THE AD LIB MUSIC SESSION AS A METAPHOR FOR MATHEMATICS CLASSROOM ACTIVITIES IN THE THEORY OF OBJECTIFICATION: A PHONETIC ANALYSIS OF LAUGHTER

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The article aims at articulating the potential impact of Radford's theory of objectification on technical research on mathematics teaching. First, instead of Radford's metaphor of mathematical activities as an orchestra, we proposed an alternative metaphor—mathematical activities as an ad lib music session. With that in mind, we focused on laughter as affective expression in class and conducted phonetic analysis of the laughter in a Japanese high-school mathematics classroom. The analysis revealed that a fun atmosphere that included laughter transformed students' treatment of a mathematical model in the lesson and that the proposed metaphor is more suitable than the original. We, thus, conclude that affective factors might determine the quality of mathematics learning in an ad lib manner.

INTRODUCTION

The theory of objectification (TO), proposed by Radford (2016b, 2018), is a promising theory for holistically capturing the endeavor of mathematics education. We have gradually become aware that teaching and learning mathematics is a complex process that can be neither simply psychological nor simply epistemological (Radford, 2016a, 2018). In the TO, Radford elaborates the concept of joint labor as "an historically produced aesthetic form of life where matter, body, movement, action, rhythm, passion, and sensation come to the fore" (Radford, 2016b, p. 200). From this point of view, teaching-and-learning is regarded "not as two separate activities, but as a single and same activity" (Radford, 2016b, pp. 200–201). Through joint labor, teachers and students engage collaboratively in producing the common work that Radford calls "the sensuous appearance of knowledge" (Radford, 2016a, p. 5). All the participants in joint labor "not only create and re-create knowledge but they also co-produce themselves as subjects" (Radford, 2016b, p. 201). This dual process comprises objectification of knowledge and subjectification of the self (Radford, 2015, 2016b). This theory is, thus, characterized as:

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> an attempt to understand learning not as the result of the individual student's deeds (as in individualist accounts of learning) but as a cultural-historical situated process, and to offer accounts of the entailed processes of knowing and becoming (Radford, 2015, p. 553).

Empirical episodes collected over several years support the validity of the TO (e.g., Radford, 2015, 2016b). We now know that we should view classroom activity holistically as joint labor, not separating the intellectual and emotional aspects of individual thinking, the teacher and students' engagement, or objectification and subjectification. However, we should be aware that the TO does not explicitly show the extent to which the psychological and epistemological perspectives are narrow. Meaning, we may take a simplified psychological or epistemological view of classroom activities for a particular educational purpose. We do not challenge the generalizability of the empirical episodes in the TO. Rather, our argument relates to the technical–political divide that problematizes the gap between technical research aiming at improving local implementation of teaching and socio-political research aiming at global social justice (Ernest, 2016). If the TO wants to say that technical research on intentionally designed classroom activities for an educational purpose—from a psychological or an epistemological perspective—tends to oversimplify classroom activities, then it must show that a theoretical perspective of the theory can contribute at least to the achievement of the original purposes of technical research. This paper aims at articulating such a potential by presenting episodes of laughter in a Japanese high-school mathematics classroom.

THEORETICAL PERSPECTIVE

In this section, to describe the basis of the TO accurately, we elaborate some theoretical concepts. First, according to the TO, *knowledge* is not an object but a process (Radford, 2013). Knowledge is a cultural codification of a potential way of practice. *Knowing*, thus, corresponds to the actualization of knowledge (Radford, 2013). Through this actualization process, learners gradually become aware of a difference in a chaotic situation in between something that they are the *objects* of and something they are the *subjects* of. The former is called *objectification* of knowledge and the latter is called *subjectification* of the self (Radford, 2013). In any situation, a learner experiences objectification and subjectification. Even if practice is repeated, actualization is not completely stable; this instability engenders new learning.

Next, we carefully reconceptualize the concept of *common work* as a product of *joint labor* in a mathematics classroom. This is our main theoretical proposal. In this regard, Radford (2016a) proposes the *orchestra* as a noteworthy metaphor for common work:

> *Common work* is the bearer of dialectic tensions because of the emotional and conceptual contradictions of which it is made. Through it, knowledge appears sensuously in the classroom (through action, perception, symbols, artifacts, gestures, language), much in the same way and, with similar aesthetic force, that

> music appears aurally in a concert hall through the common work of the members of the orchestra. (p. 5, italics in the original)

However, based on our understanding of the TO, we propose that an *ad lib music session* is a more suitable metaphor for common work in a mathematics classroom. The orchestra metaphor provides three images: (1) The members of the orchestra share a goal pregiven by a score; (2) they consciously follow their conductor to achieve it; and (3) their motivation for achieving the goal comes from the existence of the concert audience to an extent. The corresponding images of a mathematics lesson are: (1) The participants do not share a pregiven goal; (2) students do not necessarily purposefully follow their teacher; and (3) there is no external observer in many cases.

A mathematics lesson is, rather, like an ad lib session. In an ad lib session, once the first player introduces an initial phrase, the other participants freely play the subsequent phrases. Their main purpose may be to enjoy playing itself. Although the audience may evaluate the quality of the music, observers are not always present, and players are not necessarily conscious of any such observers. The same holds true in a mathematical activity. Once a teacher provides an initial mathematical task or topic, all participants freely discuss it. Their main purpose may be to learn mathematics together. Although an observer may evaluate the lessons' quality, such observers are not always present and the participants are not necessarily conscious of such observers when they are present. A mathematics lesson must be a different kind of common work than an orchestra, if students do not behave in a prescribed manner. Note that by term "ad lib," we do not mean ill-planned. Rather, we emphasize the possibility that unanticipated improvisational collaboration produces valuable mathematics learning.

The TO has an interest in the dynamic nature of a mathematics lesson *from an observer point of view*. It focuses on products in the public rather than private domains. Observers and participants may feel differently about what roles the participants play in a lesson. For example, although students may be embarrassed to err in solving a mathematical problem, their mistakes may lead to deeper understanding of the problem for themselves and their peers. Additionally, when considering learning mathematics in a public domain, both cognitive and affective elements must be taken into account. As Roth & Walshaw (2019) argue that we should not regard effect as the sole driving force toward cognitive development. Rather, as with an ad lib music session, a mathematics lesson brings observable cognitive and affective changes in participants engaged in common work in a public domain.

In this paper, we particularly focus on laughter as a kind of affective expressions in a mathematics lesson. Although laughter is known to play a crucial role for interwoven cognitive and affective development (Roth et al., 2011; Roth & Walshaw, 2019), to our best knowledge there is no research on how laughter emerges in a mathematics classroom or on the role it plays when it does emerge.

RESEARCH QUESTION

Based on the abovementioned understanding of the TO, we ask: *What role does laughter potentially play for mutually dependent cognitive and affective development in a mathematics classroom?* As Cobb (2007) argues, an insight is a key criterion for choosing theoretical perspectives. The answer to this question offers good reasons for technical researchers to refer to the TO. In addition, this inquiry is worthwhile because it provides a concrete way to fill the gap between technical and socio-political research.

METHOD

In order to locate the potential roles of laughter, we conducted a phonetic analysis of a classroom discussion.

Participants and data collection

We video recorded a tenth-grade mathematics lesson at a public high school in the Kanto region in Japan. The teacher was the third author of this paper. Thirteen students (3 males and 10 females) in an IB math class participated in the lesson. One video camera was located in the rear of the classroom. The official language of the class is English, and the mathematics textbook is written in English; all teachers and students use English primarily and their native Japanese supplementarily.

The topic of the lesson was quadratic functions. The teacher introduced the following opening problem written in the textbook: A motorcyclist Marvin attempts to jump his motorcycle a long distance from the take-off ramp; supposing his height is given as $H = -0.009x^2 + x + 6$ meters, will he safely reach the landing ramp? Figure 1 shows a photograph of the problem presentation by the teacher in the lesson.

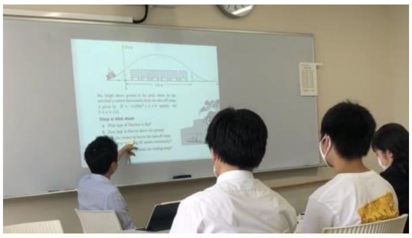

Figure 1: Presentation of the problem in the lesson

Analysis

We processed the video clip of the classroom discussion as follows. First, we transcribed it and attempted to understand what participants talked about. This analysis reveals a cognitive aspect of the discussion. Second, we cropped all but the scenes with participants' laughter. We extracted an audio clip from each scene and graphed the transitions of pitch and intensity using *Praat* speech analysis software (Boersma & Weenink, 2020). The pitch of the laughter indicates whether it was from a female or a male, and the intensity indicates loudness. This analysis reveals an affective aspect of the discussion. In this paper, we report on three scenes of laughter from the lesson.

RESULTS

The original transcript includes English and Japanese. For readability, the Japanese was translated to English and the translations are underlined. Students' names are pseudonyms.

The three scenes of laughter were extracted from a classroom discussion about the opening problem. In the first scene, the teacher drew a picture of a bike and a ramp on the whiteboard and explained the height of the landing ramp, prompting students to imagine the moment of landing.

27	S:	He might get crashed if his height is less than 6 meters.
28	T:	If I magnified this, it would look like this. (Teacher drew the stand in the picture). Thus, he would be here, a little higher than the one I drew.
29	S:	The momentum.
30	T:	Well, it is 6 meters, so here is 6, and 1.1. Like that.
31	S:	Well.
32	T:	I mean, the bike would come like that.
33	SS:	(Laughter) [about six seconds from (1) to (4) in Figure 2]
34	T:	Then, how would it be?
35	Ken:	Isn't the bike you drew too small?
36	T:	Well, it's 6 meters high. 6 meters. This should be fine.

Laughter occurs in #33 because the size of the teacher's bike looked too small from Ken's perspective in #35. However, from the teacher's perspective, the size was valid. When he noticed that the laughing students did not grasp the validity of the scale of the bike, he explained (#36). The laughter seemed to occur because of the cognitive gap between the size of the bike they anticipated and the size of the one the teacher drew.

Figure 2 shows the pitch and intensity of the classroom laughter in #33. A brief episode of laughter began at (1) in Figure 2. The following episode increased in intensity from (2) to (3). From (3) to (4), the intensity decreased. The laughter lasted until the teacher finished drawing the motorcycle. Although the intensity between (2) and (3) vacillates, we can see that it increases overall, representing the swell of the laughter. The better the students grasped the entire picture drawn by the teacher, the louder their laughter become. In addition, because it was the female students who were primarily laughing, the pitch of the laughter is higher than the male teacher's speech in #32 before (1) and in #34 after (4).

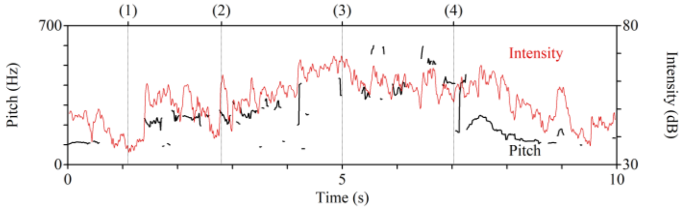

Figure 2: Pitch and intensity of the classroom laughter in the transcript #33

After the long wave of laughter occurred in #33, Ken answered the teacher's question in #34.

37 Nao: By the momentum of the bike.
38 T: By the momentum of the bike.
39 Ken: Because he flew.
40 SS: (Laughter) [about four seconds from (1) to (4) in Figure 3[a]]

Ken thought that the safety of the motorcyclist depended on how vigorously the bike jumped. The nuance of his response in #39 in Japanese manifested a comical image of a bike passing over the landing ramp, probably causing the students' laughter in #40.

Figure 3[a] shows the pitch and intensity of the classroom laughter in #40. Listening to the response from the student, the teacher's lower pitched laughter continued from (1) to (2), and the higher pitched laughter of the female students began from (2). One female student continued laughing from (3) to (4). The intensity of the classroom laughter decreased from (2) to (3). The duration of this laugher was instantaneous.

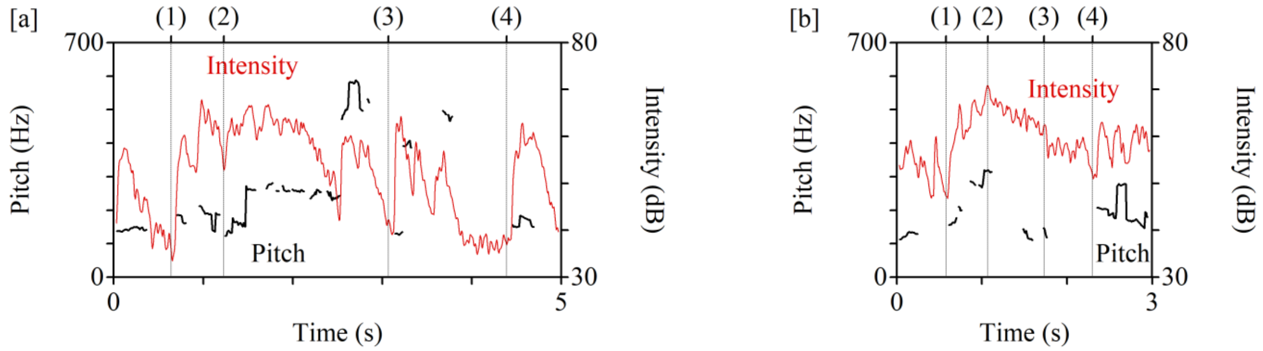

Figure 3: The pitches and the intensities of the classroom laughter in the transcripts #40 [a] and #46 [b]

Following the abovementioned second scene of laughter, the final scene of laughter occurred when the teacher and the students discussed the height of the jump.

41 T: Dangerous? Safe? What do you think?
42 Nao: It seems safe.
43 T: It seems same. Why?
44 Nao: Well, I reckon he might get crashed into the ramp if his speed was less than 6 meters.
45 T: Well, right, if it was less than 6 meters, it is out of the question.
46 SS: (Laughter) [about two seconds from (1) to (4) in Figure 3[b]]
47 Nao: Well, if it was more than 6 meters and about 1 meter (higher than 6 meters), it should be okay.

In #44, Nao argued that the motorcyclist would be safe because the formula H given in the problem indicated that he would not crash into the ramp. The teacher agreed with her in #45, and many students laughed in #46.

Figure 3[b] shows the pitch and the intensity of the classroom laughter in #46. The laughter rapidly swelled from (1) to (2). Since many students kept laughing in turns, the laughter neither vacillated nor rapidly decreased from (2) to (3). It gradually decreased from (2) to (4). The comical image of the bike crash seemed to link with a particular value of the given quadratic function.

DISCUSSION AND CONCLUSION

Figures 2 and 3 corroborate the interwoven relationships between cognition and affect. First, Figure 2 shows that the students took a while to get in a laughing mood. A cognitive factor, misunderstanding the scale, is related to an affective impact on the classroom. Second, the flying bike image provided by Ken in the second scene critically influenced how the students understood Nao's claim in the final scene. As argued in the previous section, Figures 3[a] and [b] indicate that more students were laughing in the final scene than in the second one. Since the image of the bike was referred to twice, more students might have clearly imagined it crashing by the final scene. In addition, Ken's claim characterized Nao's claim as *a necessary condition* for the motorcyclist's safety. Ken suggested that the motorcyclist might have been in danger even if he had jumped sufficiently high. If Ken had not made any claim and the students only discussed how high the motorcyclist jumped, Nao's claim might have been treated as *a sufficient condition* for the safety; students might have implicitly assumed that the motorcyclist had a reliable ability to land on the ramp. This means that the bike image prompted the students to make sense of the mathematical model differently. Our classroom episode, therefore, suggests an affective factor, that is, Ken's funny claim may have contributed to the production of an essentially different mathematical conclusion in the classroom.

Based on the TO, we regarded Ken and Nao's claims as common work in the lesson. Each claim appeared as an intermediate product of mutual engagement in the joint labor. The appearance of these claims could not have been conjectured before the lesson from a solely psychological or solely epistemological perspective. Thus, according to our definition, the classroom activity consisted of *ad lib* collaboration between the teacher and the students.

While *knowledge* of quadratic functions in the sense of the TO provided one possible way of modeling the height of the bike when jumping, the process of *knowing* was not limited to that possibility. Rather, it included dual aspects, objectification and subjectification. As an object, the model was characterized as a necessary condition for safety in practice. As a subject, each student found it socially accepted to discuss mathematical problems with humor. Although we did not capture the changes in the private domains of the students' minds, we did reveal how mutually dependent cognitive and affective elements were in a lesson as a public domain.

In conclusion, the ad lib music metaphor can be more suitable for the mathematics classroom activity than the orchestra one. Laughter in the first scene was an indicator of the students' cognitive understanding, and fun atmosphere with laughter in the second scene influenced the treatment of the mathematical model in the final scene. Therefore, as a potential answer to our research question, we argue that affective factors might determine the quality of mathematics learning in an ad lib manner. As our theoretical contribution to the TO, we also argue that (i) both cognitive and affective development should be regarded as an interwoven achievement of a mathematics lesson and, thus, that (ii) mathematics teachers need to plan their lessons more holistically for better ad lib collaboration. In this regard, using the metaphor of ad lib music as a theoretical framework, future research can seek out what impacts ad lib collaborations have on cognitive and affective development in mathematics classrooms.

In this study, our consideration is limited to laughter and does not focus on the influence of foreign and native languages on cognitive and affective development. A variety of affective and cognitive elements should be investigated in future research.

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References

Boersma, P., & Weenink, D. (2020). *Praat: Doing Phonetics by Computer* (Ver. 6.1.36). https://www.fon.hum.uva.nl/praat/

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspective. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (Vol. 1, pp. 3–38). Information Age Publishing.

Ernest, P. (2016). The unit of analysis in mathematics education: Bridging the political-technical divide? *Educational Studies in Mathematics*, *92*(1), 37–58.

Radford, L. (2013). Three Key Concepts of the Theory of Objectification: Knowledge, Knowing, and Learning. *Journal of Research in Mathematics Education*, 2(1), 7–44.

Radford, L. (2015). Methodological aspects of the theory of objectification. *Perspectivas Da Educação Matemática*, *8*(18), 547–567.

Radford, L. (2016a). Mathematics Education as a Matter of Labor. In M. A. Peters (Ed.), *Encyclopedia of Educational Philosophy and Theory* (pp. 1–6). Springer Singapore.

Radford, L. (2016b). The Theory of Objectification and Its Place among Sociocultural Research in Mathematics Education. *The RIPEM - International Journal for Research in Mathematics Education, 6*(2), 187–206.

Radford, L. (2018). A Cultural-Historical Approach to Teaching and Learning: The Theory of Objectification. In F.-J. Hsieh (Ed.), *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education* (Vol. 1, pp. 137–147).

Roth, W.-M., Ritchie, S. M., Hudson, P., & Mergard, V. (2011). A study of laughter in science lessons. *Journal of Research in Science Teaching*, *48*(5), 437–458.

Roth, W.-M., & Walshaw, M. (2019). Affect and emotions in mathematics education: Toward a holistic psychology of mathematics education. *Educational Studies in Mathematics*, *102*(1), 111–125.