

Improving Mathematics Lesson and Evaluation through Setting Inquiry Problem-Solving Activities

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Abstract: The purpose of this research is to make a theoretical framework for improving mathematics lesson and evaluation through setting inquiry problem-solving activities effectively in junior high and senior high mathematics lessons. To achieve this, we first carried out a questionnaire survey to capture how students perceive learning mathematics and, based on the results, made a framework incorporated these activities for lesson design and evaluation of student's learning. Then, sample lesson conducted in a junior and senior high school based on the framework was analyzed to investigate its effectiveness and get some practical suggestions.

1. Introduction

The 2018 revision of Course of Study for High School (Ministry of Education, 2018) placed importance on real-world capabilities such as the ability to identify and solve problems, logical thinking skills, and communication skills. In addition to the elements of interdisciplinary and comprehensive learning that had been incorporated up to that point, the revision emphasizes the perspective of inquiry learning in all school subjects more than before.

The purpose of this research is to make a theoretical framework for improving mathematics lesson and evaluation through setting inquiry problem-solving activities effectively in junior high and senior high mathematics lessons, as well as conduct actual lessons in order to get some suggestions for mathematics lesson design and practice. To achieve this, we first carried out a questionnaire survey to capture how students perceive learning mathematics and, based on the results, investigated a framework incorporated these activities for lesson design and evaluation of student's learning. Then, sample lesson conducted in middle school based on the framework was analyzed to investigate its effectiveness and get some practical suggestions.

2. Student Perceptions of Learning Mathematics

We surveyed 42 third-year junior high school students about their perceptions of learning mathematics. The survey asked them two questions regarding various approaches to learning: (I)

Do you believe this activity is important when learning mathematics? and (II) Do you actually do this when learning mathematics? On the actual survey, the students responded to the following two questions by choosing from 1 to 4 to indicate how much they agreed with each:

I : “When learning mathematics, do you think actions (1)–(12) below are important?”

II : “When learning mathematics, do you actually do actions (1)–(12)?”

- | | |
|----------------------|-------------------|
| 1. Agree | 2. Somewhat agree |
| 3. Somewhat disagree | 4. Disagree |

Table 1 summarizes the average student response score for each question, and items with average scores of more than 2.5 are marked with an asterisk. For nearly all items, the students generally perceived the approaches to learning mathematics to be “important.” However, the problem is whether they are actually taking these approaches. The results suggest that it is important to incorporate the following learning situations when designing mathematics lessons in which students independently engage with the subject at hand and are proactive about thinking deeply while working on mathematics.

- Even when a student can solve a problem with one method, he/she will try to apply other methods too.
- Even when a student is able to solve one problem, he/she will look for and think through other areas that he/she is unsure about.
- Even when a student is able to solve a problem, he/she will expand on it through other means, for example changing the conditions of the problem.
- A student will look at a set of information that is not worded as a problem and identify the problem within.

Table 1: Average student response score (survey results) N=42

items	I	II
① I read problems thoroughly.	1.12	1.68
② I use tables and figures to form my own understanding of what is written in a problem.	1.39	1.71
③ Even if I can't solve a problem immediately, I'm persistent and try to change the way I'm looking at it.	1.49	1.93
④ Even when I can solve a problem with one method, I try to apply other methods too.	2.07	2.93*
⑤ When considering something general, I test out several specific examples.	1.61	2.07
⑥ To solve a problem, I go through my past knowledge that could be relevant.	1.73	1.93
⑦ I look at a set of information that is not worded as a problem and identify the problem within.	2.05	2.61*
⑧ I compare my method for solving a problem with others'.	1.98	2.34
⑨ Even when I'm able to solve one problem, I look for and think through other areas that I'm unsure about.	2.12	2.83*
⑩ Even when I'm able to solve a problem, I expand on it through other means, for example changing the parameters.	2.07	2.85*
⑪ I use math to think about problems in my daily life.	2.27	2.85*
⑫ I apply the answers I get from math in my daily life.	2.32	3.00*

This also demonstrates the importance of activities in which students actively apply mathematics to the problems around them and interpret or investigate the results obtained by thinking mathematically in those circumstances. There needs to be more research on lesson design that fosters these approaches and ways of thinking by tweaking the parameters of learning situations in regular mathematics lessons.

3. School Subject Learning and Inquiry Learning

The 1998 revision of Course of Study for High School placed importance on real-world capabilities such as the ability to identify and solve problems, logical thinking skills, and communication skills. In addition to the elements of interdisciplinary and comprehensive learning that had been incorporated up to that point, the revision emphasized a perspective of inquiry learning in overall study time, putting in place a four-step cycle: establishing the problem, collecting information, organizing and analyzing, and summarizing and expressing (Ministry of Education, 2010). This cycle can be fitted into the curriculum-based study of mathematics by applying scenarios such as the following in lessons.

- (a) Inquiry learning activities conducted in relatively short blocks of time of about one hour
- (b) Inquiry learning activities positioned as part of a cycle of learning, application, and inquiry placed as the last unit in a lesson plan
- (c) Inquiry learning activities conducted as part of a cross-curricular lesson design that connects mathematics with other school subjects

This research will design lessons that effectively incorporate inquiry activities like the ones imagined in (a) above.

4. A Framework for Setting Inquiry Learning Activities in Mathematics Lesson

Inquiry learning activities in mathematics lesson are considered problem-solving tools that include the following dimensions.

(A) Recognizing the problem

Learners identify (recognize) a problem based on the circumstances around them or information presented to them.

(B) Making connections organically

Learners make organic connections by themselves between applicable knowledge, concepts, and methods.

(C) Reflecting and orienting toward integration/expansion

By reflecting on solution methods and results from a variety of dimensions, learners attain an integrative perspective and better expandability, and recognize new problems.

This section presents key points for designing lessons to effectively incorporate learning scenarios that encourage such aspects into mathematics lessons. Lessons are designed from the following three perspectives: 1. learners' thought process; 2. the composition of teaching materials, including the arrangement of contents; and 3. the teacher's roles in lesson.

(A) Recognizing the problem

Learners identify (recognize) a problem based on the circumstances around them or information presented to them.

A1: Contradicting existing knowledge and experience

When students encounter phenomena that conflict with their previous experiences or cannot be explained by their existing knowledge, it is expected to peak their intellectual curiosity and motivate them to learn.

A2: Synthesizing learning experiences and ascertaining readiness

Lesson design takes into account students' readiness for learning. It is important to ascertain whether they have the knowledge, skills, and level of understanding necessary to recognize a problem, and accordingly, lesson designers need to determine the appropriate amount of mental load for the approach students will take to identify a problem. The key to designing the lesson introduction is to devise a path (establish a scenario) that allows students to follow their natural thinking process to achieve an awareness of their task and a feeling that it is worth solving.

A3: Methods for presenting scenarios and situations

Although teachers are responsible for establishing scenarios and presenting information, they should leave room for students to themselves discover and focus on the problem that they need to investigate; the ideal scenario lets them gradually discover their task as they apply themselves to the subject at hand.

(B) Making connections organically

Learners make organic connections by themselves between applicable knowledge, concepts, and methods.

B1: Making connections between study contents when predicting solutions and devising strategies

In scenarios that involve working to solve a problem, students reference prior experiences, knowledge, and ways of thinking that seem applicable. By thinking through the structurally clear parts of a problem, they consider the connections between their existing circumstances and their prior experiences, knowledge, and ways of thinking. In scenarios where they make these connections, they think about not only what they have learned in the unit on the relevant subject matter but all the information and methods learned thus far. They also work to develop the strategies they need by considering various dimensions of the subject at hand, aiming to solve the problem through trial and error.

B2: Clarifying the position of past and future learning

When designing a lesson, the teacher determines what prior knowledge is required for the material being studied, as students work on the problems, and what information is related. This allows us to predict how student discussions will develop in a lesson and establish appropriate scenarios. At the same time, it also clarifies what sort of content the lessons can lead to in the future. This preparation allows teachers to review previously learned information as required and give students a glimpse of the content that will be studied in the future without pushing them beyond their limits.

B3: Establishing interactive scenarios and encouraging exchange of ideas

Lessons involving inquiry learning activities do not strongly encourage a particular strategy. Instead, teacher and students value broad thinking, such as the application of previously learned methods or the creation of new ideas, and discussions proceed in a way that enriches previous study through multifaceted thinking. To encourage students to think in these ways, scenarios where they

can exchange ideas appropriately are established in the classroom, and they refine their views and acquire a more solid understanding of information through mutual interaction.

(C) Reflecting and orienting toward integration/expansion

By reflecting on solution methods and results for a variety of dimensions, learners attain an integrative perspective and better expandability, and recognize new problems.

C1: Giving meaning to previous learning experiences

Reflecting on the problem-solving process and acquired information allows students to view things comprehensively in a way that incorporates prior knowledge and experience, as well as identify and give further consideration to new problems in an expansive way. This is an important opportunity for them to self-evaluate their mathematics learning thus far and imbue them with meaning.

C2: Accounting for the systematic nature of mathematics and adopting a cross-curricular perspective

In terms of the level of integration and the direction of expandability the lessons take, there must be some flexibility to respond to student's performance. A student's uncertainties and questions may be addressed by the class as a whole, or s/he may continue exploring them individually. Furthermore, the teacher could also select several directions for the class to proceed in and then investigate the potential problems therein. Teachers must appropriately determine how new student questions will be positioned in future lessons (whether they can be addressed immediately, whether problems can be solved with the knowledge and thinking processes learners are familiar with, how to work together with other subjects, etc.) and decide on the form of discussions and time allotment in advance.

C3: Directing reflections on learning

These scenarios need to be established as students themselves reflect on the ongoing lesson. Incorporating intentional and repeated opportunities for reflection into lessons leads to a proactive approach to solving problems with mathematics and promotes changes in attitude. The teacher's role is to actively evaluate, and value the realizations students gain and the changes they make thanks to such reflection.

Shimizu (2007) regards the problem solution process of the learner as "the transformation of a problem" that raises an equivalent, supplementary and new problem by changing the point of view in question, and shows some possibilities of the analysis of the teaching materials and the teaching and learning process. In the sense, the inquiry learning activities in a mathematics lesson includes a process of transformation where a problem is focused according to the progress of students' consideration and discussion after their independent activity, and students recognize a new problem by looking back on their problem-solving activities.

The goal of inquiry activities in mathematics lessons is to facilitate deeper learning while progressing through the cycle. Furthermore, these activities can be used not only for lessons on specific topics using special materials but also in standard-format mathematics lessons that focus on textbooks.

5. Case Analysis of Mathematics Lessons in Junior and Senior High School

We used a case study to investigate the setting of inquiry learning activities in mathematics lessons. The case below is founded on an understanding of basic information that might be included in a textbook, but it also intentionally incorporates inquiry learning activities that lead to deeper learning. The lesson design example below is structured to accommodate the dimensions of three activities described in the previous section:

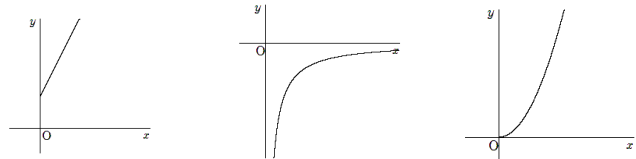
- (A) Recognizing the problem
- (B) Making connections organically
- (C) Reflecting and orienting toward integration/expansion

Case study of mathematics lesson in Junior high school

Subjects: Third-year junior high school students

Unit: function $y = ax^2$

Content: Rate of change (taught over a two-hour period)

Learning Activities	
1. Identifying and describing the characteristics of functions	<ul style="list-style-type: none">• Describe the characteristics of the graphs in terms of linear functions, inverse proportions, and functions proportional to the square (focusing on multiple subjects' commonalities and differences).
<p>How would you explain the characteristics of the following three function graphs?</p> 	
2. Focusing on and discussing the problem	<ul style="list-style-type: none">(1) The differences between straight lines and curves<ul style="list-style-type: none">• Focus on the rate of change and express that the line is not straight.• Understand that reducing the width of an interval makes a group of broken lines resemble the actual graph more closely.(2) The differences between concave and convex lines<ul style="list-style-type: none">• Understand that a certain interval's rate of change is represented by the slope of a broken line that connects two points.• Explain differences in the positive slope of a parabola and hyperbola by focusing on changes in the rate of change.
3. Summarizing lesson content	<ul style="list-style-type: none">• Summarize the properties of the rate of change in the function $y = ax^2$ and associate them with the characteristics of the graphs. (Practice problems)• Touch upon connections with the content of future lessons.

(A) Recognizing the problem

The lesson introduction focused the students' attention on shared characteristics and differences by comparing multiple subjects. Having them share and process what they have realized in groups established a scenario that allowed them to recognize the problem. In this scenario, they identified a characteristic that multiple graphs share—rising to the right—which made them aware that the clear differences between the graphs need to be explained.

(B) Making connections organically

All the students shared the task of deciding how to describe the differences between straight lines and curves, and how to explain the differences in the positive slope, but the scenario encouraged them to make connections between previously learned information, methods, and ways of thinking. At this stage, they focused on key lines of inquiry as they compared the newly learned functions proportional to the square with the previously learned ones, highlighting their similarities and differences.

For instance, they recalled their experience in the second year of junior high school—that the graph of a linear function is a straight line—and attempted to connect those methods and ways of thinking to the content of the ongoing lesson. Specifically, based on clues like letter usage in general discussions or the focus on the rate of change and slope of a straight line, the students deepened their discussions through their own thinking and the interactive exchange of resulting ideas.

Additionally, when discussing straight lines, the students had the realization that, although the rate of change for straight lines is constant regardless of the interval width, in functions proportional to the square, the slope of a line segment changes depending on the interval, and the line gets closer to the curve in the graph as the interval width is decreased (Figure 1).

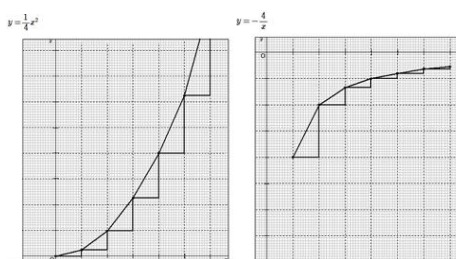


Figure 1: Graphs and Rate of Change

(C) Reflecting and orienting toward integration/expansion

This step established a scenario that promotes integrative thinking and an expansive perspective by giving the students opportunities to reflect on learned content and the problem-solving process. In class, once it became clear that the rate of change is not constant in functions proportional to the square, the students were given specific practice problems that asked them to find an interval's rate of change (based on the situation, they might find it for different intervals). These problems used the rate of change to highlight differences from linear functions, and allowed the students to simultaneously check their understanding and achieve a common perspective of understanding

functions through rate of change. Afterward, the rate of change was kept the same, and the students were encouraged to change the interval (Figure 2).

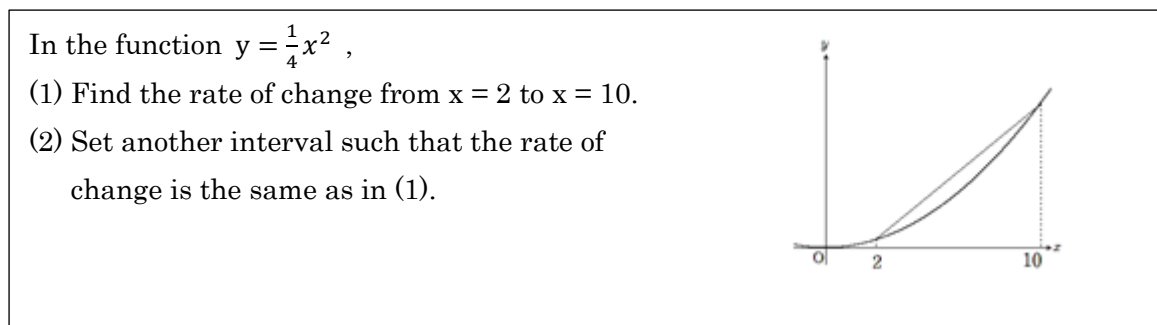


Figure 2: Practice Problem Presented to the Students

Then, while reviewing the mathematical expression of scenarios studied using letters, the students realized that the same value should be brought inward (or moved outward). Based on this, the teacher could end by touching upon the existence of tangent lines, as well as give the students a glimpse at the thinking behind derivatives based on the rate of change; they could also bring up connections with other scientific subjects.

6. Evaluating Aspects of Inquiry Learning Activities in Mathematics Lesson

Lessons must provide balanced evaluations of academic abilities, organized around the three pillars presented in Course of Study for School Mathematics (Ministry of Education, 2019). In addition to existing evaluation methods, the “reflection sheet” shown in Figure 3 is intended to obtain information on aspects of students’ inquiry learning activities, including their thinking processes, and ensure that evaluations prove useful in educational guidance.

The sheet can be positioned appropriately in lesson plans for classes that include problem solving, and the information gleaned therein should be applied effectively in future lessons. The sheet, which comprises three questions, can be handed out during the last few minutes of lesson, and students are instructed to actively reflect on and write about their studies, even if they keep the descriptions brief.

First, the question “Were you able to actively engage with the content of the lesson?” encourages learners to give a comprehensive self-evaluation of their independent study. Next, the sheet shows how inquiry thinking is presented by getting students to reflect on the methods and information they learned in that time and discuss their realizations or doubts. The responses below were given by second-year junior high school and first-year senior high school students. The former wrote about a unit on parallelograms and their properties, while the latter about a unit on the power of a point theorem. The text inside parentheses was added by the author, and that in square brackets indicates a student’s school year.

<p>Reflection Sheet</p> <p>1. Were you able to actively engage with the content of the lesson? (A, B, or C)</p> <p>2. How did you feel about the methods you learned? (It worked well, it didn't work well, I realized..., etc.)</p> <p>3. How did you feel about the information you learned? (I learned something new, I didn't understand, I realized..., I want to learn more about..., etc.)</p>

Figure 3: Reflection Sheet

(1) “How did you feel about the methods you learned?”

Among the different ways students make connections—an act that is at the center of inquiry learning activities—this question elicits information about methods they have learned. The students were asked to describe not only successful experiences of problem solving in class but also methods and ways of thinking that did not lead to a solution. This allowed them to notice the connections between previous strategies and ways of thinking by solving problems, actively applying those, and becoming aware of comprehensive ways of thinking. The responses here refer to mutual relationships and expandability in student learning activities, including the learning process.

- (When I was thinking of a proof,) I was able to calculate backward from my conclusion and create a proof successfully. [First-year senior high school student]
- I was able to estimate based on the part that seemed significant. I feel like the intersection point of the two circles could be some kind of a hint. [First-year senior high school student]
- I think the way I look at problems changes based on additional lines. [First-year senior high school student]
- Instead of creating proofs all in one go, I can write them more easily when I imagine the broader framework first. [Second-year junior high school student]

These demonstrate that students make adjustments to the way they study as they go by, for example, looking at problems from different angles or thinking things through in a methodical way.

Meanwhile, there are also responses that show some reflection on the problem-solving process but do not go beyond the surface level, only discussing whether or not the student was successful.

- I had trouble using theorems I'd already learned. [First-year senior high school student]
- I wouldn't be able to think of it on my own. Someone would have to teach me for me to understand. [First-year senior high school student]
- Using the alternate segment theorem and similarity made writing the proof easy. [First-year senior high school student]

(2). “How did you feel about the information you learned?”

Here, the students reflected on their own learning process and wrote about the mathematical

content. This question made them go beyond simply writing what they had learned and pushed them to actively describe what they had realized, what they had not understood, and what they would like to think about further. Their responses reveal references to connections with existing knowledge and the direction of expansive thinking.

- The power of a point (theorem) still applies when one is a tangent line. What about other theorems about circles? [First-year senior high school student]
- Just changing a condition creates a whole new problem. [First-year senior high school student]
- A parallelogram has a lot of conditions to be called one, so I think there are different kinds of proofs to discover there. [Second-year junior high school student]

Meanwhile, responses like those below only provide surface-level reflections on the results of lessons or state facts, and it is difficult to discern much about inquiry learning activities from them.

- The last problem was hard. [First-year senior high school student]
- I learned that squares, rectangles, and rhombuses are all parallelograms. [Second-year junior high school student]

7. Suggestions for Setting Inquiry Learning Activities in Mathematics Lesson

(1) Designing mathematics lesson

When designing mathematics lessons that take place over a relatively short span, teachers can center them on inquiry learning by incorporating the following elements into a problem-solving lesson: recognizing the problem, making connections organically between knowledge and strategies, and using reflection to orient toward expansion. These frameworks can be applied to not only basic content that follows the textbook structure but also expansive lessons that involve solving problems using such basic content. In class, it is important to get students to voluntarily participate in scenarios where they first zero in on the problem to be explored. Participating in the decision-making process gives a student a sense of ownership over the problem developed due to the entire lesson, including the student himself or herself, working together (even if a lone student has identified the problem). Additionally, the key to approaching the problem is to establish a scenario that encourages connections between knowledge and strategies and the development of expansion through individual work and mutual interaction.

Furthermore, scenarios for reflection provided at the end of a lesson are an important opportunity for students to encourage integrative viewpoints and ways of thinking, and to generate new questions. Even in scenarios where students solve similar problems on their own, the scenario itself may play a larger role than simple practice problems. This can produce expansive material such as broader ways of thinking and new questions as well as, we expect, cycles of inquiry.

(2) Reflecting on learning and evaluating what learned

In the time allotted for summarizing at the end of lesson, it is effective to reserve some for students to reflect on their own learning, even if briefly. By explicitly stating in the reflection sheet their realizations and the new questions discovered, they can recognize changes in their learning and begin a new cycle of inquiry that deepens their comprehension.

Still, when students reflect on their studies, they tend to focus on activities and thoughts that led to success; however, ideas that did not contribute to solving the problem or methods that led nowhere also contain material that can deepen learning or expand perspective. Additionally, thinking about why a method or way of thinking was not helpful can lead students to reevaluate their mathematical thinking from one level higher, and we expect that reflecting on these so-called “experiences of failure” contributes to a systematic understanding of mathematics. From the perspective of inquiry learning activities, it is important to make use of these student responses to ensure lesson improvements that facilitate deeper learning. However, because lessons time is limited, efficient formats and well-considered methods that make meaningful reflection possible need to be incorporated into daily lessons effectively to appropriately balance the time needed for problem solving.

8. Conclusion

Moving forward, there is a need for qualified people who identify problems that need solving by independently engaging with the subject at hand and take the initiative to solve them. Subject-based education can contribute by using daily lessons to encourage students to be proactive about thinking deeply and adjust the way they learn as they advance their thinking process. To achieve that, it is important to use problem-solving scenarios for inquiry learning in mathematics lessons, and design lessons that promote cooperative and proactive thinking among students.

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