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Estimation of Precautionary Savings in the U.S. and Japan

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by

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ABSTRACT OF THE DISSERTATION

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The first chapter provides a theoretical deviation of a closed-form solution to the consumer intertemporal optimization problem when labor income is stochastic. Cantor-Lam algorithm is modified to allow for a nonstationary labor income. An ARCH structure for the error term of the labor income process is introduced into the algorithm. It is shown that precautionary savings can be empirically estimated based on an optimal consumption function derived in this way.

The second chapter shows the results of estimating precautionary savings in the U.S. Consumption "puzzles" of excess smoothness and sensitivity are discussed based upon this estimation. Cyclical implications of precautionary savings are investigated as well.

The third chapter reports the estimation of Japanese precautionary savings. The results here are compared to the U.S. The possibility of partial explanations of Japanese savings behavior based on a new factor - precautionary savings is attempted.

CHAPTER I

Optimal Consumption and Precautionary Savings

Introduction

This is the theoretical part of a project seeking to estimate precautionary savings(Leland(1968)) as a consumer's behavior to prepare for uncertainty of future income. We will try to estimate the increase in savings due to increasing future income uncertainty based on a specific econometric method called the ARCH model(Engle(1982)). In a recent version of this model, the conditional variance of a stochastic variable is allowed to depend on elements of the information set, and can be used as one of explanatory variables to capture the effect of uncertainty in a system of equations(Baba, Engle, Kraft and Kroner(1987)). This econometric model is chosen not only because of the systematic estimation schema it provides, but also because it can be incorporated into the process of deriving an explicit closed form solution to the consumer's intertemporal optimization problem with stochastic labor income. Such an optimal consumption theory was introduced by Cantor(1985) and Lam(1987)(the latter is an extension of the former), but it needs some modification because the model was constructed on the assumption that the labor income process is stationary, and perhaps it is not true. We instead assume that labor income follows a nonstationary stochastic process with a unit root, and try to apply the Cantor-Lam algorithm. It will be shown that once this alternative assumption is accomodated, it is straightforward to introduce the conditional variance as an "ARCH in mean"(ARCH-M) term(Engle, Lilien, and Robins(1984)) into the consumption function to capture the effect of the demand for precautionary savings.

Using such a consumption function, we will be able to make an empirical attempt to test for the effectiveness of introducing precautionary savings in orderto solve consumption "puzzles" of the excess sensitivity and smoothness(Flavin(1981), Deaton(1986), Campbell and Deaton(1987)). Also, based upon a time series of precautionary savings, we can try to characterize the properties of consumer's response toward uncertainty in the context of business cycles. Other possibilities of direct reach by our model are: we can see the applicability of the same optimization model to other countries, and compare the share of precautionary savings among different countries(expecting to be able to attribute to it a part of the cross national difference in the saving rate). The comovement of precautionary savings among different countries might be observed by allowing a certain period of time lag. This would lead to the idea of an international economic link in the dimension of the second moment.

In the following study, part I discusses the derivation of the exact solution to the representative consumer's intertemporal optimization problem when labor income is stochastic(the real interest rate is assumed to be constant throughout the paper). The derivation stems from Cantor(1985) and Lam(1987) in which they treated labor income as a stationary stochastic process -an assumption that is perhaps not true. Instead we assume that the labor income is nonstationary, having a unit root, and show that their algorithm survives under this alternative assumption. We also assume that the disturbance term of the labor income process follows an ARCH or generalized ARCH(GARCH)(Bollerslev(1986)) process, then the optimal consumption policy that parameterizes the precautionary savings motive in terms of conditional variances can be derived.

Part II studies the possibility of approximate solutions when the utility function is not of the negative exponential type(Cantor(1985), Caballero(1987), and Lam(1987) assume this utility function). When the utility function is not of this type, an explicit closed-form solution is not available(see Caballero(1987) and Lam(1987), for example). We characterize the relation between future income uncertainty(as measured by the conditional variance) and the current consumption when utility is the logarithmic and the power function that are special cases of the most commonly utilized HARA(hyperbolic absolute risk aversion, ie. $U(C_t)=\frac{1-\sigma}{\sigma}\left(\frac{ac_t}{1-\sigma}+b\right)^{\sigma}, b>0$) utility function(the negative exponential utility function is also a special case of this function). The literature showed that when utility is nonquadratic with a positive third derivative, the optimal consumption policy exhibits precautionary savings (Zeldes(1984)). The above utility functions satisfy this property. An approximation is made by a linear Taylor series expansion of the Euler equation. After finding the approximate solution, a second order Taylor expansion is considered. The inverse relation between current consumption and the conditional variance of future income is investigated based on this equation. Any attempt(by a simulation method, for instance) to show the plausibility of these approximate solutions is not within the scope of the current paper, however.

Part I

The initial papers on precautionary savings are Leland(1968), Sandmo(1970), and Dreze and Modigliani(1972). They studied two period models in which utility was not necessarily time separable. Although they did not derive a closed-form solution, when utility is time separable, their models imply that a necessary and sufficient condition for the existence of precautionary savings is that the third derivative of the utility function be positive($u''' > 0$)(Zeldes(1984)). In the absence of the quadratic utility function, the extension to the multi-period model becomes difficult when facing a stochastic labor income(see Zeldes(1984), Cantor(1985), Caballero(1987), and Lam(1987), for example). Most of the literature is characterized by the complex stochastic dynamic programming method(Merton(1971), Sibley(1975), Miller(1976), Schechtman and Escudero(1977), Levhari, Mirman and Zilcha(1980)).

In contrast, Cantor(1985)(finite horizon, normally distributed errors), Caballero(1987)(finite horizon, iid or bilinear errors), and Lam(1987)(infinite horizon, no distribution assumption on the errors being made) assumed the negative exponential utility function, and thereby simplified the analysis. Cantor(1985) and Lam(1987), however, assumed labor income to be stationary. We first review Lam's model, and introduce a nonstationary labor income process with a unit root while assuming that the errors are subject to an ARCH process.

The replacement of the conditional expectation by the unconditional expectation operator in the relevant Euler equation is commonly made based upon the assumption that the errors are of an iid process(letting f and ψ_t be the density function of the disturbance ϵ_t , and the information set at time t, $f(\epsilon_{t+1}|\psi_t)=f(\epsilon_{t+1})$ holds if ϵ_t is an iid random variable). When the ARCH assumption is made, however, this property no longer holds.

We will have to work with the conditional distribution of the disturbances throughout the operation.

1. Lam's model

The infinite horizon optimal consumption model is formulated as

$$\max \; E_t \sum_{i=0}^{i=\infty} \beta^i U(C_{t+i}) \qquad (1)$$

subject to

$$A_{t+1} = (1 + R)(A_t + Y_t - C_t) \qquad (2),$$

where the notation is as follows:

E_t = mathematical expectation conditional on all information available at t,

β = the subjective time discount factor,

C_t = consumption,

$U(.)$ = period utility function,

Y_t = stochastic labor income,

R = constant real net interest rate,

A_t = financial asset.

The representative consumer knows A_t , Y_t , β , R, and the stochastic process generating Y_t at time t, and must decide upon the current consumption to maximize his lifetime utility. The negative exponential utility function with constant absolute risk aversion(CARA) is assumed:

$$U(C_t) = -z_0 exp(-zC_t) \qquad (3)$$

$$z_0 , z > 0,$$

where z is the degree of absolute risk aversion. Labor income follows the stationary stochastic process:

$$Y_t = A(L)\epsilon_t, \quad A(L) = \sum_{i=0}^{i=\infty} a_i L^i, \quad a_0=1 \qquad (4),$$

where A(L) is a lag polynomial. The innovations in income, ϵ_t , are assumed to be iid with zero mean and constant variance, and satisfying the regularity condition:

$$E\left\{ exp(-zR[\sum_{i=0}^{i=\infty} a_i/(1+R)^{i+1}]\epsilon)\right\} < \infty \qquad (5).$$

Then, he showed the solution to the maximization problem to be:

$$C_t = -[1/zR]log[Eexp(k\epsilon_t)] - [1/zR]log[\beta(1+R)] + \frac{R}{(1+R)}[A_t+H_t] \qquad (6),$$

where

$$H_t = E_t \sum_{i=0}^{i=\infty} Y_{t+i}/(1+R)^i \qquad (7),$$

and

$$k = -z[R/(1+R)]\left[\sum_{i=0}^{i=\infty}\frac{a_i}{(1+R)^i}\right]$$

2. Labor income as a random walk

To see what the solution to (1) will be when labor income is nonstationary, we assume that income is an I(1) process(with a unit root). Especially, when income follows a random walk, it can be shown that the solution is rather simplified: Suppose the equation (4) is replaced by

$$Y_t = Y_{t-1} + \epsilon_t$$

$$\epsilon_t \ \tilde{} N(0,\ h_t|\ \psi_{t-1}) \qquad (4)',$$

where ψ_{t-1} is the information set as of time (t-1), and h_t is the conditional variance of ϵ_t . ϵ_t is assumed to follow a generalized ARCH process(Bollerslev(1986)), ie. the conditional variance equation of the GARCH(p,q) process is

$$h_t = \alpha_0 + \sum_{i=1}^{i=q}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{i=p}\beta_i h_{t-i} = \alpha_0 + B_0(L)\epsilon_t^2 + B_1(L)h_t \qquad (4)'',$$

$$p \geqslant 0,\ \ q \geqslant 0,\ \ \alpha_0 \geqslant 0,\ \ \alpha_i \geqslant 0,\ \ i=1,2,\dots q,\ \ \ \beta_i \geqslant 0,\ \ i=1,2,\dots p.$$

When $B_0(1) + B_1(1) < 1$, the unconditional distribution of ϵ_t is wide sense stationary(Bollerslev(1986)). For example, let ϵ_t follow a GARCH(1,1) process so that where v_t is an iid standard normal variable. Then, $E(\epsilon_t) = E[E_{t-1}(\epsilon_t)] = 0$, $E(\epsilon_t\epsilon_{t-j}) = E[E_{t-1}(\epsilon_t\epsilon_{t-j})] = 0, \forall j \geqslant 1$, and when $\alpha_1+\beta_1 < 1$,

$$E(h_t)=\frac{\alpha_0}{1-\beta_1} + \frac{\alpha_1}{1-\beta_1}Var(\epsilon_t)$$

so that

$$Var(\epsilon_t) = E(\epsilon_t^2) = E[\alpha_0 + \alpha_1\epsilon_{t-1}^2 + \beta_1 h_{t-1}] = \alpha_0+\alpha_1 Var(\epsilon_t) + \frac{\alpha_0\beta_1}{1-\beta_1} + \frac{\alpha_1\beta_1}{1-\beta_1}Var(\epsilon_t).$$

Thus $Var(\epsilon_t) = \dfrac{\alpha_0}{1-\alpha_1-\beta_1}$. However, the unconditional marginal distribution of ϵ_t will not be normal. Also, ϵ_t will not be independently distributed, for $\epsilon_t = v_t(\alpha_0+\alpha_1\epsilon_{t-1}^2+\beta_1 h_{t-1})^{1/2}$, and therefore the conditional density $f(\epsilon_t|\,\epsilon_{t-1})$ will not be equal to $f(\epsilon_t)$. In the rest of the paper, the stationarity condition $B_0(1)+B_1(1) < 1$ is assumed.

Then, human wealth (7) is, by the law of iterative expectations:

$$H_t = E_t\sum_{i=0}^{i=\infty}\frac{Y_{t+i}}{(1+R)^i} = Y_t + E_t\left[\frac{Y_t+\epsilon_{t+1}}{(1+R)}+\frac{Y_t+\epsilon_{t+1}+\epsilon_{t+2}}{(1+R)^2}+\cdots\right]$$

$$= Y_t\left[1+\frac{1}{(1+R)}+\frac{1}{(1+R)^2}+\cdots\right] = Y_t\left(\frac{(1+R)}{R}\right) \qquad (8).$$

The optimal consumption policy needs to satisfy the Bellman equation. Guessing the value function to be $V=-\gamma_0 exp[-\gamma(A_t+H_t)]$, the optimal policy must satisfy the following:

$$-\gamma_0 exp[-\gamma(A_t + H_t)] = \max_{C_t}\left\{-z_0 exp(-zC_t) + E_t\beta[-\gamma_0\text{exp}-\gamma(A_{t+1} + H_{t+1})]\right\} \qquad (9),$$

where the relationship

$$(1+R)H_t + (\frac{1+R}{R})\epsilon_{t+1} = (1+R)Y_t + H_{t+1}$$

holds in the right-hand side because

$$(1+R)H_t = (1+R)Y_t + E_t(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots),$$

and

$$H_{t+1} = E_{t+1}(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots).$$

Therefore,

$$(1+R)H_t = (1+R)Y_t + H_{t+1} + E_t(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots) - E_{t+1}(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots),$$

but the relations

$$E_t(Y_{t+1}) - E_{t+1}(Y_{t+1}) = E_t(Y_t + \epsilon_{t+1}) - E_{t+1}(Y_t + \epsilon_{t+1}) = -\epsilon_{t+1},$$

$$E_t\left(\frac{Y_{t+2}}{1+R}\right) - E_{t+1}\left(\frac{Y_{t+2}}{1+R}\right) = E_t\left[\frac{1}{1+R}(Y_t + \epsilon_{t+1} + \epsilon_{t+2})\right]$$

$$- E_{t+1}\left[\frac{1}{1+R}(Y_t + \epsilon_{t+1} + \epsilon_{t+2})\right] = -\left(\frac{1}{1+R}\right)\epsilon_{t+1},$$

and

$$E_t\left(\frac{Y_{t+3}}{(1+R)^2}\right) - E_{t+1}\left(\frac{Y_{t+3}}{(1+R)^2}\right)$$

$$= E_t\left[\frac{1}{(1+R)^2}(Y_t + \epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3})\right] - E_{t+1}\left[\frac{1}{(1+R)^2}(Y_t + \epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3})\right] = -\left[\frac{1}{(1+R)^2}\right]\epsilon_{t+1}$$

. . . .

. . . .

hold, thus

$$E_t\left(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots\ldots\right) - E_{t+1}\left(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots\ldots\right)$$

$$= -\epsilon_{t+1} - \frac{1}{1+R}\epsilon_{t+1} - \frac{1}{(1+R)^2}\epsilon_{t+1} - \ldots\ldots$$

$$= -\left(\sum_{i=0}^{i=\infty}\frac{1}{(1+R)^i}\right)\epsilon_{t+1} = -\left(\frac{1+R}{R}\right)\epsilon_{t+1}.$$

Using this result, equation (9) is rewritten as

$$-\gamma_0 exp[-\gamma(A_t + H_t)] = \max_{C_t}\left\{-z_0 exp(-zC_t) - \beta\gamma_0 exp[-\gamma(1+R)(A_t + H_t + \gamma(1+R)C_t] E_t exp[-\gamma(\frac{1+R}{R})\epsilon_{t+1}]\right\} \quad (10).$$

Solving the maximization problem of the right-hand side yields

$$C_t = \left[\frac{1}{-(z+\gamma(1+R))}\right]\left\{\log\left[\frac{1}{z_0 z}\beta\gamma_0\gamma(1+R)\right] + [-\gamma(1+R)(A_t + H_t)] + \log E_t \exp\left[-\gamma(\frac{1+R}{R})\epsilon_{t+1}\right]\right\} \quad (11)$$

Substituting (11) back into the Bellman equation, the right-hand side of (10) becomes

$$-z_0\left\{[\frac{1}{z_0 z}\beta\gamma_0\gamma(1+R)]^{\frac{z}{z+\gamma(1+R)}}\exp\left[\frac{-z\gamma(1+R)(A_t + H_t)}{z+\gamma(1+R)}\right]\left[E_t exp(-\gamma(\frac{1+R}{R})\epsilon_{t+1})\right]^{\frac{z}{z+\gamma(1+R)}}\right\}$$

$$-\ \beta\gamma_0 exp(-\gamma(1+R)(A_t+H_t))$$

$$*\left[[\frac{1}{z_0 z}\beta\gamma_0\gamma(1+R)]^{\frac{-\gamma(1+R)}{[z+\gamma(1+R)]}} * \exp\left[\frac{\gamma^2(1+R)^2(A_t+H_t)}{z+\gamma(1+R)}\right] * \left[E_t exp(-\gamma(\frac{1+R}{R})\epsilon_{t+1})\right]^{\frac{-\gamma(1+R)}{z+\gamma(1+R)}}\right]$$

$*E_t exp(-\gamma(\frac{1+R}{R})\epsilon_{t+1})$ Rearranging terms and comparing it with the left-hand side yields

$$\gamma = \frac{zR}{1+R},$$

and

$$\gamma_0 = z_0(1+R^{-1})^{\frac{1+R}{R}}[R\beta E_t exp(-z\epsilon_{t+1})]^{1/R}.$$

Substituting these values into (11) yields

$$C_t = \frac{-1}{z(1+R)}\left\{(\frac{1+R}{R})log[\beta(1+R)] + \frac{1}{R}\log E_t exp(-z\epsilon_{t+1}) - zR(A_t + H_t) + \log E_t exp(-z\epsilon_{t+1})\right\}$$

$$= -\left(\frac{1}{zR}\right)\log E_t exp(-z\epsilon_{t+1}) - \left(\frac{1}{zR}\right)log[\beta(1+R)] + \left(\frac{R}{1+R}\right)(A_t + H_t) \qquad (12)$$

Since we assume that ϵ_{t+1} is subject to an ARCH process(obviously this assumption is stronger than Lam's regularity condition (5)): ϵ_{t+1} $\tilde{}N(0, h_{t+1}| \psi_t)$, the distribution of $-z\epsilon_{t+1}$ is $-z\epsilon_{t+1}$ $\tilde{}N(0, z^2h_{t+1}| \psi_t)$. Let $x_{t+1} = exp(-z\epsilon_{t+1})$, then $\log x_{t+1} = -z\epsilon_{t+1}$, so that x_{t+1} is conditionally lognormally distributed, and $E_t(x_{t+1}) = exp(\frac{1}{2}z^2h_{t+1})$. Thus, we have $log[E_t exp(-z\epsilon_{t+1})] = \frac{1}{2}z^2h_{t+1}$. Substituting this into (12), the consumption function becomes

$$C_t = -\left(\frac{z}{2R}\right)h_{t+1} - \left(\frac{1}{zR}\right)log[\beta(1+R)] + \left(\frac{R}{1+R}\right)[A_t + H_t] \qquad (13).$$

Both in (6)(when ϵ_t is normally distributed) and in (13), we find that the degree of absolute risk aversion(z) positively affects precautionary savings.

Caballero(1987) showed that the dependence of precautionary savings on the degree of absolute risk aversion is negative. It is possible to see that the direction of this dependence is not uniformly determined by using an example based on Zeldes(1984)'s intuitive explanation of the derivation of precautionary savings: when the third derivative of the utility function is positive(U''' >0), the marginal utility(MU) is convex. Beginning in a certainty situation (Y_1^e , Y_2^e) in the two period model(Y_i^e, i=1,2 is an expected value of labor income in period i), the optimal solution is given by $MU(C_1) = MU[E_1(C_2)]$. Let us imagine that there are two possible states for Y_i with an expected value Y_2^e . Expected utility maximization means that $MU(C_1)=E_1[MU(C_2)]$, but the convexity of MU implies that $E_1[MU(C_2)] > MU[E_1(C_2)]$, therefore first period consumption must be smaller at the new optimum.

In such an explanation, the size of precautionary savings depends on the difference $E_1[MU(C_2)] - MU[E_1(C_2)]$, or the degree of the convexity of MU. The ratio $-\frac{U'''}{U''}$ does not seem to serve as a measure of the degree of convexity because the concept equivalent to the risk premium is difficult to define here. The curvature is also not appropriate for the same reason as in Pratt(1964). But we can show by an example the relation between the degree of risk aversion and precautionary savings using the definition of the convexity of MU: let C_2^1 and C_2^2 be two possible consumptions in second period, and let $C_2^1 < C_2^2$. Given $0 < \lambda_1 , \lambda_2 < 1$, $C_2^* = \lambda_1 C_2^1 + \lambda_2 C_2^2$, the (strict) convexity of MU means

$$\lambda_1 MU(C_2^1) + \lambda_2 MU(C_2^2) > MU(\lambda_1 C_2^1 + \lambda_2 C_2^2) \qquad (D-1)$$

We compare the degree of the convexity of the logarithmic and the power utility functions(with degrees of absolute risk aversion $\frac{1}{C_t}$ and $\frac{\sigma}{C_t}$,$0 <\sigma< 1$, respectively). Let D_l and D_p be the differences between the left and right-hand side of (D-1)(corresponding to those utility functions). Writing $D_l - D_p$ as

$$D_l - D_p = \lambda_1\left(\frac{1}{C_2^1} - \frac{1}{(C_2^1)^\sigma}\right) + \lambda_2\left(\frac{1}{C_2^2} - \frac{1}{(C_2^2)^\sigma}\right) + \left(\frac{1}{[\lambda_1 C_2^1 + \lambda_2 C_2^2]^\sigma} - \frac{1}{[\lambda_1 C_2^1 + \lambda_2 C_2^2]}\right),$$

if the sign of this difference is not definite, we do not have a unique relationship between the degree of risk aversion and the size of precautionary savings. Let $C_2^1 = 1$, $C_2^2 = 4$,$\sigma = 1/2$, $\lambda_1 = \lambda_2 = 1/2$, then $D_l - D_p = .105$. When $C_2^1 = 4$, and C_2^2 is very large, we have $D_l - D_p \approx -.125$,so the sign of $D_l - D_p$ depends on the values of C_2^1 and C_2^2 . In general, a smaller C_2^1 tends to imply a positive $(D_l - D_p)$ for a given C_2^2, and a larger C_2^2 implies a negative $(D_l - D_p)$ for a given C_2^1 .

For the negative exponential utility function $U(C_t) = -z_0 exp(-zC_t)$, letting $z_1 < z_2$, the difference corresponding to $D_l - D_p$ can be written as

$$\lambda_1 \left\{ exp(-z_2 C_2^1) - exp(-z_1 C_2^1) \right\} + \lambda_2 \left\{ exp(-z_2 C_2^2) - exp(-z_1 C_2^2) \right\}$$

$$+ \left\{ exp[-z_1(\lambda_1 C_2^1 + \lambda_2 C_2^2)] - exp[-z_2(\lambda_1 C_2^1 + \lambda_2 C_2^2)] \right\},$$

where the first two terms are negative, but the third term is positive. As a consequence, the sign of this expression is difficult to determine.

3. Labor income as an AR(p)

When labor income is subject to an AR(p) process with a unit root, the first difference of income is a stationary series:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \epsilon_t,$$

or

$$(1 - \phi_1 L - \phi_2 L^2 - ..... - \phi_p L^p) Y_t = A(L) Y_t = \epsilon_t.$$

Let $(1-L)A^*(L) Y_t = \epsilon_t$, then $(1-L) Y_t = \left[A^*(L)\right]^{-1} \epsilon_t = B(L)\epsilon_t$, where

$(1-L)A^*(L) = A(L)$, and $B(L) = \sum_{i=0}^{i=\infty} b_i L^i = \left[A^*(L)\right]^{-1}$ are lag polynomials, and

$$\sum_{i=0}^{i=\infty} b_i^2 < \infty, \quad \sum_{i \neq j}^{i,j=\infty} b_i b_j < \infty\,, \quad \sum_{i=0}^{i=\infty} b_i < \infty$$

hold. Under this nonstationarity assumption for labor income, it can be shown that human wealth is finite given Y_t and its past realizations :

$$H_t = E_t \sum_{i=0}^{i=\infty} \frac{Y_{t+i}}{(1+R)^i} = Y_t + E_t \sum_{i=1}^{i=\infty} \frac{Y_{t+i}}{(1+R)^i}$$

$$= Y_t + E_t \left[\frac{Y_{t+1}}{1+R} + \frac{Y_{t+2}}{(1+R)^2} + \frac{Y_{t+3}}{(1+R)^3} + \ldots\ldots \right]$$

$$= Y_t + E_t \left[\frac{Y_t + B(L)\epsilon_{t+1}}{(1+R)} + \frac{Y_t + B(L)(\epsilon_{t+1} + \epsilon_{t+2})}{(1+R)^2} + \frac{Y_t + B(L)(\epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3})}{(1+R)^3} + \ldots\ldots\ldots \right]$$

$$= Y_t(\frac{1+R}{R}) + E_t \left[\frac{B(L)\epsilon_{t+1}}{(1+R)} + \frac{B(L)(\epsilon_{t+1} + \epsilon_{t+2})}{(1+R)^2} + \frac{B(L)(\epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3})}{(1+R)^3} + \cdots \right]$$

Taking conditional expectations in the square brackets, only the current and past realizations of the error terms remain, and they are finite. Let G^u be the least upper bound of the terms such as

$$\left(\sum_{i=1}^{i=\infty} b_i L^{i-1} \right) \epsilon_t, \quad \left(\sum_{i=2}^{i=\infty} b_i L^{i-2} \right) \epsilon_t, \quad \left(\sum_{i=3}^{i=\infty} b_i L^{i-3} \right) \epsilon_t\,, \ldots,$$

then the inequality

$$E_t\left[\frac{B(L)\epsilon_{t+1}}{(1+R)}+\frac{B(L)(\epsilon_{t+1}+\epsilon_{t+2}))}{(1+R)^2}+\frac{B(L)(\epsilon_{t+1}+\epsilon_{t+2}+\epsilon_{t+3})}{(1+R)^3}+\ldots\right]$$

$$\leqslant \frac{G^u}{(1+R)}+\frac{2G^u}{(1+R)^2}+\frac{3G^u}{(1+R)^3}+\ldots$$

$$= G^u\left(\frac{1+R}{R^2}\right) < \infty$$

shows that human wealth can be defined.

On the other hand, the fundamental operation on the law of motion is as follows:

$$(1+R)H_t = (1+R)Y_t + H_{t+1} + E_t\left(Y_{t+1}+\frac{Y_{t+2}}{(1+R)}+\frac{Y_{t+3}}{(1+R)^2}+\ldots\right) - E_{t+1}\left(Y_{t+1}+\frac{Y_{t+2}}{(1+R)}+\frac{Y_{t+3}}{(1+R)^2}+\ldots\right)$$

The difference between the two expected terms("revision") in the right-hand side becomes

$$-b_0\epsilon_{t+1}-\frac{1}{(1+R)}(b_0+b_1)\epsilon_{t+1}-\frac{1}{(1+R)^2}(b_0+b_1+b_2)\epsilon_{t+1}-\cdots=-\left\{\sum_{i=0}^{i=\infty}\left[\frac{\sum_{j=0}^{j=i}b_j}{(1+R)^i}\right]\right\}\epsilon_{t+1}.$$

Let M be such that $b_j \leqslant M,\ \forall\ j \geqslant 0$, then

$$\left\{\sum_{i=0}^{i=\infty}\left[\frac{\sum_{j=0}^{j=i}b_j}{(1+R)^i}\right]\right\} \leqslant \left\{\sum_{i=0}^{i=\infty}\left[\frac{(i+1)M}{(1+R)^i}\right]\right\} = \left(\frac{1+R}{R}\right)^2 M.$$

Thus, the expression $-\sum_{i=0}^{i=\infty}\left[\frac{\sum_{j=0}^{j=i}b_j}{(1+R)^i}\right]$ has a finite sum. Then, letting

$$k = -z\left[\frac{R}{1+R}\right]\left[\sum_{i=0}^{i=\infty}\left(\frac{\sum_{j=0}^{j=i} b_j}{(1+R)^i}\right)\right] \quad (14),$$

$$\gamma_0 = z_0[1 + R^{-1}]^{\frac{1+R}{R}}[R\beta E_t \exp(k\epsilon_{t+1})]^{1/R} \quad (15),$$

and

$$\gamma = \frac{zR}{1+R} \quad (16),$$

the optimal consumption policy in this case is

$$C_t = -(\frac{k^2}{2zR})h_{t+1} - (\frac{1}{zR})\log[\beta(1+R)] + (\frac{R}{1+R})(A_t + H_t) \quad (17).$$

4. Labor income as an ARMA(p,q)

When labor income follows an ARMA(p,q) process with a unit root, the same argument as the case of AR(p) process applies: let the labor income process be

$$Y_t = \phi_1 Y_{t-1} + \ldots\ldots + \phi_p Y_{t-p} + \epsilon_t + \lambda_1\epsilon_{t-1} + \ldots.. + \lambda_q\epsilon_{t-q},$$

and

$$(1-L)A_1(L)Y_t = \Lambda(L)\epsilon_t,$$

where

$$(1-L)A_1(L) = 1 - \phi_1 L - \ldots - \phi_p L^p,$$

and

$$\Lambda(L) = 1 + \lambda_1 L + ... + \lambda_q L^q.$$

Then, the first difference of income can be written as $(1-L)Y_t = A_1^*(L)\Lambda(L)\epsilon_t = G^*(L)\epsilon_t$, where $A_1^*(L) = A_1(L)^{-1}$, and $G^*(L) = A_1^*(L)\Lambda(L)$. Letting

$$G^*(L) = \sum_{j=0}^{j=\infty} g_j L^j \quad (18),$$

the optimal consumption policy under this income process(because the algorithm is the same as in the AR(p) case) is represented by equation (17) with the b_j replaced by g_j in (14), and γ_0 and γ remain unchanged.

Part II.

We have seen in part I that following relations hold, corresponding to each different income process:

$$(1+R)H_t = (1+R)Y_t + H_{t+1} - (\frac{1+R}{R})\epsilon_{t+1} \qquad (19) \text{ (random walk)},$$

$$(1+R)H_t = (1+R)Y_t + H_{t+1} - \left\{ \sum_{i=0}^{i=\infty} \left[\frac{\sum_{j=0}^{j=i} b_j}{(1+R)^i} \right] \right\} \epsilon_{t+1} \qquad (20) \quad (AR(p)),$$

$$(1+R)H_t = (1+R)Y_t + H_{t+1} - \left\{ \sum_{i=0}^{i=\infty} \left[\frac{\sum_{j=0}^{j=i} g_j}{(1+R)^i} \right] \right\} \epsilon_{t+1} \qquad (21) \ (ARMA(p,q)).$$

Let Z_{t+1} denote the last term of the right-hand side of (19) or (20) or (21), and then the law of motion (2) can be written as

$$A_{t+1} + H_{t+1} = (1+R)(A_t + H_t - C_t) + Z_{t+1} \qquad (22),$$

or equivalently

$$W_{t+1} = (1+R)(W_t - C_t) + Z_{t+1} \quad (23),$$

where $W_t = A_t + H_t$. Let $\hat{u}_t = W_t - C_t$ be the control variable, and W_t and R be the state variables, then the Bellman equation corresponding to the maximization problem (1) can be written as

$$V(W_t, R) = \max_{\hat{u}_t} \left\{ U(W_t - \hat{u}_t) + \beta E_t V((1+R)\hat{u}_t + Z_{t+1}, R) \right\}.$$

The Euler equation for maximizing the right-hand side is:

$$U'(C_t) = \beta(1+R)E_t U'(C_{t+1}) \qquad (24).$$

1. Logarithmic utility function

If we assume the logarithmic utility function $U(C_t) = \log C_t$, (24) becomes

$$\frac{1}{C_t} = \beta(1+R)E_t\frac{1}{C_{t+1}} \qquad (25).$$

We need to guess a specific formula for consumption in order to derive the optimal policy. Our guess is $C_t = \gamma^* W_t$. Substituting this into (25)(using (23)), we get

$$\frac{1}{\gamma^* W_t} = \beta(1+R)E_t\left[\frac{1}{\gamma^* W_{t+1}}\right] = \beta(1+R)E_t\left\{\frac{1}{\gamma^*[(1+R)(W_t - C_t) + Z_{t+1}]}\right\}$$

$$= \beta(1+R)E_t\left\{\frac{1}{\gamma^*[(1+R)(W_t - \gamma^* W_t) + Z_{t+1}]}\right\}$$

$$= \beta(1+R)E_t\left\{\frac{1}{\gamma^*[(1-\gamma^*)(1+R)W_t + Z_{t+1}]}\right\} \qquad (26).$$

A linear Taylor series approximation(around $E_t\epsilon_{t+1} = 0$) of the right-hand side gives

$$\frac{1}{\gamma^* W_t} = \frac{\beta(1+R)}{\gamma^*(1-\gamma^*)(1+R)W_t},$$

from which $\gamma^* = 1-\beta$ obtains($C_t = (1-\beta) W_t$). But, taking up to the second order term of the Taylor series expansion, we have

$$\beta(1+R)E_t\left\{\frac{1}{\gamma^*(1-\gamma^*)(1+R)W_t} - \frac{\gamma^*}{[\gamma^*(1-\gamma^*)(1+R)W_t]^2}Z_{t+1} + \frac{2\gamma^*}{[\gamma^*(1-\gamma^*)(1+R)W_t]^3}Z_{t+1}^2\right\}$$

$$= \beta(1+R)\left\{\frac{1}{\gamma^*(1-\gamma^*)(1+R)W_t} + \frac{2\gamma^*}{[\gamma^*(1-\gamma^*)(1+R)W_t]^3}E_tZ_{t+1}^2\right\} \qquad (27),$$

where $E_tZ_{t+1}^2$ equals

$$Q_ih_{t+1},\ i=1,2,3\ :Q_1=\left(\frac{1+R}{R}\right)^2 \text{ (random walk)},\quad Q_2=\left\{\sum_{i=0}^{i=\infty}\left[\frac{\sum_{j=0}^{j=i} b_j}{(1+R)^i}\right]\right\}^2 \quad (AR(p)),$$

$$\text{and } Q_3=\left\{\sum_{i=0}^{i=\infty}\left[\frac{\sum_{j=0}^{j=i} g_j}{(1+R)^i}\right]\right\}^2 \quad (ARMA(p,q)),$$

corresponding to each income process. Using (27), (26) can be rewritten as

$$\gamma^*(1-\gamma^*)^2(1-\gamma^*-\beta) = \frac{2\beta}{[(1+R)W_t]^2}E_tZ_{t+1}^2 \qquad (28).$$

Taking the derivative of the left-hand side with respect to γ^* at $\gamma^*=1-\beta$, we have

$$(1-\gamma^*)^2(1-\gamma^*-\beta)+2\gamma^*(1-\gamma^*)(1-\gamma^*-\beta)-\gamma^*(1-\gamma^*)^2=-\gamma^*(1-\gamma^*)^2,$$

which is negative, implying that when h_{t+1} increases, γ^* must decrease, and therefore current consumption becomes smaller. We express this precautionary savings motive additively in the consumption function:

$$C_t = \gamma^* W_t - Q\frac{h_{t+1}}{W_t^2} \qquad (29),$$

where $Q = \dfrac{\alpha_1 Q_i \beta}{(1+R)^2} > 0$, i=1,2,3, and $\alpha_1 > 0$ is some adjustment parameter for the approximation.

In (29), precautionary savings will be smaller, as the wealth W_t gets larger. This negative dependence of the precautionary motive on the level of wealth was first pointed out by Zeldes(1984).

We call the equation (29) an approximate solution to the optimization problem (1) when the utility function is assumed to be logarithmic.

2. Power utility function

When the power utility function, ie. $C_t = \dfrac{C_t^{1-s}}{1-s}$, $0 < s < 1$ is applied, the Euler equation takes the form

$$\frac{1}{C_t^s} = \beta(1+R)E_t\left(\frac{1}{C_{t+1}^s}\right) \qquad (30).$$

Guessing the optimal policy to be $C_t = \gamma^* W_t$, this becomes

$$\frac{1}{(\gamma^* W_t)^s} = \beta(1+R)E_t\frac{1}{(\gamma^* W_{t+1})^s}$$

$$= \beta(1+R)E_t\left(\frac{1}{\gamma^*[(1-\gamma^*)(1+R)W_t + Z_{t+1}]^s}\right) \qquad (31).$$

A linear approximation to the right-hand side renders $\gamma^* = 1 - \beta^{1/s}(1+R)^{\frac{1-s}{s}}$

(Hayashi(1982) refers to this case). The condition for $\gamma^{*} > 0$ is that $\beta < \frac{1}{(1+R)^{1-s}}$.

Rearranging (31) yields

$$(\gamma^{*})^{2}((1-\gamma^{*})^{s+2}[(1-\gamma)^{s}-\beta(1+R)^{1-s}] = \frac{\beta(1+R)^{1-s}s(s+1)}{[(1+R)W_{t}]^{2}}E_{t}Z_{t+1}^{2}.$$

Again the differentiation of the left-hand side at $\gamma^{*} = 1 - \beta^{1/s}(1+R)^{\frac{(1-s)}{s}}$ yields

$$-s(\gamma^{*})^{2}(1-\gamma^{*})^{s+2}(1-\gamma^{*})^{s-1},$$

which is negative. Therefore as in the logarithmic utility case, the second order Taylor series approximation gives

$$C_{t} = \gamma^{*}W_{t} - Q\frac{h_{t+1}}{W_{t}^{2}} \qquad (32)$$

where now

$$Q = \frac{\alpha_{2}Q_{i}\beta(1+R)^{1-s}s(s+1)}{(1+R)^{2}}, \quad \alpha_{2} > 0, \quad i = 1, 2, 3.$$

3. Concluding remarks

The relation between equation (17) and the permanent income hypothesis is straightforward. $R(A_t + H_t)$ is the permanent income which is defined as the annuity value of net worth(Friedman(1957), Flavin(1981), Hayashi(1982)), and the equation (17) implies that we need to take into account the precautionary savings in formulating a consumption function in accord with the PIH.

The problem involved in estimating (17) is that the human wealth H_t is not observable. We follow Hayashi(1982) to get around this: letting $\alpha = \left\{ \sum_{i=0}^{i=\infty} \left[\frac{\sum_{j=0}^{j=i} b_j}{(1+R)^i} \right] \right\}$, the stochastic difference equation (20) (for example) can be written as

$$H_t = (1+R)(H_{t-1} - Y_{t-1}) + \alpha\epsilon_t \qquad (33).$$

Multiplying C_{t-1} by (1 + R), subtracting it from C_t while using (33), we get

$$C_t = (1+R)C_{t-1} - \left(\frac{k^2}{2zR}\right)h_{t+1} + (1+R)\left(\frac{k^2}{2zR}\right)h_t$$

$$+ (1/z)log[\beta(1+R)] + R[(\frac{1}{1+R})A_t - (A_{t-1} + Y_{t-1})] + v_t \quad (34),$$

where $v_t = (\frac{\alpha R}{1+R})\epsilon_t$ consists of the innovation of labor income that is known before a consumer makes his consumption decision in period t. In estimating (34), this property of v_t will require a specific consideration on the error term of the consumption function.

Given other expenditure items in the national income accounts, the estimation of (34) will provide time series for C_t and h_t that can be compared to the business cycle chronology. Based on this comparison, precautionary savings will be given a cyclical implication(stabilizing, destabilizing, or neutral).

On the other hand, by including policy instrument variables such as tax rates, the discount rate, and government expenditures into the variance equation (4)''(this is possible, Baba, Engle, Kraft, and Kroner(1987)), how the changes in these variables affect consumption through precautionary savings can be estimated.

The Lucas(1976) critique as the parametric shift due to the optimizing behavior after announced policy changes can not be treated in such a way, however. Caballero(1987) showed that the derivative of the precautionary savings parameter with respect to the parameter of the income generating process is nonzero implying that the consumption process is not immune to the critique. This simply corresponds to the changes in the value of k in (34).

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CHAPTER II

Precautionary savings in the U.S.

1. Consumption function.

The first literature that referred to the possible combination of precautionary savings and the ARCH method is perhaps Baba (1984, chap.1). While developing the method of estimating a system of equations with "ARCH-in-Mean" (ARCH-M) terms, he reported no empirical result. Caballero (1988) contains some results from applying a univariate ARCH-M model to the change in consumption. His parameter estimate, however, was not statistically significant. In addition to a lack of precision, the sign condition in his consumption function is hard to understand.

Our strategy is to derive the exact solution to the consumer's intertemporal optimization problem, and to incorporate it with the conditional variances of stochastic labor income in a natural way, i.e. the conditional variance of the ARCH model is embedded into the solution process. It was shown in Chapter I that Cantor (1985) and Lam (1987)'s algorithm can be modified so as to conform with a nonstationary labor income process with a unit root. While their algorithm heavily depends on the assumption of the negative exponential utility function as well as in Caballero (1987, 1988), it better fits the application of the ARCH model with respect to the way that the ARCH process is introduced into the consumption function. The result shows a large deviation from Caballero's consumption function.

It was also shown that the optimal consumption function takes the form (symbols are common with Chapter I)

$$C_t = \left(\frac{k^2}{2zR}\right) h_{t+1} - \left(\frac{1}{zR}\right) \log\left[\beta(1+R)\right] + \left(\frac{R}{1+R}\right)\left(A_t + H_t\right) \tag{1}$$

where the value of k will be different depending on the time series structure of labor income. The determination of this structure is regarded as a purely empirical matter in this paper, and later it will be shown that the usual statistical procedures lead us to a random walk with drift as the labor income process for both the U.S. and Japan.(1) Then, $k^2 = z^2$ holds, so that the coefficient of the one period ahead conditional variance of labor income becomes $-\left(\frac{z}{2R}\right)$. In the above paper, the human wealth in equation (1) was eliminated using Hayashi's method (Hayashi (1982)) to get around the difficulty that it is unobservable. Now the consumption function takes the form.

$$C_t = \left(\frac{1}{z}\right) \log\left[\beta(1+R)\right] - \left(\frac{z}{2R}\right) h_{t+1} + (1+R)\left(\frac{z}{2R}\right) h_t \tag{2}$$

$$+ (1+R)\, C_{t-1} + R\left[\left(\frac{1}{1+R}\right) A_t - \left(A_{t-1} + Y_{t-1}\right)\right] + \epsilon_t$$

We will estimate a system of the labor income process and the consumption function by the multivariate simultaneous generalized autoregressive conditional heteroscedasticity (M GARCH) model (Baba, Engle, Kraft, and Kroner

(1987)) which is a full information maximum likelihood estimation method with a special structure for the conditional variance equation that is called a positive definite parameterization. In such a system estimation method, all the relevant information including identities must be fully incorporated into the model. In this context, the consumption function (1) is further rewritten based on the financial asset transition equation

$$A_{t+1} = (1+R)(A_t + Y_t - C_t) \qquad (2)',$$

and becomes

$$C_t = \left(\frac{1}{z}\right) \log \left[\beta(1+R)\right] - \left(\frac{z}{2R}\right) h_{t+1} + (1+R)\left(\frac{z}{2R}\right) h_t \qquad (3)$$

$$+ C_{t-1} + \epsilon_t$$

This consumption function can be seen as a random walk with drift derived by Hall (1978) and Caballero (1987, 1988). But the drift term here is influenced by the two conditional variances in an immediately estimable way. The final term ϵ_t is the current innovation of labor income, and we treat this as one of the explanatory variables because (as being obvious from the process of deriving this solution to the consumer's optimization problem) when a consumer makes a consumption decision, current labor income is assumed to be known as one of state variables implying that ϵ_t is also known. It turns out, therefore, that the theoretical consumption function (3) does not have an error term.(2) As in Flavin (1981), and Campbell and Deaton (1987), we introduce an error term w_t into the consumption function (3) by arguing that w_t represents either an unexpected stochastic deviation of consumer's behavior from the planned time path expressed by the equation (3), or a measurement error that can originate from using aggregate data for a representative consumer's behavior, or both. At any rate, w_t is assumed to be serially uncorrelated and uncorrelated with the explanatory variables.

2. Excess smoothness and excess sensitivity

The problem of actual consumption being too smooth, i.e., smoother than the permanent income hypothesis (PIH) predicts, was first pointed out by Deaton (1986). It was identified as the problem that the labor income innovation has too little influence on changes in consumption to be justified by the PIH, thus the actual smoothness of consumption (which was the original motivation to introduce the PIH) is not fully explained by the hypothesis. The excess sensitivity phenomenon, on the other, was first reported by Flavin (1981), and was attributed to the well known liquidity constraint. Campbell and Deaton (1987) presented empirical evidence on both of them. Although they confirmed the excess smoothness, they needed two conditions to fully reconcile their results with Flavin's excess sensitivity concept: (1) that savings Granger causes income, and (2) that (the first difference of logarithmic) labor income follows an AR(1) process. The first condition gives the equivalence between tests for excess smoothness and sensitivity in the sense of orthogonality between the current change in consumption and all lagged information, and by the second condition, the "under-sensitivity" of consumption to the current income change was interpreted as a violation of the orthogonality condition. Therefore, if the income process is characterized by a random walk with drift, as will be seen in our model, the second channel does not work, and a part of Flavin's definition of excess sensitivity (consumption responds to the current income change more than justified by the PIH, Flavin (1981, p. 1006)) is no longer compatible with the excess smoothness property. When we estimate equation (3), we will fix the parameter of ϵ_t at one in one case, and will allow it to be free in the other case. If, in the latter case, the estimate takes on a value less than one, and the difference from one is statistically significant on the conventional level, then we would say that there is the excess smoothness. Although the full investigation of the excess sensitivity phenomenon is not in our scope, it will be shown that such a confirmation of the excess smoothness leads to doubts about part of the excess sensitivity and the relevance of the liquidity constraint as its economic explanation; If a significant portion of the population is really under a liquidity constraint, then instead of lagged changes, it will be to the change in current income that changes in consumption are most sensitive. Caballero (1988)'s analytical study of the empirical implications of introducing precautionary savings claims that this new factor can solve the aforementioned consumption puzzles in the sense that the theory predicts that consumption reacts too little to income innovations and too much (from conventional criterion) to anticipated (lagged) labor income changes. We can not say a lot about the latter, but our empirical work has some significant implications for the former for both the U.S. and Japan.[3]

3. Labor income process.

Our consumption function was derived based on the Cantor-Lam algorithm which was modified to allow for a nonstationary labor income process with a unit root. The real per capita U.S. labor income[4] was tested for the existence of a unit root by the augmented Dickey-Fuller test; the first difference of labor income was regressed on a constant, the previous level, and four lagged differences. The Dickey-Fuller t-statistic was about -0.94, strongly accepting the null hypothesis of unit root. Further, the first 20 autocorrelations of the first differences were computed. They were 0.113, 0.109, 0.141,...(other values were mostly in the range (-0.1, 0.1). This implies that labor income will be an I(1) series. Thus, taking the first difference of labor income, and regressing it on a constant, and lagged differences, the OLS estimator is asymptotically normally distributed. Based upon this result, an F test was used to see whether the lagged variables had a significant explanatory power. The F statistic was 1.77 against a 5% critical value of 2.37 for 150 observations. After deleting other lagged variables, the t statistic of the previous difference was 1.47. These results lead us to a random walk with drift as the U.S. labor income process:[5]

$$Y_t = \underset{(3.4)}{0.0083} + Y_{t-1}, \qquad \sigma_Y = 0.03 \tag{4}$$

where σ_Y is the standard error of the regression, and the t-statistic is in the parenthesis. The residual from (4) was regressed on a constant and four lagged residuals for the Lagrange multiplier test. A $TR^2 \cong 7.1$ was obtained against the 5% critical value 9.5 of χ^2 variable of the four degrees of freedom. This provides further empirical support for (4). Assuming that the error term for this structure follows a GARCH process, we will estimate our consumption function (3).

4. Multivariate simultaneous GARCH model

While Engle (1982)'s ARCH model was extended to allow for an ARMA type of specification of the conditional variance equation by Bollerslev (1986), an application of the ARCH model to multivariate (bivariate) models was made by Engle, Granger and Kraft (1984) and Bollerslev, Engle and Wooldridge (1988) using the "vech" parameterization for the conditional variance equation. This specification allows conditional variance-covariances to depend on the vectors of squared past residuals and their cross products, and the vectors of past conditional variances and covariances in an intuitively appealing way. A possible drawback of this specification, however, is that the conditional covariance matrix is not assured to be positive definite. Baba, Engle, Kraft and Kroner (1987) proved that there is a new parameterization of the conditional variance equation that embodies the restriction for the positive definiteness. They also formulated the estimation algorithm for a simultaneous equations system in which the conditional covariance matrix follows a GARCH structure. Let the simultaneous equations system with a ARCH-M term be

$$\Gamma y_t + B x_t + \Lambda h_t = \eta_t \tag{5}$$

$$\eta_t \mid \psi_{t-1} \sim N(0,\, H_t^*)$$

$$vech\ H_t^* \equiv h_t$$

where y_t is a vector of endogenous variables, x_t a vector of weakly exogenous and lagged dependent variables. Γ, B and Λ are parameter matrices. h_t is defined as the conditional variance-covariance vector formed by stacking the lower triangle portion of the conditional covariance matrix H_t^* which is positive definite, and is expressed by the multivariate GARCH process (for the case of no exogenous influences)

$$H_t^* = C_o' C_o + \sum_{k=1}^{l} \sum_{i=1}^{q} A_{ik}' \eta_{t-i} \eta_{t-i}' A_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{p} G_{jk}' H_{t-j} G_{jk} \qquad (6),$$

where C_o, A_{ik} and G_{jk} are $n \times n$ parameter matrices, and C_0 is defined to be symmetric. Then, the log-likelihood function to be maximized with respect to parameters in the system of equations (5) and (6) is

$$l = \sum_{t=1}^{T} l_t \qquad (7),$$

where $l_t = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log|H_t^*| - \frac{1}{2}\eta_t' H_t^{*-1}\eta_t$, T is sample size, and n is the number of endogenous variables. To reduce the computational burden, they use the numerical optimization technique developed by Berndt, Hall, Hall and Hausman (1974) (BHHH) to maximize (7). The BHHH method approximates the Hessian of the Gauss-Newton numerical optimization technique by

$$\sum_{t=1}^{T} \frac{\partial l_t}{\partial \psi} \frac{\partial l_t}{\partial \psi'} \tag{8}$$

where ψ is the parameter vector, therefore, the parameter estimate updating schema becomes

$$\psi^* = \hat{\psi} + \lambda \left[\sum_{t=1}^{T} \frac{\partial l_t}{\partial \psi} \cdot \frac{\partial l_t}{\partial \psi'} \right]^{-1} \sum_{t=1}^{T} \frac{\partial l_t}{\partial \psi} \tag{9}$$

where $\hat{\psi}$ is the current parameter estimate, ψ^* the updated parameter value, and λ the step size obtained by a line search at each iteration. In practice, this iteration takes place through a series of OLS regressions: let $s_{ij} = \frac{\partial l_i}{\partial \psi_j}$ be the (i, j)th element of matrix S, and y be a vector that consists of T of ones. Then, the expression $\left[\sum_{t=1}^{T} \frac{\partial l_t}{\partial \psi} \cdot \frac{\partial l_t}{\partial \psi'} \right]^{-1} \sum_{t=1}^{T} \frac{\partial l_t}{\partial \psi}$ in (9) is identical with the parameter estimate in regressing y on S, i.e. $(S'S)^{-1}S'y$. As the iteration process approaches a maximum point of the likelihood function, and as the gradient vector $\frac{\partial l_t}{\partial \psi}$, t=1,...,T becomes smaller, the R^2 of the auxiliary regression will become smaller. In the actual computation on the simultaneous GARCH model, a certain small fraction of this R^2 serves as a convergence criterion, and, as in the case of other numerical optimization techniques, the estimator generated from this algorithm is asymptotically efficient and normally distributed (Baba, Engle, Kraft and Kroner (1987), Harvey (1981)).

As our theoretical model of consumer's behavior depends on the time series property of labor income, it is necessary to have a stationarity condition for a vector of error terms $\eta_t = (\epsilon_{t+1},\ w_t)'$. For the multivariate GARCH(1, 1) case (that this is often a very parsimonious way of specifying the conditional variance equation will be shown later), using the column stacking operator "vec" and the unconditional expectation operator E, this condition can be expressed as follows:[7] Let $Q \equiv E\ vecH_t^* = E\ vecH_{t-1}^*$, and

$$H_t^* = \Omega + A'\ \eta_{t-1}\,\eta_{t-1}'\ A + G'\ H_{t-1}^*\ G \tag{10}$$

where Ω is a constant matrix, and A and G are as defined in the previous section. Then, taking vec and the unconditional expectation of (10) yields

$$E(vec\ H_t^*) = vec\ \Omega + (A' \otimes A')E\ vec\ (\eta_{t-1}\,\eta_{t-1}')$$

$$+ (G' \otimes G')E\ vec\ H_{t-1}^*$$

$$= vec\ \Omega + \Big[(A' \otimes A') + (G' \otimes G')\Big] E\ vec\ H_{t-1}^* \tag{11}$$

Using Q, this can be written as

$$Q = vec\ \Omega + \Big[(A' \otimes A') + (G' \otimes G')\Big] Q \qquad (12),$$

or equivalently,

$$Q = \left\{I - \Big[(A \otimes A)' + (G \otimes G)'\Big]\right\}^{-1} vec\ \Omega \qquad (13),$$

and therefore Q, as defined above, exists if and only if the inverse matrix

$$\left\{ I - \left[(A \otimes A)' + (G \otimes G)' \right] \right\}^{-1}$$

exists. This condition, in turn, is equivalent to the absolute value of all the eigenvalues of the matrix

$$D \equiv \left[(A \otimes A)' + (G \otimes G)' \right] \tag{14}$$

being less than one. We will apply this condition to our estimated model.

5. Estimation

The equation system we are going to estimate in this paper is

$$Y_{t+1} = \gamma_o + Y_t + \epsilon_{t+1} \tag{15}$$

$$C_t = \gamma_1 - \gamma_2 \, h_{t+1} + \gamma_3 \, h_t + C_{t-1} + \epsilon_t + w_t$$

$$\gamma_2 > 0 \ , \ \ \gamma_3 > 0 \ ,$$

where the parameters of the consumption function in (15) correspond to (3). Comparing (3) to (1), we can see that γ_2 is the relevant parameter when we attempt to capture the precautionary savings empirically. A natural question regarding this feature of the model can be raised: why do we not count the uncertainty of the further distant future? The one period ahead second moment stems from the "revision" of expectation $(E_t - E_{t+1}) \sum_{i=1}^{\infty} \left(\frac{Y_{t+i}}{(1+R)^{i-1}} \right)$ which is an important part of the solution algorithm. Interpreting this in the context of how to implement the consumption plan leads to the idea that the Cantor-Lam algorithm implies the continual renewal of the consumption plan over time as new information becomes available. In this situation, the information status of the two-period ahead and more distant future is unaltered as a matter of expectation while the consumer proceeds ahead one period of time. The implication of this behavioral pattern is that only the second moment of information (ϵ_{t+1}) coming in between the two consumption plannings matters. If the labor income innovation is assumed to be i.i.d. as is usually assumed in optimal consumption literature, such a problem does not become explicit. The introduction of heteroscedastic innovations in the sense of Engle (1982), however, puts some value on waiting and collecting information in responding to the uninsured uncertainty of labor income. The cost involved in the case of a sudden adjustment would tend to be cancelled by low premiums in other periods. Precautionary savings will be rendered a typical short run behavior under such a consideration. This short run property seems to make it reasonable to think that the share of precautionary savings in total personal saving can not be very large. As is shown later, this intuition finds empirical support in our study.

We set the labor income process one period ahead of the consumption equation in accord with the theoretical assumption that Y_t is one of the state variables in the optimization problem, and therefore is known when a consumer makes a consumption decision, implying ϵ_t is also known. Thus, if ϵ_t is in the error vector, the conditional likelihood function can not be formed. This information assumption, however, makes it possible to use the one period ahead conditional variance of labor income h_{t+1} as one of the ARCH-M terms in (15) because h_{t+1} depends on elements of the information set which now includes ϵ_t. As has already been observed, γ_2 in (15) is the parameter of precautionary savings and therefore this information assumption is crucial in our model.

Taking a glance at the model (15), it is obvious that labor income depends only on its past value while the second moment of the labor income process affects the current value of consumption. This aspect of the model can be a suggestion of a two step estimation, i.e, estimating the income process, and getting conditional variances first, and estimating the consumption function is estimated in the second step. Nevertheless, we choose a system estimation method because it provides an efficiency gain when there is a correlation between ϵ_{t+1} and w_t. We assume that the conditional expectation is taken after Y_t flows in and before the consumption decision is made, and the correlation between the error terms is taken to be an empirical matter. The results of estimating the model are reported in the next section.

6. Estimated model

The equations system (10) was estimated by the multivariate simultaneous GARCH method for the two cases, i.e. the parameter of ϵ_t being fixed at one, and it being allowed to vary.[8] Table-U1 shows the former case, and Table-U2 the latter. In this estimation, the value of ϵ_t in consumption function of (10) was obtained using the income process as

$$\epsilon_t = Y_t - \gamma_o - Y_{t-1} \qquad (16),$$

therefore, the consumption function estimated is

$$C_t = (\gamma_1 - \gamma_0) - \gamma_2 h_{t+1} + \gamma_3 h_t + C_{t-1} + (Y_t - Y_{t-1}) + w_t \qquad (17).$$

When the parameter of ϵ_t, thus, is allowed to vary, this equation takes the form

$$C_t = (\gamma_1 - \theta\gamma_0) - \gamma_2 h_{t+1} + \gamma_3 h_t + C_{t-1} + \theta(Y_t - Y_{t-1}) + w_t \qquad (17)',$$

where θ stands for the parameter of ϵ_t.[9]

In Table-U1, γ_2 and γ_3 have the correct sign, and their asymptotic t-statistics are significant on the conventional level. However, if the real interest rate is to be nonnegative, the relative size of γ_2 and γ_3 should be opposite. The significance of the difference $\gamma_2 - \gamma_3$ was, therefore, tested using a standard error of γ_2. The t-ratio 1.53 was obtained, implying that the difference is not statistically significant.[10] The 5% confidence interval for γ_2 is (5.33, 19.8). For instance, the value $\gamma_2 = 6.85$ is compatible with the 1% quarterly real interest rate given the estimate of γ_3, and it lies in the confidence interval. A similar thing happens in Table-U2 where the parameter θ is allowed to vary, although γ_2 is now significant either on the two-tailed 10% level or on the one-tailed 5% level. The likelihood ratio test for imposing the restriction θ=1, however, yields a χ^2=176.6 against the

5% critical values of 3.84 implying a strong rejection of the null. Also, it will be shown later that the model with free θ is better for a consistent international comparison. Therefore, we will discuss the implication of our model based upon the model with free θ.

If so, that both models satisfy the stationarity condition can be seen by getting the relevant matrix D in (14): when θ is fixed, we have diagonal matrix

$$D = \begin{bmatrix} 0.41 & & & \\ & 0.24 & & \\ & & 0.24 & \\ & & & 0.994 \end{bmatrix},$$

the eigenvalues of which are diagonal elements that are all less than one. On the other hand, allowing θ to vary yields

$$D = \begin{bmatrix} 0.88 & & & \\ & -0.64 & & \\ & & -0.64 & \\ & & & 0.936 \end{bmatrix},$$

which again implies the conditional variance-covariance structure is stationary.

7. Excess smoothness and sensitivity

The result in Table-U1 represents the estimates when the influence of the labor income innovation on the change in consumption is fixed at one as the theory predicts. Then, the excess smoothness problem can be interpreted that the parameter θ takes on a much smaller value than one. When Deaton (1986), and Campbell and Deaton (1987) discussed this problem, precautionary savings were that introduction of this concept can explain both the excess smoothness and sensitivity phenomena in the context of the PIH. Table-U2 shows that this claim is not empirically supported. The point estimate of θ is 0.32 which is decisively different from one, and making it free improves the fit of the model significantly as the likelihood ratio test indicates. The actual consumption path is still too smooth to be explained by the PIH even after the new empirical concept of precautionary savings is explicitly taken into account.

On the other hand, a part of Flavin (1981)'s excess sensitivity, i.e. the response of the change in consumption to current income innovation that signals changes in permanent income is not really observed. In our model, the income innovation ϵ_t should affect the change in consumption by the same amount. The empirical estimate shows that this influence is around one-third the theoretical prediction. Thus, we have here a rather "undersensitive," but excessively smooth consumption.

Our innovation term ϵ_t consists of the change in current labor income minus the constant, therefore, we can express the "undersensitivity" alternatively in terms of a current income change which can not be related to lagged changes in income, unlike in Campbell and Deaton (1987). If the liquidity constraint is to be an explanation for the excess sensitivity, "undersensitive" consumption in our model seems to reduce its explanatory power considerably, for obvious reason. One warning about these arguments is urgently due, however, because our labor income contains the proprietors' income to make the international comparison consistent, and the proprietors' income will perhaps not have as much propensity to consume as other labor income components. Our arguments above fall under this qualification. Japan's labor income, as announced by the EPA (Economic Planning Agency), contains a part of the proprietors' income which is regarded as wages and salaries counterpart in the proprietors' income. But the definition necessary in such a separation is hardly available, and moreover, it is not clear if the parallel division can be implemented on the U.S. data. Therefore, we decided to include all the proprietors' income into labor income. This is definitely not an ideal procedure, but it is a possible way under a situation we are facing.

8. Time series of precautionary savings

In this section, we present the numerical values of precautionary savings as a time series of its ratio to real per capita personal saving. Also a time series of conditional variances is presented, and we look into the possibility of relating it to business cycle phases.

Fig.-U2 is the precautionary savings ratio with θ free, and Fig.-U3 is that ratio for the case of fixed θ. Although it is quite clear that the two series show a similar variation, imposing the theoretical restriction, $\theta = 1$, tends to lead to higher estimates of precautionary savings, i.e. in Fig.-U3, precautionary savings often exceed 10% of personal saving, but most of observations lie in the area within

10% in Fig.-U2. Table-U3 represents the quarterly numerical values corresponding to Fig.-U2. Fig.-U1 is a plot of conditional variances(11) against peaks and troughs of the business cycles. Eight cyclical phases are shown, and each of them is denoted by a pair of p_i and t_i ($i=1, \cdots, 8$) where p_i stands for a cyclical peak in the ith phase, and t_i the ith trough. Table-U7 shows the exact timing of peaks and troughs on a quarterly basis.

If the conditional variance rises in the neighborhood of troughs, and declines in the neighborhood of peaks, we will say that precautionary savings have a destabilizing effect on the cyclical movement of the economy. When these tendencies are reversed, precautionary savings are stabilizing. In making such a comparison, peaks and troughs may not always coincide with the tops and bottoms of the fluctuation of the conditional variance. Then, we interpret their relationship based on whether the peaks and troughs are located on downward or upward sloping parts of the cycle. Table-U8 reports the results of such a classification. The result in the upper cells means that the conditional variances do not tend to be consistently low or high in the neighborhood of peaks.(12) Lower cells show, on the other, that some destabilizing tendency of precautionary savings can be read in the neighborhood of the troughs. Especially, if the observation that the spikes right after t_1 and t_6 would have hampered recovery from the troughs are added to the results in Table-8, the possibility of this tendency is enhanced.

It will then potentially erode the effects of common built-in stabilizing factors, and especially the "ratchet" effect in the context of the consumption function. In general, the correspondence between fluctuations of the conditional variance and cyclical phases is not really universal and decisive, but it seems to be economically meaningful to be able to make such a comparison.

CHAPTER III

Precautionary savings in Japan

1. Saving rate in Japan

Japan's saving rate is often referred to as being relatively high compared to other nations. Table-U5 and Table-J1 provide some idea how true this is by comparing the U.S. and Japan. The difference is quite obvious. While the high saving rate has made Japan's investment possible over a long time period, foreign observers (especially in the U.S.) have sometimes pointed out the combination between high saving and export drive (i.e. a low domestic demand and a large trade surplus).

The usual reasons given for why Japan's saving rate is high are: (1) the bonus payment system, (2) social security, (3) the tax credit on savings, (4) the Confucianism tradition (Nakatani (1987)).[18] Besides these factors, soaring land prices in Japan in recent years has touched off the argument that relates the motivation to save to the difficulty of getting private residential housing. Although this argument would not be able to explain the recent decline in the saving rate (since 1979), it seems to be reasonable to argue that there may be an ever lasting fundamental savings in Japan to prepare for future housing construction.

In this chapter, we try to add a new factor, precautionary savings, to the explanation of Japan's saving behavior. Using the same consumer intertemporal optimization model as in Chapter III, and the simultaneous GARCH model, we estimate a reduced form of Japan's consumption function to get the numerical estimate of precautionary savings. This time series is compared to the U.S. to see if this factor can be any help to explain Japan's relatively high saving rate.

2. Labor income

As in Chapter III, we start from specifying the time series structure of labor income. The data we have on Japan for the purpose of this study are not seasonally adjusted. Seasonally adjusted labor income is quite difficult to collect. In some sense, however, it would be of some interest to estimate the model based on seasonally nonadjusted data, and compare the result with the one obtained from the seasonally adjusted data of the U.S. At any rate, our strategy to cope with the seasonal variation contained in Japanese data is to use seasonal dummy variables in the regression. In so doing, we can avoid losing a few sample observation due to seasonal adjustment. The relevant Japanese data available for our study is far from being abundant (the number of observations is 64), therefore the efficient use of information is not a trivial matter here.(14) Then, the empirical test for the existence of a unit root needs some special treatment because the distribution of the relevant parameter in the Dickey-Fuller test is not clear when there are seasonal dummy variables. Hylleberg, Engle, Granger, and Yoo (1988) provides a great help in handling this problem. They showed that the distribution of the parameter of the previous level variable in the augmented Dickey-Fuller test when seasonal dummy variables exist is exactly the same as the usual Dickey-Fuller "t-distribution." Based upon this result, we regressed the first difference of Japan's labor income on a constant, three seasonal dummy variables, the previous level of labor income, and four lagged first differences. The "t-statistic" of the previous level was -2.59, leading to the acceptance of the null hypothesis of the existence of a unit root. As a check on the possibility of a second unit root, we calculated the autocorrelation function for the variable $(1-L)(1-L^4)Y_t$ for the past 15 periods. They are 0.074, -0.059, -0.29, -0.41, -0.259,...(most of the rest take on negligible values) supporting the view that labor income is I(1). Next, an F-test was made to see whether the lagged first differences are significant explanatory variables in the labor income process. The F-statistic turns out to be about 1.51 against a 5% critical value 2.5, implying that Japan's labor income can be well described as a random walk with drift just as in the case of the U.S. labor income, although our drift term here shows seasonal variation. This specification can be supported by the LM test, too, i.e. regressing the residual from a random walk with drift on a constant, three seasonal dummies, and four lagged residuals. A $TR^2 = 5.6$ was obtained against the 5% critical value of 9.49. Based on these test procedures, we will use a random walk with drift as Japan's labor income process.(15)

3. Estimated model

Including seasonal dummy variables both into the labor income and consumption equations, the system of equations to be estimated for Japan becomes

$$Y_{t+1} = \gamma_o + \gamma_1\, D2 + \gamma_2\, D3 + \gamma_3\, D4 + Y_t + \epsilon_{t+1}$$

$$C_t = \gamma_4 + \gamma_5\, D2^* + \gamma_6\, D3^* + \gamma_7\, D4^* + -\gamma_8\, h_{t+1}$$

$$+ \gamma_9\, h_t + C_{t-1} + \epsilon_t + w_t$$

The dummy variable Di takes on the value of one for the ith quarter, and zero in the other periods. In actual estimation, the relations that $D2^* = D3$ and $D3^* = D4$ were used. These relations hold (must be used) because of the difference in timing across the two equations. $D4^*$ is the dummy variable for the fourth quarter of the consumption equation. As in (17) and (17)$'$, ϵ_t is written using the income equation as

$$\epsilon_t = Y_t - \gamma_0 - \gamma_1\, D2^* - \gamma_2\, D3^* - \gamma_3\, D4^* - Y_{t-1}$$

and is substituted into the consumption function. Japan's consumption equation is thus

$$C_t = (\gamma_4 - \gamma_0) + (\gamma_5 - \gamma_1) D2^* + (\gamma_6 - \gamma_2) D3^* + (\gamma_7 - \gamma_3) D4^* \qquad (19)$$

$$- \gamma_8\, h_{t+1} + \gamma_9\, h_t + C_{t-1} + \theta(Y_t - Y_{t-1}) + w_t$$

The estimates shown in Table-J2 represent this version.

Using the simultaneous GARCH method, we first tried to estimate the model for the case that the parameter of ϵ_t in the consumption equation is fixed at one. The model's performance in this case was quite unsatisfactory: convergence was attained only when the criterion was raised from ordinary 0.001 to 0.0033. Besides, the estimates of γ_8 and γ_9 were of the wrong signs, and their t-statistics were 1.07 and 0.13, respectively. This poor performance of the theoretical restriction is part of the reason for selecting the nonrestricted model to make an international comparison across the two countries. Thus, we only show the estimated model for the case of a free θ on Table-J2. The absolute values of γ_8 and γ_9 are in a similar relationship to the U.S. case, but the 95% confidence interval of γ_8 is (0.001, 0.00472) which can easily contain γ_9, thus these estimates can be compatible with a realistic real rate of interest. The t-statistics of γ_8 and γ_9 are very similar to the corresponding case of the U.S., therefore a similar significance level must be applied. The stationarity condition also needs a similar statistical justification here. The matrix D defined in (14) becomes

$$D = \begin{bmatrix} 1.2 & & & \\ & -0.12 & & \\ & & -0.12 & \\ & & & 0.02 \end{bmatrix},$$

therefore, one of eigenvalues exceeds one violating the stationarity condition. However, the A(1,1) element 0.82 of the variance equation has a standard error 0.2. Using this statistic, $A(1,1)$ is not significantly different from 0.6, for instance. This number is sufficiently small to attain the stationary criterion ($D(1,1)$=0.89 in this case).

The deviation of θ from theoretical prediction is even greater than in the U.S. The point estimate of θ is around one-tenth of the U.S. estimate, implying that Japan's consumption path is much smoother: The excess smoothness puzzle deepens in Japan. There can be some partial explanations of this: the bonus payment system, consumption used here being nondurable plus services, and the lifetime employment habit. But these factors do not seem to be able to explain the too smooth Japanese consumption behavior fully (Although the last factor might be very important). Precautionary savings are also unable to, because they have been taken into account in obtaining this result. We do not pursue this problem further in this paper, but our empirical finding on the excess smoothness in the U.S. and Japan seems to be a suggestion of some reasonable reconciliation being due between the PIH and the reality. As for the excess sensitivity, Japan's very low estimate might reflect the overall fact that loaning for consumption is much less popular in Japan than in the U.S., although we do not see empirical support for this concept here in the case of Japan, too.

4. Time series of precautionary savings

Fig.-J1 and Table-J3 present the time series of the conditional variance of Japan's labor income process. Based upon Japan's business cycle chronology (Table-J6), we attempted to classify the stabilizing or destabilizing property of precautionary savings. Table-J7 is in vivid contrast to Table-U8 of the U.S., i.e. Japan's precautionary savings tend to be in favor of expansion by inducing consumers to save less in the neighborhood of most of peaks and troughs.

Table-J4 and Table-J5 show the numerical order of Japan's precautionary savings as a ratio to personal saving. Quarterly figures on Table-J4 suffer from seasonal variation which reflects the seasonality of personal saving. Thus we present Table-J5 which is free from the seasonality. In any case, the implication of these figures is that the aforementioned cyclical property of precautionary savings is hardly a primary factor in Japan's economic expansion, although it could be interpreted as one aspect of the growth oriented characteristics of the Japanese economy.

5. Significance of precautionary savings and their comovement

Table-J1 shows that there was a prominent reduction in Japan's saving rate in 1979 (around a three point drop). Table-J5, on the other hand, tells us that the share of precautionary savings in the personal saving starts decreasing in 1977, and ever declines until the tide changes in 1984. Comparing the corresponding periods of Table-J5 and Table-U4, it can be recognized that the Japanese share is almost always (with one exception) higher than the one of the U.S. up until 1977. Since then, the Japanese share never exceeded that of the U.S. On the other hand, although Japan's annual saving rate remains much higher than the U.S. throughout our observation period, its downward shift in recent years is quite obvious. Combining the two findings, we can observe that the share of precautionary savings foreran the decrease in Japan's saving rate, and the absolute and relative reduction in the share of Japan's precautionary savings signals the decrease in the margin of the Japanese-U.S. annual saving rate (see

Table-J8).

Table-J3 and Fig.-J1 show that the conditional variance of Japan's labor income dropped in 1977, and continued to decline for a long time since then, depriving precautionary savings of its numerical importance. But the symptom of an upswing in this series near the end of the observation period might be encouraging in evaluating this variable.

After all we are led to the idea that the second moment of the labor income process can help explaining a part of the nation's saving rate.

Before ending the section, we briefly make an international comparison of the time series of conditional variances of the U.S. and Japan to see the possibility of their consistent comovement. Fig.-U4 gives some idea about this comparison; the comovement over the entire sample period is unlikely due to their different development in the latter part. This can be confirmed by correlation coefficients calculated for various combinations among the current Japanese conditional variance (level and first differences) and the lagged U.S. conditional variances (level and first differences). For the entire sample period, the maximum correlations are observed at the two period difference (0.37 for the first differences, and 0.39 for the level). Fig.-U4, however, seems to suggest more significant comovement in the former part of the period, and numerical correlations (based on first 23 observations) tend to support this visual identification: The fifth lag for the first difference recorded the correlation 0.71, and 0.79 for the level. The observation

24-62, on the other, produce 0.58 for the first lag of the first difference, and 0.76 for the third lag of the level. These results will be interpreted that there exist nonnegligible orders of comovement of the conditional variances across the two nations, and that the time lag in the comovement has been becoming shorter, and the quantitative difference has been widening. The reason for the comovement can be either that the economies of both countries were operating under the influence of some common factors, or that "when the U.S. sneezes, Japan catches cold, but less badly in these days", but no further effort is made to detect the plausible cause of the relationship.

6. Concluding remarks

Our primary objective in this study was to get the explicit numerical estimates of precautionary savings based on the empirical concept of ARCH. We made such an attempt both on the U.S. and Japanese data, obtaining similar statistical significance for precautionary savings parameters. These results were used to try to reconcile the consumption puzzles with actual data, and to explain the relatively high Japanese savings rate. Unlike Caballero (1987, 1988)'s theoretical prediction, the derivation of the PIH consumption function with explicit characterization of precautionary savings does not seem to help the PIH from the "puzzle" as far as our empirical estimates based on the ARCH model are concerned.

The explanation of Japan's saving behavior was more successful than this: The marked decline in precautionary savings in Japan preceded a considerable decrease in the overall saving rate in 1979, and the time series of precautionary savings seem to well match the development of the margin between the Japanese and American saving rate, although the shares of precautionary savings in both countries are mostly less than 10% of personal saving.

Several limitations are now due: Our concept of labor income contains the proprietors' income for the purpose of international comparison. Its exclusion for the U.S., and taking only its wages and salaries component for Japan could have led to somewhat different result from ours.

Our discussion of consumption behavior was limited to the concept of nondurables plus services. This is because the PIH is commonly analyzed by excluding durables (Flavin (1981), Hayashi(1982). Using total consumption, but setting $\theta = 0$, some estimations were made (although not reported). The relative size of precautionary savings in this case tended to be significantly higher than the results reported here. However, when $\theta \neq 0$ was introduced, the maximum likelihood estimation turns into being very difficult. This might not be the case if the concept of labor income is modified as mentioned above.

Finally, the requirement of data series in estimating our model is not really demanding. This can be an encouragement to extend the cross-national empirical study of precautionary savings to more than the two countries. Such a work will certainly improve our understanding of consumer's behavior and precautionary savings.

Appendix

Whether the form of the solution to the consumer's intertemporal optimization problem changes when there is a constant in labor income process depends on two things: (1) The human wealth, H_t, can be defined. (2) The fundamental first order stochastic difference equation takes the same form.

Let the three cases of the labor income process be

$$Y_t = \delta_0 + Y_{t-1} + \epsilon_t \tag{A-1}$$

$$Y_t = \delta_1 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \epsilon_t \tag{A-2}$$

$$Y_t = \delta_2 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \epsilon_t + \lambda_1 \epsilon_t + \lambda_1 \epsilon_{t-1} + \cdots + \lambda_q \epsilon_{t-q} \text{(A-3)},$$

then it is easy to see that the human wealth, $H_t = E_t \sum_{i=o}^{\infty} \frac{Y_{t+i}}{(1+R)^i}$, becomes $Y_t\left(\frac{1+R}{R}\right) + \delta_0\left(\frac{1+R}{R^2}\right)$ for the case of random walk with drift, and that the terms $\delta_1^*\left(\frac{1+R}{R^2}\right)$ and $\delta_2^*\left(\frac{1+R}{R^2}\right)$ are added to the previous sum for other cases where letting $A_1^*(L)$ and $A_2^*(L)$ be such that the $(A-2)$ and $(A-3)$ equations can be written as $(1-L)A_1^*(L)\,Y_t = \delta_1 + \epsilon_t$ and $(1-L)A_2^*(L)\,Y_t = \delta_2 + \epsilon_t + \lambda_1 \epsilon_{t-1} + \cdots + \lambda_q \epsilon_{t-q}$ respectively, then the relations $\delta_1^* = \frac{\delta_1}{A_1^*(1)}$ and $\delta_2^* = \frac{\delta_2}{A_2^*(1)}$ hold. These results imply that the human wealth is finite under the income processes with a drift term.

On the other hand, because the effects of adding a drift term to the "revision" of expectation $(E_t - E_{t+1})$ $(Y_{t+1} + \frac{Y_{t+2}}{(1+R)} + \frac{Y_{t+3}}{(1+R)^2} + \ldots)$ cancel out, the fundamental relations ((19), (20), and (21) in Ginama(1988)) are unaltered. Therefore, the solutions to the consumers' problem stay the same.

Data

This section consists of four subsections explaining the concepts and the sources of the U.S. and Japan's data, and figures and tables.

1. The U.S. data.

The U.S. data were all obtained from the "Citibank Database" over the period 1947,I-1985,IV on a quarterly basis. Using seasonally adjusted series, the nominal labor income was calculated as wages and salaries plus other labor income plus transfers to persons plus proprietors' income minus personal tax and nontax payments minus contributions for social insurance. This is very similar to Hayashi (1982)'s definition of labor income except for proprietors' income and transfers to foreigners. The latter is a negligible amount, and the number of observation is smaller than the other series, therefore, we excluded this item from our definition. The former was included for the purpose of making international comparisons consistent.

Real per capita labor income is obtained after adjusting this nominal value by the resident population and the CPI (all items, 1967=100). Nominal consumption is the sum of nondurables and services. The same population and CPI were used to get real per capita consumption. Real per capita figures are expressed in terms of thousands of dollars.

2. The Japanese data.

Quarterly Japanese data come from EPA's national income accounts on labor income and consumption, and from the "Toyo Keizai" long run business time series on population and the CPI (1967=100) (originally from the Statistics Bureau of the Prime Minister's Office).

The division of consumption items into durables, nondurables, and services is available only from 1970, and our observation period is 1970-1985. Japan's labor income is defined as wages and salaries plus proprietors' income plus pension receipts and other transfer aid minus tax and non tax payments including social insurance contributions. Japan's proprietors' income is divided into wages and salaries, and proprietors' operating surpluses. Instead of doing the same calculation (which is not readily known) on the U.S. data, we simply add them up, and include in the concept of labor income. Certainly this procedure will not be entirely satisfactory, but it can avoid arbitrariness involved in the separation.

Consumption is the sum of nondurables and services. Population is the total Japanese population, and the CPI is the all item index. All series were collected on a quarterly basis. Using the population and the CPI, labor income and consumption are expressed in terms of thousands of 1967 yen.

3. Figures

Fig.-U1 depicts the time series of the U.S. labor income obtained in the model where θ is free to move. The vertical lines show the peaks and troughs of the business cycles over the period 1947,III-1985,IV. p_i and t_i correspond to specific quarters in Table-U7.

Fig.-U2 is the U.S. precautionary savings as a ratio to personal saving over the period 1947,II-1985,III. The absolute values of precautionary savings were obtained using conditional variances in Table-U1. This series was divided by real per capita personal savings.

Fig.-U3 was calculated in the same manner as in Fig.-U2 except conditional variances here come from the model in which θ is fixed. Observation period is 1947,II-1985,III.

Fig.-U4 describes the time series of the conditional variances of the U.S. and Japan over the period 1970.III-1985.IV. The U.S. values are the same as in Fig.-U1, and Japanese values come from the model with free θ, thus, a direct comparison can be made.

Fig.-J1 is the conditional variance of Japanese labor income obtained from the model with free θ. This was used in Fig.-U4 above. The period of observation is, thus, 1970.III-1985.IV. Vertical lines stand for the peaks and troughs of Japan's business cycles. These phases can be read from Table-J6.

Fig.-J2 is shown only for reference. Due to the seasonality in Japanese real per capita personal saving, this series also shows seasonal variation (1970.II-1985.III).

4. Tables

Table-U1 reports the results of estimating the model (15) with θ=1. The period of observation of Y_{t+1} is 1947.III-1985.IV.

Table-U2 shows the results of estimation with θ being free.

Table-U3 represents the numerical values of the U.S. precautionary savings as a ratio to real per capita savings. The period of observation is 1947.II-1985.III.

Table-U4 was calculated based on the quarterly figures of precautionary savings and personal savings. An annual ratio of precautionary savings is needed because the corresponding Japanese ratio is also annual (to avoid the influence of bothersome seasonal variation). The period of observation is 1947-1985.

Table-U5 is the U.S. annual saving rate as a ratio of personal savings to disposable income. This series is used to show that the Japanese saving rate is relatively higher than the U.S. The period of observation is 1947-1985.

Table-U6 is numerical values of the U.S. conditional variances obtained when θ is allowed to move. Fig.-U1 is thus a plot of this table.

Table-U7 provides the U.S. business cycle chronology. This was obtained from the appendix of the Citibank database.

Table-U8 represents a classification of the cyclical characteristics of the conditional variances of the U.S. p_i and t_i stand for the peak and trough in the ith cycle.

Table-U9 is the time series of the U.S. real per capita consumption of non-durables plus services in terms of thousands of dollars. The period of observation is 1947.I-1985.IV.

Table-U10 is the time series of the U.S. real per capita labor income in terms of thousands of dollars. The period of observation is 1947.I-1985.IV.

Table-J1 is the Japanese annual saving rate to be compared to Table-U5 of the U.S. The period of observation is 1965-1985.

Table-J2 reports the result of estimating the Japanese model (18) with free θ. The period of observation of Y_{t+1} here is 1970.III-1985.IV.

Table-J3 shows the numerical values of the conditional variances of Japanese labor income. This is depicted in Fig.-J1. The period of observation is 1970.III-1985.IV.

Table-J4 shows Japanese precautionary savings as a ratio to per capita saving on a quarterly basis. The period of observation is 1970.II-1985.III.

Table-J5 shows that annual ratio of Japanese precautionary savings to personal savings. The period of observation is 1970-1985.

Table-J6 shows Japan's business cycle chronology as prepared by the EPA.

Table-J7 gives the result of classifying the properties of the Japanese conditional variances of labor income. It shows whether a particular variation of the conditional variance at each point of time can be regarded as being stabilizing or destabilizing in terms of p_i and t_i.

Table-J8 gives the time series of the difference between the Japanese and American annual savings rates. The period of observation is 1965-1985.

Table-J9 is the time series of Japanese real per capita consumption of non-durables plus services in terms of thousands of Yen. The period of observation is 1970.I-1985.IV.

Table-J10 is the time series of Japanese real per capita labor income in terms of thousands of Yen. The period of observation is 1970.I-1985.IV.

Table-U1

The U.S. model with $\theta = 1.0$

Parameter	income	t-statistic	consumption	t-stat	variance	t-stat
γ_0	0.0032	1.36				
γ_1			0.0064	1.65		
$-\gamma_2$			-12.58	3.40		
γ_3			6.92	2.37		
θ			1.0			
$C(1, 1)$					-0.025	16.87
$C(2, 1)$					0.0022	0.33
$C(2, 2)$					0.0021	0.53
$A(1, 1)$					0.64	7.80
$A(2, 2)$					0.38	4.87
$B(1, 1)$						
$B(2, 2)$					0.92	31.25

number of observations = 154
convergence R^2 = 0.0007
Log likelihood = 848.9

Table-U2

The U.S. model with free θ

Parameter	income	t-stat	consumption	t-stat	variance	t-stat
γ_0	0.011	5.84				
γ_1			0.0079	4.50		
$-\gamma_2$			-4.30	1.84		
γ_3			3.11	1.44		
θ			0.32	9.12		
$C(1, 1)$					-0.011	3.54
$C(2, 1)$						
$C(2, 2)$					0.0027	1.31
$A(1, 1)$					0.53	5.88
$A(2, 2)$					0.19	2.33
$B(1, 1)$					-0.78	10.31
$B(2, 2)$					0.95	17.34

number of observations = 154
convergence R^2 = 0.0000046
Log likelihood = 937.2

Table-U3(quarterly)

The U.S. precautionary savings

1947,II	0.169481203
	0.053624012
	0.103104927
1948,I	0.051302310
	0.037757851
	0.023335762
	0.020087341
	0.064585552
	0.073959820
	0.050450489
	0.044907164
	0.073844992
	0.087215938
	0.126699045
	0.037479784
1951,I	0.049109627
	0.018788211
	0.015453861
	0.016456714
	0.017735068
	0.017417835
	0.015372972
	0.017270736
	0.016323978
	0.012351949
	0.015982145
	0.018192450
1954,I	0.017808514
	0.022391072
	0.018218087
	0.018392079
	0.018174710
	0.022028031
	0.019233834
	0.015598897
	0.012572700
	0.010790119
	0.011041299
	0.009884581
1957,I	0.012497430
	0.012587980
	0.012542149
	0.018910510

	0.026336737
	0.023308005
	0.021406326
	0.016029613
	0.015279314
	0.013989868
	0.025104640
	0.020773511
1960,I	0.015802933
	0.015318370
	0.015809467
	0.023984816
	0.017091503
	0.014927647
	0.011744501
	0.011896311
	0.010800472
	0.010155860
	0.011130870
	0.012809164
1963,I	0.011607561
	0.010870718
	0.011219808
	0.010384457
	0.013952608
	0.022184025
	0.018022317
	0.012646094
	0.012105038
	0.010305718
	0.020993240
	0.020151921
1966,I	0.015988408
	0.013633494
	0.011456762
	0.009216605
	0.008080122
	0.007623566
	0.006895635
	0.006409697
	0.009923860
	0.011612738
	0.016185356
	0.012623998
1969,I	0.019955035
	0.014217571
	0.014130730
	0.011781166
	0.012723657

	0.010438331
	0.008160238
	0.014055735
	0.015283340
	0.014483880
	0.012087638
	0.010373748
1972,I	0.009744823
	0.012695090
	0.012161345
	0.041597918
	0.028680488
	0.017795447
	0.012742811
	0.008366657
	0.022519529
	0.029869070
	0.020592749
	0.022627369
1975,I	0.032808132
	0.076098800
	0.078192420
	0.048725825
	0.035718724
	0.024010325
	0.017481485
	0.013926274
	0.016381603
	0.013597534
	0.015457300
	0.012712034
1978,I	0.009177910
	0.011920277
	0.011968123
	0.009362535
	0.008438447
	0.020576436
	0.022693289
	0.039866820
	0.041275568
	0.079582162
	0.056155141
	0.033703052
1981,I	0.033697430
	0.040760577
	0.030106436
	0.027427405
	0.024172891
	0.017342681

	0.013477650
	0.014579662
	0.012606316
	0.012570838
	0.012428843
	0.018119667
1984,I	0.020551963
	0.018185452
	0.012798222
	0.012716617
1985,I	0.018133376
	0.026775917
	0.053064700

Table-U4(annual)

The U.S. precautionary savings

1947	0.0916
	0.0302543
	0.0589121
	0.0734118
	0.0216523
1952	0.0168923
	0.0156983
	0.0191256
	0.0186716
	0.0110248
1957	0.0140601
	0.0215789
	0.0184016
	0.0175948
	0.0137622
1962	0.0111636
	0.0110040
	0.0167319
	0.0162061
	0.0124552
1967	0.0072424
	0.0123974
	0.0146894
	0.0112823
	0.0131200
1972	0.0204196
	0.0162437
	0.0238185
	0.0607152
	0.0232142
1977	0.0144921
	0.0105882
	0.0220899
	0.0523470
	0.0327934
1982	0.0175054
	0.0140494
	0.0161296
	0.0306

Table-U5(annual)

The U.S. saving rate

1947	3.037487
	5.887043
	3.962766
	6.061701
	7.303679
1952	7.266472
	7.221950
	6.321949
	5.748363
	7.183667
1957	7.229874
	7.486918
	6.340224
	5.809418
	6.648829
1962	6.544239
	5.933987
	6.983441
	7.051150
	6.840654
1967	8.032735
	6.967970
	6.426314
	8.066658
	8.541452
1972	7.307269
	9.370640
	9.310444
	9.155346
	7.645944
1977	6.575680
	7.102451
	6.825114
	7.134105
	7.494360
1982	6.807655
	5.491935
	6.462183
	4.612281

Table-U6(quarterly)

The U.S. conditional variances

1947,III	0.00092
	0.00076
1948,I	0.00103
	0.00079
	0.00088
	0.00071
	0.00057
	0.00129
	0.00110
	0.00082
	0.00064
	0.00277
	0.00235
	0.00163
1951,I	0.00112
	0.00093
	0.00071
	0.00057
	0.00058
	0.00059
	0.00052
	0.00056
	0.00053
	0.00050
	0.00043
	0.00054
1954,I	0.00064
	0.00060
	0.00062
	0.00050
	0.00053
	0.00045
	0.00059
	0.00058
	0.00048
	0.00042
	0.00039
	0.00041
1957,I	0.00038
	0.00045
	0.00048
	0.00047
	0.00066

	0.00095
	0.00081
	0.00083
	0.00065
	0.00053
	0.00051
	0.00073
1960,I	0.00065
	0.00052
	0.00045
	0.00049
	0.00068
	0.00055
	0.00050
	0.00044
	0.00045
	0.00041
	0.00038
	0.00040
1963,I	0.00041
	0.00038
	0.00036
	0.00035
	0.00037
	0.00051
	0.00095
	0.00073
	0.00057
	0.00048
	0.00043
	0.00102
1966,I	0.00090
	0.00067
	0.00058
	0.00050
	0.00044
	0.00043
	0.00039
	0.00037
	0.00035
	0.00051
	0.00062
	0.00068
1969,I	0.00055
	0.00073
	0.00057
	0.00071
	0.00059
	0.00062

	0.00060
	0.00049
	0.00084
	0.00093
	0.00095
	0.00076
1972,I	0.00059
	0.00051
	0.00061
	0.00064
	0.00268
	0.00186
	0.00127
	0.00094
	0.00070
	0.00177
	0.00200
	0.00138
1975,I	0.00165
	0.00198
	0.00654
	0.00522
	0.00327
	0.00228
	0.00150
	0.00104
	0.00076
	0.00075
	0.00071
	0.00091
1978,I	0.00069
	0.00055
	0.00069
	0.00070
	0.00056
	0.00052
	0.00120
	0.00125
	0.00201
	0.00228
	0.00443
	0.00314
1981,I	0.00202
	0.00198
	0.00226
	0.00184
	0.00169
	0.00133
	0.00098

	0.00072
	0.00071
	0.00060
	0.00049
	0.00053
1984,I	0.00087
	0.00120
	0.00094
	0.00073
1985,I	0.00065
	0.00073
	0.00136
	0.00164

Table-U7(quarterly)

The U.S. business cycles rate

TROUGH	PEAK
IV,1949(t_1)	IV,1948(p_1)
II,1954(t_2)	II,1953(p_2)
II,1958(t_3)	III,1957(p_3)
I,1961(t_4)	II,1960(p_4)
IV,1970(t_5)	IV,1969(p_5)
I,1975(t_6)	IV,1973(p_6)
III,1980(t_7)	I,1980(p_7)
IV,1982(t_8)	III,1981(p_8)

Table-U8

Cyclical characteristics

stabilizing	destabilizing
p_3, p_5, p_7, p_8	p_1, p_2, p_4, p_6
t_1, t_5, t_8	t_2, t_3, t_4, t_6, t_7

p_i: Peaks
t_i: Troughs

Table-U9(quarterly)

The U.S. real per capita consumption

1947,I	1.48011
	1.48684
	1.48161
	1.45590
	1.44641
	1.44924
	1.43389
	1.45145
	1.45384
	1.44498
	1.43596
	1.44758
	1.45675
	1.47137
	1.49676
	1.47588
	1.48649
	1.47290
	1.49375
	1.49786
	1.49429
	1.51667
	1.53402
	1.55766
	1.57060
	1.57377
	1.56967
	1.56200
	1.57323
	1.58360
	1.60128
	1.62046
	1.63241
	1.64617
	1.65596
	1.67945
	1.69664
	1.69828
	1.69734
	1.69932
1957,I	1.70218
	1.69802
	1.71029

	1.70790
	1.69184
	1.69735
	1.71998
	1.72699
	1.74846
	1.76511
	1.77612
	1.78528
	1.79102
	1.80602
	1.80250
	1.80314
	1.81101
	1.82688
	1.82178
	1.84351
	1.85307
	1.86625
	1.87492
	1.88917
	1.89243
	1.89995
	1.91700
	1.92306
	1.94903
	1.97524
	2.00239
	2.00829
	2.02392
	2.04433
	2.07179
	2.11734
	2.13186
	2.14763
	2.15790
	2.15990
1967,I	2.17886
	2.19166
	2.19801
	2.20183
	2.23546
	2.26432
	2.28462
	2.28965
	2.30169
	2.31057
	2.31490
	2.32447

	2.33267
	2.32958
	2.34035
	2.34466
	2.35025
	2.36364
	2.36893
	2.39175
	2.41607
	2.45161
	2.47770
	2.51720
	2.53108
	2.53131
	2.53966
	2.53407
	2.52318
	2.52968
	2.51963
	2.49096
	2.49395
	2.52798
	2.53661
	2.54313
	2.57559
	2.59494
	2.61609
	2.64883
1977,I	2.66743
	2.67102
	2.69268
	2.72291
	2.73969
	2.76388
	2.76519
	2.77201
	2.77609
	2.75922
	2.75214
	2.75564
	2.72161
	2.67350
	2.69880
	2.70354
	2.69298
	2.69384
	2.66663
	2.66367
	2.67368

2.66767
2.67009
2.71239
2.73962
2.77220
2.78616
2.80333
2.81699
2.84988
2.84924
2.85804
2.87506
2.88339
2.88933
2.90881

Table-U10(quarterly)

The U.S. real per capital labor income

1947,I	1.55915
	1.50488
	1.53224
	1.50386
	1.50387
	1.54662
	1.57036
	1.57537
	1.53314
	1.51836
	1.52099
	1.52491
	1.62559
	1.59282
	1.62194
	1.63100
	1.62139
	1.64214
	1.64989
	1.64208
	1.63412
	1.63458
	1.66672
	1.69204
	1.71662
	1.72507
	1.71343
	1.69934
	1.69347
	1.68426
	1.69403
	1.72375
	1.73738
	1.77426
	1.80375
	1.81778
	1.83065
	1.83974
	1.83890
	1.85108
1957,I	1.84433
	1.83969
	1.83772

	1.81980
	1.79259
	1.78437
	1.82322
	1.84349
	1.85578
	1.88222
	1.86186
	1.85672
	1.87099
	1.87768
	1.87197
	1.85328
	1.85961
	1.88328
	1.89658
	1.92274
	1.93531
	1.94731
	1.94599
	1.94682
	1.95436
	1.96507
	1.97516
	1.99740
	2.03189
	2.08598
	2.10636
	2.11525
	2.12803
	2.14696
	2.20547
	2.24017
	2.25006
	2.24970
	2.27032
	2.28507
1967,I	2.30796
	2.32031
	2.32852
	2.33917
	2.37496
	2.41190
	2.39888
	2.40406
	2.38477
	2.39816
	2.43864
	2.44032

	2.42990
	2.45957
	2.47304
	2.44624
	2.49006
	2.53138
	2.52890
	2.53993
	2.54198
	2.57838
	2.61214
	2.71106
	2.74259
	2.76297
	2.76222
	2.77260
	2.71844
	2.67658
	2.67486
	2.63719
	2.59340
	2.74067
	2.68830
	2.70309
	2.73944
	2.74809
	2.75687
	2.77066
1977,I	2.75841
	2.79152
	2.83830
	2.85527
	2.86545
	2.90542
	2.89337
	2.90020
	2.89782
	2.85764
	2.83151
	2.77997
	2.73376
	2.64341
	2.68980
	2.69944
	2.66376
	2.61759
	2.59391
	2.56545
	2.55154

2.54998
2.56139
2.59544
2.61925
2.63206
2.66232
2.71221
2.76776
2.76237
2.76287
2.75727
2.74185
2.80630
2.76858
2.78548

Table-J1(annual)

Japanese saving rate

1965	15.767791
	15.050321
	15.498103
	16.709318
	17.353218
1970	18.230455
	17.949787
	18.233158
	20.906475
	23.721058
1975	22.115925
	22.394760
	21.047821
	20.548443
	17.850569
1980	17.904911
	18.293875
	16.519670
	16.313227
	16.137892
1985	16.045830

Table-J2

The Japanese model with free θ

Parameter	income	t-stat	consumption	t-stat	variance	t-stat
γ_0	-53.3	92.7				
γ_1	81.1	86.8				
γ_2	46.9	24.0				
γ_2	87.5	103.2				
γ_4			-5.8	9.5		
γ_5			6.08	6.1		
γ_6			9.7	13.1		
γ_7			9.6	9.6		
$-\gamma_8$			-0.00231	1.88		
γ_9			0.00173	1.47		
θ			0.0309	2.4		
$C(1, 1)$					0.45	1.1
$C(2, 1)$						
$C(2, 2)$					0.74	9.2
$A(1, 1)$					0.82	4.1
$A(2, 2)$					-0.15	0.56
$B(1, 1)$					0.73	7.38
$B(2, 2)$						

number of observations = 62
convergence R^2 = 0.0007
Log likelihood = -204.2

Table-J3(quarterly)

Japanese conditional variances

1970,III	118.32164
	143.92831
1971,I	114.77527
	187.66262
	296.39444
	241.28179
	179.88869
	248.61482
	235.11594
	202.44624
	107.28600
	90.91726
	199.88744
	258.47416
1974,I	155.69793
	1319.71811
	701.43014
	421.35547
	249.03956
	133.13132
	392.45835
	293.59294
	991.18801
	535.88528
	329.97332
	198.30643
1977,I	105.39324
	56.68866
	56.73170
	66.25065
	35.60603
	22.37636
	37.13302
	40.28162
	21.79017
	12.23035
	15.36585
	15.48992
1980,I	8.83322
	15.12697
	16.63984
	9.04783
	4.98871

	3.21003
	2.43722
	1.64882
	1.23760
	0.87608
	0.71151
	0.57946
1983,I	0.55792
	0.93333
	2.58993
	1.67998
	4.57213
	2.78171
	5.32669
	3.15083
	4.38207
	2.99028
	20.58886
1985,IV	21.73045

Table-J4(quarterly)

Japanese precautionary savings

1970,II	0.020030349
	0.020392725
	0.008007147
1971,I	0.063431464
	0.050212711
	0.032163985
	0.013528556
	0.120001309
	0.034206100
	0.027798770
	0.006444629
	0.038138300
	0.025219448
	0.023493016
	0.007954723
1974,I	0.495765507
	0.057903323
	0.034989733
	0.012771746
	0.046914835
	0.036219671
	0.027649552
	0.050586980
	0.138454184
	0.027408466
	0.020873033
	0.005250855
1977,I	0.029039685
	0.005148476
	0.007002564
	0.001737208
	0.007433841
	0.003091975
	0.004338686
	0.001123385
	0.011260932
	0.001427255
	0.001863428
	0.000473772
1980,I	0.017146628
	0.001451624
	0.001108160
	0.000269752

	0.001882195
	0.000196563
	0.000205105
	0.000068243
	0.000806393
	0.000063979
	0.000078157
	0.000032321
1983.I	0.006216536
	0.000211563
	0.000208314
	0.000272058
	-0.042146057
	0.000440516
	0.000377701
	0.000255673
1985,I	0.008845385
	0.001587846
	0.002843238

Table-J5(annual)

Japanese precautionary savings

1970	0.0138
	0.0305203
	0.0241356
	0.0172429
	0.0581110
1975	0.0412523
	0.0257148
	0.0050148
	0.0027821
	0.0013378
1980	0.0011743
	0.0002119
	0.0000739
	0.0002623
	0.0004170
1985	0.0022

Table-J6(quarterly)

Japanese business cycles

TROUGH	PEAK
IV,1971(t_1)	IV,1973(p_1)
I,1975(t_2)	I,1977(p_2)
IV,1977(t_3)	I,1980(p_3)
I,1983(t_4)	II,1985(p_4)

Table-J7

Cyclical characteristics

stabilizing	destabilizing
p_1	p_2, p_3, p_4
t_1, t_2, t_4	t_3

p_i: Peaks
t_i: Troughs

Table-J8(annual)

Difference in saving rates

1965	8.717
	8.21
	7.465
	9.741
	10.927
1970	10.163
	9.41
	10.93
	11.535
	14.41
1975	12.961
	14.75
	14.472
	13.446
	11.025
1980	10.771
	10.8
	9.712
	10.821
	9.676
	11.434

Table-J9(quarterly)

Japanese real per capita consumption

1970,I	55.59314
	57.49810
	62.00795
	65.23530
	58.44082
	60.33413
	62.85198
	67.95373
	62.43388
	64.75287
	69.01488
	74.22378
	67.65031
	67.37944
	71.43155
	76.73022
	63.90483
	66.04576
	70.38811
	74.35010
1975,I	67.90360
	69.21875
	73.59182
	77.40318
	69.30695
	70.38363
	74.86702
	79.16708
	71.72138
	72.66901
	76.76080
	81.40956
	75.41971
	76.13753
	80.80383
	86.36153
	79.35249
	80.76552
	84.74539
	89.31451
1980,I	80.77272
	81.34749
	84.80631

	89.26825
	81.40295
	82.09627
	85.82100
	91.10539
	83.49673
	85.23384
	88.86243
	94.59286
	87.27557
	87.15231
	91.26151
	96.23463
	88.25484
	89.54910
	92.94926
	98.26347
1985,I	90.71288
	91.87241
	95.11697
	100.9834

Table-J10(quarterly)

Japanese real per capita labor income

1970,I	62.90751
	77.68880
	82.26780
	108.9278
	69.30156
	79.95768
	84.74849
	110.1733
	71.91158
	87.26862
	91.62541
	125.9115
	79.67881
	122.3874
	131.0336
	170.5097
	74.39407
	99.94724
	102.1828
	130.1861
1975,I	78.19861
	127.7536
	132.6434
	131.6766
	82.46474
	101.9130
	101.4398
	134.9753
	80.60483
	102.0658
	102.9895
	137.9245
	86.82471
	108.4593
	107.5793
	142.4280
	88.24268
	112.3846
	109.2598
	142.6589
1980,I	85.44013
	109.6357
	103.5095

	137.7175
	83.65579
	110.4940
	103.6324
	138.3262
	84.83786
	112.3088
	105.9350
	140.4060
	86.28009
	115.6832
	108.9051
	140.8389
	87.02840
	117.0826
	111.1417
	143.4168
1985,I	89.25988
	122.2647
	111.9166
	146.5572

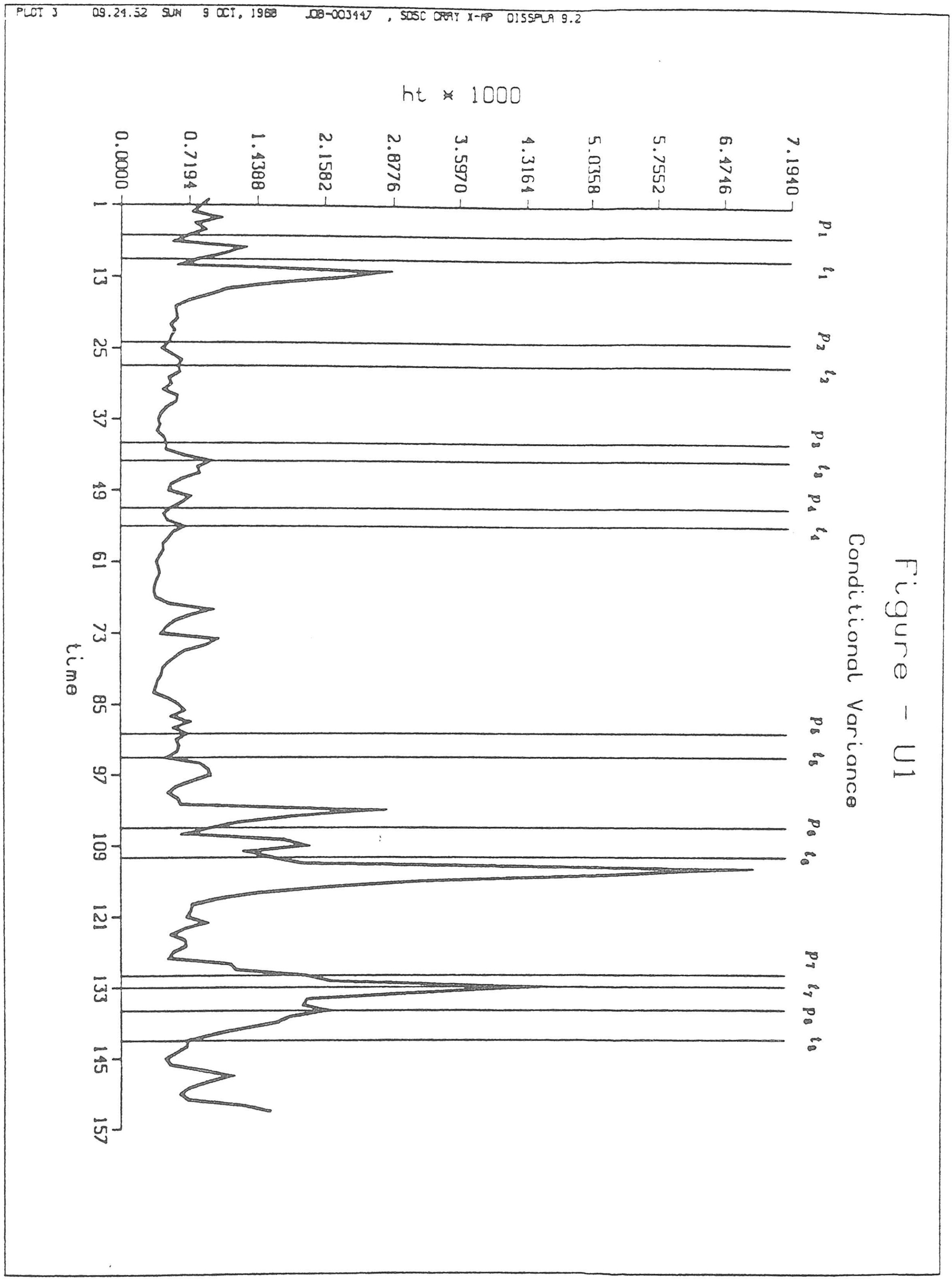


Figure - U1
Conditional Variance

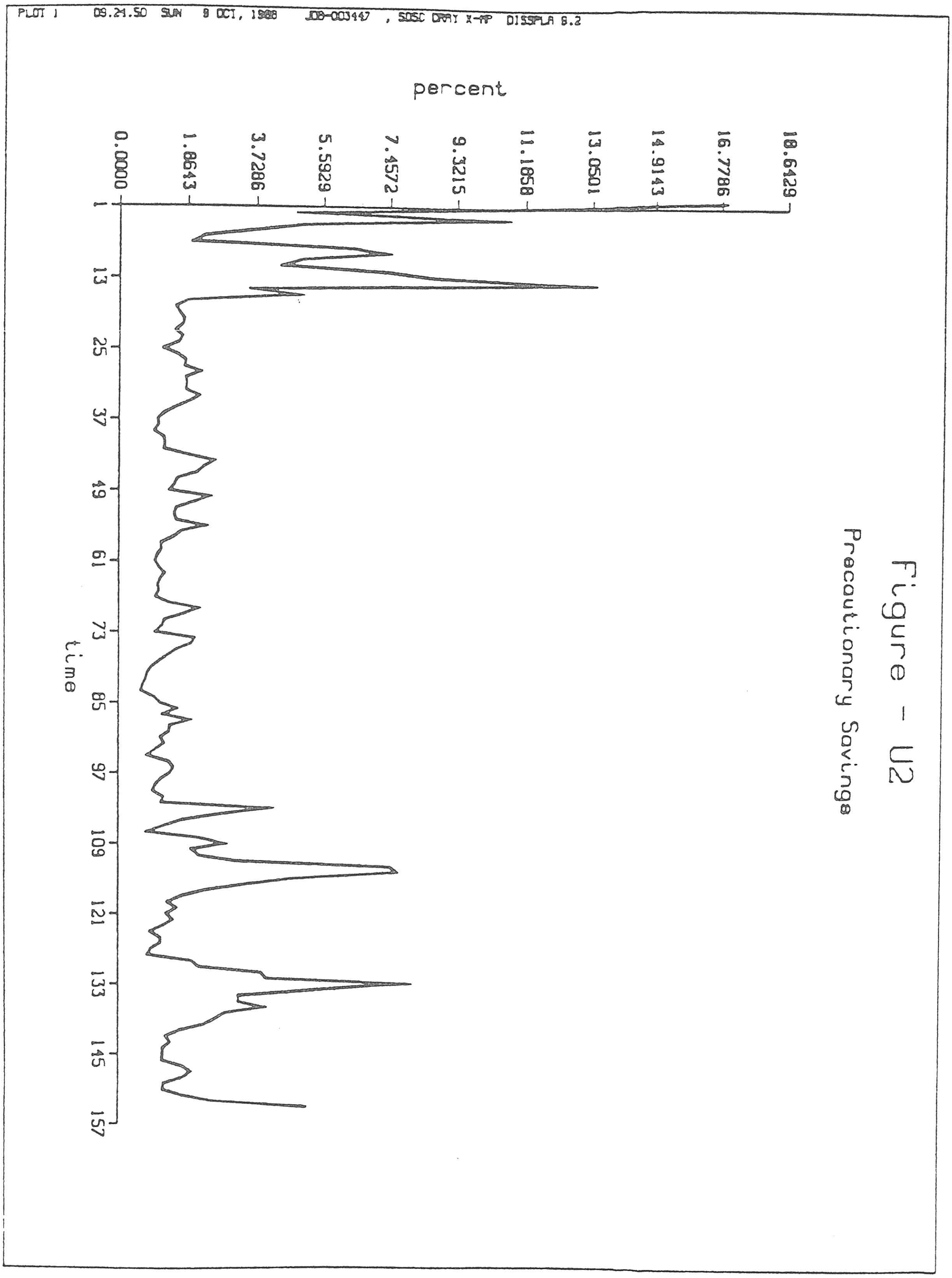


Figure - U2
Precautionary Savings

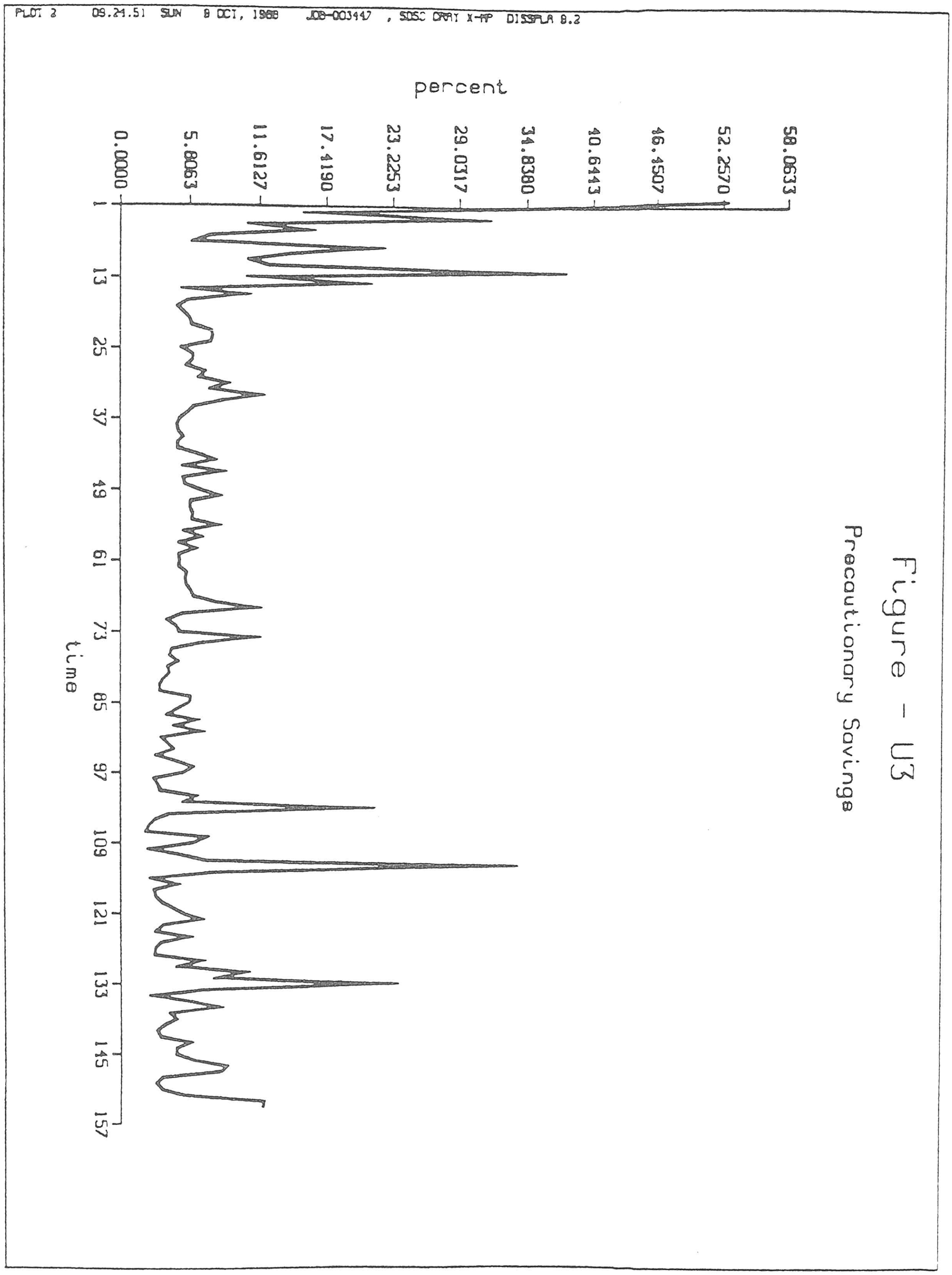
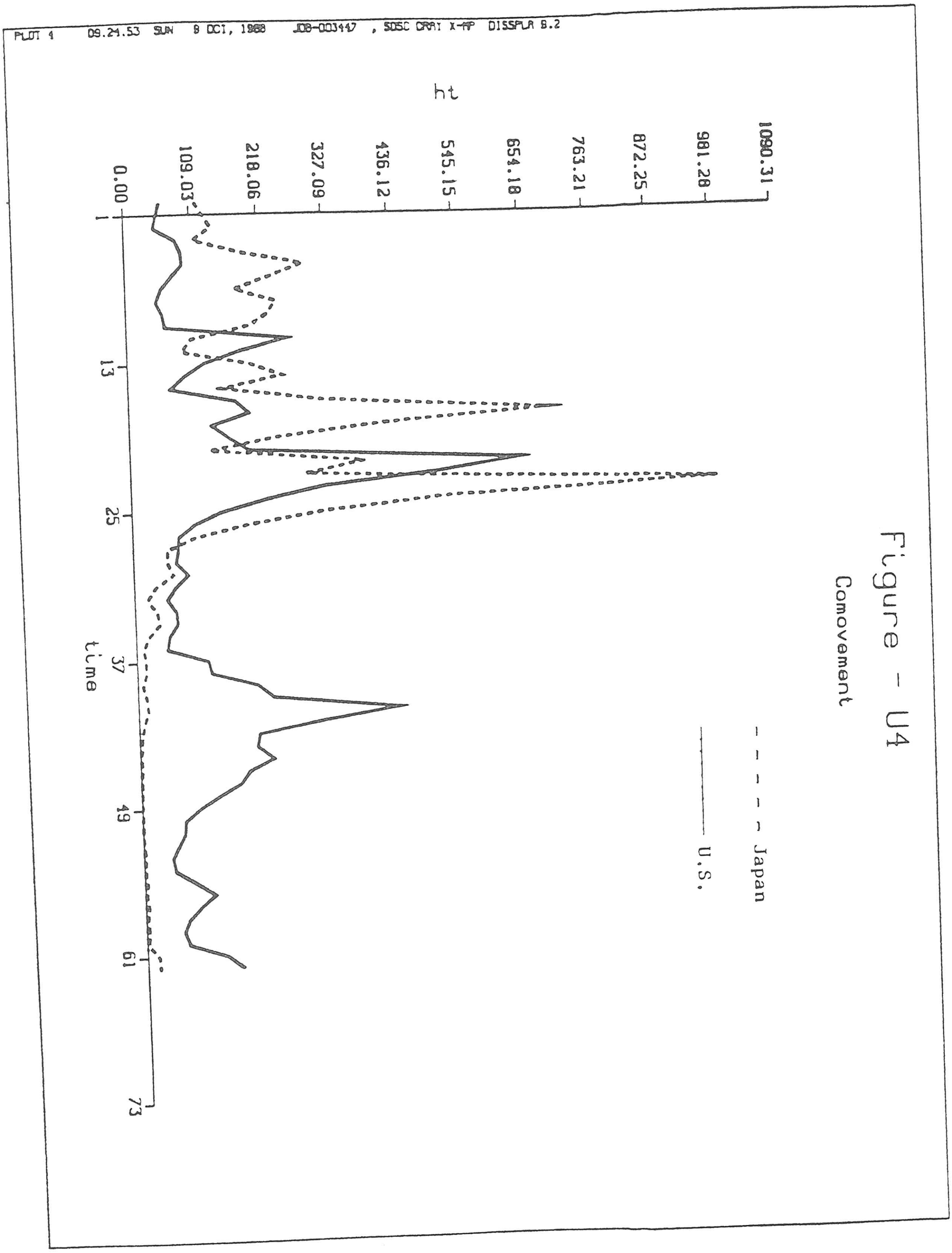


Figure - U3
Precautionary Savings


PLOT 1 09.24.53 SUN 9 OCT, 1988 JOB-003447 , SDSC CRAY X-MP DISSPLA 8.2
Figure - U4
Comovement
Japan
U.S.
ht
1090.31
981.28
872.25
763.21
654.18
545.15
436.12
327.09
218.06
109.03
0.00
time
1
13
25
37
49
61
73

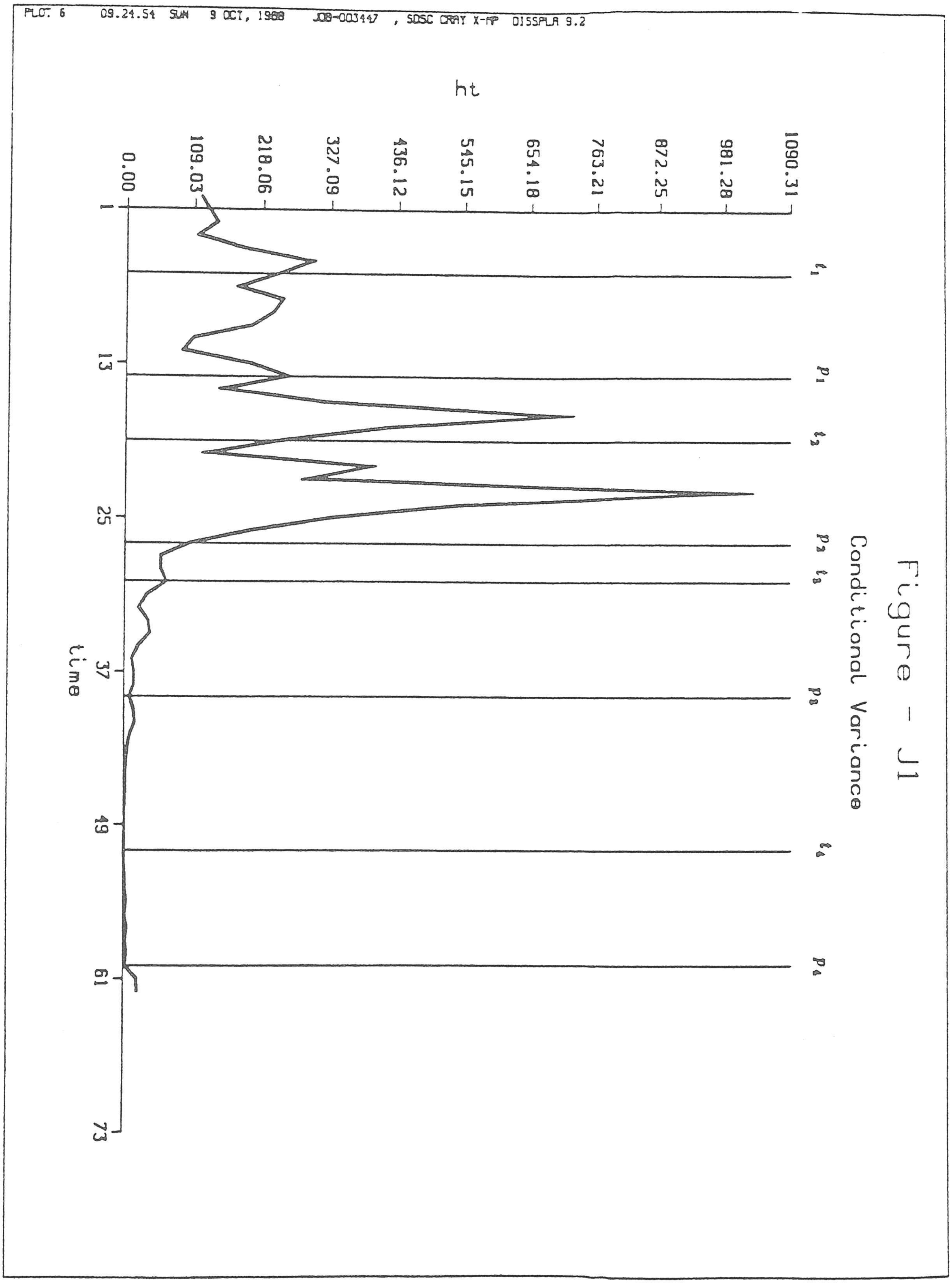


Figure - J1
Conditional Variance

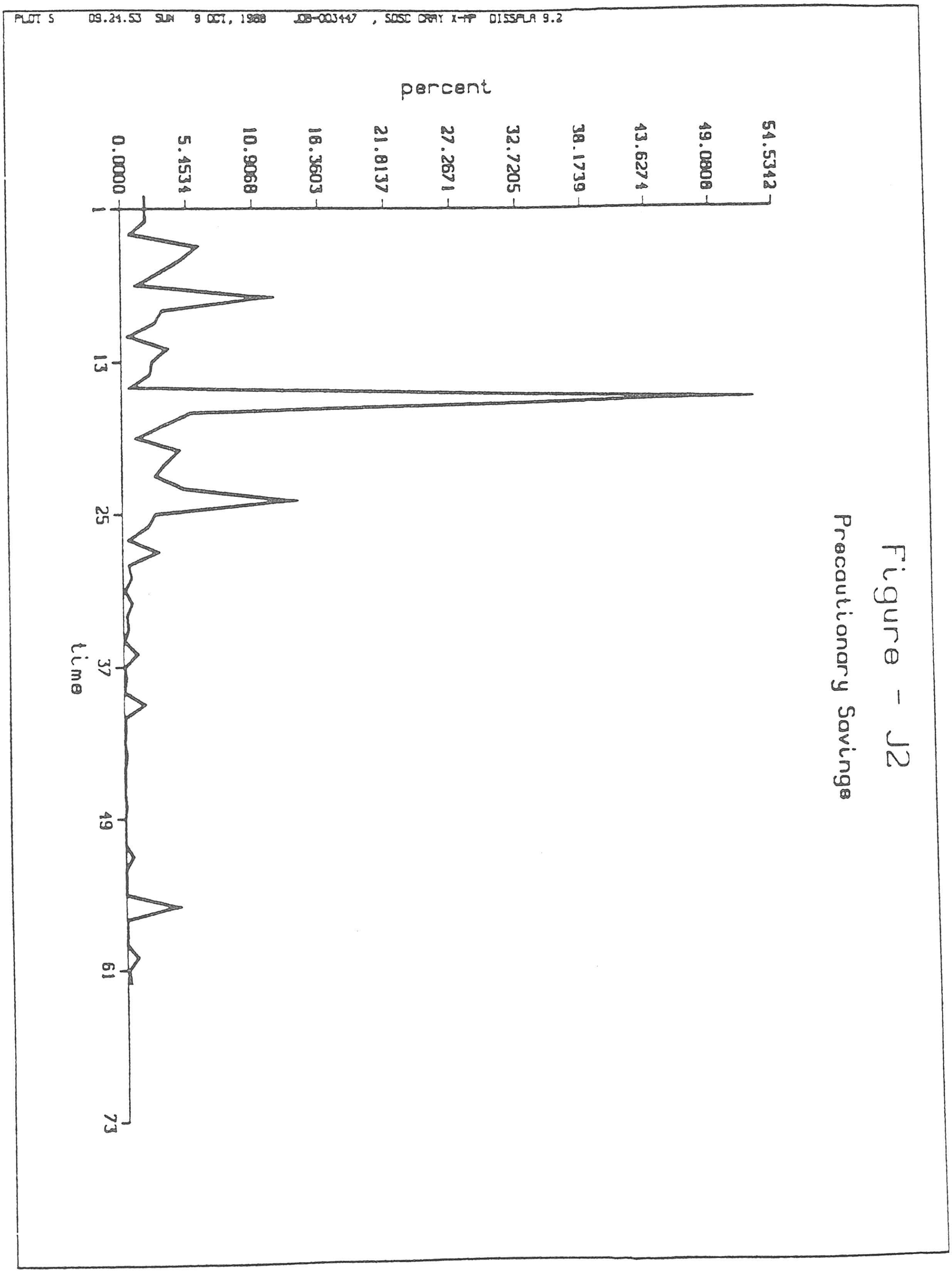


Figure - J2
Precautionary Savings
percent
0.0000
5.4534
10.9068
16.3603
21.8137
27.2671
32.7205
38.1739
43.6274
49.0808
54.5342
time
1
13
25
37
49
61
73

Footnotes

(1) The existence of a drift term in the labor income process does not cause any problem in the solution algorithm (See Appendix A).

(2) ϵ_t can not be treated as an error term in forming the likelihood function because it is assumed to be known. However, this assumption makes it possible to use the one period ahead conditional variance in the ARCH model.

(3) One intuitive difficulty seems to exist interpreting his measure of precautionary savings, the drift term which increases as the uncertainty of the labor income increases. In contrast to equations (1) and (3), this drift term being positive implies that precautionary savings raises the change in consumption, but decreases the level of consumption (when it is expressed using a permanent income with the coefficient of one). The former explains "persistent growth of consumption" while the latter captures the precautionary motive toward future labor income uncertainty. His point estimate of this term from an univariate ARCH-M model is positive (although not significant), implying that the consumption growth is to be sustained by the increased uncertainty. On the contrary, we need to get a

negative estimate for the coefficient of h_{t+1} in equations (1) and (3). Thus if an attempt is to be made in our model to explain persistent consumption growth in terms of the precautionary motive, it must be done by decreased uncertainty as measured by the conditional variance.

(4) See data section in the end for the concept of labor income.

(5) The same procedure leads to an AR(1) model if a constant is not included:

$$\Delta Y_t = \underset{(2.29)}{0.18}\, \Delta Y_{t-1} \quad , \quad \sigma_Y = 0.03 \; .$$

Campbell and Deaton's (1987)'s reconciliation of their finding with the Flavin (1981)'s finding was based on such a formulation (although their equation was in terms of differences in logarithms).

(6) It allows the conditional variance of the zero mean error term from this process to depend on the past information.

(7) I owe this presentation to a helpful discussion with Ken Kroner. Actually, it could have been included in Baba, Engle, Kraft and Kroner (1987).

(8) The ordinary convergence criterion for R^2 of the auxiliary OLS regression is 0.001. See Kroner (1987).

(9) Engle and Granger (1987) indicated the possibility of misspecification of an equation which is expressed in terms of first differences when the variables are co-integrated. Equations (17), therefore, will be misspecified if consumption and labor income are co-integrated. We ran a series of co-integration regressions to test for such a possibility. The augmented Dickey-Fuller test was applied to residuals from the regression fo real per capita nondurables plus services consumption on labor income. The "t-statistic" was around -0.70 indicating that the two variables are not co-integrated (the regression of labor income on consumption generates the "t-statistic" of -1.03). Because we will use the same specification for the Japanese consumption function later, it is convenient for us to report here the results of testing for co-integration in Japanese variables. Using seasonal dummy variables in the testing regression (for a unit root) with residuals from regressing consumption on labor income, the "t-statistic" of -0.28 was obtained. The regression of labor income on consumption generates the "t-statistic" of -1.85. All of these results uniformly show that (nondurables plus services) consumption and labor income are not co-integrated, therefore the possibility of misspecification due to the absence of the error correction term in the equation (17)′ and (19) can be statistically rejected.

(10) The t-ratio of this difference when using the standard error of γ_3 is 1.94. This is not quite significant yet on the 5% level for 154 observations, but close.

(11) The absolute values of precautionary savings are obtained by multiplying them by the estimate of γ_2 on Table-U2 in real per capita terms (thousands of dollars).

(12) p_3 could be counted as destabilizing because the value of the conditional variance is quite small, and the upswing of the fluctuation is not really straight forward.

(13) As Nakatani discusses, it is difficult to raise empirical evidence for (4), although the older generation might be under such an influence. The relevance of (2) has never been very clear. (3) has been lifted in the last year, but certainly would have had positive effect. Chronical payments of bonuses would make people regard it as a part of permanent income, thus making (1) weaker.

(14) The separation of consumption into durables, nondurables, and services before 1975 is not possible as of now.

(15) The data on Japan were collected over the period 1965-1985, and for the entire period, labor income needs to be represented by AR(4) in first differences including dummy variables. Our data on consumption are available only since 1970. Since then, the U.S. and Japan show similar time series properties for labor income, i.e. a random walk with drift.

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