Thesis Summary

Relativistic Effects in Gravitational Quantum States
(重力場中の量子状態における相対論的効果)

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In this thesis, we revisited the nature of a bouncing particle in a gravitational field above a fixed floor of an ideal mirror. Such a system refers to as the quantum bouncer, which exhibits discrete energy levels with the normalizable wave functions. From the viewpoint of the equivalence principle of relativity, we investigated the relativistic effects in gravitational quantum states by discussing the quantum bouncer problem for the Klein–Gordon and Dirac equations in Rindler coordinates under appropriate mirror boundary conditions.

For the Klein–Gordon equation, the Dirichlet boundary condition of the vanishing field at the mirror surface can be applied to investigate the bound system. However, the mirror boundary condition for the Dirac equation is not trivial. Imposing the vanishing wave function at the mirror surface leads the Dirac wave function to vanish everywhere trivially. As an alternative, we used the boundary condition given in the MIT bag model, which ensures the vanishing of normal probability current and scalar densities at the mirror surface. From the Dirac equation, we discussed the quantum bouncer problem for Dirac and Majorana bouncing particles under the same mirror boundary condition. For the case of the Majorana particle, we used an additional condition. Namely, the charge conjugation of the Majorana wave function is the same as itself. Along with this analysis, we investigated the energy levels and transition frequencies between two energy eigenstates of relativistic bouncing particles. In the nonrelativistic limit, the results reduce to those of the Schrödinger equation in a linear gravitational potential with an ideal mirror. The results show that the energy levels of a Klein–Gordon bouncer are higher than its nonrelativistic limit. The energy levels of Dirac and Majorana bouncers give the same results; both are lower than their nonrelativistic limits for the lowest few states and shift to be higher than their nonrelativistic limits for the sufficiently large states. Interestingly, the transition frequencies of all relativistic bouncers are higher than their nonrelativistic limits. The results also show that the relativistic corrections to the transition frequencies of Dirac and Majorana bouncers are higher than those of a Klein–Gordon bouncer.

The behaviors of eigen-energies of bouncing particles under appropriate boundary conditions may relate to the behaviors of their wave functions, in particular around the mirror surface. The probability densities of both nonrelativistic and Klein–Gordon bouncers vanish at the mirror surface, as required by the Dirichlet boundary condition. However, the probability density of a Dirac bouncer does not vanish at the mirror surface. For the probability density of a Majorana bouncer, there is a factor that corresponds to the *Zitterbewegung* and arises from the interference between the positive- and negative-energy components of the Majorana wave function. This factor depends on the spin orientation and oscillates in the function of time. The *Zitterbewegung* also appears in the normal probability current density of a Majorana bouncer. The normal probability current density of a Dirac bouncer and the scalar density of a Majorana bouncer vanish everywhere, while the scalar density of a Dirac bouncer does not. The scalar density of a Dirac bouncer vanishes at the mirror surface for all states. In some specific spin orientations, both the probability and normal probability current densities of a Majorana bouncer coincide with those of a Dirac bouncer.

The obtained results mentioned above indicate that boundary conditions play essential roles. To understand the roles of the boundary conditions in more detail, we also revisited the system of a Dirac particle confined in a 1D box in the absence of a gravitational field. We used the chiral MIT boundary conditions to describe the properties of the walls or mirrors. These boundary conditions are general forms of the boundary condition given in the MIT bag model by including the contribution of the chiral angle. The results show that the system exhibits discrete momenta and energy levels, which are generally obtained in the function of the chiral angle. In the nonrelativistic limit, the results reduce to the well-known features of the Schrödinger equation in a 1D box.

We also discussed how the spin state of a Dirac particle in the confinement system behaves owing to the reflections at the mirrors under the chiral MIT boundary conditions. We found that the spin state is in a consistent state to repeat the reflections at both mirrors for the allowed discrete momenta: the direction of the reflected spin state owing to the reflection at the second mirror is the same as that of the incident spin state for the reflection at the first mirror. We also investigated how the density functions of a Dirac particle inside the box behave under the boundary conditions and initial spin orientation. We found that the probability and scalar densities can be asymmetric. In contrast, the normal probability current density vanishes everywhere.