

A hierarchical structural interpretation of 1-dimensional 2-state number conserving cellular automata

(1次元2状態保存的セルオートマトンの階層構造による解釈)

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Abstract

A cellular automaton (CA), introduced by Von Neumann as a self-reproducing model, is a discrete dynamics system that evolves in discrete space and discrete time. A CA consists of a grid (finite dimension) of cells of which states are finite numbers. Each cell evolves by a local function of which arguments are its neighborhood cells. CA is widely used as a modeling tool for a wide variety of fields, especially for physical modeling. CAs for conserving mass or any quantity have also been studied. One of them, the Number-Conserving CA(NCCA), can be interpreted as a model for particle interaction. The state number of each cell is regarded as the number of particles in the cell. The evolution of the NCCA is described by the particle movements between cells. In addition, a motion representation which expressed NCCA as the movement of particles was introduced. Unlike the rule table expressing CA, the motion representation more intuitively represents the movement of the particles by the NCCA. In the first part of this thesis, we propose a hierarchical motion representation (HMR) that can be summarized and expressed more simply according to the complexity of each motion (pattern length, number of 1) in a motion representation. The relation between n -cell NCCA and $(n-1)$ -cell NCCA, one of the main principles of HMR, shows that NCCAs of different sizes can be efficiently expressed through HMR. Through this, we propose an HMR tree that can express all NCCAs for one neighborhood size at once. Any two-state NCCA with the state set, $\{0,1\}$ keeps the number of 1s on the configuration constant. In other words, all the 1s on the configuration move without disappearing or appearing at any time step. When 1s on a configuration are moved by motions defined in a motion representation of a two-state NCCA, these motions are determined by the related argument patterns of its local function of which value is 1. In the second part of this thesis, we define a bundle quad (length $n-2$) meaning 4 length n patterns and a bundle pair (length $n-1$) meaning 2 length n patterns. Using that structure, we showed that there are NCCAs that have perfectly identical evolution between neighborhood size n and $n-2$, and show some properties according to the structure.