Time-Reversal Symmetries in Reversible Elementary Square and Triangular Partitioned Cellular Automata, and Their Data

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Abstract

Time-reversal symmetry (T-symmetry) in a reversible cellular automaton (CA) is the property in which forward and backward evolutions of configurations are governed by the same local transition function. We show that the framework of partitioned cellular automata (PCAs) is useful to study T-symmetries of reversible CAs. Here, we investigate reversible elementary square PCAs (ESPCAs) and reversible elementary triangular PCAs (ETPCAs), and prove that a large number of reversible ESPCAs and all reversible ETPCAs are T-symmetric under some kinds of simple transformations on configurations. As applications, these results are used to find and analyse backward evolution processes in reversible PCAs. For example, for a given functional module implemented in a reversible PCA, such as a reversible logic element, we can obtain its inverse functional module very easily using its T-symmetry.

1 Introduction

The notion of time-reversal symmetry (T-symmetry, for short) comes from physics. It is the symmetry of an evolution law of a dynamical system under the transformation of reversal of time (see, e.g., a survey paper [5]). For example, in the classical mechanics, its law for the negative time direction is exactly the same as the one for the positive time direction. Assume a classical mechanical system starts to evolve from a given initial state. At some time, if we transform the momentum vector $\mathbf{p}$ of every particle to $-\mathbf{p}$ simultaneously, the whole evolving process is exactly traced back. Namely, it goes back to the initial state by the same evolution law.

There are various kinds of dynamical systems having such a property. A cellular automaton (CA) is a discrete dynamical system, in which configurations (i.e., whole states of the cellular space) evolve by applying a local transition function to all the cells in parallel. A reversible CA is one whose evolution process can be traced back uniquely (but not necessarily by the same local function). In [2, 4, 6, 17], it is argued that some kinds of reversible CAs have T-symmetry, i.e., the backward transition is performed by the same local function. For example, the ‘block CA’ of Margolus is known to be T-symmetric [2, 6]. In fact, by applying a simple transformation to a configuration, the block CA evolves to the reverse direction by the same local function. On the other hand, it is also known that there are reversible CAs that are not T-symmetric [2].

Here, we pose the question: Which reversible CAs are T-symmetric? We study this problem using the framework of two-dimensional reversible partitioned cellular automata (PCAs). A PCA was introduced as a special subclass of CAs for making it easy to design a reversible CA [15]. Each cell of a PCA is divided into several parts, whose number is equal to the neighbourhood size. The next state of a cell is determined by the present states of the corresponding parts of the neighbour cells, not by the states of the whole neighbour cells. It has been shown that in a PCA injectivity of a local function is equivalent to injectivity of the global function that determines evolutions of configurations [15, 8] (see Lemmas 2.4 and 5.4). By this property, we can obtain a reversible PCA very easily by designing a PCA so that its local function is injective.

The framework of reversible PCAs also has an advantage for studying T-symmetry. Each part of a cell can be regarded as an output port to the corresponding neighbour cell, and thus its state is interpreted as a signal moving to the neighbour cell. Therefore, reversing the moving directions of signals, which corresponds to changing the momentum vector of each particle from $\mathbf{p}$ to $-\mathbf{p}$ in the classical mechanics, is easily performed by a simple transformation on configurations (defined by H^{rev} in Sections 3.1 and 5.3). By this, T-symmetries for reversible PCAs are naturally defined.

In this paper, we investigate T-symmetries of reversible four-neighbour elementary square partitioned CAs (ESPCAs), which are rotation-symmetric and each part of a cell has only two states. We also extend the results on T-symmetries of reversible three-neighbour elementary triangular partitioned CAs (ETPCAs) given in [13]. Here, we define two sorts of T-symmetries for these reversible PCAs. The first one is strict T-symmetry. If a PCA is strictly T-symmetric, then its backward evolution of configurations is governed by exactly the same local function for the forward evolution. The

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second one is a weaker version of T-symmetry, where the backward evolution of configuration is governed by a local function that is 'similar' to the forward local function. As 'similar' ones, the local functions obtained by taking a mirror image, taking 0-1 complementation, and taking both of these operations to the forward local function are used. Hence, we consider three kinds of weaker T-symmetries here.

In the following, we show that a large number of reversible ESPCAs and all reversible ETPCAs are strictly or weakly T-symmetric in the above sense. These results are useful for finding or analysing their backward evolution processes. In particular, it makes it easy to design an 'inverse functional module' that undoes the forward function of a given module. We give several examples of applications of T-symmetries.

In Section 2, definitions on ESPCAs are given. In Section 3, T-symmetries in reversible ESPCAs are defined and their properties are studied. In Section 4, several applications of T-symmetries in ESPCAs are shown. In Section 5, ETPCAs are defined, and their T-symmetries are clarified. In Section 6, applications of T-symmetries of ETPCAs are shown. Section 7 gives concluding remarks and open problems. In Appendix, all 1536 reversible ESPCAs are listed, and the data on their T-symmetries are given.

2 Elementary Square Partitioned Cellular Automata (ESPCAs)

In this section, we give basic definitions on a four-neighbour *square partitioned cellular automaton* (SPCA). Figure 1 **(a)** is the cellular space of SPCA. Its square cell is divided into four parts. In SPCA, a cell changes its state depending on the top part of the south-neighbour cell, the right part of the west cell, the bottom part of the north cell, and the left part of the east cell as shown in Figure 1 **(b)**. Each part of a cell can be interpreted as an 'output port' to the corresponding neighbour cell, and thus the state in the part is regarded as a signal moving to the cell. In this section, we investigate the simplest subclass of SPCAs called an *elementary SPCA* (ESPCA). It is a rotation-symmetric SPCA, and each part a cell has only two states.

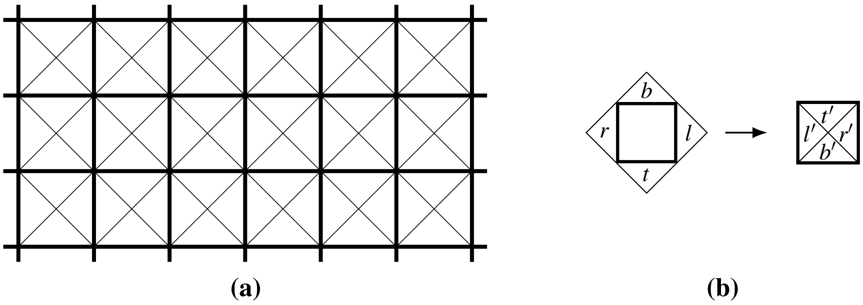

Figure 1: **(a)** Cellular space of a four-neighbour SPCA, and **(b)** a local transition rule that represents $f(t,r,b,l) = (t',r',b',l')$

2.1 Definitions on ESPCAs

Definition 2.1 *A* four-neighbour square partitioned cellular automaton *(SPCA) is a system defined by*

$$P = (\mathbb{Z}^2, (T,R,B,L), ((0,-1),(-1,0),(0,1),(1,0)), f).$$

Here, $\mathbb{Z}^2$ is the set of all points with integer coordinates where cells are placed. The items $T,R,B,$ and L are non-empty finite sets of states of the top, right, bottom, and left parts of a cell. The set of states of a cell is thus $Q = T\times R\times B\times L$. The quadruple $((0,-1),(-1,0),(0,1),(1,0))$ is a neighbourhood *of each cell, and $f : Q \to Q$ is a* local (transition) function.

If $f(t,r,b,l) = (t',r',b',l')$ holds for $(t,r,b,l),(t',r',b',l') \in Q$, this relation is called a *local transition rule* of P. It is also indicated as in Figure 1 **(b)**. The local function f is thus defined by a set of local transition rules.

Definition 2.2 *Let $P = (\mathbb{Z}^2, (T,R,B,L), ((0,-1),(-1,0),(0,1),(1,0)), f)$ be a four-neighbour SPCA. A* configuration *of P is a function $\alpha : \mathbb{Z}^2 \to Q$. The set of all configurations of P is denoted by* $\mathrm{Conf}(P)$*, i.e.,* $\mathrm{Conf}(P) = \{\alpha \mid \alpha : \mathbb{Z}^2 \to Q\}$*. Let $\mathrm{pr}_T : Q \to T$ be the* projection function *that satisfies $\mathrm{pr}_T(t,r,b,l) = t$ for all $(t,r,b,l) \in Q$. The projection functions $\mathrm{pr}_R : Q \to R$, $\mathrm{pr}_B : Q \to B$ and $\mathrm{pr}_L : Q \to L$ are defined similarly. The* global function *$F : \mathrm{Conf}(P) \to \mathrm{Conf}(P)$ of P is defined as the one that satisfies the following.*

$$\begin{array}{l} \forall \alpha \in \mathrm{Conf}(P), \forall x \in \mathbb{Z}^2 : \\ \quad F(\alpha)(x) \;=\; f(\mathrm{pr}_T(\alpha(x+(0,-1))), \mathrm{pr}_R(\alpha(x+(-1,0))), \mathrm{pr}_B(\alpha(x+(0,1))), \mathrm{pr}_L(\alpha(x+(1,0)))) \end{array}$$

In this paper, reversibility of an SPCA is defined as follows. Note that a detailed discussion on the definition of a reversible CA is found in Section 10.3 of [8].

Definition 2.3 *An SPCA P is called* reversible *if its global function is injective.*

The next Lemma shows that, in a PCA, injectivity of the global function is equivalent to injectivity of the local function [8, 15]. By this, we can easily obtain a reversible CA by giving a PCA whose local function is injective.

Lemma 2.4 *Let P be an SPCA. Its global function F is injective if and only if its local function f is injective.*

Next, we define a subclass of SPCAs such that its local function is rotation-symmetric, and each of four parts has only two states. It is called an *elementary* SPCA (ESPCA) as in the case of a one-dimensional *elementary cellular automaton* (ECA) [20]. We first define the notion of rotation-symmetry.

Definition 2.5 *Let* $P = (\mathbb{Z}^2, (T,R,B,L), ((0,-1),(-1,0),(0,1),(1,0)), f)$ *be an SPCA. The SPCA P is called* rotation-symmetric *(or* isotropic*) if the following conditions (1) and (2) hold.*

(1) $T = R = B = L$

(2) $\forall (t,r,b,l),(t',r',b',l') \in T \times R \times B \times L:$
$f(t,r,b,l) = (t',r',b',l') \Rightarrow f(r,b,l,t) = (r',b',l',t')$

Definition 2.6 *Let* $P = (\mathbb{Z}^2, (T,R,B,L), ((0,-1),(-1,0),(0,1),(1,0)), f)$ *be an SPCA. We say P is an* elementary triangular partitioned cellular automaton *(ESPCA), if* $T = R = B = L = \{0,1\}$*, and it is rotation-symmetric.*

Since an ESPCA is rotation-symmetric, its local function $f : \{0,1\}^4 \to \{0,1\}^4$ is defined by only six local transition rules that are described by the following six values.

$$f(0,0,0,0),\ f(0,0,1,0),\ f(0,0,1,1),\ f(1,0,1,0),\ f(0,1,1,1),\ f(1,1,1,1)$$

Here, $f(0,0,1,0), f(0,0,1,1), f(0,1,1,1) \in \{0,1\}^4$. On the other hand, $f(1,0,1,0) \in \{(0,0,0,0),(0,1,0,1),(1,0,1,0),(1,1,1,1)\}$ and $f(0,0,0,0), f(1,1,1,1) \in \{(0,0,0,0),\ (1,1,1,1)\}$, since it is rotation-symmetric. Hence, there are $16^3 \times 4 \times 2^2 = 65,536$ ESPCAs in total.

Reading the 4-bit values of $f(0,0,0,0)$, $f(0,0,1,0)$, $f(0,0,1,1)$, $f(1,0,1,0)$, $f(0,1,1,1)$, $f(1,1,1,1)$ as six binary numbers, we express an ESPCA by a 6-digit hexadecimal identification (ID) number *uvwxyz* as in Figure 2. An ESPCA with the ID number *uvwxyz* is denoted by ESPCA-*uvwxyz*. Its local and global functions are denoted by f_{uvwxyz} and F_{uvwxyz}, respectively. For example, Figure 3 shows the six local transition rules of ESPCA-01c57f, which define f_{01c57f}.

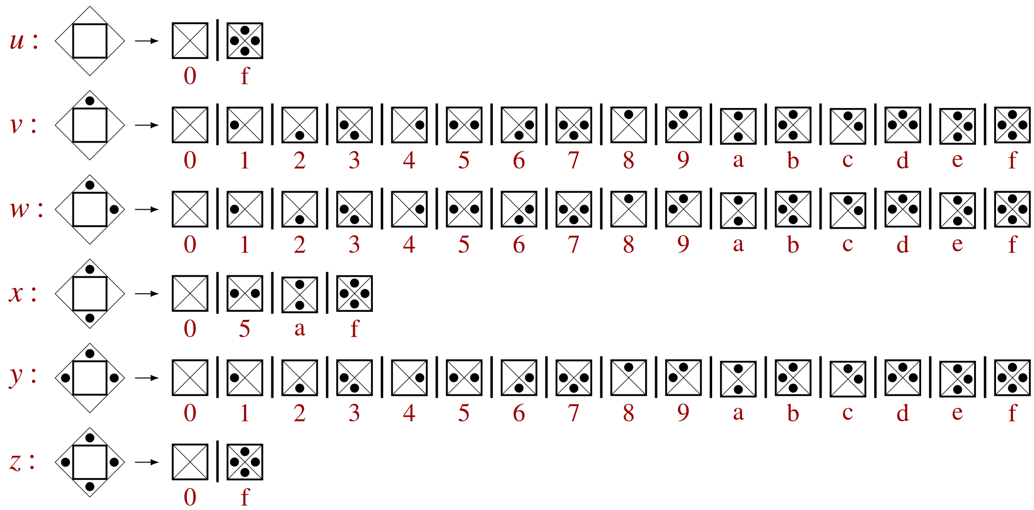

Figure 2: Expressing an ESPCA by a 6-digit hexadecimal ID number *uvwxyz*. States 0 and 1 are represented by a blank and •. Vertical bars indicate alternatives of the right-hand side of each local transition rule

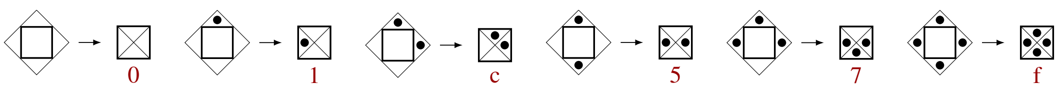

Figure 3: Local function f_{01c57f} of ESPCA-01c57f defined by the six local transition rules

Definition 2.7 *An ESPCA P is called* conservative *if the number of state 1's (i.e., particles) is conserved in each of its local transition rules.*

Conservativeness of an ESPCA is an analog of various conservation laws in physics such as conservation of mass, energy, and so on. From Definitions 2.3 and 2.7, it is easy to see the following proposition.

Proposition 2.8 *Let P be an ESPCA with an ID number uvwxyz.*

(1) P is reversible if and only if the following condition holds.
$(u,z) \in \{(0,\mathrm{f}),(\mathrm{f},0)\} \wedge x \in \{5,\mathrm{a}\} \wedge (v,w,y) \in (A{\times}B{\times}C \cup A{\times}C{\times}B \cup B{\times}A{\times}C \cup B{\times}C{\times}A \cup C{\times}A{\times}B \cup C{\times}B{\times}A)$, *where* $A=\{1,2,4,8\}, B=\{3,6,9,\mathrm{c}\}, C=\{7,\mathrm{b},\mathrm{d},\mathrm{e}\}$

(2) P is conservative if and only if the following condition holds.
$u=0 \wedge v \in \{1,2,4,8\} \wedge w \in \{3,5,6,9,\mathrm{a},\mathrm{c}\} \wedge x \in \{5,\mathrm{a}\} \wedge y \in \{7,\mathrm{b},\mathrm{d},\mathrm{e}\} \wedge z=\mathrm{f}$

(3) P is reversible and conservative if and only if the following condition holds.
$u=0 \wedge v \in \{1,2,4,8\} \wedge w \in \{3,6,9,\mathrm{c}\} \wedge x \in \{5,\mathrm{a}\} \wedge y \in \{7,\mathrm{b},\mathrm{d},\mathrm{e}\} \wedge z=\mathrm{f}$

From the above proposition, we can see the total numbers of reversible, conservative, and reversible and conservative ESPCAs are 1536, 192, and 128, respectively.

2.2 Dualities in ESPCA

We consider two kinds of dualities among ESPCAs, which are the ones under *reflection* and *complementation*. These notions are given in [19] for one-dimensional elementary cellular automata (ECAs). The dual ESPCAs are essentially the same as the original one in the sense that any evolution process is simulated in the dual ESPCA after taking a simple transformation to the initial configuration.

Definition 2.9 *Let P be an ESPCA and* $f:\{0,1\}^4 \to \{0,1\}^4$ *be its local function. Define* $f^{\mathrm{r}}:\{0,1\}^4 \to \{0,1\}^4$ *as follows.*

$$\begin{array}{l}\forall (t,r,d,l),(t',r',d',l') \in \{0,1\}^4 : \\ \quad f(t,r,d,l)=(t',r',d',l') \Leftrightarrow f^{\mathrm{r}}(t,l,d,r)=(t',l',d',r')\end{array}$$

Then, the ESPCA P^{r} *having the local function* f^{r} *is called the* dual ESPCA of P under reflection.

From this definition, we can see that the local transition rules of P^{r} are the mirror images of those of P. It means that any evolution process in P is simulated in P^{r} in a straightforward manner by taking the mirror image of the initial configuration (see Lemma 3.5). Note that, in the above definition, the mirror images are taken with respect to the vertical axis (i.e., r and l, and r' and l' are exchanged). However, since ESPCA P is rotation-symmetric (Definition 2.5), it is equivalent to the case where the mirror images are taken with respect to the horizontal axis.

Definition 2.10 *Let P be an ESPCA and* $f:\{0,1\}^4 \to \{0,1\}^4$ *be its local function. For* $x \in \{0,1\}$*, let* $\bar{x}=1-x$*, i.e.,* $\bar{x}$ *is the complement of x. Define* $f^{\mathrm{c}}:\{0,1\}^4 \to \{0,1\}^4$ *as follows.*

$$\begin{array}{l}\forall (t,r,d,l),(t',r',d',l') \in \{0,1\}^4 : \\ \quad f(t,r,d,l)=(t',r',d',l') \Leftrightarrow f^{\mathrm{c}}(\bar{t},\bar{r},\bar{d},\bar{l})=(\overline{t'},\overline{r'},\overline{d'},\overline{l'})\end{array}$$

Then, the ESPCA P^{c} *having the local function* f^{c} *is called the* dual ESPCA of P under complementation.

From this definition, we can see that the local transition rules of P^{c} are obtained from those of P by exchanging 0 and 1. Therefore, any evolution process in P is simulated in P^{c} in a straightforward manner by taking the complement of the initial configuration (see Lemma 3.7).

For an ESPCA P with a local function f, there is an ESPCA P^{rc} whose local function is $(f^{\mathrm{r}})^{\mathrm{c}}=(f^{\mathrm{c}})^{\mathrm{r}}$. It can also be regarded as a kind of a dual ESPCA. We write the local function of P^{rc} by f^{rc} shortly.

Let P_{uvwxyz} be a reversible ESPCA. We denote the ID numbers of f^{r}_{uvwxyz}, f^{c}_{uvwxyz}, f^{rc}_{uvwxyz}, and f^{-1}_{uvwxyz} by $\mathrm{r}(uvwxyz)$, $\mathrm{c}(uvwxyz)$, $\mathrm{rc}(uvwxyz)$, and $\mathrm{inv}(uvwxyz)$, respectively. Namely, $f^{\mathrm{r}}_{uvwxyz}=f_{\mathrm{r}(uvwxyz)}$, $f^{\mathrm{c}}_{uvwxyz}=f_{\mathrm{c}(uvwxyz)}$, $f^{\mathrm{rc}}_{uvwxyz}=f_{\mathrm{rc}(uvwxyz)}$, and $f^{-1}_{uvwxyz}=f_{\mathrm{inv}(uvwxyz)}$.

Table 1 shows the list of ID numbers of local functions (f) of 128 reversible and conservative ESPCAs, their dual ones (f^{r}, f^{c} and f^{rc}), and their inverses (f^{-1}). We included inverse local functions besides dual ones in the table, since they will be used in Section 3. For example, if we consider ESPCA-01357f, then $f^{\mathrm{r}}_{01357\mathrm{f}}=f_{\mathrm{r}(01357\mathrm{f})}=f_{04357\mathrm{f}}$, $f^{\mathrm{c}}_{01357\mathrm{f}}=f_{\mathrm{c}(01357\mathrm{f})}=f_{0235\mathrm{bf}}$, $f^{\mathrm{rc}}_{01357\mathrm{f}}=f_{\mathrm{rc}(01357\mathrm{f})}=f_{0235\mathrm{ef}}$ and $f^{-1}_{01357\mathrm{f}}=f_{\mathrm{inv}(01357\mathrm{f})}=f_{04357\mathrm{f}}$ (Figure 4). Note that, since the total number of all reversible ESPCAs is 1536, their complete list is given in Appendix A.

Table 1: Identification numbers of 128 reversible and conservative ESPCAs, their dual ones (under reflection, complementation, and both) and inverses. In each ESPCA, the IDs of local functions among $f, f^{\mathrm{r}}, f^{\mathrm{c}}$ and f^{rc} that are equal to f^{-1} are marked by $*$. It means that the ESPCA is T-symmetric under the corresponding involutions (see Section 3)

f	f^{r}	f^{c}	f^{rc}	f^{-1}
01357f	04357f*	0235bf	0235ef	04357f
0135bf	0435ef*	0135bf	0435ef*	0435ef
0135df	0435df*	0835bf	0835ef	0435df
0135ef	0435bf*	0435bf*	0135ef	0435bf
013a7f	043a7f*	023abf	023aef	043a7f
013abf	043aef*	013abf	043aef*	043aef
013adf	043adf*	083abf	083aef	043adf
013aef	043abf*	043abf*	013aef	043abf
01657f	04957f*	0265bf	0295ef	04957f
0165bf	0495ef*	0165bf	0495ef*	0495ef
0165df	0495df*	0865bf	0895ef	0495df
0165ef	0495bf*	0465bf	0195ef	0495bf
016a7f	049a7f*	026abf	029aef	049a7f
016abf	049aef*	016abf	049aef*	049aef
016adf	049adf*	086abf	089aef	049adf
016aef	049abf*	046abf	019aef	049abf
01957f	04657f*	0295bf	0265ef	04657f
0195bf	0465ef*	0195bf	0465ef*	0465ef
0195df	0465df*	0895bf	0865ef	0465df
0195ef	0465bf*	0495bf	0165ef	0465bf
019a7f	046a7f*	029abf	026aef	046a7f
019abf	046aef*	019abf	046aef*	046aef
019adf	046adf*	089abf	086aef	046adf
019aef	046abf*	049abf	016aef	046abf
01c57f	04c57f*	02c5bf	02c5ef	04c57f
01c5bf	04c5ef*	01c5bf	04c5ef*	04c5ef
01c5df	04c5df*	08c5bf	08c5ef	04c5df
01c5ef	04c5bf*	04c5bf*	01c5ef	04c5bf
01ca7f	04ca7f*	02cabf	02caef	04ca7f
01cabf	04caef*	01cabf	04caef*	04caef
01cadf	04cadf*	08cabf	08caef	04cadf
01caef	04cabf*	04cabf*	01caef	04cabf
02357f*	02357f*	02357f*	02357f*	02357f
0235bf	0235ef*	01357f	04357f	0235ef
0235df*	0235df*	08357f	08357f	0235df
0235ef	0235bf*	04357f	01357f	0235bf
023a7f*	023a7f*	023a7f*	023a7f*	023a7f
023abf	023aef*	013a7f	043a7f	023aef
023adf*	023adf*	083a7f	083a7f	023adf
023aef	023abf*	043a7f	013a7f	023abf
02657f	02957f*	02657f	02957f*	02957f
0265bf	0295ef*	01657f	04957f	0295ef
0265df	0295df*	08657f	08957f	0295df
0265ef	0295bf*	04657f	01957f	0295bf
026a7f	029a7f*	026a7f	029a7f*	029a7f
026abf	029aef*	016a7f	049a7f	029aef
026adf	029adf*	086a7f	089a7f	029adf
026aef	029abf*	046a7f	019a7f	029abf
02957f	02657f*	02957f	02657f*	02657f
0295bf	0265ef*	01957f	04657f	0265ef
0295df	0265df*	08957f	08657f	0265df
0295ef	0265bf*	04957f	01657f	0265bf
029a7f	026a7f*	029a7f	026a7f*	026a7f
029abf	026aef*	019a7f	046a7f	026aef
029adf	026adf*	089a7f	086a7f	026adf
029aef	026abf*	049a7f	016a7f	026abf
02c57f*	02c57f*	02c57f*	02c57f*	02c57f
02c5bf	02c5ef*	01c57f	04c57f	02c5ef
02c5df*	02c5df*	08c57f	08c57f	02c5df
02c5ef	02c5bf*	04c57f	01c57f	02c5bf
02ca7f*	02ca7f*	02ca7f*	02ca7f*	02ca7f
02cabf	02caef*	01ca7f	04ca7f	02caef
02cadf*	02cadf*	08ca7f	08ca7f	02cadf
02caef	02cabf*	04ca7f	01ca7f	02cabf

f	f^{r}	f^{c}	f^{rc}	f^{-1}
04357f	01357f*	0235ef	0235bf	01357f
0435bf	0135ef*	0135ef*	0435bf	0135ef
0435df	0135df*	0835ef	0835bf	0135df
0435ef	0135bf*	0435ef	0135bf*	0135bf
043a7f	013a7f*	023aef	023abf	013a7f
043abf	013aef*	013aef*	043abf	013aef
043adf	013adf*	083aef	083abf	013adf
043aef	013abf*	043aef	013abf*	013abf
04657f	01957f*	0265ef	0295bf	01957f
0465bf	0195ef*	0165ef	0495bf	0195ef
0465df	0195df*	0865ef	0895bf	0195df
0465ef	0195bf*	0465ef	0195bf*	0195bf
046a7f	019a7f*	026aef	029abf	019a7f
046abf	019aef*	016aef	049abf	019aef
046adf	019adf*	086aef	089abf	019adf
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04957f	01657f*	0295ef	0265bf	01657f
0495bf	0165ef*	0195ef	0465bf	0165ef
0495df	0165df*	0895ef	0865bf	0165df
0495ef	0165bf*	0495ef	0165bf*	0165bf
049a7f	016a7f*	029aef	026abf	016a7f
049abf	016aef*	019aef	046abf	016aef
049adf	016adf*	089aef	086abf	016adf
049aef	016abf*	049aef	016abf*	016abf
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04ca7f	01ca7f*	02caef	02cabf	01ca7f
04cabf	01caef*	01caef*	04cabf	01caef
04cadf	01cadf*	08caef	08cabf	01cadf
04caef	01cabf*	04caef	01cabf*	01cabf
08357f*	08357f*	0235df	0235df	08357f
0835bf	0835ef*	0135df	0435df	0835ef
0835df*	0835df*	0835df*	0835df*	0835df
0835ef	0835bf*	0435df	0135df	0835bf
083a7f*	083a7f*	023adf	023adf	083a7f
083abf	083aef*	013adf	043adf	083aef
083adf*	083adf*	083adf*	083adf*	083adf
083aef	083abf*	043adf	013adf	083abf
08657f	08957f*	0265df	0295df	08957f
0865bf	0895ef*	0165df	0495df	0895ef
0865df	0895df*	0865df	0895df*	0895df
0865ef	0895bf*	0465df	0195df	0895bf
086a7f	089a7f*	026adf	029adf	089a7f
086abf	089aef*	016adf	049adf	089aef
086adf	089adf*	086adf	089adf*	089adf
086aef	089abf*	046adf	019adf	089abf
08957f	08657f*	0295df	0265df	08657f
0895bf	0865ef*	0195df	0465df	0865ef
0895df	0865df*	0895df	0865df*	0865df
0895ef	0865bf*	0495df	0165df	0865bf
089a7f	086a7f*	029adf	026adf	086a7f
089abf	086aef*	019adf	046adf	086aef
089adf	086adf*	089adf	086adf*	086adf
089aef	086abf*	049adf	016adf	086abf
08c57f*	08c57f*	02c5df	02c5df	08c57f
08c5bf	08c5ef*	01c5df	04c5df	08c5ef
08c5df*	08c5df*	08c5df*	08c5df*	08c5df
08c5ef	08c5bf*	04c5df	01c5df	08c5bf
08ca7f*	08ca7f*	02cadf	02cadf	08ca7f
08cabf	08caef*	01cadf	04cadf	08caef
08cadf*	08cadf*	08cadf*	08cadf*	08cadf
08caef	08cabf*	04cadf	01cadf	08cabf

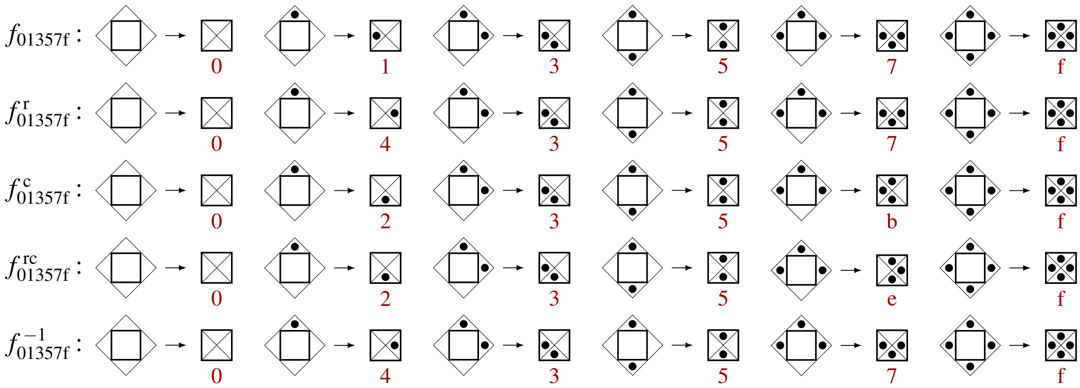


Figure 4: Local function $f_{01357\mathrm{f}}$, and its duals and inverse

3 Time-Reversal Symmetries in Reversible ESPCAs

In this section we define two sorts of time-reversal symmetries (T-symmetries) in reversible ESPCAs. The first one is a stronger version of T-symmetry where the backward evolution is governed by exactly the same law as the forward one. The second is a weaker version of T-symmetry where the backward evolution is governed by a 'similar' law. As we shall see in Section 4, the both versions of T-symmetries are useful for finding or anlysing the backward evolution process for a given evolution process.

3.1 Basic property of reversible ESPCAs

Before defining T-symmetries on ESPCAs, we first show a basic property on their backward evolution. Let $\mathrm{Conf_E} = \{\alpha \mid \alpha : \mathbb{Z}^2 \to \{0,1\}^4\}$ denote the set of all configurations of ESPCA. A function H is called an *involution* if $H \circ H$ is an identity function. We define an involution $H^{\mathrm{rev}} : \mathrm{Conf_E} \to \mathrm{Conf_E}$ by the reversible and conservative ESPCA-08cadf:

$$H^{\mathrm{rev}} = F_{\mathrm{08cadf}}$$

As it is seen from Figure 5, H^{rev} can be interpreted as the one that *reverses the moving directions* of all the particles in the cellular space. In the classical mechanics, the operation H^{rev} corresponds to the transformation of the momentum vector $\mathbf{p}$ of each particle to $-\mathbf{p}$. It is easy to verify that H^{rev} is an involution.

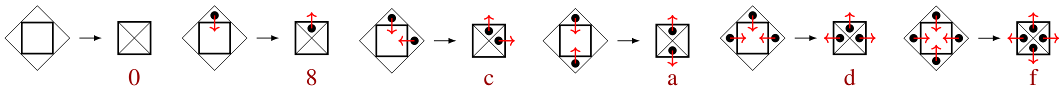


Figure 5: Local function of ESPCA-08cadf by which H^{rev} is defined. The involution H^{rev} reverses the moving directions of all the particles in a configuration

The following lemma shows that a backward evolution of reversible ESPCA-*uvwxyz* is performed by $F_{\mathrm{inv}(uvwxyz)}$ applying H^{rev} just before and after $F_{\mathrm{inv}(uvwxyz)}$. However, it does not mean T-symmetry of the ESPCA, since $F_{\mathrm{inv}(uvwxyz)}$ may be very different from F_{uvwxyz}. In the special case where $F_{\mathrm{inv}(uvwxyz)} = F_{uvwxyz}$ holds, we can say that the backward evolution is carried out by exactly the same global function as the forward one, which we call strict T-symmetry given in Section 3.2. This lemma is also used for defining weaker T-symmetries as discussed in Section 3.3. Note that this lemma is proved in a similar manner as in the case of reversible elementary triangular PCAs (ETPCA) [13].

Lemma 3.1 *Let P be a reversible ESPCA-uvwxyz with the local function f_{uvwxyz} and the global function F_{uvwxyz}. Let P' be a reversible ESPCA with the ID number* $\mathrm{inv}(uvwxyz)$. *The local and global functions of P' are thus $f_{\mathrm{inv}(uvwxyz)} = f^{-1}_{uvwxyz}$ and $F_{\mathrm{inv}(uvwxyz)}$, respectively. Then the following holds.*

$$F^{-1}_{uvwxyz} = H^{\mathrm{rev}} \circ F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}$$

Proof. Let $\alpha_1 \in \mathrm{Conf_E}$ be any configuration, and $(x_0, y_0) \in \mathbb{Z}^2$ be any point. Let $(t_1, r_1, b_1, l_1) \in \{0,1\}^4$ be as follows: $\alpha_1(x_0, y_0) = (t_1, r_1, b_1, l_1)$. See Figure 6 that shows the process of state-changes by the operations given below. First, we can see the following relations.

$$\begin{array}{rcl}
\mathrm{pr}_T(H^{\mathrm{rev}}(\alpha_1)(x_0, y_0 - 1)) & = & b_1 \\
\mathrm{pr}_R(H^{\mathrm{rev}}(\alpha_1)(x_0 - 1, y_0)) & = & l_1 \\
\mathrm{pr}_B(H^{\mathrm{rev}}(\alpha_1)(x_0, y_0 + 1)) & = & t_1 \\
\mathrm{pr}_L(H^{\mathrm{rev}}(\alpha_1)(x_0 + 1, y_0)) & = & r_1
\end{array}$$

Assume $f^{-1}_{uvwxyz}(t_1,r_1,b_1,l_1) = (t_0,r_0,b_0,l_0)$ (i.e., $f_{uvwxyz}(t_0,r_0,b_0,l_0) = (t_1,r_1,b_1,l_1)$). Then, the following holds, since f_{uvwxyz} (and thus f^{-1}_{uvwxyz}) is rotation-symmetric.

$$(F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}(\alpha_1))(x_0,y_0) = (b_0,l_0,t_0,r_0)$$

Let $\alpha_0 = F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}(\alpha_1)$. Then, the following relations hold.

$$\begin{array}{rcl}
\mathrm{pr}_T(H^{\mathrm{rev}}(\alpha_0)(x_0,y_0-1)) & = & t_0 \\
\mathrm{pr}_R(H^{\mathrm{rev}}(\alpha_0)(x_0-1,y_0)) & = & r_0 \\
\mathrm{pr}_B(H^{\mathrm{rev}}(\alpha_0)(x_0,y_0+1)) & = & b_0 \\
\mathrm{pr}_L(H^{\mathrm{rev}}(\alpha_0)(x_0+1,y_0)) & = & l_0
\end{array}$$

Hence,

$$(F_{uvwxyz} \circ H^{\mathrm{rev}}(\alpha_0))(x_0,y_0) = (t_1,r_1,b_1,l_1) = \alpha_1(x_0,y_0).$$

By above, the following holds for all $(x_0,y_0) \in \mathbb{Z}^2$.

$$(F_{uvwxyz} \circ H^{\mathrm{rev}} \circ F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}(\alpha_1))(x_0,y_0) \quad = \quad \alpha_1(x_0,y_0)$$

Thus, $F_{uvwxyz} \circ H^{\mathrm{rev}} \circ F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}(\alpha_1) = \alpha_1$ for all $\alpha_1 \in \mathrm{Conf_E}$. Therefore,

$$F^{-1}_{uvwxyz} = H^{\mathrm{rev}} \circ F_{\mathrm{inv}(uvwxyz)} \circ H^{\mathrm{rev}}.$$

This completes the proof. □

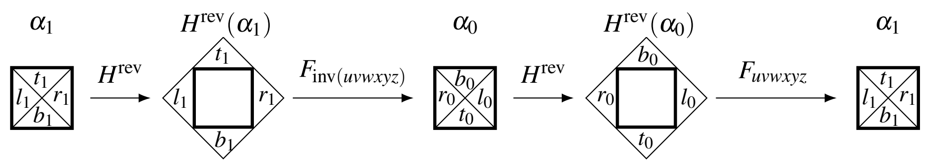


Figure 6: Process of the state-changes around the cell (x_0,y_0) in Lemma 3.1

3.2 Strict T-symmetry

We now define the notion of strict T-symmetry for reversible ESPCAs. It basically follows the definition given in [2].

Definition 3.2 *Let P be a reversible ESPCA whose global function is F. If $F^{-1} = H^{\mathrm{rev}} \circ F \circ H^{\mathrm{rev}}$, then P is called* strictly time-reversal symmetric *(or* strictly T-symmetric *for short).*

The above definition means that in a strictly T-symmetric reversible ESPCA its backward transition is carried out by exactly the same global function as the one for the forward evolution provided that the moving directions of all the particles are reversed before and after the global function is applied.

By Lemma 3.1, we have the following theorem.

Theorem 3.3 *A reversible ESPCA with the ID number uvwxyz is strictly T-symmetric, if* $\mathrm{inv}(uvwxyz) = uvwxyz$. *In this case, the following holds.*

$$F^{-1}_{uvwxyz} = H^{\mathrm{rev}} \circ F_{uvwxyz} \circ H^{\mathrm{rev}}$$

From Theorem 3.3 and Table 1, we can see that 16 reversible and conservative ESPCAs are strictly T-symmetric.

Example 1 *We consider ESPCA-02c5df (Figure 7). It is reversible and conservative. Since* $\mathrm{inv}(\mathrm{02c5df}) = \mathrm{02c5df}$ *as shown in Table 1, it is strictly T-symmetric by Theorem 3.3. Thus the following holds.*

$$F^{-1}_{\mathrm{02c5df}} = H^{\mathrm{rev}} \circ F_{\mathrm{02c5df}} \circ H^{\mathrm{rev}}$$

The diagram given in Figure 8 illustrates this relation. It shows that the backward transition from a configuration $\alpha(t)$ to $\alpha(t-1)$ is performed by the global function F_{02c5df} for the forward transition. Only the additional operation H^{rev}, which reverses the moving direction of all particles, is needed before and after applying F_{02c5df}.

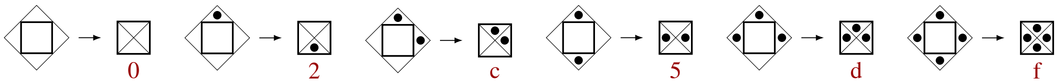


Figure 7: Local function f_{02c5df} of ESPCA-02c5df

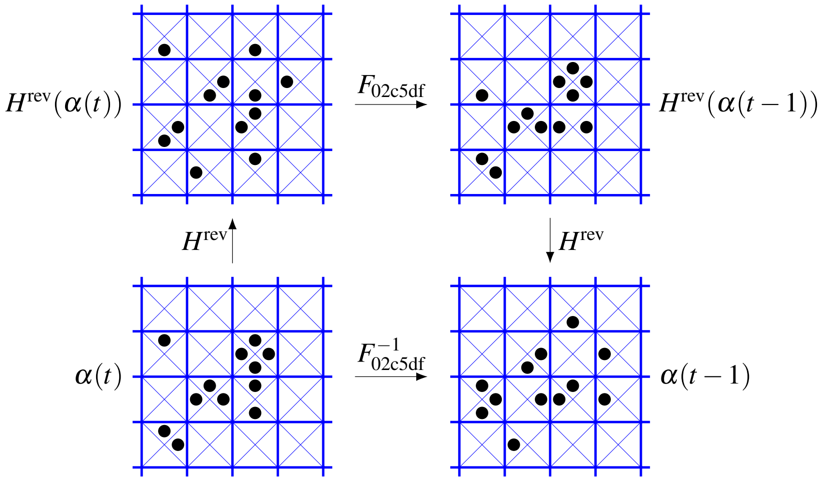


Figure 8: Diagram that illustrates strict T-symmetry of ESPCA-02c5df

3.3 T-symmetry under a general involution H

Next, we define a weaker version of T-symmetry by replacing the particular involution H^{rev} in Definition 3.2 by an arbitrary involution H. Note that the notion of 'weak' T-symmetry in this definition is expressed by the phrase 'under the involution H'.

Definition 3.4 *Let P be a reversible ESPCA whose global function is F. If there is an involution $H : \text{Conf}_{\text{E}} \to \text{Conf}_{\text{E}}$ that satisfies $F^{-1} = H \circ F \circ H$, then P is called* time-reversal symmetric under the involution H *(or* T-symmetric under H *for short).*

We do not restrict the involution H in this definition. However, in the following, we consider the case where H is expressed by $H = H^{\text{rev}} \circ H' = H' \circ H^{\text{rev}}$ for some involution H'. In this case, the backward evolution is performed by $H' \circ F \circ H'$ applying H^{rev} just before and after $H' \circ F \circ H'$. If H' is a simple involution, we can say that the backward evolution is carried out by a 'similar' law as the forward one.

We show that ESPCA-$uvwxyz$ satisfying $\text{inv}(uvwxyz) = \text{r}(uvwxyz)$ is T-symmetric under a certain simple involution. First, define a function $\text{refl}_4 : \{0,1\}^4 \to \{0,1\}^4$ as follows: $\text{refl}_4(t,r,b,l) = (t,l,b,r)$ for any $(t,r,b,l) \in \{0,1\}^4$. Next define an involution $H^{\text{refl}} : \text{Conf}_{\text{E}} \to \text{Conf}_{\text{E}}$ as follows. For all $\alpha \in \text{Conf}_{\text{E}}$ and $(x_0,y_0) \in \mathbb{Z}^2$:

$$H^{\text{refl}}(\alpha)(x_0,y_0) = \text{refl}_4(\alpha(-x_0,y_0))$$

It gives the *mirror image* of a configuration with respect to the y-axis.

Lemma 3.5 *The next relation holds for any ESPCA-$uvwxyz$.*

$$F_{\text{r}(uvwxyz)} = H^{\text{refl}} \circ F_{uvwxyz} \circ H^{\text{refl}}$$

Proof. First, we show $F_{uvwxyz} = H^{\text{refl}} \circ F_{\text{r}(uvwxyz)} \circ H^{\text{refl}}$. Let $\alpha \in \text{Conf}_{\text{E}}$ be any configuration, and $(x_0,y_0) \in \mathbb{Z}^2$ be any point. Let $(t_0,r_0,b_0,l_0) \in \{0,1\}^4$ be as follows.

$$\begin{aligned}
\text{pr}_T(\alpha(x_0,y_0-1)) &= t_0 \\
\text{pr}_R(\alpha(x_0-1,y_0)) &= r_0 \\
\text{pr}_B(\alpha(x_0,y_0+1)) &= b_0 \\
\text{pr}_L(\alpha(x_0+1,y_0)) &= l_0
\end{aligned}$$

See Figure 9 that shows the process of state-changes by the operations given below. In the next step, we apply H^{refl}, and have the following.

$$\begin{aligned}
\text{pr}_T(H^{\text{refl}}(\alpha)(-x_0,y_0-1)) &= t_0 \\
\text{pr}_R(H^{\text{refl}}(\alpha)(-x_0-1,y_0)) &= l_0 \\
\text{pr}_B(H^{\text{refl}}(\alpha)(-x_0,y_0+1)) &= b_0 \\
\text{pr}_L(H^{\text{refl}}(\alpha)(-x_0+1,y_0)) &= r_0
\end{aligned}$$

Here we assume $f_{uvwxyz}(t_0, r_0, b_0, l_0) = (t_1, r_1, b_1, l_1)$. Since $f_{\mathrm{r}(uvwxyz)}(t_0, l_0, b_0, r_0) = (t_1, l_1, b_1, r_1)$,

$$(F_{\mathrm{r}(uvwxyz)} \circ H^{\mathrm{refl}}(\alpha))(-x_0, y_0) = (t_1, l_1, b_1, r_1).$$

Finally, we have the following relation for all $\alpha \in \mathrm{Conf_E}$ and (x_0, y_0).

$$(H^{\mathrm{refl}} \circ F_{\mathrm{r}(uvwxyz)} \circ H^{\mathrm{refl}}(\alpha))(x_0, y_0) = (t_1, r_1, b_1, l_1) = F_{uvwxyz}(\alpha)(x_0, y_0)$$

Therefore, $F_{uvwxyz} = H^{\mathrm{refl}} \circ F_{\mathrm{r}(uvwxyz)} \circ H^{\mathrm{refl}}$ holds, and thus

$$H^{\mathrm{refl}} \circ F_{uvwxyz} \circ H^{\mathrm{refl}} = H^{\mathrm{refl}} \circ H^{\mathrm{refl}} \circ F_{\mathrm{r}(uvwxyz)} \circ H^{\mathrm{refl}} \circ H^{\mathrm{refl}} = F_{\mathrm{r}(uvwxyz)}.$$

This completes the proof. □

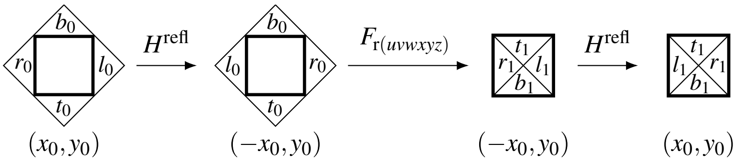


Figure 9: Process of the state-changes around the cells (x_0, y_0) and $(-x_0, y_0)$ in Lemma 3.5

By Lemmas 3.1 and 3.5, we have the following theorem, since it is easy to see $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} = H^{\mathrm{refl}} \circ H^{\mathrm{rev}}$.

Theorem 3.6 *A reversible ESPCA with the ID number uvwxyz is T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$, *if* $\mathrm{inv}(uvwxyz) = \mathrm{r}(uvwxyz)$. *In this case, the following holds.*

$$\begin{aligned} F^{-1}_{uvwxyz} &= H^{\mathrm{rev}} \circ F_{\mathrm{r}(uvwxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ F_{uvwxyz} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Theorem 3.6 and Table 1, we can see that *all* the 128 reversible and conservative ESPCAs are T-symmetric under the involution $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$.

Example 2 *We consider ESPCA-02c5bf (Figure 10). It is reversible and conservative. Since* $\mathrm{inv}(02\mathrm{c}5\mathrm{bf}) = \mathrm{r}(02\mathrm{c}5\mathrm{bf})$ *as shown in Table 1, it is T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$ *(Theorem 3.6). Thus the following holds.*

$$F^{-1}_{02\mathrm{c}5\mathrm{bf}} = H^{\mathrm{rev}} \circ F_{\mathrm{r}(02\mathrm{c}5\mathrm{bf})} \circ H^{\mathrm{rev}} = H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ F_{02\mathrm{c}5\mathrm{bf}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}}$$

The diagram given in Figure 11 illustrates this relation. Namely, the backward transition from a configuration $\alpha(t)$ *to* $\alpha(t-1)$ *is performed by the global function* $F_{02\mathrm{c}5\mathrm{bf}}$ *for the forward transition, provided that the operation* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$ *is applied before and after* $F_{02\mathrm{c}5\mathrm{bf}}$. *The diagram can also be interpreted that the backward transition is performed by the 'similar' global function* $F_{\mathrm{r}(02\mathrm{c}5\mathrm{bf})}$, *provided that* H^{rev} *is applied before and after* $F_{\mathrm{r}(02\mathrm{c}5\mathrm{bf})}$. *Here, 'similar' means that each local transition rule for* $F_{\mathrm{r}(02\mathrm{c}5\mathrm{bf})}$ *is a mirror image of the corresponding rule for* $F_{02\mathrm{c}5\mathrm{bf}}$.

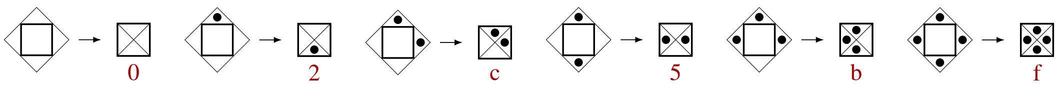


Figure 10: Local function $f_{02\mathrm{c}5\mathrm{bf}}$ of ESPCA-02c5bf

Next, we show Lemma 3.7 stating that ESPCA-*uvwxyz* satisfying $\mathrm{inv}(uvwxyz) = \mathrm{c}(uvwxyz)$ is T-symmetric under a certain simple involution. First, define a function $\mathrm{comp}_4 : \{0,1\}^4 \to \{0,1\}^4$ as follows: $\mathrm{comp}_4(t, r, b, l) = (\bar{t}, \bar{r}, \bar{b}, \bar{l})$ for any $(t, r, b, l) \in \{0,1\}^4$. Next define an involution $H^{\mathrm{comp}} : \mathrm{Conf_E} \to \mathrm{Conf_E}$ as follows. For all $\alpha \in \mathrm{Conf_E}$ and $(x_0, y_0) \in \mathbb{Z}^2$:

$$H^{\mathrm{comp}}(\alpha)(x_0, y_0) = \mathrm{comp}_4(\alpha(x_0, y_0))$$

The involution H^{comp} gives the *complement image* of a configuration.

Lemma 3.7 *The next relation holds for any ESPCA-uvwxyz.*

$$F_{\mathrm{c}(uvwxyz)} = H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}}$$

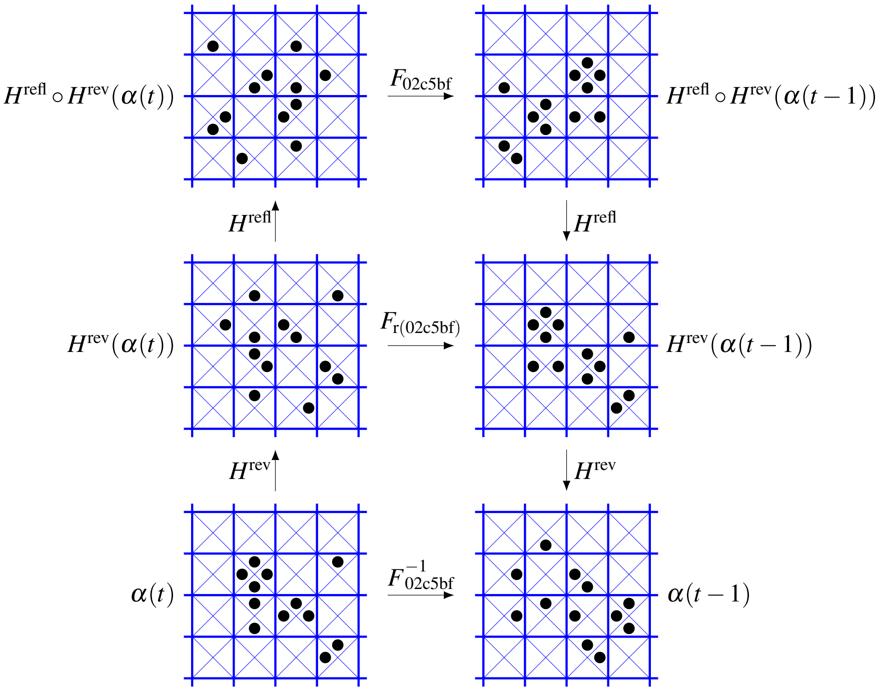


Figure 11: Diagram that illustrates T-symmetry of ESPCA-02c5bf under the involution $H^{\text{rev}} \circ H^{\text{refl}}$

Proof. First, we show $F_{uvwxyz} = H^{\text{comp}} \circ F_{\text{c}(uvwxyz)} \circ H^{\text{comp}}$. Let $\alpha \in \text{Conf}_{\text{E}}$ be any configuration, and $(x_0, y_0) \in \mathbb{Z}^2$ be any point. Let $(t_0, r_0, b_0, l_0) \in \{0,1\}^4$ be as follows.

$$\begin{array}{rcl} \text{pr}_T(\alpha(x_0, y_0-1)) & = & t_0 \\ \text{pr}_R(\alpha(x_0-1, y_0)) & = & r_0 \\ \text{pr}_B(\alpha(x_0, y_0+1)) & = & b_0 \\ \text{pr}_L(\alpha(x_0+1, y_0)) & = & l_0 \end{array}$$

See Figure 12 that shows the process of state-changes by the operations given below. In the next step, we apply H^{comp}, and have the following.

$$\begin{array}{rcl} \text{pr}_T(H^{\text{comp}}(\alpha)(x_0, y_0-1)) & = & \overline{t_0} \\ \text{pr}_R(H^{\text{comp}}(\alpha)(x_0-1, y_0)) & = & \overline{r_0} \\ \text{pr}_B(H^{\text{comp}}(\alpha)(x_0, y_0+1)) & = & \overline{b_0} \\ \text{pr}_L(H^{\text{comp}}(\alpha)(x_0+1, y_0)) & = & \overline{l_0} \end{array}$$

Here we assume $f_{uvwxyz}(t_0, r_0, b_0, l_0) = (t_1, r_1, b_1, l_1)$. Since $f_{\text{c}(uvwxyz)}(\overline{t_0}, \overline{r_0}, \overline{b_0}, \overline{l_0}) = (\overline{t_1}, \overline{r_1}, \overline{b_1}, \overline{l_1})$,

$$(F_{\text{c}(uvwxyz)} \circ H^{\text{comp}}(\alpha))(x_0, y_0) = (\overline{t_1}, \overline{r_1}, \overline{b_1}, \overline{l_1}).$$

Finally, we have the following relation for all $\alpha \in \text{Conf}_{\text{E}}$ and (x_0, y_0).

$$(H^{\text{comp}} \circ F_{\text{c}(uvwxyz)} \circ H^{\text{comp}}(\alpha))(x_0, y_0) = (t_1, r_1, b_1, l_1) = F_{uvwxyz}(\alpha)(x_0, y_0)$$

Therefore, $F_{uvwxyz} = H^{\text{comp}} \circ F_{\text{c}(uvwxyz)} \circ H^{\text{comp}}$ holds, and thus

$$H^{\text{comp}} \circ F_{uvwxyz} \circ H^{\text{comp}} = F_{\text{c}(uvwxyz)}.$$

This completes the proof. □

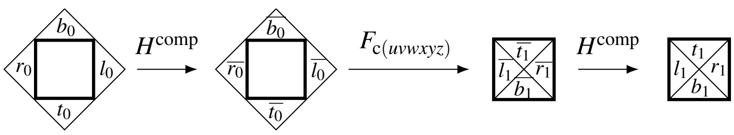


Figure 12: Process of the state-changes around the cell (x_0, y_0) in Lemma 3.7

By Lemmas 3.1 and 3.7, we have the following theorem, since it is easy to see $H^{\mathrm{rev}} \circ H^{\mathrm{comp}} = H^{\mathrm{comp}} \circ H^{\mathrm{rev}}$.

Theorem 3.8 *A reversible ESPCA with the ID number uvwxyz is T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, *if* $\mathrm{inv}(uvwxyz) = \mathrm{c}(uvwxyz)$. *In this case, the following holds.*

$$\begin{aligned} F_{uvwxyz}^{-1} &= H^{\mathrm{rev}} \circ F_{\mathrm{c}(uvwxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Theorem 3.8 and Table 1, we can see that 16 reversible and conservative ESPCAs are T-symmetric under the involution $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$.

Combining Lemmas 3.5 and 3.7, we also obtain the next lemma.

Lemma 3.9 *The next relation holds for any ESPCA-uvwxyz.*

$$F_{\mathrm{rc}(uvwxyz)} = H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}}$$

Proof. By Lemma 3.7, we have

$$F_{\mathrm{c}(uvwxyz)} = H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}}.$$

Therefore, by Lemma 3.5, we have

$$F_{\mathrm{r}(\mathrm{c}(uvwxyz))} = H^{\mathrm{refl}} \circ F_{\mathrm{c}(uvwxyz)} \circ H^{\mathrm{refl}} = H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}}.$$

Since $F_{\mathrm{rc}(uvwxyz)} = F_{\mathrm{r}(\mathrm{c}(uvwxyz))}$, the lemma holds. □

By Lemmas 3.1 and 3.9, we have the following theorem, since it is easy to see $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}} = H^{\mathrm{comp}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}}$.

Theorem 3.10 *A reversible ESPCA with the ID number uvwxyz is T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$, *if* $\mathrm{inv}(uvwxyz) = \mathrm{rc}(uvwxyz)$. *In this case, the following holds.*

$$\begin{aligned} F_{uvwxyz}^{-1} &= H^{\mathrm{rev}} \circ F_{\mathrm{rc}(uvwxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{uvwxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Theorem 3.10 and Table 1, we can see that 32 reversible and conservative ESPCAs are T-symmetric under the involution $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$.

Table 1 shows all reversible and conservative ESPCAs. We can see every reversible and conservative ESPCA is T-symmetric under the corresponding involution. On the other hand, there are many reversible but non-conservative ESPCAs. A complete list of all reversible ESPCAs is given in Appendix A. Since the number of such ESPCAs is large, we give here the total numbers of T-symmetric reversible (but may not be conservative) ESPCAs under H^{rev}, $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$, $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, and $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$ in Table 2. The number of non-T-symmetric reversible ESPCAs under these involutions is also given in this table. It is not known whether each of these 640 ESPCAs becomes T-symmetric under some other involution H.

Table 2: Total numbers of reversible ESPCAs, four types of T-symmetric ones, and non-T-symmetric ones

Types of ESPCAs	Numbers
Reversible ESPCAs	1536
T-symmetric reversible ESPCAs under H^{rev} (i.e., strictly T-symmetric)	128
T-symmetric reversible ESPCAs under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$	448
T-symmetric reversible ESPCAs under $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$	128
T-symmetric reversible ESPCAs under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$	448
Non-T-symmetric reversible ESPCAs under the above involutions	640

4 Applications of T-Symmetries in Reversible ESPCAs

A reversible ESPCA was first proposed in [16]. There, computational universality of ESPCA-02c5df and ESPCA-02c5bf was studied. It was shown that both in these ESPCAs, a *switch gate* and an *inverse switch gate*, which are reversible logic gates, are realisable in their cellular spaces [16, 8]. Since a *Fredkin gate*, a universal reversible logic gate, can be composed of these gates [1], these ESPCAs and their dual ones are computationally universal. It means that any reversible Turing machine can be realised in these reversible ESPCAs [8]. Although T-symmetry is not mentioned in [16, 8], construction of an inverse switch gate has been, in fact, done in a symmetric way as that of a switch gate. In Sections 4.1 and 4.2, we show that such construction of the inverse functional module is explained using T-symmetries of these ESPCAs.

In [14], ESPCA-01caef was investigated, and its computational universality was shown by implementing a reversible logic element with 1-bit memory (RLEM). In ESPCA-01caef, various kinds of space-moving patterns exist. In Section 4.3 we show that a backward evolution process for a given process on space-moving patterns is easily obtained by using T-symmetry of the ESPCA.

We now give the following lemma to make it easy to show application examples of T-symmetries.

Lemma 4.1 *Let P be a reversible ESPCA with the global function F_{uvwxyz}. Assume P is T-symmetric under an involution H, i.e., $F^{-1}_{uvwxyz} = H \circ F_{uvwxyz} \circ H$. Then the following holds for any $n \in \{1,2,\ldots\}$.*

$$(F^{-1}_{uvwxyz})^n = H \circ (F_{uvwxyz})^n \circ H$$

Proof. It is easily proved by a mathematical induction. The case $n = 1$ is obvious. Assume it holds for $n = k$. Then, $(F^{-1}_{uvwxyz})^{k+1} = H \circ (F_{uvwxyz})^k \circ H \circ H \circ F_{uvwxyz} \circ H = H \circ (F_{uvwxyz})^k \circ F_{uvwxyz} \circ H = H \circ (F_{uvwxyz})^{k+1} \circ H$. □

4.1 ESPCA-02c5df: strictly T-symmetric

Consider ESPCA-02c5df given in Example. 1. Its local function is shown in Figure 7. It is reversible and conservative. Note that it is isomorphic to the block-update function of Margolus' CA [6]. Here, we explain how an inverse switch gate is obtained from a switch gate by using strict T-symmetry of ESPCA-02c5df.

A *switch gate* is a 2-input 3-output reversible gate having the logical function $f_S(c,x) = (c, cx, \bar{c}x)$. We can interpret it as the operation where the input c switches the output port of the input x. It is reversible in the sense that $f_S : \{0,1\}^2 \to \{0,1\}^3$ is an injection.

An *inverse switch gate* is a 3-input 2-output reversible gate having the partial logical function $f_S^{-1}(y_1,y_2,y_3) = (c,x)$, where $c = y_1$, and $x = y_2 + y_3$ under the assumption of $(y_2 \Rightarrow y_1) \wedge (y_3 \Rightarrow \overline{y_1})$. Namely, it is defined only on the set $\{(0,0,0),(0,0,1),(1,0,0),(1,1,0)\}$.

In [16, 8], a switch gate is implemented in ESPCA-02c5df in the following way. First, a signal, which represents the logical value 1, is given by a space-moving pattern consisting of two particles shown in Figure 13.

Colliding two signals as in Figure 14 their trajectories are changed. By this, a kind of logical operation is performed.

A left-turn of a signal is realised by a reflector composed of two blocks as shown in Figure 15, where a *block* is a stable pattern consisting of eight particles. Note that a right-turn of a signal is possible by mirror images of the configurations in Figure 15.

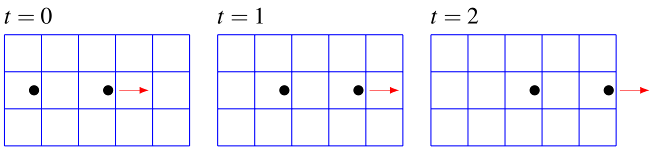


Figure 13: Signal in ESPCA-02c5df. It consists of two particles

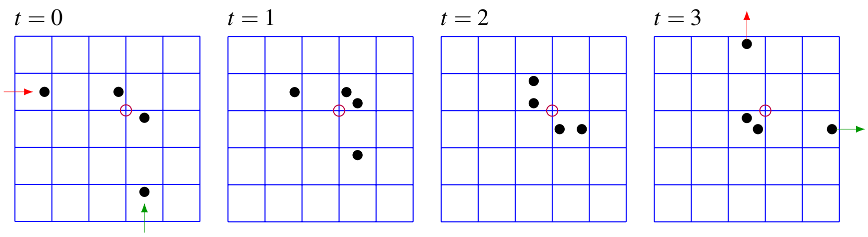


Figure 14: Collision of two signals in ESPCA-02c5df [16, 8]. A small circle shows the virtual collision point

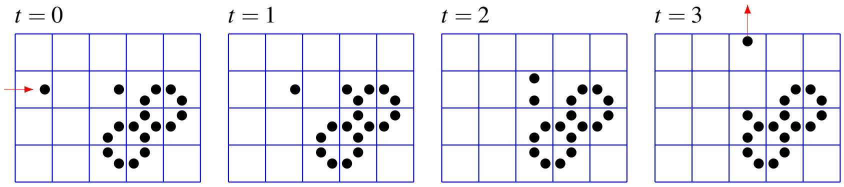


Figure 15: Left-turn of a signal by a reflector composed of two blocks in ESPCA-02c5df [16, 8]

Using these phenomena, we can compose a switch gate. The configuration $\alpha(0)$ in Figure 16 is a switch gate pattern [16, 8]. In this figure, input signals are given to both c and x at $t=0$. In this case, the signals come out from the output ports c and cx as in $\alpha(28)$ at $t=28$. If only one signal is given to the input port c (or x, respectively), it comes out from c (or $\bar{c}x$). Thus, the pattern correctly simulates a switch gate.

From Lemma 4.1 we can easily obtain an inverse switch gate. Since ESPCA-02c5df is strictly T-symmetric, the following relation holds by Lemma 4.1.

$$(F_{02c5df}^{-1})^{28}(\alpha(28)) = H^{rev} \circ (F_{02c5df})^{28} \circ H^{rev}(\alpha(28))$$

As shown in Figure 16, this relation means that a 28-step backward evolution starting from $\alpha(28)$ is simulated by a 28-step forward evolution starting from $H^{rev}(\alpha(28))$. Its resulting configuration is $(F_{02c5df})^{28} \circ H^{rev}(\alpha(28)) = H^{rev}(\alpha(0))$. Since blocks do not change the patterns by H^{rev}, the configurations $H^{rev}(\alpha(28))$ and $H^{rev}(\alpha(0))$ are obtained from $\alpha(28)$ and $\alpha(0)$ by reversing the move directions of signals. Thus, the switch gate pattern itself works as an inverse switch gate by simply swapping the roles of input and output ports.

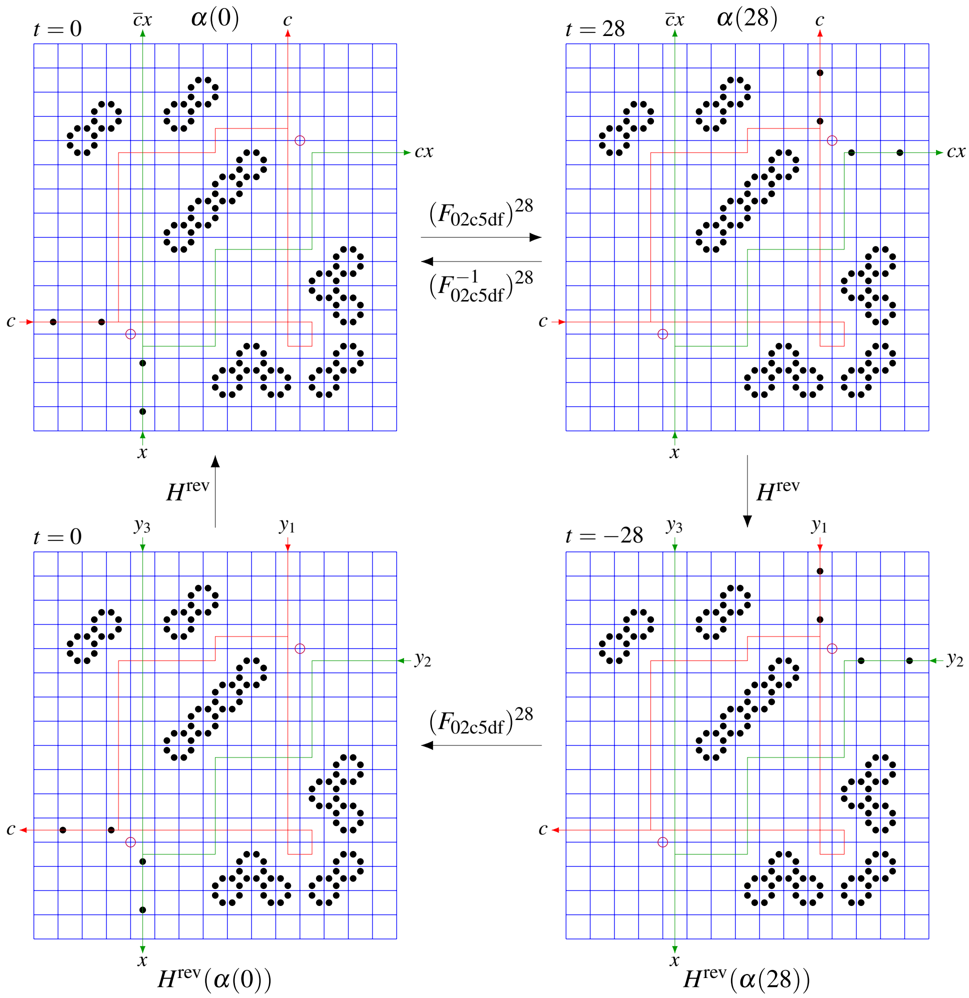


Figure 16: We can make an inverse switch gate ($H^{rev}(\alpha(28))$) from a switch gate ($\alpha(0)$) in ESPCA-02c5df based on its strict T-symmetry

4.2 ESPCA-02c5bf: T-symmetric under $H^{\text{rev}} \circ H^{\text{refl}}$

Next, consider ESPCA-02c5bf given in Example. 2. Its local function is in Figure 10. It is reversible and conservative. A signal (Figure 13), and collision of two signals (Figure 14) are exactly the same as in ESPCA-02c5df (Section 4.1). But, a left-turn of a signal is different. As shown in Figure 17, it is performed by a single block. However, a right-turn is not possible by a single block. If we start from a mirror image of the configuration of $t = 0$ in Figure 17, then the signal and the block will be broken (Figure 18). Hence, a right-turn should be implemented by three left-turns.

Using the above phenomena, we obtain a switch gate in ESPCA-02c5bf as in the configuration $\beta(0)$ [16, 8] in Figure 19. This configuration shows that input signals are given to both c and x at $t = 0$. Then, they will come out from the output ports c and cx at $t = 25$. It is also easy to verify other cases, and thus the pattern correctly simulates a switch gate.

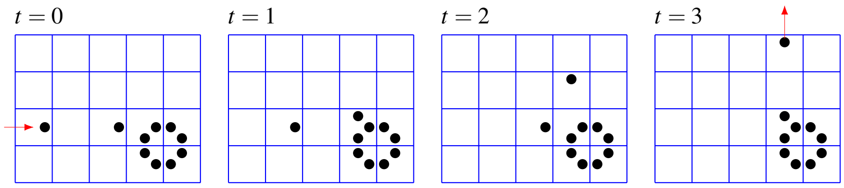


Figure 17: Left-turn of a signal by a reflector composed of a block in ESPCA-02c5bf [16, 8]

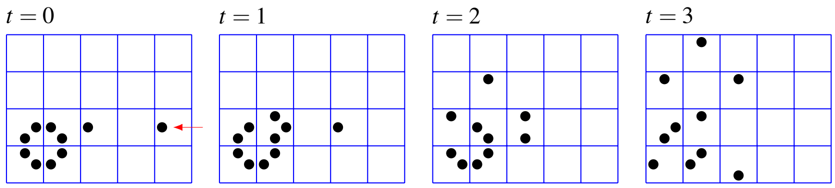


Figure 18: Right-turn of a signal by a block is not possible in ESPCA-02c5bf [8]

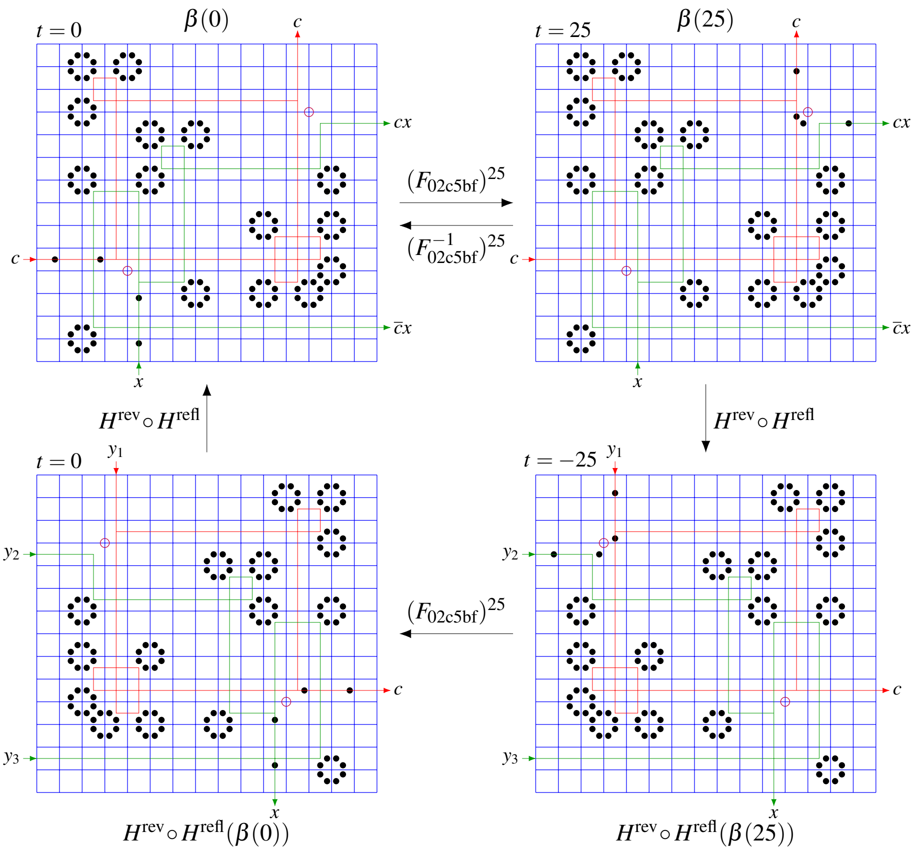


Figure 19: We can make an inverse switch gate ($H^{\text{rev}} \circ H^{\text{refl}}(\beta(25))$) from a switch gate ($\beta(0)$) in ESPCA-02c5bf based on its T-symmetry under $H^{\text{rev}} \circ H^{\text{refl}}$

From Lemma 4.1 we have the following relation, since ESPCA-02c5bf is T-symmetric under $H^{\text{rev}} \circ H^{\text{refl}}$.

$$(F_{02c5bf}^{-1})^{25}(\beta(25)) = H^{\text{rev}} \circ H^{\text{refl}} \circ (F_{02c5bf})^{25} \circ H^{\text{rev}} \circ H^{\text{refl}}(\beta(25))$$

As shown in Figure 19, it means that a 25-step backward evolution starting from $\beta(25)$ is simulated by a 25-step forward evolution starting from $H^{\text{rev}} \circ H^{\text{refl}}(\beta(25))$. Its resulting configuration is $(F_{02c5bf})^{25} \circ H^{\text{rev}} \circ H^{\text{refl}}(\beta(25)) = H^{\text{rev}} \circ H^{\text{refl}}(\beta(0))$. Note that $H^{\text{rev}} \circ H^{\text{refl}}(\beta(25))$ and $H^{\text{rev}} \circ H^{\text{refl}}(\beta(0))$ are obtained from $\beta(25)$ and $\beta(0)$ by taking mirror images of them, and reversing the move directions of signals. Therefore, the mirror image of the switch gate pattern works as an inverse switch gate by swapping the roles of input and output ports.

Note that ESPCA-02c5df (Section 4.1) is T-symmetric under $H^{\text{rev}} \circ H^{\text{refl}}$, as well as T-symmetric under H^{rev} (see Table 1 and Theorems 3.3 and 3.6). Therefore, $H^{\text{rev}} \circ H^{\text{refl}}(\alpha(28))$ also works as an inverse switch gate in ESPCA-02c5df.

4.3 ESPCA-01caef: T-symmetric under $H^{\text{rev}} \circ H^{\text{refl}}$ and $H^{\text{rev}} \circ H^{\text{comp}}$

We consider ESPCA-01caef, whose local function is shown in Figure 20. It is a reversible and conservative ESPCA studied in [11, 14]. It was shown that any *reversible logic element with memory* (RLEM), which is a kind of a reversible finite automaton, is implemented in its cellular space. Since reversible Turing machines (RTMs) can be composed of RLEMs, we can see ESPCA-01caef is computationally universal. Construction of RTMs becomes much simpler by using RLEMs than by using reversible logic gates [11, 14].

Note that, despite the simplicity of its local function, evolution processes of ESPCA-01caef are generally very complex, and thus it is difficult to follow them by paper and pencil. We created an emulator for ESPCA-01caef on the general purpose CA simulator *Golly* [18] for viewing evolution processes. The emulator file and pattern files are available in [10].

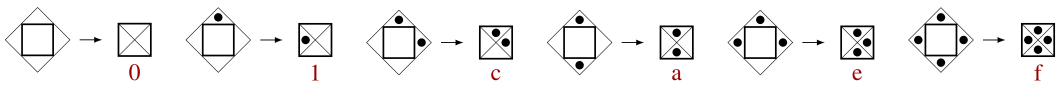


Figure 20: Local function f_{01caef} of ESPCA-01caef

First, we give a simple example of using its T-symmetry. In ESPCA-01caef, there exist many kinds of space-moving patterns [11]. Figure 21 is one such example having period 12, which we call here a *glider-12*. It flies in the cellular space in a diagonal direction. Figure 22 is another example of a space-moving pattern called a *glider-44*, which is of period 44. It moves horizontally or vertically. Here we consider a process that transforms the former glider to the latter. From it, we can obtain two kinds of inverse transformation processes by two kinds of T-symmetries of ESPCA-01caef.

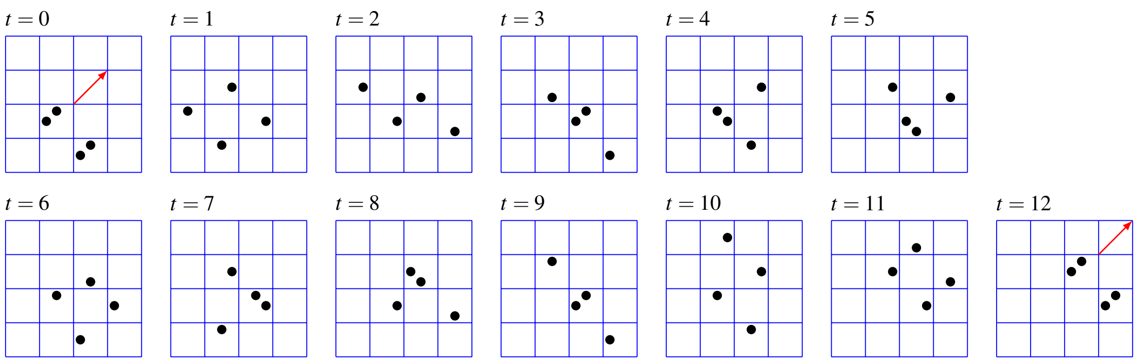


Figure 21: Glider-12, a space-moving pattern of period 12 in ESPCA-01caef [11, 14]

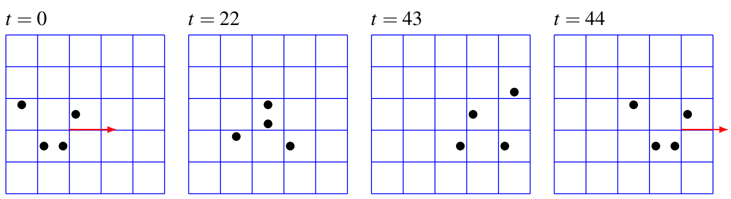


Figure 22: Glider-44, a space-moving pattern of period 44 in ESPCA-01caef

Consider the configuration $\gamma(0)$ in Figure 23. There are a glider-12 moving to the north-east and a particle. They interact, and produce a glider-44 moving to the east and a particle as shown in $\gamma(145)$. Thus, a glider-12 is converted into a glider-44 by colliding it with a particle.

Applying $H^{\text{rev}} \circ H^{\text{refl}}$ to $\gamma(145)$, we obtain a configuration that gives the inverse process of the above. In $H^{\text{rev}} \circ H^{\text{refl}}(\gamma(145))$ there are a glider-44 moving to the east and a particle. Note that the pattern of the glider-44 is the same as the one at $t = 43$ in Figure 22. Namely, the phase of the glider-44 is shifted by the application of $H^{\text{rev}} \circ H^{\text{refl}}$. These two objects interact in ESPCA-01caef, and finally produce a glider-12 moving to the south-east direction and a particle as shown in $H^{\rm rev} \circ H^{\rm refl}(\gamma(0))$ of Figure 23. Thus, a glider-44 is converted into a glider-12. Note that the pattern of the glider-12 is the one obtained by rotating the one at $t = 11$ in Figure 21 by 90 degrees clockwise. Namely, the phase and the direction of the glider-12 are changed by the application of $H^{\rm rev} \circ H^{\rm refl}$.

Since ESPCA-01caef is T-symmetric under $H^{\rm rev} \circ H^{\rm comp}$ as well as $H^{\rm rev} \circ H^{\rm refl}$ (see Table 1), we can obtain a backward evolution process also by this involution, where the glider-12 and glider-44 are represented by 'holes', as shown in Figure 24. In this case, however, the resulting configuration is infinite (i.e., it contains an infinite number of non-blank cells). If we want to have a finite configuration that undoes the evolution process of a given finite configuration, this method is not usable.

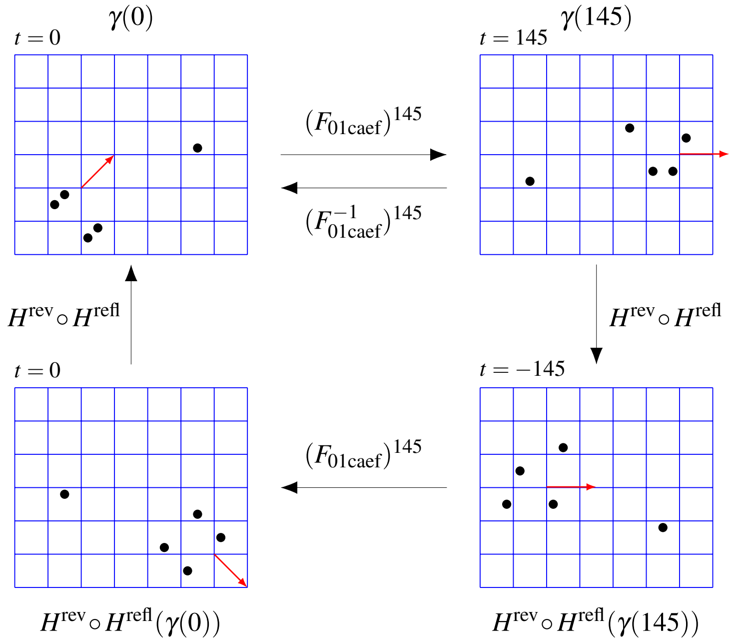


Figure 23: Using the process of converting a glider-12 ($\gamma(0)$) to a glider-44 ($\gamma(145)$), we can convert a glider-44 ($H^{\rm rev} \circ H^{\rm refl}(\gamma(145))$) to a glider-12 ($H^{\rm rev} \circ H^{\rm refl}(\gamma(0))$) in ESPCA-01caef. It is based on its T-symmetry under $H^{\rm rev} \circ H^{\rm refl}$

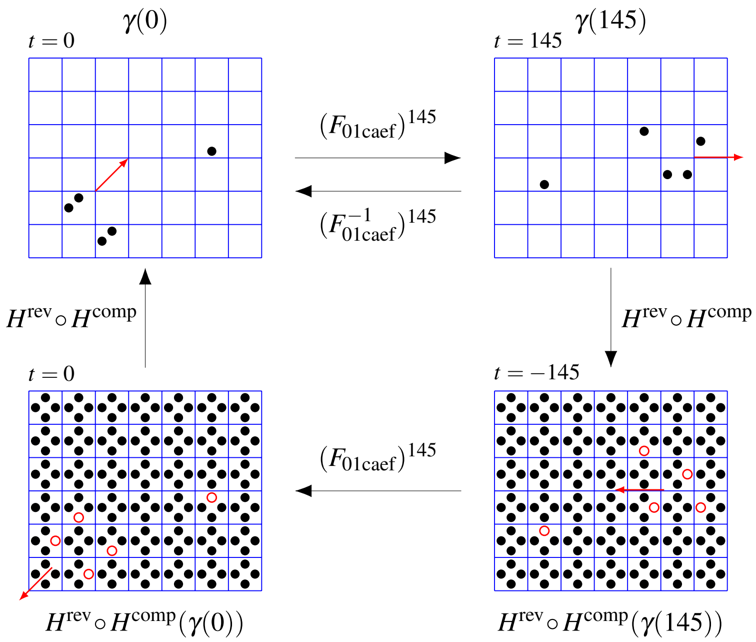


Figure 24: Using the process of converting a glider-12 ($\gamma(0)$) to a glider-44 ($\gamma(145)$), we can convert a complemented glider-44 (shown in $H^{\rm rev} \circ H^{\rm comp}(\gamma(145))$ as 'holes') to a complemented glider-12 ($H^{\rm rev} \circ H^{\rm comp}(\gamma(0))$) in ESPCA-01caef. It is based on its T-symmetry under $H^{\rm rev} \circ H^{\rm comp}$. Note that, in $H^{\rm rev} \circ H^{\rm comp}(\gamma(0))$ and $H^{\rm rev} \circ H^{\rm comp}(\gamma(145))$, the state 0 is represented by a small circle

Next, we give another example. We show that for a given patten that simulates an RLEM, a pattern that simulates its *inverse* RLEM is easily obtained by using T-symmetry of ESPCA-01caef.

Here, we make some preparations (see [8, 14] for the details). A *sequential machine* is a kind of a finite automaton having output symbols as well as input symbols. It is defined by $M = (Q, \Sigma, \Gamma, \delta)$, where Q is a finite set of states, Σ and Γ are finite sets of input and output symbols, and $\delta : Q \times \Sigma \to Q \times \Gamma$ is a move function. If δ is injective, it is calld a *reversible sequential machine* (RSM). A *reversible logic element with memory* (RLEM) is an RSM that satisfies $|\Sigma| = |\Gamma|$. If $n = |Q|$ and $k = |\Sigma| = |\Gamma|$, it is called an n-state k-symbol RLEM. 2-state RLEMs are particularly important, since it is known that any 2-state k-symbol RLEM is universal if $k \geq 3$, i.e., any RSM can be composed only of it [8].

We consider a 2-state RLEM No. 2-3, where '2' stands for 2-symbol and '3' is the serial number in the class of 2-state 2-symbol RLEMs. It is defined by $M_{2\text{-}3} = (\{0,1\}, \{a,b\}, \{x,y\}, \delta_{2\text{-}3})$, where $\delta_{2\text{-}3}$ is as follows.

$$\delta_{2\text{-}3}(0,a) = (0,x),\ \delta_{2\text{-}3}(0,b) = (1,x),\ \delta_{2\text{-}3}(1,a) = (1,y),\ \delta_{2\text{-}3}(1,b) = (0,y)$$

A 2-state RLEM No. 2-4 is defined by $M_{2\text{-}4} = (\{0,1\}, \{a,b\}, \{x,y\}, \delta_{2\text{-}4})$, where $\delta_{2\text{-}4}$ is as follows.

$$\delta_{2\text{-}4}(0,a) = (0,x),\ \delta_{2\text{-}4}(0,b) = (1,y),\ \delta_{2\text{-}4}(1,a) = (0,y),\ \delta_{2\text{-}4}(1,b) = (1,x)$$

It is easy to see that $\delta_{2\text{-}4}$ is isomorphic to $\delta_{2\text{-}3}^{-1}$. In this sense, RLEM 2-4 is the *inverse* of RLEM 2-3. It has been shown that the set {RLEM 2-3, RLEM 2-4} is universal, though each of RLEMs 2-3 and 2-4 is non-universal [8]. Namely, any RSM can be constructed out of RLEMs 2-3 and 2-4.

These RLEMs are implemented in ESPCA-01caef using a glider-12 (Figure 21) and a periodic patten called a *blinker* (Figure 25 **(a)**) [14]. Here, a glider-12 is used as a signal. Note that a stable pattern called a *block* (Figure 25 **(b)**) is also used for writing comments and indicating a border of a logic element in the cellular space. Hence, a block has no functional role for composing the RLEMs.

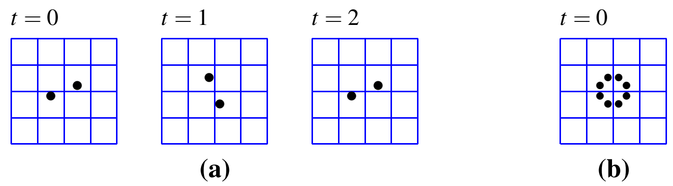


Figure 25: **(a)** Blinker, a peridic pattern of period 2, and **(b)** block, a stable pattern, in ESPCA-01caef

By colliding a glider-12 with a blinker appropriately, a right-turn and a U-turn of a glider-12, and shifting a blinker is possible. First, colliding a glider-12 with a blinker as in Figure 26, a right-turn of a glider-12 is realised.

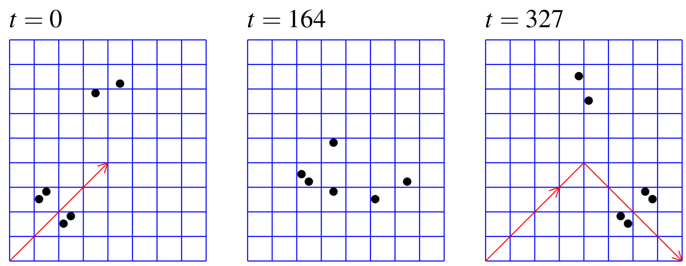


Figure 26: Right-turn of a glider-12 in ESPCA-01caef [11, 14]

A U-turn of a glider-12 is performed as in Figure 27. It is used to test if a blinker exists or not at a specified position. It is also used to reversibly merge two signal paths into one (it is explained later).

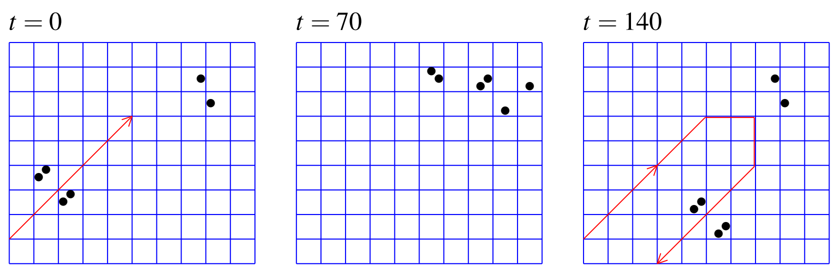


Figure 27: U-turn of a glider-12 in ESPCA-01caef [11, 14]

Finally, colliding a glider-12 with a blinker as in Figure 28, the position of the blinker is shifted by 6 cells, and the glider-12 makes a right-turn. Using this phenomenon, a kind of memory device is realized, where the memory states are kept by the positions of the blinker. At the same time, it can test if a blinker exists at a specified position, and can merge two signal paths into one.

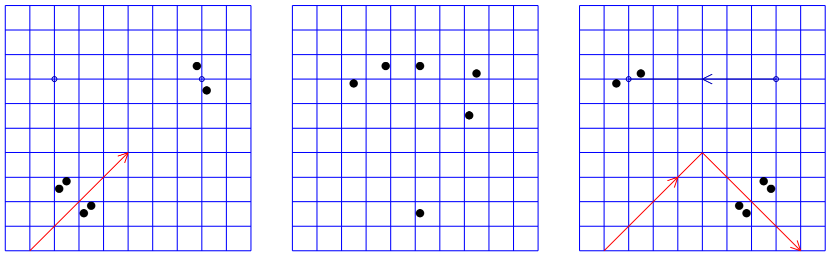


Figure 28: Shifting a blinker by a glider-12 in ESPCA-01caef [11, 14]

The pattern shown in Figure 29 simulates RLEM 2-3. There are many blinkers in this pattern. One is used as a *position marker* for keeping the memory state 0 or 1, while others are used for turning a signal. Two small circles near the center of the pattern show possible positions of the position marker. If the marker is at the left (right, respectively) position, we regard that the RLEM is in the state 0 (1).

First, consider the case where the state is 0 and an input signal is given to the port *a* The signal makes a U-turn at the U-turn gadget U_1 since the state is 0. Then it goes to the gadget U_2, and again makes a U-turn passing through Q. Note that U_2 is used to reversibly merge the path with that of the second case. Finally the signal goes out from the port *x*.

Second, consider the case where the state is 0 and an input signal is given to the port *b*. At P the signal shifts the position marker to the right, and makes a right-turn. Thus, the state changes to 1. Then, the signal goes out from the output port *x* via the point Q. This signal path is merged with that of the first case at Q.

Third, consider the case where the state is 1 and an input signal is given to the port *a*. In this case, the signal goes out from the output port *y* via S and R without interacting the position marker.

Fourth, consider the case where the state is 1 and an input signal is given to the port *b*. The signal goes straight ahead at the point P. Then, it shifts the position marker to the left and makes a right-turn at R. Thus, the state changes to 0. Finally it goes out from *y*. This signal path is merged with that of the third case at R.

Note that, in an RLEM, an incoming signal interacts with the state of the RLEM, not with other signals. Therefore, there is no need of synchronizing two or more signals as in the case of logic gates. Therefore, it greatly simplifies implementation of RLEMs and connecting them in ESPCA-01caef.

The pattern shown in Figure 30 simulates RLEM 2-4. It is obtained by applying the involution $H^{\text{rev}} \circ H^{\text{refl}}$ to the pattern of RLEM 2-4 given in Figure 29. By the T-symmetry of ESPCA-01caef, the pattern for RLEM 2-4 *undoes* the operations of the pattern for RLEM 2-3. As in the case of RLEM 2-3, one blinker near the center of the pattern is used as a position marker for keeping the memory state 0 or 1. If the marker is in the right (left, respectively) small circle, we regard that the RLEM is in the state 0 (1).

First, consider the case where the state is 0 and an input signal is given to the port *a*. The signal makes a U-turn at U_2. Then it goes to U_1, and again makes a U-turn passing through T. Finally the signal goes out from the port *x*.

Second, consider the case where the state is 1 and an input signal is given to the port *b*. At R the signal goes straight ahead. Then it passes through the points S and T. Finally it goes out from the port *x*. This signal path is merged with that of the first case at T.

Third, consider the case where the state is 1 and an input signal is given to the port *a*. The signal goes straight ahead at Q. Then, at P it shifts the position marker to the right, and makes a right-turn. By this, the state changes to 0. Finally it goes out from the port *y*.

Fourth, consider the case where the state is 0 and an input signal is given to the port *b*. At R the signal shifts the position marker to the left, and makes a right-turn. By this, state changes to 1. Then, it passes through P, and finally goes out from *y*. In this case, the signal path is merged with that of the third case at P.

In [14], reversible Turing machines are constructed using the above patterns that simulate RLEMs 2-3 and 2-4. Their whole computing processes in the ESPCA space can be seen on the CA simulator *Golly* [18] using the emulator file and the pattern files given in [10].

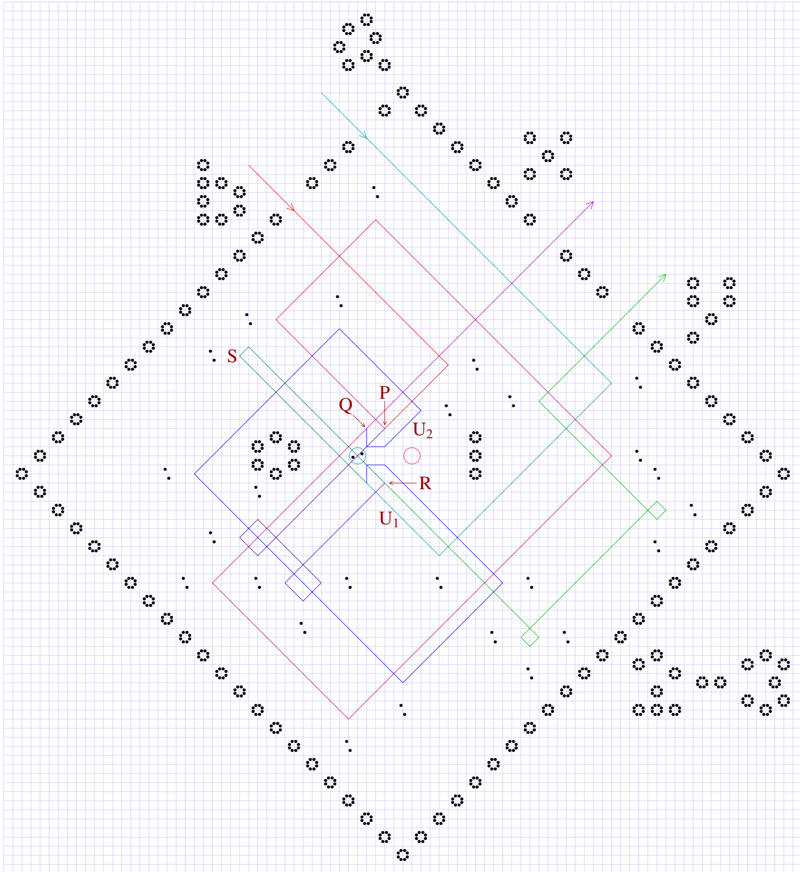


Figure 29: RLEM 2-3 implemented in ESPCA-01caef [14]

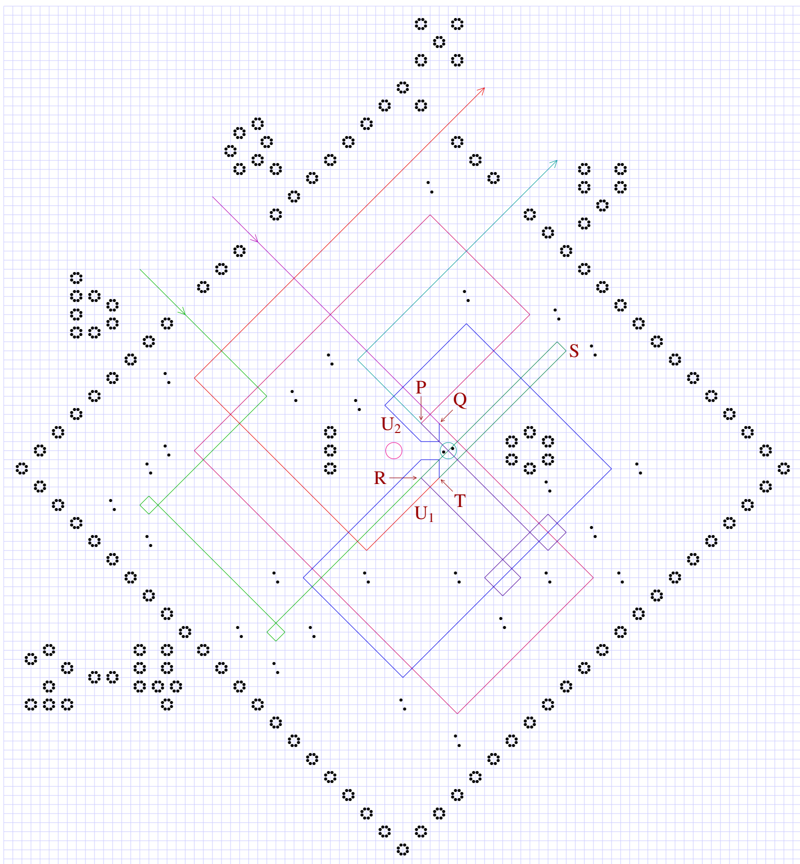


Figure 30: RLEM 2-4 implemented in ESPCA-01caef [14]. It is obtained by applying the involution $H^{\text{rev}} \circ H^{\text{refl}}$ to the pattern given in Figure 29 (except the comment part written by blocks)

5 Elementary Triangular Partitioned Cellular Automata (ETPCAs) and Their T-symmetries

A three-neighbour triangular partitioned cellular automaton (TPCA) proposed in [3] is a PCA whose cell is triangular and is divided into three parts. Since its cell has only three neighbour cells, its 'elementary' version, which is rotation-symmetric and each part of whose cell has only two states, is simpler than ESPCA. Despite their simplicity, several kinds of reversible ETPCAs have been known to be computationally universal, i.e., any reversible Turing machine can be constructed in their cellular spaces [3, 8, 13].

T-symmetries in reversible ETPCAs were first investigated in [13]. In this section, we describe the previous results, and add some results that have not been given before. We compare T-symmetries in reversible ETPCAs with those in reversible ESPCAs, and show examples of their applications.

5.1 Definitions on ETPCAs

The cellular space of a TPCA is shown in Fig 31. All the cells are identical in their logical functions. However, there are two kinds of directions, i.e., upward and downward (Figure 32). Therefore, the neighbour cells of an up-triangle cell are different from those of a down-triangle cell. A local transition rule for an up-triangle cell is depicted in Figure 33. For a down-triangle cell, the rule obtained by rotating both sides of the rule given in Figure 33 by 180 degrees is used. In the following, we assume up-triangle (down-triangle, respectively) cells are placed at a coordinates $(x,y) \in \mathbb{Z}^2$ such that $x+y$ is even (odd).

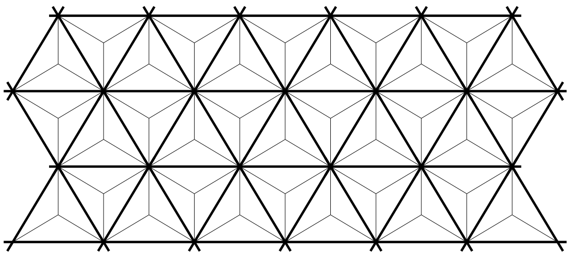
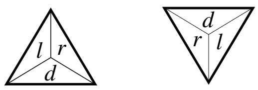


Figure 31: Cellular space of a triangular partitioned cellular automaton (TPCA)



(a) **(b)**

Figure 32: **(a)** An up-triangle cell, and **(b)** a down-triangle cell of TPCA

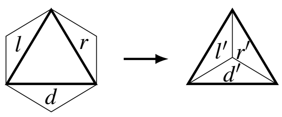


Figure 33: Local transition rule of TPCA. It depicts the relation $f(l,d,r) = (l',d',r')$

Definition 5.1 *A* three-neighbour triangular partitioned cellular automaton *(TPCA) is a system defined by*

$$P = (\mathbb{Z}^2, (L,D,R), ((-1,0),(0,-1),(1,0)), ((1,0),(0,1),(-1,0)), f).$$

Here, $\mathbb{Z}^2$ is the set of all two-dimensional points with integer coordinates at which cells are placed, and L, D and R are non-empty finite sets of states of the left, downward and right parts of a cell. The state set Q of a cell is thus $Q = L \times D \times R$. The triplet $((-1,0),(0,-1),(1,0))$ is a neighbourhood *for up-triangle cells, and $((1,0),(0,1),(-1,0))$ is a neighbourhood for down-triangle cells. The item $f : Q \to Q$ is a* local (transition) function.

If $f(l,d,r) = (l',d',r')$ holds for $(l,d,r),(l',d',r') \in Q$, then this relation is called a *local transition rule* of the TPCA *P*. It is written pictorially as in Figure 33. The local function f is thus defined by a set of local transition rules.

Configurations of a TPCA, and the global function are defined as below.

Definition 5.2 *Let* $P = (\mathbb{Z}^2, (L,D,R), ((-1,0),(0,-1),(1,0)), ((1,0),(0,1),(-1,0)), f)$ *be a TPCA. A* configuration *of P is a function* $\alpha : \mathbb{Z}^2 \to Q$*. The set of all configurations of P is denoted by* $\mathrm{Conf}(P)$*, i.e.,* $\mathrm{Conf}(P) = \{\alpha \mid \alpha : \mathbb{Z}^2 \to Q\}$*. Let* $\mathrm{pr}_L : Q \to L$ *be the* projection function *such that* $\mathrm{pr}_L(l,d,r) = l$ *for all* $(l,d,r) \in Q$*. The projection functions* $\mathrm{pr}_D : Q \to D$ *and* $\mathrm{pr}_R : Q \to R$ *are defined similarly. The* global function $F : \mathrm{Conf}(P) \to \mathrm{Conf}(P)$ *of P is defined as the one that satisfies the following.*

$$\forall \alpha \in \mathrm{Conf}(T), \forall (x,y) \in \mathbb{Z}^2 :$$
$$F(\alpha)(x,y) = \begin{cases} f(\mathrm{pr}_L(\alpha(x-1,y)), \mathrm{pr}_D(\alpha(x,y-1)), \mathrm{pr}_R(\alpha(x+1,y))) & \text{if } x+y \text{ is even} \\ f(\mathrm{pr}_L(\alpha(x+1,y)), \mathrm{pr}_D(\alpha(x,y+1)), \mathrm{pr}_R(\alpha(x-1,y))) & \text{if } x+y \text{ is odd} \end{cases}$$

Reversibility of TPCA is defined similarly to the case of SPCA (Definition 2.3).

Definition 5.3 *A TPCA P is called* reversible *if its global function F is injective.*

As in the case of SPCA (Lemma 2.4), injectivity of the global function is equivalent to injectivity of the local function in a TPCA [8].

Lemma 5.4 *Let P be a TPCA. Its global function F is injective if and only if its local function f is injective.*

An elementary triangular partitioned cellular automaton (ETPCA) is also defined similarly as in the case of ESPCA (Definition 2.6).

Definition 5.5 *Let* $P = (\mathbb{Z}^2, (L,D,R), ((-1,0),(0,-1),(1,0)), ((1,0),(0,1),(-1,0)), f)$ *be a TPCA. The TPCA P is called* rotation-symmetric *(or* isotropic*) if the following conditions (1) and (2) holds.*

(1) $L = D = R$

(2) $\forall (l,d,r), (l',d',r') \in L \times D \times R : \; f(l,d,r) = (l',d',r') \Rightarrow f(d,r,l) = (d',r',l')$

Definition 5.6 *Let* $P = (\mathbb{Z}^2, (L,D,R), ((-1,0),(0,-1),(1,0)), ((1,0),(0,1),(-1,0)), f)$ *be a TPCA. The TPCA P is called an* elementary triangular partitioned cellular automaton *(ETPCA), if* $L = D = R = \{0,1\}$*, and it is rotation-symmetric.*

Since an ETPCA is rotation-symmetric, its local function $f : \{0,1\}^3 \to \{0,1\}^3$ is described by only four local transition rules, which are obtained by giving the following four values.

$$f(0,0,0),\; f(0,1,0),\; f(1,0,1),\; f(1,1,1)$$

Here, $f(0,1,0), f(1,0,1) \in \{0,1\}^3$, but $f(0,0,0), f(1,1,1) \in \{(0,0,0),(1,1,1)\}$ since it is rotation-symmetric.

Reading the values of $f(0,0,0), f(0,1,0), f(1,0,1)$ and $f(1,1,1)$ as four binary numbers, we can express an ETPCA by a 4-digit octal identification (ID) number *wxyz* as shown in Figure 34. Thus there are 256 ETPCAs in total.

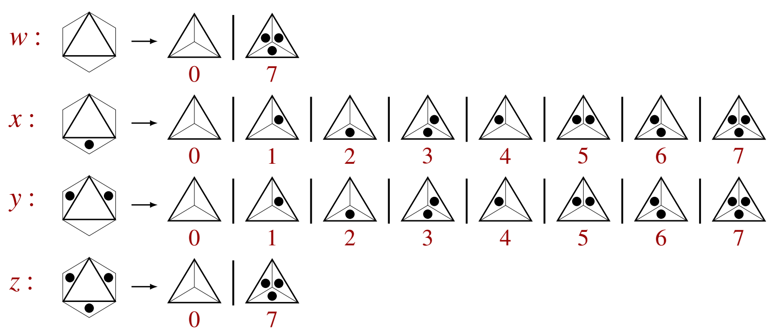


Figure 34: Expressing an ETPCA by a 4-digit octal ID number *wxyz*. Vertical bars indicate alternatives of the right-hand side of each local transition rule

An ETPCA with the ID number *wxyz* is denoted by ETPCA-*wxyz*. Its local function and global function are represented by f_{wxyz} and F_{wxyz}, respectively. Figure 35 shows the set of local transition rules of ETPCA-0137.

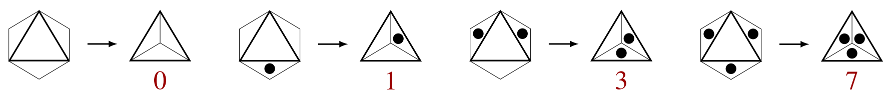


Figure 35: Local transition rules of ETPCA-0137, which define the local function f_{0137}

Definition 5.7 *An ETPCA P is called* conservative *if the number of state 1's is conserved in each local transition rule.*

It is easy to see the following proposition as in the case of ESPCAs (Proposition 2.8).

Proposition 5.8 *Let P be an ETPCA with an ID number wxyz.*

(1) P is reversible if and only if the following condition holds.
$(w,z) \in \{(0,7),(7,0)\} \wedge (x,y) \in \{1,2,4\} \times \{3,5,6\} \cup \{3,5,6\} \times \{1,2,4\}$

(2) P is conservative if and only if the following condition holds.
$w = 0 \wedge x \in \{1,2,4\} \wedge y \in \{3,5,6\} \wedge z = 7$

From the above proposition, we can see that in the case of ETPCAs, conservative ETPCAs are all reversible. Note that it is not the case in ESPCA (Proposition 2.8). The total numbers of reversible, conservative, and reversible and conservative ETPCAs are 36, 9, and 9, respectively.

5.2 Dualities in ETPCA

As in the case of ESPCA, we consider two kinds of dualities in ETPCA.

Definition 5.9 *Let P be an ETPCA and $f : \{0,1\}^3 \to \{0,1\}^3$ be its local function. Define $f^{\mathrm{r}} : \{0,1\}^3 \to \{0,1\}^3$ as follows.*

$$\forall (l,d,r),(l',d',r') \in \{0,1\}^3 : \ f(l,d,r) = (l',d',r') \Leftrightarrow f^{\mathrm{r}}(r,d,l) = (r',d',l')$$

Then, the ETPCA P^{r} having the local function f^{r} is called the dual ETPCA of *P* under reflection.

Definition 5.10 *Let P be an ETPCA and $f : \{0,1\}^3 \to \{0,1\}^3$ be its local function. Let $\overline{x} = 1 - x$ be the complement of x. Define $f^{\mathrm{c}} : \{0,1\}^3 \to \{0,1\}^3$ as follows.*

$$\forall (l,d,r),(l',d',r') \in \{0,1\}^3 : \ f(l,d,r) = (l',d',r') \Leftrightarrow f^{\mathrm{c}}(\overline{l},\overline{d},\overline{r}) = (\overline{l'},\overline{d'},\overline{r'})$$

Then, the ETPCA P^{c} having the local function f^{c} is called the dual ETPCA of *P* under complementation.

As in the case of ESPCA, for an ESTCA P with a local function f, there is an ETPCA P^{rc} whose local function is $(f^{\mathrm{r}})^{\mathrm{c}} = (f^{\mathrm{c}})^{\mathrm{r}}$. We write the local function of P^{rc} by f^{rc} shortly.

We denote the ID numbers of f^{r}_{wxyz}, f^{c}_{wxyz}, f^{rc}_{wxyz}, and f^{-1}_{wxyz} by $\mathrm{r}(wxyz)$, $\mathrm{c}(wxyz)$, $\mathrm{rc}(wxyz)$, and $\mathrm{inv}(wxyz)$, respectively. Namely, $f^{\mathrm{r}}_{wxyz} = f_{\mathrm{r}(wxyz)}$, $f^{\mathrm{c}}_{wxyz} = f_{\mathrm{c}(wxyz)}$, $f^{\mathrm{rc}}_{wxyz} = f_{\mathrm{rc}(wxyz)}$, and $f^{-1}_{wxyz} = f_{\mathrm{inv}(wxyz)}$.

Table 3 shows the list of ID numbers of local functions (f) of 36 reversible ETPCAs, their dual ones (f^{r}, f^{c} and f^{rc}), and their inverses (f^{-1}).

Table 3: Identification numbers of 36 reversible ETPCAs, their dual ones (under reflection, complementation, and both), and their inverses. In each ETPCA, the IDs of local functions among $f, f^{\mathrm{r}}, f^{\mathrm{c}}$ and f^{rc} that are equal to f^{-1} are marked by $*$. It means that the ETPCA is T-symmetric under the corresponding involutions

f	f^{r}	f^{c}	f^{rc}	f^{-1}	f	f^{r}	f^{c}	f^{rc}	f^{-1}
0137	0467*	0467*	0137	0467	7130	7460*	7460*	7130	7460
0157	0457*	0267	0237	0457	7150	7450*	7260	7230	7450
0167	0437*	0167	0437*	0437	7160	7430*	7160	7430*	7430
0237	0267*	0457	0157	0267	7230	7260*	7450	7150	7260
0257*	0257*	0257*	0257*	0257	7250*	7250*	7250*	7250*	7250
0267	0237*	0157	0457	0237	7260	7230*	7150	7450	7230
0317*	0647	0647	0317*	0317	7310*	7640	7640	7310*	7310
0327	0627	0547	0517*	0517	7320	7620	7540	7510*	7510
0347	0617*	0347	0617*	0617	7340	7610*	7340	7610*	7610
0437	0167*	0437	0167*	0167	7430	7160*	7430	7160*	7160
0457	0157*	0237	0267	0157	7450	7150*	7230	7260	7150
0467	0137*	0137*	0467	0137	7460	7130*	7130*	7460	7130
0517	0547	0627	0327*	0327	7510	7540	7620	7320*	7320
0527*	0527*	0527*	0527*	0527	7520*	7520*	7520*	7520*	7520
0547	0517	0327	0627*	0627	7540	7510	7320	7620*	7620
0617	0347*	0617	0347*	0347	7610	7340*	7610	7340*	7340
0627	0327	0517	0547*	0547	7620	7320	7510	7540*	7540
0647*	0317	0317	0647*	0647	7640*	7310	7310	7640*	7640

5.3 T-symmetries in reversible ETPCAs

Let $\mathrm{Conf}_{\mathrm{E3}} = \{\alpha \,|\, \alpha : \mathbb{Z}^2 \to \{0,1\}^3\}$ denote the set of all configurations of ETPCA. We define the involution $H_3^{\mathrm{rev}} : \mathrm{Conf}_{\mathrm{E3}} \to \mathrm{Conf}_{\mathrm{E3}}$ by the reversible ETPCA-0257:

$$H_3^{\mathrm{rev}} = F_{0257}$$

As shown in Figure 36, the involution H_3^{rev} is interpreted as the one that *reverses the moving directions* of all the particles in the cellular space. Though H_3^{rev} is different from H^{rev} for ESPCAs defined in Section 3, it has a similar meaning with the latter. Therefore, in the following, we use the notation H^{rev} in place of H_3^{rev}, since no confusion occurs.

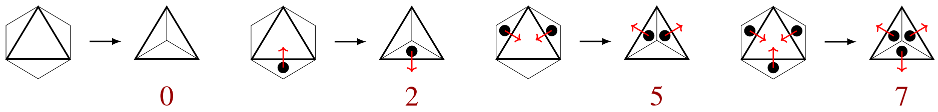


Figure 36: Local function of ETPCA-0257 by which H_3^{rev} for ETPCAs is defined. The involution H_3^{rev} makes every particle turn backward. Hereafter, we use the notation H^{rev} in place of H_3^{rev}

Lemma 5.11 *[13] Let P be a reversible ETPCA-wxyz with the local function f_{wxyz} and the global function F_{wxyz}. Let P' be a reversible ETPCA having the ID number* $\mathrm{inv}(wxyz)$*. Hence, the local and global functions of P' are $f_{\mathrm{inv}(wxyz)} = f_{wxyz}^{-1}$ and $F_{\mathrm{inv}(wxyz)}$, respectively. Then, the following holds.*

$$F_{wxyz}^{-1} = H^{\mathrm{rev}} \circ F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}$$

Proof. Let $\alpha_1 \in \mathrm{Conf}_{\mathrm{E3}}$ be any configuration. Let $(x_0,y_0) \in \mathbb{Z}^2$ be any point, and $(l_1,d_1,r_1) \in \{0,1\}^3$ be as follows: $\alpha_1(x_0,y_0) = (l_1,d_1,r_1)$. See Figure 37 that shows the process of state-changes by the operations given below. We consider only the case where x_0+y_0 is even, since the other case is similar. First, we can see the following relations.

$$\begin{array}{rcl} \mathrm{pr}_L(H^{\mathrm{rev}}(\alpha_1)(x_0-1,y_0)) & = & l_1 \\ \mathrm{pr}_D(H^{\mathrm{rev}}(\alpha_1)(x_0,y_0-1)) & = & d_1 \\ \mathrm{pr}_R(H^{\mathrm{rev}}(\alpha_1)(x_0+1,y_0)) & = & r_1 \end{array}$$

Assume $f_{wxyz}^{-1}(l_1,d_1,r_1) = (l_0,d_0,r_0)$ (thus, $f_{wxyz}(l_0,d_0,r_0) = (l_1,d_1,r_1)$). Then,

$$(F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}(\alpha_1))(x_0,y_0) \quad = \quad (l_0,d_0,r_0).$$

Let $\alpha_0 = F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}(\alpha_1)$. Then, the following relations hold.

$$\begin{array}{rcl} \mathrm{pr}_L(H^{\mathrm{rev}}(\alpha_0)(x_0-1,y_0)) & = & l_0 \\ \mathrm{pr}_D(H^{\mathrm{rev}}(\alpha_0)(x_0,y_0-1)) & = & d_0 \\ \mathrm{pr}_R(H^{\mathrm{rev}}(\alpha_0)(x_0+1,y_0)) & = & r_0 \end{array}$$

Hence,

$$(F_{wxyz} \circ H^{\mathrm{rev}}(\alpha_0))(x_0,y_0) = (l_1,d_1,r_1) = \alpha_1(x_0,y_0).$$

By above, the following holds for all $(x_0,y_0) \in \mathbb{Z}^2$.

$$(F_{wxyz} \circ H^{\mathrm{rev}} \circ F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}(\alpha_1))(x_0,y_0) \quad = \quad \alpha_1(x_0,y_0)$$

Thus, $F_{wxyz} \circ H^{\mathrm{rev}} \circ F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}(\alpha_1) = \alpha_1$ for all $\alpha_1 \in \mathrm{Conf}_{\mathrm{E3}}$. Therefore,

$$F_{wxyz}^{-1} \quad = \quad H^{\mathrm{rev}} \circ F_{\mathrm{inv}(wxyz)} \circ H^{\mathrm{rev}}.$$

This completes the proof. □

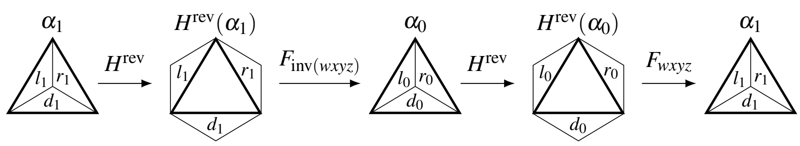


Figure 37: Process of the state-changes around the cell at (x_0,y_0) in Lemma 5.11

Definition 5.12 *Let P be a reversible ETPCA-wxyz whose global function is F_{wxyz}. If $F_{wxyz}^{-1} = H^{\mathrm{rev}} \circ F_{wxyz} \circ H^{\mathrm{rev}}$, then P is called* strictly T-symmetric.

From Lemma 5.11 we have the following theorem.

Theorem 5.13 *[13] A reversible ETPCA with the ID number wxyz is strictly T-symmetric, if* $\mathrm{inv}(wxyz) = wxyz$*. In this case, the following holds.*

$$F_{wxyz}^{-1} = H^{\mathrm{rev}} \circ F_{wxyz} \circ H^{\mathrm{rev}}$$

From Table 3 we can see the following.

Corollary 5.14 *[13] The 8 reversible ETPCAs w25z, w31z, w52z and w64z are strictly T-symmetric, where* $(w,z) \in \{(0,7),(7,0)\}$.

We now define a weaker version of T-symmetry for reversible ETPCAs.

Definition 5.15 *Let P be a reversible ETPCA-wxyz whose global function is F_{wxyz}. If there is an involution $H : \mathrm{Conf}_{\mathrm{E3}} \to \mathrm{Conf}_{\mathrm{E3}}$ that satisfies $F_{wxyz}^{-1} = H \circ F_{wxyz} \circ H$, then P is called* T-symmetric under the involution H.

Define a function $\mathrm{refl}_3 : \{0,1\}^3 \to \{0,1\}^3$ as follows: $\mathrm{refl}_3(l,d,r) = (r,d,l)$ for any $(l,d,r) \in \{0,1\}^3$. Next define an involution $H_3^{\mathrm{refl}} : \mathrm{Conf}_{\mathrm{E3}} \to \mathrm{Conf}_{\mathrm{E3}}$ as follows: $H_3^{\mathrm{refl}}(\alpha)(x_0,y_0) = \mathrm{refl}_3(\alpha(-x_0,y_0))$ for all $\alpha \in \mathrm{Conf}_{\mathrm{E3}}$ and $(x_0,y_0) \in \mathbb{Z}^2$. The involution H_3^{refl} gives the *mirror image* of a configuration with respect to the y-axis. As in the case of H_3^{rev}, we hereafter use the notation H^{refl} in place of H_3^{refl}.

Lemma 5.16 *[13] The next relation holds for any ETPCA-wxyz.*

$$F_{\mathrm{r}(wxyz)} = H^{\mathrm{refl}} \circ F_{wxyz} \circ H^{\mathrm{refl}}$$

Proof. First, we show $F_{wxyz} = H^{\mathrm{refl}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{refl}}$. Let $\alpha \in \mathrm{Conf}_{\mathrm{E3}}$ be any configuration, and $(x_0,y_0) \in \mathbb{Z}^2$ be any point. We consider only the case where $x_0 + y_0$ is even. Let $(l_0,d_0,r_0) \in \{0,1\}^3$ be as follows.

$$\begin{array}{rcl} \mathrm{pr}_L(\alpha(x_0-1,y_0)) & = & l_0 \\ \mathrm{pr}_D(\alpha(x_0,y_0-1)) & = & d_0 \\ \mathrm{pr}_R(\alpha(x_0+1,y_0)) & = & r_0 \end{array}$$

See Figure 38 that shows the process of state-changes by the operations given below. In the next step, we have the following.

$$\begin{array}{rcl} \mathrm{pr}_L(H^{\mathrm{refl}}(\alpha)(-x_0-1,y_0)) & = & r_0 \\ \mathrm{pr}_D(H^{\mathrm{refl}}(\alpha)(-x_0,y_0-1)) & = & d_0 \\ \mathrm{pr}_R(H^{\mathrm{refl}}(\alpha)(-x_0+1,y_0)) & = & l_0 \end{array}$$

Assume $f_{wxyz}(l_0,d_0,r_0) = (l_1,d_1,r_1)$. Since $f_{\mathrm{r}(wxyz)}(r_0,d_0,l_0) = (r_1,d_1,l_1)$,

$$(F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{refl}}(\alpha))(-x_0,y_0) \quad = \quad (r_1,d_1,l_1).$$

Finally, we have the following relation for all α and (x_0,y_0).

$$(H^{\mathrm{refl}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{refl}}(\alpha))(x_0,y_0) = (l_1,d_1,r_1) = F_{wxyz}(\alpha)(x_0,y_0)$$

Therefore, $F_{wxyz} = H^{\mathrm{refl}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{refl}}$ holds, and thus

$$H^{\mathrm{refl}} \circ F_{wxyz} \circ H^{\mathrm{refl}} = H^{\mathrm{refl}} \circ H^{\mathrm{refl}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{refl}} \circ H^{\mathrm{refl}} = F_{\mathrm{r}(wxyz)}.$$

This completes the proof. □

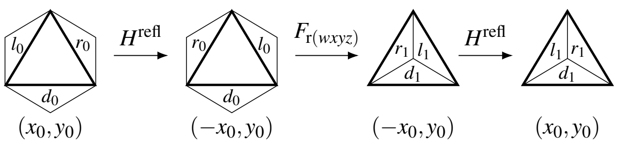


Figure 38: Process of the state-changes around the cells at (x_0,y_0) and $(-x_0,y_0)$ in Lemma 5.16

From Lemmas 5.16 and 5.16 we have the following.

Theorem 5.17 *[13] A reversible ETPCA with the ID number wxyz is T-symmetric under the involution $H^{\mathrm{refl}} \circ H^{\mathrm{rev}}$, if* $\mathrm{inv}(wxyz) = \mathrm{r}(wxyz)$. *In this case, the following holds.*

$$\begin{aligned} F_{wxyz}^{-1} &= H^{\mathrm{rev}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ F_{wxyz} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Table 3 we can see the following.

Corollary 5.18 *[13] The 24 reversible ETPCAs $w13z$, $w15z$, $w16z$, $w23z$, $w25z$, $w26z$, $w34z$, $w43z$, $w45z$, $w46z$, $w52z$ and $w61z$ are T-symmetric under the involution $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$, where $(w,z) \in \{(0,7),(7,0)\}$.*

Next, define a function $\mathrm{comp}_3 : \{0,1\}^3 \to \{0,1\}^3$ as follows: $\mathrm{comp}_3(l,d,r) = (\bar{l},\bar{d},\bar{r})$ for any $(l,d,r) \in \{0,1\}^3$. Define an involution $H_3^{\mathrm{comp}} : \mathrm{Conf}_{\mathrm{E3}} \to \mathrm{Conf}_{\mathrm{E3}}$ as follows. For all $\alpha \in \mathrm{Conf}_{\mathrm{E3}}$ and $(x_0,y_0) \in \mathbb{Z}^2$:

$$H_3^{\mathrm{comp}}(\alpha)(x_0,y_0) = \mathrm{comp}_3(\alpha(x_0,y_0))$$

The involution H_3^{comp} gives the *complement image* of a configuration. Hereafter, we use the notation H^{comp} in place of H_3^{comp}.

Lemma 5.19 *The next relation holds for any ETPCA-wxyz.*

$$F_{\mathrm{c}(wxyz)} = H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}}$$

Proof. First, we show $F_{wxyz} = H^{\mathrm{comp}} \circ F_{\mathrm{c}(wxyz)} \circ H^{\mathrm{comp}}$. Let $\alpha \in \mathrm{Conf}_{\mathrm{E3}}$ be any configuration, and $(x_0,y_0) \in \mathbb{Z}^2$ be any point. We consider only the case where $x_0 + y_0$ is even. Let $(l_0,d_0,r_0) \in \{0,1\}^3$ be as follows.

$$\begin{aligned} \mathrm{pr}_L(\alpha(x_0-1,y_0)) &= l_0 \\ \mathrm{pr}_D(\alpha(x_0,y_0-1)) &= d_0 \\ \mathrm{pr}_R(\alpha(x_0+1,y_0)) &= r_0 \end{aligned}$$

See Figure 38 that shows the process of state-changes by the operations given below. In the next step, we have the following.

$$\begin{aligned} \mathrm{pr}_L(H^{\mathrm{comp}}(\alpha)(x_0-1,y_0)) &= \overline{l_0} \\ \mathrm{pr}_D(H^{\mathrm{comp}}(\alpha)(x_0,y_0-1)) &= \overline{d_0} \\ \mathrm{pr}_R(H^{\mathrm{comp}}(\alpha)(x_0+1,y_0)) &= \overline{r_0} \end{aligned}$$

Assume $f_{wxyz}(l_0,d_0,r_0) = (l_1,d_1,r_1)$. Since $f_{\mathrm{c}(wxyz)}(\overline{l_0},\overline{d_0},\overline{r_0}) = (\overline{l_1},\overline{d_1},\overline{r_1})$,

$$(F_{\mathrm{c}(wxyz)} \circ H^{\mathrm{comp}}(\alpha))(x_0,y_0) = (\overline{l_1},\overline{d_1},\overline{r_1}).$$

Finally, we have the following relation for all α and (x_0,y_0).

$$(H^{\mathrm{comp}} \circ F_{\mathrm{r}(wxyz)} \circ H^{\mathrm{comp}}(\alpha))(x_0,y_0) = (l_1,d_1,r_1) = F_{wxyz}(\alpha)(x_0,y_0)$$

Therefore, $F_{wxyz} = H^{\mathrm{comp}} \circ F_{\mathrm{c}(wxyz)} \circ H^{\mathrm{comp}}$ holds, and thus

$$H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}} = F_{\mathrm{c}(wxyz)}.$$

This completes the proof. □

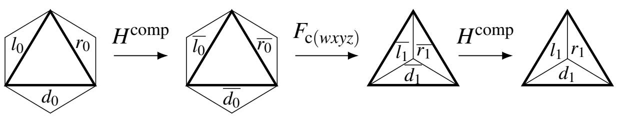


Figure 39: Process of the state-changes around the cell (x_0,y_0) in Lemma 5.19

From Lemmas 5.11 and 5.19 we have the following.

Theorem 5.20 *A reversible ETPCA with the ID number wxyz is T-symmetric under the involution $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, if* $\mathrm{inv}(wxyz) = \mathrm{c}(wxyz)$. *In this case, the following holds.*

$$\begin{aligned} F_{wxyz}^{-1} &= H^{\mathrm{rev}} \circ F_{\mathrm{c}(wxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Table 3 we can see the following.

Corollary 5.21 *The 8 reversible ETPCAs* $w13z$, $w25z$, $w46z$ *and* $w52z$ *are T-symmetric under* $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, *where* $(w,z) \in \{(0,7),(7,0)\}$.

Combining Lemmas 5.16 and 5.19, we also obtain the next lemma.

Lemma 5.22 *The next relation holds for any ETPCA-wxyz.*

$$F_{\mathrm{rc}(wxyz)} = H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}}$$

Proof. By Lemma 5.19, we have

$$F_{\mathrm{c}(wxyz)} = H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}}.$$

Therefore, by Lemma 5.16, we have

$$F_{\mathrm{r}(\mathrm{c}(wxyz))} = H^{\mathrm{refl}} \circ F_{\mathrm{c}(wxyz)} \circ H^{\mathrm{refl}} = H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}}.$$

Since $F_{\mathrm{rc}(wxyz)} = F_{\mathrm{r}(\mathrm{c}(wxyz))}$, the lemma holds. □

From Lemmas 5.11 and 5.22 we have the following.

Theorem 5.23 *A reversible ETPCA with the ID number wxyz is T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$, *if* $\mathrm{inv}(wxyz) = \mathrm{rc}(wxyz)$. *In this case, the following holds.*

$$\begin{aligned} F_{wxyz}^{-1} &= H^{\mathrm{rev}} \circ F_{\mathrm{rc}(wxyz)} \circ H^{\mathrm{rev}} \\ &= H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}} \circ F_{wxyz} \circ H^{\mathrm{comp}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{rev}} \end{aligned}$$

From Table 3 we can see the following.

Corollary 5.24 *The 24 reversible ETPCAs* $w16z$, $w25z$, $w31z$, $w32z$, $w34z$, $w43z$, $w51z$, $w52z$, $w54z$, $w61z$, $w62z$ *and* $w64z$ *are T-symmetric under the involution* $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$, *where* $(w,z) \in \{(0,7),(7,0)\}$.

From Corollaries 5.14, 5.18, 5.21 and 5.24, we can see that 'every' reversible ETPCA is T-symmetric under either of the the involutions H^{rev} (i.e., strictly T-symmetric), $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$, $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, or $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$.

6 Applications of T-Symmetries in Reversible ETPCAs

It has been shown that in reversible ETPCAs 0137, 0157 and 0347 a switch gate and an inverse switch gate are realised, and then a Fredkin gate is composed of them [3, 8, 12]. Hence, these ETPCAs and their dual ones are computationally universal. An inverse switch gate is obtained from a switch gate by using T-symmetry of these ETPCAs. But, since the method is similar to the one given in Section 4.2, we do not describe it here. Instead, we give some other examples for finding a backward evolution process for a given process.

The following lemma is the ETPCA version of Lemma 4.1. It is also easily proved.

Lemma 6.1 *Let P be a reversible ETPCA with the global function* F_{wxyz}. *Assume P is T-symmetric under an involution H, i.e.,* $F_{wxyz}^{-1} = H \circ F_{wxyz} \circ H$. *Then the following holds for any* $n \in \{1,2,\ldots\}$.

$$(F_{wxyz}^{-1})^n = H \circ (F_{wxyz})^n \circ H$$

6.1 ETPCA-0527: strictly T-symmetric

Consider ETPCA-0527, whose local function is shown in Figure 40. It is reversible, but not conservative. By Corollary 5.14, it is strictly T-symmetric.

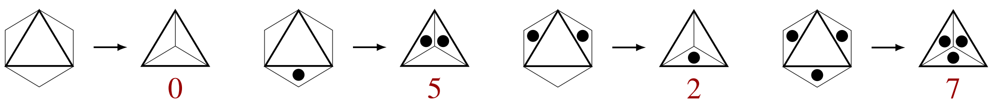


Figure 40: Local function of reversible and non-conservative ETPCA-0527

If we start from only one particle in ETPCA-0527, an expanding hexagonal pattern is created as shown in Figure 41. At $t = 6$ an isolated particle appears again inside the hexagon, and hence a new hexagon is created every 6 steps to form concentric hexagons as shown in $\delta(18)$ in Figure 42.

The process of generating indefinite number of concentric hexagons can be reversed by simply applying H^{rev} to a configuration. As in Figure 42, the configuration $H^{\mathrm{rev}}(\delta(18))$ will become a single particle by applying $(F_{0527})^{18}$. From this, we can see that a one-particle pattern $\delta(0)$ generates concentric hexagons both in the positive and the negative time directions. Note that, since $H^{\mathrm{rev}}(\delta(0))$ is the rotated configuration of $\delta(0)$ by 180 degrees, at $t = -18$ the rotated configuration of $H^{\mathrm{rev}}(\delta(18))$ by 180 degrees appears.

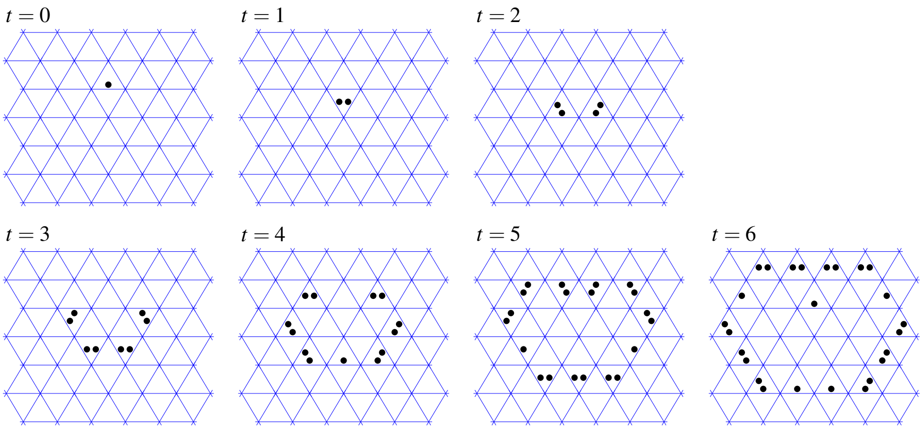


Figure 41: From a one-particle pattern an expanding hexagonal pattern appears in ETPCA-0527. This process is repeated indefinitely, and a large number of concentric hexagons are generated as in $\delta(18)$ of Figure 42

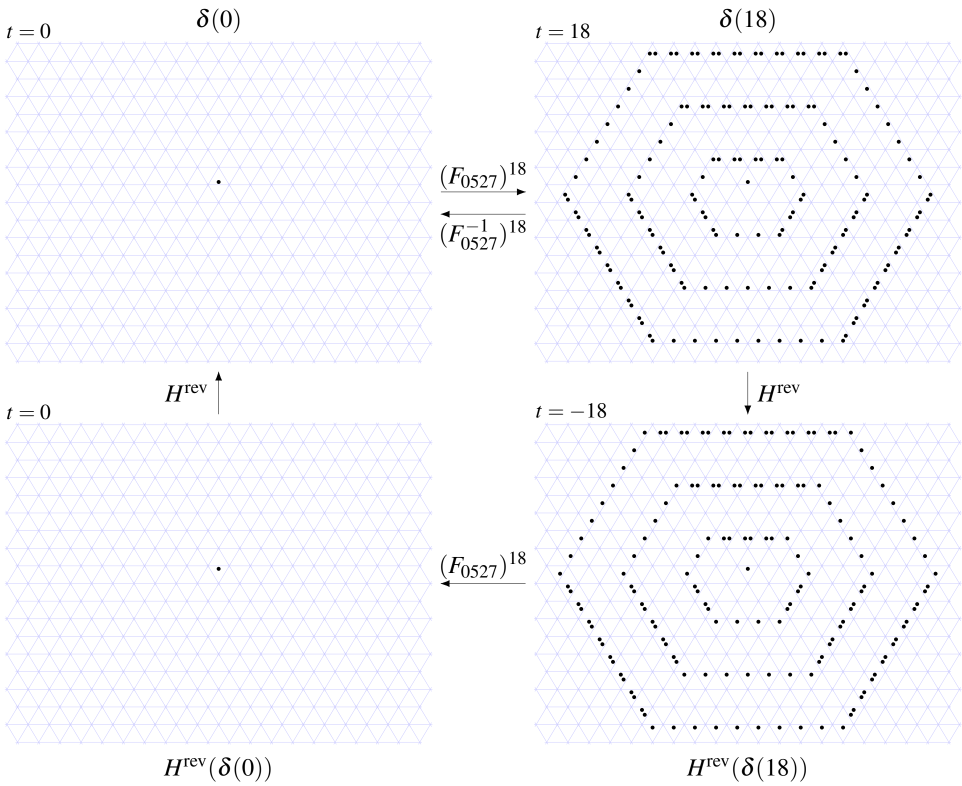


Figure 42: Using the generating process of concentric hexagons ($\delta(18)$) from a one-particle pattern ($\delta(0)$), we can shrink the concentric hexagons ($H^{\mathrm{rev}}(\delta(18))$) to a one-particle pattern ($H^{\mathrm{rev}}(\delta(0))$) in ETPCA-0527. It is based on its strict T-symmetry

6.2 ETPCA-0347: T-symmetric under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$

Consider ETPCA-0347, whose local function is shown in Figure 43. It is reversible, but not conservative. By Corollary 5.18, it is T-symmetric under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$. It was investigated in [9, 13]. In this cellular space, there is a space-moving pattern called a *glider* of period 6 (Figure 44). In ETPCA-0347, interactions of gliders and other patterns show fascinating phenomena. It has also been shown that any reversible Turing machine can be realised in its cellular space. Its emulator that works on *Golly* [18] is available in [7].

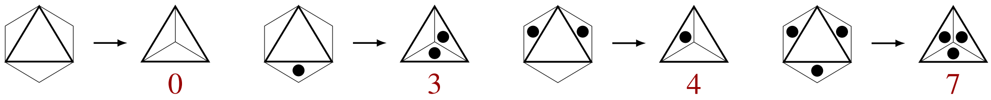


Figure 43: Local function of reversible and non-conservative ETPCA-0347

Here, we first consider an evolution process of colliding gliders and its inverse. If we collide two gliders as in $\zeta(0)$ of Figure 45, three gliders are generated after 30 steps ($\zeta(30)$). By this, the number of gliders is increased by one. Its inverse process is obtained by T-symmetry under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$. Namely, colliding three gliders as in $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\zeta(30))$, we get two gliders ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\zeta(0))$). By this, the number of gliders is decreased by one.

Using these symmetric phenomena, a *glider gun*, which generates gliders periodically, and a *glider absorber*, which reversibly erases gliders periodically, can be constructed [13]. Furthermore, the glider gun and the glider absorber themselves are symmetrically composed by using T-symmetry of ETPCA-0347.

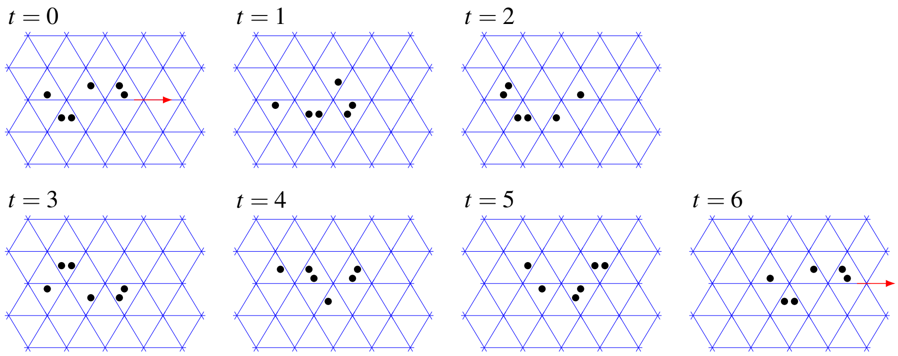


Figure 44: Glider of period 6 in ETPCA-0347

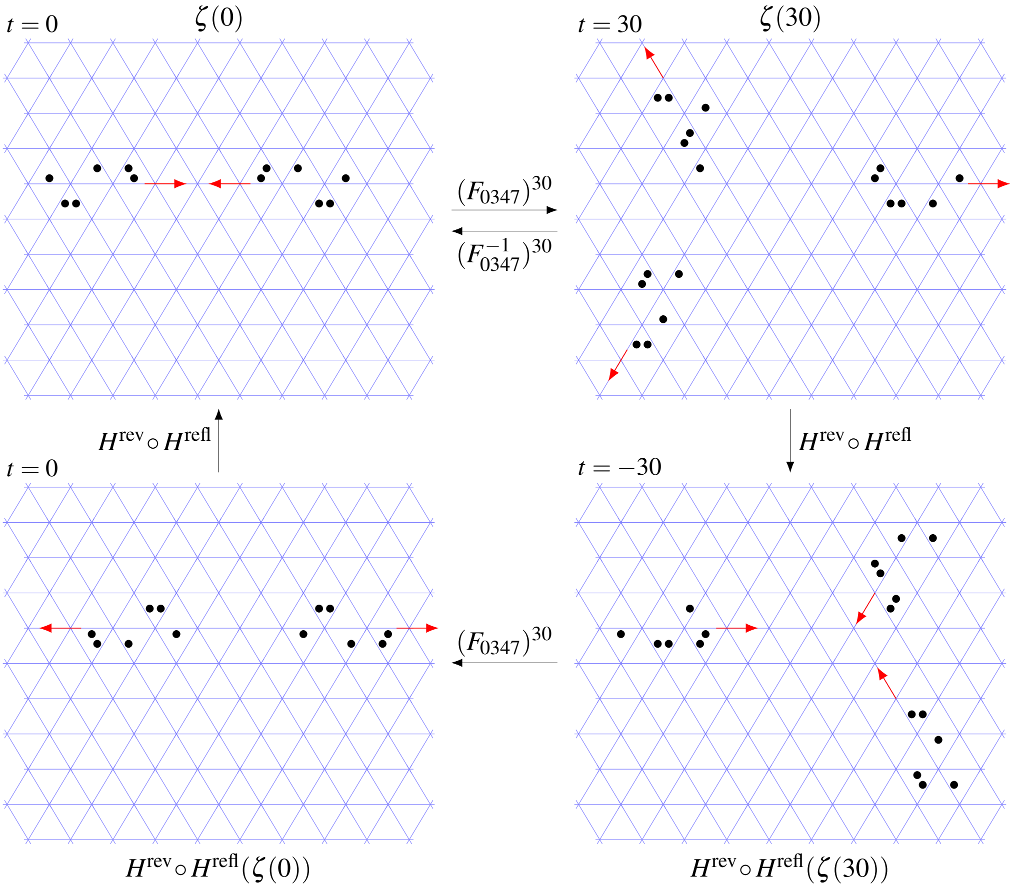


Figure 45: Using the process of generating three gliders ($\zeta(30)$) from two ($\zeta(0)$), we can generate two gliders ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\zeta(0))$) from three ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\zeta(30))$) in ETPCA-0347. Thus, both increasing and decreasing the number of gliders are possible. It is based on its T-symmetry under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$

Next, we consider an evolution process starting from a one-particle pattern in ETPCA-0347. Figure 46 shows that, if we start from a configuration containing only one particle ($t = 0$), then a disordered pattern appears ($t = 63$), and it expands bigger and bigger ($t = 320$) as if an explosion occurs.

However, if we apply $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$ to any configuration in the explosion process, it immediately starts to shrink. As seen in Figure 47, the configuration $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\eta(64))$ goes to the one-particle configuration $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\eta(0))$ after 64 steps. Therefore, in ETPCA-0347, both the explosion process and the implosion process from/to a one-particle pattern exist.

We can also observe that a one-particle pattern $\eta(0)$ generates random-like patterns both in the positive and the negative time directions. In fact, if we go to the negative time direction from $\eta(0)$, then at $t = -64$ the configuration obtained by rotating $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\eta(64))$ clockwise by 60 degrees will appear.

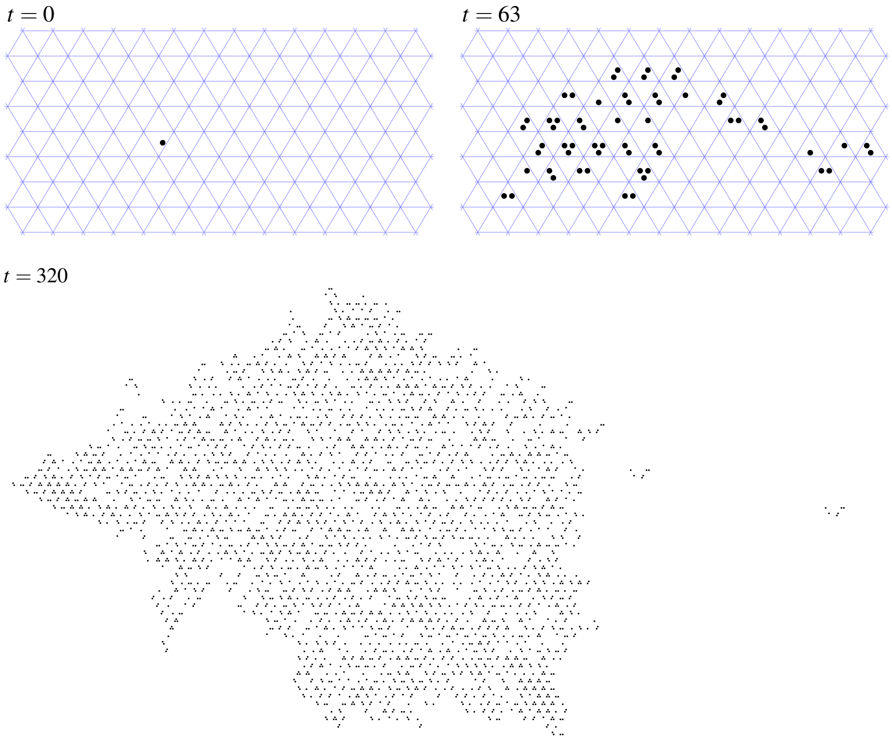


Figure 46: Evolution process like an explosion starting from a one-particle pattern in ETPCA-0347

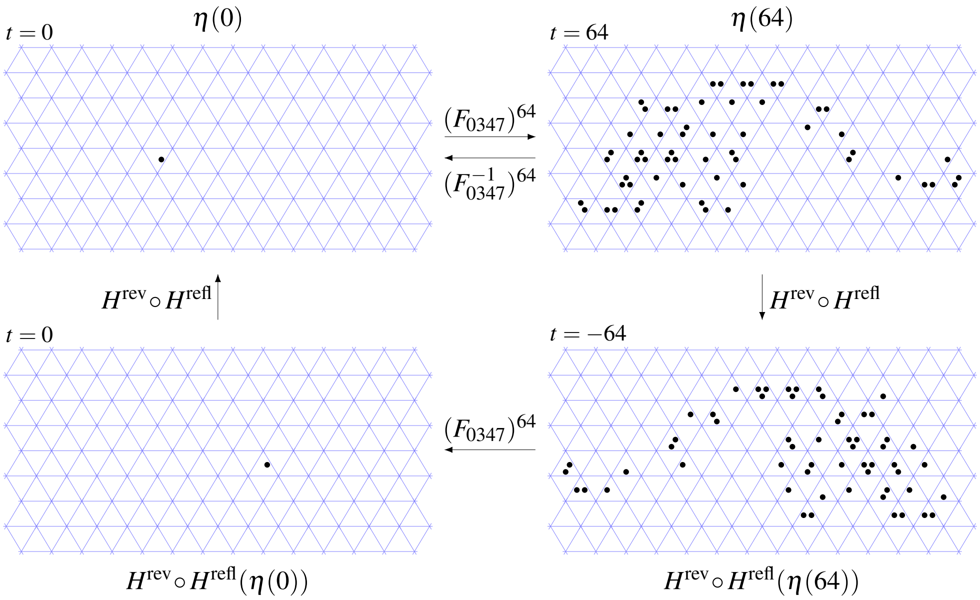


Figure 47: Using the expanding process from a one-particle pattern ($\eta(0)$) to a disordered pattern ($\eta(64)$), we can shrink the disordered pattern ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\eta(64))$) to a one-particle pattern ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}(\eta(0))$) in ETPCA-0347. It is based on its T-symmetry under $H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$

7 Concluding Remarks

In this paper, we investigated T-symmetries in reversible ESPCAs and reversible ETPCAs. The framework of PCAs is useful for formalising T-symmetries in reversible CAs. This is because the operation corresponding to the transformation of a momentum vector from $\mathbf{p}$ to $-\mathbf{p}$ in classical mechanics is simply expressed in reversible PCAs (see H^{rev} in Sections 3.1 and 5.3). We have shown that a large number of reversible ESPCAs (except 640 ESPCAs) and all reversible ETPCAs are T-symmetric under simple involutions. As applications, the results are used for finding and analysing backward evolution processes of them. It is open whether the remaining 640 ESPCAs are T-symmetric under some involutions.

In this paper, we investigated only reversible PCAs such that they have specific neighbourhoods, and each part of a cell has two states. To investigate the cases where the neighbourhood is different, or each part has more than two states is left for the future study.

References

[1] Fredkin, E., Toffoli, T.: Conservative logic. Int. J. Theoret. Phys. **21**, 219–253 (1982). doi:10.1007/BF01857727

[2] Gajardo, A., Kari, J., Moreira, M.: On time-symmetry in cellular automata. J. Comput. Syst. Sci. **78**, 1115–1126 (2012). doi:10.1016/j.jcss.2012.01.006

[3] Imai, K., Morita, K.: A computation-universal two-dimensional 8-state triangular reversible cellular automaton. Theoret. Comput. Sci. **231**, 181–191 (2000). doi:10.1007/s00354-018-0034-6

[4] Kari, J.: Reversible cellular automata: From fundamental classical results to recent developments. New Generation Computing **36**, 145–172 (2018). doi:10.1007/s00354-018-0034-6

[5] Lamb, J., Roberts, A.: Time-reversal symmetry in dynamical systems: A survey. Physica D **112**, 1–39 (1998). doi:10.1016/S0167-2789(97)00199-1

[6] Margolus, N.: Physics-like model of computation. Physica D **10**, 81–95 (1984). doi:10.1016/0167-2789(84)90252-5

[7] Morita, K.: Reversible world : Data set for simulating a reversible elementary triangular partitioned cellular automaton on Golly (2017). http://ir.lib.hiroshima-u.ac.jp/00042655

[8] Morita, K.: Theory of Reversible Computing. Springer, Tokyo (2017). doi:10.1007/978-4-431-56606-9

[9] Morita, K.: A universal non-conservative reversible elementary triangular partitioned cellular automaton that shows complex behavior. Natural Computing **18**(3), 413–428 (2019). doi:10.1007/s11047-017-9655-9

[10] Morita, K.: Data set for simulating a reversible elementary square partitioned cellular automaton with the ID number 01caef on Golly (ver. 2). Hiroshima University Institutional Repository, http://ir.lib.hiroshima-u.ac.jp/00051974 (2021)

[11] Morita, K.: Computing in a simple reversible and conservative cellular automaton. In: Proc. First Asian Symposium on Cellular Automata Technology (eds. S. Das, G.J. Martinez) AISC 1425, Springer, pp. 3–16 (2022). doi:10.1007/978-981-19-0542-1_1

[12] Morita, K.: Fredkin gates in simple reversible cellular automata. Int. J. Parallel, Emergent, & Distributed Systems **37**, 249–272 (2022). doi:10.1080/17445760.2022.2052871

[13] Morita, K.: Gliders in the Game of Life and in a reversible cellular automaton. In: The Mathematical Artist: A Tribute To John Horton Conway (eds. S. Das, S. Roy, K. Bhattacharjee), pp. 105–138. Springer, Cham (2022). doi:10.1007/978-3-031-03986-7_5

[14] Morita, K.: Making reversible computing machines in a reversible cellular space. Bulletin of EATCS (to appear)

[15] Morita, K., Harao, M.: Computation universality of one-dimensional reversible (injective) cellular automata. Trans. IEICE **E72**, 758–762 (1989). http://ir.lib.hiroshima-u.ac.jp/00048449

[16] Morita, K., Ueno, S.: Computation-universal models of two-dimensional 16-state reversible cellular automata. IEICE Trans. Inf. & Syst. **E75-D**, 141–147 (1992). http://ir.lib.hiroshima-u.ac.jp/00048451

[17] Toffoli, T., Margolus, N.: Invertible cellular automata: a review. Physica D **45**, 229–253 (1990). doi:10.1016/0167-2789(90)90185-R

[18] Trevorrow, A., Rokicki, T., Hutton, T., et al.: Golly: an open source, cross-platform application for exploring Conway's Game of Life and other cellular automata. http://golly.sourceforge.net/ (2005)

[19] Wolfram, S.: Theory and Applications of Cellular Automata. World Scientific Publishing (1986)

[20] Wolfram, S.: A New Kind of Science. Wolfram Media Inc. (2002)

Appendix

A List of All Reversible ESPCAs, Their Dual Ones, and Inverses

In this appendix, we give identification numbers of all 1536 reversible ESPCAs, their dual ones (under reflection, complementation, and both), and their inverses. In each ESPCA, the IDs of local functions among $f, f^{\mathrm{r}}, f^{\mathrm{c}}$ and f^{rc} that are equal to f^{-1} are marked by $*$. As shown in Theorems 3.3, 3.6, 3.8, and 3.10, if $f^{-1} = f$ ($f^{-1} = f^{\mathrm{r}}$, $f^{-1} = f^{\mathrm{c}}$, $f^{-1} = f^{\mathrm{rc}}$, respectively), then the ESPCA is T-symmetric under the involution H^{rev} ($H^{\mathrm{rev}} \circ H^{\mathrm{refl}}$, $H^{\mathrm{rev}} \circ H^{\mathrm{comp}}$, $H^{\mathrm{rev}} \circ H^{\mathrm{refl}} \circ H^{\mathrm{comp}}$). If there is no such function, then the ID of f^{-1} is marked by #. The mark c means that it is a conservative ESPCA. See also Table 2.

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
01357f	04357f*	0235bf	0235ef	04357f	c
0135bf	0435ef*	0135bf	0435ef*	0435ef	c
0135df	0435df*	0835bf	0835ef	0435df	c
0135ef	0435bf*	0435bf*	0135ef	0435bf	c
013a7f	043a7f*	023abf	023aef	043a7f	c
013abf	043aef*	013abf	043aef*	043aef	c
013adf	043adf*	083abf	083aef	043adf	c
013aef	043abf*	043abf*	013aef	043abf	c
01657f	04957f*	0265bf	0295ef	04957f	c
0165bf	0495ef*	0165bf	0495ef*	0495ef	c
0165df	0495df*	0865bf	0895ef	0495df	c
0165ef	0495bf*	0465bf	0195ef	0495bf	c
016a7f	049a7f*	026abf	029aef	049a7f	c
016abf	049aef*	016abf	049aef*	049aef	c
016adf	049adf*	086abf	089aef	049adf	c
016aef	049abf*	046abf	019aef	049abf	c
01753f	04b56f	0325bf	0615ef	04753f#	
01756f	04b53f*	0625bf	0315ef	04b53f	
01759f	04b5cf	0925bf	0c15ef	04e53f#	
0175cf	04b59f	0c25bf	0915ef	04d53f#	
017a3f	04ba6f	032abf	061aef	047a3f#	
017a6f	04ba3f*	062abf	031aef	04ba3f	
017a9f	04bacf	092abf	0c1aef	04ea3f#	
017acf	04ba9f	0c2abf	091aef	04da3f#	
01957f	04657f*	0295bf	0265ef	04657f	c
0195bf	0465ef*	0195bf	0465ef*	0465ef	c
0195df	0465df*	0895bf	0865ef	0465df	c
0195ef	0465bf*	0495bf	0165ef	0465bf	c
019a7f	046a7f*	029abf	026aef	046a7f	c
019abf	046aef*	019abf	046aef*	046aef	c
019adf	046adf*	089abf	086aef	046adf	c
019aef	046abf*	049abf	016aef	046abf	c
01b53f	04756f*	0315bf	0625ef	04756f	
01b56f	04753f	0615bf	0325ef	04b56f#	
01b59f	0475cf	0915bf	0c25ef	04e56f#	
01b5cf	04759f	0c15bf	0925ef	04d56f#	
01ba3f	047a6f*	031abf	062aef	047a6f	
01ba6f	047a3f	061abf	032aef	04ba6f#	
01ba9f	047acf	091abf	0c2aef	04ea6f#	
01bacf	047a9f	0c1abf	092aef	04da6f#	
01c57f	04c57f*	02c5bf	02c5ef	04c57f	c
01c5bf	04c5ef*	01c5bf	04c5ef*	04c5ef	c
01c5df	04c5df*	08c5bf	08c5ef	04c5df	c
01c5ef	04c5bf*	04c5bf*	01c5ef	04c5bf	c
01ca7f	04ca7f*	02cabf	02caef	04ca7f	c
01cabf	04caef*	01cabf	04caef*	04caef	c
01cadf	04cadf*	08cabf	08caef	04cadf	c
01caef	04cabf*	04cabf*	01caef	04cabf	c
01d53f	04e56f	0385bf	0645ef	0475cf#	
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01d5cf	04e59f	0c85bf	0945ef	04d5cf#	
01da3f	04ea6f	038abf	064aef	047acf#	
01da6f	04ea3f	068abf	034aef	04bacf#	
01da9f	04eacf*	098abf	0c4aef	04eacf	
01dacf	04ea9f	0c8abf	094aef	04dacf#	
01e53f	04d56f	0345bf	0685ef	04759f#	
01e56f	04d53f	0645bf	0385ef	04b59f#	
01e59f	04d5cf	0945bf	0c85ef	04e59f#	
01e5cf	04d59f*	0c45bf	0985ef	04d59f	
01ea3f	04da6f	034abf	068aef	047a9f#	
01ea6f	04da3f	064abf	038aef	04ba9f#	
01ea9f	04dacf	094abf	0c8aef	04ea9f#	
01eacf	04da9f*	0c4abf	098aef	04da9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
02357f*	02357f*	02357f*	02357f*	02357f	c
0235bf	0235ef*	01357f	04357f	0235ef	c
0235df*	0235df*	08357f	08357f	0235df	c
0235ef	0235bf*	04357f	01357f	0235bf	c
023a7f*	023a7f*	023a7f*	023a7f*	023a7f	c
023abf	023aef*	013a7f	043a7f	023aef	c
023adf*	023adf*	083a7f	083a7f	023adf	c
023aef	023abf*	043a7f	013a7f	023abf	c
02657f	02957f*	02657f	02957f*	02957f	c
0265bf	0295ef*	01657f	04957f	0295ef	c
0265df	0295df*	08657f	08957f	0295df	c
0265ef	0295bf*	04657f	01957f	0295bf	c
026a7f	029a7f*	026a7f	029a7f*	029a7f	c
026abf	029aef*	016a7f	049a7f	029aef	c
026adf	029adf*	086a7f	089a7f	029adf	c
026aef	029abf*	046a7f	019a7f	029abf	c
02753f*	02b56f	03257f	06157f	02753f	
02756f	02b53f*	06257f	03157f	02b53f	
02759f	02b5cf	09257f	0c157f	02e53f#	
0275cf	02b59f	0c257f	09157f	02d53f#	
027a3f*	02ba6f	032a7f	061a7f	027a3f	
027a6f	02ba3f*	062a7f	031a7f	02ba3f	
027a9f	02bacf	092a7f	0c1a7f	02ea3f#	
027acf	02ba9f	0c2a7f	091a7f	02da3f#	
02957f	02657f*	02957f	02657f*	02657f	c
0295bf	0265ef*	01957f	04657f	0265ef	c
0295df	0265df*	08957f	08657f	0265df	c
0295ef	0265bf*	04957f	01657f	0265bf	c
029a7f	026a7f*	029a7f	026a7f*	026a7f	c
029abf	026aef*	019a7f	046a7f	026aef	c
029adf	026adf*	089a7f	086a7f	026adf	c
029aef	026abf*	049a7f	016a7f	026abf	c
02b53f	02756f*	03157f	06257f	02756f	
02b56f*	02753f	06157f	03257f	02b56f	
02b59f	0275cf	09157f	0c257f	02e56f#	
02b5cf	02759f	0c157f	09257f	02d56f#	
02ba3f	027a6f*	031a7f	062a7f	027a6f	
02ba6f*	027a3f	061a7f	032a7f	02ba6f	
02ba9f	027acf	091a7f	0c2a7f	02ea6f#	
02bacf	027a9f	0c1a7f	092a7f	02da6f#	
02c57f*	02c57f*	02c57f*	02c57f*	02c57f	c
02c5bf	02c5ef*	01c57f	04c57f	02c5ef	c
02c5df*	02c5df*	08c57f	08c57f	02c5df	c
02c5ef	02c5bf*	04c57f	01c57f	02c5bf	c
02ca7f*	02ca7f*	02ca7f*	02ca7f*	02ca7f	c
02cabf	02caef*	01ca7f	04ca7f	02caef	c
02cadf*	02cadf*	08ca7f	08ca7f	02cadf	c
02caef	02cabf*	04ca7f	01ca7f	02cabf	c
02d53f	02e56f	03857f	06457f	0275cf#	
02d56f	02e53f	06857f	03457f	02b5cf#	
02d59f	02e5cf*	09857f	0c457f	02e5cf	
02d5cf*	02e59f	0c857f	09457f	02d5cf	
02da3f	02ea6f	038a7f	064a7f	027acf#	
02da6f	02ea3f	068a7f	034a7f	02bacf#	
02da9f	02eacf*	098a7f	0c4a7f	02eacf	
02dacf*	02ea9f	0c8a7f	094a7f	02dacf	
02e53f	02d56f	03457f	06857f	02759f#	
02e56f	02d53f	06457f	03857f	02b59f#	
02e59f*	02d5cf	09457f	0c857f	02e59f	
02e5cf	02d59f*	0c457f	09857f	02d59f	
02ea3f	02da6f	034a7f	068a7f	027a9f#	
02ea6f	02da3f	064a7f	038a7f	02ba9f#	
02ea9f*	02dacf	094a7f	0c8a7f	02ea9f	
02eacf	02da9f*	0c4a7f	098a7f	02da9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
03157f	06257f*	02b53f	02756f	06257f	
0315bf	0625ef*	01b53f	04756f	0625ef	
0315df	0625df*	08b53f	08756f	0625df	
0315ef	0625bf*	04b53f	01756f	0625bf	
031a7f	062a7f*	02ba3f	027a6f	062a7f	
031abf	062aef*	01ba3f	047a6f	062aef	
031adf	062adf*	08ba3f	087a6f	062adf	
031aef	062abf*	04ba3f	017a6f	062abf	
03257f*	06157f	02753f	02b56f	03257f	
0325bf	0615ef	01753f	04b56f	0325ef#	
0325df*	0615df	08753f	08b56f	0325df	
0325ef	0615bf	04753f	01b56f	0325bf#	
032a7f*	061a7f	027a3f	02ba6f	032a7f	
032abf	061aef	017a3f	04ba6f	032aef#	
032adf*	061adf	087a3f	08ba6f	032adf	
032aef	061abf	047a3f	01ba6f	032abf#	
03457f	06857f	02e53f	02d56f	09257f#	
0345bf	0685ef	01e53f	04d56f	0925ef#	
0345df	0685df	08e53f	08d56f	0925df#	
0345ef	0685bf	04e53f	01d56f	0925bf#	
034a7f	068a7f	02ea3f	02da6f	092a7f#	
034abf	068aef	01ea3f	04da6f	092aef#	
034adf	068adf	08ea3f	08da6f	092adf#	
034aef	068abf	04ea3f	01da6f	092abf#	
03751f	06b54f	0b253f	0e156f	0e253f#	
03752f	06b52f	07253f*	07156f	07253f	
03754f	06b51f	0e253f	0b156f	0b253f#	
03758f	06b58f	0d253f*	0d156f	0d253f	
037a1f	06ba4f	0b2a3f	0e1a6f	0e2a3f#	
037a2f	06ba2f	072a3f*	071a6f	072a3f	
037a4f	06ba1f	0e2a3f	0b1a6f	0b2a3f#	
037a8f	06ba8f	0d2a3f*	0d1a6f	0d2a3f	
03857f	06457f	02d53f	02e56f	0c257f#	
0385bf	0645ef	01d53f	04e56f	0c25ef#	
0385df	0645df	08d53f	08e56f	0c25df#	
0385ef	0645bf	04d53f	01e56f	0c25bf#	
038a7f	064a7f	02da3f	02ea6f	0c2a7f#	
038abf	064aef	01da3f	04ea6f	0c2aef#	
038adf	064adf	08da3f	08ea6f	0c2adf#	
038aef	064abf	04da3f	01ea6f	0c2abf#	
03b51f	06754f	0b153f	0e256f*	0e256f	
03b52f	06752f	07153f	07256f*	07256f	
03b54f	06751f	0e153f	0b256f*	0b256f	
03b58f	06758f	0d153f	0d256f*	0d256f	
03ba1f	067a4f	0b1a3f	0e2a6f*	0e2a6f	
03ba2f	067a2f	071a3f	072a6f*	072a6f	
03ba4f	067a1f	0e1a3f	0b2a6f*	0b2a6f	
03ba8f	067a8f	0d1a3f	0d2a6f*	0d2a6f	
03d51f	06e54f	0b853f	0e456f	0e25cf#	
03d52f	06e52f	07853f	07456f	0725cf#	
03d54f	06e51f	0e853f	0b456f	0b25cf#	
03d58f	06e58f	0d853f	0d456f	0d25cf#	
03da1f	06ea4f	0b8a3f	0e4a6f	0e2acf#	
03da2f	06ea2f	078a3f	074a6f	072acf#	
03da4f	06ea1f	0e8a3f	0b4a6f	0b2acf#	
03da8f	06ea8f	0d8a3f	0d4a6f	0d2acf#	
03e51f	06d54f	0b453f	0e856f	0e259f#	
03e52f	06d52f	07453f	07856f	07259f#	
03e54f	06d51f	0e453f	0b856f	0b259f#	
03e58f	06d58f	0d453f	0d856f	0d259f#	
03ea1f	06da4f	0b4a3f	0e8a6f	0e2a9f#	
03ea2f	06da2f	074a3f	078a6f	072a9f#	
03ea4f	06da1f	0e4a3f	0b8a6f	0b2a9f#	
03ea8f	06da8f	0d4a3f	0d8a6f	0d2a9f#	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
04357f	01357f*	0235ef	0235bf	01357f	c
0435bf	0135ef*	0135ef*	0435bf	0135ef	c
0435df	0135df*	0835ef	0835bf	0135df	c
0435ef	0135bf*	0435ef	0135bf*	0135bf	c
043a7f	013a7f*	023aef	023abf	013a7f	c
043abf	013aef*	013aef*	043abf	013aef	c
043adf	013adf*	083aef	083abf	013adf	c
043aef	013abf*	043aef	013abf*	013abf	c
04657f	01957f*	0265ef	0295bf	01957f	c
0465bf	0195ef*	0165ef	0495bf	0195ef	c
0465df	0195df*	0865ef	0895bf	0195df	c
0465ef	0195bf*	0465ef	0195bf*	0195bf	c
046a7f	019a7f*	026aef	029abf	019a7f	c
046abf	019aef*	016aef	049abf	019aef	c
046adf	019adf*	086aef	089abf	019adf	c
046aef	019abf*	046aef	019abf*	019abf	c
04753f	01b56f	0325ef	0615bf	01753f#	
04756f	01b53f*	0625ef	0315bf	01b53f	
04759f	01b5cf	0925ef	0c15bf	01e53f#	
0475cf	01b59f	0c25ef	0915bf	01d53f#	
047a3f	01ba6f	032aef	061abf	017a3f#	
047a6f	01ba3f*	062aef	031abf	01ba3f	
047a9f	01bacf	092aef	0c1abf	01ea3f#	
047acf	01ba9f	0c2aef	091abf	01da3f#	
04957f	01657f*	0295ef	0265bf	01657f	c
0495bf	0165ef*	0195ef	0465bf	0165ef	c
0495df	0165df*	0895ef	0865bf	0165df	c
0495ef	0165bf*	0495ef	0165bf*	0165bf	c
049a7f	016a7f*	029aef	026abf	016a7f	c
049abf	016aef*	019aef	046abf	016aef	c
049adf	016adf*	089aef	086abf	016adf	c
049aef	016abf*	049aef	016abf*	016abf	c
04b53f	01756f*	0315ef	0625bf	01756f	
04b56f	01753f	0615ef	0325bf	01b56f#	
04b59f	0175cf	0915ef	0c25bf	01e56f#	
04b5cf	01759f	0c15ef	0925bf	01d56f#	
04ba3f	017a6f*	031aef	062abf	017a6f	
04ba6f	017a3f	061aef	032abf	01ba6f#	
04ba9f	017acf	091aef	0c2abf	01ea6f#	
04bacf	017a9f	0c1aef	092abf	01da6f#	
04c57f	01c57f*	02c5ef	02c5bf	01c57f	c
04c5bf	01c5ef*	01c5ef*	04c5bf	01c5ef	c
04c5df	01c5df*	08c5ef	08c5bf	01c5df	c
04c5ef	01c5bf*	04c5ef	01c5bf*	01c5bf	c
04ca7f	01ca7f*	02caef	02cabf	01ca7f	c
04cabf	01caef*	01caef*	04cabf	01caef	c
04cadf	01cadf*	08caef	08cabf	01cadf	c
04caef	01cabf*	04caef	01cabf*	01cabf	c
04d53f	01e56f	0385ef	0645bf	0175cf#	
04d56f	01e53f	0685ef	0345bf	01b5cf#	
04d59f	01e5cf*	0985ef	0c45bf	01e5cf	
04d5cf	01e59f	0c85ef	0945bf	01d5cf#	
04da3f	01ea6f	038aef	064abf	017acf#	
04da6f	01ea3f	068aef	034abf	01bacf#	
04da9f	01eacf*	098aef	0c4abf	01eacf	
04dacf	01ea9f	0c8aef	094abf	01dacf#	
04e53f	01d56f	0345ef	0685bf	01759f#	
04e56f	01d53f	0645ef	0385bf	01b59f#	
04e59f	01d5cf	0945ef	0c85bf	01e59f#	
04e5cf	01d59f*	0c45ef	0985bf	01d59f	
04ea3f	01da6f	034aef	068abf	017a9f#	
04ea6f	01da3f	064aef	038abf	01ba9f#	
04ea9f	01dacf	094aef	0c8abf	01ea9f#	
04eacf	01da9f*	0c4aef	098abf	01da9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
06157f*	03257f	02b56f	02753f	06157f	
0615bf	0325ef	01b56f	04753f	0615ef#	
0615df*	0325df	08b56f	08753f	0615df	
0615ef	0325bf	04b56f	01753f	0615bf#	
061a7f*	032a7f	02ba6f	027a3f	061a7f	
061abf	032aef	01ba6f	047a3f	061aef#	
061adf*	032adf	08ba6f	087a3f	061adf	
061aef	032abf	04ba6f	017a3f	061abf#	
06257f	03157f*	02756f	02b53f	03157f	
0625bf	0315ef*	01756f	04b53f	0315ef	
0625df	0315df*	08756f	08b53f	0315df	
0625ef	0315bf*	04756f	01b53f	0315bf	
062a7f	031a7f*	027a6f	02ba3f	031a7f	
062abf	031aef*	017a6f	04ba3f	031aef	
062adf	031adf*	087a6f	08ba3f	031adf	
062aef	031abf*	047a6f	01ba3f	031abf	
06457f	03857f	02e56f	02d53f	09157f#	
0645bf	0385ef	01e56f	04d53f	0915ef#	
0645df	0385df	08e56f	08d53f	0915df#	
0645ef	0385bf	04e56f	01d53f	0915bf#	
064a7f	038a7f	02ea6f	02da3f	091a7f#	
064abf	038aef	01ea6f	04da3f	091aef#	
064adf	038adf	08ea6f	08da3f	091adf#	
064aef	038abf	04ea6f	01da3f	091abf#	
06751f	03b54f	0b256f	0e153f*	0e153f	
06752f	03b52f	07256f	07153f*	07153f	
06754f	03b51f	0e256f	0b153f*	0b153f	
06758f	03b58f	0d256f	0d153f*	0d153f	
067a1f	03ba4f	0b2a6f	0e1a3f*	0e1a3f	
067a2f	03ba2f	072a6f	071a3f*	071a3f	
067a4f	03ba1f	0e2a6f	0b1a3f*	0b1a3f	
067a8f	03ba8f	0d2a6f	0d1a3f*	0d1a3f	
06857f	03457f	02d56f	02e53f	0c157f#	
0685bf	0345ef	01d56f	04e53f	0c15ef#	
0685df	0345df	08d56f	08e53f	0c15df#	
0685ef	0345bf	04d56f	01e53f	0c15bf#	
068a7f	034a7f	02da6f	02ea3f	0c1a7f#	
068abf	034aef	01da6f	04ea3f	0c1aef#	
068adf	034adf	08da6f	08ea3f	0c1adf#	
068aef	034abf	04da6f	01ea3f	0c1abf#	
06b51f	03754f	0b156f	0e253f	0e156f#	
06b52f	03752f	07156f*	07253f	07156f	
06b54f	03751f	0e156f	0b253f	0b156f#	
06b58f	03758f	0d156f*	0d253f	0d156f	
06ba1f	037a4f	0b1a6f	0e2a3f	0e1a6f#	
06ba2f	037a2f	071a6f*	072a3f	071a6f	
06ba4f	037a1f	0e1a6f	0b2a3f	0b1a6f#	
06ba8f	037a8f	0d1a6f*	0d2a3f	0d1a6f	
06d51f	03e54f	0b856f	0e453f	0e15cf#	
06d52f	03e52f	07856f	07453f	0715cf#	
06d54f	03e51f	0e856f	0b453f	0b15cf#	
06d58f	03e58f	0d856f	0d453f	0d15cf#	
06da1f	03ea4f	0b8a6f	0e4a3f	0e1acf#	
06da2f	03ea2f	078a6f	074a3f	071acf#	
06da4f	03ea1f	0e8a6f	0b4a3f	0b1acf#	
06da8f	03ea8f	0d8a6f	0d4a3f	0d1acf#	
06e51f	03d54f	0b456f	0e853f	0e159f#	
06e52f	03d52f	07456f	07853f	07159f#	
06e54f	03d51f	0e456f	0b853f	0b159f#	
06e58f	03d58f	0d456f	0d853f	0d159f#	
06ea1f	03da4f	0b4a6f	0e8a3f	0e1a9f#	
06ea2f	03da2f	074a6f	078a3f	071a9f#	
06ea4f	03da1f	0e4a6f	0b8a3f	0b1a9f#	
06ea8f	03da8f	0d4a6f	0d8a3f	0d1a9f#	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
07153f	07256f	03b52f	06752f*	06752f	
07156f	07253f	06b52f*	03752f	06b52f	
07159f	0725cf	09b52f	0c752f	06e52f#	
0715cf	07259f	0cb52f	09752f	06d52f#	
071a3f	072a6f	03ba2f	067a2f*	067a2f	
071a6f	072a3f	06ba2f*	037a2f	06ba2f	
071a9f	072acf	09ba2f	0c7a2f	06ea2f#	
071acf	072a9f	0cba2f	097a2f	06da2f#	
07253f	07156f	03752f*	06b52f	03752f	
07256f	07153f	06752f	03b52f*	03b52f	
07259f	0715cf	09752f	0cb52f	03e52f#	
0725cf	07159f	0c752f	09b52f	03d52f#	
072a3f	071a6f	037a2f*	06ba2f	037a2f	
072a6f	071a3f	067a2f	03ba2f*	03ba2f	
072a9f	071acf	097a2f	0cba2f	03ea2f#	
072acf	071a9f	0c7a2f	09ba2f	03da2f#	
07351f	07354f	0b352f	0e352f*	0e352f	
07352f*	07352f*	07352f*	07352f*	07352f	
07354f	07351f	0e352f	0b352f*	0b352f	
07358f	07358f	0d352f*	0d352f*	0d352f	
073a1f	073a4f	0b3a2f	0e3a2f*	0e3a2f	
073a2f*	073a2f*	073a2f*	073a2f*	073a2f	
073a4f	073a1f	0e3a2f	0b3a2f*	0b3a2f	
073a8f	073a8f	0d3a2f*	0d3a2f*	0d3a2f	
07453f	07856f	03e52f	06d52f	09752f#	
07456f	07853f	06e52f	03d52f	09b52f#	
07459f	0785cf	09e52f*	0cd52f	09e52f	
0745cf	07859f	0ce52f	09d52f*	09d52f	
074a3f	078a6f	03ea2f	06da2f	097a2f#	
074a6f	078a3f	06ea2f	03da2f	09ba2f#	
074a9f	078acf	09ea2f*	0cda2f	09ea2f	
074acf	078a9f	0cea2f	09da2f*	09da2f	
07651f	07954f	0b652f	0e952f*	0e952f	
07652f	07952f*	07652f	07952f*	07952f	
07654f	07951f	0e652f	0b952f*	0b952f	
07658f	07958f	0d652f	0d952f*	0d952f	
076a1f	079a4f	0b6a2f	0e9a2f*	0e9a2f	
076a2f	079a2f*	076a2f	079a2f*	079a2f	
076a4f	079a1f	0e6a2f	0b9a2f*	0b9a2f	
076a8f	079a8f	0d6a2f	0d9a2f*	0d9a2f	
07853f	07456f	03d52f	06e52f	0c752f#	
07856f	07453f	06d52f	03e52f	0cb52f#	
07859f	0745cf	09d52f	0ce52f*	0ce52f	
0785cf	07459f	0cd52f*	09e52f	0cd52f	
078a3f	074a6f	03da2f	06ea2f	0c7a2f#	
078a6f	074a3f	06da2f	03ea2f	0cba2f#	
078a9f	074acf	09da2f	0cea2f*	0cea2f	
078acf	074a9f	0cda2f*	09ea2f	0cda2f	
07951f	07654f	0b952f	0e652f*	0e652f	
07952f	07652f*	07952f	07652f*	07652f	
07954f	07651f	0e952f	0b652f*	0b652f	
07958f	07658f	0d952f	0d652f*	0d652f	
079a1f	076a4f	0b9a2f	0e6a2f*	0e6a2f	
079a2f	076a2f*	079a2f	076a2f*	076a2f	
079a4f	076a1f	0e9a2f	0b6a2f*	0b6a2f	
079a8f	076a8f	0d9a2f	0d6a2f*	0d6a2f	
07c51f	07c54f	0bc52f	0ec52f*	0ec52f	
07c52f*	07c52f*	07c52f*	07c52f*	07c52f	
07c54f	07c51f	0ec52f	0bc52f*	0bc52f	
07c58f	07c58f	0dc52f*	0dc52f*	0dc52f	
07ca1f	07ca4f	0bca2f	0eca2f*	0eca2f	
07ca2f*	07ca2f*	07ca2f*	07ca2f*	07ca2f	
07ca4f	07ca1f	0eca2f	0bca2f*	0bca2f	
07ca8f	07ca8f	0dca2f*	0dca2f*	0dca2f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
08357f*	08357f*	0235df	0235df	08357f	c
0835bf	0835ef*	0135df	0435df	0835ef	c
0835df*	0835df*	0835df*	0835df*	0835df	c
0835ef	0835bf*	0435df	0135df	0835bf	c
083a7f*	083a7f*	023adf	023adf	083a7f	c
083abf	083aef*	013adf	043adf	083aef	c
083adf*	083adf*	083adf*	083adf*	083adf	c
083aef	083abf*	043adf	013adf	083abf	c
08657f	08957f*	0265df	0295df	08957f	c
0865bf	0895ef*	0165df	0495df	0895ef	c
0865df	0895df*	0865df	0895df*	0895df	c
0865ef	0895bf*	0465df	0195df	0895bf	c
086a7f	089a7f*	026adf	029adf	089a7f	c
086abf	089aef*	016adf	049adf	089aef	c
086adf	089adf*	086adf	089adf*	089adf	c
086aef	089abf*	046adf	019adf	089abf	c
08753f*	08b56f	0325df	0615df	08753f	
08756f	08b53f*	0625df	0315df	08b53f	
08759f	08b5cf	0925df	0c15df	08e53f#	
0875cf	08b59f	0c25df	0915df	08d53f#	
087a3f*	08ba6f	032adf	061adf	087a3f	
087a6f	08ba3f*	062adf	031adf	08ba3f	
087a9f	08bacf	092adf	0c1adf	08ea3f#	
087acf	08ba9f	0c2adf	091adf	08da3f#	
08957f	08657f*	0295df	0265df	08657f	c
0895bf	0865ef*	0195df	0465df	0865ef	c
0895df	0865df*	0895df	0865df*	0865df	c
0895ef	0865bf*	0495df	0165df	0865bf	c
089a7f	086a7f*	029adf	026adf	086a7f	c
089abf	086aef*	019adf	046adf	086aef	c
089adf	086adf*	089adf	086adf*	086adf	c
089aef	086abf*	049adf	016adf	086abf	c
08b53f	08756f*	0315df	0625df	08756f	
08b56f*	08753f	0615df	0325df	08b56f	
08b59f	0875cf	0915df	0c25df	08e56f#	
08b5cf	08759f	0c15df	0925df	08d56f#	
08ba3f	087a6f*	031adf	062adf	087a6f	
08ba6f*	087a3f	061adf	032adf	08ba6f	
08ba9f	087acf	091adf	0c2adf	08ea6f#	
08bacf	087a9f	0c1adf	092adf	08da6f#	
08c57f*	08c57f*	02c5df	02c5df	08c57f	c
08c5bf	08c5ef*	01c5df	04c5df	08c5ef	c
08c5df*	08c5df*	08c5df*	08c5df*	08c5df	c
08c5ef	08c5bf*	04c5df	01c5df	08c5bf	c
08ca7f*	08ca7f*	02cadf	02cadf	08ca7f	c
08cabf	08caef*	01cadf	04cadf	08caef	c
08cadf*	08cadf*	08cadf*	08cadf*	08cadf	c
08caef	08cabf*	04cadf	01cadf	08cabf	c
08d53f	08e56f	0385df	0645df	0875cf#	
08d56f	08e53f	0685df	0345df	08b5cf#	
08d59f	08e5cf*	0985df	0c45df	08e5cf	
08d5cf*	08e59f	0c85df	0945df	08d5cf	
08da3f	08ea6f	038adf	064adf	087acf#	
08da6f	08ea3f	068adf	034adf	08bacf#	
08da9f	08eacf*	098adf	0c4adf	08eacf	
08dacf*	08ea9f	0c8adf	094adf	08dacf	
08e53f	08d56f	0345df	0685df	08759f#	
08e56f	08d53f	0645df	0385df	08b59f#	
08e59f*	08d5cf	0945df	0c85df	08e59f	
08e5cf	08d59f*	0c45df	0985df	08d59f	
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08ea6f	08da3f	064adf	038adf	08ba9f#	
08ea9f*	08dacf	094adf	0c8adf	08ea9f	
08eacf	08da9f*	0c4adf	098adf	08da9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
09157f	0c257f	02b59f	0275cf	06457f#	
0915bf	0c25ef	01b59f	0475cf	0645ef#	
0915df	0c25df	08b59f	0875cf	0645df#	
0915ef	0c25bf	04b59f	0175cf	0645bf#	
091a7f	0c2a7f	02ba9f	027acf	064a7f#	
091abf	0c2aef	01ba9f	047acf	064aef#	
091adf	0c2adf	08ba9f	087acf	064adf#	
091aef	0c2abf	04ba9f	017acf	064abf#	
09257f	0c157f	02759f	02b5cf	03457f#	
0925bf	0c15ef	01759f	04b5cf	0345ef#	
0925df	0c15df	08759f	08b5cf	0345df#	
0925ef	0c15bf	04759f	01b5cf	0345bf#	
092a7f	0c1a7f	027a9f	02bacf	034a7f#	
092abf	0c1aef	017a9f	04bacf	034aef#	
092adf	0c1adf	087a9f	08bacf	034adf#	
092aef	0c1abf	047a9f	01bacf	034abf#	
09457f*	0c857f	02e59f	02d5cf	09457f	
0945bf	0c85ef	01e59f	04d5cf	0945ef#	
0945df*	0c85df	08e59f	08d5cf	0945df	
0945ef	0c85bf	04e59f	01d5cf	0945bf#	
094a7f*	0c8a7f	02ea9f	02dacf	094a7f	
094abf	0c8aef	01ea9f	04dacf	094aef#	
094adf*	0c8adf	08ea9f	08dacf	094adf	
094aef	0c8abf	04ea9f	01dacf	094abf#	
09751f	0cb54f	0b259f	0e15cf	0e453f#	
09752f	0cb52f	07259f	0715cf	07453f#	
09754f	0cb51f	0e259f	0b15cf	0b453f#	
09758f	0cb58f	0d259f	0d15cf	0d453f#	
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097a2f	0cba2f	072a9f	071acf	074a3f#	
097a4f	0cba1f	0e2a9f	0b1acf	0b4a3f#	
097a8f	0cba8f	0d2a9f	0d1acf	0d4a3f#	
09857f	0c457f*	02d59f	02e5cf	0c457f	
0985bf	0c45ef*	01d59f	04e5cf	0c45ef	
0985df	0c45df*	08d59f	08e5cf	0c45df	
0985ef	0c45bf*	04d59f	01e5cf	0c45bf	
098a7f	0c4a7f*	02da9f	02eacf	0c4a7f	
098abf	0c4aef*	01da9f	04eacf	0c4aef	
098adf	0c4adf*	08da9f	08eacf	0c4adf	
098aef	0c4abf*	04da9f	01eacf	0c4abf	
09b51f	0c754f	0b159f	0e25cf	0e456f#	
09b52f	0c752f	07159f	0725cf	07456f#	
09b54f	0c751f	0e159f	0b25cf	0b456f#	
09b58f	0c758f	0d159f	0d25cf	0d456f#	
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09ba2f	0c7a2f	071a9f	072acf	074a6f#	
09ba4f	0c7a1f	0e1a9f	0b2acf	0b4a6f#	
09ba8f	0c7a8f	0d1a9f	0d2acf	0d4a6f#	
09d51f	0ce54f	0b859f	0e45cf*	0e45cf	
09d52f	0ce52f	07859f	0745cf*	0745cf	
09d54f	0ce51f	0e859f	0b45cf*	0b45cf	
09d58f	0ce58f	0d859f	0d45cf*	0d45cf	
09da1f	0cea4f	0b8a9f	0e4acf*	0e4acf	
09da2f	0cea2f	078a9f	074acf*	074acf	
09da4f	0cea1f	0e8a9f	0b4acf*	0b4acf	
09da8f	0cea8f	0d8a9f	0d4acf*	0d4acf	
09e51f	0cd54f	0b459f	0e85cf	0e459f#	
09e52f	0cd52f	07459f*	0785cf	07459f	
09e54f	0cd51f	0e459f	0b85cf	0b459f#	
09e58f	0cd58f	0d459f*	0d85cf	0d459f	
09ea1f	0cda4f	0b4a9f	0e8acf	0e4a9f#	
09ea2f	0cda2f	074a9f*	078acf	074a9f	
09ea4f	0cda1f	0e4a9f	0b8acf	0b4a9f#	
09ea8f	0cda8f	0d4a9f*	0d8acf	0d4a9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
0b153f	0e256f	03b51f	06754f*	06754f	
0b156f	0e253f	06b51f	03754f	06b54f#	
0b159f	0e25cf	09b51f	0c754f	06e54f#	
0b15cf	0e259f	0cb51f	09754f	06d54f#	
0b1a3f	0e2a6f	03ba1f	067a4f*	067a4f	
0b1a6f	0e2a3f	06ba1f	037a4f	06ba4f#	
0b1a9f	0e2acf	09ba1f	0c7a4f	06ea4f#	
0b1acf	0e2a9f	0cba1f	097a4f	06da4f#	
0b253f	0e156f	03751f	06b54f	03754f#	
0b256f	0e153f	06751f	03b54f*	03b54f	
0b259f	0e15cf	09751f	0cb54f	03e54f#	
0b25cf	0e159f	0c751f	09b54f	03d54f#	
0b2a3f	0e1a6f	037a1f	06ba4f	037a4f#	
0b2a6f	0e1a3f	067a1f	03ba4f*	03ba4f	
0b2a9f	0e1acf	097a1f	0cba4f	03ea4f#	
0b2acf	0e1a9f	0c7a1f	09ba4f	03da4f#	
0b351f	0e354f*	0b351f	0e354f*	0e354f	
0b352f	0e352f	07351f	07354f*	07354f	
0b354f*	0e351f	0e351f	0b354f*	0b354f	
0b358f	0e358f	0d351f	0d354f*	0d354f	
0b3a1f	0e3a4f*	0b3a1f	0e3a4f*	0e3a4f	
0b3a2f	0e3a2f	073a1f	073a4f*	073a4f	
0b3a4f*	0e3a1f	0e3a1f	0b3a4f*	0b3a4f	
0b3a8f	0e3a8f	0d3a1f	0d3a4f*	0d3a4f	
0b453f	0e856f	03e51f	06d54f	09754f#	
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0b459f	0e85cf	09e51f	0cd54f	09e54f#	
0b45cf	0e859f	0ce51f	09d54f*	09d54f	
0b4a3f	0e8a6f	03ea1f	06da4f	097a4f#	
0b4a6f	0e8a3f	06ea1f	03da4f	09ba4f#	
0b4a9f	0e8acf	09ea1f	0cda4f	09ea4f#	
0b4acf	0e8a9f	0cea1f	09da4f*	09da4f	
0b651f	0e954f*	0b651f	0e954f*	0e954f	
0b652f	0e952f	07651f	07954f*	07954f	
0b654f	0e951f	0e651f	0b954f*	0b954f	
0b658f	0e958f	0d651f	0d954f*	0d954f	
0b6a1f	0e9a4f*	0b6a1f	0e9a4f*	0e9a4f	
0b6a2f	0e9a2f	076a1f	079a4f*	079a4f	
0b6a4f	0e9a1f	0e6a1f	0b9a4f*	0b9a4f	
0b6a8f	0e9a8f	0d6a1f	0d9a4f*	0d9a4f	
0b853f	0e456f	03d51f	06e54f	0c754f#	
0b856f	0e453f	06d51f	03e54f	0cb54f#	
0b859f	0e45cf	09d51f	0ce54f*	0ce54f	
0b85cf	0e459f	0cd51f	09e54f	0cd54f#	
0b8a3f	0e4a6f	03da1f	06ea4f	0c7a4f#	
0b8a6f	0e4a3f	06da1f	03ea4f	0cba4f#	
0b8a9f	0e4acf	09da1f	0cea4f*	0cea4f	
0b8acf	0e4a9f	0cda1f	09ea4f	0cda4f#	
0b951f	0e654f*	0b951f	0e654f*	0e654f	
0b952f	0e652f	07951f	07654f*	07654f	
0b954f	0e651f	0e951f	0b654f*	0b654f	
0b958f	0e658f	0d951f	0d654f*	0d654f	
0b9a1f	0e6a4f*	0b9a1f	0e6a4f*	0e6a4f	
0b9a2f	0e6a2f	079a1f	076a4f*	076a4f	
0b9a4f	0e6a1f	0e9a1f	0b6a4f*	0b6a4f	
0b9a8f	0e6a8f	0d9a1f	0d6a4f*	0d6a4f	
0bc51f	0ec54f*	0bc51f	0ec54f*	0ec54f	
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0bc54f*	0ec51f	0ec51f	0bc54f*	0bc54f	
0bc58f	0ec58f	0dc51f	0dc54f*	0dc54f	
0bca1f	0eca4f*	0bca1f	0eca4f*	0eca4f	
0bca2f	0eca2f	07ca1f	07ca4f*	07ca4f	
0bca4f*	0eca1f	0eca1f	0bca4f*	0bca4f	
0bca8f	0eca8f	0dca1f	0dca4f*	0dca4f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
0c157f	09257f	02b5cf	02759f	06857f#	
0c15bf	0925ef	01b5cf	04759f	0685ef#	
0c15df	0925df	08b5cf	08759f	0685df#	
0c15ef	0925bf	04b5cf	01759f	0685bf#	
0c1a7f	092a7f	02bacf	027a9f	068a7f#	
0c1abf	092aef	01bacf	047a9f	068aef#	
0c1adf	092adf	08bacf	087a9f	068adf#	
0c1aef	092abf	04bacf	017a9f	068abf#	
0c257f	09157f	0275cf	02b59f	03857f#	
0c25bf	0915ef	0175cf	04b59f	0385ef#	
0c25df	0915df	0875cf	08b59f	0385df#	
0c25ef	0915bf	0475cf	01b59f	0385bf#	
0c2a7f	091a7f	027acf	02ba9f	038a7f#	
0c2abf	091aef	017acf	04ba9f	038aef#	
0c2adf	091adf	087acf	08ba9f	038adf#	
0c2aef	091abf	047acf	01ba9f	038abf#	
0c457f	09857f*	02e5cf	02d59f	09857f	
0c45bf	0985ef*	01e5cf	04d59f	0985ef	
0c45df	0985df*	08e5cf	08d59f	0985df	
0c45ef	0985bf*	04e5cf	01d59f	0985bf	
0c4a7f	098a7f*	02eacf	02da9f	098a7f	
0c4abf	098aef*	01eacf	04da9f	098aef	
0c4adf	098adf*	08eacf	08da9f	098adf	
0c4aef	098abf*	04eacf	01da9f	098abf	
0c751f	09b54f	0b25cf	0e159f	0e853f#	
0c752f	09b52f	0725cf	07159f	07853f#	
0c754f	09b51f	0e25cf	0b159f	0b853f#	
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0c7a1f	09ba4f	0b2acf	0e1a9f	0e8a3f#	
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0c7a4f	09ba1f	0e2acf	0b1a9f	0b8a3f#	
0c7a8f	09ba8f	0d2acf	0d1a9f	0d8a3f#	
0c857f*	09457f	02d5cf	02e59f	0c857f	
0c85bf	0945ef	01d5cf	04e59f	0c85ef#	
0c85df*	0945df	08d5cf	08e59f	0c85df	
0c85ef	0945bf	04d5cf	01e59f	0c85bf#	
0c8a7f*	094a7f	02dacf	02ea9f	0c8a7f	
0c8abf	094aef	01dacf	04ea9f	0c8aef#	
0c8adf*	094adf	08dacf	08ea9f	0c8adf	
0c8aef	094abf	04dacf	01ea9f	0c8abf#	
0cb51f	09754f	0b15cf	0e259f	0e856f#	
0cb52f	09752f	0715cf	07259f	07856f#	
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0cba2f	097a2f	071acf	072a9f	078a6f#	
0cba4f	097a1f	0e1acf	0b2a9f	0b8a6f#	
0cba8f	097a8f	0d1acf	0d2a9f	0d8a6f#	
0cd51f	09e54f	0b85cf	0e459f	0e85cf#	
0cd52f	09e52f	0785cf*	07459f	0785cf	
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0cd58f	09e58f	0d85cf*	0d459f	0d85cf	
0cda1f	09ea4f	0b8acf	0e4a9f	0e8acf#	
0cda2f	09ea2f	078acf*	074a9f	078acf	
0cda4f	09ea1f	0e8acf	0b4a9f	0b8acf#	
0cda8f	09ea8f	0d8acf*	0d4a9f	0d8acf	
0ce51f	09d54f	0b45cf	0e859f*	0e859f	
0ce52f	09d52f	0745cf	07859f*	07859f	
0ce54f	09d51f	0e45cf	0b859f*	0b859f	
0ce58f	09d58f	0d45cf	0d859f*	0d859f	
0cea1f	09da4f	0b4acf	0e8a9f*	0e8a9f	
0cea2f	09da2f	074acf	078a9f*	078a9f	
0cea4f	09da1f	0e4acf	0b8a9f*	0b8a9f	
0cea8f	09da8f	0d4acf	0d8a9f*	0d8a9f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
0d153f	0d256f	03b58f	06758f*	06758f	
0d156f	0d253f	06b58f*	03758f	06b58f	
0d159f	0d25cf	09b58f	0c758f	06e58f#	
0d15cf	0d259f	0cb58f	09758f	06d58f#	
0d1a3f	0d2a6f	03ba8f	067a8f*	067a8f	
0d1a6f	0d2a3f	06ba8f*	037a8f	06ba8f	
0d1a9f	0d2acf	09ba8f	0c7a8f	06ea8f#	
0d1acf	0d2a9f	0cba8f	097a8f	06da8f#	
0d253f	0d156f	03758f*	06b58f	03758f	
0d256f	0d153f	06758f	03b58f*	03b58f	
0d259f	0d15cf	09758f	0cb58f	03e58f#	
0d25cf	0d159f	0c758f	09b58f	03d58f#	
0d2a3f	0d1a6f	037a8f*	06ba8f	037a8f	
0d2a6f	0d1a3f	067a8f	03ba8f*	03ba8f	
0d2a9f	0d1acf	097a8f	0cba8f	03ea8f#	
0d2acf	0d1a9f	0c7a8f	09ba8f	03da8f#	
0d351f	0d354f	0b358f	0e358f*	0e358f	
0d352f	0d352f	07358f*	07358f*	07358f	
0d354f	0d351f	0e358f	0b358f*	0b358f	
0d358f*	0d358f*	0d358f*	0d358f*	0d358f	
0d3a1f	0d3a4f	0b3a8f	0e3a8f*	0e3a8f	
0d3a2f	0d3a2f	073a8f*	073a8f*	073a8f	
0d3a4f	0d3a1f	0e3a8f	0b3a8f*	0b3a8f	
0d3a8f*	0d3a8f*	0d3a8f*	0d3a8f*	0d3a8f	
0d453f	0d856f	03e58f	06d58f	09758f#	
0d456f	0d853f	06e58f	03d58f	09b58f#	
0d459f	0d85cf	09e58f*	0cd58f	09e58f	
0d45cf	0d859f	0ce58f	09d58f*	09d58f	
0d4a3f	0d8a6f	03ea8f	06da8f	097a8f#	
0d4a6f	0d8a3f	06ea8f	03da8f	09ba8f#	
0d4a9f	0d8acf	09ea8f*	0cda8f	09ea8f	
0d4acf	0d8a9f	0cea8f	09da8f*	09da8f	
0d651f	0d954f	0b658f	0e958f*	0e958f	
0d652f	0d952f	07658f	07958f*	07958f	
0d654f	0d951f	0e658f	0b958f*	0b958f	
0d658f	0d958f*	0d658f	0d958f*	0d958f	
0d6a1f	0d9a4f	0b6a8f	0e9a8f*	0e9a8f	
0d6a2f	0d9a2f	076a8f	079a8f*	079a8f	
0d6a4f	0d9a1f	0e6a8f	0b9a8f*	0b9a8f	
0d6a8f	0d9a8f*	0d6a8f	0d9a8f*	0d9a8f	
0d853f	0d456f	03d58f	06e58f	0c758f#	
0d856f	0d453f	06d58f	03e58f	0cb58f#	
0d859f	0d45cf	09d58f	0ce58f*	0ce58f	
0d85cf	0d459f	0cd58f*	09e58f	0cd58f	
0d8a3f	0d4a6f	03da8f	06ea8f	0c7a8f#	
0d8a6f	0d4a3f	06da8f	03ea8f	0cba8f#	
0d8a9f	0d4acf	09da8f	0cea8f*	0cea8f	
0d8acf	0d4a9f	0cda8f*	09ea8f	0cda8f	
0d951f	0d654f	0b958f	0e658f*	0e658f	
0d952f	0d652f	07958f	07658f*	07658f	
0d954f	0d651f	0e958f	0b658f*	0b658f	
0d958f	0d658f*	0d958f	0d658f*	0d658f	
0d9a1f	0d6a4f	0b9a8f	0e6a8f*	0e6a8f	
0d9a2f	0d6a2f	079a8f	076a8f*	076a8f	
0d9a4f	0d6a1f	0e9a8f	0b6a8f*	0b6a8f	
0d9a8f	0d6a8f*	0d9a8f	0d6a8f*	0d6a8f	
0dc51f	0dc54f	0bc58f	0ec58f*	0ec58f	
0dc52f	0dc52f	07c58f*	07c58f*	07c58f	
0dc54f	0dc51f	0ec58f	0bc58f*	0bc58f	
0dc58f*	0dc58f*	0dc58f*	0dc58f*	0dc58f	
0dca1f	0dca4f	0bca8f	0eca8f*	0eca8f	
0dca2f	0dca2f	07ca8f*	07ca8f*	07ca8f	
0dca4f	0dca1f	0eca8f	0bca8f*	0bca8f	
0dca8f*	0dca8f*	0dca8f*	0dca8f*	0dca8f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
0e153f	0b256f	03b54f	06751f*	06751f	
0e156f	0b253f	06b54f	03751f	06b51f#	
0e159f	0b25cf	09b54f	0c751f	06e51f#	
0e15cf	0b259f	0cb54f	09751f	06d51f#	
0e1a3f	0b2a6f	03ba4f	067a1f*	067a1f	
0e1a6f	0b2a3f	06ba4f	037a1f	06ba1f#	
0e1a9f	0b2acf	09ba4f	0c7a1f	06ea1f#	
0e1acf	0b2a9f	0cba4f	097a1f	06da1f#	
0e253f	0b156f	03754f	06b51f	03751f#	
0e256f	0b153f	06754f	03b51f*	03b51f	
0e259f	0b15cf	09754f	0cb51f	03e51f#	
0e25cf	0b159f	0c754f	09b51f	03d51f#	
0e2a3f	0b1a6f	037a4f	06ba1f	037a1f#	
0e2a6f	0b1a3f	067a4f	03ba1f*	03ba1f	
0e2a9f	0b1acf	097a4f	0cba1f	03ea1f#	
0e2acf	0b1a9f	0c7a4f	09ba1f	03da1f#	
0e351f*	0b354f	0b354f	0e351f*	0e351f	
0e352f	0b352f	07354f	07351f*	07351f	
0e354f	0b351f*	0e354f	0b351f*	0b351f	
0e358f	0b358f	0d354f	0d351f*	0d351f	
0e3a1f*	0b3a4f	0b3a4f	0e3a1f*	0e3a1f	
0e3a2f	0b3a2f	073a4f	073a1f*	073a1f	
0e3a4f	0b3a1f*	0e3a4f	0b3a1f*	0b3a1f	
0e3a8f	0b3a8f	0d3a4f	0d3a1f*	0d3a1f	
0e453f	0b856f	03e54f	06d51f	09751f#	
0e456f	0b853f	06e54f	03d51f	09b51f#	
0e459f	0b85cf	09e54f	0cd51f	09e51f#	
0e45cf	0b859f	0ce54f	09d51f*	09d51f	
0e4a3f	0b8a6f	03ea4f	06da1f	097a1f#	
0e4a6f	0b8a3f	06ea4f	03da1f	09ba1f#	
0e4a9f	0b8acf	09ea4f	0cda1f	09ea1f#	
0e4acf	0b8a9f	0cea4f	09da1f*	09da1f	
0e651f	0b954f	0b654f	0e951f*	0e951f	
0e652f	0b952f	07654f	07951f*	07951f	
0e654f	0b951f*	0e654f	0b951f*	0b951f	
0e658f	0b958f	0d654f	0d951f*	0d951f	
0e6a1f	0b9a4f	0b6a4f	0e9a1f*	0e9a1f	
0e6a2f	0b9a2f	076a4f	079a1f*	079a1f	
0e6a4f	0b9a1f*	0e6a4f	0b9a1f*	0b9a1f	
0e6a8f	0b9a8f	0d6a4f	0d9a1f*	0d9a1f	
0e853f	0b456f	03d54f	06e51f	0c751f#	
0e856f	0b453f	06d54f	03e51f	0cb51f#	
0e859f	0b45cf	09d54f	0ce51f*	0ce51f	
0e85cf	0b459f	0cd54f	09e51f	0cd51f#	
0e8a3f	0b4a6f	03da4f	06ea1f	0c7a1f#	
0e8a6f	0b4a3f	06da4f	03ea1f	0cba1f#	
0e8a9f	0b4acf	09da4f	0cea1f*	0cea1f	
0e8acf	0b4a9f	0cda4f	09ea1f	0cda1f#	
0e951f	0b654f	0b954f	0e651f*	0e651f	
0e952f	0b652f	07954f	07651f*	07651f	
0e954f	0b651f*	0e954f	0b651f*	0b651f	
0e958f	0b658f	0d954f	0d651f*	0d651f	
0e9a1f	0b6a4f	0b9a4f	0e6a1f*	0e6a1f	
0e9a2f	0b6a2f	079a4f	076a1f*	076a1f	
0e9a4f	0b6a1f*	0e9a4f	0b6a1f*	0b6a1f	
0e9a8f	0b6a8f	0d9a4f	0d6a1f*	0d6a1f	
0ec51f*	0bc54f	0bc54f	0ec51f*	0ec51f	
0ec52f	0bc52f	07c54f	07c51f*	07c51f	
0ec54f	0bc51f*	0ec54f	0bc51f*	0bc51f	
0ec58f	0bc58f	0dc54f	0dc51f*	0dc51f	
0eca1f*	0bca4f	0bca4f	0eca1f*	0eca1f	
0eca2f	0bca2f	07ca4f	07ca1f*	07ca1f	
0eca4f	0bca1f*	0eca4f	0bca1f*	0bca1f	
0eca8f	0bca8f	0dca4f	0dca1f*	0dca1f	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f13570	f43570*	f235b0	f235e0	f43570	
f135b0	f435e0*	f135b0	f435e0*	f435e0	
f135d0	f435d0*	f835b0	f835e0	f435d0	
f135e0	f435b0*	f435b0*	f135e0	f435b0	
f13a70	f43a70*	f23ab0	f23ae0	f43a70	
f13ab0	f43ae0*	f13ab0	f43ae0*	f43ae0	
f13ad0	f43ad0*	f83ab0	f83ae0	f43ad0	
f13ae0	f43ab0*	f43ab0*	f13ae0	f43ab0	
f16570	f49570*	f265b0	f295e0	f49570	
f165b0	f495e0*	f165b0	f495e0*	f495e0	
f165d0	f495d0*	f865b0	f895e0	f495d0	
f165e0	f495b0*	f465b0	f195e0	f495b0	
f16a70	f49a70*	f26ab0	f29ae0	f49a70	
f16ab0	f49ae0*	f16ab0	f49ae0*	f49ae0	
f16ad0	f49ad0*	f86ab0	f89ae0	f49ad0	
f16ae0	f49ab0*	f46ab0	f19ae0	f49ab0	
f17530	f4b560	f325b0	f615e0	f47530#	
f17560	f4b530*	f625b0	f315e0	f4b530	
f17590	f4b5c0	f925b0	fc15e0	f4e530#	
f175c0	f4b590	fc25b0	f915e0	f4d530#	
f17a30	f4ba60	f32ab0	f61ae0	f47a30#	
f17a60	f4ba30*	f62ab0	f31ae0	f4ba30	
f17a90	f4bac0	f92ab0	fc1ae0	f4ea30#	
f17ac0	f4ba90	fc2ab0	f91ae0	f4da30#	
f19570	f46570*	f295b0	f265e0	f46570	
f195b0	f465e0*	f195b0	f465e0*	f465e0	
f195d0	f465d0*	f895b0	f865e0	f465d0	
f195e0	f465b0*	f495b0	f165e0	f465b0	
f19a70	f46a70*	f29ab0	f26ae0	f46a70	
f19ab0	f46ae0*	f19ab0	f46ae0*	f46ae0	
f19ad0	f46ad0*	f89ab0	f86ae0	f46ad0	
f19ae0	f46ab0*	f49ab0	f16ae0	f46ab0	
f1b530	f47560*	f315b0	f625e0	f47560	
f1b560	f47530	f615b0	f325e0	f4b560#	
f1b590	f475c0	f915b0	fc25e0	f4e560#	
f1b5c0	f47590	fc15b0	f925e0	f4d560#	
f1ba30	f47a60*	f31ab0	f62ae0	f47a60	
f1ba60	f47a30	f61ab0	f32ae0	f4ba60#	
f1ba90	f47ac0	f91ab0	fc2ae0	f4ea60#	
f1bac0	f47a90	fc1ab0	f92ae0	f4da60#	
f1c570	f4c570*	f2c5b0	f2c5e0	f4c570	
f1c5b0	f4c5e0*	f1c5b0	f4c5e0*	f4c5e0	
f1c5d0	f4c5d0*	f8c5b0	f8c5e0	f4c5d0	
f1c5e0	f4c5b0*	f4c5b0*	f1c5e0	f4c5b0	
f1ca70	f4ca70*	f2cab0	f2cae0	f4ca70	
f1cab0	f4cae0*	f1cab0	f4cae0*	f4cae0	
f1cad0	f4cad0*	f8cab0	f8cae0	f4cad0	
f1cae0	f4cab0*	f4cab0*	f1cae0	f4cab0	
f1d530	f4e560	f385b0	f645e0	f475c0#	
f1d560	f4e530	f685b0	f345e0	f4b5c0#	
f1d590	f4e5c0*	f985b0	fc45e0	f4e5c0	
f1d5c0	f4e590	fc85b0	f945e0	f4d5c0#	
f1da30	f4ea60	f38ab0	f64ae0	f47ac0#	
f1da60	f4ea30	f68ab0	f34ae0	f4bac0#	
f1da90	f4eac0*	f98ab0	fc4ae0	f4eac0	
f1dac0	f4ea90	fc8ab0	f94ae0	f4dac0#	
f1e530	f4d560	f345b0	f685e0	f47590#	
f1e560	f4d530	f645b0	f385e0	f4b590#	
f1e590	f4d5c0	f945b0	fc85e0	f4e590#	
f1e5c0	f4d590*	fc45b0	f985e0	f4d590	
f1ea30	f4da60	f34ab0	f68ae0	f47a90#	
f1ea60	f4da30	f64ab0	f38ae0	f4ba90#	
f1ea90	f4dac0	f94ab0	fc8ae0	f4ea90#	
f1eac0	f4da90*	fc4ab0	f98ae0	f4da90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f23570*	f23570*	f23570*	f23570*	f23570	
f235b0	f235e0*	f13570	f43570	f235e0	
f235d0*	f235d0*	f83570	f83570	f235d0	
f235e0	f235b0*	f43570	f13570	f235b0	
f23a70*	f23a70*	f23a70*	f23a70*	f23a70	
f23ab0	f23ae0*	f13a70	f43a70	f23ae0	
f23ad0*	f23ad0*	f83a70	f83a70	f23ad0	
f23ae0	f23ab0*	f43a70	f13a70	f23ab0	
f26570	f29570*	f26570	f29570*	f29570	
f265b0	f295e0*	f16570	f49570	f295e0	
f265d0	f295d0*	f86570	f89570	f295d0	
f265e0	f295b0*	f46570	f19570	f295b0	
f26a70	f29a70*	f26a70	f29a70*	f29a70	
f26ab0	f29ae0*	f16a70	f49a70	f29ae0	
f26ad0	f29ad0*	f86a70	f89a70	f29ad0	
f26ae0	f29ab0*	f46a70	f19a70	f29ab0	
f27530*	f2b560	f32570	f61570	f27530	
f27560	f2b530*	f62570	f31570	f2b530	
f27590	f2b5c0	f92570	fc1570	f2e530#	
f275c0	f2b590	fc2570	f91570	f2d530#	
f27a30*	f2ba60	f32a70	f61a70	f27a30	
f27a60	f2ba30*	f62a70	f31a70	f2ba30	
f27a90	f2bac0	f92a70	fc1a70	f2ea30#	
f27ac0	f2ba90	fc2a70	f91a70	f2da30#	
f29570	f26570*	f29570	f26570*	f26570	
f295b0	f265e0*	f19570	f46570	f265e0	
f295d0	f265d0*	f89570	f86570	f265d0	
f295e0	f265b0*	f49570	f16570	f265b0	
f29a70	f26a70*	f29a70	f26a70*	f26a70	
f29ab0	f26ae0*	f19a70	f46a70	f26ae0	
f29ad0	f26ad0*	f89a70	f86a70	f26ad0	
f29ae0	f26ab0*	f49a70	f16a70	f26ab0	
f2b530	f27560*	f31570	f62570	f27560	
f2b560*	f27530	f61570	f32570	f2b560	
f2b590	f275c0	f91570	fc2570	f2e560#	
f2b5c0	f27590	fc1570	f92570	f2d560#	
f2ba30	f27a60*	f31a70	f62a70	f27a60	
f2ba60*	f27a30	f61a70	f32a70	f2ba60	
f2ba90	f27ac0	f91a70	fc2a70	f2ea60#	
f2bac0	f27a90	fc1a70	f92a70	f2da60#	
f2c570*	f2c570*	f2c570*	f2c570*	f2c570	
f2c5b0	f2c5e0*	f1c570	f4c570	f2c5e0	
f2c5d0*	f2c5d0*	f8c570	f8c570	f2c5d0	
f2c5e0	f2c5b0*	f4c570	f1c570	f2c5b0	
f2ca70*	f2ca70*	f2ca70*	f2ca70*	f2ca70	
f2cab0	f2cae0*	f1ca70	f4ca70	f2cae0	
f2cad0*	f2cad0*	f8ca70	f8ca70	f2cad0	
f2cae0	f2cab0*	f4ca70	f1ca70	f2cab0	
f2d530	f2e560	f38570	f64570	f275c0#	
f2d560	f2e530	f68570	f34570	f2b5c0#	
f2d590	f2e5c0*	f98570	fc4570	f2e5c0	
f2d5c0*	f2e590	fc8570	f94570	f2d5c0	
f2da30	f2ea60	f38a70	f64a70	f27ac0#	
f2da60	f2ea30	f68a70	f34a70	f2bac0#	
f2da90	f2eac0*	f98a70	fc4a70	f2eac0	
f2dac0*	f2ea90	fc8a70	f94a70	f2dac0	
f2e530	f2d560	f34570	f68570	f27590#	
f2e560	f2d530	f64570	f38570	f2b590#	
f2e590*	f2d5c0	f94570	fc8570	f2e590	
f2e5c0	f2d590*	fc4570	f98570	f2d590	
f2ea30	f2da60	f34a70	f68a70	f27a90#	
f2ea60	f2da30	f64a70	f38a70	f2ba90#	
f2ea90*	f2dac0	f94a70	fc8a70	f2ea90	
f2eac0	f2da90*	fc4a70	f98a70	f2da90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f31570	f62570*	f2b530	f27560	f62570	
f315b0	f625e0*	f1b530	f47560	f625e0	
f315d0	f625d0*	f8b530	f87560	f625d0	
f315e0	f625b0*	f4b530	f17560	f625b0	
f31a70	f62a70*	f2ba30	f27a60	f62a70	
f31ab0	f62ae0*	f1ba30	f47a60	f62ae0	
f31ad0	f62ad0*	f8ba30	f87a60	f62ad0	
f31ae0	f62ab0*	f4ba30	f17a60	f62ab0	
f32570*	f61570	f27530	f2b560	f32570	
f325b0	f615e0	f17530	f4b560	f325e0#	
f325d0*	f615d0	f87530	f8b560	f325d0	
f325e0	f615b0	f47530	f1b560	f325b0#	
f32a70*	f61a70	f27a30	f2ba60	f32a70	
f32ab0	f61ae0	f17a30	f4ba60	f32ae0#	
f32ad0*	f61ad0	f87a30	f8ba60	f32ad0	
f32ae0	f61ab0	f47a30	f1ba60	f32ab0#	
f34570	f68570	f2e530	f2d560	f92570#	
f345b0	f685e0	f1e530	f4d560	f925e0#	
f345d0	f685d0	f8e530	f8d560	f925d0#	
f345e0	f685b0	f4e530	f1d560	f925b0#	
f34a70	f68a70	f2ea30	f2da60	f92a70#	
f34ab0	f68ae0	f1ea30	f4da60	f92ae0#	
f34ad0	f68ad0	f8ea30	f8da60	f92ad0#	
f34ae0	f68ab0	f4ea30	f1da60	f92ab0#	
f37510	f6b540	fb2530	fe1560	fe2530#	
f37520	f6b520	f72530*	f71560	f72530	
f37540	f6b510	fe2530	fb1560	fb2530#	
f37580	f6b580	fd2530*	fd1560	fd2530	
f37a10	f6ba40	fb2a30	fe1a60	fe2a30#	
f37a20	f6ba20	f72a30*	f71a60	f72a30	
f37a40	f6ba10	fe2a30	fb1a60	fb2a30#	
f37a80	f6ba80	fd2a30*	fd1a60	fd2a30	
f38570	f64570	f2d530	f2e560	fc2570#	
f385b0	f645e0	f1d530	f4e560	fc25e0#	
f385d0	f645d0	f8d530	f8e560	fc25d0#	
f385e0	f645b0	f4d530	f1e560	fc25b0#	
f38a70	f64a70	f2da30	f2ea60	fc2a70#	
f38ab0	f64ae0	f1da30	f4ea60	fc2ae0#	
f38ad0	f64ad0	f8da30	f8ea60	fc2ad0#	
f38ae0	f64ab0	f4da30	f1ea60	fc2ab0#	
f3b510	f67540	fb1530	fe2560*	fe2560	
f3b520	f67520	f71530	f72560*	f72560	
f3b540	f67510	fe1530	fb2560*	fb2560	
f3b580	f67580	fd1530	fd2560*	fd2560	
f3ba10	f67a40	fb1a30	fe2a60*	fe2a60	
f3ba20	f67a20	f71a30	f72a60*	f72a60	
f3ba40	f67a10	fe1a30	fb2a60*	fb2a60	
f3ba80	f67a80	fd1a30	fd2a60*	fd2a60	
f3d510	f6e540	fb8530	fe4560	fe25c0#	
f3d520	f6e520	f78530	f74560	f725c0#	
f3d540	f6e510	fe8530	fb4560	fb25c0#	
f3d580	f6e580	fd8530	fd4560	fd25c0#	
f3da10	f6ea40	fb8a30	fe4a60	fe2ac0#	
f3da20	f6ea20	f78a30	f74a60	f72ac0#	
f3da40	f6ea10	fe8a30	fb4a60	fb2ac0#	
f3da80	f6ea80	fd8a30	fd4a60	fd2ac0#	
f3e510	f6d540	fb4530	fe8560	fe2590#	
f3e520	f6d520	f74530	f78560	f72590#	
f3e540	f6d510	fe4530	fb8560	fb2590#	
f3e580	f6d580	fd4530	fd8560	fd2590#	
f3ea10	f6da40	fb4a30	fe8a60	fe2a90#	
f3ea20	f6da20	f74a30	f78a60	f72a90#	
f3ea40	f6da10	fe4a30	fb8a60	fb2a90#	
f3ea80	f6da80	fd4a30	fd8a60	fd2a90#	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f43570	f13570*	f235e0	f235b0	f13570	
f435b0	f135e0*	f135e0*	f435b0	f135e0	
f435d0	f135d0*	f835e0	f835b0	f135d0	
f435e0	f135b0*	f435e0	f135b0*	f135b0	
f43a70	f13a70*	f23ae0	f23ab0	f13a70	
f43ab0	f13ae0*	f13ae0*	f43ab0	f13ae0	
f43ad0	f13ad0*	f83ae0	f83ab0	f13ad0	
f43ae0	f13ab0*	f43ae0	f13ab0*	f13ab0	
f46570	f19570*	f265e0	f295b0	f19570	
f465b0	f195e0*	f165e0	f495b0	f195e0	
f465d0	f195d0*	f865e0	f895b0	f195d0	
f465e0	f195b0*	f465e0	f195b0*	f195b0	
f46a70	f19a70*	f26ae0	f29ab0	f19a70	
f46ab0	f19ae0*	f16ae0	f49ab0	f19ae0	
f46ad0	f19ad0*	f86ae0	f89ab0	f19ad0	
f46ae0	f19ab0*	f46ae0	f19ab0*	f19ab0	
f47530	f1b560	f325e0	f615b0	f17530#	
f47560	f1b530*	f625e0	f315b0	f1b530	
f47590	f1b5c0	f925e0	fc15b0	f1e530#	
f475c0	f1b590	fc25e0	f915b0	f1d530#	
f47a30	f1ba60	f32ae0	f61ab0	f17a30#	
f47a60	f1ba30*	f62ae0	f31ab0	f1ba30	
f47a90	f1bac0	f92ae0	fc1ab0	f1ea30#	
f47ac0	f1ba90	fc2ae0	f91ab0	f1da30#	
f49570	f16570*	f295e0	f265b0	f16570	
f495b0	f165e0*	f195e0	f465b0	f165e0	
f495d0	f165d0*	f895e0	f865b0	f165d0	
f495e0	f165b0*	f495e0	f165b0*	f165b0	
f49a70	f16a70*	f29ae0	f26ab0	f16a70	
f49ab0	f16ae0*	f19ae0	f46ab0	f16ae0	
f49ad0	f16ad0*	f89ae0	f86ab0	f16ad0	
f49ae0	f16ab0*	f49ae0	f16ab0*	f16ab0	
f4b530	f17560*	f315e0	f625b0	f17560	
f4b560	f17530	f615e0	f325b0	f1b560#	
f4b590	f175c0	f915e0	fc25b0	f1e560#	
f4b5c0	f17590	fc15e0	f925b0	f1d560#	
f4ba30	f17a60*	f31ae0	f62ab0	f17a60	
f4ba60	f17a30	f61ae0	f32ab0	f1ba60#	
f4ba90	f17ac0	f91ae0	fc2ab0	f1ea60#	
f4bac0	f17a90	fc1ae0	f92ab0	f1da60#	
f4c570	f1c570*	f2c5e0	f2c5b0	f1c570	
f4c5b0	f1c5e0*	f1c5e0*	f4c5b0	f1c5e0	
f4c5d0	f1c5d0*	f8c5e0	f8c5b0	f1c5d0	
f4c5e0	f1c5b0*	f4c5e0	f1c5b0*	f1c5b0	
f4ca70	f1ca70*	f2cae0	f2cab0	f1ca70	
f4cab0	f1cae0*	f1cae0*	f4cab0	f1cae0	
f4cad0	f1cad0*	f8cae0	f8cab0	f1cad0	
f4cae0	f1cab0*	f4cae0	f1cab0*	f1cab0	
f4d530	f1e560	f385e0	f645b0	f175c0#	
f4d560	f1e530	f685e0	f345b0	f1b5c0#	
f4d590	f1e5c0*	f985e0	fc45b0	f1e5c0	
f4d5c0	f1e590	fc85e0	f945b0	f1d5c0#	
f4da30	f1ea60	f38ae0	f64ab0	f17ac0#	
f4da60	f1ea30	f68ae0	f34ab0	f1bac0#	
f4da90	f1eac0*	f98ae0	fc4ab0	f1eac0	
f4dac0	f1ea90	fc8ae0	f94ab0	f1dac0#	
f4e530	f1d560	f345e0	f685b0	f17590#	
f4e560	f1d530	f645e0	f385b0	f1b590#	
f4e590	f1d5c0	f945e0	fc85b0	f1e590#	
f4e5c0	f1d590*	fc45e0	f985b0	f1d590	
f4ea30	f1da60	f34ae0	f68ab0	f17a90#	
f4ea60	f1da30	f64ae0	f38ab0	f1ba90#	
f4ea90	f1dac0	f94ae0	fc8ab0	f1ea90#	
f4eac0	f1da90*	fc4ae0	f98ab0	f1da90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f61570*	f32570	f2b560	f27530	f61570	
f615b0	f325e0	f1b560	f47530	f615e0	#
f615d0*	f325d0	f8b560	f87530	f615d0	
f615e0	f325b0	f4b560	f17530	f615b0	#
f61a70*	f32a70	f2ba60	f27a30	f61a70	
f61ab0	f32ae0	f1ba60	f47a30	f61ae0	#
f61ad0*	f32ad0	f8ba60	f87a30	f61ad0	
f61ae0	f32ab0	f4ba60	f17a30	f61ab0	#
f62570	f31570*	f27560	f2b530	f31570	
f625b0	f315e0*	f17560	f4b530	f315e0	
f625d0	f315d0*	f87560	f8b530	f315d0	
f625e0	f315b0*	f47560	f1b530	f315b0	
f62a70	f31a70*	f27a60	f2ba30	f31a70	
f62ab0	f31ae0*	f17a60	f4ba30	f31ae0	
f62ad0	f31ad0*	f87a60	f8ba30	f31ad0	
f62ae0	f31ab0*	f47a60	f1ba30	f31ab0	
f64570	f38570	f2e560	f2d530	f91570	#
f645b0	f385e0	f1e560	f4d530	f915e0	#
f645d0	f385d0	f8e560	f8d530	f915d0	#
f645e0	f385b0	f4e560	f1d530	f915b0	#
f64a70	f38a70	f2ea60	f2da30	f91a70	#
f64ab0	f38ae0	f1ea60	f4da30	f91ae0	#
f64ad0	f38ad0	f8ea60	f8da30	f91ad0	#
f64ae0	f38ab0	f4ea60	f1da30	f91ab0	#
f67510	f3b540	fb2560	fe1530*	fe1530	
f67520	f3b520	f72560	f71530*	f71530	
f67540	f3b510	fe2560	fb1530*	fb1530	
f67580	f3b580	fd2560	fd1530*	fd1530	
f67a10	f3ba40	fb2a60	fe1a30*	fe1a30	
f67a20	f3ba20	f72a60	f71a30*	f71a30	
f67a40	f3ba10	fe2a60	fb1a30*	fb1a30	
f67a80	f3ba80	fd2a60	fd1a30*	fd1a30	
f68570	f34570	f2d560	f2e530	fc1570	#
f685b0	f345e0	f1d560	f4e530	fc15e0	#
f685d0	f345d0	f8d560	f8e530	fc15d0	#
f685e0	f345b0	f4d560	f1e530	fc15b0	#
f68a70	f34a70	f2da60	f2ea30	fc1a70	#
f68ab0	f34ae0	f1da60	f4ea30	fc1ae0	#
f68ad0	f34ad0	f8da60	f8ea30	fc1ad0	#
f68ae0	f34ab0	f4da60	f1ea30	fc1ab0	#
f6b510	f37540	fb1560	fe2530	fe1560	#
f6b520	f37520	f71560*	f72530	f71560	
f6b540	f37510	fe1560	fb2530	fb1560	#
f6b580	f37580	fd1560*	fd2530	fd1560	
f6ba10	f37a40	fb1a60	fe2a30	fe1a60	#
f6ba20	f37a20	f71a60*	f72a30	f71a60	
f6ba40	f37a10	fe1a60	fb2a30	fb1a60	#
f6ba80	f37a80	fd1a60*	fd2a30	fd1a60	
f6d510	f3e540	fb8560	fe4530	fe15c0	#
f6d520	f3e520	f78560	f74530	f715c0	#
f6d540	f3e510	fe8560	fb4530	fb15c0	#
f6d580	f3e580	fd8560	fd4530	fd15c0	#
f6da10	f3ea40	fb8a60	fe4a30	fe1ac0	#
f6da20	f3ea20	f78a60	f74a30	f71ac0	#
f6da40	f3ea10	fe8a60	fb4a30	fb1ac0	#
f6da80	f3ea80	fd8a60	fd4a30	fd1ac0	#
f6e510	f3d540	fb4560	fe8530	fe1590	#
f6e520	f3d520	f74560	f78530	f71590	#
f6e540	f3d510	fe4560	fb8530	fb1590	#
f6e580	f3d580	fd4560	fd8530	fd1590	#
f6ea10	f3da40	fb4a60	fe8a30	fe1a90	#
f6ea20	f3da20	f74a60	f78a30	f71a90	#
f6ea40	f3da10	fe4a60	fb8a30	fb1a90	#
f6ea80	f3da80	fd4a60	fd8a30	fd1a90	#

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f71530	f72560	f3b520	f67520*	f67520	
f71560	f72530	f6b520*	f37520	f6b520	
f71590	f725c0	f9b520	fc7520	f6e520	#
f715c0	f72590	fcb520	f97520	f6d520	#
f71a30	f72a60	f3ba20	f67a20*	f67a20	
f71a60	f72a30	f6ba20*	f37a20	f6ba20	
f71a90	f72ac0	f9ba20	fc7a20	f6ea20	#
f71ac0	f72a90	fcba20	f97a20	f6da20	#
f72530	f71560	f37520*	f6b520	f37520	
f72560	f71530	f67520	f3b520*	f3b520	
f72590	f715c0	f97520	fcb520	f3e520	#
f725c0	f71590	fc7520	f9b520	f3d520	#
f72a30	f71a60	f37a20*	f6ba20	f37a20	
f72a60	f71a30	f67a20	f3ba20*	f3ba20	
f72a90	f71ac0	f97a20	fcba20	f3ea20	#
f72ac0	f71a90	fc7a20	f9ba20	f3da20	#
f73510	f73540	fb3520	fe3520*	fe3520	
f73520*	f73520*	f73520*	f73520*	f73520	
f73540	f73510	fe3520	fb3520*	fb3520	
f73580	f73580	fd3520*	fd3520*	fd3520	
f73a10	f73a40	fb3a20	fe3a20*	fe3a20	
f73a20*	f73a20*	f73a20*	f73a20*	f73a20	
f73a40	f73a10	fe3a20	fb3a20*	fb3a20	
f73a80	f73a80	fd3a20*	fd3a20*	fd3a20	
f74530	f78560	f3e520	f6d520	f97520	#
f74560	f78530	f6e520	f3d520	f9b520	#
f74590	f785c0	f9e520*	fcd520	f9e520	
f745c0	f78590	fce520	f9d520*	f9d520	
f74a30	f78a60	f3ea20	f6da20	f97a20	#
f74a60	f78a30	f6ea20	f3da20	f9ba20	#
f74a90	f78ac0	f9ea20*	fcda20	f9ea20	
f74ac0	f78a90	fcea20	f9da20*	f9da20	
f76510	f79540	fb6520	fe9520*	fe9520	
f76520	f79520*	f76520	f79520*	f79520	
f76540	f79510	fe6520	fb9520*	fb9520	
f76580	f79580	fd6520	fd9520*	fd9520	
f76a10	f79a40	fb6a20	fe9a20*	fe9a20	
f76a20	f79a20*	f76a20	f79a20*	f79a20	
f76a40	f79a10	fe6a20	fb9a20*	fb9a20	
f76a80	f79a80	fd6a20	fd9a20*	fd9a20	
f78530	f74560	f3d520	f6e520	fc7520	#
f78560	f74530	f6d520	f3e520	fcb520	#
f78590	f745c0	f9d520	fce520*	fce520	
f785c0	f74590	fcd520*	f9e520	fcd520	
f78a30	f74a60	f3da20	f6ea20	fc7a20	#
f78a60	f74a30	f6da20	f3ea20	fcba20	#
f78a90	f74ac0	f9da20	fcea20*	fcea20	
f78ac0	f74a90	fcda20*	f9ea20	fcda20	
f79510	f76540	fb9520	fe6520*	fe6520	
f79520	f76520*	f79520	f76520*	f76520	
f79540	f76510	fe9520	fb6520*	fb6520	
f79580	f76580	fd9520	fd6520*	fd6520	
f79a10	f76a40	fb9a20	fe6a20*	fe6a20	
f79a20	f76a20*	f79a20	f76a20*	f76a20	
f79a40	f76a10	fe9a20	fb6a20*	fb6a20	
f79a80	f76a80	fd9a20	fd6a20*	fd6a20	
f7c510	f7c540	fbc520	fec520*	fec520	
f7c520*	f7c520*	f7c520*	f7c520*	f7c520	
f7c540	f7c510	fec520	fbc520*	fbc520	
f7c580	f7c580	fdc520*	fdc520*	fdc520	
f7ca10	f7ca40	fbca20	feca20*	feca20	
f7ca20*	f7ca20*	f7ca20*	f7ca20*	f7ca20	
f7ca40	f7ca10	feca20	fbca20*	fbca20	
f7ca80	f7ca80	fdca20*	fdca20*	fdca20	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f83570*	f83570*	f235d0	f235d0	f83570	
f835b0	f835e0*	f135d0	f435d0	f835e0	
f835d0*	f835d0*	f835d0*	f835d0*	f835d0	
f835e0	f835b0*	f435d0	f135d0	f835b0	
f83a70*	f83a70*	f23ad0	f23ad0	f83a70	
f83ab0	f83ae0*	f13ad0	f43ad0	f83ae0	
f83ad0*	f83ad0*	f83ad0*	f83ad0*	f83ad0	
f83ae0	f83ab0*	f43ad0	f13ad0	f83ab0	
f86570	f89570*	f265d0	f295d0	f89570	
f865b0	f895e0*	f165d0	f495d0	f895e0	
f865d0	f895d0*	f865d0	f895d0*	f895d0	
f865e0	f895b0*	f465d0	f195d0	f895b0	
f86a70	f89a70*	f26ad0	f29ad0	f89a70	
f86ab0	f89ae0*	f16ad0	f49ad0	f89ae0	
f86ad0	f89ad0*	f86ad0	f89ad0*	f89ad0	
f86ae0	f89ab0*	f46ad0	f19ad0	f89ab0	
f87530*	f8b560	f325d0	f615d0	f87530	
f87560	f8b530*	f625d0	f315d0	f8b530	
f87590	f8b5c0	f925d0	fc15d0	f8e530	#
f875c0	f8b590	fc25d0	f915d0	f8d530	#
f87a30*	f8ba60	f32ad0	f61ad0	f87a30	
f87a60	f8ba30*	f62ad0	f31ad0	f8ba30	
f87a90	f8bac0	f92ad0	fc1ad0	f8ea30	#
f87ac0	f8ba90	fc2ad0	f91ad0	f8da30	#
f89570	f86570*	f295d0	f265d0	f86570	
f895b0	f865e0*	f195d0	f465d0	f865e0	
f895d0	f865d0*	f895d0	f865d0*	f865d0	
f895e0	f865b0*	f495d0	f165d0	f865b0	
f89a70	f86a70*	f29ad0	f26ad0	f86a70	
f89ab0	f86ae0*	f19ad0	f46ad0	f86ae0	
f89ad0	f86ad0*	f89ad0	f86ad0*	f86ad0	
f89ae0	f86ab0*	f49ad0	f16ad0	f86ab0	
f8b530	f87560*	f315d0	f625d0	f87560	
f8b560*	f87530	f615d0	f325d0	f8b560	
f8b590	f875c0	f915d0	fc25d0	f8e560	#
f8b5c0	f87590	fc15d0	f925d0	f8d560	#
f8ba30	f87a60*	f31ad0	f62ad0	f87a60	
f8ba60*	f87a30	f61ad0	f32ad0	f8ba60	
f8ba90	f87ac0	f91ad0	fc2ad0	f8ea60	#
f8bac0	f87a90	fc1ad0	f92ad0	f8da60	#
f8c570*	f8c570*	f2c5d0	f2c5d0	f8c570	
f8c5b0	f8c5e0*	f1c5d0	f4c5d0	f8c5e0	
f8c5d0*	f8c5d0*	f8c5d0*	f8c5d0*	f8c5d0	
f8c5e0	f8c5b0*	f4c5d0	f1c5d0	f8c5b0	
f8ca70*	f8ca70*	f2cad0	f2cad0	f8ca70	
f8cab0	f8cae0*	f1cad0	f4cad0	f8cae0	
f8cad0*	f8cad0*	f8cad0*	f8cad0*	f8cad0	
f8cae0	f8cab0*	f4cad0	f1cad0	f8cab0	
f8d530	f8e560	f385d0	f645d0	f875c0	#
f8d560	f8e530	f685d0	f345d0	f8b5c0	#
f8d590	f8e5c0*	f985d0	fc45d0	f8e5c0	
f8d5c0*	f8e590	fc85d0	f945d0	f8d5c0	
f8da30	f8ea60	f38ad0	f64ad0	f87ac0	#
f8da60	f8ea30	f68ad0	f34ad0	f8bac0	#
f8da90	f8eac0*	f98ad0	fc4ad0	f8eac0	
f8dac0*	f8ea90	fc8ad0	f94ad0	f8dac0	
f8e530	f8d560	f345d0	f685d0	f87590	#
f8e560	f8d530	f645d0	f385d0	f8b590	#
f8e590*	f8d5c0	f945d0	fc85d0	f8e590	
f8e5c0	f8d590*	fc45d0	f985d0	f8d590	
f8ea30	f8da60	f34ad0	f68ad0	f87a90	#
f8ea60	f8da30	f64ad0	f38ad0	f8ba90	#
f8ea90*	f8dac0	f94ad0	fc8ad0	f8ea90	
f8eac0	f8da90*	fc4ad0	f98ad0	f8da90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
f91570	fc2570	f2b590	f275c0	f64570	#
f915b0	fc25e0	f1b590	f475c0	f645e0	#
f915d0	fc25d0	f8b590	f875c0	f645d0	#
f915e0	fc25b0	f4b590	f175c0	f645b0	#
f91a70	fc2a70	f2ba90	f27ac0	f64a70	#
f91ab0	fc2ae0	f1ba90	f47ac0	f64ae0	#
f91ad0	fc2ad0	f8ba90	f87ac0	f64ad0	#
f91ae0	fc2ab0	f4ba90	f17ac0	f64ab0	#
f92570	fc1570	f27590	f2b5c0	f34570	#
f925b0	fc15e0	f17590	f4b5c0	f345e0	#
f925d0	fc15d0	f87590	f8b5c0	f345d0	#
f925e0	fc15b0	f47590	f1b5c0	f345b0	#
f92a70	fc1a70	f27a90	f2bac0	f34a70	#
f92ab0	fc1ae0	f17a90	f4bac0	f34ae0	#
f92ad0	fc1ad0	f87a90	f8bac0	f34ad0	#
f92ae0	fc1ab0	f47a90	f1bac0	f34ab0	#
f94570*	fc8570	f2e590	f2d5c0	f94570	
f945b0	fc85e0	f1e590	f4d5c0	f945e0	#
f945d0*	fc85d0	f8e590	f8d5c0	f945d0	
f945e0	fc85b0	f4e590	f1d5c0	f945b0	#
f94a70*	fc8a70	f2ea90	f2dac0	f94a70	
f94ab0	fc8ae0	f1ea90	f4dac0	f94ae0	#
f94ad0*	fc8ad0	f8ea90	f8dac0	f94ad0	
f94ae0	fc8ab0	f4ea90	f1dac0	f94ab0	#
f97510	fcb540	fb2590	fe15c0	fe4530	#
f97520	fcb520	f72590	f715c0	f74530	#
f97540	fcb510	fe2590	fb15c0	fb4530	#
f97580	fcb580	fd2590	fd15c0	fd4530	#
f97a10	fcba40	fb2a90	fe1ac0	fe4a30	#
f97a20	fcba20	f72a90	f71ac0	f74a30	#
f97a40	fcba10	fe2a90	fb1ac0	fb4a30	#
f97a80	fcba80	fd2a90	fd1ac0	fd4a30	#
f98570	fc4570*	f2d590	f2e5c0	fc4570	
f985b0	fc45e0*	f1d590	f4e5c0	fc45e0	
f985d0	fc45d0*	f8d590	f8e5c0	fc45d0	
f985e0	fc45b0*	f4d590	f1e5c0	fc45b0	
f98a70	fc4a70*	f2da90	f2eac0	fc4a70	
f98ab0	fc4ae0*	f1da90	f4eac0	fc4ae0	
f98ad0	fc4ad0*	f8da90	f8eac0	fc4ad0	
f98ae0	fc4ab0*	f4da90	f1eac0	fc4ab0	
f9b510	fc7540	fb1590	fe25c0	fe4560	#
f9b520	fc7520	f71590	f725c0	f74560	#
f9b540	fc7510	fe1590	fb25c0	fb4560	#
f9b580	fc7580	fd1590	fd25c0	fd4560	#
f9ba10	fc7a40	fb1a90	fe2ac0	fe4a60	#
f9ba20	fc7a20	f71a90	f72ac0	f74a60	#
f9ba40	fc7a10	fe1a90	fb2ac0	fb4a60	#
f9ba80	fc7a80	fd1a90	fd2ac0	fd4a60	#
f9d510	fce540	fb8590	fe45c0*	fe45c0	
f9d520	fce520	f78590	f745c0*	f745c0	
f9d540	fce510	fe8590	fb45c0*	fb45c0	
f9d580	fce580	fd8590	fd45c0*	fd45c0	
f9da10	fcea40	fb8a90	fe4ac0*	fe4ac0	
f9da20	fcea20	f78a90	f74ac0*	f74ac0	
f9da40	fcea10	fe8a90	fb4ac0*	fb4ac0	
f9da80	fcea80	fd8a90	fd4ac0*	fd4ac0	
f9e510	fcd540	fb4590	fe85c0	fe4590	#
f9e520	fcd520	f74590*	f785c0	f74590	
f9e540	fcd510	fe4590	fb85c0	fb4590	#
f9e580	fcd580	fd4590*	fd85c0	fd4590	
f9ea10	fcda40	fb4a90	fe8ac0	fe4a90	#
f9ea20	fcda20	f74a90*	f78ac0	f74a90	
f9ea40	fcda10	fe4a90	fb8ac0	fb4a90	#
f9ea80	fcda80	fd4a90*	fd8ac0	fd4a90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
fb1530	fe2560	f3b510	f67540*	f67540	
fb1560	fe2530	f6b510	f37540	f6b540	#
fb1590	fe25c0	f9b510	fc7540	f6e540	#
fb15c0	fe2590	fcb510	f97540	f6d540	#
fb1a30	fe2a60	f3ba10	f67a40*	f67a40	
fb1a60	fe2a30	f6ba10	f37a40	f6ba40	#
fb1a90	fe2ac0	f9ba10	fc7a40	f6ea40	#
fb1ac0	fe2a90	fcba10	f97a40	f6da40	#
fb2530	fe1560	f37510	f6b540	f37540	#
fb2560	fe1530	f67510	f3b540*	f3b540	
fb2590	fe15c0	f97510	fcb540	f3e540	#
fb25c0	fe1590	fc7510	f9b540	f3d540	#
fb2a30	fe1a60	f37a10	f6ba40	f37a40	#
fb2a60	fe1a30	f67a10	f3ba40*	f3ba40	
fb2a90	fe1ac0	f97a10	fcba40	f3ea40	#
fb2ac0	fe1a90	fc7a10	f9ba40	f3da40	#
fb3510	fe3540*	fb3510	fe3540*	fe3540	
fb3520	fe3520	f73510	f73540*	f73540	
fb3540*	fe3510	fe3510	fb3540*	fb3540	
fb3580	fe3580	fd3510	fd3540*	fd3540	
fb3a10	fe3a40*	fb3a10	fe3a40*	fe3a40	
fb3a20	fe3a20	f73a10	f73a40*	f73a40	
fb3a40*	fe3a10	fe3a10	fb3a40*	fb3a40	
fb3a80	fe3a80	fd3a10	fd3a40*	fd3a40	
fb4530	fe8560	f3e510	f6d540	f97540	#
fb4560	fe8530	f6e510	f3d540	f9b540	#
fb4590	fe85c0	f9e510	fcd540	f9e540	#
fb45c0	fe8590	fce510	f9d540*	f9d540	
fb4a30	fe8a60	f3ea10	f6da40	f97a40	#
fb4a60	fe8a30	f6ea10	f3da40	f9ba40	#
fb4a90	fe8ac0	f9ea10	fcda40	f9ea40	#
fb4ac0	fe8a90	fcea10	f9da40*	f9da40	
fb6510	fe9540*	fb6510	fe9540*	fe9540	
fb6520	fe9520	f76510	f79540*	f79540	
fb6540	fe9510	fe6510	fb9540*	fb9540	
fb6580	fe9580	fd6510	fd9540*	fd9540	
fb6a10	fe9a40*	fb6a10	fe9a40*	fe9a40	
fb6a20	fe9a20	f76a10	f79a40*	f79a40	
fb6a40	fe9a10	fe6a10	fb9a40*	fb9a40	
fb6a80	fe9a80	fd6a10	fd9a40*	fd9a40	
fb8530	fe4560	f3d510	f6e540	fc7540	#
fb8560	fe4530	f6d510	f3e540	fcb540	#
fb8590	fe45c0	f9d510	fce540*	fce540	
fb85c0	fe4590	fcd510	f9e540	fcd540	#
fb8a30	fe4a60	f3da10	f6ea40	fc7a40	#
fb8a60	fe4a30	f6da10	f3ea40	fcba40	#
fb8a90	fe4ac0	f9da10	fcea40*	fcea40	
fb8ac0	fe4a90	fcda10	f9ea40	fcda40	#
fb9510	fe6540*	fb9510	fe6540*	fe6540	
fb9520	fe6520	f79510	f76540*	f76540	
fb9540	fe6510	fe9510	fb6540*	fb6540	
fb9580	fe6580	fd9510	fd6540*	fd6540	
fb9a10	fe6a40*	fb9a10	fe6a40*	fe6a40	
fb9a20	fe6a20	f79a10	f76a40*	f76a40	
fb9a40	fe6a10	fe9a10	fb6a40*	fb6a40	
fb9a80	fe6a80	fd9a10	fd6a40*	fd6a40	
fbc510	fec540*	fbc510	fec540*	fec540	
fbc520	fec520	f7c510	f7c540*	f7c540	
fbc540*	fec510	fec510	fbc540*	fbc540	
fbc580	fec580	fdc510	fdc540*	fdc540	
fbca10	feca40*	fbca10	feca40*	feca40	
fbca20	feca20	f7ca10	f7ca40*	f7ca40	
fbca40*	feca10	feca10	fbca40*	fbca40	
fbca80	feca80	fdca10	fdca40*	fdca40	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
fc1570	f92570	f2b5c0	f27590	f68570	#
fc15b0	f925e0	f1b5c0	f47590	f685e0	#
fc15d0	f925d0	f8b5c0	f87590	f685d0	#
fc15e0	f925b0	f4b5c0	f17590	f685b0	#
fc1a70	f92a70	f2bac0	f27a90	f68a70	#
fc1ab0	f92ae0	f1bac0	f47a90	f68ae0	#
fc1ad0	f92ad0	f8bac0	f87a90	f68ad0	#
fc1ae0	f92ab0	f4bac0	f17a90	f68ab0	#
fc2570	f91570	f275c0	f2b590	f38570	#
fc25b0	f915e0	f175c0	f4b590	f385e0	#
fc25d0	f915d0	f875c0	f8b590	f385d0	#
fc25e0	f915b0	f475c0	f1b590	f385b0	#
fc2a70	f91a70	f27ac0	f2ba90	f38a70	#
fc2ab0	f91ae0	f17ac0	f4ba90	f38ae0	#
fc2ad0	f91ad0	f87ac0	f8ba90	f38ad0	#
fc2ae0	f91ab0	f47ac0	f1ba90	f38ab0	#
fc4570	f98570*	f2e5c0	f2d590	f98570	
fc45b0	f985e0*	f1e5c0	f4d590	f985e0	
fc45d0	f985d0*	f8e5c0	f8d590	f985d0	
fc45e0	f985b0*	f4e5c0	f1d590	f985b0	
fc4a70	f98a70*	f2eac0	f2da90	f98a70	
fc4ab0	f98ae0*	f1eac0	f4da90	f98ae0	
fc4ad0	f98ad0*	f8eac0	f8da90	f98ad0	
fc4ae0	f98ab0*	f4eac0	f1da90	f98ab0	
fc7510	f9b540	fb25c0	fe1590	fe8530	#
fc7520	f9b520	f725c0	f71590	f78530	#
fc7540	f9b510	fe25c0	fb1590	fb8530	#
fc7580	f9b580	fd25c0	fd1590	fd8530	#
fc7a10	f9ba40	fb2ac0	fe1a90	fe8a30	#
fc7a20	f9ba20	f72ac0	f71a90	f78a30	#
fc7a40	f9ba10	fe2ac0	fb1a90	fb8a30	#
fc7a80	f9ba80	fd2ac0	fd1a90	fd8a30	#
fc8570*	f94570	f2d5c0	f2e590	fc8570	
fc85b0	f945e0	f1d5c0	f4e590	fc85e0	#
fc85d0*	f945d0	f8d5c0	f8e590	fc85d0	
fc85e0	f945b0	f4d5c0	f1e590	fc85b0	#
fc8a70*	f94a70	f2dac0	f2ea90	fc8a70	
fc8ab0	f94ae0	f1dac0	f4ea90	fc8ae0	#
fc8ad0*	f94ad0	f8dac0	f8ea90	fc8ad0	
fc8ae0	f94ab0	f4dac0	f1ea90	fc8ab0	#
fcb510	f97540	fb15c0	fe2590	fe8560	#
fcb520	f97520	f715c0	f72590	f78560	#
fcb540	f97510	fe15c0	fb2590	fb8560	#
fcb580	f97580	fd15c0	fd2590	fd8560	#
fcba10	f97a40	fb1ac0	fe2a90	fe8a60	#
fcba20	f97a20	f71ac0	f72a90	f78a60	#
fcba40	f97a10	fe1ac0	fb2a90	fb8a60	#
fcba80	f97a80	fd1ac0	fd2a90	fd8a60	#
fcd510	f9e540	fb85c0	fe4590	fe85c0	#
fcd520	f9e520	f785c0*	f74590	f785c0	
fcd540	f9e510	fe85c0	fb4590	fb85c0	#
fcd580	f9e580	fd85c0*	fd4590	fd85c0	
fcda10	f9ea40	fb8ac0	fe4a90	fe8ac0	#
fcda20	f9ea20	f78ac0*	f74a90	f78ac0	
fcda40	f9ea10	fe8ac0	fb4a90	fb8ac0	#
fcda80	f9ea80	fd8ac0*	fd4a90	fd8ac0	
fce510	f9d540	fb45c0	fe8590*	fe8590	
fce520	f9d520	f745c0	f78590*	f78590	
fce540	f9d510	fe45c0	fb8590*	fb8590	
fce580	f9d580	fd45c0	fd8590*	fd8590	
fcea10	f9da40	fb4ac0	fe8a90*	fe8a90	
fcea20	f9da20	f74ac0	f78a90*	f78a90	
fcea40	f9da10	fe4ac0	fb8a90*	fb8a90	
fcea80	f9da80	fd4ac0	fd8a90*	fd8a90	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
fd1530	fd2560	f3b580	f67580*	f67580	
fd1560	fd2530	f6b580*	f37580	f6b580	
fd1590	fd25c0	f9b580	fc7580	f6e580	#
fd15c0	fd2590	fcb580	f97580	f6d580	#
fd1a30	fd2a60	f3ba80	f67a80*	f67a80	
fd1a60	fd2a30	f6ba80*	f37a80	f6ba80	
fd1a90	fd2ac0	f9ba80	fc7a80	f6ea80	#
fd1ac0	fd2a90	fcba80	f97a80	f6da80	#
fd2530	fd1560	f37580*	f6b580	f37580	
fd2560	fd1530	f67580	f3b580*	f3b580	
fd2590	fd15c0	f97580	fcb580	f3e580	#
fd25c0	fd1590	fc7580	f9b580	f3d580	#
fd2a30	fd1a60	f37a80*	f6ba80	f37a80	
fd2a60	fd1a30	f67a80	f3ba80*	f3ba80	
fd2a90	fd1ac0	f97a80	fcba80	f3ea80	#
fd2ac0	fd1a90	fc7a80	f9ba80	f3da80	#
fd3510	fd3540	fb3580	fe3580*	fe3580	
fd3520	fd3520	f73580*	f73580*	f73580	
fd3540	fd3510	fe3580	fb3580*	fb3580	
fd3580*	fd3580*	fd3580*	fd3580*	fd3580	
fd3a10	fd3a40	fb3a80	fe3a80*	fe3a80	
fd3a20	fd3a20	f73a80*	f73a80*	f73a80	
fd3a40	fd3a10	fe3a80	fb3a80*	fb3a80	
fd3a80*	fd3a80*	fd3a80*	fd3a80*	fd3a80	
fd4530	fd8560	f3e580	f6d580	f97580	#
fd4560	fd8530	f6e580	f3d580	f9b580	#
fd4590	fd85c0	f9e580*	fcd580	f9e580	
fd45c0	fd8590	fce580	f9d580*	f9d580	
fd4a30	fd8a60	f3ea80	f6da80	f97a80	#
fd4a60	fd8a30	f6ea80	f3da80	f9ba80	#
fd4a90	fd8ac0	f9ea80*	fcda80	f9ea80	
fd4ac0	fd8a90	fcea80	f9da80*	f9da80	
fd6510	fd9540	fb6580	fe9580*	fe9580	
fd6520	fd9520	f76580	f79580*	f79580	
fd6540	fd9510	fe6580	fb9580*	fb9580	
fd6580	fd9580*	fd6580	fd9580*	fd9580	
fd6a10	fd9a40	fb6a80	fe9a80*	fe9a80	
fd6a20	fd9a20	f76a80	f79a80*	f79a80	
fd6a40	fd9a10	fe6a80	fb9a80*	fb9a80	
fd6a80	fd9a80*	fd6a80	fd9a80*	fd9a80	
fd8530	fd4560	f3d580	f6e580	fc7580	#
fd8560	fd4530	f6d580	f3e580	fcb580	#
fd8590	fd45c0	f9d580	fce580*	fce580	
fd85c0	fd4590	fcd580*	f9e580	fcd580	
fd8a30	fd4a60	f3da80	f6ea80	fc7a80	#
fd8a60	fd4a30	f6da80	f3ea80	fcba80	#
fd8a90	fd4ac0	f9da80	fcea80*	fcea80	
fd8ac0	fd4a90	fcda80*	f9ea80	fcda80	
fd9510	fd6540	fb9580	fe6580*	fe6580	
fd9520	fd6520	f79580	f76580*	f76580	
fd9540	fd6510	fe9580	fb6580*	fb6580	
fd9580	fd6580*	fd9580	fd6580*	fd6580	
fd9a10	fd6a40	fb9a80	fe6a80*	fe6a80	
fd9a20	fd6a20	f79a80	f76a80*	f76a80	
fd9a40	fd6a10	fe9a80	fb6a80*	fb6a80	
fd9a80	fd6a80*	fd9a80	fd6a80*	fd6a80	
fdc510	fdc540	fbc580	fec580*	fec580	
fdc520	fdc520	f7c580*	f7c580*	f7c580	
fdc540	fdc510	fec580	fbc580*	fbc580	
fdc580*	fdc580*	fdc580*	fdc580*	fdc580	
fdca10	fdca40	fbca80	feca80*	feca80	
fdca20	fdca20	f7ca80*	f7ca80*	f7ca80	
fdca40	fdca10	feca80	fbca80*	fbca80	
fdca80*	fdca80*	fdca80*	fdca80*	fdca80	

f	f^{r}	f^{c}	f^{rc}	f^{-1}	c
fe1530	fb2560	f3b540	f67510*	f67510	
fe1560	fb2530	f6b540	f37510	f6b510	#
fe1590	fb25c0	f9b540	fc7510	f6e510	#
fe15c0	fb2590	fcb540	f97510	f6d510	#
fe1a30	fb2a60	f3ba40	f67a10*	f67a10	
fe1a60	fb2a30	f6ba40	f37a10	f6ba10	#
fe1a90	fb2ac0	f9ba40	fc7a10	f6ea10	#
fe1ac0	fb2a90	fcba40	f97a10	f6da10	#
fe2530	fb1560	f37540	f6b510	f37510	#
fe2560	fb1530	f67540	f3b510*	f3b510	
fe2590	fb15c0	f97540	fcb510	f3e510	#
fe25c0	fb1590	fc7540	f9b510	f3d510	#
fe2a30	fb1a60	f37a40	f6ba10	f37a10	#
fe2a60	fb1a30	f67a40	f3ba10*	f3ba10	
fe2a90	fb1ac0	f97a40	fcba10	f3ea10	#
fe2ac0	fb1a90	fc7a40	f9ba10	f3da10	#
fe3510*	fb3540	fb3540	fe3510*	fe3510	
fe3520	fb3520	f73540	f73510*	f73510	
fe3540	fb3510*	fe3540	fb3510*	fb3510	
fe3580	fb3580	fd3540	fd3510*	fd3510	
fe3a10*	fb3a40	fb3a40	fe3a10*	fe3a10	
fe3a20	fb3a20	f73a40	f73a10*	f73a10	
fe3a40	fb3a10*	fe3a40	fb3a10*	fb3a10	
fe3a80	fb3a80	fd3a40	fd3a10*	fd3a10	
fe4530	fb8560	f3e540	f6d510	f97510	#
fe4560	fb8530	f6e540	f3d510	f9b510	#
fe4590	fb85c0	f9e540	fcd510	f9e510	#
fe45c0	fb8590	fce540	f9d510*	f9d510	
fe4a30	fb8a60	f3ea40	f6da10	f97a10	#
fe4a60	fb8a30	f6ea40	f3da10	f9ba10	#
fe4a90	fb8ac0	f9ea40	fcda10	f9ea10	#
fe4ac0	fb8a90	fcea40	f9da10*	f9da10	
fe6510	fb9540	fb6540	fe9510*	fe9510	
fe6520	fb9520	f76540	f79510*	f79510	
fe6540	fb9510*	fe6540	fb9510*	fb9510	
fe6580	fb9580	fd6540	fd9510*	fd9510	
fe6a10	fb9a40	fb6a40	fe9a10*	fe9a10	
fe6a20	fb9a20	f76a40	f79a10*	f79a10	
fe6a40	fb9a10*	fe6a40	fb9a10*	fb9a10	
fe6a80	fb9a80	fd6a40	fd9a10*	fd9a10	
fe8530	fb4560	f3d540	f6e510	fc7510	#
fe8560	fb4530	f6d540	f3e510	fcb510	#
fe8590	fb45c0	f9d540	fce510*	fce510	
fe85c0	fb4590	fcd540	f9e510	fcd510	#
fe8a30	fb4a60	f3da40	f6ea10	fc7a10	#
fe8a60	fb4a30	f6da40	f3ea10	fcba10	#
fe8a90	fb4ac0	f9da40	fcea10*	fcea10	
fe8ac0	fb4a90	fcda40	f9ea10	fcda10	#
fe9510	fb6540	fb9540	fe6510*	fe6510	
fe9520	fb6520	f79540	f76510*	f76510	
fe9540	fb6510*	fe9540	fb6510*	fb6510	
fe9580	fb6580	fd9540	fd6510*	fd6510	
fe9a10	fb6a40	fb9a40	fe6a10*	fe6a10	
fe9a20	fb6a20	f79a40	f76a10*	f76a10	
fe9a40	fb6a10*	fe9a40	fb6a10*	fb6a10	
fe9a80	fb6a80	fd9a40	fd6a10*	fd6a10	
fec510*	fbc540	fbc540	fec510*	fec510	
fec520	fbc520	f7c540	f7c510*	f7c510	
fec540	fbc510*	fec540	fbc510*	fbc510	
fec580	fbc580	fdc540	fdc510*	fdc510	
feca10*	fbca40	fbca40	feca10*	feca10	
feca20	fbca20	f7ca40	f7ca10*	f7ca10	
feca40	fbca10*	feca40	fbca10*	fbca10	
feca80	fbca80	fdca40	fdca10*	fdca10	