$-\frac{g_L}{2\cos\theta_W}\sum_{\alpha,\beta=1}^{4}\overline{(\hat{d}^m)^\alpha}\gamma^\mu\left[(g_V)_d^{\alpha\beta} - (g_A)_d^{\alpha\beta}\gamma^5\right](\hat{d}^m)^\beta Z_\mu, \tag{8.9}$$

where,

$$(g_V)_u^{\alpha\beta} = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} - (\kappa_{T_R})^{\alpha\beta}\right) - 2\kappa Q_u\delta^{\alpha\beta}, \tag{8.10}$$
$$(g_A)_u^{\alpha\beta} = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} + (\kappa_{T_R})^{\alpha\beta}\right), \tag{8.11}$$
$$(g_V)_d^{\alpha\beta} = -\frac{1}{2}\left((\kappa_{B_L})^{\alpha\beta} - (\kappa_{B_R})^{\alpha\beta}\right) - 2\kappa Q_d\delta^{\alpha\beta}, \tag{8.12}$$
$$(g_A)_d^{\alpha\beta} = -\frac{1}{2}\left((\kappa_{B_L})^{\alpha\beta} + (\kappa_{B_R})^{\alpha\beta}\right), \tag{8.13}$$
$$(\kappa_{T_L})^{\alpha\beta} = (\cos\theta - \sin\theta_W\tan\theta_R\sin\theta)(\mathcal{Z}^{\text{all}}_{T_L})^{\alpha\beta}, \tag{8.14}$$
$$(\kappa_{T_R})^{\alpha\beta} = \frac{\sin\theta_W\sin\theta}{\sin\theta_R\cos\theta_R}(\mathcal{Z}^{\text{all}}_{T_R})^{\alpha\beta}, \tag{8.15}$$
$$(\kappa_{B_L})^{\alpha\beta} = (\cos\theta - \sin\theta_W\tan\theta_R\sin\theta)(\mathcal{Z}^{\text{all}}_{B_L})^{\alpha\beta}, \tag{8.16}$$
$$(\kappa_{B_R})^{\alpha\beta} = \frac{\sin\theta_W\sin\theta}{\sin\theta_R\cos\theta_R}(\mathcal{Z}^{\text{all}}_{B_R})^{\alpha\beta}, \tag{8.17}$$
$$\kappa = \sin^2\theta_W\cos\theta - \sin\theta_W\tan\theta_R\sin\theta. \tag{8.18}$$

The matrix forms of 4×4 unitary matrices $\mathcal{Z}^{\text{all}}_{T_L}, \mathcal{Z}^{\text{all}}_{B_L}, \mathcal{Z}^{\text{all}}_{T_R}$, and $\mathcal{Z}^{\text{all}}_{B_R}$ are given as follows,

$$\mathcal{Z}^{\text{all}}_{T_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{T_L}\end{pmatrix}, \mathcal{Z}^{\text{all}}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{T_R}\end{pmatrix}, \mathcal{Z}^{\text{all}}_{B_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{B_L}\end{pmatrix}, \mathcal{Z}^{\text{all}}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{B_R}\end{pmatrix}, \tag{8.19}$$

where I_2 and 0_2 are 2×2 unit matrix and zero matrix respectively. The 2×2 submatrix $\mathcal{Z}_{T_L}, \mathcal{Z}_{B_L}, \mathcal{Z}_{T_R}$, and $\mathcal{Z}_{B_R}$ are given in Eqs.(4.84),(4.85),(4.88),(4.89) respectively and $Q_u = 2/3$, $Q_d = -1/3$ are the electric charge of up-type and down-type quarks respectively.

8.2.1 Up Sector

In this part, we analyze the interaction between Z-boson with the up sector in our model. From Eq. (8.9), it reads as,

$$\begin{aligned}\mathcal{L}^{Z}_{\bar{q}q} \supset \mathcal{L}^{Z}_{t} = -\frac{g_L}{2\cos\theta_W}&\Bigg\{\overline{(\hat{u}^m)^1}\gamma^\mu\Big[(g_V)^{11}_u - (g_A)^{11}_u\gamma^5\Big](\hat{u}^m)^1 \\ &+\overline{(\hat{u}^m)^2}\gamma^\mu\Big[(g_V)^{22}_u - (g_A)^{22}_u\gamma^5\Big](\hat{u}^m)^2 + \bar{t}\gamma^\mu\Big[(g_V)^{33}_u - (g_A)^{33}_u\gamma^5\Big]t \\ &+\bar{t}\gamma^\mu\Big[(g_V)^{34}_u - (g_A)^{34}_u\gamma^5\Big]t' + \bar{t'}\gamma^\mu\Big[(g_V)^{43}_u - (g_A)^{43}_u\gamma^5\Big]t \\ &+\bar{t'}\gamma^\mu\Big[(g_V)^{44}_u - (g_A)^{44}_u\gamma^5\Big]t'\Bigg\} Z_\mu, \end{aligned} \tag{8.20}$$

where the vector coupling $(g_V)_u$ and axial-vector coupling $(g_A)_u$ are defined in Eqs.(8.10)-(8.11) respectively. By using the definition of $\kappa_{T_L}, \kappa_{T_R}$ and κ which are written in Eqs.(8.14),(8.15), and (8.18) respectively, we obtain

$$(\kappa_{T_L})^{11} = (\kappa_{T_L})^{22} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{8.21}$$

$$(\kappa_{T_R})^{11} = (\kappa_{T_R})^{22} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{8.22}$$

$$(\kappa_{T_L})^{33} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{8.23}$$

$$(\kappa_{T_R})^{33} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{M_T^2}{m_{u_R}^2}, \tag{8.24}$$

$$(\kappa_{T_L})^{34} = (\kappa_{T_L})^{43} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{u_L}M_T}{m_{u_R}^2}, \tag{8.25}$$

$$(\kappa_{T_R})^{34} = (\kappa_{T_R})^{43} = -\frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{M_T}{m_{u_R}}, \tag{8.26}$$

$$(\kappa_{T_L})^{44} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{u_L}^2M_T^2}{m_{u_R}^4}, \tag{8.27}$$

$$(\kappa_{T_R})^{44} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{8.28}$$

$$\kappa = \cos\theta\left(\sin^2\theta_W - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right). \tag{8.29}$$

The suppression due to the small mixing angle θ is represented as $\mathcal{O}(v_L^2/v_R^2)$. The exact form of the mixing angle θ is given in Eq.(5.19). From Eqs.(8.25) and (8.26), the κ_{T_L} and κ_{T_R}, which are related to the Z-boson FCNC process with the top and heavy-top quarks are suppressed by $\mathcal{O}(v_L M_T/v_R^2)$ and $\mathcal{O}(v_L^2 M_T/v_R^3)$, respectively. This indicates that the Z-mediated FCNC process in the up sector is suppressed within our model. In addition, the interaction between Z-boson and heavy top quark is also suppressed. Moreover, the deviation of the SM-like terms in $(\kappa_{T_L})^{ii}$ and κ, with $i \in \{1,2,3\}$ are suppressed by a factor $\mathcal{O}(v_L^2/v_R^2)$.

8.2.2 Down Sector

In this part, we analyze the interaction between Z-boson and the down sector in our model. From Eq. (8.9), we extract,

$$\begin{aligned}
\mathcal{L}^Z_{\bar{q}q} \supset \mathcal{L}^Z_b = -\frac{g_L}{2\cos\theta_W} & \left\{ \overline{(\hat{d}^m)^1}\gamma^\mu \left[(g_V)^{11}_d - (g_A)^{11}_d \gamma^5 \right] (\hat{d}^m)^1 \right. \\
& + \overline{(\hat{d}^m)^2}\gamma^\mu \left[(g_V)^{22}_d - (g_A)^{22}_d \gamma^5 \right] (\hat{d}^m)^2 + \bar{b}\gamma^\mu \left[(g_V)^{33}_d - (g_A)^{33}_d \gamma^5 \right] b \\
& + \bar{b}\gamma^\mu \left[(g_V)^{34}_d - (g_A)^{34}_d \gamma^5 \right] b' + \overline{b'}\gamma^\mu \left[(g_V)^{43}_d - (g_A)^{43}_d \gamma^5 \right] b \\
& \left. + \overline{b'}\gamma^\mu \left[(g_V)^{44}_d - (g_A)^{44}_d \gamma^5 \right] b' \right\} Z_\mu,
\end{aligned} \tag{8.30}$$

where the vector coupling $(g_V)_d$ and axial-vector coupling $(g_A)_d$ are defined in Eqs.(8.12)-(8.13) respectively. By using the definition of $\kappa_{B_L}, \kappa_{B_R}$ and κ written in Eqs.(8.16),(8.17), and (8.18) respectively, we get

$$(\kappa_{B_L})^{11} = (\kappa_{B_L})^{22} = \cos\theta \left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \right), \tag{8.31}$$

$$(\kappa_{B_R})^{11} = (\kappa_{B_R})^{22} = \frac{\sin\theta_W \cos\theta}{\sin\theta_R \cos\theta_R} \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{8.32}$$

$$(\kappa_{B_L})^{33} = \cos\theta \left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \right), \tag{8.33}$$

$$(\kappa_{B_R})^{33} = \frac{\sin\theta_W \cos\theta}{\sin\theta_R \cos\theta_R} \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{8.34}$$

$$(\kappa_{B_L})^{34} = (\kappa_{B_L})^{43} = \cos\theta \left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \right) \frac{m_{d_L}}{M_B}, \tag{8.35}$$

$$(\kappa_{B_R})^{34} = (\kappa_{B_R})^{43} = -\frac{\sin\theta_W \cos\theta}{\sin\theta_R \cos\theta_R} \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \frac{m_{d_R}}{M_B}, \tag{8.36}$$

$$(\kappa_{B_L})^{44} = \cos\theta \left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \right) \frac{m_{d_L}^2}{M_B^2}, \tag{8.37}$$

$$(\kappa_{B_R})^{44} = \frac{\sin\theta_W \cos\theta}{\sin\theta_R \cos\theta_R} \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right) \frac{m_{d_R}^2}{M_B^2}, \tag{8.38}$$

$$\kappa = \cos\theta \left(\sin^2\theta_W - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right). \tag{8.39}$$

The FCNC process in the down sector is suppressed, similar to the up sector. As shown in Eqs.(8.35) and (8.36), the κ_{B_L} and κ_{B_R} are suppressed by factor $\mathcal{O}(v_L/M_B)$ and $\mathcal{O}(v_L^2/v_R M_B)$, respectively. In addition, the interaction between Z-boson and heavy bottom quark is also suppressed. Furthermore, the deviation of the SM-like terms in $(\kappa_{B_L})^{ii}$ and κ, with $i \in \{1, 2, 3\}$ are suppressed by a factor $\mathcal{O}(v_L^2/v_R^2)$.

Chapter 9

Summary

We have presented a systematic analysis of the quark sector of the universal seesaw model. We derived the Lagrangian of the model, including the quark sector, Higgs sector, and kinetic terms of the gauge fields. We start by writing the Lagrangian which is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)_{Y'}}$. After $\mathrm{SU(2)_R}$ Higgs doublet acquires non-zero vev, we obtain the Lagrangian, which is invariant under SM gauge symmetry. Furthermore, the SM gauge group is broken into $\mathrm{U(1)_{em}}$ after $\mathrm{SU(2)_L}$ Higgs doublet acquires non-zero vev. In the gauge interactions sector, we classify the terms based on the number of fields, such as linear, quadratic, cubic, and quartic interactions. Additionally, without fixing the gauge, we found that the massless Nambu-Goldstone bosons mix to form χ_Z and $\chi_{Z'}$. We have clearly shown that the degrees of freedom χ_Z and $\chi_{Z'}$ become the longitudinal components of the massive Z and Z' bosons, respectively.

Our model focuses on the third family of quark sector. Within this framework, we explain the hierarchy between the top and bottom quark masses by mixing with the heavy Vector-Like Quarks (VLQs). We use the direct measurement of the top quark mass and the running mass of the bottom quark. Additionally, the lower bounds on the heavy top and heavy bottom quark masses also serve as constraints. The lower mass limit of the Z'-boson, which is linked to the W_R boson mass, imposes a significant constraint on v_R. By setting g_R and the Yukawa couplings equal to 1, the lower limit of v_R in this model is 10 TeV. We found that the heavy top quark mass is in the order of v_R ($m_{t'} = 7.13$ TeV) and the heavy bottom mass is in the order of M_B ($m_{b'} = 293.82$ TeV). We confirmed that the hierarchy of VLQ's mass parameters, v_L, and v_R in our model is $v_L < M_T < v_R \ll M_B$.

Moreover, the presence of VLQs in the model induces the flavor-changing neutral currents (FCNCs) at the tree level. In the SM, the FCNC processes at tree-level are absent. In our model, we have shown that the Z-boson mediated FCNC process is suppressed for both (up and down) sectors. The deviations from the SM values are suppressed by $\mathcal{O}(v_L^2/v_R^2)$, which result from the small mixing in the lighter mass eigenstate Z from Z_R. On the other hand, Higgs mediated FCNCs of the $\bar{b}_L b'_R$ and $\bar{b}'_R b_L$ type are not suppressed when $Y^3_{d_L} \simeq 1$.

Acknowledgements

Throughout my Ph.D. journey at Hiroshima University, I have received much support and encouragement from many people. I want to express my heartfelt gratitude to all those who have contributed to this significant achievement:

First of all, I would like to express my deepest gratitude to my Lord and Savior, Jesus Christ. His grace, blessing, and guidance have been my foundation throughout this journey. I owe everything to Him.

I would also like to thank my beloved family: my parents, Yoachim Agus Tridiatno and Fredeswinda Nur Widayati, my parents-in-law, Emanuel Bangkit Dami Arsa and Benedecta Runding Irianna, my grandmother, Catarina Yamini, my brothers, Yohanes Paulus Jujur Kinanthi and Yohanes Paulus Adil Pinasti, my sister, Benedicta Anggi Mutiarani Purba from Indonesia, who always support and pray for me.

I am also deeply thankful to my wife, Claudia Nada Pingkan Larasati, and our two sons, Kristoforus Narayana Narendra and Hieronimus Ranadhira Sagara. Your love and encouragement have made my journey in Saijo not just bearable but truly joyful. Because of your presence, I am always enthusiastic and happy during this journey of my study.

I am profoundly thankful to my Ph.D. supervisor, Takuya Morozumi, for his invaluable guidance, insightful feedback, and support from the first day I set foot in Japan. His expertise in field theory and theoretical particle physics has deepened my enthusiasm for studying and conducting research in this field more and more. Thank you for your patience and dedication to our research collaboration.

I would like to thank my research collaborators and discussion partners: Apriadi Salim Adam, Yuta Kawamura, Yusuke Shimizu, Hironori Takei, Kei Yamamoto, Hiroyuki Umeeda, Satoshi Tanaka, Bayu Dirgantara, Lalu Zamakhsyari, and Reinard Primulando for the fruitful discussions. Hopefully, we will continue our discussions in the future. In addition, I am thankful to Mirza Satriawan, my former supervisor, for allowing me to refresh my knowledge of particle physics by entering his class while I was preparing for my studies. It was very helpful for me, as I graduated from a master's course almost ten years ago.

I would like to express my gratitude to the other staff members in the Theoretical Elementary Particle and Hadron Physics Group (Sohaken): Chiho Nonaka, Tomohiro Inagaki, Ken-Ichi Ishikawa, Azumi Sakai, and Nicholas J. Benoit for their discussions and comments during my study, also, to all the students who are members of Sohaken. Thank you for the discussions and, most importantly, for our friendship during my study. A special thank you goes to Nicholas, who not only guided me as a new student in Japan but also took my family and me to various festivals, helping us feel at home. I am also deeply grateful to Sakai for assisting me in translating numerous documents. Your help was truly appreciated.

I would like to thanks to Apriadi Salim Adam (BRIN) and Ar Rohim (University of Indonesia) during my preparation of study. I also want to express my gratitude to the Indonesian Student Association in Hiroshima (PPI Hiroshima) for their support during my family and my stay in Saijo. Also, to my Indonesian colleagues in Sohaken, Abdi and Azzam, thank you for your support. I hope we can do some study and collaboration together in the future. I also thank to the Student Support Office for the Fields of Science and Physics Office for the help, guidance, and support during my study.

Finally, I would like to thank the Japanese Government Ministry of Education, Culture, Sports, Science, and Technology (MEXT) for the financial support for my studies. In addition, I thank Rectorate of Sanata Dharma University and "Pengurus Yayasan Sanata Dharma" for their support and prayers during my studies. Additionally, I would like to thank the Dean of the Faculty of Teacher Training and Education, the Head of the Physics Education Study Program, and all my colleagues at Sanata Dharma University for their continued support.

Bibliography

[1] G. Aad *et al.* (ATLAS Collaboration). Phys. Lett. B **716**:1-29 (2012) [arXiv:1207.7214[hep-ex]].

[2] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**:30-61 (2012) [arXiv:1207.7235[hep-ex]].

[3] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1562 (1998) [arXiv:9807003[hep-ex]].

[4] Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **89**, 011301 (2002) [arXiv:0204008[nucl-ex]].

[5] T. Kajita, Rev. Mod. Phys. **88**, 030501 (2016).

[6] K. Abe *et al.* (Super-Kamiokande Collaboration), Phys, Rev. D **97** 072001 (2018) [arXiv:1710.09126[hep-ex]].

[7] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, Astrophys. J. Lett. **648** L109-L113 (2006) [arXiv:0608407[astro-ph]].

[8] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **87**, 091802 (2001) [arXiv:0107061[hep-ex]].

[9] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. Lett. **96**, 251802 (2006) [arXiv:0603053[hep-ex]].

[10] M. Fukugita and T. Yanagida, Phys. Lett. B **174** 45-47 (1986)

[11] S. Navas *et al.* (Particle Data Group), to be published in Phys. Rev. D **110**, 030001 (2024).

[12] P. Minkowski, Phys. Lett. B **67** 421-428 (1977).

[13] T. Yanagida, Conf. Proc. C **7902131** 95-99 (1979).

[14] M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C **790927** 315-321 (1979) [arXiv:1306.4669[hep-th]].

[15] T. Yanagida, Prog. Theor. Phys. **64**, 3 (1980).

[16] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** 912 (1980).

[17] D. Wyler and L. Wolfenstein, Nucl. Phys. B **218** 205-214 (1983).

[18] R. Foot, H. Lew, X.-G. He, and G. C. Joshi, Z. Phys. C **44**, 441 (1989).

[19] V. Brdar, A. J. Helmboldt, S. Iwamoto, and K. Schmitz, Phys. Rev. D **100** 075029 (2019) [arXiv:1905.12634[hep-ph]].

[20] Z. G. Berezhiani, Phys. Lett. B **129** 99-102 (1983).

[21] Z. G. Berezhiani, Phys. Lett. B **150** 177-181 (1985).

[22] S. Rajpoot, Phys. Lett. B **191** 122-126 (1987).

[23] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58** 1600 (1987).

[24] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59** 393 (1987).

[25] Z. G. Berezhiani and R. Rattazi, Phys. Lett. B **279** 124-130 (1992).

[26] Z. G. Berezhiani and R. Rattazi, Nucl. Phys. B **407** 249-270 (1993) [arXiv:9212245[hep-ph]].

[27] Y. Koide and H. Fusaoka, Z. Phys. C **71**, 459-468 (1996)[arXiv:9505201[hep-ph]].

[28] T. Morozumi, T. Satou, M. N. Rebelo, and M. Tanimoto, Phys. Lett. B **410**, 233-240 (1997) [arXiv:9703249[hep-ph]].

[29] Y. Kiyo, T. Morozumi, P. Parada, M. N. Rebelo, and M. Tanimoto, Prog. Theor. Phys. **101**, 3 (1999)[arXiv:9809333[hep-ph]].

[30] R. Kawasaki, T. Morozumi, and H. Umeeda, Phys. Rev. D **88**, 033019 (2013) [arXiv:1306.5080[hep-ph]].

[31] A. Patra and S. K. Rai, Phys. Rev. D **98**, 015033 (2018) [arXiv:1711.00627[hep-ph]].

[32] A. E. C. Hernandez, J. M. Gonzalez, and U. J. Saldana-Salazar, Phys. Rev. D **100** 035024 (2019) [arXiv:1904.09993[hep-ph]].

[33] S.-P. Chen and P.-H. Gu, Nucl. Phys. B **986**, 116057 (2023) [arXiv:2211.01906[hep-ph]].

[34] R. Dcruz and K. S. Babu, Phys. Rev. D **108**, 095011 (2023) [arXiv:2212.09697[hep-ph]].

[35] R. Dcruz, Nucl. Phys. B **1001** 116519 [arXiv:2301.10786[hep-ph]].

[36] T. Morozumi, A. S. Adam, Y. Kawamura, A. H. Panuluh, Y. Shimizu and K. Yamamoto, J. Phys.: Conf. Ser. **2446** 012046 (2023) [arXiv:2211.02360[hep-ph]].

[37] B. Belfatto, R. Beradze, Z. Berezhiani, Eur. Phys. J. C **80**, 149 (2020) [arXiv:1906.02714[hep-ph]].

[38] G. C. Branco, J. T. Penedo, Pedro M. F. Pereira, M. N. Rebelo, J. I. Silva-Marcos, J. High Energy Phys. **07**, 099 (2021) [arXiv:2103.13409[hep-ph]].

[39] F. Albergaria, G. C. Branco, J. F. Bastos, and J. I. Silva-Marcos, J. Phys. G: Nucl. Part. Phys. **50** 055001 (2023) [arXiv:2210.14248[hep-ph]].

[40] S. Balaji, J. High Energy Phys. **05**, 015 (2022) [arXiv:2110.05473[hep-ph]].

[41] A. K. Alok and S. Gangal, Phys. Rev. D **86** 114009 (2012) [arXiv:1209.1987[hep-ph]].

[42] F. Albergaria, G. C. Branco, J. F. Bastos, and J. I. Silva-Marcos, J. Phys. G: Nucl. Part. Phys. **50** 055001 (2023) [arXiv:2210.14248[hep-ph]].

[43] A. H. Panuluh and T. Morozumi, J. Phys.: Conf. Ser. **2446** 012049 (2023) [arXiv:2211.02359[hep-ph]].

[44] T. Morozumi, Y. Shimizu, S. Takahashi, and H. Umeeda, Prog. Theor. Exp. Phys. **2018**, 043B10 (2018) [arXiv:1801.05268[hep-ph]].

[45] J. M. Alves, G. C. Branco, A. L. Cherchiglia, C. C. Nishi, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Rept. **1057** 1-69 (2024) [arXiv:2304.10561[hep-ph]].

[46] T. Morozumi and A. H. Panuluh, Prog. Theor. Exp. Phys. (2024) DOI: 10.1093/ptep/ptae127 [arXiv:2407.00732[hep-ph]].

[47] A. Pich, hep-ph/0502010 (2005) [arXiv:0502010[hep-ph]].

[48] G. Altarelli, hep-ph/0510281 (2005) [arXiv:0510281[hep-ph]].

[49] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[50] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[51] S. L. Glashow, Nucl. Phys. **22**, 579-588 (1961).

[52] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[53] A. Salam, Conf. Proc. C **680519**, 367-377 (1968) DOI:10.1142/9789812795915_0034.

[54] H. Bethe and R. Peierls, Nature **133**, 532 (1934).

[55] C. L. Cowan, F. Reines, F. B. Harrison, H. W. Kruse and A. D. McGuire, Science **124**, 103-104 (1956).

[56] B. T. Cleveland, T. Daily, R. Davis, Jr., J. R. Distel, K. Lande, C. K. Lee, P. S. Wildenhain and J. Ullman, Astrophys. J. **496**, 505-526 (1998).

[57] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975).

[58] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).

Appendix A

Weak-basis of Yukawa interaction

In this appendix, we show how to obtain the Yukawa interaction which is written in Eq.(3.3). We start from the general Yukawa interaction terms,

$$\begin{aligned}\mathcal{L}_{\rm YM} = &-\overline{q^i_L} y^i_{u_L} \tilde{\phi}_L T_R - \overline{T_L} y^{i*}_{u_R} \tilde{\phi}^\dagger_R q^i_R - \overline{T_L} M_T T_R - h.c. \\ &- \overline{q^i_L} y^i_{d_L} \phi_L B_R - \overline{B_L} y^{i*}_{d_R} \phi^\dagger_R q^i_R - \overline{B_L} M_B B_R - h.c.\end{aligned} \tag{A.1}$$

The Yukawa couplings are general complex vectors in $\mathbb{C}^3$ with following parameterization,

$$y^i_{u_{L(R)}} = \mathbf{y}_{u_{L(R)}} = \begin{pmatrix} \sin\theta^u_{L(R)} \sin\phi^u_{L(R)} e^{i\alpha^1_{u_{L(R)}}} \\ \sin\theta^u_{L(R)} \cos\phi^u_{L(R)} e^{i\alpha^2_{u_{L(R)}}} \\ \cos\theta^u_{L(R)} e^{i\alpha^3_{u_{L(R)}}} \end{pmatrix} Y^3_{u_{L(R)}}, \tag{A.2}$$

$$y^i_{d_{L(R)}} = \mathbf{y}_{d_{L(R)}} = \begin{pmatrix} \sin\theta^d_{L(R)} \sin\phi^d_{L(R)} e^{i\alpha^1_{d_{L(R)}}} \\ \sin\theta^d_{L(R)} \cos\phi^d_{L(R)} e^{i\alpha^2_{d_{L(R)}}} \\ \cos\theta^d_{L(R)} e^{i\alpha^3_{d_{L(R)}}} \end{pmatrix} Y^3_{d_{L(R)}}, \tag{A.3}$$

where $Y^3_{u_{L(R)}}$ and $Y^3_{d_{L(R)}}$ are real positive numbers. Define following weak-basis transformation (WBT) as follows,

$$(q'_L)^i = e^{-i\alpha^i_{u_L}} q^i_L, \tag{A.4}$$

$$(q'_R)^i = e^{-i\alpha^i_{u_R}} q^i_R. \tag{A.5}$$

Applying this WBT into Eq.(A.1), we obtain

$$\mathcal{L}_{\rm YM} = -\overline{(q'_L)^i} (y'_{u_L})^i \tilde{\phi}_L T_R - \overline{T_L} (y'_{u_R})^{i*} \tilde{\phi}^\dagger_R (q'_R)^i - \overline{T_L} M_T T_R - h.c.$$

$$- \overline{(q'_L)^i} y^i_{d_L} \phi_L B_R - \overline{B_L} y^{i*}_{d_R} \phi^\dagger_R (q'_R)^i - \overline{B_L} M_B B_R - h.c., \tag{A.6}$$

where

$$(y'_{u_L})^i = y^i_{u_L} e^{-i\alpha^i_{u_L}}, \tag{A.7}$$
$$(y'_{u_R})^i = y^i_{u_R} e^{-i\alpha^i_{u_R}} \tag{A.8}$$

are real vectors. On the other hand, $y^i_{d_L}$ and $y^i_{d_R}$ remain complex vectors with the redefined phases.

Next we write the $(y'_{u_L})^i$ Yukawa coupling explained above as,

$$\begin{aligned}(y'_{u_L})^i &= \begin{pmatrix} \sin\theta^u_L \sin\phi^u_L \\ \sin\theta^u_L \cos\phi^u_L \\ \cos\theta^u_L \end{pmatrix} Y^3_{u_L} \\ &= \mathbf{e}^u_{L_3} Y^3_{u_L} \end{aligned} \tag{A.9}$$

and defining another WBT,

$$(q'_L)^i = (V_{u_L})^{ij} (q''_L)^j, \tag{A.10}$$

where in general V_{u_L} is 3×3 unitary matrix which formed by three orthonormal vectors with the third column is chosen as $\mathbf{e}^u_{L_3}$ in Eq.(A.9),

$$V_{u_L} = \begin{pmatrix} \mathbf{e}^u_{L_1} & \mathbf{e}^u_{L_2} & \mathbf{e}^u_{L_3} \end{pmatrix}. \tag{A.11}$$

which leads the product $(V^\dagger_{u_L})^{ji} (y'_{u_L})^i = \delta^{j3} Y^3_{u_L}$.

For the $(y'_{u_R})^i$ Yukawa coupling can be derived similarly by changing $L \to R$ in Eq.(A.9) - (A.11). For the down-sector, product Eq.(A.11) and the down-type Yukawa coupling yield down-type Yukawa coupling in another basis. For example, $(V^\dagger_{u_L})^{ji} (y_{d_L})^i = (y''_{d_L})^j$. Therefore, the Lagrangian in Eq.(A.6) become,

$$\begin{aligned}\mathcal{L}_{\rm YM} = &-Y^3_{u_L} \overline{(q''_L)^3} \tilde{\phi}_L T_R - Y^3_{u_R} \overline{T_L} \tilde{\phi}^\dagger_R (q''_R)^3 - \overline{T_L} M_T T_R - h.c. \\ &- \overline{(q''_L)^i} (y''_{d_L})^i \phi_L B_R - \overline{B_L} (y''_{d_R})^{i*} \phi^\dagger_R (q''_R)^i - \overline{B_L} M_B B_R - h.c. \end{aligned} \tag{A.12}$$

and it has form the Yukawa couplings of up-type quark doublet ($Y^3_{u_L}$ and $Y^3_{u_R}$) are real positive numbers while the Yukawa couplings of down-type quark are general complex vectors as written in Eq.(3.3).

Appendix B

Parameterization of V_{d_R} and V_{d_L}

In this appendix, we explain more details of how to parameterize and remove the unphysical phases of V_{d_R} and V_{d_L}. Both V_{d_R} and V_{d_L} have the following form

$$V = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{pmatrix}, \tag{B.1}$$

where the third column is related to either y_{d_R} or y_{d_L} and is parameterized by

$$\mathbf{v}_3 = \begin{pmatrix} \sin\theta \sin\phi e^{i\alpha_1} \\ \sin\theta \cos\phi e^{i\alpha_2} \\ \cos\theta e^{i\alpha_3} \end{pmatrix}. \tag{B.2}$$

Since V is a unitary matrix, the column vector satisfy $\mathbf{v_i}^\dagger \cdot \mathbf{v_j} = \delta_{ij}$ and has matrix form as follow,

$$V = (\alpha_1, \alpha_2, \alpha_3) R_{12}(\phi) R_{23}(\theta)(0, \delta, 0) R_{12}(\alpha)(\rho, \sigma, 0), \tag{B.3}$$

where $(\alpha_1, \alpha_2, \alpha_3) = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$; $(0, \delta, 0) = \mathrm{diag}(1, e^{i\delta}, 1)$;$(\rho, \sigma, 0) = \mathrm{diag}(e^{i\rho}, e^{i\sigma}, 1)$ and

$$R_{12}(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{23}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix},$$

$$R_{12}(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4}$$

We have the freedom to rotate V by U(2) transformations from the both sides. As shown in Eqs.(4.18) and (4.57), we can remove the unphysical phases and angles in Eq.(B.3) by following,

$$\widetilde{V} = \widetilde{U}^\dagger V \widetilde{W}, \tag{B.5}$$

where $\widetilde{U}$ and $\widetilde{W}$ are 3×3 unitary matrices which have following expressions,

$$\begin{aligned} \widetilde{U}^\dagger &= (0, \frac{\alpha_3}{2}, 0) R_{12}^{-1}(\phi)(-\alpha_1, -\alpha_2, 0), \\ \widetilde{W} &= (-\rho, -\sigma, 0) R_{12}^{-1}(\alpha)(0, -\delta, 0)(0, -\frac{\alpha_3}{2}, 0). \end{aligned} \tag{B.6}$$

Thus we obtain,

$$\widetilde{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{i\frac{\alpha_3}{2}} \\ 0 & -\sin\theta e^{i\frac{\alpha_3}{2}} & \cos\theta e^{i\alpha_3} \end{pmatrix}. \tag{B.7}$$

Appendix C

Diagonalization of quark mass matrix

In this appendix, we derive the exact mass eigenvalues of the top-bottom SM quarks and the heavy VLQ partners, as well as the matrices used for the diagonalization procedure. We will show the diagonalization procedure for the top sector. The bottom sector can be done similarly because the form of $\mathbb{M}_b$ is the same as $\mathbb{M}_t$. We start from Eq.(4.66), explicitly writing the $(W_{T_R})^{43}$ and $(W_{T_R})^{44}$ values,

$$\mathbb{M}_t \equiv \begin{pmatrix} -\frac{Y^3_{u_L} Y^3_{u_R} v_L v_R}{2m_{u_4}} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} \frac{M_T}{m_{u_4}} \\ 0 & m_{u_4} \end{pmatrix} = \begin{pmatrix} -m_{t_1} & m_{t_2} \\ 0 & m_{u_4} \end{pmatrix}, \tag{C.1}$$

where m_{t_1} and m_{t_2} in Eq.(C.1) are not mass eigenvalues but are defined as follows,

$$m_{t_1} = \frac{Y^3_{u_L} Y^3_{u_R} v_L v_R}{2m_{u_4}}, \qquad m_{t_2} = Y^3_{u_L} \frac{v_L}{\sqrt{2}} \frac{M_T}{m_{u_4}}. \tag{C.2}$$

The top quark mass matrix in Eq.(C.1) can be diagonalized by bi-unitary transformation, which gives,

$$K^\dagger_{T_L} \mathbb{M}_t K_{T_R} = (m_t^{\text{diag}}) = \text{diag}(m_t, m_{t'}). \tag{C.3}$$

Initially, we transform $\mathbb{M}_t$ into a real symmetric matrix by multiplying it on the left side by an orthogonal matrix S_t, which yields

$$\mathbb{M}'_t = S_t \mathbb{M}_t, \tag{C.4}$$

where,

$$S_t = \begin{pmatrix} \cos\phi_{T_l} & -\sin\phi_{T_l} \\ \sin\phi_{T_l} & \cos\phi_{T_l} \end{pmatrix}. \tag{C.5}$$

$\mathbb{M}'_t$ becomes a real symmetric matrix with the following expression

$$\mathbb{M}'_t = \begin{pmatrix} -m_{t_1}\cos\phi_{T_l} & -m_{t_1}\sin\phi_{T_l} \\ -m_{t_1}\sin\phi_{T_l} & m_{t_2}\sin\phi_{T_l} + m_{u_4}\cos\phi_{T_l} \end{pmatrix} \tag{C.6}$$

if the mixing angle satisfies the following condition:

$$\tan\phi_{T_l} = \frac{m_{t_2}}{m_{u_4} - m_{t_1}}. \tag{C.7}$$

Then, a real symmetric matrix can be diagonalized by multiplying from both sides another 2×2 orthogonal matrix R_t and its transpose,

$$R_t\,\mathbb{M}'_t R_t^T = \mathrm{diag}(-m_t, m_{t'}), \tag{C.8}$$

where,

$$R_t = \begin{pmatrix} \cos\phi_{T_R} & \sin\phi_{T_R} \\ -\sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix}. \tag{C.9}$$

The minus sign inside the diagonal matrix on the right-hand side of Eq.(C.8) arises because the determinant of the top quark mass matrix $\mathbb{M}_t$ is negative. Since m_t is lighter than m'_t, we assign the minus sign to m_t. However, we could eliminate the minus sign by multiplying Eq.(C.8) by $-\tau_3$ on the right side, where τ_3 is the third component of the Pauli matrices. The mixing angle can then be obtained as:

$$\tan 2\phi_{T_R} = \frac{2m_{t_1}m_{t_2}}{m_{u_4}^2 + m_{t_2}^2 - m_{t_1}^2}. \tag{C.10}$$

The eigenvalues of Eq.(C.8) can be computed using the following equation,

$$\lambda^2 - (\mathrm{tr}\mathbb{M}'_t)\lambda + \mathrm{det}\mathbb{M}'_t = 0. \tag{C.11}$$

After performing the calculations, we obtain

$$\lambda_1 = -m_t = \frac{\sqrt{m_{t_2}^2 + (m_{u_4} - m_{t_1})^2}}{2} - \frac{\sqrt{m_{t_2}^2 + (m_{u_4} + m_{t_1})^2}}{2}, \tag{C.12}$$

$$\lambda_2 = m_{t'} = \frac{\sqrt{m_{t_2}^2 + (m_{u_4} - m_{t_1})^2}}{2} + \frac{\sqrt{m_{t_2}^2 + (m_{u_4} + m_{t_1})^2}}{2}. \tag{C.13}$$

We can also equivalently express the explicit mass eigenvalues in the following form,

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}, \tag{C.14}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}, \tag{C.15}$$

where,

$$m_{u_R} = Y^3_{u_R}\frac{v_R}{\sqrt{2}}, \qquad m_{u_L} = Y^3_{u_L}\frac{v_L}{\sqrt{2}}. \tag{C.16}$$

Finally, we can summarize all the matrix transformations explained above as,

$$R_t S_t \mathbb{M}_t R_t^T(-\tau_3) = \text{diag}(m_t, m_{t'}). \tag{C.17}$$

Additionally, the product of two orthogonal matrices is also an orthogonal matrix. Then we can define O_t as,

$$O_t = R_t S_t = \begin{pmatrix} \cos\phi_{T_L} & \sin\phi_{T_L} \\ -\sin\phi_{T_L} & \cos\phi_{T_L} \end{pmatrix} \tag{C.18}$$

with $\phi_{T_L} = \phi_{T_R} - \phi_{T_l}$. Hence, by comparing Eq.(C.17) and Eq.(C.3) we obtain the expression for the mixing matrices as follows.

$$K^\dagger_{T_L} = \begin{pmatrix} \cos\phi_{T_L} & \sin\phi_{T_L} \\ -\sin\phi_{T_L} & \cos\phi_{T_L} \end{pmatrix}, \tag{C.19}$$

$$K_{T_R} = \begin{pmatrix} \cos\phi_{T_R} & -\sin\phi_{T_R} \\ \sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos\phi_{T_R} & -\sin\phi_{T_R} \\ -\sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix}. \tag{C.20}$$

For the bottom sector, we can derive the results similarly by replacing t with b, T with B, and u with d. Thus, we write the mass eigenvalues and the mixing matrices for the bottom sector as follows,

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}, \tag{C.21}$$

$$m_{b'} = \frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}, \tag{C.22}$$

where,

$$m_{d_R} = Y^3_{d_R}\frac{v_R}{\sqrt{2}}, \qquad m_{d_L} = Y^3_{d_L}\frac{v_L}{\sqrt{2}}, \tag{C.23}$$

$$K^{\dagger}_{B_L} = \begin{pmatrix} \cos\phi_{B_L} & \sin\phi_{B_L} \\ -\sin\phi_{B_L} & \cos\phi_{B_L} \end{pmatrix}, \tag{C.24}$$

$$K_{B_R} = \begin{pmatrix} \cos\phi_{B_R} & -\sin\phi_{B_R} \\ \sin\phi_{B_R} & \cos\phi_{B_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos\phi_{B_R} & -\sin\phi_{B_R} \\ -\sin\phi_{B_R} & \cos\phi_{B_R} \end{pmatrix}. \tag{C.25}$$

While the approximate masses are already written in Eqs.(7.9)-(7.12), the approximate mixing angles are given as follows,

$$\sin\phi_{T_L} \simeq -\frac{m_{u_L} M_T}{M_T^2 + m_{u_R}^2}, \quad \cos\phi_{T_L} \simeq 1, \quad \sin\phi_{T_R} \simeq \frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2 + m_{u_R}^2)^2}, \quad \cos\phi_{T_R} \simeq 1$$

$$\sin\phi_{B_L} \simeq -\frac{m_{d_L}}{M_B}, \quad \cos\phi_{B_L} \simeq 1, \quad \sin\phi_{B_R} \simeq \frac{m_{d_L}^2 m_{d_R}}{M_B^3}, \quad \cos\phi_{B_R} \simeq 1. \tag{C.26}$$

Using the approximate angles, one can write the approximate form for the matrices as follows,

$$K^{\dagger}_{T_L} \simeq \begin{pmatrix} 1 & -\frac{m_{u_L} M_T}{M_T^2+m_{u_R}^2} \\ \frac{m_{u_L} M_T}{M_T^2+m_{u_R}^2} & 1 \end{pmatrix}, \qquad K_{T_R} \simeq \begin{pmatrix} 1 & -\frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2+m_{u_R}^2)^2} \\ -\frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2+m_{u_R}^2)^2} & 1 \end{pmatrix} \tag{C.27}$$

$$K^{\dagger}_{B_L} \simeq \begin{pmatrix} 1 & -\frac{m_{d_L}}{M_B} \\ \frac{m_{d_L}}{M_B} & 1 \end{pmatrix}, \qquad K_{B_R} \simeq \begin{pmatrix} -1 & -\frac{m_{d_L}^2 m_{d_R}}{M_B^3} \\ -\frac{m_{d_L}^2 m_{d_R}}{M_B^3} & 1 \end{pmatrix}. \tag{C.28}$$

Appendix D

CKM Matrices

In this appendix, we will discuss CKM-like matrices in this model and the rephasing of the CKM-like matrices. CKM-like matrix, which appears for the first time in Section 4, is an "intermediate" right-handed CKM-like matrix which has explicit form as follows,

$$V_R^{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & s_{\theta_R^d} c_{\theta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_R^d} s_{\theta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} \\ 0 & -c_{\theta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & c_{\theta_{T_R}} c_{\theta_R^d} c_{\theta_{B_R}} e^{i\alpha^3_{d_R}} & c_{\theta_{T_R}} c_{\theta_R^d} s_{\theta_{B_R}} e^{i\alpha^3_{d_R}} \\ 0 & -s_{\theta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_{T_R}} c_{\theta_R^d} c_{\theta_{B_R}} e^{i\alpha^3_{d_R}} & s_{\theta_{T_R}} c_{\theta_R^d} s_{\theta_{B_R}} e^{i\alpha^3_{d_R}} \end{pmatrix}, \tag{D.1}$$

where,

$$\begin{aligned} c_{\theta_R^d} &= \cos\theta_R^d, \quad s_{\theta_R^d} = \sin\theta_R^d, \quad c_{\theta_{T_R}} = \cos\theta_{T_R}, \\ s_{\theta_{T_R}} &= \sin\theta_{T_R}, \quad c_{\theta_{B_R}} = \cos\theta_{B_R}, \quad s_{\theta_{B_R}} = \sin\theta_{B_R}. \end{aligned} \tag{D.2}$$

After Step 6 is done, we have the final expression of the left-handed CKM-like matrix and right-handed CKM-like matrix, which are defined in Eq.(4.73) and Eq.(4.74), respectively. The matrix forms of the left-handed CKM-like matrix and right-handed CKM-like matrix are as follows,

$$\mathcal{V}_L^{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_L^d} & s_{\theta_L^d} c_{\phi_{B_L}} e^{i\frac{\alpha^3_{d_L}}{2}} & -s_{\theta_L^d} s_{\phi_{B_L}} e^{i\frac{\alpha^3_{d_L}}{2}} \\ 0 & -c_{\phi_{T_L}} s_{\theta_L^d} e^{i\frac{\alpha^3_{d_L}}{2}} & c_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} e^{i\alpha^3_{d_L}} & -c_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} e^{i\alpha^3_{d_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta_L^d} e^{i\frac{\alpha^3_{d_L}}{2}} & -s_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} e^{i\alpha^3_{d_L}} & s_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} e^{i\alpha^3_{d_L}} \end{pmatrix}, \tag{D.3}$$

where

$$
\begin{aligned}
&c_{\theta_L^d} = \cos\theta_L^d, \quad s_{\theta_L^d} = \sin\theta_L^d, \quad c_{\phi_{T_L}} = \cos\phi_{T_L}, \\
&s_{\phi_{T_L}} = \sin\phi_{T_L}, \quad c_{\phi_{B_L}} = \cos\phi_{B_L}, \quad s_{\phi_{B_L}} = \sin\phi_{B_L}.
\end{aligned} \tag{D.4}
$$

and

$$
\mathcal{V}_R^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & -s_{\theta_R^d} c_{\beta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_R^d} s_{\beta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & c_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\alpha^3_{d_R}} & -c_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\alpha^3_{d_R}} \\ 0 & -s_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & -s_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\alpha^3_{d_R}} & s_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\alpha^3_{d_R}} \end{pmatrix}, \tag{D.5}
$$

where

$$
\begin{aligned}
&c_{\theta_R^d} = \cos\theta_R^d, \quad s_{\theta_R^d} = \sin\theta_R^d, \quad c_{\beta_{T_R}} = \cos\beta_{T_R}, \\
&s_{\beta_{T_R}} = \sin\beta_{T_R}, \quad c_{\beta_{B_R}} = \cos\beta_{B_R}, \quad s_{\beta_{B_R}} = \sin\beta_{B_R}, \\
&\beta_{T_R} = \theta_{T_R} - \phi_{T_R}, \quad \beta_{B_R} = \theta_{B_R} - \phi_{B_R}.
\end{aligned} \tag{D.6}
$$

Recall the mass terms in the diagonal mass basis (including the massless two lightest quark fields) as follows,

$$
\begin{aligned}
\mathcal{L}_q \supset \mathcal{L}_{\rm mass} = & -\overline{(u_L^m)^\alpha} (m_t^{\rm diag})^\alpha (u_R^m)^\alpha - h.c. \\
& - \overline{(d_L^m)^\alpha} (m_b^{\rm diag})^\alpha (d_R^m)^\alpha - h.c.
\end{aligned} \tag{D.7}
$$

We have the freedom to rephase the quark fields with following transformations,

$$
(u^m_{L(R)})^\alpha = (\theta_{u_{L(R)}})^\alpha \delta^{\alpha\beta} (\hat{u}^m_{L(R)})^\beta, \tag{D.8}
$$

$$
(d^m_{L(R)})^\alpha = (\theta_{d_{L(R)}})^\alpha \delta^{\alpha\beta} (\hat{d}^m_{L(R)})^\beta, \tag{D.9}
$$

where $\theta_{u_{L(R)}} = {\rm diag}(e^{i\theta_{u_{L(R)1}}}, e^{i\theta_{u_{L(R)2}}}, e^{i\theta_{u_3}}, e^{i\theta_{u_4}})$ and $\theta_{d_{L(R)}} = {\rm diag}(e^{i\theta_{d_{L(R)1}}}, e^{i\theta_{d_{L(R)2}}}, e^{i\theta_{d_3}}, e^{i\theta_{d_4}})$. One can show that Eq.(D.7) is invariant under transformation in Eq.(D.8)-(D.9).

We apply this rephasing transformation into the $\mathcal{L}_q$. The left-handed and right-handed CKM-like matrices are rephased and become,

$$
\hat{\mathcal{V}}_L^{\rm CKM} = \theta^\dagger_{u_L} \mathcal{V}_L^{\rm CKM} \theta_{d_L}, \qquad \hat{\mathcal{V}}_R^{\rm CKM} = \theta^\dagger_{u_R} \mathcal{V}_R^{\rm CKM} \theta_{d_R}, \tag{D.10}
$$

By choosing proper phase and phase difference, we could rephase the left-handed and right-handed CKM-like matrices and they become following matrix forms,

$$\hat{\mathcal{V}}_L^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_L^d} & s_{\theta_L^d} c_{\phi_{B_L}} & -s_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & -c_{\phi_{T_L}} s_{\theta_L^d} & c_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & -c_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta_L^d} & -s_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & s_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \end{pmatrix}, \tag{D.11}$$

$$\hat{\mathcal{V}}_R^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & -s_{\theta_R^d} c_{\beta_{B_R}} e^{i\frac{\delta}{2}} & s_{\theta_R^d} s_{\beta_{B_R}} e^{i\frac{\delta}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & c_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & -c_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \\ 0 & -s_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & -s_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & s_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \end{pmatrix}, \tag{D.12}$$

where we redefine the phase difference as $\delta = \alpha^3_{d_R} - \alpha^3_{d_L}$. Therefore, in this model, we have one CP violating phase δ and in our choice, it is included in the right-handed CKM-like matrix as shown in Eq.(D.12).

Moreover, the mixing angle β_{T_R} and β_{B_R} can be expressed in the approximate form as,

$$\sin\beta_{T_R} \simeq \frac{m_{u_R}}{\sqrt{M_T^2 + m_{u_R}^2}}, \quad \cos\beta_{T_R} \simeq \frac{M_T}{\sqrt{M_T^2 + m_{u_R}^2}}, \quad \sin\beta_{B_R} \simeq \frac{m_{d_R}}{M_B}, \quad \cos\beta_{B_R} \simeq 1. \tag{D.13}$$