Doctoral Dissertation

Essays on the effects of fiscal and monetary policy

September, 2021

Graduate School of Social Sciences

Hiroshima University

SHUO LIU

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Abstract

Japan's economy has been suffering from sluggish economic growth and recession since the early 1990s when the stock market bubble burst. Some economists, like Paul Krugman (1999) have argued that Japan's Great Recession exemplifies liquidity traps. Hence, policymakers are struggling to stimulate aggregate demand using conventional monetary stimulus, such as reducing short-term interest rates given to the zero lower bound constraint. From the classical Keynesian viewpoint, the effective stimulus for liquidity trap is expansionary fiscal policy. However, one of the major issues of the expansionary fiscal policy is that, even if it is effective in alleviating economic slowdowns, it increases the government's budget deficit and hence raises the debt-to-GDP ratio. This thesis simulates the effects of fiscal and monetary policy by employing a DSGE model built by Eggersson and Krugman (2011), a calibrated macroeconomic model from Ball (2006) and a medium scale DSGE model based on Christiano et al. (2005) in an attempt to investigate the effectiveness of fiscal and monetary policy. The main findings are as follows. Fiscal expansions can suppress the increase in the debt-to-GDP ratio when the short-term nominal interest rate is zero and the negative economic shock is temporary, due to its ability of raising the inflation rate. With respect to the effectiveness of monetary policy, the simulation shows that the monetary shocks still affect the economic variables and further stimulate the economy even though the effects are not as significant as without the zero lower bound constraint on nominal interest rates.

Keywords: Liquidity trap, Fiscal policy, Monetary policy, Debt-to-GDP ratio.

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CHAPTER I

Introduction

1.1 Background

Liquidity traps can be broadly defined as phenomena in which an increased money supply fails to lower interest rates. In a liquidity trap, bonds are nearly equivalent to cash due to low or close-to-zero short-term nominal interest rates which result in conventional monetary policies have no effect on stimulating economic growth. Liquidity trap literature is typically linked to Keynes (1936) and Hicks (1937) who firstly proposed that there was a positive floor to interest rates. This floor can be interpreted as a shift in Keynes's model, in which the long–term interest rate is related to opportunity costs in money demand, with the expectation that long-term rates are inelastic. In an IS-LM model, a monetary expansion has no effect on equilibrium interest rates and output while fiscal expansion increases the level of output without changing interest rates.

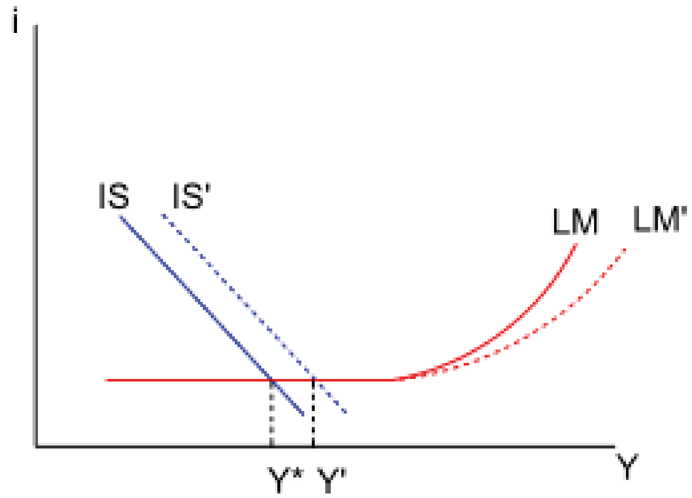

FIGURE 2.1 THE LIQUIDITY TRAP DIAGRAM

However, in the 1930s and 1940s, many neoclassical economists have pursued the solution of mitigating the effect of liquidity trap conditions. Such as Patinkin (1948) and Metzler (1951), they referred to the hypothesis that the incentive of output and employment that resulted from inducing consumption due to an increase in real balances of wealth so called “Pigou effect”, in which the “wealth effect” would self-correct the economy to falls in aggregate demand through a rise in current real balances. As a result, stimulate the economy by conducting monetary policy would be possible even in liquidity trap conditions. In addition, modern monetary theorists such as Friedman, Schwartz, Meltzer and others argued that monetary policies such as quantitative easing, publicly committing to nominal income targets and charging on excess bank reserves could be effective even when the economy is in a liquidity trap. In their view, liquidity traps are more related to low nominal GDP growth rather than low inflation and they doubted whether liquidity traps really existed as any interest rate different from zero could be a sufficient condition to obviate the existence of a liquidity trap.

Nevertheless, a period of prolonged slumps in Japan led to a revival in liquidity trap theory at the end of the last century. After the collapse of asset price bubble burst in the early 1990s, Japan’s economy experienced a deep prolonged slump and deflation so called “lost decade”. Real GDP growth unceasingly dropped, from a peak of 6.8% in 1988 to -0.5 in 1993(Figure 1.2), averaging 1.0% per year during 1991-2002, compared to the G7 average of 2.4%. Krugman (1999) holds that Japan’s Great Recession is an example of a liquidity trap. A slump in anticipated growth rate caused a large imbalance between saving and investment which in further led to a negative real interest rate. Moreover, expectations of inflation are low due to the belief that central bank will take action against inflation, as a result, the monetary stimulus fails to reduce the real interest rate enough far to recover the economy.

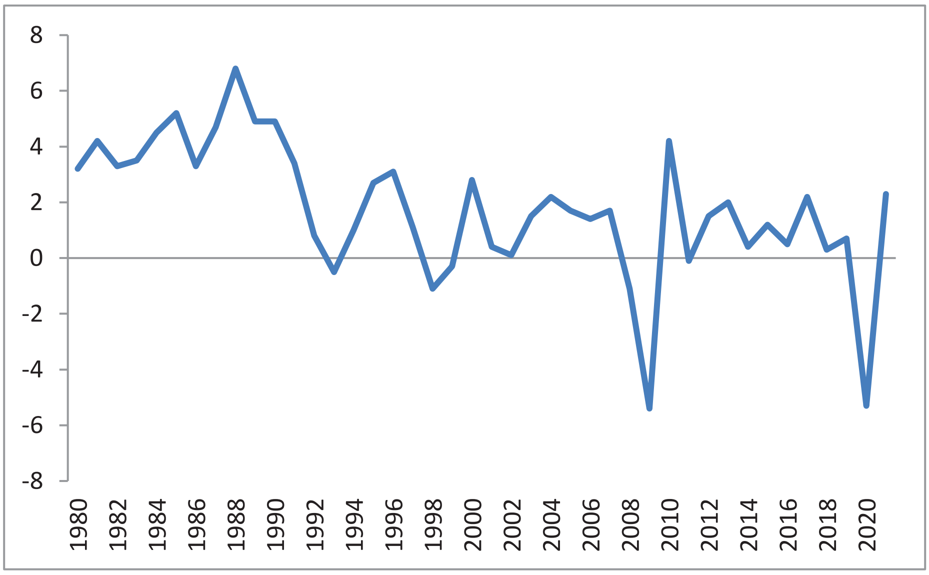

FIGURE 1.2 REAL GDP GROWTH OF JAPAN (ANNUAL PERCENTAGE CHANGE)

Source: International Monetary Fund

The concept of liquidity traps in the 1990s mainly referenced the presence of zero interest rates, since it was believed that interest rates could not drop below zero as banks would hold onto cash instead of paying a fee to deposit it. However, this no longer holds true since liquidity traps again occurred in the wake of the 2008 financial crisis and ensuing Great Recession, especially in western economies. In 2009 and 2010, Sweden and, in 2012, Denmark instituted negative interest rates to stem hot money flows into their economies. The European Central Bank in 2014 resorted to a negative interest rate policy that only applied to bank deposits to prevent European economies from economic stagnation. Further, the Bank of Japan (BoJ) followed in January 2016, employed a negative interest rate policy in which the central bank charges commercial banks a fee of 0.1 percent on a portion of their excess reserves intended to lift consumer prices and mitigate the effects of the recession. Therefore, a liquidity trap currently refers to the circumstance in which the nominal short-term interest rate is at or near zero, and is otherwise known as the Zero Lower Bound Problem.

The classic Keynesian response to a liquidity trap is fiscal expansion. In a textbook IS-LM model, expansionary fiscal policy leads to higher aggregate demand and economic growth and creates inflation as well as shown in Figure 1.2. The main debate about the effectivity of fiscal policy is on crowding out. Monetarists reject conventional wisdom that government borrowing doesn’t increase overall economic activity and would push up interest rates and in further resulting in crowding out private sector investment. Keynesians respond by arguing that an increase in government spending would not cause crowding out effect in a liquidity trap because there is no change in the interest rates associated with the change in government spending, therefore no investment cut off. On the opposite, in a liquidity trap, the excess increase in savings indicates that instead of being invested the private sector resources are being saved, hence by encouraging economic activity the private investment would be “crowded in”.

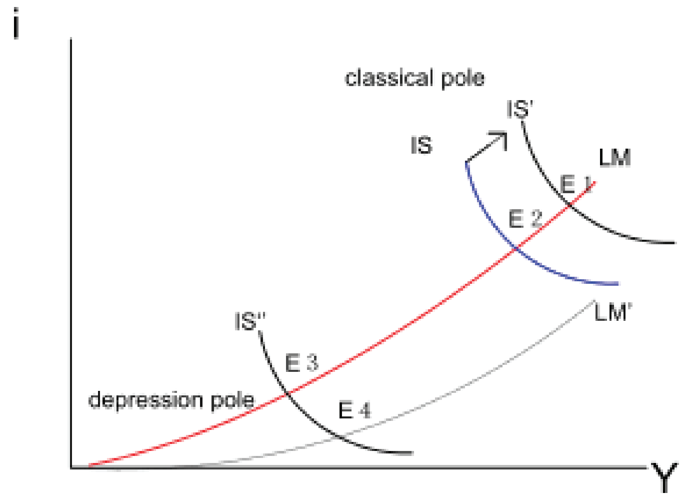

FIGURE 1.3 CLASSICAL SOLUTION FOR A LIQUIDITY TRAP

However, there are two main reasons that make fiscal expansion a hesitant decision for Japan's policymakers. The first reason is Japan's severe fiscal deficit issue. The pattern depicted in Figure 1.3 shows national government debt undergoing severe deterioration, government debt as percentage of GDP has been growing constantly and sharply since last 1990s. It rose from 66.9% in 1990 to 157% in 2003 and is continuously climbing. At the end of 2020, the general government gross debt ratio reached around 266.2% of GDP. Compared to other major advanced economies (those of the G7), Japan's general government gross debt is twice more than the major advanced countries average and about 100% more than the second worst country, Italy.

More recently, given that economic activity and prices are projected to remain under downward pressure for a prolonged period due to the unpredictable impact of COVID-19, forcing the Japanese government to implement a series of emerging economic packages to stimulate the economy since February 2020. This extra government spending is likely to exacerbate the country's fiscal deficit further.

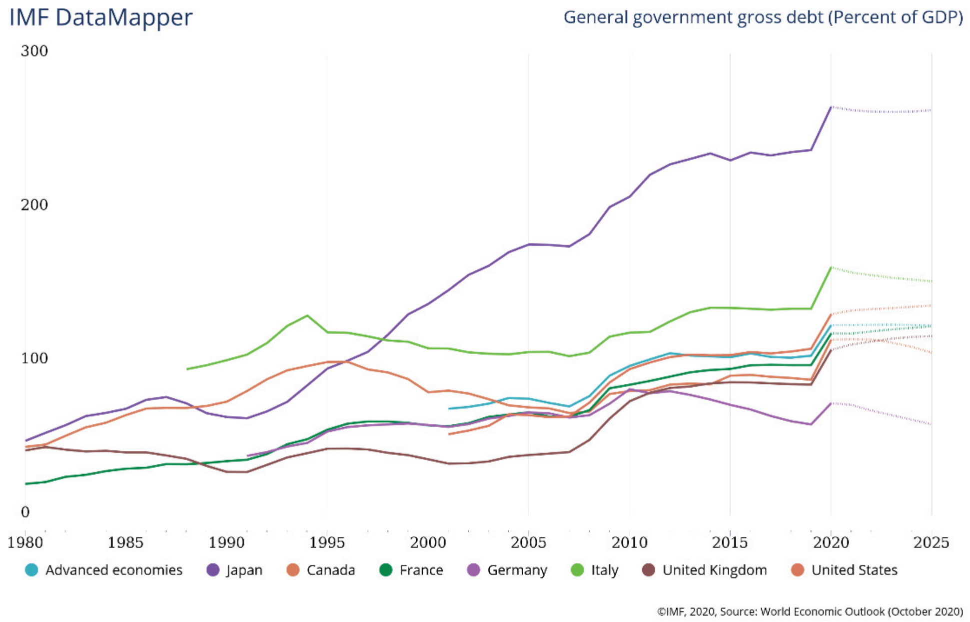

FIGURE 1.4 MAJOR ADVANCED ECONOMICS (G7) GENERAL GOVERNMENT GROSS DEBT AS PERCENTAGE OF GDP

The second reason is the debate on the effectiveness of expansionary fiscal policies. Unlike most of the other advanced economies in which the fiscal stimulus packages were adopted after the global great recession in 2008, the Japanese government had implemented substantial fiscal stimulus since 1992 including a large policy tax reduction and changes in public investment. Nevertheless, the fiscal stimulus policy of the 1990s failed to return the economy to earlier pace of growth and achieve continuing positive inflation. Japan's experience in the 1990s is cited by some economists as an empirical evidence of fiscal ineffectiveness. For instance, Morsink and Bayoumi (2001), Miyazaki (2009) argued that the effect of fiscal stimulus on output that adopted by Japanese government is limited.

Whereas, in the debate over the effectiveness of fiscal policy, there is another hypothesis that the economic decline would have even deteriorated without fiscal stimulus package. Kuttner and Posen (2001) estimated multipliers for fiscal policy in Japan and found that fiscal stimulus is not as harmful as policymakers think. This idea was further alluded to in Krugman' paper (1998), which proposed that expansionary fiscal policy could effectively help Japan's economy get rid of liquidity trap.

In addition to the debate about the feasibility of fiscal policy, some literature has put forward unconventional monetary policy ideas such as quantitative easing, forward guidance, and collateral adjustments to cope with economic crisis that caused by liquidity trap. One of the more discussed unconventional monetary policies is quantitative easing (QE), which aims to reduce long-term interest rates by purchasing long-term securities and paying for them by increasing reserve balances. Empirical evidence suggests that QE has indeed been effective. For instance, Gagnon, Raskin, Remache, and Sack (2010) present an event-study of QE that documents large reductions in interest rates on dates associated with positive QE announcements. Swanson (2010) shows confirming event study evidence from the 1961 Operation Twist, where the Fed/Treasury purchased a substantial quantity of long-term Treasuries. Although the effects of QE have been empirically confirmed, its transmission mechanism is less clear. The literature has concentrated on two main channels of transmission. One is a signaling channel, which works through lowering market expectations about future policy rates (Bauer and Rudebusch (2011), Christensen and Rudebusch (2012)). Another is a supply-induced portfolio balance channel arising from reductions in the supply of the purchased asset available to market participants (Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011)).

1.2 Purposes

This research aims to examine the effectiveness of fiscal and monetary stimulus when the economy is chronically trapped in a liquidity trap by employing three different approaches based on Ball (2006), Eggersson & Krugman (2011) and a classical medium scale DSGE model.

The first model uses a standard classical IS-LM framework in which the assumptions and value of parameters came from Ball (2006) and Jinushi, Kuroki and Miyao (2002), with data based on the circumstances of Japan’s economy in 2003 to explore the effect of fiscal expansion on economic recovery under a zero lower bound condition. Furthermore, to investigate the robustness of fiscal expansion under the various circumstances, the initial data is updated to 2013.

The second approach considers a generalization of a dynamic general equilibrium model (DSGE) with both “impatient” agents and “patient” agents at each point in time in an environment of low interest rates and deflationary pressures. Different from the chapter II, the natural interest rates are assumed as an endogenous valuable in the model and the interest rate transmission mechanism caused by the deleveraging shock can be clearly observed through the model.

The last approach employs a medium scale DSGE model with sticky prices, sticky wage setting and adjustment costs of investment to investigate the impact of monetary shocks. The monetary policies are assumed to be two types, one follows typical Taylor rules and another one is money growth rule. Furthermore, to analyze the effectiveness of monetary policies under the ZLB constraint, I employ Holden and Paetz's (2012) algorithm (the HP algorithm) to deal with the ZLB condition and compare it with the result without ZLB constraint.

1.3 Related literature

1.3.1 Studies on effects of fiscal and monetary policy in Japan

On the issue of fiscal policy for Japan's economic recovery, Kuttner and Posen (2001) found that the fiscal policy had significant expansionary effects on macroeconomic activity in Japan. The results of their paper indicate that tax cuts have mainly been associated with spending increases, and GDP generally increases by more than twice the amount of the spending and tax impact. Furthermore, spending shocks in Japan have had smaller effects than tax cuts but have been more consistent: the four-year cumulative effect of a 10% spending shock is nearly 33%. Thus, fiscal expansion in the form of both tax cuts and of spending increases on public works is effective. Even if Ricardian effects are present, they are not large enough to neutralize the effects of fiscal stimulus even in a country like Japan with a rapidly rising public debt.

However, some literature argues that fiscal policy generates limited effects on boosting the Japan's economy. Perri (2001) built a DSGE model for the case of Japanese economy and found that the net fiscal expansionary effects were not significant. This idea was further alluded by Iwata (2009), who employed a medium scale DSGE model to investigate the effectiveness of fiscal stimulus. His research indicated that the government spending indeed had a positive impact on output but only had appeared in a short term.

For discussions on how monetary policy should best be used in response to liquidity trap, Bernanke and Gertler (1999) imply that inflation-targeted monetary policy will deliver the desired results. Seeing as money, unlike other forms of government debt, pays zero interest and possesses infinite maturity, money can be issued as much as necessary and the created money could be used to acquire indeterminate quantities of goods and assets. Bernanke (2000) further suggests that numerous aggressive policies, including targeting long-term interest rates, currency depreciation, an inflation target of 3%-4%, and a money-financed fiscal expansion.

Reischneider and Williams (2000) use the FRS/US model with aggregate demand growth, inflation rate growth and a zero lower bound Taylor rule in order to quantify the bound's effect on macroeconomic stability. Their simulations demonstrate that in a low inflation scenario, if policy follows the Taylor Rule, the zero bound could constitute a substantial constraint on policy. In addition, they disagree that the zero bound has a significant impact on inflation variability, even with a 0% inflation target. Krugman (1998) studied Japan's experience and argued that even when nominal interest rates are zero, monetary policy can still affect long-term real rates, and hence aggregate demand and output. Moreover, Jinushi, Kuroki and Miyao (2000) examined a time-series analysis using Svensson's (1997) model of inflation forecast targeting to demonstrate that Japanese monetary policy of the late 1980s destabilized the real economy and unnecessarily intensified business fluctuations. Hence, they suggest that instrument independence should be avoided and that a flexible framework which allocates proper consideration to output stability is advisable.

1.3.2 Review of the determination of optimal monetary policy

Before presenting the estimation models, this subsection reviews the question of how central banks can implement optimal monetary policy in order to maintain economic stability. The mainstream manner by which central banks conduct policy does not focus on achieving a target growth rate for money stock, but on changing the short-term nominal interest rate in response to various disturbances. Taylor rules are simple monetary policy rules proposed by Taylor (1993) that prescribe how a central bank should adjust its regular interest rate policy in response to changes in inflation, output and other macroeconomic activities. More precisely, the Taylor Principle explicates how, each one percentage point increase in inflation should be met by central banks with a greater than one percentage point increase in the normal interest rate. Taylor's proposed rule is linear both in inflation and the output's percentile departure from its natural rate.

$$i - \pi_t = a + b\pi_t + c(lnY_t - ln\bar{Y}) \quad (1.1)$$

Assuming $\bar{r}$ is the real interest rate which is constant over time, and prevails when $Y_t = \bar{Y}$, then the formula is equivalent to

$$i - \pi_t = \bar{r} + b(\pi_t - \pi^*) + c(lnY_t - ln\bar{Y}) \quad (1.2)$$

Where $\pi^* = (\bar{r} - a)/b$. So as to respond to the situation in which inflation and output exceed their targets, the central bank should increase the real interest rate above its long-run equilibrium level.

Due to their flexibility and wide range of alternative monetary policies, Taylor rules have been used to examine a variety of policy schemes, including money growth targeting and inflation targeting.

On the question of how inflation, output and the natural rate should be measured, Orphanides (2003) considers the case for applying the basic Taylor rule with Taylor's coefficients to data on inflation and output. He estimates $\bar{Y}$, which was available to policymakers in the 1970s.

Moreover, Svensson (1996) and Ball (1997) consider proposals concerning policy strategies which are available for analysis. Ball (1997) considers several implications of monetary policy using Taylor rules.

The economy is described by two equations:

The aggregate demand equation:

$$y_t = -\beta r_{t-1} + \rho y_{t-1} + \varepsilon_t^D, \qquad \beta > 0, 0 < \rho < 1 \quad (1.3)$$

where y_t is the gap between output and potential output, r is the difference between the real interest rate and its equilibrium level. The natural rate of output and the long-term real interest rate are assumed to be zero.

The aggregate supply equation:

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + \varepsilon_t^s, \quad \alpha > 0 \quad (1.4)$$

where π_t is the difference between inflation and its average level. ε_t^D and ε_t^s are shocks that engender disorder for aggregate demand and aggregate supply, and are assumed to be independent from each other.

The aggregate demand equation conveys that output is negatively affected by the previous period's real interest rate. The aggregate supply equation states that a change in inflation is positively affected by the previous period's output. Due to lags in policy effects, a change in the real interest rate could not impact output until the following period and hence would not impact inflation until the period after that.

The interest rate affects the economy because it determines the expected output for the following period, taking the expectation of y in period t and π in period t+1:

$$E_t[y_{t+1}] = -\beta r_t + \rho y_t \tag{1.5}$$

$$E_t[\pi_{t+1}] = \pi + \alpha y_t \tag{1.6}$$

Furthermore, the paths of inflation and output are dependent on $E_t[y_{t+1}]$, $E_t[\pi_{t+1}]$ and future shocks. Thus, optimal policy takes the formula:

$$E_t[y_{t+1}] = -qE_t[\pi_{t+1}] \tag{1.7}$$

where the value of q is to be determined.

In period t+1, aggregate demand can be written as:

$$y_{t+1} = -\beta r_t + \rho y_t + \varepsilon_{t+1}^D \tag{1.8}$$

Taking the expectation for the squares of both sides of aggregate demand in period t+1, and since $E_t[\varepsilon_{t+1}^D] = 0$, it yields

$$E_t[y_{t+1}] = -\beta r_t + \rho y_t \tag{1.9}$$

Substituting (1.8) into (1.9):

$$y_{t+1} = E_t[y_{t+1}] + \varepsilon_{t+1}^D \tag{1.10}$$

Similarly, in period t+1, the equation for aggregate supply can be written as:

$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}^s \tag{1.11}$$

Taking the expectation for the squares of both sides of aggregate supply in period t+1, and $E_t \varepsilon_{t+1}^S = 0$ results in:

$$E_t[\pi_{t+1}] = \pi_t + \alpha y_t \tag{1.12}$$

Substituting equation (1.12) into (1.11):

$$\pi_{t+1} = E_t[\pi_{t+1}] + \varepsilon_{t+1}^s \tag{1.13}$$

Equation (8) and (11) imply that in period t, y_t and π_t equal:

$$y_t = E_{t-1}[y_t] + \varepsilon_t^D, \quad \pi_t = E_{t-1}[\pi_t] + \varepsilon_t^s \tag{1.14}$$

Substituting y_t and π_t into (1.12):

$$E_t[\pi_{t+1}] = E_{t-1}[\pi_t] + \alpha E_{t-1}[y_t] + \varepsilon_t^s + \alpha \varepsilon_t^D$$

Furthermore, from equation (1.7) we know that: $E_{t-1}[y_t] = -qE_{t-1}[\pi_t]$ substituting this into equation (1.14):

$$E_t[\pi_{t+1}] = (1 - \alpha q)E_{t-1}[\pi_t] + \varepsilon_t^s + \alpha\varepsilon_t^D \tag{1.15}$$

Taking expectations for the squares of both sides of (1.8):

$$E[(E_t[\pi_{t+1}])^2] = (1 - \alpha q)^2 E[(E_{t-1}[\pi_t])^2] + \sigma_S^2 + \alpha^2\sigma_D^2$$

σ_S^2 and σ_D^2 are the variances of ε_t^D and ε_t^s. In the long run the distribution of $E_{t-1}[\pi_t]$ will remain constant over time and independent of the economy's initial conditions. Therefore, the expectations of $(E_t[\pi_{t+1}])^2$ and of $(E_{t-1}[\pi_t])^2$ are equal. Thus,

$$E[(E_t[\pi_{t+1}])^2] = E[(E_{t-1}[\pi_t])^2]$$

$$= (1 - \alpha q)^2 E[(E_{t-1}[\pi_t])^2] + \sigma_S^2 + \alpha^2\sigma_D^2$$

$$E[(E_{t-1}[\pi_t])^2] = E[(\pi_t - \varepsilon_t^s)]^2 = \frac{\sigma_S^2+\alpha^2\sigma_D^2}{\alpha q(2-\alpha q)} \tag{1.16}$$

Given that $\pi_t = E_{t-1}[\pi_t] + \varepsilon_t^s$, the expectation for $E_{t-1}[\pi_t]$ can be written as:

$$E[\pi_t^2] = \sigma_S^2 + E[(E_{t-1}[\pi_t])^2] \tag{1.17}$$

Substituting (1.16) into (1.17):

$$E[\pi_t^2] = \frac{\sigma_S^2 + \alpha^2\sigma_D^2}{\alpha q(2 - \alpha q)} + \sigma_S^2 \tag{1.18}$$

Similarly, as $y_t = E_{t-1}[y_t] + \varepsilon_t^D$, $E_{t-1}[y_t] = -qE_{t-1}[\pi_t]$ and given that the mean of y is zero and y^* is the preferred level of output. Then,

$$E[(y - y^*)^2] = y^{*2} + q^2 E[(E_{t-1}[\pi_t])^2] + \sigma_D^2$$

$$= y^{*2} + q^2 \frac{\sigma_S^2 + \alpha^2\sigma_D^2}{\alpha q(2 - \alpha q)} + \sigma_D^2 \tag{1.19}$$

To find q which minimizes $E[(y - y^*)^2] + \lambda E[\pi_t^2]$ where λ is a positive parameter reflecting the relative weighting placed on inflation. Substituting (18) and (19) into $E[(y - y^*)^2] + \lambda E[\pi_t^2]$:

$$E[(y - y^*)^2] + \lambda E[\pi_t^2] = y^{*2} + q^2 \frac{\sigma_S^2 + \alpha^2\sigma_D^2}{\alpha q(2 - \alpha q)} + \sigma_D^2 + \lambda[\frac{\sigma_S^2 + \alpha^2\sigma_D^2}{\alpha q(2 - \alpha q)} + \sigma_S^2]$$

$$= y^{*2} + \frac{(q^2 + \lambda)(\sigma_S^2 + \alpha^2\sigma_D^2)}{\alpha q(2 - \alpha q)} \sigma_D^2 + \lambda\sigma_S^2$$

Differentiating the above function with respect to q:

$$\frac{d(E[(y - y^*)^2] + \lambda E[\pi_t^2])}{dq} = 0 \tag{1.20}$$

One solution can be excluded with:

$$\frac{d(E[(y - y^*)^2] + \lambda E[\pi_t^2])}{dq} = \frac{2\alpha(q^2 + \alpha\lambda q - \lambda)(\sigma_S^2 + \alpha^2\sigma_D^2)}{(2\alpha q - \alpha^2 q^2)^2}$$

Since $\alpha > 0, \sigma_S^2 + \alpha^2\sigma_D^2 > 0$ and $(2\alpha q - \alpha^2 q^2)^2 \geq 0$ then, in order to make $\frac{d(E[(y-y^*)^2]+\lambda E[\pi_t^2])}{dq} = 0$, $q^2 + \alpha\lambda q - \lambda$ must equal to zero.

$$q^* = \frac{\pm\sqrt{\alpha^2\lambda^2 + 4\lambda} - \alpha\lambda}{2} \tag{1.21}$$

Because a negative q causes variation in y and π to be infinite, the remaining solution is

$$q^* = \frac{\sqrt{\alpha^2\lambda^2 + 4\lambda} - \alpha\lambda}{2} \tag{1.22}$$

Equation (1.22) demonstrates how the optimum level of q varies with respect to λ, and how much weighting the central bank places on stabilizing inflation. Because q^* is directly proportional to λ, equation (1.22) implies that as the central bank allocates a greater weighting to inflation stability, output further deviates from its natural rate level in order to return inflation to its optimal level. In addition, when λ approaches infinity, q^*approaches $\frac{1}{\alpha}$. If policy takes advantage of this relationship between λ and q^*, inflation can quickly be brought back down to zero following a shock. Even if the central bank is only concerned with inflation, this measure will ensure output remains close to its natural rate level so as to protect against sizeable movements in inflation.

In determining exactly how central bank policy take into account the interest rate, Ball (1999) claims that optimal policy can be explained as a form of inflation targeting and hence follows Taylor rules, since, $E_{t+1}[\pi_{t+2}] = (1 - \alpha q)E_t[\pi_{t+1}] + \varepsilon_{t+1}^S + \alpha\varepsilon_{t+1}^D$, $E_t[\pi_{t+2}]$ is equal to $(1 - \alpha q)E_t[\pi_{t+1}]$. Because q is between 0 and $\frac{1}{\alpha}$, $(1 - \alpha q)$is hence between 0 and 1 and therefore, this form of expected inflation can be expressed as:

$$E_t[\pi_{t+2}] = \phi E_t[\pi_{t+1}] \tag{1.23}$$

where ϕ is between 0 and 1. Given the range of q, this expression can range from zero to one. Equation (1.23) is a partial-adjustment variation on an inflation target, implying that the more the central bank is concerned with inflation, the faster it can undo changes in inflation.

It makes sense that a partial-adjustment rule would constitute optimal policy. Policymakers try to return inflation to its target level, but in changing inflation, they face the problem of a deviation in output. Since policymakers who value the output stability attach little value to the parameter q, equation (1.23) implies that they gradually adapt inflation to bring it closer to its target level and as the variance of inflation increases, the variance of output decreases.

In addition, Ball (1999) proposes that inflation target is an efficient policy, based on the premise that inflation target allows partial adjustment.

As $E_t[y_{t+1}] = -\beta r_t + \rho y_t, E_t[y_{t+1}] = -qE_t[\pi_{t+1}]$, the following relationship can be obtained:

$$r_t = \frac{1}{\beta}(\rho y_t + q^* E_t[\pi_{t+1}])$$

$$= \frac{\rho + q^*\alpha}{\beta} y_t + \frac{q^*}{\beta}\pi \quad (1.24)$$

This equation is a Taylor rule: it shows that the real interest rate responds positively to output and inflation, and does not rely on any other variable.

However, many economists advocate nominal-income targeting. Economists like McCallum (1993) and Hall and Mankiw (1994) suggest that a policy featuring nominal GDP targeting would engender a better outcome and more stable output than is possible with inflation targeting. In Ball's paper (1999), this conclusion is not reached, instead he argues that policy targeting nominal income is grossly inefficient.

Income-growth policy minimizes variance in income growth, taking the assumption that expected income growth for the next period is equal to a fixed target. Since this type of policy affects output with a period lag, income growth in one period can hence be influenced. Furthermore, in deviating from the trend, income growth is the sum of output growth $y_{t+1} - y$, and inflation π_{t+1}. Therefore, this policy can be defined by:

$$E[y_{t+1} - y + \pi_{t+1}] = 0 \quad (1.25)$$

Substituting equation (1) and (2) into (24):

$$E[y_{t+1} - y + \pi_{t+1}] = E[-\beta r_t + \rho y_t + \varepsilon_{t+1}^D - y_t] + E[\pi_t + \alpha y_t + \varepsilon_{t+1}^s]$$

$$= E[-\beta r_t + (\rho - 1)y_t + \varepsilon_{t+1}^D] + E[\pi_t + \alpha y_t + \varepsilon_{t+1}^s] = 0$$

Since $E[y_{t+1}] = -\beta r_t + \rho y_t, E[\pi_{t+1}] = \pi_t + \alpha y_t$

Thus, the value of r can be derived as:

$$r_t = \left(\frac{\alpha + \lambda - 1}{\beta}\right) y_t + \frac{1}{\beta}\pi_t \quad (1.26)$$

Even though the output coefficient need not be positive, this policy is a Taylor rule variant. Equation (1.26) implies that the interest rate rule dictates how output and inflation behave. Substituting (1.26) into (1) yields:

$$y_t = (1 - \alpha)y_{t-1} - \pi_{t-1} + \varepsilon_t^D \quad (1.27)$$

Utilizing equation (1.27) and the Phillips curve (1.2) to define a vector AR-1 process for output and inflation yields:

$$Y = CY_{t-1} + E \tag{1.28}$$

where $Y = [y\ \pi]'$, $E = [\varepsilon_t^D\ \varepsilon_t^S]'$, C is a 2× 2 matrix with elements $c_{11} = 1 - \alpha, c_{12} = -1, c_{21} = \alpha$ and $c_{22} = 1$.

The eigenvalues for C come from solving:

$$\begin{vmatrix} c_{11} - \lambda & c_{12} \\ c_{21} & c_{22} - \lambda \end{vmatrix} = 0$$

Substituting the value of each element into the above equation:

$$\begin{vmatrix} 1 - \alpha - \lambda & -1 \\ \alpha & 1 - \lambda \end{vmatrix} = 0$$

Thus, the eigenvalues satisfy:

$$\lambda_1 + \lambda_2 = trC = 2 - \alpha, \quad \lambda_1\lambda_2 = |C| = 1$$

Solving the value of λ_1 and λ_2 :

$$\lambda_1 + (\alpha - 2)\lambda_1 + 1 = 0$$

$$\lambda = \frac{2 - \alpha \pm \sqrt{\alpha^2 - 4\alpha}}{2}$$

The behavior of y and π is determined by the eigenvalues for C. If $\alpha < 2$, the eigenvalues are complex and lie on the unit circle. If $\alpha > 4$, one eigenvalue is less than -1, so then the processes involved are explosive. Hence, output and inflation are unstable and their variances are infinite. This analysis proves that nominal income targeting is disastrous and highly inefficient.

However, this analysis fails to account for two relevant issues. Firstly, the tradeoff between output and inflation may fluctuate with the pace of adaptation. Secondly, the analysis pays no attention to the problem of possible parameter unpredictability. Rather, the model provides a way of analyzing which factors policymakers should consider and the optimal monetary policy.

1.3.3 Overview of Japan's monetary and fiscal policy responses to recession

1.3.3.1 Japan's monetary policy responses to recession

In response to Japan's recession, the monetary strategies adopted by the BoJ can be generalized as zero interest-rate policies and quantitative easing. According to the specific practices of policy operations, the recession can be divided into certain distinct periods:

1. The zero interest-rate policy period (1999.2-2001.3)

To combat economic stagnation and serious issues with bad debt, in February 1999, the BOJ implemented what was equivalent to a zero interest-rate policy. Furthermore, the BOJ governor announced that the zero rate will remain policy at least “until deflationary concerns subside”. In 2000, this policy was working and the economy experienced brief signs of recovery, however, after 2001, Japanese economy was not continuing to recover but shockingly began deteriorating significantly. The consumer prices index (CPI) was down to -0.7 and the 2002 GDP growth rate to -0.3. The bank’s capital losses soared and companies’ equipment investment declined. To prevent further deterioration of the economy, the BOJ decided to cut the official discount rate from 0.5% to 0.355%.

2. The “quantitative easing” period (2001.3-2006.3)

In March 2001, The BOJ decided to adopt looser monetary policy. The specific measures included: (1) Changing the monetary policy target from interest rates to money supply overnight, in particular, targeting the outstanding balance of banks’ current accounts of held at the BoJ and increasing financial institutions’ deposit account surpluses through open market operations to increase money supply and provide the market with liquidity. (2) Raising the central bank’s current account from 4 trillion to 5 trillion yen. (3) Continuing to implement loose monetary policy until the CPI registers a stable zero percent or at least a year on year increase.

3. A temporary exit from the zero-interest rate and quantitative easing policy (2006.3-2008.12)

In 2005, Japan’s macroeconomic performance was looking good, it grew more than 5% in three of the year’s quarters, higher than the rate in the United States and the European Union for the same period. The core consumer price index was also growing from November 2005 onwards, the Japanese economy temporarily exited its deflationary state. As a result, in March 2006, BOJ announced an end to its five-year-old quantitative easing policy. Furthermore, in July 2006, the BOJ announced that the zero interest rate policy was to end and raised the benchmark interest rate to 0.25%.

4. The return of the zero-interest rate and quantitative easing policy (2008.12-2016.1)

In 2008, Japan’s economy failed to sustain its momentum of growth from 2005, achieving negative growth at a rate of 1.6%, while the domestic CPI index further remained at a low level. In the context of the global financial crisis, the BOJ cut interest rates again, this time to 0.1% and encouraged the uncollateralized overnight call rate to stay below 0.1%, in effect adopting a zero interest rate policy once again.

5. The negative interest rate policy period (2016.1-)

In January 2016, the BOJ took the unexpected decision of introducing negative interest rates in a fresh bid to spur lending and investment. The BOJ joined the ECB, Denmark, Sweden and Switzerland as the only central banks pushing interest rates below the zero bound.

From the practice of monetary policies in Japan, this is not a surprise since the BOJ began its zero-bound policy in 1999, although it continuously adjusted its basic approach, it had since maintained a substantive zero-bound policy and quantitative easing until the BOJ introduced policy below the zero bound.

The BOJ's monetary policies are mainly concerned with three effects: first of all, the asset allocation rebalancing effect; the central bank provides non-interest-bearing assets which have a high level of security for financial institutions, expecting that financial institutions could actively use them for loans, bonds or securities investments in an effort to comprehensively stimulate enterprise production and household consumption. Secondly, the expectation effect; the increase in money invested is likely to give rise to economic revival, helping people shake off long-term gloom. Thirdly, the announcement effects; the government promises to keep constant zero bound and quantitative easing policies for a prolonged period. According to the theory of interest rate term structure, long-term interest rates are equal to the average short-term interest rate forecast plus a risk premium. Thus, in accordance with the government's promise, reducing the long-term interest rate forecast and interest rate risk reward would further diminish long-term interest rates, thereby realizing the goal of raising asset prices and promoting both production and household consumption.

6. Special countermeasures for COVID-19 shocks (2020.6-)

To counter the impact of unpredictable recession caused by COVID-19, BoJ has implemented the following monetary stimulus policies:

First, BoJ announced to apply a negative interest rate of minus 0.1 percent to the Policy-Rate Balances in current accounts held by financial institutions at the Bank of Japan and purchase a necessary amount of Japanese government bonds (JGBs) without setting an upper limit so that 10-year JGB yields will remain at around zero percent, aiming to provide more monetary liquidity and stimulate the economic recovery.

Further, BoJ continues with “Quantitative and Qualitative Monetary Easing (QQE) with Yield Curve Control”, intending to achieve the price stability target of 2 percent. The expansion is expected to be consistently applied until the year-on-year rate of increase in the observed consumer price index (CPI) exceeds 2 percent and stays above the target in a stable range.

In general, Japanese monetary policies after the economy collapsed in recession were not as fit-for-purpose as hoped, especially because before the bank’s bad debts were figured out, a significant amount of excess reserves did not enter the lending market. However, although the expected effects were not well achieved, monetary policy implementation did succeed in stabilizing expectation. Moreover, the announcement effects played a role in keeping the interest rate on 10-year treasury bonds below 2%, lowering the expectation for long-term interest rates. These policies have prevented the Japanese economy from sliding further into its liquidity trap.

1.3.3.2 Japan's fiscal policy responses to recession

In response to the recession, Japan’s fiscal stimulus policies were mainly focused in two periods: August 1992 to September 1995 and April 1998 to October 2000.

Given the severe economic downturn during the period from 1990 to 1995, the Japanese government began announcing fiscal stimulus packages including a sizable tax reduction to against the low pace of the real GDP growth rate in 1992. The magnitude of the stimulus efforts with the effect of the depression on tax revenue can be observed in the increase in government debt, which has climbed as much as twice as a share of GDP over the 1990s. In the phase of fiscal expansion, the government focused the expenditure effort on public investment with a majority of the announced spending for infrastructure projects. The tax policy was less bold until later in the 1990s. In 1994, the government announced an expansionary stimulus including a large tax reduction policy resulting in the reduction of income taxes by 5.9 trillion yen. This losses in revenue were anticipated to be made up by higher value-added taxes (VAT) in the future. As a result of the expectation of higher future consumption taxes, a short-run stimulation achieved—output growth averaged 4.5 percent at an annual rate in the two quarters preceding the VAY increase.

After 2000, due to the skyrocketing financial deficit issue, the further fiscal expansionary policy was waived. As the economy started to observably recover since 2006, sharp cuts in government spending and a recovery in revenues have resulted in a sharp narrowing of the deficit.

Key measures include additional loan support for companies and business, financial support for healthcare system and consumption promotion campaign. The total scale of the package announced on 7th April 2020 is 117 trillion yen which is equivalent to 22 percent of the GDP.

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CHAPTER II

Analysis on the fiscal policy for Japan's recovery: Ball's (2006) approach

2.1 Introduction

This chapter will investigate the effects of the expansionary fiscal policy in a liquidity trap and predict Japan's economic trends by using a calibrated textbook-style macroeconomic model designed by Ball (2006) and the data collected for 2003 and 2013. The first section of the chapter derives the model employing standard IS-LM equations. Section 3.2 calibrates the parameters of the model while section 3.3 presents the results of the simulation. Section 3.4 concludes.

2.2 Model

First of all, consider a dynamic IS curve:

$$\frac{(\mathrm{Y_t} - \mathrm{Y_t^*})}{\mathrm{Y_t^*}} = \lambda \left[\frac{(\mathrm{Y_{t-1}} - \mathrm{Y_{t-1}^*})}{\mathrm{Y_{t-1}^*}}\right] - \beta(\mathrm{r_{t-1}} - \mathrm{r_{t-1}^*}) + \delta \frac{\mathrm{G_{t-1}}}{\mathrm{Y_{t-1}^*}} \tag{2.1}$$

$$\lambda > 0, \beta > 0, \ \delta > 0$$

where Y is real output, $Y^*$is potential real output, assumed to grow by g percent per year and $\frac{(\mathrm{Y_t}-\mathrm{Y_t^*})}{\mathrm{Y_t^*}}$ is the real output gap. $\mathrm{r_t}$ is the real rate that equals to $i_t - \pi_t$ according to the Fisher effect, where i_t is the normal rate and π_t is inflation. $\mathrm{r_t^*}$ is the "neutral" interest rate. G is the real transfer from the government; the nominal transfer is PG where P is the price level. Equation (2.1) explains that the output gap is determined by the lagged gap, the lagged real interest rate and lagged transfers.

Equation (2.2) is an accelerated Phillips curve. The inflation rate is determined by:

$$\pi_t = \pi_t^e + \alpha \left[\frac{(\mathrm{Y_{t-1}}-\mathrm{Y_{t-1}^*})}{\mathrm{Y_{t-1}^*}}\right]$$

$$\alpha > 0 \tag{2.2}$$

where π_t^e is expected inflation and permanently equals π_{t-1} unless the inflation is negative. When π_{t-1} is negative, π_t^e is 0, which means the level of inflation is determined solely by output. Equation (2.2) explains that this year's output has no effect on this year's inflation. The price level is given by

$$P_t = (1 + \pi_t)P_{t-1} \tag{2.3}$$

From equation (2.1), (2.2) and (2.3): $\mathrm{Y_{t-1}}$ determines π_t, and π_t determines P_t. To finish the model, I follow the assumption of Ball (2006), assuming that the BoJ chooses a target interest rate i_t^T, and follows a Taylor rule, which is that there is dependency on the output gap and inflation until the interest rate hits zero. Let potential output Y^* be the BOJ's real output target and π^* be the inflation target. The nominal interest rate target i_t^T is captured by:

$$i_t^T = r^* + \pi_t + a\left[\frac{(\mathrm{Y_{t-1}} - \mathrm{Y_{t-1}^*})}{\mathrm{Y_{t-1}^*}}\right] + b(\pi_t - \pi^*), \qquad a > 0, b > 0 \tag{2.4}$$

The central bank sets the interest rate target $i_t^T = 0$, if the right-hand side of equation the above equation is negative, and sets it as i_t^T if i_t^T is positive. With positive policy parameters a and b, the BOJ reduces the interest rate target when output falls below its target level, and raises the interest rate target if inflation rises above its target level. The real interest rate i_t is determined by the supply and demand for money. The central bank controls the stock of base money by conducting open market operations and the demand for base money is given by:

$$\ln\left(\frac{M_t^D}{P_t Y_t}\right) = k - \gamma i_t$$

$$\frac{M_t^D}{P_t Y_t} = \exp(k - \gamma i_t)$$

$$\gamma > 0, i_t \geq 0 \tag{2.5}$$

Let M_t be the high-powered money supply, money supply has effect on the interest rate i_t and so the real interest rate r_t, which taking into consideration equation (2.1), (2.2) and (2.3) respectively, affects next year's output, inflation and the price level.

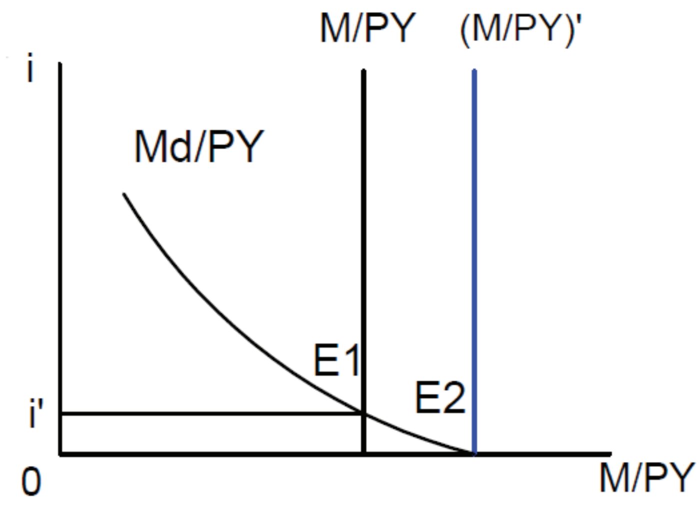

FIGURE 2.1 MONEY DEMAND AND MONEY SUPPLY

The real interest rate i is determined by the intersection of the Md/PY curve and the vertical M/PY line. When M/PY=0, the central bank increases M/PY, the vertical M/PY line shifts right in Figure 2.1 and the intersection i falls below the Md/PY curve. Thus, M/PY effects the intersection i until i hits zero. The value of i_t is given by:

$$i_t = \frac{k - \ln\frac{M_t}{P_t Y_t}}{\gamma}, \qquad i_t \geq 0 \tag{2.6}$$

and to obtain $i_t \geq 0$, the M/PY is given by:

$$\frac{M_t}{P_t Y_t} = \exp(k - \gamma i_t), \quad i_t \geq 0 \tag{2.7}$$

To make $i_t = 0$, the value of $\frac{M_t}{P_t Y_t}$ must equals exp(k). For any $\frac{M_t}{P_t Y_t}$ larger than exp(k), i_t will remain at 0, because lenders will not lend when i_t is below zero. Thus, the central bank adapts the real money supply M_t using open market operations to make the real interest rate i_t equal the interest rate target i_t^T.

The money supply equation can be described by:

$$M_t = M_{t-1} + Z_t \tag{2.8}$$

where Z_t is central-bank purchases of government bonds. When Z_t is below zero, it means that the government sells bonds.

Ball (2006) neglects the separate balance sheets of the government and central bank because Japan's fiscal problem is measured using privately held debt, which is not included in the debt held by the BOJ. The nominal debt D_t is given by:

$$D_t = D_{t-1} + i_{t-1}D_{t-1} + P_tG_t - Z_t - \theta(P_tY_t - P_tY_t^*) \quad (2.9)$$

The nominal debt consists of four parts: the past debt's interest payment; current nominal transfers; open market purchases and the government's primary surplus in the absence of transfers, which is assumed to be zero when output equals potential output. It ignores the situation when the government's primary surplus is negative.

2.3 Calibration

In the Phillips equation, the value of the Phillips curve slope α is 0.2, which is estimated based on the BOJ's study.

In the interest rate target equation, to decide coefficients a and b, the Taylor rule implies that the purpose of policy is to let inflation return to its target level at a fixed rate. Thus, in choosing coefficients a and b, one must assume that inflation closes in on its target at a pace of 50% each period. As the result, a=1.1, b=2.5. The inflation target π^* is assumed to be 2%, which is in accordance with most countries' targets.

In the money-demand equation, the interest rate semi-elasticity γ is 0.1, based on the work of Jinushi, Kuroki and Miyao (2002). The parameter k is set to $\ln(0.1)$, based on historical evidence using Japanese economic data from 1993 to 2003. In 1998, when the interest rate became zero, the monetary base's value was 0.1, thus $k = \ln(0.1)$.

In the debt equation, the primary surplus's effect on output θ is 0.25, which also comes from Kuttner and Posen's estimates.

There is a discussion on the value of Japan's neutral real interest rate. As Ball's perspective, the neutral real interest rate r^* is usually negative but will remain so in perpetuity. In the early 2000s, the real interest rate r was 1%, but output Y was far below its potential rate Y^* . As r^* makes $Y = Y^*$ when there is no fiscal expansion, this implies that r^* must be below 1%. To satisfy the situation when output remains at a low level, Ball assumes an initial $r^*$of -2 percent, which means $r - r^* = 3$ percent and the initial output gap will be -7.5 percent which satisfies the condition. However, r^*will not remain negative forever. The fall in r^*in the 1990s caused the IS equation to shift in and demand to rise. The end result was economic recovery, the IS equation returning to its natural position and r^*finally becoming positive. Therefore, I assume r^* to eventually rise to +2 percent, and that this linear progression -2% to +2% will occur over ten years.

Parameter	Value
IS	
β	1
λ	0.6
δ	1.25
θ	0.25
Phillips Curve	
α	0.2
Interest Rate Target	
a	1.1
b	2.5
π^*	0.02
Money Demand	
ϒ	0.1
k	ln(0.1)
Potential Output	
g	0.02
Neutral rate	
r^{*1}	-0.02
r^{*2}	0.02

TABLE 2.1 THE VALUES FOR THE PARAMETERS IN EACH EQUATION IN CHAPTER II

2.4 The results

2.4.1 Baseline

The initial conditions for simulations are based on Japan's circumstances in 2003. It is presented in Table 2.2.

	Circumstances
Output gap	–7.5%
Inflation	–1.0%
Nominal interest rate	0
Base/GDP	0.20
Debt/GDP	0.79

TABLE 2.2 INITIAL CIRCUMSTANCE

Figures 2.2 showcase the paths of important variables: the output gap, i, and the ratios of Z, M and D to GDP.

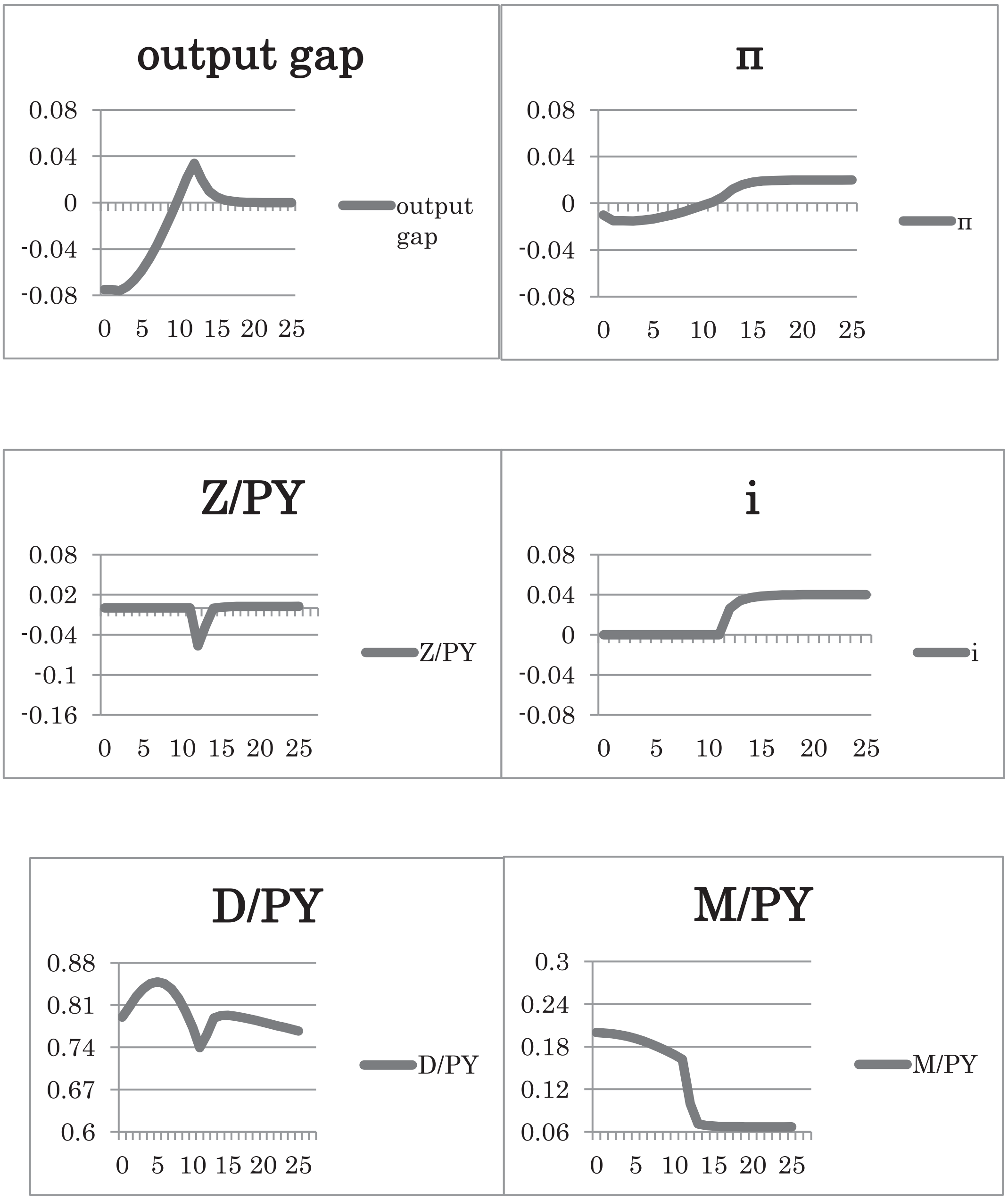

FIGURE 2.2 THE BASELINE FOR EACH MACROECONOMIC VARIABLES

In period zero, the output gap starts at -7.5% and recovers slowly, eventually reaching zero and then becoming positive from year ten onwards.

Inflation falls in the first few years then starts to rise from year four, it reaches zero in year eleven and remains positive after then. Because of the no negative interest rate assumption, the nominal interest rate in Figure 2.2 remains at zero until year twelve. The recovery forces the Taylor rule interest rate to become positive and from when the rule comes into effect, inflation reaches its 2% target at a steady pace, resulting in an output gap greater than zero.

While the interest rate is zero, money stock is constant, but because output grows faster than the price level falls, the monetary base line is not horizontal but slightly downward sloping. When the interest rate becomes positive in year twelve, the money-GDP ratio falls sharply. This phenomenon is the result of open market purchases Z being -6% of GDP as shown in Figure 2.2. This action is explained by the level of monetary base being high in period zero. Even if the money-GDP ratio declines greatly in year eleven, it will still be above the position required to yield a positive interest rate.

The ratio of debt-GDP increases at first, reaching its peak in year five. But as the economy recovers, it begins to fall, reaching its minimum in year twelve. This is because of large monetary shrinking. Then, the debt-GDP ratio rises because the BoJ sells government bonds, causing privately held debts to rise. Next, the debt-GDP ratio falls smoothly and since $r = i - \pi$ equals 25t, which is equivalent to output potential growth g, interest payments and income are finally balanced.

2.4.2 Data update

This section updates the 2003 data used in the previous section and revalidates the model using 2013 data.

Compared with the 2003 situation, Japan's economy was experiencing recovery in 2013, the output gap was still but had risen from -7.5 percent to -2.23 percent, with inflation becoming positive. However, government debt underwent constant deterioration in 2013. The initial circumstances of 2013 are summarized in Table 2.3.

Circumstances	
Output gap	-2.23%
Inflation	0.17%
Nominal interest rate	0
Base/GDP	0.17
Debt/GDP	1.4

TABLE 2.3 INITIAL CIRCUMSTANCES IN 2013

The results are shown in Figure 2.3.

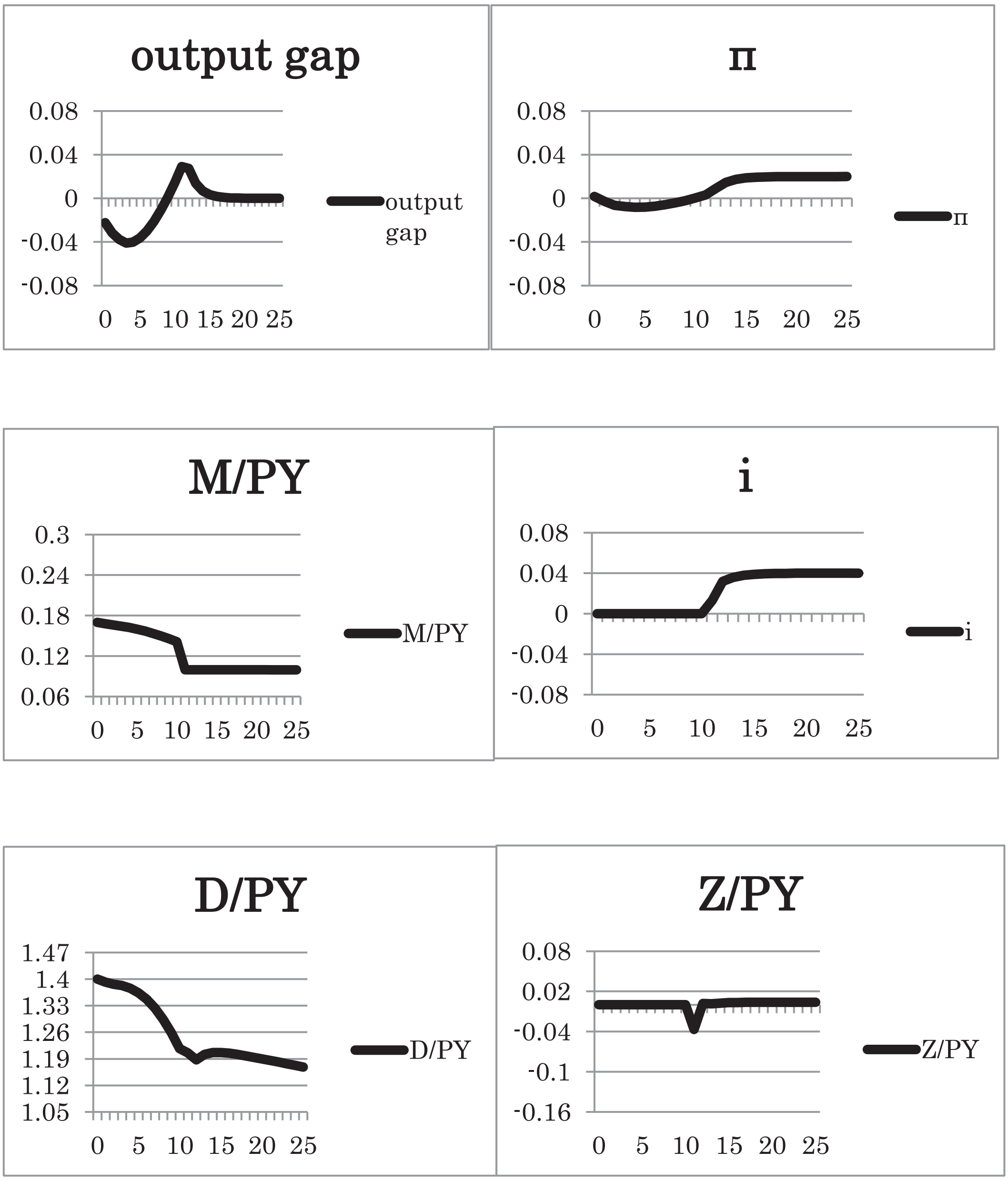

FIGURE 2.3 THE BASELINE FOR EACH MACROEONOMICS VARIABLES IN 2013

As shown in the above figure, the trends are consistent with those of 2003, with recovery at a mildly faster pace than that of 2003.

In period zero, the output gap starts at -2.3 percent and begins to rise. It reaches zero and becomes positive in year nine. Inflation falls in the first few years then starts to rise, reaching zero in year ten and becoming positive from then on. When the interest rate becomes positive in year eleven, the money-GDP ratio falls sharply, just as it did in the simulation for 2003. The ratio of debt-GDP firstly rises but since the economy is recovering, it reaches its minimum in year twelve. Then, the debt-GDP ratio falls smoothly. These results show that even when the data is changed, a fiscal expansion remains a viable option, spurring economic recovery and even reducing debt.

2.5 Conclusion

This chapter used a calibrated textbook-style macroeconomic model designed by Ball (2006) to examine the Japanese economy and predicted Japanese economic trends. The assumptions and value of parameters came from Ball' (2006) and Jinushi, Kuroki and Miyao (2002), with data based on the circumstances in 2003. The simulation results tended to concur with the case for fiscal expansion. In assuming that monetary policy follows a Taylor rule once the interest rate turn positive, potential GDP rises, and the interest rate also become positive. After recovery, the Taylor rule leads the economy on a path of steady potential output and inflation. Furthermore, because of high growth and inflation, the debt-income ratio declines. Even after updating the data to 2013, the conclusion remained the same. The results mostly supported Kuttner and Posen' s view that fiscal stimuli would not worse public debt and would in fact provide relief for the government debt problem.

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CHAPTER III

Analysis on the fiscal policy in a liquidity trap: under the assumption of Eggersson and Krugman (2012)

3.1 Introduction

When the economy is stuck at zero lower bound, a negative demand shock such as the financial crisis will cause the economic recession and increase unemployment rate. As the short-term nominal interest rate cannot be lowered anymore, the conventional monetary policies have limited effect on stimulating economic growth. Therefore, in this case, the expansionary fiscal policies are expected to provide stimulus to generate a robust economic recovery. However, the weakness of the fiscal expansion that it may cause the increase in public debt to GDP ratio has been hotly argued for a long time. Hence, policymakers are facing a trade-off between unemployment rate and public debt to GDP ratio.

This chapter aims to examine whether there is a trade-off between unemployment and public debt to GDP ratio. The result indicates that in the case of temporary negative demand shocks such as the financial crisis, the trade-off relationship between unemployment and debt to GDP ratio does not significantly exist.

The reason that the debt-to-GDP ratio does not rise even if the unemployment rate is lowered by fiscal expansion is as follows: Some empirical studies have reported that deflation raises the debt-to-GDP ratio. Suppose that the economy with zero inflation and stable debt to GDP ratio was hit by a negative demand shock, in the case that fiscal expansion policy is not adopted, the economy will fall into recession and the unemployment rate will increase. Furthermore, Inflation will decline from zero to negative which in further resulted in the rise of the public debt to GDP ratio. In contrast, the effectiveness of implementing expansionary fiscal policies is significant: the economic downturn will ease and the unemployment rate will not increase prominently. In addition, fiscal expansion will curb the decline in inflation, reduce the extent of deflation and the increase in debt to GDP ratio is unconspicuous.

Thus, compared to the case without fiscal expansion, by avoiding the decline in deflation, expansionary fiscal stimulus has impact on reducing both the rise in the unemployment rate and the rise in public debt to GDP ratio. In other words, in this case, the trade-off between unemployment and debt to GDP ratio does not occur.[1]

3.2 Model

An economy is in a liquidity trap if the short-term interest rate is near its effective lower bound and the economy is still in a recession. It appears that many industrialized countries were in a liquidity trap after the 2008 financial crisis. Figure 1 shows the short-term nominal interest rate and output gap in the euro area, Japan, the United Kingdom, and the United States. The output gaps in these economies were negative in the early 2010s even though the short-term interest rates were near zero.

The rationale for a liquidity trap relies on two conditions. The first is that the natural real interest rate falls below zero. The second is nominal rigidities (i. e., price and/or wage stickiness).[2] In recent years, there has been numerous discussion and studies on macroeconomic policies under the circumstance of zero lower bound, see, for example, Krugman (1998), DeLong and Summers (2012), Eggertsson and Woodford (2003), Blanchard (2019), and Lukasz and Summers (2019), among others. In this study, we establish the model based on Eggertsson and Krugman (2012) with the endogenousized natural rate of interest to estimate the debt-to-GDP ratio.[3]

3.2.1 Households

Households are divided into two types: the borrower and the saver. Assume that the number of households in the entire household is set to 1, the number of borrowers is $\chi_b = \chi$, while the number of the saver is $\chi_s = 1 - \chi$. First of all, Borrower's household seeks to maximize a discounted intertemporal objective function:

$$E_0 \sum_{t=0}^{\infty} \delta^t \left[u^b \left(C_t^b \right) - v^b \left(h_t^b \right) \right] \tag{3.1}$$

[1] End, Tapsoba, Terrier, and Duplay (2015) provides empirical evidence on the impact of deflation on public debt ratios. For a discussion of Japanese economy, see Ball (2006) and Blanchard and Tashiro (2019).
[2] Eggertsson and Egiev (2020).
[3] See also Curdia and Woodford (2010, 2016).

Maximize the budget constraints of borrowers which are given by

$$B_t = \left(1 + i_{t-1}^b\right)B_{t-1} + P_t C_t^b - P_t W_t h_t^b - \int I_t^b(i)di - \int Z_t^b(i)di - P_t F_t^b + P_t T_t^b \tag{3.2}$$

where B_t is the borrower's debt (when B_t is positive it means in debt), i_t^b is the nominal interest rate on borrowing from banks, C_t^b is the quantity consumption of borrowers, h_t^b represents labor and W_t is real wages. δ is the discount rate of borrowers and $0 < \delta < 1$. Furthermore, $I_t^b(i)$ is profits from the financial intermediaries distributed to the borrowers, $Z_t^b(i)$ is profits from the firms, F_t^b is revenues from "fraud" and T_t^b represents taxes.

Similarly, the saver's utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u^s(C_t^s) - v^s(h_t^s)] \tag{3.3}$$

The budget constraints are given by

$$D_t = \left(1 + i_{t-1}^d\right)D_{t-1} - P_t C_t^s + P_t W_t h_t^s + \int I_t^s(i)di + \int Z_t^s(i)di + P_t F_t^s - P_t T_t^s \tag{3.4}$$

where D_t is deposits (a positive D_t means holding the deposits), i_t^d is the interest rate the household gets on deposits it has at "banks". The rest of the notation is symmetric to the borrowers' and β is the discount factor of savers where $0 < \delta < \beta < 1$. Here, Ricardian equivalence does not hold because liquidity constraints invalidate the assumed lifetime income hypothesis.

The relationship between deposit rates and lending rates is

$$1 + i_t^b = \left(1 + i_t^d\right)(1 + \omega_t) \tag{3.5}$$

The spread ω_t is given by

$$\omega_t = \omega\left(\frac{B_t}{P_t}, \frac{M_t}{P_t}, (\frac{b}{Y})^j\right) \quad j = high, low \tag{3.6}$$

where B_t the individual saver's debt, M_t represents aggregate private nominal debt that the agents treat as exogenous value to their private decisions. The spread between deposit rates and lending rates ω_t is an increasing function on $b_t \equiv \frac{B_t}{P_t}$ and $m_t \equiv \frac{M_t}{P_t}$, scilicet $\omega_b > 0$ and $\omega_m > 0$. Additionally, we assume that $\omega_{bw} = 0$ for simplicity. The shock in this model is an unexpected fall from the debt levels that $(\frac{b}{Y})^{high}$ to debt levels that are considered safe $(\frac{b}{Y})^{low}$. A fall in debt levels which are considered safe will result in higher interest rate differentials ω_t.

Optimizing the utility function of borrowers implies the following necessary conditions for a rational-expectations equilibrium:

$$\mathcal{L}_0^b = E_0 \sum_{t=0}^{\infty} \delta^t \, [u^b(C_t^b) - v^b(h_t^b)$$

$$+ \lambda_t^b \left\{ B_t - (1 + i_{t-1}^d)\left(1 + \omega\left(\frac{B_{t-1}}{P_{t-1}}, \frac{M_{t-1}}{P_{t-1}}, b^{H,L}\right)\right) B_{t-1} - P_t C_t^b + P_t W_t h_t^b \right.$$

$$\left. + \int I_t^b(i) di + \int Z_t^b(i) di + P_t F_t^b - P_t T_t^b \right\}]$$

$$\frac{\partial \mathcal{L}_t^b}{\partial C_t^b} = u_c^b(C_t^b) - P_t \lambda_t^b = 0 \Rightarrow u_c^b(C_t^b) = P_t \lambda_t^b$$

$$\frac{\partial \mathcal{L}_t^b}{\partial h_t^b} = -v_h^b(h_t^b) + P_t W_t \lambda_t^b = 0 \Rightarrow v_h^b(h_t^b) = P_t W_t \lambda_t^b$$

Differentiate the equation with respect to B_t

$$\frac{\partial \mathcal{L}_t^b}{\partial B_t} = \frac{\partial \delta^t \{\lambda_t^b [B_t - (1 + i_{t-1}^d)(1 + \omega_{t-1}(\cdot) B_{t-1})]\} + \delta^{t+1}\{\lambda_{t+1}^b [B_{t+1} - (1 + i_t^d)(1 + \omega_t(\cdot) B_t)]\}}{\partial B_t}$$

$$= \delta^t \lambda_t^b + \delta^{t+1} \lambda_{t+1}^b B_t \left(-(1 + i_t^d)\frac{\omega_b}{P_t}\right) + \delta^{t+1} \lambda_{t+1}^b [-(1 + i_t^d)(1 + \omega_t)] = 0$$

$$\Longrightarrow \lambda_t^b - \delta E_t \lambda_{t+1}^b (1 + i_t^d)[\frac{B_t \omega_b}{P_t} + (1 + \omega_t)] = 0$$

$$\Longrightarrow \lambda_t^b = \delta E_t \lambda_{t+1}^b (1 + i_t^d)[\frac{B_t \omega_b}{P_t} + (1 + \omega_t)]$$

Combining the first and third equations:

$$\begin{cases} u_c^b(C_t^b) = P_t \lambda_t^b \\ \lambda_t^b = \delta E_t \lambda_{t+1}^b (1 + i_t^d)[\frac{B_t \omega_b}{P_t} + (1 + \omega_t)] \end{cases}$$

$$\frac{u_c^b(C_t^b)}{u_c^b(C_{t+1}^b)} = \frac{P_t \lambda_t^b}{P_{t+1} \lambda_{t+1}^b} = \frac{\lambda_t^b}{\Pi_{t+1} \lambda_{t+1}^b}$$

$$\frac{u_c^b(C_t^b)}{u_c^b(C_{t+1}^b)} = \frac{\delta E_t \lambda_{t+1}^b (1 + i_t^d)[\frac{B_t \omega_b}{P_t} + (1 + \omega_t)]}{\Pi_{t+1} \lambda_{t+1}^b}$$

Hence,

$$u_c^b(C_t^b) = \delta E_t u_c^b(C_{t+1}^b) \frac{1 + i_t^b}{\Pi_{t+1}} \left[\frac{B_t}{P_t} \omega^b + (1 + \omega_t)\right] \tag{3.7}$$

Similarly,

$$\mathcal{L}_0^s = E_0 \sum_{t=0}^{\infty} \beta^t \, [u^s(C_t^s) - v^s(h_t^s)$$

$$+ \lambda_t^s \Big\{ D_t - \left(1 + i_{t-1}^d\right) D_{t-1} + P_t C_t^s - P_t W_t h_t^s$$

$$- \int I_t^s(i) di - \int Z_t^s(i) di - P_t F_t^s + P_t T_t^s \Big\}]$$

$$\frac{\partial \mathcal{L}_t^s}{\partial C_t^s} = u_c^s(C_t^s) + P_t \lambda_t^s = 0 \Rightarrow u_c^s(C_t^s) = -P_t \lambda_t^s$$

$$\frac{\partial \mathcal{L}_t^s}{\partial h_t^s} = -v_h^s(h_t^s) - P_t W_t \lambda_t^s = 0 \Rightarrow v_h^s(h_t^s) = -P_t W_t \lambda_t^s$$

$$\frac{\partial \mathcal{L}_t^s}{\partial D_t} = \lambda_t^s - \beta E_t \lambda_{t+1}^s \left(1 + i_t^d\right) = 0 \Rightarrow \lambda_t^s = \beta E_t \lambda_{t+1}^s \left(1 + i_t^d\right)$$

$$\frac{u_c^s(C_t^s)}{u_c^s(C_{t+1}^s)} = \frac{-P_t \lambda_t^s}{-P_{t+1} \lambda_{t+1}^s} = \frac{\beta E_t \lambda_{t+1}^s (1 + i_t^d)}{\Pi_{t+1} \lambda_{t+1}^s} = \frac{\beta E_t (1 + i_t^d)}{\Pi_{t+1}}$$

Thus,

$$u_c^s(C_t^s) = \frac{\beta E_t \left(1 + i_t^d\right) u_c^s(C_{t+1}^s)}{\Pi_{t+1}} \qquad (3.8)$$

The optimal labor supply for each type of households is given by

$$u_c^b\left(C_t^b\right) = P_t \lambda_t^b$$

$$v_h^b\left(h_t^b\right) = P_t W_t \lambda_t^b$$

$$W_t = \frac{v_h^b\left(h_t^b\right)}{u_c^b\left(C_t^b\right)} \qquad (3.9)$$

and

$$u_c^s(C_t^s) = -P_t \lambda_t^s$$

$$v_h^s(h_t^s) = -P_t W_t \lambda_t^s$$

$$W_t = \frac{v_h^s\left(h_t^s\right)}{u_c^s\left(C_t^s\right)} \qquad (3.10)$$

3.2.2 Firms

The C_t of each type refers to the Dixit-Stiglitz aggregator of preference for goods is given by

$$C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}} \qquad (3.11)$$

with c_t(i) representing the quantity of good i consumed by the household in period t. By maximizing C_t for any given level of expenditures $\int_0^1 p_t(i) c_t(i) di$, it shows that each firm faces a demand equation:

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t \tag{3.12}$$

The firms maximize profits given by

$$Z_t(i) = (1-\tau)p_t(i)y_t(i) - P_t W_t h_t(i) \tag{3.13}$$

where τ represents subsidy. Each good i has a production function

$$y_t(i) = A_t h_t(i)$$

where A_t is a exogenous technology factor. Knowledge grows at a constant rate:

$$A_t = A_0 e^{gt}$$

where g is the rate of technological progress.

The price adjustment follows Calvo's (1983) assumption, that in each period, each firm has an equal probability of reconsidering its price with a probability $(1-\alpha)$ where$0 < \alpha < 1$ and the ability to reconsider its price is independent across firms and period. The adjusted price is set to p_t^*. In addition, firms are assumed to be owned and managed by the savers. Each firm maximizes profits by choosing the optimal price p_t^*. This assumption implies that

$$P_t = \left[(1-\alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.14}$$

We also assume that the firms are controlled by the savers. Each firm chooses a price p_t^* to maximize profits

$$maxE_0\left\{\sum_{t=0}^{t}(\alpha\beta)^t\lambda_t^s\left[(1-\tau)p_t^*Y_t(\tfrac{p_t^*}{P_t})^{-\theta} - P_tW_tY_t(\tfrac{p_t^*}{P_t})^{-\theta}\right]\right\} \tag{3.15}$$

The first order condition of the above function implies that an optimal price $\frac{p_t^*}{P_t}$ given by

FOC:

$$\frac{\partial\,\cdot}{\partial p_t^*} = E_0\sum_{t=0}^{t}(\alpha\beta)^t\,\lambda_t^s\left[(1-\tau)(1-\theta){p_t^*}^{-\theta}{P_t}^{\theta-1}Y_t + \theta W_tY_t{p_t^*}^{-\theta-1}{P_t}^{\theta}\right] = 0$$

$${p_t^*}^{-\theta}E_0\sum_{t=0}^{t}(\alpha\beta)^t\,\lambda_t^s(1-\tau)(\theta-1){P_t}^{\theta-1}Y_t = \ {p_t^*}^{-\theta-1}E_0\sum_{t=0}^{t}(\alpha\beta)^t\,\lambda_t^s\theta W_tY_t{P_t}^{\theta}$$

$$p_t^*E_0\sum_{t=0}^{t}(\alpha\beta)^t\,\lambda_t^s(1-\tau)(\theta-1){P_t}^{\theta-1}Y_t = E_0\sum_{t=0}^{t}(\alpha\beta)^t\,\lambda_t^s\theta W_tY_t{P_t}^{\theta}$$

Divide both sides by P_t:

$$\frac{p_t^*}{P_t} = \frac{E_0 \sum_{t=0}^{t} (\alpha\beta)^t\, u_c^s(C_t^s) (\frac{P_t}{P_0})^{\theta} Y_t \frac{\theta}{\theta-1} W_t}{E_0 \sum_{t=0}^{t} (\alpha\beta)^t\, u_c^s(C_t^s)(1-\tau)(\frac{P_t}{P_0})^{\theta-1} Y_t} \tag{3.16}$$

Let $\Delta_t \equiv \int (\frac{p_t(i)}{P_t})^{-\theta} di$, the Calvo's assumption denotes that

$$\Delta_t \equiv \int (\frac{p_t(i)}{P_t})^{-\theta} di$$
$$= (1-\alpha)\, (\frac{p_t^*}{P_t})^{-\theta} + \alpha \int (\frac{p_t(i)}{P_t})^{-\theta} di$$

(the second integral is over firms in the set of nonadjusting firms, of which there are a measure α)

Since $\int (\frac{p_t(i)}{P_t})^{-\theta} di = \int (\frac{P_{t-1}}{P_t} \frac{p_{t-1}(i)}{P_{t-1}})^{-\theta} di = (\frac{p_{t-1}}{P_t})^{-\theta} \Delta_{t-1}$ (because for these firms $P_t(i) = P_{t-1}(i)$)

Thus,

$$\Delta_t = (1-\alpha)(\frac{p_t^*}{P_t})^{-\theta} + \alpha\Delta_{t-1}(\frac{p_{t-1}}{P_t})^{-\theta}$$
$$= (1-\alpha)(\frac{p_t^*}{P_t})^{-\theta} + \alpha\Delta_{t-1}\Pi_t^{\theta} \tag{3.17}$$

3.2.3 Aggregation

The aggregate asset constraint is given by

$$Y_t = \chi_b C_t^b + \chi_s C_t^s + G_t \tag{3.18}$$

where $\chi_b + \chi_s = 1$, G_t is government spending.

The aggregate labor supply is given by

$$h_t = \chi_b h_t^b + \chi_s h_t^s = \int y_t\,(i) di = \int Y_t \frac{p_t(i)}{P_t})^{-\theta} di = Y_t \Delta_t \tag{3.19}$$

We assume that all the firms, banks and fraud profits are given out by the savers. The real value of aggregate debt $b_t = \frac{B_t}{P_t}$ is

$$b_t = \left(1 + i_{t-1}^b\right) b_{t-1} \frac{1}{\Pi_t} + C_t^b - W_t h_t^b + T_t^b \tag{3.20}$$

Finally, we assume that the central bank conducts monetary policy by following "Taylor rule":

$$i_t^d = max\left(0, f(X_t)\right) \tag{3.21}$$

where the vector X_t implies all the exogenous and endogenous variables in the economy. With the above, the model has been interpreted with 13 endogenous variables and 13 equations.

3.2.4 Steady state and equilibrium dynamics

In this model, there is 13 endogenous variables $\left\{i_t^d, i_t^b, \omega_t, b_t, C_t^b, C_t^s, h_t^b, h_t^s, Y_t, W_t, \frac{p_t^*}{P_t}, \Pi_t, \Delta_t\right\}$and 13 equations. The equilibrium conditions are obtained from these equations, and linear approximations are performed around the equilibriums. Monetary and fiscal policies are supposed to be operated with the aim of achieving zero inflation ($\bar{\Pi} = 1$). As a result, from equation (3.14) the steady state is $\overline{\left(\frac{p^*}{P}\right)} = 1$, and from equation (3.8) $\bar{\imath}^d = \beta^{-1} - 1$ holds. Furthermore, the steady state debt $\bar{b}$ from equation (3.7) can be written as the following equation by applying $\bar{b} = \bar{m}$

$$\delta^{-1}\beta = 1 + \omega(\bar{b}, \bar{b}) + \bar{b}\omega_b(\bar{b}, \bar{b})$$

Moreover, the equations $\bar{\imath}^b = \beta^{-1}(1 + \bar{\omega}) - 1$ and $\bar{W} = (1 - \tau)\frac{\theta - 1}{\theta}$ can be acquired from (3.5) and (3.16). For the simplification of signs, we set $\tau = \frac{1}{1-\theta}$ and $\bar{W} = 1$.

The balanced-growth-path values of $Y_t, C_t^b, C_t^s, G_t, T_t^b$ and b_t are $A_t\bar{Y}, A_t\bar{C}^b, A_t\bar{C}^s, A_t\bar{G}, A_t\bar{T}^b$. $A_t\bar{b}$, respectively. We set $\bar{Y} = 1$ for normalization.

As a result of the unexpected shock, the real debt has fallen from the initial equilibrium $(\frac{b}{Y})^{high}$ to the new equilibrium $(\frac{b}{Y})^{low}$. Here the linear approximation is computed around the new equilibrium given by $\overline{\left(\frac{b}{Y}\right)} = (\frac{b}{Y})^{high}$.

3.2.5 Linearization

The relationship between $\hat{\imath}_t^b$ and $\hat{\imath}_t^d$ implies

$$\hat{\imath}_t^b = \hat{\imath}_t^d + \widehat{\omega}_t \tag{3.22}$$

where $\hat{\imath}_t^b \equiv \frac{i_t^b - \bar{\imath}^b}{1+\bar{\imath}^b}$, $\hat{\imath}_t^d \equiv \frac{i_t^d - \bar{\imath}^d}{1+\bar{\imath}^d}$, $\widehat{\omega}_t \equiv \frac{\omega_t - \bar{\omega}}{1+\bar{\omega}}$. Due to $b_t = m_t$, combining with equation (2.6) $\omega_t = \omega(b_t, m_t)$ implies the following equation:

$$\widehat{\omega}_t = \theta\hat{b}_t \tag{3.23}$$

where $\theta \equiv \frac{\omega_b + \omega_m}{1+\bar{\omega}}\bar{b}$, $\hat{b}_t \equiv \frac{b_t - A_t\bar{b}}{A_t\bar{b}} = \frac{\left(\frac{b_t}{A_t}\right) - \bar{b}}{\bar{b}}$.

From equation (3.7) we obtain

$$\hat{C}_t^b = E_t\hat{C}_{t+1}^b - \sigma\left(\hat{\imath}_t^b - E_t\pi_{t+1} + \lambda\widehat{\omega}_t\right) \tag{3.24}$$

Similarly,

$$\hat{C}_t^s = E_t\hat{C}_{t+1}^s - \sigma\left(\hat{\imath}_t^d - E_t\pi_{t+1}\right) \tag{3.25}$$

can be derived from equation (2.8), where

$\lambda \equiv \frac{\omega_b}{\omega_b+\omega_m}\left[\delta\beta^{-1}(1+\bar{\omega}) - \delta\beta^{-1}\bar{b}(\omega_b+\omega_m) + \delta\beta^{-1}\bar{b}\frac{\omega_{bb}}{\omega_b}(1+\bar{\omega})\right]$, $\sigma = -\frac{u_c^b}{u_{cc}^b\bar{Y}} = -\frac{u_c^s}{u_{cc}^s\bar{Y}}$,

$\hat{C}_t^b \equiv \frac{\left(\frac{C_t^b}{A_t}\right)-\bar{C}^b}{\bar{Y}}$, $\hat{C}_t^s \equiv \frac{\left(\frac{C_t^s}{A_t}\right)-\bar{C}^s}{\bar{Y}}$ and $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

The labor supply function (3.9) and (3.10) imply

$$\widehat{W}_t = \nu\hat{h}_t^b + \sigma^{-1}\hat{C}_t^b \tag{3.26}$$

$$\widehat{W}_t = \nu\hat{h}_t^s + \sigma^{-1}\hat{C}_t^s \tag{3.27}$$

where $\nu = \frac{v_{hh}^b\bar{Y}}{v_h^b} = \frac{v_{hh}^s\bar{Y}}{v_h^s}$.

$$\hat{\Delta}_t = 0$$

$$\hat{p}_t^* = \frac{\alpha}{1-\alpha}\pi_t$$

where $\hat{p}_t^* = \frac{p_t^*}{P_t} - 1$.

The New Keynesian Phillips curve can be derived by

$$\pi_t = \gamma\widehat{W}_t + \beta E_t\pi_{t+1} \tag{3.28}$$

where $\gamma \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.[4]

The aggregate asset constraints (3.18) imply

$$\hat{Y}_t = \chi_b\hat{C}_t^b + \chi_s\hat{C}_t^s + \hat{G}_t \tag{3.29}$$

where $\hat{G}_t \equiv \frac{\left(\frac{G_t}{A_t}\right)-\bar{G}}{\bar{Y}}$ and $\hat{Y}_t \equiv \frac{\left(\frac{Y_t}{A_t}\right)-\bar{Y}}{\bar{Y}}$.

Furthermore, the aggregate labor and $\hat{\Delta}_t = 0$ indicate

$$\hat{Y}_t = \chi_b\hat{h}_t^b + \chi_s\hat{h}_t^s \tag{3.30}$$

The budget constraint of borrowers is given by

$$b_y\hat{b}_t = b_y(1+\bar{\imath}^b)\hat{\imath}_{t-1}^b + (1+\bar{\imath}^b)b_y\hat{b}_{t-1} - b_y(1+\bar{\imath}^b)\pi_t + \hat{C}_t^b - \bar{h}^b\widehat{W}_t - \hat{h}_t^b + \hat{T}_t^b \tag{3.31}$$

where $b_y \equiv \frac{\bar{b}}{\bar{Y}}$ and $\hat{T}_t \equiv \frac{\left(\frac{T_t}{A_t}\right)-\bar{T}}{\bar{Y}}$.

Regarding to the monetary policy reaction function, the zero lower bound $i_t^d \geq 0$ means $\hat{\imath}_t^d \geq \frac{-\bar{\imath}^d}{1+\bar{\imath}^d} = \beta - 1$ and

[4] See, for example, Woodford (2003).

$$\hat{i}_t^d = max(\beta - 1, \phi_x \hat{X}_t) \qquad (3.32)$$

Heretofore, endogenous variables and equations are reduced by 2 to 11. The endogenous variables are $\{i_t^d, i_t^b, \omega_t, b_t, C_t^b, C_t^s, h_t^b, h_t^s, Y_t, \pi_t, W_t\}$ and the equations are (3.22)-(3.32). The shock is presented by $\hat{b}_{-1} = \frac{(\frac{b}{Y})_{-1} - \overline{(\frac{b}{Y})}}{\overline{(\frac{b}{Y})}} = \frac{(\frac{b}{Y})^{\text{high}} - (\frac{b}{Y})^{\text{low}}}{(\frac{b}{Y})^{\text{low}}} > 0$, and $(\frac{b}{Y})$, converges to $(\frac{b}{Y})^{\text{low}}$ in the long run.

3.3 Calibration

3.3.1 Calibration of each variables

The parameters on the left side of table 3.1 choose standard values in the related literature. One period of the model is quarterly. The discount factor of the saver, β, is 0.995. The intertemporal elasticity of substitution in consumption, σ, and the inverse of the Frisch elasticity of labor supply, ν, are one. The Phillips curve slope, κ, is 0.02.

To simulate the numerical calculation of Eggertsson and Krugman (2012), the parameters on the right side of the table are set to almost the same values as in the same paper. We assume that $b_y = 4$ and $\hat{b}_{-1} = 0.3$, that is, steady state debt is 100 percent of annual income and the initial value of the debt is 130 percent of annual income.[5] We assume that $i^d = 0.005$ and $i^b = 0.02$ for the steady state interest rates, and the interest rate spread doubles at the time of impact. This implies that $\beta = 0.995$, $1 + i^b = 1.02$, and $\vartheta = 0.049$. The rate of technological progress, g, is 0.005. The parameter of the borrowers' Euler equation, λ, is discussed in Section 5.

Parameter	Value	Parameter	Value
β	0.995	b_y	4
δ	$\delta < \beta$	χ	0.5
κ	0.02	$1 + i^b$	1.02
ν	1	ϑ	0.049
σ	1	λ	1.32
		$\hat{b}_{-1}$	0.3

[5] See Table I in Eggertsson and Krugman (2012).

	γ	0.010
	$\bar{h}^b$	0.9

TABLE 3.1 THE VALUES OF EACH PARAMETERS IN CHAPTER III

3.3.2 Natural rate of interest

It should be noted that natural rate of interest is

$$\hat{r}_t^e = -\chi_b(1+\lambda)\hat{\omega}_t = -\chi_b(1+\lambda)\vartheta\hat{b}_t \tag{3.33}$$

The natural rate of interest is determined endogenously rather than being given exogenously.[6] If individual suddenly considers that the debt level is too high at some point, the interest rate difference between the borrowing rate and the deposit rate will rise. When the interest rate spread becomes large and the borrowing interest rate rises, the borrower begins to repay the debt which means the decrease in the expenditure of the borrower. With the intention of offsetting the decrease in the expenditure of the borrower, it requires expansion of the save's expenditure. However, to achieve this intention, the natural rate of interest would have to descend. As the result of the debt repayment of the borrower gradually progresses, the interest rate spread narrows, the borrower's expenditure recovers, and the natural rate of interest begins to rise to the original level.

3.4 A baseline case

In this section, it describes the economic fluctuation in the absence of the fiscal expansion policy. The economy suffers a deleverage shock during the zero period.

3.4.1 The fiscal policy

[6] The details of the derivation of natural interest rate $\hat{r}_t^e$ are:
the following two equations can be derived from equation (16) and (17)

$$\chi_b\hat{C}_t^b = \chi_b E_t\hat{C}_{t+1}^b - \sigma(\chi_b\hat{\imath}_t^b - \chi_b E_t\pi_{t+1} + \chi_b\lambda\hat{\omega}_t)$$

$$\chi_s\hat{C}_t^s = \chi_s E_t\hat{C}_{t+1}^{sc} - \sigma(\chi_s\hat{\imath}_t^d - \chi_s E_t\pi_{t+1}).$$

When $\hat{G}_t = 0$, by substituting $\hat{i}_t^b = \hat{i}_t^d + \hat{\omega}_t$, the equation is obtained as follows

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t^d - E_t\pi_{t+1} + \chi_b(1+\lambda)\hat{\omega}_t).$$

The budget constraint of the government spending is

$$\frac{B_t^g}{P_t} = (1 + i_{t-1})\frac{P_{t-1}}{P_t}\frac{B_{t-1}^g}{P_{t-1}} + G_t - \chi_s T_t^s - \chi_b T_t^b \tag{3.34}$$

where B_t^g is sovereign debts that the maturity is one period, T_t^s is the tax incomes from the savers, while T_t^b is the tax incomes from the borrowers. As $t = 1, \dots, t'$, $T_t^s = A_t\bar{T}^s$, $T_t^b = A_t\bar{T}^b$, and $A_t\bar{G} - \chi_s A_t\bar{T}^s - \chi_b A_t\bar{T}^b = 0$, it indicates that in equilibrium, the primary balance is zero. Let $i_t = i_t^d$, the following equation obtains[7]:

$$\frac{B_t^g}{P_t} = \left(1 + i_{t-1}^d\right)\frac{P_{t-1}}{P_t}\frac{B_{t-1}^g}{P_{t-1}} + G_t - A_t\bar{G}$$

Consequently,

$$B_t^g = \left(1 + i_{t-1}^d\right)B_{t-1}^g + P_t(G_t - A_t\bar{G})$$

Since $\hat{G}_t \equiv \frac{\left(\frac{G_t}{A_t}\right) - \bar{G}}{\bar{Y}} = 0$

$$B_t^g = \left(1 + i_{t-1}^d\right)B_{t-1}^g \tag{3.35}$$

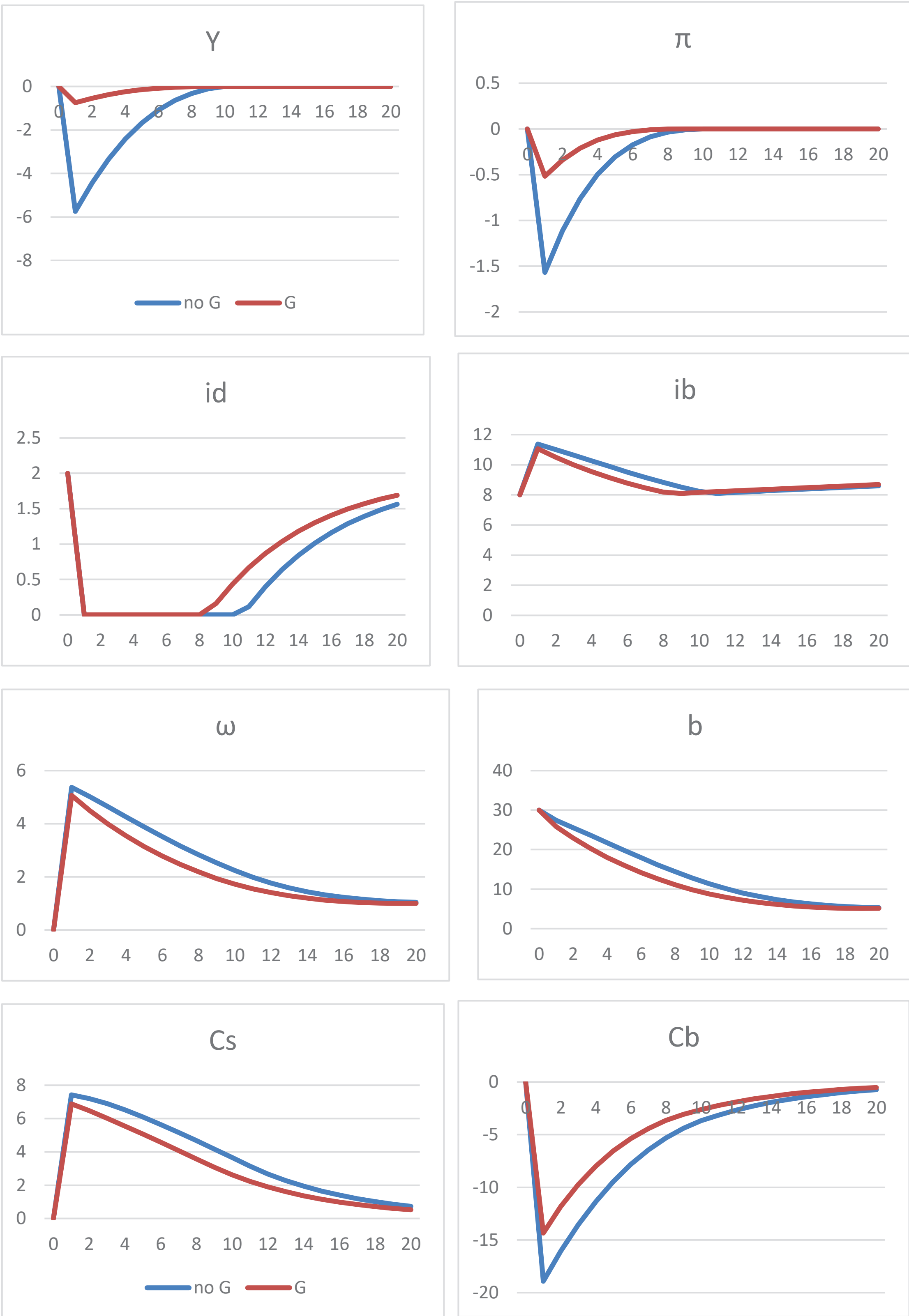
Moreover, the sovereign debts are only held by the savers.

3.4.2 The results

Figure 2.1 shows the effect of deleveraging shock on major economic variables under zero interest rates. That is output gap, $\pi, i^d, i^b, \hat{\omega}, \hat{b}, \hat{C}^s, \hat{C}^b, \hat{h}^s,$ and $\hat{h}^b$.

The output gap drops to -5.7 percent shortly after the shock and then recovers over the years. Inflation falls to -1.6 percent and consistently retains low for nine periods. The zero interest rates will be perpetuating for ten periods. Initially, the borrower's debt is 30 percent above equilibrium and the borrower repays this debt by increasing working hours and reducing consumption. As the borrower's debt repayment progresses, the debt balance $\hat{b}$ decreases. As a consequence, the interest rate spread $\hat{\omega}$ between the borrowing rate and the deposit rate will narrow, and the negative output gap and deflation will be eliminated.

[7] The government budget constraint is assumed to be satisfied by adjusting T_t^s.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-4
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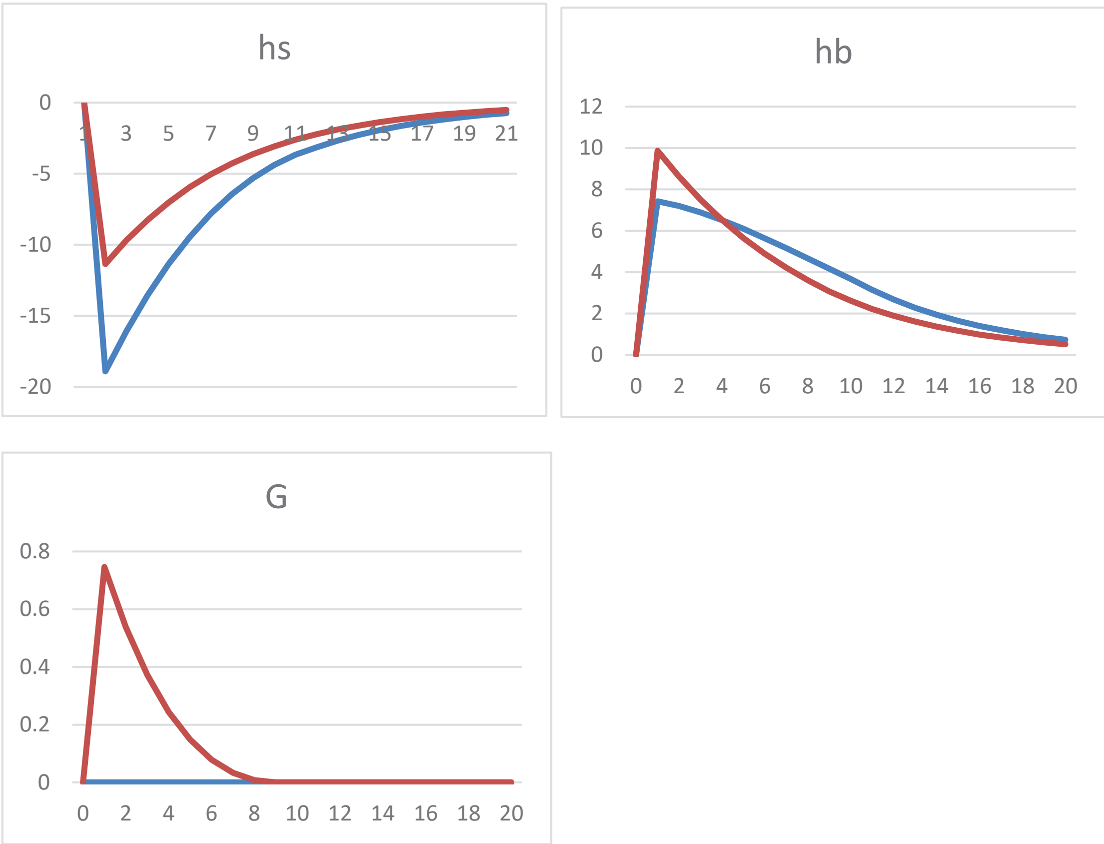

FIGURE 3.1 RESPONSES OF MACROECONOMIC VARIABLES TO DELEVERAGING SHOCK UNDER ZERO LOWER BOUND

3.4.3 The debt to GDP ratio

Public debt in most countries was running high by historical standards even before the coronavirus outbreak. As table 3.2 shows, some countries have gross public debt about 200 percent of the GDP. In our simulations, we assume that the initial public debt is 200 percent of annual GDP.[8]

Japan	248
Greece	177
Italy	133

TABLE3.2 GROSS PUBLIC DEBT TO GDP RATIO (2015)

Source: International Monetary Fund

[8] When B_0^g is twice the annual nominal GDP, $B_0^g = 8P_0Y_0$.

Figure 3.2 shows the path of the debt-to-GDP ratio. After the deleveraging shock, the debt ratio sharply jumped from 200 percent to 213 percent. The reason is that the nominal GDP which is the denominator of the debt-to-GDP ratio has decreased due to deflation and the decrease of real GDP. The debt balance which is the numerator of the debt-to-GDP ratio remains almost unchanged because there is no additional fiscal spending and $i_t^d = 0$ until the tenth period. After the debt ratio rose to 213 percent, then falls as the economy recovers. The debt-income ratio falls slowly afterwards, since $i_t^d - \pi_t < g$. In steady state, the ratio is constant, as $i_t^d - \pi_t = g$.

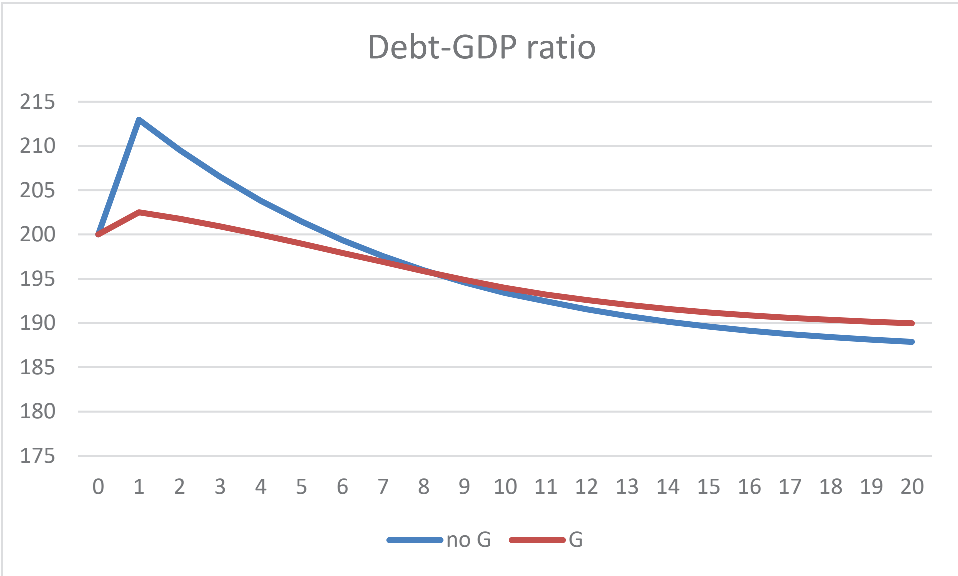

FIGURE 3.2 RESPONSES OF DEBT TO GDP RATIO TO WITH AND WITHOUT GOVERNMENT SHOCK

3.5 Government expenditure

This section examines how a fiscal expansion changes output, inflation, and debt.

3.5.1 Fiscal policy

We assume that fiscal policy is used to close a negative output gap. For an output gap $\hat{Y}_t = \frac{1}{2}\left(\hat{C}_t^b + \hat{C}_t^s\right) + \hat{G}_t$, policymakers will set a fiscal rule as

$$\hat{G}_t = max\left[0, -f\left(\hat{C}_t^b + \hat{C}_t^s\right)\right] \qquad (3.36)$$

Here the coefficient f is positive. If $\frac{1}{2}\left(\hat{C}_t^b + \hat{C}_t^s\right)$ is negative, the government spending $\hat{G}_t$ is

$-f\left(\hat{C}_t^b + \hat{C}_t^s\right)$. In this case the output gap $\hat{Y}_t$ is $\left(\frac{1}{2} - f\right)\left(\hat{C}_t^b + \hat{C}_t^s\right)$. Here the public understands fiscal policy will be conducted according to equation (3.36).

3.5.2 Effects of the government spending

We assume that the coefficient f in equation (3.36) is 0.4. Figure 3.1 shows that in period one policymakers increase government spending by 0.75 percent.

The red lines in Figure 3.1 represent the effects of government spending. The spending leads to -0.7 percent output gap and -0.5 percent inflation in period one. It is worth noting that the fiscal expansion mitigates the situation of severe deflation. The zero bound is now binding for eight quarters instead of ten quarters.

Figure 3.2 shows that, despite this fiscal expansion, the debt-income ratio rises less rapidly than in the baseline case. The debt-income ratio is 203 percent, compared to 213 percent in the baseline case. This modest increase in the debt-income ratio can be explained by the mitigation of recessionary deflation (deflation combined with a negative output gap). The expansionary fiscal policy leads to a less severe drop in nominal GDP (the denominator in the ratio), while the government debt (the numerator in the ratio) increases due to an increase in government spending and interest payments. Figure 2.2 illustrates that fiscal expansions can suppress the increase in the debt-income ratio.[9]

3.6 Robustness: speed of leverage adjustment

The parameter λ, which is a function of several structural parameters, represents how fast a borrower wants to pay down his own debt. The parameter λ increases when the borrower wants to pay down his debt faster.

Figure 3.3 shows the paths of the output gap and $\hat{b}$ for three alternative values of λ. We assume that the base value of λ is 1.32, as in Eggertsson and Krugman (2012). If λ increases, this implies that the borrower wants to pay off his debt faster and will cut spending more, which will

[9] After two years, the path of the ratio is lower in the baseline case. This is because $i_t^d - \pi_t$ is lower in the baseline case.

make the recession worse. Panel A of figure 4 shows that output drops more on impact as λ increases: $\hat{Y}_1 = -0.045$ for $\lambda = 1$, $\hat{Y}_1 = -0.059$ for $\lambda = 1.32$, and $\hat{Y}_1 = -0.136$ for $\lambda = 3$.

Observe that a deeper recession due to rapid deleveraging does not necessarily make it a short one. For output to close the gap between actual and potential output and return to its balanced-growth-path value, it takes the same 10 quarters regardless of λ. This is because, when the borrowers are willing to pay down their debt faster and that causes deflation, it will increase the real value of debt. Panel B of figure 4 indicates that there is no clear relationship between recession severity and the path of $\hat{b}$. This figure represents a variant of the Fisher's 1933 debt-deflation analysis.[10]

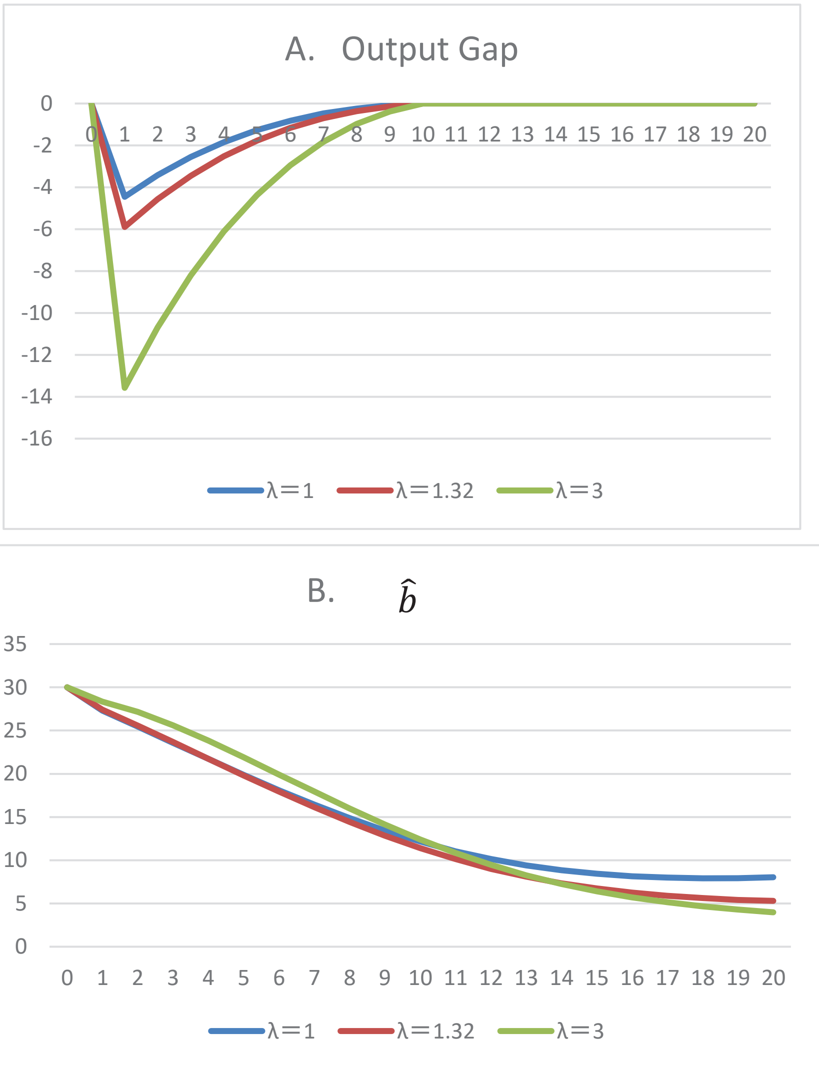

FIGURE 3.3 RESPONSES OF OUTPUT GAP AND DEBT UNDER DIFFERENT VALUES OF λ

[10] Fischer (1933). Repaying nominally fixed debt can cause a fall in prices and thus increase the real value of debt.

3.7 Discussion

This chapter has examined a controversial relationship between unemployment and the debt-to-GDP ratio. There appears to be a trade-off between unemployment and the ratio: Expansionary fiscal policies reduce unemployment but lead to a higher debt-to-GDP ratio. This study has shown that the trade-off relationship may not exist if the short-term nominal interest rate is zero and if a negative economic shock is temporary.

The expansionary fiscal policy leads to a less severe drop in nominal GDP (the denominator in the ratio). Therefore, fiscal expansions can suppress the increase in the debt-income ratio.

This paper has not discussed the effects of business cycles on the primary balance. Even if there is no additional government spending, a recession will produce primary deficits, which will result in a higher level of debt.[11] This is a subject for further analysis.

[11] If the primary balance tends to worsen during a recession, it will strengthen the main conclusion of this study.

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APPENDIX 3A

This appendix aims to explain the derivation of linearized system of equations.

The relationship between i_t^b and i_t^d implies:

$$1 + i_t^b = (1 + i_t^d)(1 + \omega_t)$$

$$i_t^b - \bar{\imath}_t^b = (1 + \bar{\omega})\left(i_t^d - \bar{\imath}_t^d\right) + (1 + \bar{\imath}_t^d)(\omega_t - \bar{\omega})$$

Since in steady state: $1 + \bar{\imath}_t^b = \left(1 + \bar{\imath}_t^d\right)(1 + \bar{\omega})$

Thus,

$$\frac{i_t^b - \bar{\imath}_t^b}{1 + \bar{\imath}_t^b} = \frac{(1 + \bar{\omega})\left(i_t^d - \bar{\imath}_t^d\right)}{\left(1 + \bar{\imath}_t^d\right)(1 + \bar{\omega})} + \frac{(1 + \bar{\imath}_t^d)(\omega_t - \bar{\omega})}{\left(1 + \bar{\imath}_t^d\right)(1 + \bar{\omega})}$$

$$\frac{i_t^b - \bar{\imath}_t^b}{1 + \bar{\imath}_t^b} = \frac{\left(i_t^d - \bar{\imath}_t^d\right)}{\left(1 + \bar{\imath}_t^d\right)} + \frac{(\omega_t - \bar{\omega})}{(1 + \bar{\omega})}$$

$$\hat{\imath}_t^b = \hat{\imath}_t^d + \widehat{\omega}_t \tag{3.27}$$

where $\hat{\imath}_t^b \equiv \frac{i_t^b - i^b}{1 + i^b}$, $\hat{\imath}_t^d \equiv \frac{i_t^d - i^d}{1 + i^d}$, $\widehat{\omega}_t \equiv \frac{\omega_t - \omega}{1 + \omega}$

$$\omega_t = \omega\left(\frac{B_t}{P_t}, \frac{M_t}{P_t}, b^j\right)$$

Linearize:

$$1 + \omega_t = 1 + \omega(b_t, m_t, b^j)$$

$$(1 + \bar{\omega}) + (\omega_t - \bar{\omega}) = (1 + \bar{\omega}) + [\omega_b\left(b_t - \bar{b}\right) + \omega_m(m_t - \bar{m})]$$

$\omega_t - \bar{\omega} = (\omega_b + \omega_m)(\, b_t - \bar{b})$ (because we assumed $b_t = m_t$, $\bar{b} = \bar{m}$)

$$\frac{\omega_t - \bar{\omega}}{1 + \bar{\omega}} = \frac{(\omega_b + \omega_m)}{1 + \bar{\omega}}\bar{b} \cdot \frac{(\, b_t - \bar{b})}{\bar{b}}$$

$$\widehat{\omega}_t = \theta \hat{b}_t \tag{3.28}$$

where $\theta \equiv \frac{\omega_b + \omega_m}{1 + \omega}\bar{b}$ and $\hat{b}_t \equiv \frac{b_t - \bar{b}}{\bar{b}}$.

The consumption Euler equations of borrowers is

$$u_c^b\left(C_t^b\right) = \delta E_t u_c^b\left(C_{t+1}^b\right)\frac{1 + i_t^b}{\Pi_{t+1}}\left[\frac{B_t}{P_t}\omega^b + (1 + \omega_t)\right]$$

Linearize the above equation：

$$\text{SS: } u_c(\bar{C}^b) = \delta u_c(\bar{C}^b)(1 + \bar{\imath}^b)\left(1 + \bar{b}\frac{\omega_b}{1 + \bar{\omega}}\right)$$

$$\rightarrow \delta(1 + \bar{\imath}^b)\left(1 + \bar{b}\frac{\omega_b}{1 + \bar{\omega}}\right) = 1$$

$$\frac{\omega_t-\bar{\omega}}{1+\bar{\omega}}=\frac{\omega_b+\omega_m}{1+\bar{\omega}}(b_t-\bar{b})$$

$$b_t-\bar{b}=\frac{\omega_t-\bar{\omega}}{\omega_b+\omega_m}$$

$$1+\bar{\imath}^b=(1+\bar{\imath}^d)(1+\bar{\omega})=\beta^{-1}(1+\bar{\omega})$$

LHS: $u_{cc}(C_t^b-\bar{C})$

RHS:

C_{t+1}^b :

①$\delta E_t u_{cc}(1+\bar{\imath}^b)\left(1+\bar{b}\frac{\omega_b}{1+\bar{\omega}}\right)\left(C_{t+1}^b-\bar{C}\right)=u_{cc}E_t\left(C_{t+1}^b-\bar{C}\right)$

i_t^b :

②$\delta u_c\left(1+\bar{b}\frac{\omega_b}{1+\bar{\omega}}\right)\left(i_t^b-\bar{\imath}^b\right)=u_c\frac{i_t^b-\bar{\imath}^b}{1+\bar{\imath}^b}$

Π_{t+1}:

③$-\delta E_t u_c(1+\bar{\imath}^b)\left(1+\bar{b}\frac{\omega_b}{1+\bar{\omega}}\right)\pi_{t+1}=-u_cE_t\pi_{t+1}$

b_t, m_t :

$$④\delta u_c(1+\bar{\imath}^b)[\tfrac{(\omega_b+\bar{b}\omega_{bb})(1+\bar{\omega})-\bar{b}\omega_b\cdot\omega_b}{(1+\bar{\omega})^2}\left(b_t-\bar{b}\right)+\tfrac{\bar{b}\omega_{bm}(1+\bar{\omega})-\bar{b}\omega_b\cdot\omega_m}{(1+\bar{\omega})^2}(m_t-\bar{m})]$$

$$=\delta u_c(1+\bar{\imath}^b)\frac{(1+\bar{\omega})[\omega_b+\bar{b}(\omega_{bb}+\omega_{bm})]-\bar{b}\omega_b(\omega_b+\omega_m)}{(1+\bar{\omega})^2}\left(b_t-\bar{b}\right)$$

$$=\delta u_c(1+\bar{\imath}^b)\frac{(1+\bar{\omega})[\omega_b+\bar{b}(\omega_{bb}+\omega_{bm})]-\bar{b}\omega_b(\omega_b+\omega_m)}{(1+\bar{\omega})^2}\frac{\omega_t-\bar{\omega}}{\omega_b+\omega_m}$$

$$=\delta u_c(1+\bar{\imath}^b)[\omega_b+\bar{b}\omega_{bb}-\frac{\bar{b}\omega_b(\omega_b+\omega_m)}{1+\bar{\omega}}]\frac{\omega_t-\bar{\omega}}{1+\bar{\omega}}\frac{1}{\omega_b+\omega_m}$$

$$=\delta u_c(1+\bar{\imath}^b)[\omega_b+\bar{b}\omega_{bb}-\frac{\bar{b}\omega_b(\omega_b+\omega_m)}{1+\bar{\omega}}]\frac{\widehat{\omega}_t}{\omega_b+\omega_m}$$

（$i^b+1=\beta^{-1}(1+\omega)$）

$$=\delta\beta^{-1}(1+\bar{\omega})\left(\frac{\omega_b+\bar{b}\omega_{bb}}{\omega_b+\omega_m}-\frac{\bar{b}\omega_b}{1+\bar{\omega}}\right)\widehat{\omega}_t$$

$$=\delta\beta^{-1}[\frac{\omega_b+\bar{b}\omega_{bb}}{\omega_b+\omega_m}(1+\bar{\omega})-\bar{b}\omega_b]\widehat{\omega}_t$$

Combine ①~④:

$$u_{cc}\left(C_t^b-\bar{C}\right)=u_{cc}E_t\left(C_{t+1}^b-\bar{C}\right)+u_c\frac{i_t^b-\bar{\imath}^b}{1+\bar{\imath}^b}-u_cE_t\pi_{t+1}$$
$$+\delta\beta^{-1}\left[\frac{\omega_b+\bar{b}\omega_{bb}}{\omega_b+\omega_m}(1+\bar{\omega})-\bar{b}\omega_b\right]\widehat{\omega}_t$$

$$\frac{C_t^b-\bar{C}}{\bar{Y}}=E_t\frac{C_{t+1}^b-\bar{C}}{\bar{Y}}+\frac{u_c}{u_{cc}\bar{Y}}\left(\frac{i_t^b-\bar{\imath}^b}{1+\bar{\imath}^b}-E_t\pi_{t+1}\right)+\frac{u_c}{u_{cc}\bar{Y}}\delta\beta^{-1}\left[\frac{\omega_b+\bar{b}\omega_{bb}}{\omega_b+\omega_m}(1+\bar{\omega})-\bar{b}\omega_b\right]\widehat{\omega}_t$$

$$\hat{C}_t^b = E_t\hat{C}_{t+1}^b - \sigma_b{}^{-1}\left(\hat{\iota}_t^b - E_t\pi_{t+1} + \lambda\widehat{\omega}_t\right) \tag{3.29}$$

where$\lambda \equiv \delta\beta^{-1}\left[\frac{\bar{b}\omega_{bb}+\omega_b}{\omega_b+\omega_m}(1+\bar{\omega}) - \bar{b}\omega_b\right]$, $\sigma_b = -\frac{u_{cc}^b\bar{Y}}{u_c^b}$, $\hat{C}_t^b \equiv \frac{C_t^b-\bar{C}^b}{\bar{Y}}$, and $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

$$u_c^s(C_t^s) = \frac{\beta E_t(1+i_t^d)u_c^s(C_{t+1}^s)}{\Pi_{t+1}}$$

Linearization:

$$u_{cc}(C_t^s - \bar{C}) = \beta E_t[u_c(i_t^d - \bar{\iota}^d) - (1+\bar{\iota}^d)u_c\pi_{t+1} + (1+\bar{\iota}^d)u_{cc}(C_{t+1}^s - \bar{C})]$$

$$\frac{C_t^s - \bar{C}}{\bar{Y}} = \frac{u_c}{u_{cc}\bar{Y}}\left(\frac{i_t^d - \bar{\iota}^d}{1+\bar{\iota}^d} - E_t\pi_{t+1}\right) + E_t\frac{C_{t+1}^s - \bar{C}}{\bar{Y}}$$

$$\hat{C}_t^s = E_t\hat{C}_{t+1}^s - \sigma_s{}^{-1}\left(\hat{\iota}_t^d - E_t\pi_{t+1}\right) \tag{3.30}$$

Where $\hat{C}_t^s \equiv \frac{C_t^s-\bar{C}^s}{\bar{Y}}$, $\sigma_s \equiv -\frac{u_{cc}^s\bar{Y}}{u_c^s}$.

The optimal labor supply of each type is

$$W_t = \frac{v_h^b(h_t^b)}{u_c^b(C_t^b)}$$

$$W_t = \frac{v_h^s(h_t^s)}{u_c^s(C_t^s)}$$

$$\bar{W}log\frac{W_t}{\bar{W}} \approx \frac{v_{hh}}{u_c}\left(h_t^b - \bar{h}\right) - \frac{v_h u_{cc}}{u_c^2}(C_t^b - \bar{C}^b)$$

$$\frac{v_h}{u_c}log\frac{W_t}{\bar{W}} = \frac{v_{hh}}{u_c}\left(h_t^b - \bar{h}\right) - \frac{v_h u_{cc}}{u_c^2}(C_t^b - \bar{C}^b)$$

$$log\frac{W_t}{\bar{W}} = \frac{v_{hh}}{v_h}\left(h_t^b - \bar{h}\right) - \frac{u_{cc}}{u_c}(C_t^b - \bar{C}^b)$$

$$log\frac{W_t}{\bar{W}} = \frac{v_{hh}\bar{Y}}{v_h}\frac{\left(h_t^b - \bar{h}\right)}{\bar{Y}} - \frac{u_{cc}\bar{Y}}{u_c}\frac{(C_t^b - \bar{C}^b)}{\bar{Y}}$$

$$\widehat{W}_t = \nu\hat{h}_t^b + \sigma_b\hat{C}_t^b \tag{3.31}$$

where $\widehat{W}_t \equiv log\frac{W_t}{\bar{W}}$ $\nu \equiv \frac{v_{hh}\bar{Y}}{v_h}$.

Similarly, we get

$$\widehat{W}_t = \nu\hat{h}_t^s + \sigma_s\hat{C}_t^s \tag{3.32}$$

Let $\widehat{\Delta}_t = -\theta\int[\hat{p}_t(i) - \hat{p}_t]di$, equation () implies that

$$\widehat{\Delta}_t = 0$$[12]

And equation $P_t = \left[(1-\alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}$ implies that

$$\hat{p}_t^* = \frac{\alpha}{1-\alpha}\pi_t \tag{3.33}$$

[12] To a first order approximation $\int\hat{p}_t(i)di = \hat{p}_t$, thus $\widehat{\Delta}_t$ around the steady state is approximately equal to zero.

where $\hat{p}_t^* \equiv \frac{p_t^*}{p_t} - 1$

We can use this and equation (2.33) to derive the New Keynesian Phillips curve

$$\pi_t = \kappa \hat{Y}_t + \beta E_t \pi_{t+1} \tag{3.34}$$

where $\equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.

The aggregate asset constraint $Y_t = \chi_b C_t^b + \chi_s C_t^s + G_t$ is

$$Y_t - \bar{Y} = \chi_b\left(C_t^b - \bar{C}^b\right) + \chi_s(C_t^s - \bar{C}^s) + (G_t - \bar{G})$$

Divide by $\bar{Y}$ for both sides:

$$\frac{Y_t - \bar{Y}}{\bar{Y}} = \chi_b\left(\frac{C_t^b - \bar{C}^b}{\bar{Y}}\right) + \chi_s\left(\frac{C_t^s - \bar{C}^s}{\bar{Y}}\right) + (\frac{G_t - \bar{G}}{\bar{Y}})$$

$$\hat{Y}_t = \chi_b \hat{C}_t^b + \chi_s \hat{C}_t^s + \hat{G}_t \tag{3.35}$$

where $\hat{Y}_t \equiv \frac{Y_t - \bar{Y}}{\bar{Y}}$ and $\hat{G}_t \equiv \frac{G_t - \bar{G}}{\bar{Y}}$.

The labor supply constraint is

$$h_t = \chi_b h_t^b + \chi_s h_t^s = Y_t \Delta_t$$

Linearize:

$$\bar{\Delta}(Y_t - \bar{Y}) + \bar{Y}(\Delta_t - \bar{\Delta}) = \chi_s(h_t^s - \bar{h}_t^s) + \chi_b(h_t^b - \bar{h}_t^b)$$

Since $\bar{\Delta} = 1, \hat{\Delta}_t = 0$

$$Y_t - \bar{Y} = \chi_s(h_t^s - \bar{h}_t^s) + \chi_b(h_t^b - \bar{h}_t^b)$$

Divide by $\bar{Y}$

$$\frac{Y_t - \bar{Y}}{\bar{Y}} = \chi_s(\frac{h_t^s - \bar{h}_t^s}{\bar{Y}}) + \chi_b(\frac{h_t^b - \bar{h}_t^b}{\bar{Y}})$$

$$\hat{Y}_t = \chi_b \hat{h}_t^b + \chi_s \hat{h}_t^s \text{[13]} \tag{3.36}$$

The borrowers' budget constraint is

$$b_t = \left(1 + i_{t-1}^b\right) b_{t-1} \frac{1}{\Pi_t} + C_t^b - W_t h_t^b + T_t^b$$

Linearize:

$$b_t - \bar{b} = \bar{b}\left(i_{t-1}^b - \bar{\iota}^b\right) + (1 + \bar{\iota}^b)\left(b_{t-1} - \bar{b}\right) - (1 + \bar{\iota}^b)\bar{b}\pi_t + \left(C_t^b - \bar{C}^b\right) - h^b \bar{W} log \frac{W_t}{\bar{W}}$$
$$- \bar{W}\left(h_t^b - \bar{h}^b\right) + (T_t^b - \bar{T})$$

Divide by $\bar{Y}$:

[13] Since $\hat{\Delta}_t = 0$.

$$\frac{\bar{b}}{\bar{Y}}\frac{b_t-\bar{b}}{\bar{b}}=\frac{\bar{b}}{\bar{Y}}(1+\bar{\imath}^b)\left(\frac{i_{t-1}^b-\bar{\imath}^b}{1+\bar{\imath}^b}\right)+(1+\bar{\imath}^b)\left(\frac{b_{t-1}-\bar{b}}{\bar{Y}}\right)-\frac{\bar{b}}{\bar{Y}}(1+\bar{\imath}^b)\pi_t+\left(\frac{C_t^b-\bar{C}^b}{\bar{Y}}\right)$$
$$-\frac{h^b}{\bar{h}}log\frac{W_t}{\bar{W}}-\left(\frac{h_t^b-\bar{h}^b}{\bar{Y}}\right)+(\frac{T_t^b-\bar{T}}{\bar{Y}})$$
$$b_y\hat{b}_t=b_y(1+\bar{\imath}^b)\left(\hat{\imath}_{t-1}^b+\hat{b}_{t-1}-\pi_t\right)+\hat{C}_t^b-h^b\widehat{W}_t-\hat{h}_t^b+\hat{T}_t^b \tag{3.37}$$

where $b_y\equiv\frac{\bar{b}}{\bar{Y}}$, $h^b\equiv\frac{h^b}{\bar{h}}$, $\hat{T}_t^b\equiv\frac{T_t{}^b-\bar{T}^b}{\bar{Y}}$.

Finally, the monetary policy rule yields

$$i_t^d=max(0,f(X_t))$$

$i_t^d\geq 0$ implies that $\hat{\imath}_t^d\geq-\frac{i^d}{1+i^d}=\beta-1$

When $i_t^d>0$

$$i_t^d=f(X_t)=f(\Pi_t,Y_t)$$

Linearize:

$$\hat{\imath}_t^d=f_\pi(\Pi_t-\bar{\Pi})+f_y(Y_t-\bar{Y})$$
$$\hat{\imath}_t^d=f_\pi\pi_t+f_y\bar{Y}(\frac{Y_t-\bar{Y}}{\bar{Y}})$$
$$\hat{\imath}_t^d=\phi_\pi\pi_t+\phi_x\hat{Y}_t$$

where $\phi_\pi=f_\pi$, $\phi_x=f_y\bar{Y}$

Thus,

$$\hat{\imath}_t^d=max\left(\beta-1,\phi_\pi\pi_t+\phi_x\hat{Y}_t\right) \tag{3.38}$$

In the case of a Taylor rule, i_t^d is affected only by output and inflation.

APPENDIX 3B

This appendix seeks to demonstrate the derivation of the figure 3.1 from equation (3.27) - (3.38).

The equations can be summarized as follows.

$$\hat{\imath}_t^b = \hat{\imath}_t^d + \widehat{\omega}_t \tag{3.27}$$

$$\widehat{\omega}_t = \vartheta \widehat{b}_t \tag{3.28}$$

$$\hat{C}_t^b = E_t \hat{C}_{t+1}^b - \sigma(\hat{\imath}_t^b - E_t \pi_{t+1} + \lambda \widehat{\omega}_t) \tag{3.29}$$

$$\hat{C}_t^s = E_t \hat{C}_{t+1}^s - \sigma(\hat{\imath}_t^d - E_t \pi_{t+1}) \tag{3.30}$$

$$\widehat{W}_t = \nu \hat{h}_t^b + \sigma^{-1} \hat{C}_t^b \tag{3.31}$$

$$\widehat{W}_t = \nu \hat{h}_t^s + \sigma^{-1} \hat{C}_t^s \tag{3.32}$$

$$\pi_t = \gamma \widehat{W}_t + \beta E_t \pi_{t+1} \tag{3.34}$$

$$b_y \widehat{b}_t = b_y (1 + \bar{\imath}^b) \hat{\imath}_{t-1}^b + (1 + \bar{\imath}^b) b_y \widehat{b}_{t-1} - b_y (1 + \bar{\imath}^b) \pi_t + \hat{C}_t^b - h^b \widehat{W}_t - h_t^b + \hat{T}_t^b \tag{3.35}$$

$$\hat{Y}_t = \chi_b \hat{C}_t^b + \chi_s \hat{C}_t^s + \hat{G}_t \tag{3.36}$$

$$\hat{Y}_t = \chi_b \hat{h}_t^b + \chi_s \hat{h}_t^s \tag{3.37}$$

$$\hat{\imath}_t^d = max(\beta - 1, \phi_x \hat{X}_t) \tag{3.38}$$

Equations (3.27) - (3.38) can be aggregated as follows

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{\sigma(1+\lambda)} + \frac{b_y(1+\bar{\imath}^b)}{\sigma\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_{t-1}^b \\ \hat{C}_{t-1}^s \\ \pi_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} -\frac{b_y}{\sigma\vartheta(1+\lambda)} - \frac{b_y(1+\bar{\imath}^b)}{\sigma(1+\lambda)} - \frac{b_y(1+\bar{\imath}^b)}{\sigma\vartheta(1+\lambda)} - 1 + \overset{-b}{\hbar} \left(\frac{1}{\sigma} + \nu\right) \chi_b + \left(\chi_b - \frac{1-\chi_b}{\nu\sigma}\right) & \overset{-b}{\hbar} \left(\frac{1}{\sigma} + \nu\right) \chi_s + \left(1 + \frac{1}{\nu\sigma}\right) \chi_s & - \\ -\gamma \left(\frac{1}{\sigma} + \nu\right) \chi_b & -\gamma \left(\frac{1}{\sigma} + \nu\right) \chi_s & \\ 0 & 1 & \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{b_y}{\sigma\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & \sigma \end{bmatrix} \begin{bmatrix} \hat{C}_{t+1}^b \\ \hat{C}_{t+1}^s \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & -\overset{-b}{\hbar}\ \nu - 1 & 1 \\ 0 & 0 & \gamma\nu & 0 \\ -\sigma & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\imath}_t^d \\ \hat{\imath}_{t-1}^d \\ \widehat{G}_t \\ \hat{T}_t^b \end{bmatrix} \quad (*1)$$

Substituting $\sigma = \nu = 1,\ \chi_b = \frac{1}{2},\ \hat{T}_t^b = 0$ into above matrix, it can be arranged as

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} + \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_{t-1}^b \\ \hat{C}_{t-1}^s \\ \pi_{t-1} \end{bmatrix} +$$

$$+\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}-1+\bar{h}^b & \bar{h}^b+1 & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}+b_y(1+\bar{\imath}^b) \\ -\gamma & -\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_t^b \\ \hat{C}_t^s \\ \pi_t\end{bmatrix}=$$

$$=\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix}\hat{C}_{t+1}^b \\ \hat{C}_{t+1}^s \\ \pi_{t+1}\end{bmatrix}+\begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix}\hat{\imath}_t^d \\ \hat{\imath}_{t-1}^d\end{bmatrix}+\begin{bmatrix}-\bar{h}^b-1 \\ \gamma \\ 0\end{bmatrix}\hat{G}_t \qquad (*2)$$

$$\hat{\imath}_t^d=\begin{cases}-(\hat{C}_{t+1}^b-\hat{C}_t^b) & if \;\; i_t^d\geq 0 \\ \beta-1 & if \;\; i_t^d<0\end{cases}$$

As the first step, we calculate the eigenvalues of the second-order linear difference equation of $\hat{C}_t^b$ with $|\Lambda|<1$ when the zero lower bound is not binding.

For all the period t when there is no zero lower bound constraint, in (*2)

$$\hat{C}_t^s=-\hat{C}_t^b,\;\;\pi_t=0,$$

while we can obtain the equation $\hat{\imath}_t^d=-(\hat{C}_{t+1}^b-\hat{C}_t^b)$ from (44). By substituting these equations into (*2), the first row can be converted as

$$\frac{2b_y}{\vartheta(1+\lambda)}(\hat{C}_{t+1}^b-\hat{C}_t^b)=b_y(1+\bar{\imath}^b)\frac{1-\lambda}{1+\lambda}(\hat{C}_t^b-\hat{C}_{t-1}^b)+(1+\bar{\imath}^b)\frac{2b_y}{\vartheta(1+\lambda)}(\hat{C}_t^b-\hat{C}_{t-1}^b)+2\hat{C}_t^b$$

It can be rearranged as

$$\hat{C}_{t+1}^b-\left[1+\frac{\vartheta(1+\bar{\imath}^b)(1-\lambda)}{2}+(1+\bar{\imath}^b)+\frac{\vartheta(1+\lambda)}{b_y}\right]\hat{C}_t^b+\left[\frac{\vartheta(1+\bar{\imath}^b)(1-\lambda)}{2}+(1+\bar{\imath}^b)\right]\hat{C}_{t-1}^b$$
$$=0$$

So far, we acquired the second-order linear difference equation of $\hat{C}_t^b$. Substituting the values of the parameters and representing the eigenvalue with Λ, the equation of $\hat{C}_t^b$ can be presented as

$$\hat{C}_t^b=A\Lambda_1^t+B\Lambda_2^t$$

where

$$\Lambda_1,\Lambda_2$$
$$=\frac{1}{2}\left\{\left[1+\frac{\vartheta(1+\bar{\imath}^b)(1-\lambda)}{2}+(1+\bar{\imath}^b)+\frac{\vartheta(1+\lambda)}{b_y}\right]\right.$$
$$\left.\pm\sqrt{\left[1+\frac{\vartheta(1+\bar{\imath}^b)(1-\lambda)}{2}+(1+\bar{\imath}^b)+\frac{\vartheta(1+\lambda)}{b_y}\right]^2-4\left[\frac{\vartheta(1+\bar{\imath}^b)(1-\lambda)}{2}+(1+\bar{\imath}^b)\right]}\right\}$$

In the case of $|\Lambda_1|<1,\;\;|\Lambda_2|>1,$ when $A\neq 0,\;\;B=0$, $\hat{C}_t^b$ has a unique stable solution.

The arrangement of the fiscal expending rules

Households arrange their consumption strategies under the awareness of the fiscal expending rules (that the deflation will be suppressed).

The fiscal expanding rules seek to constantly eliminate GDP gap that is: $\hat{Y}_t = 0$

since $\hat{Y}_t = \frac{1}{2}(\hat{C}_t^b + \hat{C}_t^s) + \hat{G}_t = 0$, as a result, the fiscal expanding rules have to satisfy

$$\hat{G}_t = -\frac{1}{2}(\hat{C}_t^b + \hat{C}_t^s)$$

when $\hat{G}_t = 0$

$$\pi_t = \gamma\hat{W}_t + \beta\pi_{t+1} = \gamma(\hat{C}_t^b + \hat{C}_t^s) + \beta\pi_{t+1}$$

when $\hat{G}_t = -\frac{1}{2}(\hat{C}_t^b + \hat{C}_t^s)$

$$\pi_t = \gamma\hat{W}_t + \beta\pi_{t+1} = \gamma(\hat{C}_t^b + \hat{C}_t^s + \hat{G}_t) + \beta\pi_{t+1} = \frac{\gamma}{2}(\hat{C}_t^b + \hat{C}_t^s) + \beta\pi_{t+1}$$

The value of $\hat{G}_t$ is $\hat{G}_T = \hat{G}_{T+1} = \hat{G}_{T+2} = \cdots = 0$

$\hat{G}_{T-1} = -\frac{1}{2}(\hat{C}_{T-1}^b + \hat{C}_{T-1}^s)$

$\hat{G}_{T-2} = -\frac{1}{2}(\hat{C}_{T-2}^b + \hat{C}_{T-2}^s)$

$\hat{G}_{T-3} = -\frac{1}{2}(\hat{C}_{T-3}^b + \hat{C}_{T-3}^s)$

when $\hat{G}_t = -g(\hat{C}_t^b + \hat{C}_t^s)$, $\hat{G}_T = \hat{G}_{T+1} = \hat{G}_{T+2} = \cdots = 0$

$\hat{G}_{T-1} = -g(\hat{C}_{T-1}^b + \hat{C}_{T-1}^s)$

$\hat{G}_{T-2} = -g(\hat{C}_{T-2}^b + \hat{C}_{T-2}^s)$

$\hat{G}_{T-3} = -g(\hat{C}_{T-3}^b + \hat{C}_{T-3}^s)$

The second step is to identify the period T that is the period when the interest rate hits zero.

(i) Investigate whether $i_1^d \geq 0$ or not.

In period $t = 1$, (3.27)-(3.38) can be summarized as

$$\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} - 1 + \bar{h}^b & \bar{h}^b + 1 & b_y(1+\bar{\imath}^b) \\ -\gamma & -\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} \\ 0 \\ -1 \end{bmatrix} \hat{\imath}_1^d + \begin{bmatrix} 1+\bar{\imath}^b \\ 0 \\ 0 \end{bmatrix} \hat{b}_0 + \begin{bmatrix} -\bar{h}^b - 1 \\ \gamma \\ 0 \end{bmatrix} \hat{G}_1$$

$$\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-1+\bar{h}^b & \bar{h}^b+1 & b_y(1+\bar{\iota}^b) \\ -\gamma & -\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} \\ 0 \\ -1 \end{bmatrix}\hat{\iota}_1^d + \begin{bmatrix} 1+\bar{\iota}^b \\ 0 \\ 0 \end{bmatrix}\hat{b}_0 - g\begin{bmatrix} -\bar{h}^b-1 \\ \gamma \\ 0 \end{bmatrix}\left(\hat{C}_1^b+\hat{C}_1^s\right)$$

$$\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-1+\bar{h}^b-g(\bar{h}^b+1) & \bar{h}^b+1-g(\bar{h}^b+1) & b_y(1+\bar{\iota}^b) \\ -\gamma+g\gamma & -\gamma+g\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} \\ 0 \\ -1 \end{bmatrix}\hat{\iota}_1^d + \begin{bmatrix} 1+\bar{\iota}^b \\ 0 \\ 0 \end{bmatrix}\hat{b}_0 \qquad (*3)$$

and

$$\hat{\iota}_1^d = \begin{cases} -(\hat{C}_2^b - \hat{C}_1^b) & if \;\; i_1^d \geq 0 \\ \beta - 1 & if \;\; i_1^d < 0 \end{cases}$$

when $i_1^d \geq 0$, $\hat{\iota}_1^d = -\left(\hat{C}_2^b - \hat{C}_1^b\right)$ pins down (*3)

$$\begin{bmatrix} -\frac{2b_y}{\vartheta(1+\lambda)}-1+\bar{h}^b-g(\bar{h}^b+1) & \bar{h}^b+1-g(\bar{h}^b+1) & b_y(1+\bar{\iota}^b) \\ -\gamma+g\gamma & -\gamma+g\gamma & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{2b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} 1+\bar{\iota}^b \\ 0 \\ 0 \end{bmatrix}\hat{b}_0$$

In other words,

$$Q_1X_1 = R_1X_2 + \epsilon_0$$

$$X_1 = Q_1^{-1}R_1X_2 + Q_1^{-1}\epsilon_0$$

$$X_1 = A_1X_2 + B_1$$

Rewriting it as the form

$$\begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} = \begin{bmatrix} a_1^{11} & a_1^{12} & a_1^{13} \\ a_1^{21} & a_1^{22} & a_1^{23} \\ a_1^{31} & a_1^{32} & a_1^{33} \end{bmatrix}\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} b_1^1 \\ b_1^2 \\ b_1^3 \end{bmatrix}$$

As $\hat{C}_2^s = -\hat{C}_2^b$, $\pi_2 = 0$ the first row can be written as

$$\hat{C}_1^b = (a_1^{11} - a_1^{12})\hat{C}_2^b + b_1^1$$

Combining with $\hat{C}_2^b = \Lambda_1\hat{C}_1^b$, thus

$$\hat{C}_1^b = \frac{b_1^1}{1-\Lambda_1(a_1^{11}-a_1^{12})}$$

Furthermore, since $\hat{\imath}_1^d = -\left(\hat{C}_2^b - \hat{C}_1^b\right) = (1-\Lambda_1)\,\hat{C}_1^b$, the annual rate of interest is

$$i_1^d = \left[\hat{\imath}_1^d + (1-\beta)\right] \times 400$$

Here if $i_1^d \geq 0$ holds, proceed to step 3. If $i_1^d < 0$, so i_1^d equal to zero, proceed to (ii) with $\hat{\imath}_1^d = \beta - 1$.

When $i_1^d = 0$ videlicet $\hat{\imath}_1^d = \beta - 1$, by substituting it into (*3) we obtain

$$\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} - 1 + h^b - g(\bar{h}^b + 1) & h^b + 1 - g(h^b + 1) & b_y(1+\bar{\imath}^b) \\ -\gamma + g\gamma & -\gamma + g\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} \\ 0 \\ -1 \end{bmatrix} (\beta - 1) + \begin{bmatrix} 1+\bar{\imath}^b \\ 0 \\ 0 \end{bmatrix} \hat{b}_0$$

or

$$\bar{Q}_1 X_1 = \bar{R}_1 X_2 + S_1 + \epsilon_0$$

$$X_1 = \bar{Q}_1^{-1} \bar{R}_1 X_2 + \bar{Q}_1^{-1}(S_1 + \epsilon_0)$$

$$X_1 = \bar{A}_1 X_2 + \bar{B}_1$$

(ii) When $T = 2$ we investigate if $\hat{\imath}_2^d \geq 0$.

when $t = 2$, the matrix (*2) is

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} + \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} +$$

$$+ \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} - \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} - 1 + h^b & \bar{h}^b + 1 & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} + b_y(1+\bar{\imath}^b) \\ -\gamma & -\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3 \end{bmatrix} + \begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\imath}_2^d \\ \hat{\imath}_1^d \end{bmatrix} + \begin{bmatrix} -\bar{h}^b - 1 \\ \gamma \\ 0 \end{bmatrix} \hat{G}_2$$

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} + \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} +$$

$$+\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}-1+h^b & \bar{h}^b+1 & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}+b_y(1+\bar{\imath}^b) \\ -\gamma & -\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2\end{bmatrix}=$$

$$=\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix}\hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3\end{bmatrix}+\begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix}\hat{\imath}_2^d \\ \hat{\imath}_1^d\end{bmatrix}$$

$$-g\begin{bmatrix}-\bar{h}^b-1 \\ \gamma \\ 0\end{bmatrix}\left(\hat{C}_2^b+\hat{C}_2^s\right)$$

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda}+\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1\end{bmatrix}+$$

$$+\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}-1+h^b-g(\bar{h}^b+1) & \bar{h}^b+1-g(\bar{h}^b+1) & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}+b_y(1+\bar{\imath}^b) \\ -\gamma+g\gamma & -\gamma+g\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2\end{bmatrix}$$

$$=\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix}\hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3\end{bmatrix}+\begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix}\hat{\imath}_2^d \\ \hat{\imath}_1^d\end{bmatrix}$$

Substituting $\hat{\imath}_1^d=\beta-1$ into above matrix to examine the value of $\hat{\imath}_2^d$

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda}+\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1\end{bmatrix}+$$

$$+\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}-1+\bar{h}^b-g(h^b+1) & h^b+1-g(\bar{h}^b+1) & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}+b_y(1+\bar{\imath}^b) \\ -\gamma+g\gamma & -\gamma+g\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2\end{bmatrix}$$

$$=\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix}\hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3\end{bmatrix}+\begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix}-(\hat{C}_3^b-\hat{C}_2^b) \\ \beta-1\end{bmatrix}$$

It can be rearranged as

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} + \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix} +$$

$$+$$

$$\begin{bmatrix} -\frac{2b_y}{\vartheta(1+\lambda)} - \frac{b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} - 1 + \bar{h}^b - g(\bar{h}^b+1) & \bar{h}^b + 1 - g(\bar{h}^b+1) & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} + b_y(1+\bar{\imath}^b) \\ -\gamma + g\gamma & -\gamma + g\gamma & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3 \end{bmatrix} + \begin{bmatrix} \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda} - \frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 \\ 0 \end{bmatrix} (\beta - 1)$$

Also it can be written as the following form

$$PX_1 + QX_2 = RX_3 + S$$

Substituting $X_1 = \bar{A}_1 X_2 + \bar{B}_1$ into the above equation pins down

$$P(\bar{A}_1 X_2 + \bar{B}_1) + QX_2 = RX_3 + S$$

$$(P\bar{A}_1 + Q)X_2 = RX_3 + S - P\bar{B}_1$$

$$X_2 = A_2 X_3 + B_2$$

where $A_2 = (P\bar{A}_1 + Q)^{-1}R, \quad B_2 = (P\bar{A}_1 + Q)^{-1}(S - P\bar{B}_1)$.

Rewriting $X_2 = A_2 X_3 + B_2$ as the form

$$\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix} = \begin{bmatrix} a_2^{11} & a_2^{12} & a_2^{13} \\ a_2^{21} & a_2^{22} & a_2^{23} \\ a_2^{31} & a_2^{32} & a_2^{33} \end{bmatrix} \begin{bmatrix} \hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3 \end{bmatrix} + \begin{bmatrix} b_2^1 \\ b_2^2 \\ b_2^3 \end{bmatrix}$$

Since $\hat{C}_3^s = -\hat{C}_3^b, \ \pi_3 = 0$, the first row can be converted as

$$\hat{C}_2^b = (a_2^{11} - a_2^{12})\hat{C}_3^b + b_2^1$$

Additionally, combing with $\hat{C}_3^b = \Lambda_1 \hat{C}_2^b$, it pins down

$$\hat{C}_2^b = \frac{b_2^1}{1 - \Lambda_1(a_2^{11} - a_2^{12})}$$

Accordingly, the value of interest rate also can be obtained

$$\hat{\imath}_2^d = -(\hat{C}_3^b - \hat{C}_2^b) = (1 - \Lambda_1)\hat{C}_2^b$$

or in the annual rate

$$i_2^d = [\hat{\imath}_2^d + (1 - \beta)] \times 400$$

Here if $i_2^d \geq 0$, proceed to step 3. Otherwise, the value of i_2^d remains zero that is $\hat{\imath}_2^d = \beta - 1$ and move to (iii).

When $i_2^d = 0$, substituting $\hat{\imath}_2^d = \beta - 1$ into (*2)

$$\begin{bmatrix} \frac{b_y(1+\bar{\imath}^b)}{1+\lambda}+\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{C}_1^b \\ \hat{C}_1^s \\ \pi_1 \end{bmatrix}+$$

$$+\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)}-\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}-1+\bar{h}^b-g(\bar{h}^b+1) & h^b+1-g(\bar{h}^b+1) & -\frac{b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)}+b_y(1+\bar{\imath}^b) \\ -\gamma+g\gamma & -\gamma+g\gamma & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{C}_2^b \\ \hat{C}_2^s \\ \pi_2 \end{bmatrix}$$

$$=\begin{bmatrix} -\frac{b_y}{\vartheta(1+\lambda)} & 0 & -\frac{b_y}{\vartheta(1+\lambda)} \\ 0 & 0 & \beta \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{C}_3^b \\ \hat{C}_3^s \\ \pi_3 \end{bmatrix}+\begin{bmatrix} \frac{b_y}{\vartheta(1+\lambda)} & \frac{\lambda b_y(1+\bar{\imath}^b)}{1+\lambda}-\frac{b_y(1+\bar{\imath}^b)}{\vartheta(1+\lambda)} \\ 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} \beta-1 \\ \beta-1 \end{bmatrix}$$

or as the form

$$\bar{P}X_1+\bar{Q}X_2=\bar{R}X_3+\bar{S}$$

Similarly, substituting $X_1=\bar{A}_1X_2+\bar{B}_1$ into the above equation

$$\bar{P}(\bar{A}_1X_2+\bar{B}_1)+\bar{Q}X_2=\bar{R}X_3+\bar{S}$$

$$(\bar{P}\bar{A}_1+\bar{Q})X_2=\bar{R}X_3+\bar{S}-\bar{P}\bar{B}_1$$

$$X_2=\bar{A}_2X_3+\bar{B}_2$$

where $\bar{A}_2=(\bar{P}\bar{A}_1+\bar{Q})^{-1}\bar{R}$, $\quad\bar{B}_2=(\bar{P}\bar{A}_1+\bar{Q})^{-1}(\bar{S}-\bar{P}\bar{B}_1)$

(iii) when $T=3$, investigate if $\hat{\imath}_3^d\geq 0$.

Substituting $X_2=\bar{A}_2X_3+\bar{B}_2$ into $PX_2+QX_3=RX_4+S$ we obtain

$$X_3=A_3X_4+B_3$$

As a result,

$$\hat{C}_3^b=\frac{b_3^1}{1-\Lambda_1(a_3^{11}-a_3^{12})}$$

and

$$\hat{\imath}_3^d=-\left(\hat{C}_4^b-\hat{C}_3^b\right)=(1-\Lambda_1)\,\hat{C}_3^b$$

$$i_3^d=\left[\hat{\imath}_3^d+(1-\beta)\right]\times 400$$

If $i_3^d\geq 0$ proceed to step 3. If the value of i_3^d below zero so i_3^d remains zero, we move to (iv) with $\hat{\imath}_3^d=\beta-1$.

(*)

In $X_t=A_tX_{t+1}+B_t$, A_t,B_t are practiced in the case of $i_t^d\geq 0$.

In $X_t = \bar{A}_t X_{t+1} + \bar{B}_t$, $\bar{A}_t, \bar{B}_t$ are practiced in the case of $i_t^d < 0$.

In $PX_{t-1} + QX_t = RX_{t+1} + S$, P, Q, R, S are practiced in the case of $i_t^d \geq 0$.

In $\bar{P}X_{t-1} + \bar{Q}X_t = \bar{R}X_{t+1} + \bar{S}$, $\bar{P}, \bar{Q}, \bar{R}, \bar{S}$ are practiced in the case of $i_t^d < 0$.

[Step 3] Solve $\hat{C}_{T+1}^b, \hat{C}_{T+2}^b, \hat{C}_{T+3}^b$ ⋯

$$\hat{C}_{T+1}^b = \Lambda_1 \hat{C}_T^b,$$

$$\hat{C}_{T+2}^b = \Lambda_1 \hat{C}_{T+1}^b,$$

$$\hat{C}_{T+3}^b = \Lambda_1 \hat{C}_{T+2}^b, \cdots$$

[Step 4] Solve $\hat{C}_{T-1}^b, \hat{C}_{T-2}^b, \hat{C}_{T-3}^b$ ⋯

Substituting $X_T = \begin{bmatrix} \hat{C}_T^b \\ \hat{C}_T^s \\ \pi_T \end{bmatrix} = \begin{bmatrix} \hat{C}_T^b \\ -\hat{C}_T^b \\ 0 \end{bmatrix}$ into $X_{T-1} = \bar{A}_{T-1} X_T + \bar{B}_{T-1}$, X_{T-1} and $\hat{C}_{T-1}^b$ can be obtained.

Next substituting X_{T-1} into $X_{T-2} = \bar{A}_{T-2} X_{T-1} + \bar{B}_{T-2}$ pins down X_{T-2} and $\hat{C}_{T-2}^b$.

In the same way, substituting X_{T-2} into $X_{T-3} = \bar{A}_{T-3} X_{T-2} + \bar{B}_{T-3}$ pins down X_{T-3} and $\hat{C}_{T-3}^b$.

CHAPTER IV

Analysis on the effects of monetary policy in a liquidity trap: a medium scale DSGE model approach

4.1 Introduction

The problem of the Zero Lower Bound (ZLB) returned to prominence with Japan's experience during the 90's, and more recently with the financial crisis sweeping the world in 2008. Since then, the notion of an effective lower bound on policy interest rates that is lower than zero has become a concrete concern for monetary policy. While the effective lower bound for short term rates exists, it does not impose a binding constraint on the effectiveness of monetary policy. Some research has considered models with non-negative constraint on nominal interest rates in the optimal monetary policy literature. Their research implies that the economy is affected by not only the present level of the overnight rate, but the path of expected future short term interest rates. (Clarida, Gali, and Gertler 1999, Woodford 2003) In theory, if the central bank is capable to commit to future policy actions, it can work around the zero bound constraint by promising monetary easing in the future once the zero bound constraint lapses. (Krugman 1998, Reifschneider and Williams 2000, Eggertsson and Woodford 2003). Empirically, the analysis shows that the monetary policy announcements have impact on asset prices primarily through the effects on financial market expectations of future monetary policy, rather than changes in the current federal funds rate target. (Gurkaynak, Sack, and Swanson 2005) As a result, the theoretical and empirical evidences both suggest that monetary policy still has room to affect the economy even when the short term nominal interest rates is at zero.

In this chapter, I employ a medium scale DSGE model that mainly based on Christiano et al. (2005) to investigate the effectiveness of the monetary policy shocks when the zero bound is binding. In this chapter, the model consists of four sectors, including households, final goods firms, intermediate firms and the government. Also, it is assumed that households have a certain wage pricing ability thus the concept of wage stickiness could be introduced into the model. The firms in the intermediate goods produce differentiated goods and follow the Calvo’s (1983) price setting rule. The fiscal policy follows Ricardian and the central bank set monetary policy according to Taylor's (1993) rule and money growth rule. To tackle the zero bound constraint, the study employs an algorithm created by Holden and Paetz (2012). This method introduces the "shadow shocks" which hit the bounded variables every time the constraint is violated, and "push" these variables back to zero. To make sure the solution ties in with rational expectations, the shocks are expected by agents previously, so that the "shadow shocks" can be seen as endogenous news shock.

4.2 Model

4.2.1 Households

The representative household is composed of a continuum of labors, each specialized in a particular labor type indexed by $i \in (0,1)$. The households obtain the utility from the consumption $C_t(i)$ and real money balances $\frac{M_t(i)}{P_t}$ and get the disutility from the labor supply $N_t(i)$. The representative household maximizes his utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(i)^{1-\sigma}}{1-\sigma} + \frac{(\frac{M_t(i)}{P_t})^{1-\nu}}{1-\nu} - \psi \frac{N_t(i)^{1+\phi}}{1+\phi} \right) \tag{4.1}$$

Subjective to the budget constraint:

$$P_t C_t(i) + P_t I_t(i) + B_t(i) + M_t(i) \leq W_t(i) N_t(i) + r_t^k P_t K_{t-1}(i) + R_{t-1} B_{t-1}(i) + M_{t-1}(i) + D_t(i) \tag{4.2}$$

where M_t is the nominal money balance held by the household at the end of period t, P_t is the aggregate price level, $W_t(i)$ is the nominal wage rate associated with labor of type i. I_t, K_t, B_t, R_t and D_t represent the investment, capital, government bond, nominal interest rate and the dividends that households received from monopolistic firms. r_t^k is real rate of return on capital. β is the discount factor, σ and ν denote the inverse of the intertemporal elasticity of substitution and the inverse of the elasticity of money holdings with respect to the interest rate. ϕ is the Frisch elasticity of labor supply.

The capital accumulation equation is given by:

$$K_t(i) = (1-\delta) K_{t-1}(i) + I_t(i) - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right) I_t(i) \tag{4.3}$$

where δ is the depreciation rate, $S(\cdot)$ represents the adjustment cost function of the investment with $S(\gamma)=0, S'(\gamma)=0, S''(\gamma)=0$.

Optimizing the utility function implies the following necessary condition for a rational-expectations equilibrium:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(i)^{1-\sigma}}{1-\sigma} + \frac{(\frac{M_t(i)}{P_t})^{1-\nu}}{1-\nu} - \frac{N_t(i)^{1+\phi}}{1+\phi} \right)$$
$$+ \lambda_t(i) \left\{ \frac{W_t(i)N_t(i)}{P_t} + r_t^k K_{t-1}(i) + \frac{R_{t-1}B_{t-1}(i)}{P_t} + \frac{M_{t-1}(i)}{P_t} + \frac{D_t(i)}{P_t} - [C_t(i) \right.$$
$$\left. + I_t(i) + \frac{B_t(i)}{P_t} + \frac{M_t(i)}{P_t}] \right\} + \psi_t(i)\{ K_t(i)$$
$$- \left[(1-\delta)K_{t-1}(i) + I_t(i) - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right) I_t(i) \right] \}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = C_t(i)^{-\sigma} - \lambda_t(i) = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial M_t} = \frac{1}{P_t}(\frac{M_t(i)}{P_t})^{-\nu} - \frac{1}{P_t}\lambda_t(i) + \frac{1}{P_{t+1}}\, \beta E_t \lambda_{t+1}(i) = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = N_t(i)^{\phi} - \lambda_t(i)\frac{W_t(i)}{P_t} = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial I_t} = \lambda_t(i) - \psi_t(i)\left[1 - S'\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\cdot \frac{I_t(i)}{I_{t-1}(i)} - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right]$$
$$- \beta E_t \psi_{t+1}(i)\left[-S'\left(\frac{I_{t+1}(i)}{I_t(i)}\right)\left(-\frac{I_{t+1}(i)}{I_t(i)^2}\right)\cdot I_{t+1}(i)\right] = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = \psi_t(i) + \beta E_t \lambda_{t+1}(i) r_{t+1}^k - \beta E_t \psi_{t+1}(i)(1-\delta) = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial B_t} = \frac{\beta \lambda_{t+1}(i) R_t}{P_{t+1}} - \frac{\lambda_t(i)}{P_t} = 0$$

The following equations can be obtained by rearranging the first-order conditions:

$$C_t(i)^{-\sigma} = \lambda_t(i) \tag{4.4}$$

$$m_t(i)^{-\nu} = \lambda_t(i) - \frac{\beta E_t \lambda_{t+1}(i)}{\Pi_{t+1}} \tag{4.5}$$

$$N_t(i)^{\phi} = \lambda_t(i)\frac{W_t(i)}{P_t} \tag{4.6}$$

$$\lambda_t(i) = \psi_t(i)\left[1 - S'\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\cdot\frac{I_t(i)}{I_{t-1}(i)} - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right] + \beta E_t\psi_{t+1}(i)\left[S'\left(\frac{I_{t+1}(i)}{I_t(i)}\right)\left(\frac{I_{t+1}(i)}{I_t(i)}\right)^2\right] \quad (4.7)$$

$$\psi_t(i) = \beta E_t\left[\lambda_{t+1}(i)r_{t+1}^k + \psi_{t+1}(i)(1-\delta)\right] \quad (4.8)$$

$$\lambda_t(i) = \beta R_t E_t\left[\frac{\lambda_{t+1}(i)}{\Pi_{t+1}}\right] \quad (4.9)$$

where λ_t and ψ_t are Lagrange multipliers associated with the household's budget constraint and capital accumulation equation. m_t is the real money balance and $\Pi_t = \frac{P_t}{P_{t-1}}$.

The conditions can be further reduced into

$$1 = \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left(\frac{C_{t+1}(i)}{C_t(i)}\right)^{-\sigma}\right] \quad (4.10)$$

$$\frac{m_t(i)^{-\nu}}{C_t(i)^{-\sigma}} = \frac{R_t - 1}{R_t} \quad (4.11)$$

$$q_t(i) = E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^k + (1-\delta)q_{t+1}(i)\right]\right\} \quad (4.12)$$

$$q_t(i)\left[1 - S'\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\cdot\frac{I_t(i)}{I_{t-1}(i)} - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right] = 1 - E_t\left[q_{t+1}(i)\frac{\lambda_{t+1}(i)}{\lambda_t(i)}S'\left(\frac{I_{t+1}(i)}{I_t(i)}\right)\left(\frac{I_{t+1}(i)}{I_t(i)}\right)^2\right] \quad (4.13)$$

Equation (4.10) is an Euler equation that describes the household's intertemporal consumption decisions respectively. Equation (4.11) is money demand equation implying that the opportunity cost of holding cash identical to the nominal interest rate. Equation (4.12) implies the mechanism of the asset price determination while the equation (4.13) is the process of the investment related to the adjustment costs. q_t is defined as Tobin's marginal q, equals $\frac{\psi_t}{\lambda_t}$. That is, q_t represents the increased real profit for each additional unit of capital stock.

For simplicity of derivation and analysis, focus on the symmetric equilibrium, i.e. $C_t(i) = C_t, M_t(i) = M_t, K_t(i) = K_t, q_t(i) = q_t$.

Next, consider labor supply decisions of households and the wage setting. Households supply their homogenous labor to an intermediate labor union which differentiates the labor services, sets wages following Calvo's (1983) rule and offers the labor services to intermediate labor packers. The labor packers purchase the differentiated labor services and provide it to the intermediate goods producers.

The aggregate labor supply is defined as

$$N_t(j) = \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{4.14}$$

$$N_t = \int_0^1 N_t\,(j)dj \tag{4.15}$$

where $N_t(i,j)$ is the i type of labor supply to the firm j, $j \in (0,1)$. N_t is a composite consist of differentiated labor services $N_t(j)$. ϵ_w denotes the elasticity of substitution between heterogeneous labors, $\epsilon_w > 1$.

The labor packers maximize profits:

$$max_{N(i,j)} W_t N_t(j) - \int_0^1 W_t(i) N_t(i,j) di \ \ s.t. \ \ N_t(j) = \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{4.16}$$

The first-order condition is

$$W_t \frac{\epsilon_w}{\epsilon_w - 1} \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1} - 1} N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w} - 1} \frac{\epsilon_w - 1}{\epsilon_w} = W_t(i) \tag{4.17}$$

Rearranging the above function, the demand function for i type of labor by firm j can be derived by

$$N_t(i,j) = \left(\frac{W_t}{W_t(i)} \right)^{-\epsilon_w} N_t(j) \tag{4.18}$$

Substituting Equation (13) to (12) yields the wage evolution function:

$$W_t = (\int_0^1 W_t\,(i)^{1-\epsilon_w} di)^{\frac{1}{1-\epsilon_w}} \tag{4.19}$$

Under Calvo's pricing assumption, in each period, each household has an equal probability of reconsidering its price with a probability ω where $0 < \omega < 1$. Households choose the optimal nominal wage $W_t(i)$ to maximize their utility. Utility is divided into two parts: one is the disutility brought by labor; The second is the positive effect of income generated by labor, which is converted into utility using the Lagrange multiplier. Supposing that the household is able to optimally adjust its wages $W_t(i)$ in the period t, and is unable to adjust its wages in the subsequent periods, $W_{t+s}(i) = W_t(i), s > 0$. The corresponding optimization problem is:

$$max_{W_t(i)} E_t \sum_{s=0}^{\infty} (\omega\beta)^s [\lambda_{t+s}(i) \frac{W_t(i)X_{t,s}}{P_{t+s}} N_{t+s}(i) - \psi \frac{N_t(i)^{1+\phi}}{1+\phi}] \tag{4.20}$$

It is constrained by the labor demand curve

$$N_{t+s}(i) \leq \left(\frac{W_t(i)X_{t,s}}{W_{t+s}}\right)^{-\epsilon_w} N_{t+s} \tag{4.21}$$

where,

$$X_{t,s} = \begin{cases} \Pi_1 \times \Pi_2 \times \ldots \times \Pi_t & t \geq 1 \\ 1 & t = 0 \end{cases}$$

Let $\widetilde{W}_t$ denote the value of $W_t(i)$ set by a household that can reoptimize its wage rate at time t, the Lagrangian function of the optimization of wage can be written as

$$\mathcal{L} \equiv E_t \sum_{s=0}^{\infty} (\omega\beta)^s [\lambda_{t+s}(i) \frac{\widetilde{W}_t X_{t,s}}{P_{t+s}} (\frac{\widetilde{W}_t X_{t,s}}{W_{t+s}})^{-\epsilon_w} N_{t+s} - \psi \frac{((\frac{\widetilde{W}_t X_{t,s}}{W_{t+s}})^{-\epsilon_w} N_{t+s})^{1+\phi}}{1+\phi}]$$

The first-order condition associated with $\widetilde{W}_t$ is:

$$\frac{\partial \mathcal{L}}{\partial W_t(i)} = E_t \sum_{s=0}^{\infty} (\omega\beta)^s (\psi \epsilon_w (\widetilde{W}_t X_{t,s})^{-(1+\phi)\epsilon_w - 1} (W_{t+s})^{(1+\phi)\epsilon_w} N_{t+s}{}^{1+\phi}$$

$$- E_t \sum_{s=0}^{\infty} (\omega\beta)^s (\frac{\lambda_{t+s}(i)}{P_{t+s}} (\epsilon_w - 1) (\widetilde{W}_t X_{t,s})^{-\epsilon_w} (W_{t+s})^{\epsilon_w} N_{t+s}) = 0$$

Divide both sides by $(\widetilde{W}_t X_{t,s})^{-\epsilon_w}$, the first-order condition of the optimal nominal wage is

$$(\widetilde{W}_t X_{t,s})^{1+\epsilon_w} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} (\omega\beta)^s (\psi (W_{t+s})^{(1+\phi)\epsilon_w} N_{t+s}{}^{1+\phi}}{E_t \sum_{s=0}^{\infty} (\omega\beta)^s (\frac{\lambda_{t+s}(i)}{P_{t+s}} (W_{t+s})^{\epsilon_w} N_{t+s})}$$

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \lambda_{t+s}(i) [\frac{\widetilde{W}_t X_{t,s}}{P_{t+s}} - \eta_w \frac{N_{t+s}(i)}{\lambda_{t+s}(i)}] N_{t+s}(i) \tag{4.22}$$

where I define $\eta_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

The aggregate wage is given by the Dixit-Stigltz form.

$$W_t{}^{1-\epsilon_w} = \left[(1-\omega)\widetilde{W}_t{}^{1-\epsilon_w} + \omega(\pi_{t-1} W_{t-1})^{1-\epsilon_w}\right] \tag{4.23}$$

4.2.2 Final goods producers

The final good Y_t is a composite made of an infinite continuum of intermediate goods $Y_t(j)$. The final goods sector produces goods by combining the intermediate goods, packages them into Y_t and sell to consumers, investors and the government in a perfectly competitive market.

$$Y_t(j) = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{4.24}$$

Under a given production technology (4.24), the final product producer takes the final product price and intermediate product price as given, and chooses the quantity of intermediate products to maximize profit:

$$max_{Y(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j) di \;\; s.t. \;\; Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{4.25}$$

The first-order condition is:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t \tag{4.26}$$

Equation (4.26) denotes the demand function of intermediate goods. It implies that the demand for intermediate goods depends on relative prices and price demand elasticity parameters ϵ_p, and at a given price index P_t, the demand for intermediate goods $Y_t(j)$ follows a downward sloping curve, that is, it declines as the price $P_t(j)$ rises.

The profits of the final goods producer is zero due to it faces perfect competition. Substituting Equation (4.26) into (4.24) yields the aggregate price index function:

$$P_t = (\int_0^1 P_t\,(j)^{1-\epsilon_p} dj)^{\frac{1}{1-\epsilon_p}} \tag{4.27}$$

4.2.3 Intermediate goods sector

Defining the production technology of the intermediate firm j is given by:

$$Y_t(j) = K_{t-1}(j)^{\alpha} N_t(j)^{1-\alpha} \tag{4.28}$$

The firm’s profit is given by:

$$P_t(j)Y_t(j) - W_tN_t(j) - r_t^k K_{t-1}(j) \tag{4.29}$$

where r_t^k is the rental rate on capital.

The Lagrange function for the minimization problem of firm's cost is:

$$\mathcal{L}_t = P_t(j)Y_t(j) - W_tN_t(j) - r_t^k K_{t-1}(j) + \psi_t(j)\left[K_{t-1}(j)^\alpha N_t(j)^{1-\alpha} - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t\right] \tag{4.30}$$

The first-order conditions for labor and capital stock are:

$$\big(\partial N_t(j)\big):\ W_t = \psi_t(j)(1-\alpha)K_{t-1}(j)^\alpha N_t(j)^{-\alpha} \tag{4.31}$$

$$\big(\partial K_{t-1}(j)\big):\ r_t^k = \psi_t(j)\alpha K_{t-1}(j)^{\alpha-1} N_t(j)^{1-\alpha} \tag{4.32}$$

where $\psi_t(j)$ is the Lagrange multiplier associated with the production function and equals marginal cost MC_t.

Combining the first-order conditions, yields:

$$\frac{W_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{K_{t-1}(j)}{N_t(j)} \tag{4.33}$$

All firms face the same marginal cost MC_t and equal to:

$$MC_t \equiv \psi_t(j) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}{W_t}^{1-\alpha}{r_t^k}^\alpha \tag{4.34}$$

The intermediate firm's profits are:

$$[\frac{P_t(j)}{P_t} - MC_t]\, P_tY_t(j)$$

Considering that the intermediate firm is able to adjust the price in period t and is unable to adjust the price in subsequent periods with probability γ. It is assumed that the optimal price is $\tilde{P}_t$, the maximization profits problem can be written by:

$$max_{\tilde{P}_t} E_t \sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s}[\frac{\tilde{P}_t X_{t+s}}{P_{t+s}} - MC_{t+s}] P_{t+s} Y_{t+s}(j),$$

$$s.t.\ Y_{t+s}(j) = \left(\frac{\tilde{P}_t X_{t+s}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s} \tag{4.35}$$

The first-order condition is:

$$P_t^* \equiv \frac{\epsilon_p}{\epsilon_p - 1}\frac{E_t \sum_{s=0}^{\infty}(\gamma\beta)^s \lambda_{t+s} mc_t\, {P_{t+s}}^{\epsilon_p} Y_{t+s}}{E_t \sum_{s=0}^{\infty}(\gamma\beta)^s \lambda_{t+s}\, {P_{t+s}}^{\epsilon_p - 1} Y_{t+s}} \tag{4.36}$$

The aggregate price is given by the Dixit-Stiglitz type constant elasticity of substitution aggregator and it is divided into the changed price component and the unchanged price component.

$${P_t}^{1-\epsilon_p} = (1-\gamma){\tilde{P}_t}^{1-\epsilon_p} + \gamma(\pi_{t-1}P_{t-1})^{1-\epsilon_p} \tag{4.37}$$

4.2.4 Monetary and fiscal policy

Assume that the impulse responses to both the Taylor (1993) rule and the money growth rule that discussed in Christiano, et al. (2001).

As standard in the literature, the central bank sets nominal interest rates R_t by following a Taylor(1993) rule which responds to both output and inflation:

$$R_t = \phi_\pi(\pi_t - \pi) + \phi_y(Y_t - Y) + \tau_t^i \tag{4.38}$$

Y is the steady-state value of the output Y_t, ϕ_π, ϕ_y are parameters that indicate the sensitivity of interest rates to changes in inflation and output gap, and $\phi_\pi, \phi_y > 0$. ϵ_t^i is an exogenous shock.

The money growth rule satisfies an AR(1) progress:

$$\Delta logM_t = \rho_m \Delta logM_{t-1} + \tau_t^m$$

$$\Delta logM_t \equiv logM_t - logM_{t-1}$$

where ρ_m is the persistence parameter, τ_t^m is an exogenous shock.

Finally, it is assumed that the government has access to lump sum taxes and seeks a Ricardian fiscal policy. Under this assumption, fiscal policy has no effect on aggregate economic variables. Therefore, the fiscal policy is not specifically set in this model. (Sims 1994, Woodford 1994)

4.2.5 Aggregation

The aggregation of the household's budget constraint and the intermediate firm's budget constraint in i and j are:

$$P_tC_t + P_tI_t + B_t + M_t = W_tN_t + P_tr_t^kK_{t-1} + R_{t-1}B_{t-1} + M_{t-1} + D_t$$

$$D_t = P_tY_t - W_tN_t - P_tr_t^kK_{t-1}$$

Integrating the budget constraint across households and combing with the government budget constraint, the goods market clearing condition can be obtained by:

$$Y_t = C_t + I_t \tag{4.39}$$

4.3 Steady state conditions

First, from the Euler equation,

$$R = \frac{1}{\beta}$$

The capital rental rate is:

$$r^k = \bar{R} + \delta - 1$$

The optimal price equation at the steady state yields the steady state value of the marginal cost:

$$mc = \frac{\epsilon_p - 1}{\epsilon_p}$$

Accordingly, the real wage at the steady state is:

$$w = (1-\alpha)(\alpha^\alpha \frac{mc}{(r^k)^\alpha})^{\frac{1}{1-\alpha}} = (1-\alpha)(\alpha^\alpha \frac{\frac{\epsilon_p - 1}{\epsilon_p}}{(r^k)^\alpha})^{\frac{1}{1-\alpha}}$$

Further, according to the capital return function (4.33), the labor-capital ratio at the steady state is:

$$\frac{K}{N} = \frac{w}{r^k}\frac{\alpha}{1-\alpha}$$

From (4.28) it is easily obtained the steady state value of output as follows:

$$Y = K^\alpha N^{1-\alpha}$$

Combing the above two equation yields the output-capital ratio:

$$\frac{K}{Y} = (\frac{(1-\alpha)r^k}{w})^{\alpha-1}$$

Finally, the steady state value for consumption-output ratio is:

$$\frac{C}{Y} = 1 - \frac{I}{Y}$$

4.4 Linearization

In this section, I log-linearize the equations of the household's behavior (4.10)-(4.13) around the steady state.

First, taking the derivative of logarithmic function of the equation (4.10) and expanding both sides around the steady state derives:

$$1 = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} \left(\frac{C_{t+1}(i)}{C_t(i)} \right)^{-\sigma} \right] \tag{4.10}$$

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{r}_t - E_t \pi_{t+1} - \rho) \tag{4.40}$$

where $\rho \equiv ln\beta$, $\hat{r}_t \equiv \frac{R_t-\bar{R}}{\bar{R}}$, $\hat{c}_t \equiv \frac{C_t-\bar{C}}{\bar{C}}$.

Similarly, taking the derivative of logarithmic function of the equation (4.11) and substituting $\bar{R} \equiv \frac{1}{\beta}$, the linearized equation can be obtained as follows.

$$\frac{m_t(i)^{-\nu}}{C_t(i)^{-\sigma}} = \frac{R_t - 1}{R_t} \tag{4.11}$$

$$\hat{r}_t = \frac{\sigma(1-\beta)}{\beta}\hat{c}_t - \frac{\nu(1-\beta)}{\beta}\widehat{m}_t \tag{4.41}$$

where $\widehat{m}_t \equiv \frac{m_t-\bar{m}}{\bar{m}}$.

Next, consider the linearization of the asset price determination function. Equation (4.4) implies $\frac{\lambda_{t+1}}{\lambda_t} = (\frac{C_{t+1}}{C_t})^{-\sigma}$, and substituting it into (4.10) derives $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta}\frac{E_t\pi_{t+1}}{R_t}$. As a result, the (4.12) can be rearranged by:

$$q_t(i) = E_t\left\{\frac{1}{\beta}\frac{\pi_{t+1}}{R_t}\left[r_{t+1}^k + (1-\delta)q_{t+1}(i)\right]\right\}$$

The log expression can be approximated by a first-order Taylor polynomial at the steady state as:

$$\hat{\imath}_t = \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} + \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{\theta\beta}{1+\beta}\hat{q}_t \tag{4.43}$$

where $\theta \equiv \frac{1}{S''}$, $\hat{q}_t = \frac{q_t-\bar{q}}{\bar{q}}$, $\hat{\imath}_t = \frac{I_t-\bar{I}}{I}$.

Taking the derivative of logarithmic function of the equation (4.28) obtains:

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t \tag{4.44}$$

Furthermore, take the derivative of logarithm of (4.33):

$$\frac{W_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{K_{t-1}(j)}{N_t(j)} \tag{4.33}$$

$$\hat{k}_{t-1} = \widehat{w}_t - \hat{r}_t^k + \hat{n}_t \tag{4.45}$$

where $\hat{k}_t = \frac{K_t-K}{K}$, $\hat{r}_t^k = \frac{r_t^k-r^k}{r^k}$, $\hat{n}_t = \frac{N_t-N}{N}$.

Next, I focus on the derivation of the wage Philips curve (WPC).

For convenience, I define:

$$\widetilde{w}_t = \frac{\widetilde{W}_t}{W_t}\ ,w_t = \frac{W_t}{P_t}$$

Linearizing (4.22) around the steady state and rearranging it by making use of (4.18) yields:

$$\widehat{\widetilde{w}}_t + \widehat{w}_t = \sum_{t=1}^{\infty} (\beta\omega)^s (\hat{\pi}_{t+s} - \hat{\pi}_{t+s-1}) + \frac{(1-\beta\omega)(\eta_w - 1)}{2\eta_w - 1} \sum_{j=s}^{\infty} (\beta\omega)^j (\hat{n}_{t+j} + \frac{\eta_w}{\eta_w - 1} \widehat{w}_{t+j} - \hat{c}_{t+j})$$

Combining the above expression with the linearized deviation of (4.22) obtains:

$$\kappa_w \widehat{w}_t = \beta E_t \widehat{w}_{t+1} + \widehat{w}_{t-1} + \beta E_t (\hat{\pi}_{t+1} - \hat{\pi}_t) - (\hat{\pi}_t - \hat{\pi}_{t-1}) + \frac{(1-\eta_w)\sigma}{b_w \omega} \hat{c}_t - \frac{1-\eta_w}{b_w \omega} \hat{n}_t \quad (4.46)$$

where $\kappa_w \equiv \frac{b_w(1+\beta\gamma^2)-\eta_w}{b_w \omega}$, $b_w \equiv \frac{2\eta_w - 1}{(1-\omega)(1-\beta\omega)}$.

Next, move to the derivation of the New Keynesian Philips curve. Linearizing the equation (4.36) by employing Uhilig's (1999) method, the equation can be rewritten as:

$$P_t^{1-\epsilon_p} = (1-\gamma)\tilde{P}_t^{1-\epsilon_p} + \gamma(\pi_{t-1}P_{t-1})^{1-\epsilon_p} \quad (4.37)$$

$$P_t^* = \frac{\gamma}{1-\gamma}(\hat{\pi}_t - \hat{\pi}_{t-1}) \quad (4.47)$$

Log-linearizing (4.36) and (4.37) and combining the two linearized equations yield the dynamic equation of inflation:

$$\hat{\pi}_t = \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} + \frac{1}{1+\beta} \hat{\pi}_{t-1} + \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\beta)} mc_t \quad (4.48)$$

Equation (4.45) is the New Keynesian Philips curve (NKPC), which describes the supply side of the economy. The detail of the derivation of (4.48) is shown in Appendix. It shows that the effects of stickiness are on the marginal cost of intermediate firms mc_t.

Moreover, the log-linear form of real marginal cost (4.34) can be written as:

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1-\alpha)\widehat{w}_t \quad (4.49)$$

The aggregate asset constraints (4.39) can be converted into log-deviations form:

$$\hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{\imath}_t \quad (4.50)$$

Finally, the linearization of the monetary policy is

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \tau_t^i \quad (4.51)$$

The exogenous monetary policy shock τ_t^i is determined as

$$\tau_t^i = \rho_i \tau_{t-1}^i + \epsilon_t^i, \ \epsilon_t^i \sim i.i.d\left(0, \sigma_i^2\right) \quad (4.52)$$

The linearization of money growth rule is:

$$\mu_t = \rho_m \tau_{t-1}^m + \epsilon_t^m, \qquad \epsilon_t^m \sim i.i.d(0, \sigma_m^2) \quad (4.53)$$

where $\mu_t \equiv M_t - M_{t-1}$ represents the money growth rate. Since in linearized form $m_t = M_t - p_t$, thus,

$$\mu_t = m_t - m_{t-1} + \pi_t$$

In additional, the zero lower bound constraint is given by

$$\hat{r}_t \geq -\ln\left(\frac{1}{\beta}\right)$$

Heretofore, the endogenous variables are $\{\hat{c}_t, \hat{r}_t, \hat{\pi}_t, \widehat{m}_t, \hat{q}_t, \hat{r}_t^k, \hat{\imath}_t, \hat{y}_t, \hat{k}_t, \hat{n}_t, \widehat{w}_t, \mu_t, \tau_t^i\}$ and the linearization equations are (4.40) - (4.53).

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{r}_t - E_t\hat{\pi}_{t+1} - \rho) \tag{4.40}$$

$$\hat{r}_t = \frac{\sigma(1-\beta)}{\beta}\hat{c}_t - \frac{\nu(1-\beta)}{\beta}\widehat{m}_t \tag{4.41}$$

$$\hat{q}_t = E_t\hat{\pi}_{t+1} - \hat{r}_t + \frac{r^k}{1+r^k-\delta}\hat{r}_{t+1}^k + \frac{1-\delta}{1+r^k-\delta}E_t\hat{q}_{t+1} \tag{4.42}$$

$$\hat{\imath}_t = \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} + \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{\theta\beta}{1+\beta}\hat{q}_t \tag{4.43}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t \tag{4.44}$$

$$\hat{k}_{t-1} = \widehat{w}_t - \hat{r}_t^k + \hat{n}_t \tag{4.45}$$

$$\kappa_w\widehat{w}_t = \beta E_t\widehat{w}_{t+1} + \widehat{w}_{t-1} + \beta E_t(\hat{\pi}_{t+1} - \hat{\pi}_t) - (\hat{\pi}_t - \hat{\pi}_{t-1}) + \frac{(1-\eta_w)\sigma}{b_w\omega}\hat{c}_t - \frac{1-\eta_w}{b_w\omega}\hat{n}_t \tag{4.46}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\beta)}\widehat{mc}_t \tag{4.48}$$

$$\widehat{mc}_t = \alpha\hat{r}_t^k + (1-\alpha)\widehat{w}_t \tag{4.49}$$

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{\imath}_t \tag{4.50}$$

$$\hat{r}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t + \tau_t^i \tag{4.51}$$

$$\tau_t^i = \rho_i\tau_{t-1}^i + \epsilon_t^i, \tag{4.52}$$

$$\mu_t = \rho_m\tau_{t-1}^m + \epsilon_t^m \tag{4.53}$$

4.5 Calibration

As in standard literatures, the value of household's discount factor β is set to be 0.99 which means the annual risk-free rate is 4.1%[14], the capital share of the production function α equals 0.3, and the depreciation rate δ is fixed at 0.025 (on a quarterly basis) which implies an annual rate of depreciation on capital income equal to 10 percent. The parameters of the utility function

[14] According to the nominal interest rate function $i = \pi/\beta$, as the inflation at the steady state is assumed to be zero and the data frequency corresponding to the model is quarterly, so the annual nominal interest rate is $i^4 - 1 = \left(\frac{\pi}{\beta}\right)^4 - 1 = 4.1\%$.

are assumed to be distributed as follows. The elasticity of labor supply is set at 1.5, and the elasticity of substitution in money is 2. These are all standard calibrations.

I set both the value of stickiness in prices and in wages to be 0.75 which means the average price adjustment period and the average wage adjustment period are both 1 year, namely 4 quarters. In addition, elasticity of substitution between different intermediates and between heterogeneous labor are assumed to equal 11 which implies that the markup of wages and prices are 10%. The value of θ which is the inverse of quadratic differential of investment adjustment function is assumed to be 7 based on Christiano, et al. (2005).

Furthermore, the Taylor rule weight of output and inflation are set as 0.5 and 1.5 respectively. The persistence parameters of exogenous shock are both set at 0.5.

Share/Parameter	Description	Value
Steady state value		
C/Y	Consumption to GDP	0.5488
I/Y	Investment to GDP	0.4512
r^k	Rental rate on capital	0.0351
mc	Marginal cost	0.909
Parameters		
β	Discount factor	0.99
σ	Elasticity of substitution in labor	1.5
ν	Elasticity of substitution in money	2
δ	Capital depreciation rate	0.025
α	Capital share of the production function	0.3
ω	Stickiness in wages	0.75
γ	Stickiness in prices	0.75
ϵ_p	Elasticity of substitution between different intermediates	11
ϵ_w	Elasticity of substitution between heterogeneous labor	11
θ	Inverse of quadratic differential of investment adjustment function	7
ϕ_π	Inflation Taylor rule weight	1.5
ϕ_y	Output Taylor rule weight	0.5
ρ_i	Autocorrelation of money policy shock	0.5
ρ_m	Autocorrelation of money growth rule shock	0.5

TABLE 4.1 THE VALUES OF EACH PARAMETERS

4.6 Simulation

4.6.1 The HP algorithm method

First of all, I introduce the HP algorithm method to cope with the ZLB constraint.

When the ZLB constraint is binding, the nominal interest rates likely stay at zero for some terms. As agents can manage their decision given the information about the time that shock will happen, thus, the impulse responses can be treated as an anticipated shock. The structure of the HP algorithm is accommodated to replace future ZLB with anticipated shock by adding the "shadow shocks".

Introducing the shadow shock term to the Taylor rule:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \tau_t^i + \sum_{s=0}^{T^*-1} v_{t,s}^n \tag{4.54}$$

where $v_{t,s}^n$ is the shadow shock which is known at $t-s$ and occurs at period t. For instance, when $s=3$, $v_{t,s}^n$ can be written in an AR(1) process as:

$$x_t = \rho_x x_{t-1} + v_t + v_{t,3}^n$$

$$v_{t,3}^n = shadow3_t$$

$$\begin{bmatrix} shadow3_t \\ shadow2_t \\ shadow1_t \end{bmatrix} = \begin{bmatrix} shadow2_{t-1} \\ shadow1_{t-1} \\ \varepsilon \end{bmatrix}$$

When there is a zero bound constraint, the nominal interest rates have to satisfy

$$r_t^z = \max\left\{-\ln\left(\frac{1}{\beta}\right), r_t\right\} \tag{4.55}$$

According to Holden and Paetz (2012), the above function can be converted into a parameter weighted form:

$$r_t^z = U_{r,t} + \ln\left(\frac{1}{\beta}\right) + A_{r,t}\alpha \tag{4.56}$$

where U_r is the shock response of the interest rate to an unanticipated policy shock and A_r is the responses to T^*, α is a $T^* \times 1$ vector.

As a result, to deal with the ZLB constraint, the key is to find the optimal value of vector α.

To solve α, the method applies a complementary slackness condition as follows:

$$\alpha^{\intercal}\left(U_r + \ln\left(\frac{1}{\beta}\right) + A_r\alpha\right) = 0 \tag{4.57}$$

$$\alpha^* = \arg\min \alpha^{\intercal}\left(U_r + \ln\left(\frac{1}{\beta}\right) + A_r\alpha\right) \tag{4.58}$$

$$s.t.\, \alpha \geq 0, U_r + \ln\left(\frac{1}{\beta}\right) + A_r\alpha \geq 0$$

If the objective function is near zero, it regards α^* as satisfying the complementary condition.

Finally, the responses solving ZLB for each variable is:

$$U_l + A_l\alpha^* \tag{4.59}$$

4.6.2 Impulse responses under the ZLB constraint

In Figure 4.1, the black line shows the responses to the Taylor rule shock without the ZLB constraint, while the blue line implies the impulse response with the ZLB by the HP algorithm.

Under an easing monetary shock, according to the Taylor rule, the short-term nominal interest rate falls, thus bringing about an expansionary effect: the decrease in the nominal interest rate causes the increase in the labor supply, money supply, inflation, investment and output. The change range of consumption is modest, retaining approximate a horizontal trend. This is because the effect of monetary stimulus on consumption is offset by a fall in wages as the supply of labor increases. On the whole, since the ZLB constraint is not binding, the monetary policy under the Taylor rule is significantly effective.

As shown in the blue line, the effect of monetary easing policy is distinct weakening compared to the result without the constraint. The interest rate approximately keeps staying at zero for all the time and other economic indicators such as output, interest rates and labor supply have a fraction of response to the shock. The nominal interest rates cannot be reduced under zero anymore due to the ZLB constraint, which results in the decrease in the ability of easing. This absence of significant easing dampens the effectiveness of monetary policy attempts to stimulate the economy by adjusting interest rates.

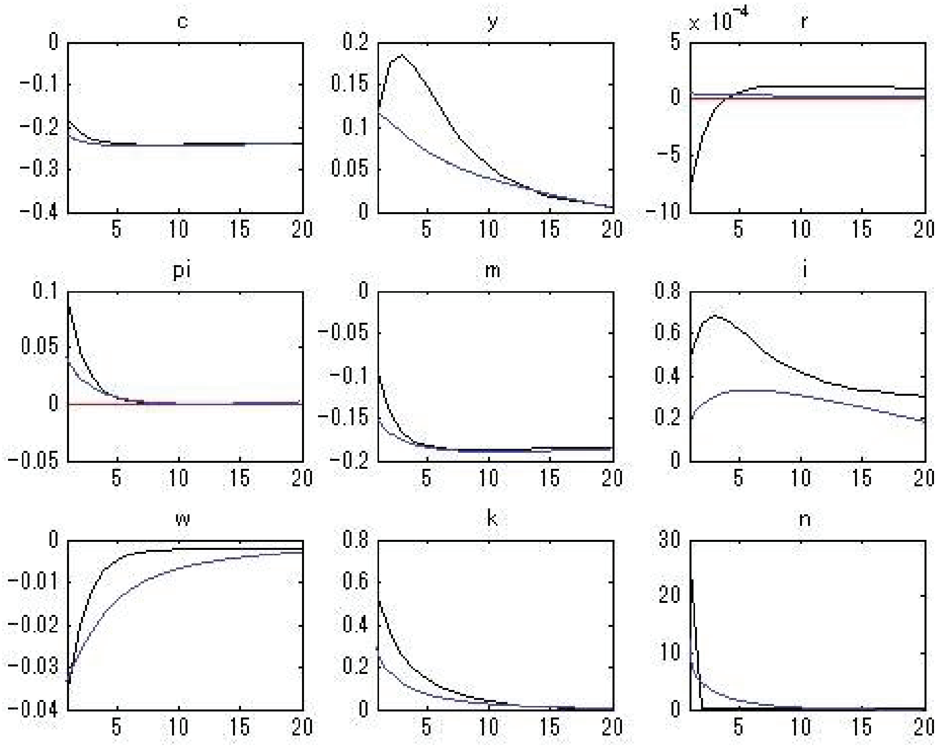

FIGURE 4.1 IMPULSE RESPONSES TO MONETARY POLICY SHOCKS UNDER THE TAYLOR RULE

4.6 Conclusion

This chapter explored the impact of monetary shock by utilizing a medium scale DSGE model with nominal rigidities in price and wage setting, real frictions such as investment adjustment costs. The model consists of two types of firms which are final good firm and intermediate good producing firm and both fiscal and monetary authorities. Since it is assumed that the government has access to lump sum taxes and seeks a Ricardian fiscal policy, which means fiscal policy does not affect aggregate economic activities, therefore the fiscal policy was not specifically set in the model. The results of simulation indicates that under the non-negative constraint, the monetary shocks still affect the economic variables even though the effects are not as significant as without the zero lower bound constraint on nominal interest rates.

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APPENDIX 4

Method

The method that implemented in this subsection primarily based on Uhilig(1999).

Define the log-deviation of variable x_t from its steady state x as:

$$\tilde{x}_t \equiv lnx_t - lnx$$

The right hand side of the above equation can be rewritten as:

$$\ln\left(\frac{x_t}{x}\right) = \ln\left(1 + \frac{x_t - x}{x}\right)$$

The log expression can be approximated by a first-order Taylor polynomial at the steady state $x_t = x$,

$$\ln\left(1 + \frac{x_t - x}{x}\right) \cong ln1 + \frac{1}{x}(x_t - x) = \frac{x_t - x}{x}$$

As a result,

$$\tilde{x}_t \approx \frac{x_t - x}{x}$$

Moreover, since $xe^{\tilde{x}_t} \approx xe^{lnx_t - lnx} = xe^{\frac{lnx_t}{lnx}} = x\frac{x_t}{x} = x_t$

Thus, $x_t \approx xe^{\tilde{x}_t}$ holds.

Linearization

Log-linearize the equations of the household's behavior (4.10)-(4.13) around the steady state. First, taking the derivative of logarithmic function of the equation (4.10) derives:

$$1 = \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left(\frac{C_{t+1}(i)}{C_t(i)}\right)^{-\sigma}\right] \tag{4.10}$$

$$ln1 = ln\beta + ln\bar{R} - E_t ln\pi_{t+1} - \sigma(E_t lnC_{t+1} - lnC_t)$$

Expanding both sides around the steady state yields

$$0 = ln\beta + \frac{R_t - \bar{R}}{\bar{R}} - E_t\left(\frac{\pi_{t+1} - \bar{\pi}}{\bar{\pi}}\right) - \sigma\left(E_t\frac{C_{t+1} - \bar{C}}{\bar{C}} - \frac{C_t - \bar{C}}{\bar{C}}\right)$$

Rewriting the above equation yields:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{r}_t - E_t\pi_{t+1} - \rho) \tag{4.40}$$

where $\rho \equiv ln\beta$, $\hat{r}_t \equiv \frac{R_t - \bar{R}}{\bar{R}}$, $\hat{c}_t \equiv \frac{C_t - \bar{C}}{\bar{C}}$.

Similarly, taking the derivative of logarithmic function of the equation (4.11):

$$\frac{m_t(i)^{-\nu}}{C_t(i)^{-\sigma}} = \frac{R_t - 1}{R_t} \quad (4.11)$$

$$-\nu ln m_t + \sigma ln C_t = \ln(R_t - 1) - ln R_t$$

$$-\nu(ln m_t + \widehat{m}_t) + \sigma(ln C_t + \hat{c}_t) = \ln(R_t - 1) + \frac{1}{R-1}(R_t - \bar{R}) - ln\bar{R} - \frac{1}{\bar{R}}(R_t - \bar{R})$$

Substituting $\bar{R} \equiv \frac{1}{\beta}$ into above equation, the linearized equation can be obtained as follows.

$$\hat{r}_t = \frac{\sigma(1-\beta)}{\beta}\hat{c}_t - \frac{\nu(1-\beta)}{\beta}\widehat{m}_t \quad (4.41)$$

where $\widehat{m}_t \equiv \frac{m_t - \bar{m}}{\bar{m}}$.

Next, consider the linearization of the asset price determination function. Equation (4.4) implies $\frac{\lambda_{t+1}}{\lambda_t} = (\frac{C_{t+1}}{C_t})^{-\sigma}$, and substituting it into (4.10) derives $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta}\frac{E_t\pi_{t+1}}{R_t}$. As a result, the (4.12) can be rearranged by:

$$q_t(i) = E_t\left\{\frac{1}{\beta}\frac{\pi_{t+1}}{R_t}\left[r_{t+1}^k + (1-\delta)q_{t+1}(i)\right]\right\}$$

Taking the logarithm on both sides results in:

$$ln q_t = -ln\beta + ln E_t\pi_{t+1} - ln R_t + \ln\left(r_{t+1}^k + (1-\delta)q_{t+1}\right)$$

The log expression can be approximated by a first-order Taylor polynomial at the steady state as:

$$ln q_t + \hat{q}_t = -ln\beta + ln E_t\pi_{t+1} - ln\bar{R} - \hat{r}_t + [\ln(r^k + (1-\delta)\bar{q}) + \frac{1}{r^k + (1-\delta)}\left(r_{t+1}^k - r^k\right)$$
$$+ \frac{1-\delta}{r^k + (1-\delta)}(q_{t+1} - \bar{q})]$$

$$\hat{q}_t = E_t\pi_{t+1} - \hat{r}_t + \frac{r^k}{1 + r^k - \delta}\hat{r}_{t+1}^k + \frac{1-\delta}{1 + r^k - \delta}E_t\hat{q}_{t+1} \quad (4.42)$$

The linearization of the equation (4.13) employs the first-order Taylor polynomial expansion. For simplicity, I assume $S(1) = S'(1) = 0$. The following relationship can be derived:

$$\hat{q}_t - S''(\hat{\imath}_t - \hat{\imath}_{t-1}) = -\beta(S''E_t\hat{\imath}_{t+1} - S''\hat{\imath}_t)$$

Arranging it results in:

$$\hat{\imath}_t = \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} + \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{\theta\beta}{1+\beta}\hat{q}_t \quad (4.43)$$

where $\theta \equiv \frac{1}{S''}$, $\hat{q}_t = \frac{q_t - \bar{q}}{\bar{q}}$, $\hat{\imath}_t = \frac{I_t - \bar{I}}{I}$.

Taking the derivative of logarithmic function of the equation (4.28) obtains:

$$ln Y_t = \alpha ln K_{t-1} + (1-\alpha) ln N_t$$

Subtracting the steady state values of logarithmic form of each variable from both sides derives:

$$lnY_t - lnY = \alpha(lnK_{t-1} - lnK) + (1-\alpha)(lnN_t - lnN)$$

As a result, the linearized production function is given by

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t \tag{4.44}$$

Furthermore, take the derivative of logarithm of (4.33):

$$\frac{W_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{K_{t-1}(j)}{N_t(j)} \tag{4.33}$$

$$lnW_t - lnr_t^k = \ln(1-\alpha) - ln\alpha + lnK_{t-1} - lnN_t$$

The steady state of the above equation is:

$$lnW - lnr^k = \ln(1-\alpha) - ln\alpha + lnK - lnN$$

Subtracting the steady state values of each variable from both sides, the above equation can be rearranged as:

$$\hat{k}_{t-1} = \hat{w}_t - \hat{r}_t^k + \hat{n}_t \tag{4.45}$$

where $\hat{k}_t = \frac{K_t - K}{K}, \hat{r}_t^k = \frac{r_t^k - r^k}{r^k}, \hat{n}_t = \frac{N_t - N}{N}$.

Next, I focus on the derivation of the wage Philips curve (WPC).

For convenience, I define:

$$\widetilde{w}_t = \frac{\widetilde{W}_t}{W_t} \quad , w_t = \frac{W_t}{P_t}$$

Linearizing (4.22) around the steady state and rearranging it by making use of (4.18) yields:

$$\hat{\widetilde{w}}_t + \hat{w}_t = \sum_{t=1}^{\infty}(\beta\omega)^s(\hat{\pi}_{t+s} - \hat{\pi}_{t+s-1}) + \frac{(1-\beta\omega)(\eta_w - 1)}{2\eta_w - 1}\sum_{j=s}^{\infty}(\beta\omega)^j(\hat{n}_{t+j} + \frac{\eta_w}{\eta_w - 1}\hat{w}_{t+j} - \hat{c}_{t+j})$$

$\hat{\widetilde{w}}_t + \hat{w}_t$ corresponds to the percentage deviation of the household's real wage rate from its steady state value and $\hat{\widetilde{w}}_t = \hat{w}_t - \hat{\pi}_t$.

Dividing by P_t and linearizing about steady state of (4.22) obtains:

$$(1-\omega)\widetilde{w}_t = \omega\hat{w}_t - \omega(\hat{w}_{t-1} - (\hat{\pi}_t - \hat{\pi}_{t-1}))$$

Combining the above expression with the linearized deviation of (4.22) obtains:

$$0 = \hat{w}_{t-1} - \frac{b_w(1+\beta\gamma^2) - \eta_w}{b_w\omega}\hat{w}_t + \beta E_t\hat{w}_{t+1} + \beta E_t(\hat{\pi}_{t+1} - \hat{\pi}_t) - (\hat{\pi}_t - \hat{\pi}_{t-1}) + \frac{(1-\eta_w)\sigma}{b_w\omega}\hat{c}_t - \frac{1-\eta_w}{b_w\omega}\hat{n}_t$$

Rearranging it yields:

$$\kappa_w \widehat{w}_t = \beta E_t \widehat{w}_{t+1} + \widehat{w}_{t-1} + \beta E_t(\widehat{\pi}_{t+1} - \widehat{\pi}_t) - (\widehat{\pi}_t - \widehat{\pi}_{t-1}) + \frac{(1-\eta_w)\sigma}{b_w\omega}\hat{c}_t - \frac{1-\eta_w}{b_w\omega}\widehat{n}_t \qquad (4.46)$$

where $\kappa_w \equiv \frac{b_w(1+\beta\gamma^2)-\eta_w}{b_w\omega}$, $b_w \equiv \frac{2\eta_w - 1}{(1-\omega)(1-\beta\omega)}$.

The detail of the derivation of the New Keynesian Philips curve will be shown in next section.

Moreover,

$$MC_t \equiv \psi_t(j) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} {W_t}^{1-\alpha} {r_t^k}^{\alpha} \qquad (4.34)$$

$$lnMC_t = -\alpha ln\alpha - (1-\alpha)\ln(1-\alpha) + (1-\alpha)lnW_t + \alpha lnr_t^k$$

At the steady state, $lnMC = -\alpha ln\alpha - (1-\alpha)\ln(1-\alpha) + (1-\alpha)lnW + \alpha lnr^k$

Subtracting the steady state values of each variable from both sides:

$$lnMC_t - lnMC = (1-\alpha)(lnW_t - lnW) + \alpha(lnr_t^k - lnr^k)$$

The log-linear form of real marginal cost (4.34) can be written as:

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1-\alpha)\widehat{w}_t \qquad (4.49)$$

The aggregate asset constraints (4.39) can be converted into log-deviations form:

$$Y(1+\hat{y}_t) = C(1+\hat{c}_t) + I(1+\hat{\imath}_t)$$

Multiply out the log-deviations equation and subtract Y on the left and $(C+I)$ on the right to obtain:

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{\imath}_t \qquad (4.50)$$

Finally, the linearization of the monetary policy is

$$\hat{r}_t = \phi_\pi \widehat{\pi}_t + \phi_y \hat{y}_t + \tau_t^i \qquad (4.51)$$

The exogenous monetary policy shock τ_t^i is determined as

$$\tau_t^i = \rho_i \tau_{t-1}^i + \epsilon_t^i,\ \ \epsilon_t^i \sim i.i.d\left(0, \sigma_i^2\right) \qquad (4.52)$$

The linearization of money growth rule is:

$$\mu_t = \rho_m \tau_{t-1}^m + \epsilon_t^m, \qquad \epsilon_t^m \sim i.i.d(0, \sigma_m^2) \qquad (4.53)$$

where $\mu_t \equiv M_t - M_{t-1}$ represents the money growth rate. Since in linearized form $m_t = M_t - p_t$, thus,

$$\mu_t = m_t - m_{t-1} + \pi_t$$

In additional, the zero lower bound constraint is given by

$$\hat{r}_t \geq -\ln\left(\frac{1}{\beta}\right)$$

Derivation of non-linear new-Keynesian Philips curve

The first-order condition of firms resulting from price setting is represented by:

$$E_t \sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s}[\frac{\tilde{P}_t X_{t+s}}{P_{t+s}} - \chi mc_{t+s}] P_{t+s} Y_{t+s}(j) = 0$$

$$Y_{t+s}(j) = \left(\frac{\tilde{P}_t X_{t+s}}{P_{t+s}}\right)^{\frac{\chi}{1-\chi}} Y_{t+s} \tag{4.35}$$

where

$\chi = \frac{\epsilon_p}{\epsilon_p - 1}$, and ϵ_p denotes an elasticity of substitution

$$\frac{\chi}{1-\chi} = -\epsilon_p$$

The FOC can be expanded in the following way:

$$E_t \sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s}[\frac{\tilde{P}_t X_{t+s}}{P_{t+s}} - \chi mc_{t+s}] \left(\frac{\tilde{P}_t X_{t+s}}{P_{t+s}}\right)^{\frac{\chi}{1-\chi}} Y_{t+s}$$

$$= \lambda_t \left[\frac{\tilde{P}_t}{P_t} - \chi mc_t\right] \left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\chi}{1-\chi}} P_t Y_t + \gamma\beta\lambda_{t+1} \left[\frac{\tilde{P}_t}{P_{t+1}} \pi_t^{\gamma} - \chi mc_{t+1}\right] \left(\pi_t^{\gamma} \frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\chi}{1-\chi}} P_{t+1} Y_{t+1}$$

$$+ (\gamma\beta)^2 \lambda_{t+2} \left[\frac{\tilde{P}_t}{P_{t+2}} \pi_t^{\gamma} \pi_{t+1}^{\gamma} - \chi mc_{t+2}\right] \left(\pi_t^{\gamma} \pi_{t+1}^{\gamma} \frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\chi}{1-\chi}} P_{t+2} Y_{t+2} + \cdots$$

$$= \lambda_t \frac{\tilde{P}_t}{P_t} \left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\chi}{1-\chi}} P_t Y_t - \lambda_t \chi mc_t \left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\chi}{1-\chi}} P_t Y_t + \gamma\beta\lambda_{t+1} \frac{\tilde{P}_t}{P_{t+1}} \left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\chi}{1-\chi}} P_{t+1} Y_{t+1}$$

$$-\gamma\beta\lambda_{t+1} \chi mc_{t+1} \left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\chi}{1-\chi}} P_{t+1} Y_{t+1} + (\gamma\beta)^2 \lambda_{t+2} \frac{\tilde{P}_t}{P_{t+2}} \left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\chi}{1-\chi}} P_{t+2} Y_{t+2}$$

$$- (\gamma\beta)^2 \lambda_{t+2} \chi mc_{t+2} \left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\chi}{1-\chi}} P_{t+2} Y_{t+2} + \cdots$$

Below I divide the above equation into two parts, first part is $\sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s} \frac{\tilde{P}_t}{P_{t+s}} X_{t+s} P_{t+s} Y_{t+s}$ and the second part is $\sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s} \chi mc_{t+s} P_{t+s} Y_{t+s}$.

First part

Denoting the first part as $P_t x_t^1$

$$P_t x_t^1 = \lambda_t (\tilde{p}_t)^{\frac{1}{1-\chi}} P_t Y_t + \gamma\beta\lambda_{t+1} \left(\pi_t^{\gamma} \frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\chi}} P_{t+1} Y_{t+1}$$

$$+ (\gamma\beta)^2 \lambda_{t+2} \left(\pi_t^{\gamma} \pi_{t+1}^{\gamma} \frac{\tilde{p}_t}{\pi_{t+1}\pi_{t+2}}\right)^{\frac{1}{1-\chi}} P_{t+2} Y_{t+2} + \cdots$$

where $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$.

Forwarding one period the equation, the equation becomes:

$$P_{t+1} x_{t+1}^1 = \lambda_{t+1} (\tilde{p}_{t+1})^{\frac{1}{1-\chi}} P_{t+1} Y_{t+1} + \gamma\beta\lambda_{t+2} \left(\pi_{t+1}^{\gamma} \frac{\tilde{p}_{t+1}}{\pi_{t+2}}\right)^{\frac{1}{1-\chi}} P_{t+2} Y_{t+2}$$

$$+ (\gamma\beta)^2 \lambda_{t+3} \left(\pi_{t+1}^{\gamma} \pi_{t+2}^{\gamma} \frac{\tilde{p}_{t+1}}{\pi_{t+2}\pi_{t+3}}\right)^{\frac{1}{1-\chi}} P_{t+3} Y_{t+3} + \cdots$$

Next, we look for an expression φ_t that meets the following condition:

$$\varphi_t \times \lambda_{t+1} (\tilde{p}_{t+1})^{\frac{1}{1-\chi}} P_{t+1} Y_{t+1} = \gamma\beta\lambda_{t+1} \left(\pi_t^{\gamma} \frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\chi}} P_{t+1} Y_{t+1}$$

As a result,

$$\varphi_t = \gamma\beta \left(\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\chi}} (\pi_t^{\gamma})^{\frac{1}{1-\chi}} \left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\chi}}$$

Thus,

$$P_t x_t^1 = \lambda_t (\tilde{p}_t)^{\frac{1}{1-\chi}} P_t Y_t + \gamma\beta \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\chi}} (\pi_t^{\gamma})^{\frac{1}{1-\chi}} \left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\chi}} P_{t+1} x_{t+1}^1$$

$$P_{t+1} x_{t+1}^1 = \lambda_{t+1} (\tilde{p}_{t+1})^{\frac{1}{1-\chi}} P_{t+1} Y_{t+1} + \gamma\beta \left(\frac{\tilde{p}_t}{\tilde{p}_{t+2}}\right)^{\frac{1}{1-\chi}} (\pi_{t+1}^{\gamma})^{\frac{1}{1-\chi}} \left(\frac{1}{\pi_{t+2}}\right)^{\frac{1}{1-\chi}} P_{t+2} x_{t+2}^1$$

Dividing by P_t gives:

$$x_t^1 = \lambda_t (\tilde{p}_t)^{\frac{1}{1-\chi}} Y_t + \gamma\beta \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\chi}} (\pi_t^{\gamma})^{\frac{1}{1-\chi}} \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\chi}{1-\chi}} x_{t+1}^1$$

Second part

Similarly, denoting the second part as $P_t x_t^2$

$$P_t x_t^2 = \lambda_t \chi mc_t (\tilde{p}_t)^{\frac{\chi}{1-\chi}} P_t Y_t + \gamma\beta\lambda_{t+1}\chi mc_{t+1}\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{\chi}{1-\chi}} P_{t+1}Y_{t+1}$$

$$+ (\gamma\beta)^2\lambda_{t+2}\chi mc_{t+2}\left(\pi_t^{\gamma}\pi_{t+1}^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}\pi_{t+2}}\right)^{\frac{\chi}{1-\chi}} P_{t+2}Y_{t+2} + \cdots$$

$$P_{t+1} x_{t+1}^2 = \lambda_{t+1} \chi mc_{t+1} (\tilde{p}_{t+1})^{\frac{\chi}{1-\chi}} P_{t+1} Y_{t+1} + \gamma\beta\lambda_{t+2}\chi mc_{t+2}\left(\pi_{t+1}^{\gamma}\frac{\tilde{p}_{t+1}}{\pi_{t+2}}\right)^{\frac{\chi}{1-\chi}} P_{t+2}Y_{t+2}$$

$$+ (\gamma\beta)^2\lambda_{t+3}\chi mc_{t+3}\left(\frac{\tilde{p}_t}{\pi_{t+2}\pi_{t+3}}\right)^{\frac{\chi}{1-\chi}} P_{t+3}Y_{t+3} + \cdots$$

We look for an expression φ_t that meets the following condition:

$$\varphi_t \times \lambda_{t+1}\chi mc_{t+1}(\tilde{p}_{t+1})^{\frac{\chi}{1-\chi}} P_{t+1}Y_{t+1} = \gamma\beta\lambda_{t+1}\chi mc_{t+1}\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{\chi}{1-\chi}} P_{t+1}Y_{t+1}$$

Thus,

$$\varphi_t = \gamma\beta\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}\tilde{p}_{t+1}}\right)^{\frac{\chi}{1-\chi}} = \left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\chi}{1-\chi}} (\pi_t^{\gamma})^{\frac{\chi}{1-\chi}} (\frac{1}{\pi_{t+1}})^{\frac{\chi}{1-\chi}}$$

$$P_t x_t^2 = \lambda_t \chi mc_t (\tilde{p}_t)^{\frac{\chi}{1-\chi}} P_t Y_t + \gamma\beta\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\chi}{1-\chi}} (\pi_t^{\gamma})^{\frac{\chi}{1-\chi}} (\frac{1}{\pi_{t+1}})^{\frac{\chi}{1-\chi}} P_{t+1}x_{t+1}^2$$

$$P_{t+1} x_{t+1}^2 = \lambda_{t+1} \chi mc_{t+1} (\tilde{p}_{t+1})^{\frac{\chi}{1-\chi}} P_{t+1} Y_{t+1} + \gamma\beta\left(\frac{\tilde{p}_{t+1}}{\tilde{p}_{t+2}}\right)^{\frac{\chi}{1-\chi}} (\pi_{t+1}^{\gamma})^{\frac{\chi}{1-\chi}} (\frac{1}{\pi_{t+2}})^{\frac{\chi}{1-\chi}} P_{t+2}x_{t+2}^2$$

Dividing by P_t yields:

$$x_t^2 = \lambda_t \chi mc_t (\tilde{p}_t)^{\frac{\chi}{1-\chi}} Y_t + \gamma\beta\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\chi}{1-\chi}} (\pi_t^{\gamma})^{\frac{\chi}{1-\chi}} (\frac{1}{\pi_{t+1}})^{\frac{1-\chi}{1-2\chi}} x_{t+1}^2$$

Next, log-linearizing the equations related to x_t^1, x_t^2 around the steady state.

$$x_t^1(1+\hat{x}_t^1) = \lambda Y(1+\hat{\lambda}_t + \frac{1}{1-\chi}\tilde{p}_t + \hat{Y}_t) + \gamma\beta x^1(1+\frac{1}{1-\chi}\left(\tilde{p}_t - \tilde{p}_{t+1}\right) + \frac{\chi}{1-\chi}\hat{\pi}_t$$
$$- \frac{\chi}{1-\chi}\hat{\pi}_{t+1} + \hat{x}_{t+1}^1$$

$$\hat{x}_t^1 = (1-\gamma\beta)\left(\hat{\lambda}_t + \frac{1}{1-\chi}\tilde{p}_t + \hat{Y}_t\right) + \gamma\beta(\frac{1}{1-\chi}\left(\tilde{p}_t - \tilde{p}_{t+1}\right) + \frac{\chi}{1-\chi}\hat{\pi}_t - \frac{\chi}{1-\chi}\hat{\pi}_{t+1} + \hat{x}_{t+1}^1)$$

The log-linearization of the second part results in:

$$\hat{x}_t^2 = (1-\gamma\beta)\left(\hat{\lambda}_t + \widehat{mc}_t + \frac{\chi}{1-\chi}\tilde{p}_t + \hat{Y}_t\right) + \gamma\beta(\frac{\chi}{1-\chi}\left(\tilde{p}_t - \tilde{p}_{t+1}\right) + \frac{\chi}{1-\chi}\hat{\pi}_t + \frac{1-2\chi}{1-\chi}\hat{\pi}_{t+1}$$
$$+ \hat{x}_{t+1}^2)$$

Linearizing the price index function (4.37) by employing Uhilig's (1999) method, the equation can be rewritten as:

$$P_t^{1-\epsilon_p} = (1-\gamma)\tilde{P}_t^{1-\epsilon_p} + \gamma(\pi_{t-1}P_{t-1})^{1-\epsilon_p} \tag{4.37}$$

$$P^{1-\epsilon_p}e^{(1-\epsilon_p)\hat{P}_t} = \gamma P^{1-\epsilon_p}e^{(1-\epsilon_p)\hat{\pi}_{t-1}\hat{P}_{t-1}} + (1-\gamma)P^{*1-\epsilon_p}e^{(1-\epsilon_p)P_t^*}$$

Since $P = P^*$ at the steady state, as a result,

$$e^{(1-\epsilon_p)\hat{P}_t} = \gamma e^{(1-\epsilon_p)\hat{\pi}_{t-1}\hat{P}_{t-1}} + (1-\gamma)e^{(1-\epsilon_p)\tilde{\hat{P}}_t}$$

Taking the first-order Taylor polynomial expansion obtains:

$$1 + (1-\epsilon_p)\hat{P}_t = \gamma(1 + (1-\epsilon_p)\hat{\pi}_{t-1}\hat{P}_{t-1}) + (1-\gamma)(1 + (1-\epsilon_p)P_t^*)$$

Regrouping the above equation we find the price determination function:

$$\hat{p}_t = \frac{\gamma}{1-\gamma}(\hat{\pi}_t - \hat{\pi}_{t-1})$$

Substituting for $\hat{x}_t^1$ and $\hat{x}_t^2$ into $\hat{x}_t^1 = \hat{x}_t^2$ results in:

$$(1-\gamma\beta)\left(\hat{\lambda}_t + \frac{1}{1-\chi}\tilde{\hat{p}}_t + \hat{Y}_t\right) + \gamma\beta\left(\frac{1}{1-\chi}(\tilde{\hat{p}}_t - \tilde{\hat{p}}_{t+1}) + \frac{\chi}{1-\chi}\hat{\pi}_t - \frac{\chi}{1-\chi}\hat{\pi}_{t+1} + \hat{x}_{t+1}^1\right)$$

$$= (1-\gamma\beta)\left(\hat{\lambda}_t + \widehat{mc}_t + \frac{\chi}{1-\chi}\tilde{\hat{p}}_t + \hat{Y}_t\right) + \gamma\beta\left(\frac{\chi}{1-\chi}(\tilde{\hat{p}}_t - \tilde{\hat{p}}_{t+1}) + \frac{\chi}{1-\chi}\hat{\pi}_t + \frac{1-2\chi}{1-\chi}\hat{\pi}_{t+1} + \hat{x}_{t+1}^2\right)$$

Summarizes to:

$$\left[(1-\gamma\beta)\frac{1}{1-\chi} - (1-\gamma\beta)\frac{\chi}{1-\chi}\right]\tilde{\hat{p}}_t - (1-\gamma\beta)\widehat{mc}_t + \gamma\beta\frac{1}{1-\chi}(\tilde{\hat{p}}_t - \tilde{\hat{p}}_{t+1})$$
$$- \gamma\beta\left(\frac{\chi}{1-\chi}(\tilde{\hat{p}}_t - \tilde{\hat{p}}_{t+1})\right) + \gamma\beta\left(\frac{1}{1-\chi}\hat{\pi}_t - \frac{\chi}{1-\chi}\hat{\pi}_{t+1}\right)$$
$$- \gamma\beta\left(\frac{\chi}{1-\chi}\hat{\pi}_t + \frac{1-2\chi}{1-\chi}\hat{\pi}_{t+1}\right) = 0$$

$$(\gamma\beta)\tilde{\hat{p}}_t - (1-\gamma\beta)\widehat{mc}_t + \gamma\beta(\tilde{\hat{p}}_t - \tilde{\hat{p}}_{t+1}) + \gamma\beta(\hat{\pi}_t - \hat{\pi}_{t+1}) = 0$$

Substituting the linearization of price index function $\hat{p}_t = \frac{\gamma}{1-\gamma}(\hat{\pi}_t - \hat{\pi}_{t-1})$into the above equation:

$$(1-\gamma\beta)\frac{1}{1-\chi}(\hat{\pi}_t - \hat{\pi}_{t-1}) - (1-\gamma\beta)\widehat{mc}_t + \gamma\beta\left(\frac{\chi}{1-\chi}(\hat{\pi}_t - \hat{\pi}_{t-1}) - \frac{\chi}{1-\chi}(\hat{\pi}_{t+1} - \hat{\pi}_t)\right)$$
$$+ \gamma\beta(\hat{\pi}_t - \hat{\pi}_{t+1}) = 0$$

Divide both sides by $\gamma\beta$:

$$\frac{(1-\gamma\beta)}{(1-\gamma)\beta}(\hat{\pi}_t - \hat{\pi}_{t-1}) + \frac{\gamma}{1-\gamma}(\hat{\pi}_t - \hat{\pi}_{t-1}) - \frac{\gamma}{1-\gamma}(\hat{\pi}_{t+1} - \hat{\pi}_t) + (\hat{\pi}_t - \hat{\pi}_{t+1}) = \frac{(1-\gamma\beta)}{\gamma\beta}\widehat{mc}_t$$

Consequently,

$$\frac{(1+\beta)}{(1-\gamma)\beta}\hat{\pi}_t = \frac{1}{1-\gamma}\hat{\pi}_{t+1} + \frac{\gamma}{(1-\gamma)\beta}\hat{\pi}_{t-1} + \frac{(1-\gamma\beta)}{\gamma\beta}\widehat{mc}_t$$

Therefore, the dynamic equation of inflation is:

$$\hat{\pi}_t = \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\beta)}\widehat{mc}_t \tag{4.48}$$

Equation (4.48) is the New Keynesian Philips curve (NKPC), which describes the supply side of the economy. It shows that the effects of stickiness are on the marginal cost of intermediate firms mc_t.

CHAPTER V

Conclusions

5.1 Summary of the study

This thesis has explained liquidity trap theory and reviewed the circumstances of Japan's economy at the time it collapsed into long-run recession, falling victim to a liquidity trap because of the stock market bubble bursting in the early 1990s. This thesis summarized monetary and fiscal policy conduct in Japan following the recession and discussed the fiscal and monetary stimulus for Japan's recovery. Previous works demonstrated that in a standard Keynesian theory, the classic solution for a liquidity trap is fiscal expansion. However in the case of Japan's economy, the implement of expansionary fiscal policy is controversial due to the fears of further increases in the government public debt.

The purpose of this study was to investigate the effectiveness of fiscal and monetary policy on stimulating Japan's economy. To achieve that, chapter II used a calibrated textbook-style macroeconomic model designed by Ball (2006) to examine the Japanese economy and predicted Japanese economic trends. The assumptions and value of parameters came from Ball (2006) and Jinushi, Kuroki and Miyao (2002), with data based on the circumstances in 2003. The simulation results tended to favor fiscal expansion. In assuming that monetary policy follows a Taylor rule once the interest rate turn positive, potential GDP rises, and the interest rate also become positive. After recovery, Taylor rule leads the economy on a path of steady potential output and inflation. Furthermore, due to high growth and inflation, the debt-income ratio declines. Even after updating the data to 2013, the conclusion remained the same. The results mostly supported the view that fiscal expansions can suppress the increase in the debt-income ratio resulted from the expansionary fiscal policy leads to a less severe drop in nominal GDP (the denominator in the ratio).

Chapter III has employed a DSGE model with both "impatient" agents so called borrowers and "patient" agents so called savers and established a situation when the economy fell in a liquidity trap caused by a sudden reduction in the quantity of debt. The main findings are as follows. First, a temporary fiscal stimulus is effective during zero lower bound periods. Second, a temporary rise in government spending would not crowd out private expenditure and it would lead to the increase in debtors with liquidity constraint. Furthermore, in this chapter, it has examined a controversial relationship between unemployment and the debt-to-GDP ratio. There appears to be a trade-off between unemployment and the ratio: Expansionary fiscal policies reduce unemployment but lead to a higher debt-to-GDP ratio. This study has shown that the trade-off relationship may not exist if the short-term nominal interest rate is zero and if a negative economic shock is temporary.

Chapter IV turned to analyze the effect of monetary policy with the zero lower bound constraint. This chapter has implemented a typical medium scale DSGE model with the stickiness of prices and wages and adjustment costs of investment. To cope with the occasionally binding constraint, I added the HP algorithm created by Holden and Paetz (2012) to the model to ensure the nominal interest rates keep being zero. With respect to the effectiveness of monetary policy under the circumstance of ZLB constraint, the results showed under the ZLB constraint, the monetary stimulus under the Taylor rule is still effective on aggregate demand even though the effects are not as significant as without the zero lower bound constraint on nominal interest rates.

5.2 Limitations and future work

Although the findings of this thesis provided some insights of policy implications for Japan's liquidity trap, there might have some limitations. For chapter II, the main criticism of the model is that the natural rate of interest is not endogenous but determined exogenously. For improvement, the natural rate of interest in the model that applied in chapter III is set as an endogenous variable. For chapter III, first of all, although the study has shown the trade-off relationship between the unemployment and debt to GDP ratio, it has not fully discussed the effects of business cycles on the primary balance. Even if there is no additional government spending, a recession will produce primary deficits, which will result in a higher level of debt. The analysis on the change in primary balance will be a subject for further analysis. Second, Second, the values of the parameters are consistent with Eggertsson and Krugman (2012). For greater rigor, the values of the parameters should be calibrated using actual data and more empirical research should be conducted in the future research.

For chapter IV, with respect to the money growth shocks, the analysis has not simulated the impulse responses to the money growth shock and adequately discussed the effect of the money growth shocks. Thus, the investigation of the effect of the money growth shock on macroeconomic variables and the comparison with the effect of the interest rate shocks would be necessary for the future work. Additionally, since aggregate demand cannot be stimulated by further interest-rate reductions, the unconventional policies such as quantitative and qualitative monetary easing (QQE) and large-scale asset purchases have been widely implemented by major central banks. The analysis on transmission mechanism of unconventional policies is relatively limited in this research. Thus, the comprehensive discussion on unconventional policy alternatives and its overall macroeconomic effects should be essential for future research.