

# 論文の要旨

氏名 杜瑞雪

論文のテーマ Essays on Trend Estimation by Penalized Least Squares  
(罰則付き最小二乗法によるトレンド推定に関する諸論)

論文の要旨

In econometric analysis, trend estimation/smoothing methods based on penalized least squares are popular. This thesis focuses on three such methods. They are (a) Whittaker–Henderson (WH) method of graduation, which include Hodrick and Prescott (1997) filter as a special case, (b)  $\ell_1$  (polynomial) trend filtering developed by Kim et al. (1999), and (c) cubic smoothing spline, which was developed by Schoenberg (1964), Reinsch (1967) and others.

This thesis consists of four chapters.

In Chapter 1, we briefly review some researches which are closely related to our studies and then present the outline of the thesis.

Chapter 2 is based on a research paper on cubic smoothing spline. Fitting a cubic smoothing spline is a typical smoothing method. In this study, we reveal a principle of duality in the penalized least squares regressions relating to the method. This is the main contribution of this study. We also provide a number of results derived from them, some of which are illustrated by a real data example.

Chapter 3 is based on a research paper on WH method of graduation. In the study, we present a modified WH method of graduation. After giving a closed-form solution, we show that it is of practical use because it provides not only a smoothed series identical to that of the WH graduation, but also an extrapolation beyond the sample limit of current data. In addition, we introduce two other penalized least squares problems and show that they provide the same results as those of the modified

WH graduation.

Chapter 4 is based on a research paper on  $\ell_1$  polynomial trend filtering, which include  $\ell_1$  trend filtering as a special case. It is also a filtering method described as an  $\ell_1$ -norm penalized least-squares problem. It is promising because it enables the estimation of a piecewise polynomial trend in a univariate economic time series without prespecifying the number and location of knots. This paper shows some theoretical results on the filtering, one of which is that a small modification of the filtering provides not only identical trend estimates as the filtering but also extrapolations of the trend beyond both sample limits.

## References

1. Hodrick, R. J. and E. C. Prescott, 1997, Postwar U.S. business cycles: An empirical investigation, *Journal of Money, Credit and Banking*, 29, 1, 1–16.
2. Kim, S., K. Koh, S. Boyd, and D. Gorinevsky, 2009,  $\ell_1$  trend filtering, *SIAM Review*, 51, 2, 339–360.
3. Reinsch, C., 1967, Smoothing by spline functions, *Numerische Mathematik*, 10, 3, 177–183.
4. Schoenberg, I. J., 1964, Spline functions and the problem of graduation, *Proceedings of the National Academy of Sciences*, 52, 947–950.