Online optimization based control methods for enhancing tracking and disturbance attenuation performance of constrained systems

(制約システムの目標値追従および外乱抑圧性能向上のための実時間最適化制御手法)

KHONJUN SURAJET

March 23, 2020

Contents

Chapter 1

Introduction

1.1 Tracking Control Problem of Constrained Systems

A tracking control problem of multi-input multi-output systems under input and state constraints arises in various industrial fields (e.g., see [28, 35, 40, 52]). One typical way to deal with such a problem is to use a linear time-invariant feedback control law which is designed so that the input state constraints will never be violated for possible reference signals. However, in this case, when the reference signal is small, the control performance would be conservative, since only the small control signal is used. As for the control problem, various control strategies that can achieve desirable tracking performance for a wide class of reference signals have been proposed, e.g., model predictive control [25], reference governor [9], switching control [10] and gain-scheduled control (see [45, 46]). In the model predictive control, an open-loop optimal control problem with constraints is resolved at each sampling time. In this approach, the computational e ort required to solve the online optimization problem tends to increase rapidly with increasing the dimensions and or the number of inputs of the controlled object. A similar problem occurs in the case of the reference governor. In switching control, several controllers have been designed o -line, and the controllers are switched online so that the control performance is improved. In the standard approach, the switching is carried out discontinuously. To improve transient responses after switching, the initial value compensation techniques are applied in [30, 28, 53]. In [45, 46], a design method of a gain-scheduled control law for input constrained systems has been proposed. In the approach, based on the information on the state of the plant, the scheduling parameter of the controller is continuously updated online so that the control performance is improved.

The method is computationally tractable since the scheduling parameter can be determined by solving a convex optimization problem with respect to a single parameter. Further, it has been shown in [47, 49] that the tracking control performance of [45, 46] can be improved by simultaneously optimizing the value of the scheduling parameter and the controller state at each sampling time. However, the control method of [47, 49] is only applicable to linear time invariant systems. In practical control systems, the system usually include uncertain parameters and hence needs to be modeled as a linear parameter-varying system. Therefore, it is required to extend the tracking control method in [47, 49] so that the method can be applied to linear parameter-varying systems.

1.2 Disturbance Attenuation Control Problem of Constrained Systems

Recently, various control algorithms which ensure closed-loop stability and optimize disturbance attenuation performance under input state constraints have been developed. In particular, online optimization based control techniques have been proposed with the aim of achieving higher control performance as compared with standard linear control law. For example, a min-max optimization based model predictive control (MPC) approach has been proposed in [36]. Also, a tube-based MPC approach has been developed in [26]. These control algorithms are developed so that feasibility and stability are guaranteed in the presence of persistent bounded disturbances. On the other hand, several MPC algorithms for l_2 disturbance attenuation have been proposed. In the MPC algorithm of Reference [31], the feedback gain is recomputed online by solving an optimization problem with LMI[3] constraints derived from the bounded real lemma. However, feasibility and dissipativity have not been discussed in [31]. In [5], an MPC algorithm which ensures dissipativity in the presence of l_2 bounded disturbances has been proposed. In this method, the dissipation constraint is introduced to ensure the dissipativity. The MPC algorithms in [31] and [5] could be regarded as an extensions of the MPC algorithm in [16] to an l_2 disturbance attenuation problem. From the experience of numerical examples, we have confirmed that the control method in [5] tends to lead conservative results. This seem to be because the state feedback control law with time-varying feedback gain is used and the open-loop control input sequence has not been applied to the system in the method.

1.3 Thesis Outline

The reminder of this thesis is organized as follows.

In Chapter 2, we will explain definition and features of a linear matrix inequality (LMI) which will be utilized as a tool for controller synthesis in this thesis.

In Chapter 3, we will present a tracking control law for a linear parameter-varying system with input constraints. Firstly, a design condition of a controller parameterized by a single scheduling parameter is introduced. The proposed controller includes an integrator to achieve the zero steady-state error in the case where a step reference signal is applied. In the proposed control algorithm, the scheduling parameter and the state of the controller are determined on-line so that the tracking control performance is improved.

In Chapter 4, we will apply the control method in Chapter 3 to a torque control problem of a permanent magnet synchronous motor under input voltage limitation. Firstly, a plant model of a PMSM is derived as a linear parameter varying system in which the rotor speed is included as the varying parameter. Secondly, we show that setpoint tracking control is achievable under the time variation of the rotor speed. Then, we show a method of constructing a control law that achieves convergence of the motor torque to a step reference signal under input voltage limitation and time variation of the rotor speed.

In Chapter 5, we will extend the control method in Chapter 4 so that the motor torque tracks a time-varying reference signal under input voltage limitation and the time variation of the rotor speed. To this end, we develop a motor torque control law that consists of the feedback control law in Chapter 4 and a reference governor. In the control law, the modified reference signal is determined online simultaneously with the controller state and the scheduling parameter. The e ectiveness of the method is shown by a numerical example and an experimental result.

In Chapter 6, we will show a model predictive control law for constrained linear systems in the presence of l_2 disturbances. In the proposed control method, the feedback gain and the controller state are updated online so that the l_2 gain of the system is minimized. The method in this chapter could be viewed as an extension of [5] so that the disturbance attenuation performance is improved by introducing the controller state as a free move. We show that both feasibility of the control algorithm and the dissipation inequality are guaranteed for all times.

In Chapter 7, we will show a method for reducing computation time required to solve the optimization problem in the control algorithm of Chapter 6. In this chapter, a control law with a scheduling parameter is designed o -line, and the scheduling parameter and the controller state is updated on-line so that the l_2-gain of the system is minimized. The problem of determining the scheduling parameter and the controller state is formulated as an optimization problem with constraints described by linear matrix inequalities. Then the optimization problem is reduced to a convex optimization problem with respect to a scalar variable.

Finally, in Chapter 8, we will summarize this thesis.

Chapter 2

Mathematical Preliminary

In this chapter, we explain definitions and features of linear matrix inequalities. A linear matrix inequality is defined as follows.

Definition 1 *Linear Matrix Inequality (LMI) [3]*

Suppose that $F(x)$ is a linear or a ne function in the variable $x \quad {}^n$ and satisfies $F(x) \quad F(x)^T$. The following inequality

$$F(x) \quad 0 \tag{2.1}$$

is called a Linear Matrix Inequality, and its standard form is described by

$$F(x) \quad F_0 \quad \overset{n}{\underset{i \; 1}{}} x_i F_i \quad 0 \tag{2.2}$$

where $F_i \quad F_i^T \quad {}^{m \; m}$.

Note that the LMI (2.1) is a convex constraint on x, namely, $\quad : \quad x \quad {}^n : F(x) \quad 0$ is a convex set. Moreover, multiple LMI conditions can be expressed as a single LMI condition, namely,

$$F^{(j)}(x) \quad 0 \quad j \quad 1 \qquad k$$
$$F(x) : \quad \text{block-diag}[F^{(1)}(x) \qquad F^{(k)}(x)] \quad 0$$

This property will be utilized in Chapter 3 to design a compensator which satisfies multiple design specifications. In [3], it is shown that wide variety of analysis conditions arising in control theory can be expressed as the LMI condition.

Most of design and analysis problems in this thesis will be formulated as the following convex optimization problem under the LMI constraint.

Convex Optimization Problem (COP):
minimize $c^T x$ subject to $F(x) \quad 0$

where $c \quad ^n$. Various types of polynomial-time interior point algorithm have been developed for solving COP [3]. In this thesis, we will utilize the so-called Projective Method which is implemented in the *Robust Control Toolbox*[1] for use with MATLAB. By using this algorithm, a global optimal solution can be computed by less than some prespecified accuracy within a polynomial-time.

Chapter 3

Constrained Tracking Control by Gain-scheduled Feedback with State Resets

In this chapter, a tracking control problem for a discrete-time linear parameter-varying system with actuator saturation is addressed. Firstly, a design condition of a controller parameterized by a single scheduling parameter is introduced. The proposed controller includes an integrator to achieve the zero steady-state error in the case where a step reference signal is applied. Then a gain-scheduled control algorithm that guarantees closed-loop stability and makes the tracking error converge to zero in the case of a step reference signal is proposed. In the proposed control algorithm, the scheduling parameter and the state of the controller are determined on-line so that the tracking control performance is improved. The scheduling parameter and the state of the controller are computed simultaneously by solving a convex optimization problem with a linear matrix inequality constraint. A numerical examples is provided to illustrate e ectiveness of the proposed control algorithm.

3.1 Introduction

In this chapter, a tracking control problem of a discrete-time linear parameter-varying (LPV) system with input saturation is addressed. Such a control problem arises in various industrial fields (e.g., see [28, 35, 40, 52]). One typical way to deal with this problem is to use a linear time-invariant feedback control law that is designed so that the control signal will never violate the input constraint for the largest reference signal. However, in this case, when the reference signal is small, the control performance would be conservative, since only the small control signal is used.

As for the control problem, various control strategies that can achieve desirable tracking performance for a wide class of reference signals have been proposed, e.g., model predictive control [25], reference governor [9], switching control [10] and gain-scheduled control (see [45, 46]). In the model predictive control, an open-loop optimal control problem with constraints is resolved at each sampling time. In this approach, the computational effort required to solve the on-line optimization problem tends to increase rapidly with increasing the dimensions and or the number of inputs of the controlled object. A similar problem occurs in the case of the reference governor. In switching control, several controllers have been designed off-line, and the controllers are switched on-line so that the control performance is improved. In the standard approach, the switching is carried out discontinuously. To improve transient responses after switching, the initial value compensation techniques are applied in [30, 28, 53]. In [45, 46], a design method of a gain-scheduled control law for input constrained systems has been proposed. In the approach, based on the information on the state of the plant, the scheduling parameter of the controller is continuously updated on-line so that the control performance is improved. The method is computationally tractable since the scheduling parameter can be determined by solving a convex optimization problem with respect to a single parameter. The effectiveness of the method has been evaluated experimentally.

In this chapter, we further consider the tracking control problem of discrete-time systems with input saturation within the framework of the gain-scheduled control. The controller in this chapter has an integrator so that the plant output tracks the step reference signal. In the proposed control approach, we attempt to enhance tracking control performance by resetting the state of the controller at each sampling time until the tracking error becomes sufficiently small. In the control algorithm, the scheduling parameter and the state of the controller are determined on-line by solving a convex optimization problem with a linear matrix inequality (LMI) [3] constraint. It is shown that feasibility of the control algorithm and stability of the control system are guaranteed. A numerical example is provided to illustrate effectiveness of the control method.

Notations: For a vector $u \in \mathbb{R}^m$ and a diagonal matrix $A = \mathrm{diag}[a_1 \cdots a_m] > 0$, we define the saturation function as $_{A}(u) : \quad (\ _{a_1}(u_1) \qquad _{a_m}(u_m))^{\mathrm{T}}$, where

$$_{a_i}(u_i) : \quad \begin{matrix} a_i \mathrm{sgn}(u_i) & u_i & a_i \\ u_i & u_i & a_i \end{matrix}$$

If $A \quad I$, we will omit it. For a positive definite matrix $P \qquad ^{n \ n}$, a vector $\qquad ^{n}$ and a positive scalar , we define $(P \quad) : \quad x \quad ^{n} : (x \quad)^{\mathrm{T}} P (x \quad) \quad$. For a matrix $F \qquad ^{m \ n}$, we define $(F) : \quad x \quad ^{n} : F^{(i)} x \quad 1 \ i \quad 1 \qquad m$, where $F^{(i)}$ denotes the ith row of F. For integers k_1 and k_2 such that $k_1 \quad k_2$, we define $I[k_1 \ k_2] : \quad [k_1 \ k_1 \quad 1 \qquad k_2]$. Let be the set of $m \quad m$ diagonal matrices whose diagonal element are either 1 or 0. There are 2^m elements in . Suppose that each element of is labeled as $\mathbf{E}_j \ j \quad 1 \ 2 \qquad 2^m$. Also, we define $\tilde{\mathbf{E}}_j : \quad I \quad \mathbf{E}_j$.

3.2 Problem Formulation and Preliminaries

In this section, we consider the system described by

$$x_p(t \quad 1) \qquad A_p(\ (t)) x_p(t) \quad B_p(\ (t)) \ \ (u(t)) \tag{3.1}$$

$$(t) \qquad C_p x_p(t) \tag{3.2}$$

where $x_p \qquad ^{n}$ is the plant state, $u \qquad ^{m}$ is the control signal and $\qquad ^{n}$ is the plant output.

For the system (3.1), (3.2), we make the following assumption.

Assumption 1 $\quad ^{L}$ *is time-varying and satisfies* $_i(t) \quad [\underline{\ }_i \ \overline{\ }_i] \quad t \quad 0 \quad i$, *where* $\underline{\ }_i$ *and* $\overline{\ }_i$ *are known constants. In addition, the matrices* $A_p(\ )$ *and* $B_p(\ )$ *are represented as* $A_p(\ ) \quad \sum_{s \ 1}^{2^L} \ _s(\ ) A_{ps}$ *and* $B_p(\ ) \quad \sum_{s \ 1}^{2^L} \ _s(\ ) B_{ps}$, *where* $_i \quad 0 \quad i \quad 0$ *and* $\sum_{s \ 1}^{2^L} \ _s \quad 1$.

We define a hyper-rectangle $\ : \qquad ^{L} : \ _i \quad \underline{\ }_i \ \overline{\ }_i$. It is clear that $\qquad$.

In this section, we consider the following problem.

Problem 1 *Consider the system (3.1), (3.2). Assume that* $(t) \qquad t \quad 0$. *Design a control law* $u(t) \qquad (x_p(t) \ r(t))$ *that ensures closed-loop stability and achieves* $\lim_t \quad (t) \quad r(t)$, *where* $r(t) \quad \bar{r} \quad t \quad 0$ *and* $\bar{r}$ *is a constant.*

3.3 Controller Design

In this section, we design a controller described by

$$x_c(t \quad 1) \qquad x_c(t) \quad e(t) \tag{3.3}$$

$$e(t) \qquad r(t) \qquad (t) \tag{3.4}$$

$$u(t) \qquad F_c(t)x_c(t) \quad F_p(t)x_p(t) \quad M(t)r(t) \tag{3.5}$$

where $x_c \qquad ^n$ is the integrator state. The design condition of the matrices $F_c(t) \quad F_p(t)$ and $M(t)$ will be introduced in this section later. In the proposed control method, by suitably resetting the controller state x_c, we attempt to improve tracking control performance. The detail of the control algorithm will be explained in Section 3.4.

From eqs. (3.1)–(3.4), an augmented system is derived as

$$x(t \quad 1) \qquad A(\ (t))x(t) \quad B(\ (t)) \ \ (u(t)) \quad Er(t) \tag{3.6}$$

$$e(t) \qquad Cx(t) \quad D \ r(t) \tag{3.7}$$

where $x : \quad [x_p^{\mathrm{T}} \ x_c]^{\mathrm{T}}$ and

$$A(\ ) \ : \quad \begin{matrix} A_p(\ ) & 0 \\ C_p & I \end{matrix} \quad B(\ ): \quad \begin{matrix} B_p(\ ) \\ 0 \end{matrix} \quad E: \quad \begin{matrix} 0 \\ I \end{matrix}$$

$$C \ : \quad C_p \ \ 0 \quad D \ : \ I$$

We also define the following matrices.

$$A_s: \quad \begin{matrix} A_{ps} & 0 \\ C_p & I \end{matrix} \quad B_s: \quad \begin{matrix} B_{ps} \\ 0 \end{matrix}$$

Note that $A(\ )$ and $B(\ )$ can be expressed as $A(\ ) \quad {}^{2^L}_{s \ 1} \ {}_s(\ )A_s$ and $B(\ ) \quad {}^{2^L}_{s \ 1} \ {}_s(\ )B_s$ respectively.

We make following assumption.

Assumption 2 *There exist matrices and ( ) that satisfy*

$$A(\ ) \qquad B(\ ) \ (\ ) \quad E \tag{3.8}$$

$$0 \qquad C \qquad D \tag{3.9}$$

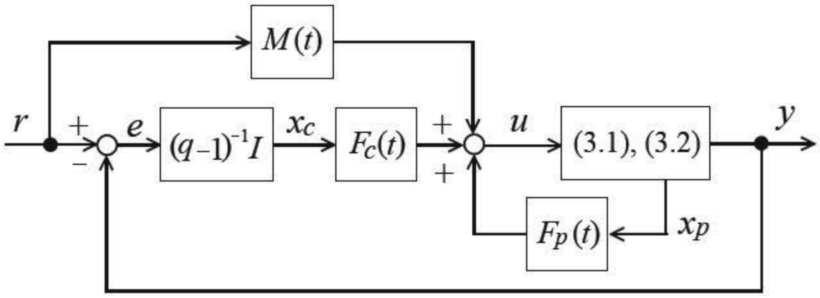

Figure 3.1: Control System

Eqs. (3.8), (3.9) could be viewed as an extension of the regulator equation [8, 24] to a class of LPV systems.

We define the following variables.

$$u_e \;:\quad u \qquad (\ )r \tag{3.10}$$

$$:\quad x \qquad r \tag{3.11}$$

From (3.6)–(3.11), the error system can be derived as

$$(t \quad 1) \quad A(\ (t))\ (t) \quad B[\ (u_e(t) \qquad (\ (t))r(t) \qquad (\ (t))r(t)] \tag{3.12}$$

Remark 1 *r in (3.11) represents the steady-state value of the state x when the plant output tracks the step reference signal r. Similarly, ( )r in (3.10) represents the steady-state value of the signal u. It should be noted that the solution matrix to (3.8), (3.9) is constant. This implies that the steady-state value of the state does not depend on the varying parameter . Note that, in this case, the error system (3.12) can be derived. This property will enable us to design a control law that achieves setpoint tracking under the time variation of .*

Remark 2 *In general, the steady-state value of the state of LPV systems changes depending on varying parameters. Hence, setpoint tracking is usually achievable only in the case where the varying parameter becomes constant.*

We make the following assumption.

Assumption 3 *The reference signal satisfies* $^{(l)}(\ )r \quad 1$.

The above assumption ensures that the tracking control is achievable under the input limitation in the steady-state.

In the following, we introduce a polytopic model of a saturation function proposed in Reference [12]. The following polytopic model will be used to design the feedback controller (3.3)–(3.5).

Lemma 1 *[12] Let* u ^{m}*. Suppose that* $_{j}$ 1 *for all* j $I[1\ m]$*, then*

$$(u) \quad co\ \mathbf{E}_j u \quad \tilde{\mathbf{E}}_j \ : j \quad I[1\ 2^m] \tag{3.13}$$

where co denotes the convex hull.

From the above Lemma, it can easily be verified that, for $_{j}$ 1, (u) can be expressed as

$$(u) \quad \overset{2^m}{\underset{j\ 1}{}} \ _j(\mathbf{E}_j u \quad \tilde{\mathbf{E}}_j\ ) \tag{3.14}$$

where 0 $_{j}$ 1 $^{2^m}_{j\ 1}$ $_{j}$ 1.

In the following, we introduce a theorem that is used to design the feedback control law (3.3)–(3.5). The control law designed based on the following theorem has a structure that a high-gain control law and a low-gain control law are interpolated by a single scheduling parameter. The scheduling parameter will be used to tune the trade o between the size of the region of attraction and tracking performance. The control algorithm will be introduced in Section 3.4.

Theorem 1 *Consider the system (3.6) and (3.7). For given positive definite matrices* **R**, **S**, *positive scalars* , $_{0}$ *and* $_{1}$ *such that* $_{0}$ $_{1}$*, assume that there exist matrices* Q_i Y_i Z_i $(i$ $0\ 1)$ *that satisfy*

$$\begin{matrix} Q_i & & & \\ \mathbf{R}^{1\ 2} Y_i & & & \\ \mathbf{S}^{1\ 2} Q_i & & _iI & 0 \\ A_s Q_i \quad B(\mathbf{E}_j Y_i \quad \tilde{\mathbf{E}}_j Z_i) & & 0 & Q_i \end{matrix}$$

$$i \quad I[0\ 1] \quad j \quad I[1\ 2^m] \quad s \quad I[1\ 2^L] \tag{3.15}$$

$$\begin{matrix} Q_i & \\ Z_i^{(l)} & \frac{2}{i} \end{matrix} \quad 0 \quad i \quad I[0\ 1] \quad l \quad I[1\ m] \tag{3.16}$$

$$Q_0 \quad Q_1 \tag{3.17}$$

where $_l$: 1 max ( )$^{(l)}r$ *and the symbol stands for symmetric block in matrix inequalities. Further, for some constant* [0 1], *we suppose that* (0) (*P*( ) 0) *where* $P(\ ):\ \ Q(\ )^{1}\ Q(\ ):\ (1\ \ )Q_0\ \ Q_1$. *Then, by applying the control law*

$$u(t)\quad F(\ )x(t)\quad M(\quad (t))r(t) \tag{3.18}$$

where $F(\ ):\ \ Y(\ )Q(\ )^{1}$, $Y(\ ):\ (1\ \ )Y_0\ \ Y_1$ *and* $M(\quad):\ \ (\ )\ \ F(\ )$ *to the system (3.6) and (3.7), the relations* (*t*) (*P*( ) 0) *t* 0, $\lim_t$ *e*(*t*) 0 *hold. Further,* $J:\ \ _{t\ 0}\ (t)^{\mathrm{T}}\mathbf{S}\ (t)\ \ u_e(t)^{\mathrm{T}}\mathbf{R}u_e(t)\ \ \ (\ )$ *holds, where* $(\ ):\ (1\ \ )\ _0\ \ \ _1$.

Proof) From (3.10), (3.11) and (3.18), we obtain u_e *F*( ) . Hence, the closed-loop system (3.12), (3.18) can be represented as

$$(t\quad 1)\quad A(\ (t))\ (t)\quad B(\ (t))\ \ (F(\ )\ (t)) \tag{3.19}$$

where (*F*( )) : (*F*( ) ( )*r*) ( )*r*. We define $H(\ ):\ \ Z(\ )Q(\ )^{1}\ \ Z(\ ):$ $(1\ \ )Z_0\ \ Z_1$ and : diag[$_1$ $_m$]. If (*H*( )) and max ( )$^{(l)}r$ 1 *l* *I*[1 *m*], then *H*( )$^{(l)}$ ( )$^{(l)}r$ 1 *l* *I*[1 *m*] . Hence, in this case, the relation (*F*( ) ( )*r*) $^{2m}_{j\ 1}\ _j\ \mathbf{E}_j(F(\ )\ \ (\ )r)\ \ \tilde{\mathbf{E}}_j(H(\ )\ \ (\ )r)$ holds from Lemma 1. Therefore, the relation (*F*( )) $^{2m}_{j\ 1}\ _j\ \mathbf{E}_jF(\ )\ \ \tilde{\mathbf{E}}_jH(\ )$ holds.

By using this relation, if (*t*) (*H*( )) and max ( )$^{(l)}r$ 1 *l* *I*[1 *m*], the closed-loop system (3.19) can be rewritten as

$$(t\quad 1)\qquad (\ (t)\ \ (t))\ (t) \tag{3.20}$$

where ( ) : $^{4}_{j\ 1}\ _j\ \ _j(\ )\ \ \ _j(\ ):\ A(\ )\ \ B(\ )\ \mathbf{E}_jF(\ )\ \ \tilde{\mathbf{E}}_jH(\ )$.

On the other hand, from (3.7) and (3.9), the signal *e*(*t*) can be expressed as

$$e(t)\quad C\ (t) \tag{3.21}$$

From (3.16), we have

$$\begin{matrix} Q(\ ) & \\ Z(\ )^{(l)}\ \frac{2}{l} & \end{matrix}\quad 0\quad l\quad I[1\ m] \tag{3.22}$$

Then, by substituting *Z*( )$^{(l)}$ *H*( )$^{(l)}$*Q*( ) for (3.22) and performing a congruence transformation with block- diag[$Q(\ )^{1}$ 1] and substituting $Q(\ )^{1}$ *P*( ), and applying Schur complement [3], we have

$$\frac{1}{\ _l^2}H(\ )^{(l)T}H(\ )^{(l)}\quad \frac{1}{\ }P(\ )\quad l\quad I[1\ m] \tag{3.23}$$

Equation (3.23) implies that $(P( ) \quad 0) \quad (H( ) \quad)$.

By carrying out the similar procedures used to derive (3.22) to (3.15), and substituting $Z( ) \quad H( )Q( )$ and $Y( ) \quad F( )Q( )$ for the resulting inequality, and performing a congruence transformation with block-diag$[Q( )^{\ 1} \ I \ I]$, and multiplying the resulting inequality by $_s(\ (t))$, and summing them up for $s \quad 1 \qquad L$, we have

$$\begin{matrix} P( ) & & \\ \mathbf{R}^{1\ 2}F( ) & (\ )I & 0 \\ \mathbf{S}^{1\ 2} & & \\ _j(\ (t)) & 0 & P( )^{\ 1} \end{matrix} \tag{3.24}$$

Further, by multiplying the inequality (3.24) by $_j(t)$, and summing them up for j $1 \qquad 2^m$, we have

$$\begin{matrix} P( ) & & \\ \mathbf{R}^{1\ 2}F( ) & (\ )I & 0 \\ \mathbf{S}^{1\ 2} & & \\ (\ (t) \quad (t)) & 0 & P( )^{\ 1} \end{matrix} \tag{3.25}$$

By applying Schur complement to (3.25), and multiplying the resulting inequality from the left by $(t)^{\mathrm{T}}$ and from the right by (t), and using (3.20) and (3.21), we have

$$V(\ (t \quad 1)) \quad V(\ (t)) \qquad \frac{1}{(\ )} \ (t)^{\mathrm{T}}\mathbf{S} \ (t) \quad u_e(t)^{\mathrm{T}}\mathbf{R}u_e(t) \tag{3.26}$$

where $V( ) : \quad {}^{\mathrm{T}}P( )$. From (3.26), we can conclude that if $(0) \qquad (P( ) \quad 0)$ then

$$V(\ (t)) \quad V(\ (0)) \qquad t \quad 0 \tag{3.27}$$

Equation (3.27) implies that $(t) \qquad (P( ) \quad 0) \quad t \quad 0$. On the other hand, it has been shown that the nonlinearity $(F( ) \ (t))$ can be represented as $(F( ) \ (t)) \qquad {}^{2^m}_{j\ 1} \ _j(t) \ \mathbf{E}_jF( )$ $\tilde{\mathbf{E}}_jH( ) \ (t)$ if $(t) \qquad (H( ) \quad)$ and max $\qquad (\ )^{(l)}r \quad 1 \quad l \quad I[1 \ m]$. From (3.23) and (3.27), we can state that if the conditions in Theorem 1 hold, the relation $(t) \qquad (H( ) \quad) \quad t$ 0 holds. From (3.26), since $(t) \quad 0 \ (t \qquad)$, $e(t) \quad 0 \ (t \qquad)$ holds. Moreover, from (3.26) and (3.27), $J \qquad _{t\ 0} \ (t)^{\mathrm{T}}\mathbf{S} \ (t) \quad u_e(t)^{\mathrm{T}}\mathbf{R}u_e(t) \qquad (\ )$ holds. **Q.E.D.**

Based on Theorem 1, we design a gain $F(1) \quad Y_1Q_1^{\ 1}$ which makes the region $(P(1) \quad 0)$ large and a gain $F(0) \quad Y_0Q_0^{\ 1}$ which achieves fast convergence of the state in $(P(0) \quad 0)$ by suitably choosing the parameters $_0 \quad _1$, $\mathbf{R}$ and $\mathbf{S}$. Then we construct the control law (3.18) by interpolating the obtained gains.

Remark 3 *When the control law (3.18) is designed based on Theorem 1, the matrix $P(\ )$ needs to be determined so that the condition (0) $(P(\ )\quad 0)$ is satisfied for some constant* $[0\ \ 1]$*. This can be achieved by solving the design problem with the constraint* (0) $(P_1\quad 0)$*. By applying Schur complement, this condition can be rewritten as the following linear matrix inequality (LMI) condition.*

$$x(0) \qquad r \quad Q_1 \qquad 0 \tag{3.28}$$

Also, the conditions (3.15)–(3.17) are LMIs with respect to the variables $Q_i\ Y_i\ Z_i\ (i\quad 0\ 1)$. Hence, the design problem of the control law (3.18) that satisfies (3.15)–(3.17) and (3.28) can be solved e ciently by a numerical optimization algorithm based on an interior point method[3].

Remark 4 *The size of the set $(P_1\quad 0)$ mainly depends on the choice of the parameter $_1$. By choosing a larger value as $_1$, the size of the set $(P_1\quad 0)$ could be expanded in general. However, the control law designed based on Theorem 1 can only ensure local stability. This implies that, when the magnitude of the reference signal r is large, there may not exist a solution that satisfies the conditions (3.15)–(3.17) and (3.28) even if a large value is chosen as $_1$.*

Remark 5 *If $A\quad A_s\quad s\quad I[1\ \ L]$, Theorem 1 is equivalent to Theorem 1 of Reference [47].*

3.4 Control Algorithm

The control law (3.18) includes the scalar . The upper bound of the cost function J is given as $(\ )$, and the function $(\ )$ takes a smaller value when a smaller value is chosen as . Hence, it can be expected that the control performance is improved by minimizing at each sampling time. Moreover, the state of the controller x_c can be used as a tuning parameter to improve the control performance. Thus, we utilize the following control algorithm.

Algorithm 1

Step 0: Set $t\quad 0$ and $\quad 1$.

Step 1: Measure $x_p(t)$ and (t).

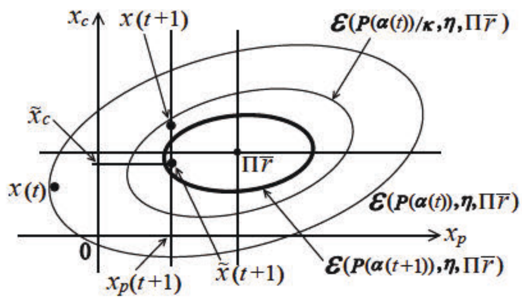

Figure 3.2: Invariant Set

Step 2: If $\alpha = 0$*, set* $\alpha(t) = 0$ *and go to Step 4.*

Step 3: For given $x_p(t)$*, solve* $\min_{\alpha\in[0,1],\ \tilde{x}_c\in\mathbb{R}} \alpha$*, s.t.*

$$\begin{bmatrix} \eta & * \\ \begin{bmatrix} x_p(t) \\ \tilde{x}_c \end{bmatrix} - \Pi r & Q(\alpha) \end{bmatrix} > 0 \tag{3.29}$$

Then, set $\alpha(t) = \alpha$ *and* $x_c(t) = \tilde{x}_c$*.*

Step 4: Apply $u(t) = \Phi(F(\alpha(t))[x_p(t)^{\mathrm{T}}, x_c(t)]^{\mathrm{T}} + M(\alpha(t), \theta(t))r)$ *to the plant (3.1), (3.2).*

Step 5: Compute $x_c(t+1)$ *by (3.3) and (3.4).*

Step 6: $t \leftarrow t+1$ *and go to Step 1.*

It should be noted that the value of the controller state x_c is reset so that the scheduling parameter α is minimized at Step 3 at each sampling time.

Remark 6 *The optimization problem of Step 3 in Algorithm 1 is an LMI optimization problem with respect to* α *and* $\tilde{x}_c$*. This optimization problem can be solved efficiently by a simple bisection algorithm. This will be explained in Appendix.*

Remark 7 *Algorithm 1 is equivalent to the control algorithm in Reference [47] except for the structure of the control law at Step 4.*

As for feasibility of Algorithm 1 and closed-loop stability, the following result holds.

Theorem 2 *Consider the system (3.1), (3.2). Assume that* $\max_{\theta\in\Theta} |\Gamma(\theta(t))^{(l)} r| < 1, \forall l \in I[1, m], \forall t \geq 0$*. Moreover, assume that there exists* $\tilde{x}_c$ *such that* $[x_p(0)^{\mathrm{T}}, \tilde{x}_c]^{\mathrm{T}} \in \mathcal{E}(P(1), \eta, \Pi r)$*. Then by applying Algorithm 1 to the system (3.1), (3.2),* $e(t)$ *converges to zero as* $t \to \infty$*.*

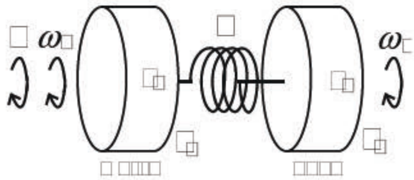

Figure 3.3: Two-mass-spring System

Theorem 2 is a slightly modified version of Theorem 2 of Reference [47] and can be readily proven by using the proof procedure of Reference [47]. Hence, we omit the detail of the proof of Theorem 2 and outline the proof briefly. We assume that $x(t)$ $(P((t))$ $r)$ holds at time t as shown in Fig.3.2. When the control signal$u(t)$ $(F((t))[x_p(t)^{\mathrm{T}}\ x_c(t)]^{\mathrm{T}}$ $M((t) (t))r)$ is applied to the system (3.1), (3.2), $[x(t)$ $r]^{\mathrm{T}}P((t))[x(t)$ $r]$ $[x(t$ $1)$ $r]^{\mathrm{T}}P((t))[x(t$ $1)$ $r]$ holds from Theorem 1. Hence, for some positive scalar 1, $x(t$ $1)$ $(P((t))$ $r)$ holds. This implies that there exists $(t$ $1)$ such that $x(t$ $1)$ $(P((t$ $1))$ $r)$ and $(t$ $1)$ (t). Hence, the scheduling parameter (t) decreases monotonically and converges to zero. Note that the convergence speed of the parameter (t) could be enhanced by resetting the integrator state x_c at each sampling time(see Fig. 3.2). After (t) becomes zero, the constant high gain feedback control law with the integral action is applied to the system. As the result, the tracking error $e(t)$ converges to zero.

3.5 Numerical Example

Consider the angular velocity control problem of a two-mass-spring system described by

$$J_m \ddot{}_m \quad c_m \dot{}_m \quad K(_m \quad _l) \quad u \tag{3.30}$$

$$J_l \ddot{}_l \quad c_l \dot{}_l \quad K(_m \quad _l) \quad 0 \tag{3.31}$$

The physical parameters are J_m J_l 0 01kgm^2, c_m 0 001Nms rad, K 50Nm rad. We assume that the parameter c_l is time-varying and satisfies 0 001Nms rad $c_l(t)$ 0 1Nms rad t 0. In addition, we assume that the limitation u 1Nm is imposed on the control signal. We choose the state of the plant as x_p $[_m \ \dot{}_m \ _l \ \dot{}_l]$. We discretize the above system using the Euler method with the sampling period T 1 ms. In this numerical example, we design the controllers such that the controlled output $_l$ converges to a step reference signal r 10 rad. The solution to the equations (3.8), (3.9) are obtained as 0 $[1\ 0\ 1\ 0\ 40]^T$. For this system, we designed a controller with $\mathbf{E}_1$ 1 $\mathbf{E}_2$ 0, 1, $_1$ 260, $_0$ 3, $\mathbf{R}$ 10^{6} and $\mathbf{S}$ diag$[10^{3}\ 10^{3}\ 10^{3}\ 10^{3} 10^{6}]$.

The numerical simulations are performed in the following three cases.

Case I: Algorithm 1 (Gain-scheduled feedback with state resets) is used.

Case II: The control algorithm in [47] (Gain-scheduled feedback) is used.

Case III: The constant low-gain control law $u(t)$ $F(1)x(t)$ $M(1)$ (t) is used.

Figures 3.4–3.7 show the responses of (t), $(u(t))$, (t) and $x_c(t)$ in Case I. Figures 3.8–3.11 show the responses of (t), $(u(t))$, (t) and $x_c(t)$ in Case II. Figures 3.12–3.14 show the responses of (t), $(u(t))$ and $x_c(t)$ in Case III. In these numerical simulations, the reference signal is $r(t)$ 10rad, t 0 and the initial state of the plant is $x_p(0)$ 0. It can be seen that the scheduling parameter (t) in Case I and II converges to zero. Further, the plant output (t) tracks the reference signal in all cases. It can be seen from Fig. 3.4 that the plant output (t) in Case I converges to the reference signal more rapidly as compared with Cases II and III.

3.6 Conclusions

In this chapter, we have proposed a tracking control law for discrete-time linear systems with input saturation. The proposed controller has an integrator to achieve the zero steady-state error in the case where a step reference signal is applied The controller includes a single scheduling parameter, and the control performance and the size of the region of attraction can be tuned by the parameter. In the proposed control algorithm, the scheduling parameter and the state of the controller are determined on-line so that the tracking control performance is improved. The problem of computing the scheduling parameter and the state of the controller is reduced to a convex optimization problem with an LMI constraint. It has been shown that feasibility of the control algorithm and stability of the control system are guaranteed. Further, the e ectiveness of the proposed method has been shown through two numerical examples.

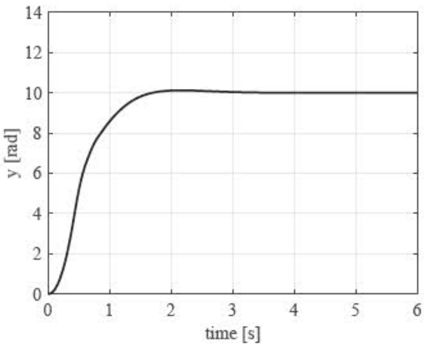

Figure 3.4: [rad s] (Case I)

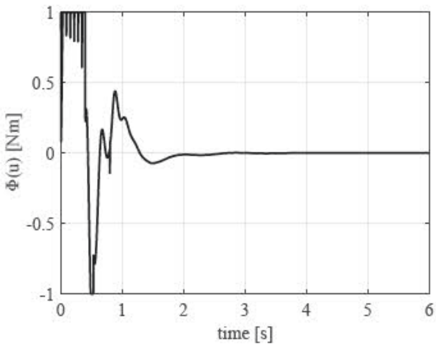

Figure 3.5: (u) [Nm] (Case I)

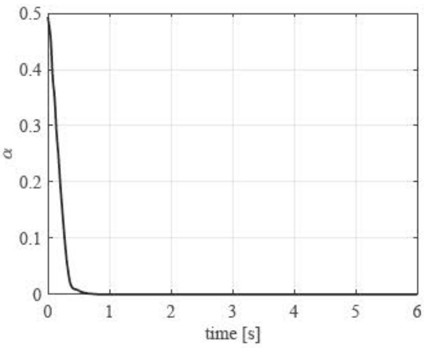

Figure 3.6: (Case I)

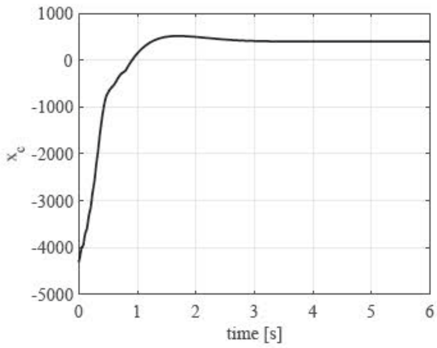

Figure 3.7: x_c (Case I)

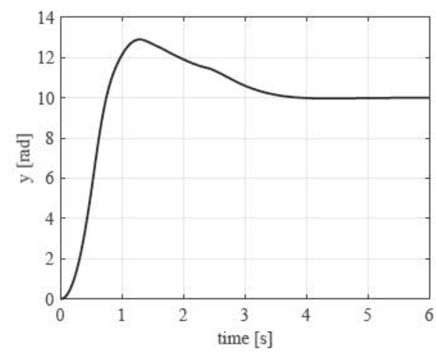

Figure 3.8: [rad s] (Case II)

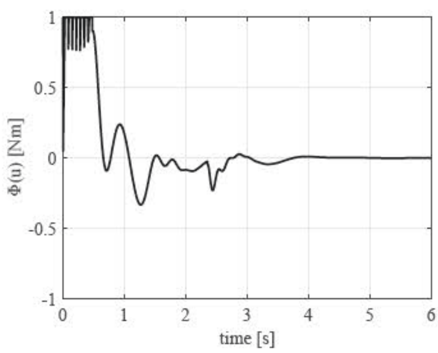

Figure 3.9: (*u*) [Nm] (Case II)

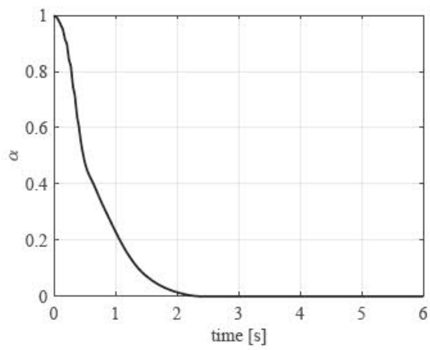

Figure 3.10: (Case II)

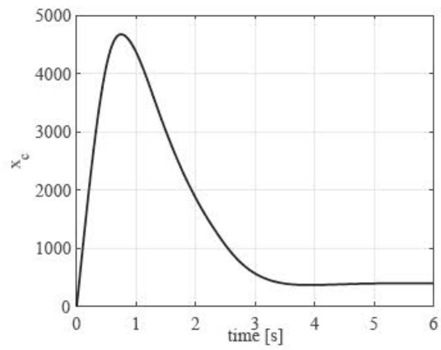

Figure 3.11: x_c (Case II)

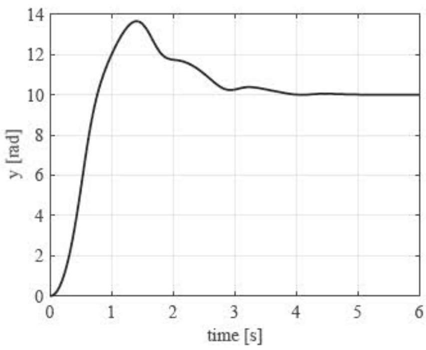

Figure 3.12: [rad s] (Case III)

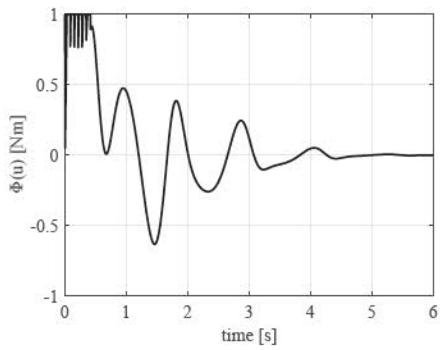

Figure 3.13: (u) [Nm] (Case III)

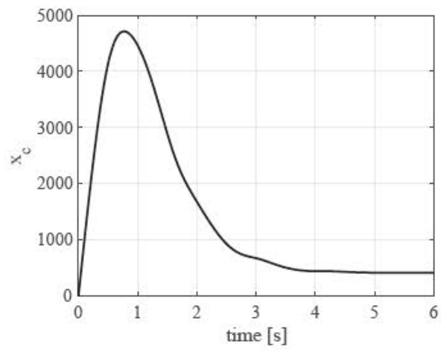

Figure 3.14: x_c (Case III)

Chapter 4

Permanent Magnet Synchronous Motor Torque Control by Gain-scheduled Feedback with State Resets

In this chapter, we apply the control method in Chapter 3 to a torque control problem of a permanent magnet synchronous motor under input voltage limitation. Firstly, a plant model of a PMSM is derived as a linear parameter varying system in which the rotor speed is included as the varying parameter. Secondly, we show that setpoint tracking control is achievable under the time variation of the rotor speed. Then, we show a method of constructing a control law that achieves convergence of the motor torque to a step reference signal under input voltage limitation and time variation of the rotor speed. The proposed control law consists of a gain-scheduled control law and a servo compensator. In the proposed control method, the scheduling parameter and the controller state are optimally updated so that the transient response is improved. The e ectiveness of the method is shown by a numerical example.

4.1 Introduction

The Permanent Magnet Synchronous Motor (PMSM) has been widely used in various industries due to its characteristics of high e ciency, high torque to inertia ratio, and fast dynamic performance. The dynamics of the PMSM includes nonlinear coupling terms between the d-axis subsystem and the q-axis subsystem. A standard approach to construct a control law for a PMSM is to use a decentralized PI controller with a decoupling compensator used to cancel the nonlinear coupling terms [23]. The controller of this type is very practical since the controller design and its implementation on the computer are fairly easy. In general, a larger control signal is required transiently to achieve higher tracking control performance, which would cause control signal saturation. When the control signal is saturated, the decoupling compensator is no longer e ective. The design problem of the controller which guarantees closed-loop stability under control signal saturation is a di cult problem due to the nonlinear characteristics of the control system.

An anti-windup scheme is one way to deal with input saturation problems and has been applied to a control problem of a PMSM in Reference [22]. It has been shown in Reference [22] that closed-loop stability can be ensured under input voltage limitation by using an appropriately designed anti-windup compensator. However, the control law of Reference [22] is designed under the assumption that the rotor speed is constant. Hence, the closed-loop performance would deteriorate when the rotor speed changes.

Recently, several control techniques based on an optimal control theory have been applied to the control problem of a PMSM. Model predictive control schemes [25] have been applied to a torque control problem of a PMSM in References [13, 54] and a velocity control problem in Reference [4]. Also, a nonlinear optimal control technique has been applied to a torque control problem of a PMSM in Reference [41]. It has been shown in these literatures that higher tracking control performance can be achieved under input voltage limitation as compared with the standard decentralized PI control approach. However, in these literatures, the controller is designed under the assumption that the rotor speed is constant. Hence, it seems that further studies are required to examine tracking performance and stability of the control system under the time variation of the rotor speed.

In Reference [47], a gain-scheduled servo control law for input constrained linear time-invariant systems has been proposed. The controller in Reference [47] consists of a servo compensator and a gain-scheduled controller. In the control method of Reference [47], the scheduling parameter and the controller state are updated at each sampling time so that the tracking control performance is improved. It has been shown in Reference [49] that the optimization problem to determine the scheduling parameter and the integrator state can be solved e ciently by a simple bisection method. However, this method cannot be directly applied to a control problem of a PMSM since the method is only applicable to linear time-invariant systems.

In this chapter, we develop a torque control method for a PMSM based on Reference [47]. Firstly, a plant model of a PMSM is derived as a linear parameter varying (LPV) system in which the rotor speed is included as the varying parameter. Secondly, we show that setpoint tracking control is achievable under the time variation of the rotor speed. Then, we show a method of constructing a torque control law that achieves set point tracking under input voltage limitation and time variation of the rotor speed. To derive the control law, we extend the control law of Reference [47] so that the LPV system can be handled. The e ectiveness of the proposed method is shown by a numerical example.

Notations: For a vector u ^{m} and a diagonal matrix A $[a_1 \quad a_m]$ 0, we define the saturation function as $_{A}(u)$: ($_{a_1}(u_1)$ $_{a_m}(u_m))^{\mathrm{T}}$, where

$$_{a_i}(u_i) : \quad \begin{matrix} a_i\mathrm{sgn}(u_i) & u_i & a_i \\ u_i & u_i & a_i \end{matrix}$$

If A I, we will omit it. For a positive definite matrix P $^{n\ n}$, a vector ^{n} and a positive scalar , we define $(P \quad)$: x ^{n} : $(x \quad)^{\mathrm{T}}P(x \quad)$. For a matrix F $^{m\ n}$, we define (F) : x ^{n} : $F^{(i)}x$ 1 i 1 m , where $F^{(i)}$ denotes the ith row of F. For integers k_1 and k_2 such that k_1 k_2, we define $I[k_1\ k_2]$: $[k_1\ k_1 \quad 1 \quad k_2]$. Let be the set of m m diagonal matrices whose diagonal element are either 1 or 0. There are 2^m elements in . Suppose that each element of is labeled as $\mathbf{E}_j$ j 1 2 2^m. Also, we define $\tilde{\mathbf{E}}_j$: I $\mathbf{E}_j$.

4.2 Problem Formulation and Preliminaries

The PMSM is composed of a permanent magnet rotor and stator windings. The PMSM stator has three coils spatially separated by 120 degree each other. A rotating magnetic field is generated by the three-phase current, and the magnetic torque is generated by the interaction of the rotating magnetic field and the flux of the permanent magnet[54]. In the control system design for PMSMs, the d-q rotating frame is commonly used, since AC signals appear as DC ones in the d-q rotating frame. Fig. 4.1 shows the relationship between the d-q rotating frame and the u- - reference frame. In this figure, is the angular position of the rotor, and p is the number of pole pairs. The dynamics of a PMSM in the d-q rotating frame is described by the following di erential equations [4, 32].

$$\frac{di_d}{dt} \quad \frac{1}{L_s}(\ Ri_d \quad L_s p \quad i_q \quad _d) \tag{4.1}$$

$$\frac{di_q}{dt} = \frac{1}{L_s}(-pL_s\omega i_d - Ri_q + v_q - p\omega\psi_{\mathrm{mg}}) \tag{4.2}$$

$$\frac{d\omega}{dt} = \frac{1}{J_s}(T_e - B\omega) \tag{4.3}$$

$$T_e = \frac{3}{2}p\psi_{\mathrm{mg}}i_q \tag{4.4}$$

where i_d, i_q[A] are stator currents in the d-q frame, and v_d, v_q[V] represent stator voltages in the same frame. ω[rad/s] is the rotor speed. T_e[Nm] is the electric magnetic torque. L_s[H] is the stator phase winding inductance. ψ_{mg}[Wb] denotes the flux of the permanent magnet. J_s[kgm^2] is the rotor moment of inertia. B [N/rad/s] is the viscous coefficient. R[Ω] denotes the stator resistance.

In this chapter, we consider the torque control problem of a PMSM. Hence, we choose $y := T_e$ as the controlled output. Also, the dynamical system described by (4.1), (4.2) is used for control system design. More specifically, the following discretized model of the dynamical system (4.1), (4.2) with the Euler method is used as the plant model for control system design.

$$x_p(t+1) = A_p(\omega(t))x_p(t) + B_p(v(t) + h(\omega(t))) \tag{4.5}$$

$$y(t) = C_p x_p(t) \tag{4.6}$$

where T_s[s] is the sampling period, $x_p := [i_d\ i_q]^{\mathrm{T}}$ is the plant state, $v := [v_d\ v_q]^{\mathrm{T}}$ is the control input and

$$A_p(\omega) := I + T_s\begin{bmatrix} -\frac{R}{L_s} & p\omega \\ -p\omega & -\frac{R}{L_s}\end{bmatrix}, \quad B_p := T_s\begin{bmatrix}\frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s}\end{bmatrix}$$

$$C_p := \begin{bmatrix} 0 & \frac{3p\psi_{\mathrm{mg}}}{2}\end{bmatrix}, \quad h(\omega) := \begin{bmatrix} 0 \\ -p\omega\psi_{\mathrm{mg}}\end{bmatrix}$$

In PMSMs, the norm constraint described by $v_d^2 + v_q^2 \le V_{\max}^2$, where $V_{\max} := V_{\mathrm{dc}}/\sqrt{3}$ is usually imposed on the input voltage [41]. To satisfy the constraint, we compute the input voltage by

$$v(t) = \sigma_{\bar{V}}(u(t)) \tag{4.7}$$

where $u \in \mathbb{R}^2$ is a new control signal, Also, $\bar{V}$ and $\nu_{\max}$ are defined by $\bar{V} := \nu_{\max}I$ and $\nu_{\max} := V_{\max}/\sqrt{2}$, respectively.

For the system (4.5), (4.6), we make the following assumption.

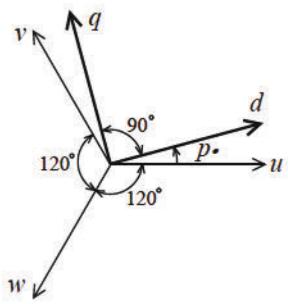

Figure 4.1: Relationship between the d-q rotating frame and the u-v-w reference frame

Assumption 4 *ω satisfies $\omega(t) \in [\underline{\omega}, \overline{\omega}], \forall t \geq 0$, where $\underline{\omega}$ and $\overline{\omega}$ are constants such that $\underline{\omega} \leq \overline{\omega}$.*

Note that, in this case, the matrix $A_p(\omega)$ can be represented as $A_p(\omega) = \sum_{s=1}^{2} \beta_s(\omega) A_{ps}$, where $A_{p1} := A_p(\underline{\omega})$, $A_{p2} := A_p(\overline{\omega})$, $\beta_1, \beta_2 \geq 0$ and $\beta_1 + \beta_2 = 1$.

In this chapter, we consider the following problem.

Problem 2 *Consider the system (4.5)–(4.7). Assume that $\omega(t) \in [\underline{\omega}, \overline{\omega}], \forall t \geq 0$. Design a control law $u(t) = \mathcal{K}(x_p(t), \omega(t), r(t))$ that ensures closed-loop stability and achieves $\lim_{t\to\infty} y(t) = r(t)$, where $r(t) = \overline{r}, \forall t \geq 0$ and $\overline{r}$ is a constant.*

In this chapter, we compute the signal $u(t)$ by

$$u(t) = \tilde{v}(t) + h(\omega(t)), \tag{4.8}$$

where $\tilde{v} \in \mathbb{R}^2$ is a new control signal. The system (4.5)–(4.8) is expressed as

$$\begin{aligned} x_p(t+1) &= A_p(\omega(t)) x_p(t) \\ &\quad + B_p[(\Phi_{\overline{V}}(\tilde{v}(t) + h(\omega(t))) - h(\omega(t))]. \end{aligned} \tag{4.9}$$

4.3 Controller Design

In this chapter, we design a controller described by

$$x_c(t+1) = x_c(t) + e(t), \tag{4.10}$$

$$e(t) = r(t) - y(t), \tag{4.11}$$

$$\tilde{v}(t) = F_c(t) x_c(t) + F_p(t) x_p(t) + M(t) r(t), \tag{4.12}$$

where x_c is the integrator state. The design condition of the matrices $F_c(t)$ $F_p(t)$ and $M(t)$ will be introduced in this section later. In the proposed control method, by suitably resetting the controller state x_c, we attempt to improve tracking control performance. The detail of the control algorithm will be explained in Section 4.4.

From eqs. (4.9)–(4.11), an augmented system is derived as

$$\begin{aligned} x(t \quad 1) \qquad & A(\ (t))x(t) \\ & B[\ \bar{v}(\tilde{\ }(t) \quad h(\ (t))) \quad h(\ (t))] \quad Er(t) \end{aligned} \tag{4.13}$$

$$e(t) \qquad Cx(t) \quad D \ r(t) \tag{4.14}$$

where x : $[x_p^{\mathrm{T}} \ x_c]^{\mathrm{T}}$ and

$$A(\ ) \ : \ \begin{matrix} A_p(\ ) & 0 \\ C_p & I \end{matrix} \qquad B : \ \begin{matrix} B_p \\ 0 \end{matrix} \qquad E : \ \begin{matrix} 0 \\ I \end{matrix}$$

$$C \ : \ \ C_p \ \ 0 \qquad D \ : \ I$$

We also define the following matrices.

$$A_s : \ \begin{matrix} A_{ps} & 0 \\ C_p & I \end{matrix} \qquad s \quad 1 \ 2$$

Note that $A(\ )$ can be expressed as $A(\ ) \quad {}^{2}_{s \ 1} \ {}_{s}(\ )A_s$.

The following result holds.

Lemma 2 *There exist matrices and ( ) that satisfy*

$$A(\ ) \qquad B \ (\ ) \quad E \tag{4.15}$$

$$0 \qquad C \qquad D \tag{4.16}$$

Proof) The solution to (4.15), (4.16) are given as

$$\begin{matrix} c_1 \\ \frac{2}{3p \ \ \mathrm{mg}} \\ c_2 \end{matrix} \qquad (\ ) \qquad \begin{matrix} c_1 R & \frac{2L_s}{3 \ \ \mathrm{mg}} \\ \frac{2R}{3p \ \ \mathrm{mg}} & c_1 p L_s \end{matrix} \tag{4.17}$$

where c_1 and c_2 are arbitrary constants. **Q.E.D.**

Remark 8 *Eqs. (4.15), (4.16) could be viewed as an extension of the regulator equation [8, 24] to a class of LPV systems.*

We define the following variables.

$$\tilde{}_e \; : \quad \tilde{} \quad (\;)r \tag{4.18}$$

$$: \quad x \quad r \tag{4.19}$$

From (4.13)–(4.19), the error system can be derived as

$$(t \quad 1) \qquad A(\;(t))\;(t) \quad B[\;\bar{_V}(\tilde{}_e(t) \qquad (\;(t))r(t)$$
$$h(\;(t))) \quad (\;(\;(t))r(t) \quad h(\;(t)))] \tag{4.20}$$

Remark 9 *r in (4.19) represents the steady-state value of the state x when the plant output tracks the step reference signal r. Similarly, ( )r in (4.18) represents the steady-state value of the signal ˜. It should be noted that the solution matrix to (4.15), (4.16) is constant. This is a crucial property of the PMSM dynamics. This implies that the steady-state value of the state does not depend on the varying parameter . Note that, thanks to this property, the error system (4.20) can be derived. Further, this property will enable us to design a control law that achieves setpoint tracking under the time variation of the rotor speed .*

Remark 10 *In general, the steady-state value of the state of LPV systems changes depending on varying parameters. Hence, setpoint tracking is usually achievable only in the case where the varying parameter becomes constant.*

We make the following assumption.

Assumption 5 *The reference signal satisfies* $^{(l)}(\;)\bar{r} \quad h^{(l)}(\;) \qquad {}_{\max} \qquad [\_ \; \bar{}\;]\; l \quad 1\; 2.$

The above assumption ensures that the tracking control is achievable under the input limitation in the steady-state.

In the following, we introduce a polytopic model of a saturation function proposed in Reference [12]. The following polytopic model will be used to design the feedback controller (4.10)–(4.12).

Lemma 3 *[12] Let u* ^{m}*. Suppose that* $_j \quad 1$ *for all j* $I[1 \;\; m]$*, then*

$$_A(u) \quad co\; \mathbf{E}_j u \quad \tilde{\mathbf{E}}_j \;:\; j \quad I[1 \;\; 2^m] \tag{4.21}$$

where co denotes the convex hull.

From the above Lemma, it can easily be verified that, for A diag$[a_1 \quad a_m]$ 0 and $_j \quad a_j$, $_A(u)$ can be expressed as

$$_A(u) \quad \sum_{j\ 1}^{2^m} \ _j(\mathbf{E}_j u \quad \tilde{\mathbf{E}}_j \ ) \tag{4.22}$$

where 0 $_j$ 1 $^{2^m}_{j\ 1}\ _j$ 1.

In the following, we introduce a theorem that is used to design the feedback control law (4.10)–(4.12). The control law designed based on the following theorem has a structure that a high-gain control law and a low-gain control law are interpolated by a single scheduling parameter. The scheduling parameter will be used to tune the trade o between the size of the region of attraction and tracking performance. The control algorithm will be introduced in Section 4.4.

Theorem 3 *Consider the system (4.13) and (4.14). Suppose that $\bar{r}$ satisfies* max $^{(l)}\bar{r}$ $h(\ )^{(l)}$ $_{\max}$ l 1 2. *For given positive definite matrices* **R**, **S**, *positive scalars* , $_0$ *and* $_1$ *such that* $_0$ $_1$, *assume that there exist matrices* $Q_i\ Y_i\ Z_i$ $(i$ 0 1) *that satisfy*

$$\begin{matrix} Q_i & & \\ \mathbf{R}^{1\ 2}Y_i & & \\ \mathbf{S}^{1\ 2}Q_i & {}_iI & 0 \\ A_sQ_i \quad B(\mathbf{E}_jY_i \quad \tilde{\mathbf{E}}_jZ_i) & 0 & Q_i \end{matrix}$$

$$i \quad I[0\ 1] \quad j \quad I[1\ 4] \quad s \quad I[1\ 2] \tag{4.23}$$

$$\begin{matrix} Q_i & \\ Z_i^{(l)} & \frac{2}{l} \end{matrix} \quad 0 \quad i \quad I[0\ 1] \quad l \quad I[1\ 2] \tag{4.24}$$

$$Q_0 \quad Q_1 \tag{4.25}$$

where $_l$: $_{\max}$ max $(\ )^{(l)}\bar{r}$ $h(\ )^{(l)}$ *and the symbol stands for symmetric block in matrix inequalities. Further, for some constant* [0 1], *we suppose that* (0) $(P(\ )$ 0) *where* $P(\ ):\ Q(\ )^{\ 1}$ $Q(\ ):\ (1\ \ )Q_0$ Q_1. *Then, by applying the control law*

$$\tilde{\ }(t) \quad F(\ )x(t) \quad M(\quad (t))r(t) \tag{4.26}$$

where $F(\ ):\ Y(\ )Q(\ )^{\ 1}$, $Y(\ ):\ (1\ \ )Y_0$ Y_1 *and* $M(\quad):\ (\ )\ F(\ )$ *to the system (4.13) and (4.14), the relations* (t) $(P(\ )$ 0) t 0, $\lim_t$ $e(t)$ 0 *hold. Further,* $J:$ $_{t\ 0}$ $(t)^{\mathrm{T}}\mathbf{S}\ (t)$ $\tilde{\ }_e(t)^{\mathrm{T}}\mathbf{R}\tilde{\ }_e(t)$ $(\ )$ *holds, where* $(\ ):\ (1\ \ )\ _0$ $_1$.

Proof) From (4.18), (4.19) and (4.26), we obtain $\tilde{\ }_e \quad F(\ )$. Hence, the closed-loop system (4.20), (4.26) can be represented as

$$(t \quad 1) \quad A(\ (t))\ (t) \quad B \ \ _{\bar{V}}(F(\ )\ (t)) \tag{4.27}$$

where $_{\bar{V}}(F(\ )\ ) : \quad _{\bar{V}}(F(\ ) \quad (\ )\bar{r} \quad h(\ )) \quad (\ (\ )\bar{r} \quad h(\ ))$. We define $H(\ ) :$ $Z(\ )Q(\ )^{1} \quad Z(\ ) : \quad (1 \quad)Z_0 \quad Z_1$ and $\ :$ diag[$_1$ $_2$]. If $\quad (H(\ ) \quad)$ and max $\quad (\ )^{(l)}\bar{r} \quad h(\ )^{(l)} \quad _{\max} \quad l \quad I[1\ 2]$, then $H(\ )^{(l)} \quad (\ )^{(l)}\bar{r} \quad h(\ )^{(l)}$ $_{\max} \quad l \quad I[1\ 2]$ [ ]. Hence, in this case, the relation $(F(\ ) \quad (\ )\bar{r} \quad h(\ )^{(l)})$ $^{4}_{j\ 1}\ _j\ \mathbf{E}_j(F(\ ) \quad (\ )r \quad h(\ ))\ \ \tilde{\mathbf{E}}_j(H(\ ) \quad (\ )\bar{r} \quad h(\ ))$ holds from Lemma 3. Therefore, the relation $_{\bar{V}}(F(\ )\ ) \quad ^{4}_{j\ 1}\ _j\ \mathbf{E}_jF(\ ) \quad \tilde{\mathbf{E}}_jH(\ )$ holds.

By using this relation, if $(t) \quad (H(\ ) \quad)$ and max $\quad (\ )^{(l)}\bar{r} \quad h(\ )^{(l)} \quad _{\max} \quad l$ $I[1\ 2]$, the closed-loop system (4.27) can be rewritten as

$$(t \quad 1) \quad (\ (t) \quad (t))\ (t) \tag{4.28}$$

where $(\quad) : \quad ^{4}_{j\ 1}\ _j\ \ _j(\ ) \quad _j(\ ) : \ A(\ ) \quad B\ \mathbf{E}_jF(\ ) \quad \tilde{\mathbf{E}}_jH(\ )$.

On the other hand, from (4.14) and (4.16), the signal $e(t)$ can be expressed as

$$e(t) \quad C\ (t) \tag{4.29}$$

From (4.24), we have

$$\begin{matrix} Q(\ ) & \\ Z(\ )^{(l)} & \frac{2}{l} \end{matrix} \quad 0 \quad l \quad I[1\ 2] \tag{4.30}$$

Then, by substituting $Z(\ )^{(l)} \quad H(\ )^{(l)}Q(\ )$ for (4.30) and performing a congruence transformation with block- diag[$Q(\ )^{1}$ 1] and substituting $Q(\ )^{1} \quad P(\ )$, and applying Schur complement [3], we have

$$\frac{1}{\ _l^2}H(\ )^{(l)T}H(\ )^{(l)} \quad \frac{1}{\ }P(\ ) \quad l \quad I[1\ 2] \tag{4.31}$$

Equation (4.31) implies that $(P(\ ) \quad 0) \quad (H(\ ) \quad)$.

By carrying out the similar procedures used to derive (4.30) to (4.23), and substituting $Z(\ ) \quad H(\ )Q(\ )$ and $Y(\ ) \quad F(\ )Q(\ )$ for the resulting inequality, and performing a congruence transformation with block-diag[$Q(\ )^{1}\ I\ I$], and multiplying the resulting inequality by $_s(\ (t))$, and summing them up for $s\quad 1\ 2$, we have

$$\begin{matrix} P(\ ) & & \\ \mathbf{R}^{1\ 2}F(\ ) & & \\ \mathbf{S}^{1\ 2} & (\ )I & 0 \\ _j(\ (t)) & 0 & P(\ )^{1} \end{matrix} \quad (4.32)$$

Further, by multiplying the inequality (4.32) by $_j(t)$, and summing them up for j 1 4, we have

$$\begin{matrix} P(\ ) & & \\ \mathbf{R}^{1\ 2}F(\ ) & & \\ \mathbf{S}^{1\ 2} & (\ )I & 0 \\ (\ (t)\quad (t)) & 0 & P(\ )^{1} \end{matrix} \quad (4.33)$$

By applying Schur complement to (4.33), and multiplying the resulting inequality from the left by $(t)^{\mathrm{T}}$ and from the right by (t), and using (4.28) and (4.29), we have

$$V(\ (t\quad 1))\quad V(\ (t)) \\ \frac{1}{(\ )}\ (t)^{\mathrm{T}}\mathbf{S}\ (t)\quad \tilde{}_e(t)^{\mathrm{T}}\mathbf{R}\tilde{}_e(t) \quad (4.34)$$

where $V(\ ) :\ \ ^{\mathrm{T}}P(\ )$. From (4.34), we can conclude that if $(0)\quad (P(\ )\quad 0)$ then

$$V(\ (t))\quad V(\ (0))\qquad t\quad 0 \quad (4.35)$$

Equation (4.35) implies that $(t)\quad (P(\ )\quad 0)\quad t\quad 0$. On the other hand, it has been shown that the nonlinearity $(F(\ )\ (t))$ can be represented as $(F(\ )\ (t))\ \ {}^{4}_{j\ 1}\ _j(t)\ \mathbf{E}_jF(\ )$ $\tilde{\mathbf{E}}_jH(\ )\ (t)$ if $(t)\quad (H(\ )\quad)$ and max $^{(l)}\bar{r}\quad h(\ )^{(l)}\quad _{\max}\quad l\quad I[1\ 2]$. From (4.31) and (4.35), we can state that if the conditions in Theorem 3 hold, the relation (t) $(H(\ )\quad)\quad t\quad 0$ holds. From (4.34), since $(t)\quad 0\ (t\quad)$, $e(t)\quad 0\ (t\quad)$ holds. Moreover, from (4.34) and (4.35), $J\quad _{t\ 0}\ (t)^{\mathrm{T}}\mathbf{S}\ (t)\quad \tilde{}_e(t)^{\mathrm{T}}\mathbf{R}\tilde{}_e(t)\qquad (\ )$ holds. **Q.E.D.**

Based on Theorem 3, we design a gain $F(1)\quad Y_1Q_1{}^{1}$ which makes the region $(P(1)\quad 0)$ large and a gain $F(0)\quad Y_0Q_0{}^{1}$ which achieves fast convergence of the state in $(P(0)\quad 0)$ by suitably choosing the parameters $_0\quad _1$, **R** and **S**. Then we construct the control law (4.26) by interpolating the obtained gains.

Remark 11 *When the control law (4.26) is designed based on Theorem 3, the matrix* $P(\ )$ *needs to be determined so that the condition* $(0)\quad (P(\ )\quad 0)$ *is satisfied for some constant*

$[0\ 1]$. *This can be achieved by solving the design problem with the constraint* (0) $(P_1\ \ 0)$. *By applying Schur complement, this condition can be rewritten as the following linear matrix inequality (LMI) condition.*

$$x(0) \quad \bar{r} \quad Q_1 \quad 0 \tag{4.36}$$

Also, the conditions (4.23)–(4.25) are LMIs with respect to the variables $Q_i\ Y_i\ Z_i\ (i\ \ 0\ 1)$. *Hence, the design problem of the control law (4.26) that satisfies (4.23)–(4.25) and (4.36) can be solved e ciently by a numerical optimization algorithm based on an interior point method[3].*

Remark 12 *The size of the set* $(P_1\ \ 0)$ *mainly depends on the choice of the parameter* $_1$. *By choosing a larger value as* $_1$, *the size of the set* $(P_1\ \ 0)$ *could be expanded in general. However, the control law designed based on Theorem 3 can only ensure local stability. This implies that, when the magnitude of the reference signal* $\bar{r}$ *is large, there may not exist a solution that satisfies the conditions (4.23)–(4.25) and (4.36) even if a large value is chosen as* $_1$.

Remark 13 *If* $A\ \ A_s\ \ s\ \ I[1\ 2]$ *and* $h(\ )\ \ 0\ \ [\_\ \bar{\ }]$, *Theorem 3 is equivalent to Theorem 1 of Reference [47].*

4.4 Control Algorithm

The control law (4.26) includes the scalar . The upper bound of the cost function J is given as ( ) , and the function ( ) takes a smaller value when a smaller value is chosen as . Hence, it can be expected that the control performance is improved by minimizing at each sampling time. Moreover, the state of the controller x_c can be used as a tuning parameter to improve the control performance. Thus, we utilize the following control algorithm.

Algorithm 2

Step 0: Set $t\ \ 0$ *and* $\ \ 1$.

Step 1: Measure $x_p(t)$ *and* (t).

Step 2: If $\ \ 0$, *set* $(t)\ \ 0$ *and go to Step 4.*

Step 3: For given $x_p(t)$, solve $\min_{[0\ 1]\ \tilde{x}_c}$ *, s.t.*

$$\begin{matrix} x_p(t) \\ \tilde{x}_c \end{matrix} \quad \bar{r} \quad Q(\ ) \qquad 0 \tag{4.37}$$

Then, set (t) *and* $x_c(t)$ $\tilde{x}_c$*.*

Step 4: Apply (t) $\bar{v}(F(\ (t))[x_p(t)^{\mathrm{T}}\ x_c(t)]^{\mathrm{T}}\ \ M(\ (t)\ \ (t))r(t)\ \ h(\ (t)))$ *to the plant (4.5), (4.6).*

Step 5: Compute $x_c(t\ \ 1)$ *by (4.10) and (4.11).*

Step 6: $t\ \ t\ \ 1$ *and go to Step 1.*

It should be noted that the value of the controller state x_c is reset so that the scheduling parameter is minimized at Step 3 at each sampling time.

Remark 14 *The optimization problem of Step 3 in Algorithm 2 is an LMI optimization problem with respect to and $\tilde{x}_c$. It has been shown in Reference [49] that the optimization problem can be solved e ciently by a simple bisection algorithm.*

Remark 15 *Algorithm 2 is equivalent to the control algorithm in Reference [47] except for the structure of the control law at Step 4.*

As for feasibility of Algorithm 2 and closed-loop stability, the following result holds.

Theorem 4 *Consider the system (4.5), (4.6). Assume that* (t) *satisfies* $(\ (t))^{(l)}\bar{r}\ \ h(\ (t))^{(l)}$ $_{\max}$ l $I[1\ 2]$ t 0*. Moreover, assume that there exists $\tilde{x}_c$ such that* $[x_p(0)^{\mathrm{T}}\ \tilde{x}_c]^{\mathrm{T}}$ $(P(1)\ \ \bar{r})$*. Then by applying Algorithm 1 to the system (4.5), (4.6),* $e(t)$ *converges to zero as* t .

Theorem 4 is a slightly modified version of Theorem 2 of Reference [47] and can be readily proven by using the proof procedure of Reference [47]. Hence, we omit the detail of the proof of Theorem 4 and outline the proof briefly. We assume that $x(t)$ $(P(\ (t))\ \ \bar{r})$ holds at time t as shown in Fig.4.2. When the control signal (t) $\bar{v}(F(\ (t))[x_p(t)^{\mathrm{T}}\ x_c(t)]^{\mathrm{T}}$ $M(\ (t)\ \ (t))r(t)\ \ h(\ (t)))$ is applied to the system (4.5), (4.6), $[x(t)\ \ \bar{r}]^{\mathrm{T}}P(\ (t))[x(t)\ \ \bar{r}]$ $[x(t\ \ 1)\ \ \bar{r}]^{\mathrm{T}}P(\ (t))[x(t\ \ 1)\ \ \bar{r}]$ holds from Theorem 3. Hence, for some positive scalar 1, $x(t\ \ 1)$ $(P(\ (t))\ \ \bar{r})$ holds. This implies that there exists $(t\ \ 1)$ such

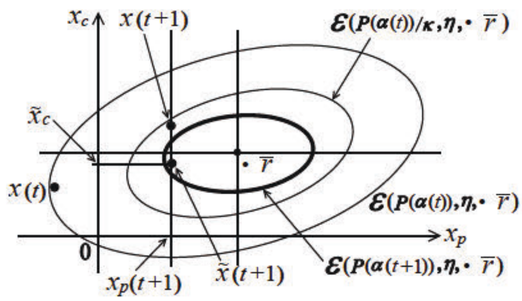

Figure 4.2: Graphical interpretation of the optimization problem at Step 3 in Algorithm 1. In this figure, the controller state x_c is reset so that $\alpha(t+1)$ is minimized at time $t+1$. As the result, the state at time $t+1$ is moved from $x(t+1)$ to $\tilde{x}(t+1)$.

that $x(t+1) \in \mathcal{E}(P(\alpha(t+1)), \eta, \Pi\bar{r})$ and $\alpha(t+1) < \alpha(t)$. Hence, the scheduling parameter $\alpha(t)$ decreases monotonically and converges to zero. Note that the convergence speed of the parameter $\alpha(t)$ could be enhanced by resetting the integrator state x_c at each sampling time(see Fig. 4.2). After $\alpha(t)$ becomes zero, the constant high gain feedback control law with the integral action is applied to the system. As the result, the tracking error $e(t)$ converges to zero.

4.5 Comparison with an existing method

A standard approach to construct a control law for a PMSM is to use a decentralized PI controller with a decoupling compensator used to cancel the nonlinear coupling terms of the PMS dynamics [23]. In this approach, firstly, the decoupling compensator defined by

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \tilde{v}_d - L_s p\omega i_q \\ \tilde{v}_q + pL_s\omega i_d + \omega p\phi_{\mathrm{mg}} \end{bmatrix} \tag{4.38}$$

is applied to the system (4.5), (4.6) to cancel the nonlinear coupling terms. Then the following decentralized PI controller is used to achieve the setpoint tracking.

$$x_c(t+1) = x_c(t) + e(t) \tag{4.39}$$

$$e(t) = r(t) - y(t) \tag{4.40}$$

$$\tilde{v}_q(t) = K_P e(t) + K_I x_c(t) \tag{4.41}$$

$$\tilde{v}_d(t) = K_F i_d(t) \tag{4.42}$$

$$v(t) = \Phi_{\bar{V}}(u(t)) \tag{4.43}$$

where K_P K_I and K_F are feedback gains. When the equality (t) $u(t)$ holds, the closed-loop system (4.5), (4.6), (4.38)–(4.43) is divided into two linear time-invariant systems. The feedback gains of the controller can be designed by solving two independent state-feedback controller design problems. In addition, the implementation of the control algorithm on the computer is fairly easy. Hence, the above control law is very practical. However, when the signal (t) is saturated, the decoupling compensator (4.38) is no longer e ective. As the result, the closed-loop stability with the control law (4.38)–(4.43) might not be guaranteed under such a situation. The analysis of the closed-loop system is di cult problem due to the nonlinear characteristics of the feedback system.

4.6 Numerical Example

The values of the physical parameters are J_s 2 35 10 ^{4} kgm^2, B 1 1 10 ^{4} N rad s, L_s 7 10 ^{3} H, R 2 98 , $_{\mathrm{mg}}$ 0 125 Wb, V_{dc} 100 V ($_{\max}$ 40 82 V) and p 2. The sampling period is chosen as T_s 0 1 ms. For this plant, we designed the control law (4.26) with **S** diag[0 1 0 1 0 01], **R** 10 $^5 I$, diag[37 46 10 38], $_1$ 60 $_0$ 0 2, 1, $\bar{r}$ 1 Nm, _ 100 rad s, ¯ 100 rad s, c_1 0 and c_2 0.

Case I: Algorithm 2 is applied.

Case II: Decentralized PI control with the nonlinear decoupling feedback compensation in Section 4.5 is applied. The feedback gains are chosen as K_P 111 5 K_I 18 82 K_F 32 02.

Figs. 4.3–4.8 show the results of the numerical simulation for the reference signal $r(t)$ 0 2 Nm, t 0. As the dynamical model of the PMSM to carried out the numerical simulation, we have used the discretized model of (4.1)-(4.4). The discretization was done using the Euler method with the sampling period T_s 0 1 ms. The initial values of i_d, i_q and were set to zero. In both cases, the maximum values of $_d$ and $_q$ are smaller than $_{\max}$ 40 82 V. Further, in both cases, the controlled output converges to the reference signal even though the rotor speed increases. In Case II, the controlled output tracks the reference signal with the 12 5% overshoot, and the settling time is 1.6 ms. In Case I, the controlled output tracks the reference signal without producing the overshoot, and the settling time is 0.5 ms.

Figs. 4.9–4.14 show the results of the numerical simulation for the reference signal $r(t)$ 1 Nm, t 0. The initial values of i_d, i_q and were set to zero. Note that, in this numerical simulation, the signal $_q$ is in the saturation region transiently. In Case II, the controlled output tracks the reference signal with the 30% overshoot, and the settling time is 2.2ms. The larger overshoot might occur since the decoupling compensator (4.38) is no longer e ective and the integrator windup occurs while the signal $_q$ is saturated. In Case I, the controlled output tracks the reference signal without producing the overshoot, and the settling time is 0.7 ms. In the proposed method, the integrator state x_c is reset until becomes zero. Hence, the integrator state x_c does not accumulate while the resets are carried out. It seems that this enables to produce the response without the overshoot.

Figs. 4.15–4.20 show the results of the numerical simulation for the reference signal $r(t)$ 1 Nm, t 0. In this numerical simulation, the initial values of was set to 70 rad s. The initial values of i_d and i_q were set to zero. It can be seen from these figures that the plant output in Case I converges to the reference signal rapidly without producing the overshoot.

All the numerical simulations were performed with the digital computer (Intel Xeon 3.6GHz, 4GB RAM), using MATLAB. The maximum computation time required to solve the optimization problem in Algorithm 2 was 0.36 ms. The computation time could be reduced by using a compiled language.

4.7 Conclusions

In this chapter, we have proposed a torque control method for a PMSM under input voltage limitation. In the proposed control method, the scheduling parameter and the controller state are updated at each sampling time so that the tracking control performance is improved. It has been shown that, by using the proposed control method, the setpoint tracking is achievable under the variation of the rotor speed and the input voltage limitation. The control method in this chapter is only applicable to the case where the reference signal is a step signal. Hence, it is required to extend the control method so that the time-varying reference signal can be handled. This extension could be done by introducing a target recalculation mechanism to the proposed control method. We will study this problem in Chapter 5.

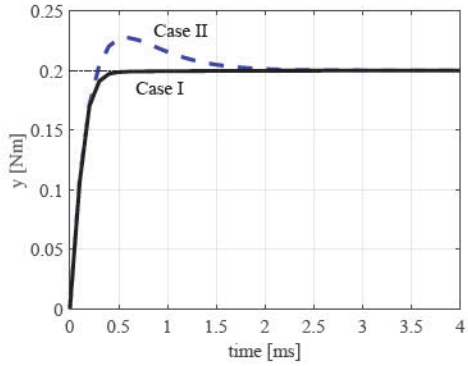

Figure 4.3: Plant output [Nm]

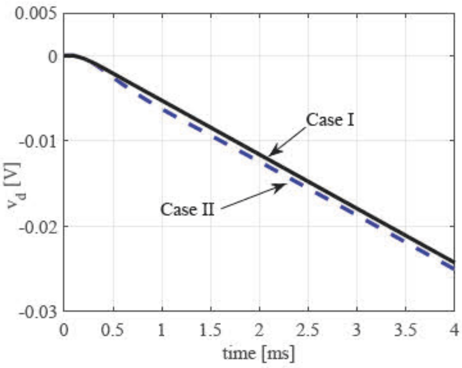

Figure 4.4: *d*-axis voltage $_{d}$ [V]

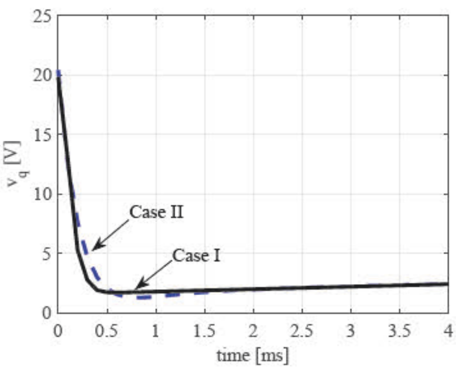

Figure 4.5: *q*-axis voltage $_{q}$ [V]

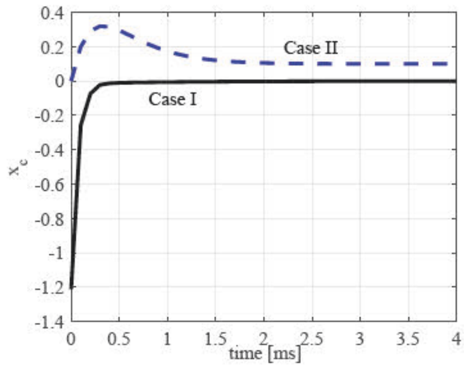

Figure 4.6: Integrator state x_c

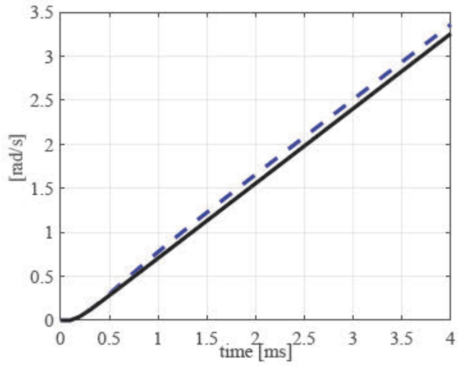

Figure 4.7: Rotor speed [rad s]

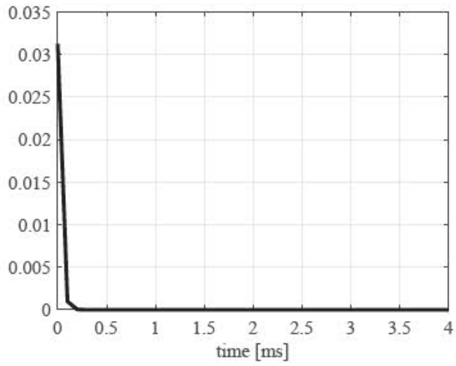

Figure 4.8: Scheduling parameter

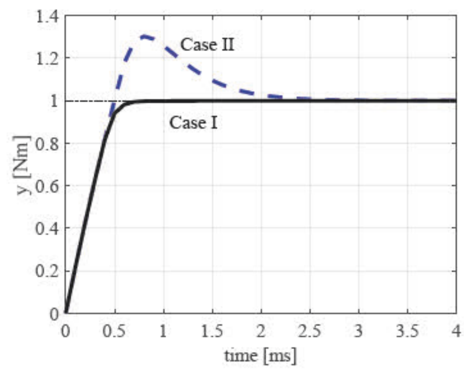

Figure 4.9: Plant output [Nm]

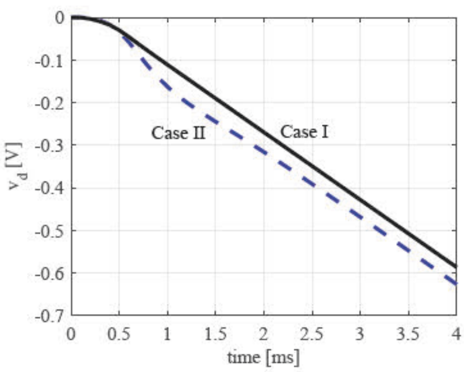

Figure 4.10: *d*-axis voltage $_d$ [V]

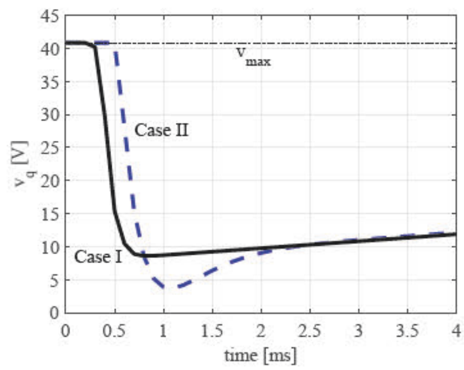

Figure 4.11: *q*-axis voltage $_q$ [V]

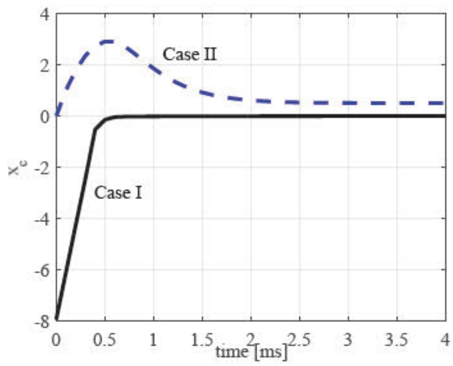

Figure 4.12: Integrator state x_c

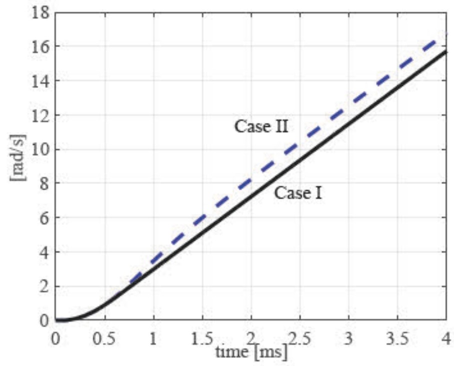

Figure 4.13: Rotor speed [rad s]

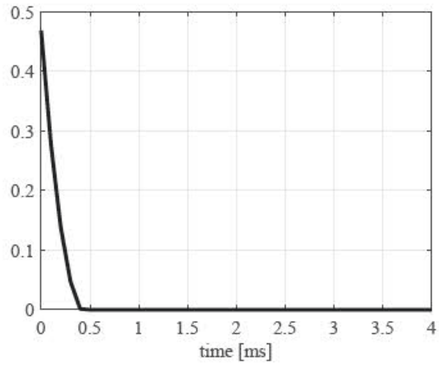

Figure 4.14: Scheduling parameter

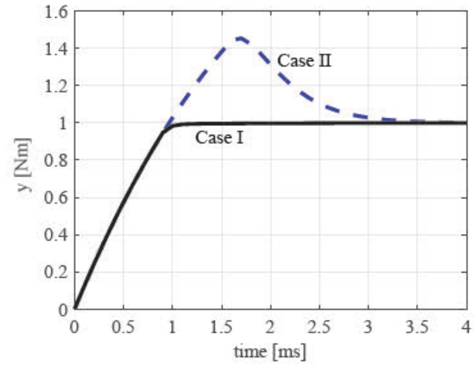

Figure 4.15: Plant output [Nm]

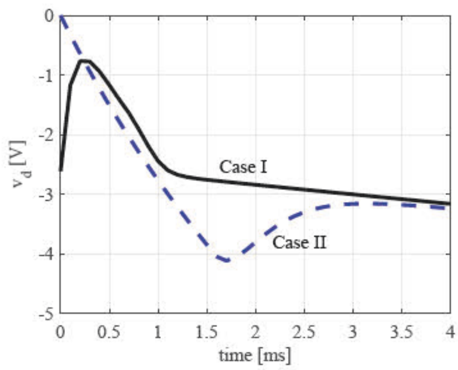

Figure 4.16: d-axis voltage $_d$ [V]

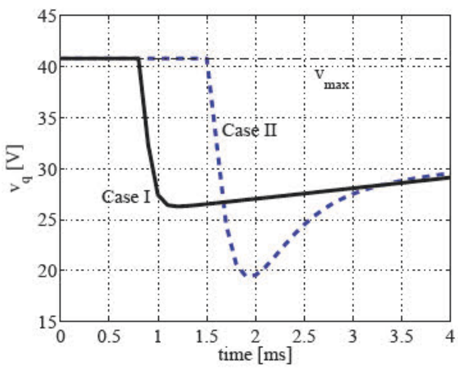

Figure 4.17: q-axis voltage $_q$ [V]

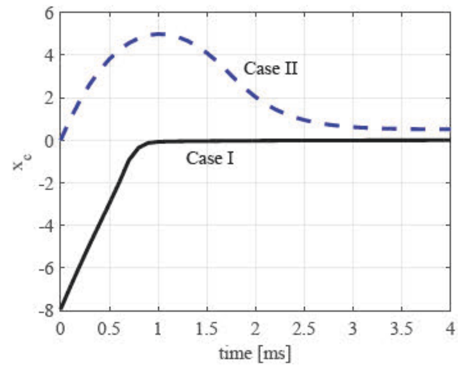

Figure 4.18: Integrator state x_c

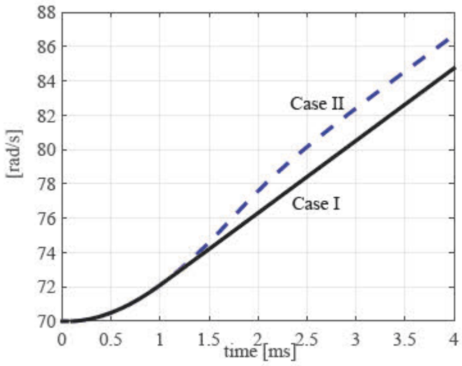

Figure 4.19: Rotor speed [rad s]

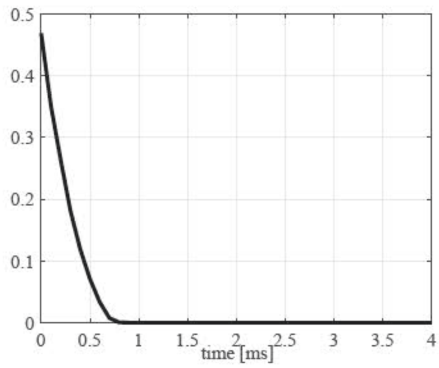

Figure 4.20: Scheduling parameter

Chapter 5

Torque Control of a PMSM Using a Reference Governor with Integrator Resets

In this chapter, we extend the control method in Chapter 4 so that the motor torque tracks a time-varying reference signal under input voltage limitation and the time variation of the rotor speed. To this end, we introduce a reference governor to ensure feasibility of the control algorithm. The proposed control law consists of a gain-scheduled control law and the reference governor. In the proposed control method, in addition to the scheduling parameter and the controller state, the modified reference signal is determined online so that the transient response is improved. The e ectiveness of the method is shown by a numerical example and an experimental result.

5.1 Introduction

The Permanent Magnet Synchronous Motor (PMSM) has been widely used in various industries due to its characteristics of high e ciency, high torque to inertia ratio, and fast dynamic performance. The dynamics of the PMSM includes nonlinear coupling terms between the d-axis subsystem and the q-axis subsystem. A standard approach to construct a control law for a PMSM is to use a decentralized PI controller with a decoupling compensator used to cancel the nonlinear coupling terms [23]. The controller of this type is very practical since the controller design and its implementation on the computer are fairly easy. In general, a larger control signal is required transiently to achieve higher tracking control performance, which would cause control signal saturation. When the control signal is saturated, the decoupling compensator is no longer effective. The design problem of the controller which guarantees closed-loop stability under control signal saturation is a difficult problem due to the nonlinear characteristics of the control system.

Recently, several control techniques based on an optimal control theory have been applied to the control problem of a PMSM. Model predictive control schemes have been applied to a torque control problem of a PMSM in [13, 54]. Also, a nonlinear optimal control technique has been applied to a torque control problem of a PMSM in Reference [41]. It has been shown in these literatures that higher tracking control performance can be achieved under input voltage limitation as compared with the standard decentralized PI control approach. However, in these literatures, the controller is designed under the assumption that the rotor speed is constant. Hence, it seems that further studies are required to examine tracking performance and stability of the control system under the time variation of the rotor speed. For this problem, in Chapter 4, we have proposed a torque control method that ensures the motor torque converges to a step reference signal under input voltage limitation and time variation of the rotor speed. In Chapter 4, a plant model of a PMSM is derived as a linear parameter varying (LPV) system in which the rotor speed is included as the varying parameter. Then, we have derived an LMI-based design condition of a feedback controller. However, when the reference signal is time-varying, feasibility and stability of the control algorithm in Chapter 4 are not guaranteed.

In this chapter, we extend the control method in Chapter 4 so that the motor torque tracks a time-varying reference signal under input voltage limitation and the time variation of the rotor speed. Such a control law is typically required in traction control of an electric vehicle. To this end, we develop a motor torque control law that consists of the feedback control law of [50] and a reference governor [51]. In the proposed control method, in addition to the scheduling parameter and the controller state, the modified reference signal is determined online so that the transient response is improved. In the proposed control method, the control signal is computed by solving a convex optimization problem at each sampling time. The effectiveness of the method is shown by a numerical example and an experimental result.

Notations: For a vector u ^{m} and a diagonal matrix A $[a_1$ $a_m]$ 0, we define the saturation function as $_A(u) : (_{a_1}(u_1) \quad _{a_m}(u_m))^{\mathrm{T}}$, where

$$_{a_i}(u_i) : \quad \begin{matrix} a_i \mathrm{sgn}(u_i) & u_i & a_i \\ u_i & u_i & a_i \end{matrix}$$

If $A \quad I$, we will omit it. For a positive definite matrix $P \quad ^{n \ n}$, a vector $\quad ^n$ and a positive scalar , we define $(P \quad) : \quad x \quad ^n : (x \quad)^{\mathrm{T}} P(x \quad) \quad$. For a matrix $F \quad ^{m \ n}$, we define $(F) : \quad x \quad ^n : F^{(i)} x \quad 1 \ i \quad 1 \quad m$, where $F^{(i)}$ denotes the ith row of F. For integers k_1 and k_2 such that $k_1 \quad k_2$, we define $I[k_1 \ k_2] : \ [k_1 \ k_1 \quad 1 \quad k_2]$. Let be the set of $m \quad m$ diagonal matrices whose diagonal element are either 1 or 0. There are 2^m elements in . Suppose that each element of is labeled as $\mathbf{E}_j \ j \quad 1 \ 2 \quad 2^m$. Also, we define $\tilde{\mathbf{E}}_j : \ I \quad \mathbf{E}_j$.

5.2 Problem Formulation and Preliminaries

The PMSM is composed of a permanent magnet rotor and stator windings. The PMSM stator has three coils spatially separated by 120 degree each other. A rotating magnetic field is generated by the three-phase current, and the magnetic torque is generated by the interaction of the rotating magnetic field and the flux of the permanent magnet[54]. In the control system design for PMSMs, the d-q rotating frame is commonly used, since AC signals appear as DC ones in the d-q rotating frame. Fig. 5.1 shows the relationship between the d-q rotating frame and the u- - reference frame. In this figure, is the angular position of the rotor, and p is the number of pole pairs. The dynamics of a PMSM in the d-q rotating frame is described by the following state equation [32].

$$x_p(t \quad 1) \quad A_p(\ (t)) x_p(t) \quad B_p(\ (t) \quad h(\ (t))) \tag{5.1}$$

$$(t) \quad C_p x_p(t) \tag{5.2}$$

where $x_p : \ [i_d \ i_q]^{\mathrm{T}}$ is the plant state, $: \ [\ _d \ \ _q]^{\mathrm{T}}$ is the control input and

$$A_p(\ ) : \ I \quad T_s \quad \begin{matrix} \frac{R}{L_s} & p \\ p & \frac{R}{L_s} \end{matrix} \qquad B_p : \ \frac{T_s}{L_s} I$$

$$C_p : \quad 0 \quad \frac{3p \ _{\mathrm{m}}}{2} \qquad h(\ ) : \quad \begin{matrix} 0 \\ p \ _{\mathrm{m}} \end{matrix}$$

T_s is the sampling period. $i_d \ i_q$ are stator currents in the d-q frame, and $_d \quad _q$ represent stator voltages in the same frame. is the rotor speed. is the electric magnetic torque. L_s

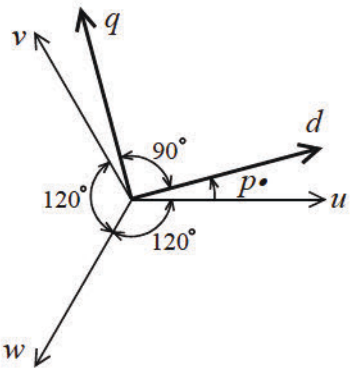

Figure 5.1: Relationship between the d-q rotating frame and the u-v-w reference frame

is the stator phase winding inductance. ϕ_{mg} denotes the flux of the permanent magnet. J_s is the rotor moment of inertia. B_v is the viscous coefficient. R denotes the stator resistance.

In PMSMs, the norm constraint described by $v_d^2 + v_q^2 \le V_{\max}^2$, where $V_{\max} := V_{\mathrm{dc}}/\sqrt{3}$ is usually imposed on the input voltage [41]. To satisfy the constraint, we compute the input voltage by

$$v(t) = \Phi_{\overline{V}}(u(t)), \tag{5.3}$$

where $u \in \mathbb{R}^2$ is a new control signal, Also, $\overline{V}$ and $v_{\max}$ are defined by $\overline{V} := v_{\max} I$ and $v_{\max} := V_{\max}/\sqrt{2}$, respectively.

For the system (5.1), (5.2), we make the following assumption.

Assumption 6 *ω satisfies $\omega(t) \in [\underline{\omega}, \overline{\omega}], \forall t \ge 0$, where $\overline{\omega}$ is a positive constant and $\underline{\omega} = -\overline{\omega}$.*

Note that, in this case, the matrix $A_p(\omega)$ can be represented as $A_p(\omega) = \sum_{s=1}^{2} \beta_s(\omega) A_{ps}$, where $A_{p1} := A_p(\underline{\omega})$, $A_{p2} := A_p(\overline{\omega})$, $\beta_1, \beta_2 \ge 0$ and $\beta_1 + \beta_2 = 1$.

In this chapter, we consider the following problem.

Problem 3 *Consider the system (5.1)–(5.3). Assume that $\omega(t) \in [\underline{\omega}, \overline{\omega}], \forall t \ge 0$. Design a control law $u(t) = \mathcal{K}(x_p(t), \omega(t), r(t))$ that minimizes the tracking error $z(t) = r(t) - y(t)$ at each sampling time and achieves $\lim_{t\to\infty} y(t) = r(t)$ if $r(t) = \overline{r}, \forall t \ge T_f$, where r is a reference signal and $\overline{r}$ is a constant.*

In this chapter, we compute the signal $u(t)$ by

$$u(t) = \tilde{v}(t) + h(\omega(t)), \tag{5.4}$$

where ˜ ^{2} is a new control signal. The system (5.1)–(5.4) is expressed as

$$x_p(t \quad 1) \quad A_p(\quad (t))x_p(t)$$
$$B_p[(\quad \bar{V}(\tilde{\ }(t) \quad h(\quad (t))) \quad h(\quad (t))] \tag{5.5}$$

5.3 Controller Design

In this chapter, we design a controller described by

$$x_c(t \quad 1) \quad x_c(t) \quad e(t) \tag{5.6}$$
$$e(t) \qquad (t) \qquad (t) \tag{5.7}$$
$$\tilde{\ }(t) \quad F_c(t)x_c(t) \quad F_p(t)x_p(t) \quad M(t) \ (t) \tag{5.8}$$

where x_c is the integrator state, is the modified reference signal. The design condition of the matrices $F_c(t)$ $F_p(t)$ and $M(t)$ will be introduced in this section later. In the proposed control method, by suitably resetting the controller state x_c, we attempt to improve tracking control performance. The detail of the control algorithm will be explained in Section 5.4.

From eqs. (5.5)–(5.7), an augmented system is derived as

$$x(t \quad 1) \quad A(\quad (t))x(t)$$
$$B[\quad \bar{V}(\tilde{\ }(t) \quad h(\quad (t))) \quad h(\quad (t))] \quad E \ (t) \tag{5.9}$$
$$e(t) \quad Cx(t) \quad D \quad (t) \tag{5.10}$$

where x : $[x_p^{\mathrm{T}} \ x_c]^{\mathrm{T}}$ and

$$A(\ ) : \quad \begin{matrix} A_p(\ ) & 0 \\ C_p & I \end{matrix} \qquad B : \quad \begin{matrix} B_p \\ 0 \end{matrix} \qquad E : \quad \begin{matrix} 0 \\ I \end{matrix}$$
$$C : \quad C_p \quad 0 \qquad D : I$$

We also define the following matrices.

$$A_s : \quad \begin{matrix} A_{ps} & 0 \\ C_p & I \end{matrix} \qquad s \quad 1 \ 2$$

Note that $A(\ )$ can be expressed as $A(\ ) \quad {}_{s \ 1}^{2} \quad {}_{s}(\ )A_s$.

The following result holds.

Lemma 4 *There exist and ( ) that satisfy*

$$A( ) \quad B ( ) \quad E \tag{5.11}$$

$$0 \quad C \quad D \tag{5.12}$$

Proof) The solution matrices to (5.11), (5.12) are given by

$$\begin{matrix} c_1 \\ \frac{1}{p \ m} \\ c_2 \end{matrix} \qquad ( ) \qquad \begin{matrix} c_1 R_p & \frac{L_s}{m} \\ \frac{R_p}{p \ m} & c_1 p \ L_s \end{matrix}$$

where c_1 c_2 are arbitrary constants. (Q.E.D.)

Remark 16 *Eqs. (5.11), (5.12) could be viewed as an extension of the regulator equation to a class of LPV systems.*

We define the variables $\tilde{}_e$: $\tilde{}$ ( ) : x . We make the following assumption.

Assumption 7 *The reference signal r satisfies* $r(t)$ $\bar{r}$ t 0 *and* $^{(l)}( )r(t)$ $h^{(l)}( )$ max [ ] l 1 2 t 0.

The above assumption ensures that the tracking control is achievable under the input limitation in the steady-state.

In the following, we introduce a theorem that is used to design the feedback control law (5.6)–(5.8). The control law designed based on the following theorem has a structure that a high-gain control law and a low-gain control law are interpolated by a single scheduling parameter. The scheduling parameter will be used to tune the trade o between the size of the region of attraction and tracking performance. The control algorithm will be introduced in Section 5.4.

Theorem 5 *Consider the system (5.9) and (5.10). Suppose that* (t) *is a step reference signal such that* $\bar{r}$ *and* $^{(l)}( )$ $h^{(l)}( )$ max [ ] l 1 2. *For given positive definite matrices* **R**, **S**, *positive scalars* , $_0$ *and* $_1$ *such that* $_0$ $_1$, *assume that there exist matrices* Q_i Y_i Z_i (i 0 1) *that satisfy*

$$\begin{matrix} Q_i & & \\ \mathbf{R}^{1\ 2} Y_i & & \\ \mathbf{S}^{1\ 2} Q_i & {}_i I & 0 \\ A_s Q_i \quad B(\mathbf{E}_j Y_i \quad \tilde{\mathbf{E}}_j Z_i) & 0 & Q_i \end{matrix}$$

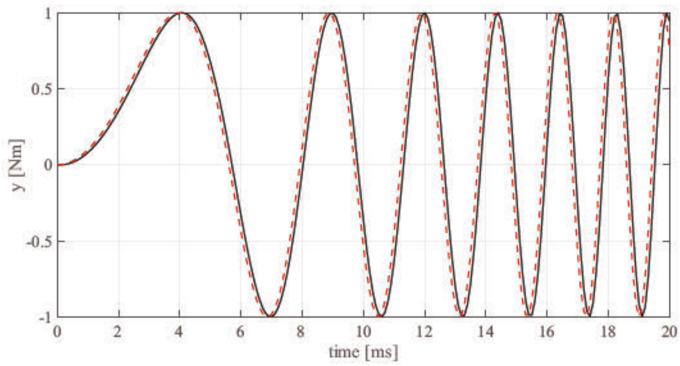

Figure 5.2: [Nm]

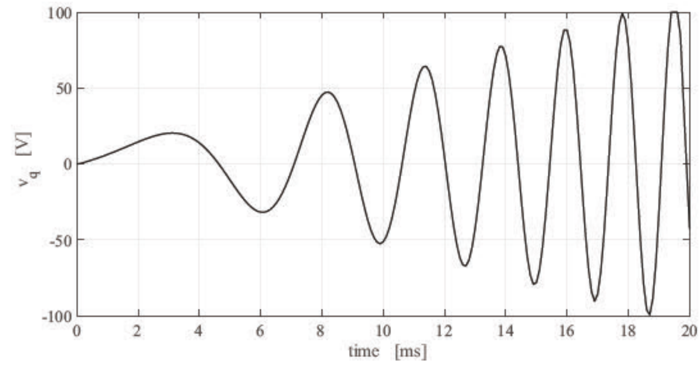

Figure 5.3: $_q$ [V]

$$i \quad I[0\ 1] \quad j \quad I[1\ 4] \quad s \quad I[1\ 2] \tag{5.14}$$

$$\begin{matrix} Q_i \\ Z_i^{(l)} \quad \frac{2}{l} \end{matrix} \quad 0 \quad i \quad I[0\ 1] \quad l \quad I[1\ 2] \tag{5.15}$$

$$Q_0 \quad Q_1 \tag{5.16}$$

where $_l$: $_{\max}$ max ( )$^{(l)}$ (t) h( )$^{(l)}$ *and the symbol stands for symmetric block in matrix inequalities. Further, for some constant* [0 1], *we suppose that* (0) (P( ) 0) *where* P( ) : Q( ) ^{1} Q( ) : (1)Q_0 Q_1. *Then, by applying the control law*

$$\tilde{}(t) \quad F(\ )x(t) \quad M(\quad (t))\ (t) \tag{5.17}$$

where F( ) : Y( )Q( ) ^{1}, Y( ) : (1)Y_0 Y_1 *and* M( ) : ( ) F( ) *to*

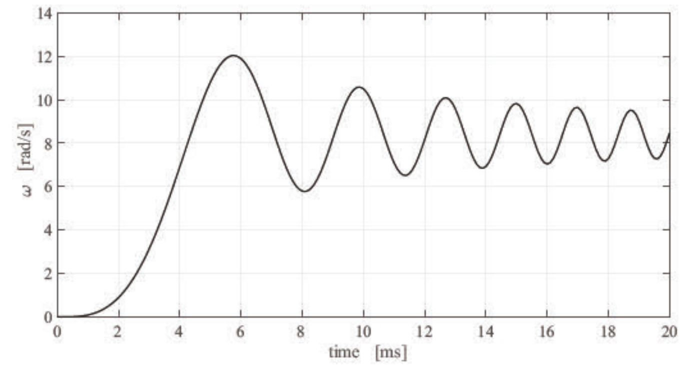

Figure 5.4: [rad s]

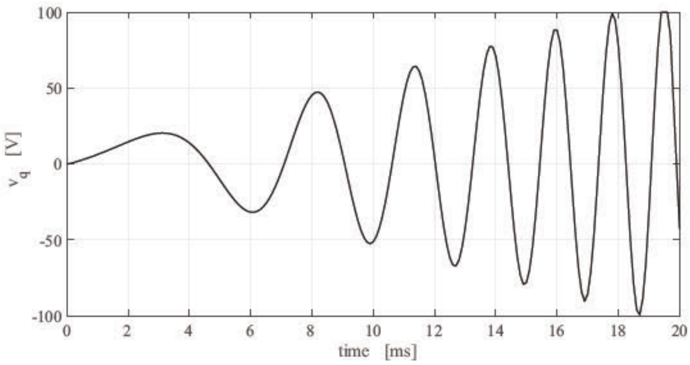

Figure 5.5: $_q$ [V]

the system (5.9) and (5.10), the relations (t) $(P(\ )\ \ 0)$ t 0, $\lim_t$ $e(t)$ 0 *hold. Further,* J : $_{t\ 0}$ $(t)^{\mathrm{T}}\mathbf{S}\ (t)$ $\tilde{}_e(t)^{\mathrm{T}}\mathbf{R}\tilde{}_e(t)$ $(\ )$ *holds, where* $(\ )$: $(1\ \ )$ $_0$ $_1$.

Based on Theorem 5, we design a gain $F(1)$ $Y_1 Q_1{}^1$ which makes the region $(P(1)\ \ 0)$ large and a gain $F(0)$ $Y_0 Q_0{}^1$ which achieves fast convergence of the state in $(P(0)\ \ 0)$ by suitably choosing the parameters $_0$ $_1$, **R** and **S**. Then we construct the control law (5.17) by interpolating the obtained gains.

5.4 Control Algorithm

The control law (5.17) includes the scalar . The upper bound of the cost function J is given as ( ) , and the function ( ) takes a smaller value when a smaller value is chosen as .

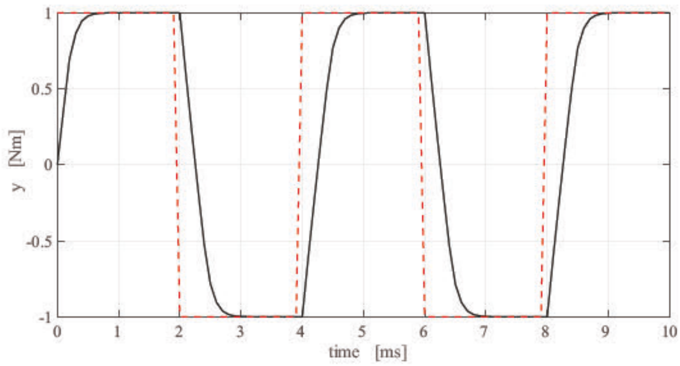

Figure 5.6: [Nm]

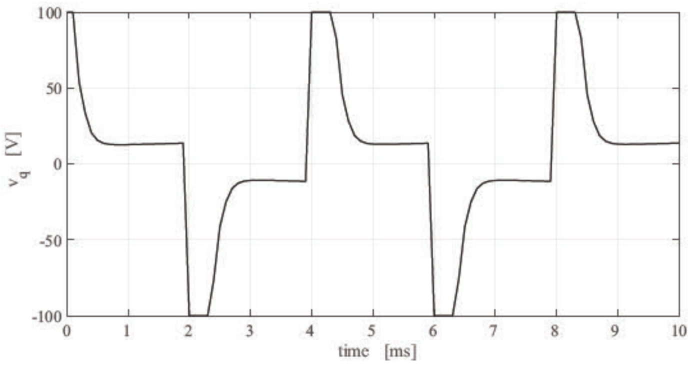

Figure 5.7: $_q$ [V]

Hence, it can be expected that the control performance is improved by minimizing at each sampling time. Moreover, the state of the controller x_c can be used as a tuning parameter to improve the control performance. Thus, we utilize the following control algorithm.

Algorithm 3

Step 0: Set t 0 *and* 1.

Step 1: Measure $x_p(t)$ *and* $r(t)$.

Step 2: If 0, *set* (t) 0 *and go to Step 4.*

Step 3: Set $\tilde{}$ 1 *and solve* $\min_{\tilde{}\ \tilde{x}_c}$ $(r(t)\ \ \tilde{})\ {}^2_2$, *subject to*

$$\begin{matrix} x_p(t) \\ \tilde{x}_c \end{matrix} \quad \tilde{}\quad Q(\tilde{}) \quad 0 \tag{5.18}$$

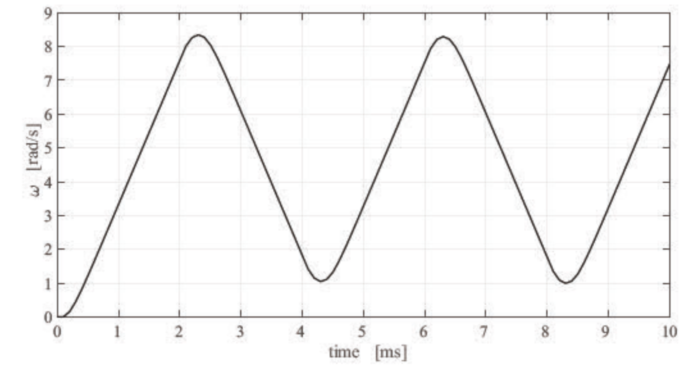

Figure 5.8: [rad s]

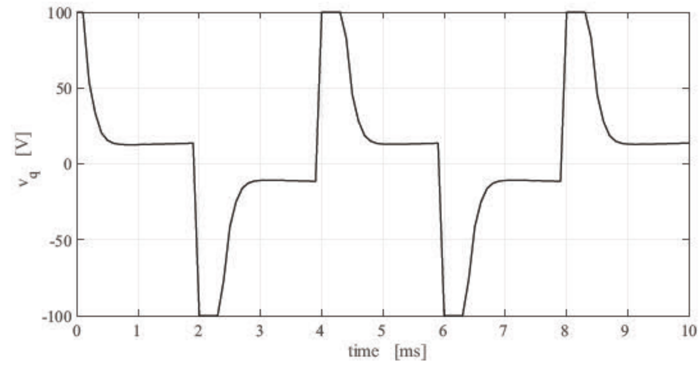

Figure 5.9: $_q$ [V]

$$(\ )^{(l)}\ \tilde{}\quad h(\ )^{(l)}\qquad \max\quad l$$

$$l\quad 1\ 2\qquad [\_\ \bar{}]$$

Then, set (t) 1 (t) $\tilde{}$, $x_c(t)$ $\tilde{x}_c$ *and go to Step 5.*

Step 4: Set $\tilde{}$ $r(t)$, *solve* $\min_{\tilde{}\ [0\ 1]\ \tilde{x}_c}$, *subject to (5.18). Then, set* (t) $\tilde{}$ (t) $r(t)$, $x_c(t)$ $\tilde{x}_c$.

Step 5: Apply (t) $\bar{v}(F(\ (t))[x_p(t)^{\mathrm{T}}\ x_c(t)]^{\mathrm{T}}\quad M(\ (t)\quad (t))\ (t)\quad h(\ (t)))$ *to the plant (5.1), (5.2).*

Step 6: Compute $x_c(t\quad 1)$ *by (5.6) and (5.7).*

Step 7: t t 1 *and go to Step 1.*

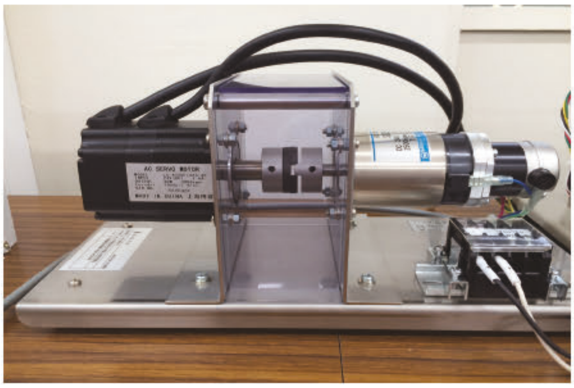

Figure 5.10: PMSM

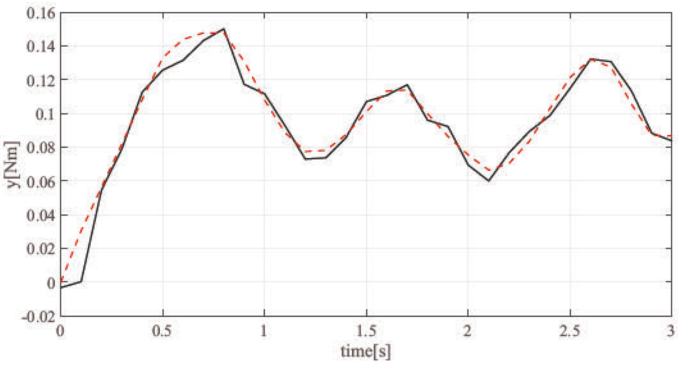

Figure 5.11:   [Nm]

The above algorithm can be considered an extension of the method in [51] to an LPV system. In the above control algorithm, the reference signal is modified at Step 3 so that the feasibility of the control algorithm is ensured. It should be noted that the value of the controller state x_c is reset so that the value of $(r(t) \quad \tilde{\ })\ {}^{2}_{2}$ is minimized. The optimization problem at Step 3 corresponds to the reference governor.

The optimization problems at Steps 3 and 4 in Algorithm 3 are convex optimization problems with LMI constraints.

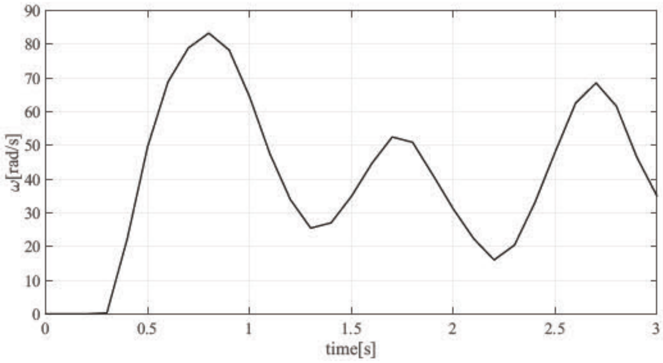

Figure 5.12: [rad s]

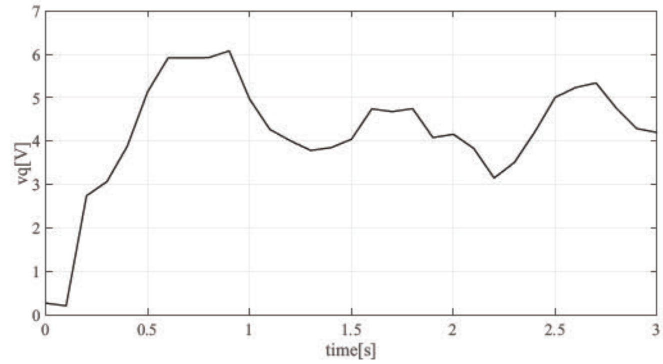

Figure 5.13: $_q$ [V]

5.5 Numerical Example

The values of the physical parameters are J_s 2 35 10 ^{4} kgm^2, B 1 1 10 ^{4} N rad s, L_s 7 10 ^{3} H, R 2 98 , $_{\rm mg}$ 0 125 Wb, $_{\rm max}$ 100 V and p 2. For this plant, we designed the control law (5.17) with **S** diag[0 1 0 1 0 01], **R** 10 $^5 I$, diag[37 46 10 38], $_1$ 60 $_0$ 0 2, 1, $\bar{r}$ 1 Nm, $\underline{\ }$ 100 rad s, $\bar{\ }$ 100 rad s, c_1 c_2 0. The sampling period is set to T_s 0 1 ms. Figs. 2 and 3 show the numerical simulation results for two di erent reference signals. The initial values of i_d, i_q and were set to zero. In Figs. 2 (a) and 3 (a), the solid lines show the plant output and the dashed lines show the reference signals. It can be seen from these figures that, in both cases, the plant output tracks the reference signal even though the rotor speed changes under the input voltage limitation.

5.6 Experiment

Fig. 4 (a) shows the experimental setup. The values of the physical parameters are J_s 2 0 10 ^{5} kgm^2, B 3 0 10 ^{4} N rad s, L_s 0 25 10 ^{3} H, R 0 25 , $_{\text{mg}}$ 0 0185 Wb, $_{\text{max}}$ 24 V and p 4. For this plant, we designed the control law (5.17) with **S** diag[0 1 0 1 0 01], **R** 10 $^5 I$, $_1$ 60 $_0$ 0 2, 1, $\bar{r}$ 1 Nm, _ 100 rad s, ¯ 100 rad s, c_1 c_2 0. In these experiments, SH7216 (32bit microprocessor) is used to calculate the control signal at each sampling time. The sampling period is set to T_s 0 1 ms. Figs. 4 (b)–4 (d) show the experimental result. The initial values of i_d, i_q and were set to zero. In Fig. 4 (b), the solid line shows the plant output and the dashed line shows the reference signal. It can be seen from these figures that, the plant output tracks the reference signal even though the rotor speed changes under the input voltage limitation.

5.7 Conclusions

In this chapter, we have proposed a torque control method for a PMSM under input voltage limitation. The proposed controller consists of a gain-scheduled feedback controller and a reference governor. In the proposed control algorithm, when the tracking error is large, the reference governor generates a modified reference signal that can be tracked under the input limitation. The main feature of the proposed method is that the controller state is reset at each sampling time so that the tracking control performance is improved. The e ectiveness of the method has been shown by a numerical example and an experimental result.

Chapter 6

Online Optimization of l_2 Gain Performance for Constrained Linear Systems by Model Predictive Control with State Resets

In the preceding chapters, we have studied tracking control methods for input constrained systems and their application to torque control of a PMSM. In this chapter, we consider disturbance attenuation problem of constrained control systems. In particular, we address a model predictive control problem for constrained linear systems in the presence of l_2 disturbances. In the proposed control method, the feedback gain and the controller state are updated online so that the l_2 gain of the system is minimized. The problem of determining the feedback gain and the controller state is formulated as a convex optimization problem with linear matrix inequality constraints. We show that both feasibility of the control algorithm and the dissipation inequality are guaranteed for all times.

6.1 Introduction

Model predictive control(MPC) is an e ective means to deal with constrained control problems[25]. In the standard MPC approach, in order to improve transient response, a control signal is computed by solving a finite horizon optimal control problem at each sampling time. Recently, various types of MPC algorithms which are robust against disturbances have been developed. For example, a min-max optimization based MPC approach has been proposed in Reference [36]. Also, a tube-based MPC approach has been developed in Reference [26].

These control algorithms are developed so that feasibility and stability are guaranteed in the presence of persistent bounded disturbances. On the other hand, several MPC algorithms for l_2 disturbance attenuation have been proposed. In the MPC algorithm of Reference [31], the feedback gain is recomputed online by solving an optimization problem with LMI[3] constraints derived from the bounded real lemma. However, feasibility and dissipativity have not been discussed in Reference [31]. In Reference [5], an MPC algorithm which ensures dissipativity in the presence of l_2 bounded disturbances has been proposed. In this method, the dissipation constraint is introduced to ensure the dissipativity. The MPC algorithms in References [31] and [5] could be regarded as an extensions of the MPC algorithm in Reference [16] to an l_2 disturbance attenuation problem.

The use of the state reset to improve control performance has been studied in the control engineering community since 1950s [6, 2]. In this approach, the controller state is usually reset to zero when its input equals zero. In Reference [48], it has been shown that the tracking control performance of MPC can be improved by resetting the controller state so that the cost function is minimized. In Reference [34], an analysis condition of L_2 performance of a system with state resets has been derived. Also, in the literature, a design condition of a feedback controller and a reset condition has been developed based on the analysis condition.

In this chapter, we propose an MPC algorithm which optimizes l_2 gain performance under input constraints. In the proposed control method, both the feedback gain and the controller state are recomputed at each sampling time so that the l_2 gain is minimized. It will be shown that the feasibility and the dissipativity are guaranteed for all times. To ensure the dissipativity, we introduce a modified version of the dissipation constraint of Reference [5]. We modify the dissipation constraint of Reference [5] so that the dissipativity still holds even when the state reset occurs. In the proposed control approach, the feedback gain and the controller state are computed by solving an LMI optimization problem[3]. A numerical example is provided to illustrate e ectiveness of the proposed method.

Notations: For a vector $\ ^n$, we denote its Euclidean norm as $\ _2 :\ (\ )^{1\ 2}$. For a signal (k) defined on $[0\ )$, we define its l_2 norm as $\ _{l_2} :\ (\ _{k\ 0}\ (k)\ (k))^{1\ 2}$. For a positive definite matrix $P\ \ ^{n\ n}$, we denote $(P\ ) :\ x\ \ ^n : x\ Px\ $. For a matrix $H\ \ ^{m\ n}$, we denote the ith row of H as $H^{(i)}$. For integers k_1 and k_2 such that $k_1\ \ k_2$, we define $I[k_1\ k_2] :\ [k_1\ k_1\ \ 1\ \ k_2]$.

6.2 Problem Formulation and Preliminaries

Let us consider the system described by

$$x(t \quad 1) \qquad Ax(t) \quad B_1u(t) \quad B_2 \ \ (t) \tag{6.1}$$

$$z(t) \qquad Cx(t) \quad D_1u(t) \quad D_2 \ \ (t) \tag{6.2}$$

where $x \qquad ^n$ is a state vector, $u \qquad ^m$ is an input vector and $z \qquad ^q$ is an output vector. $\qquad ^p$ is a disturbance that satisfies $\qquad : \qquad ^p : \ \ ^2_{l_2} \qquad$. We assume that the above system consists of a plant and dynamics of a controller. Further, we assume that the structure of the state x is $x \quad [x_p \ \ x_c\ ]$, where $x_p \qquad ^{n_p}$ is the plant state and $x_c \qquad ^{n_c}$ is the controller state.

The following constraint is assumed to be imposed on the control signal and the state.

$$_x x(t) \qquad _u u(t) \tag{6.3}$$

where $_x \qquad ^{n_s \ n}$, $_u \qquad ^{n_s \ m}$ and $\qquad ^{n_s}$ are constant matrices. n_s is the number of constraints. Note that (6.3) is an element-wise inequality. For example, in the case of $m \quad 1$, $u(t) \quad 1$ can be rewritten in the form of (6.3) with $_x \quad 0$, $_u \quad [1 \quad 1]$ and $\quad [1 \ \ 1]$.

The purpose of this chapter is to design a control algorithm in which the feedback gain and the controller state of the following control law

$$u(t) \quad F(t)x(t) \tag{6.4}$$

are updated at each sampling time so that the l_2 gain from to z of the system (6.1), (6.2) and (6.4) is minimized under the constraint (6.3).

We initially introduce the following theorem.

Theorem 6 *Consider the system (6.1), (6.2). For given positive scalars $\qquad$, assume that there exist matrices $Q \ \ Y$ and a positive constant $\hat{}$ that satisfy*

$$\begin{matrix} Q & & & & \\ 0 & I & & & \\ CQ \quad D_1Y & D_2 & \hat{} I & & \\ AQ \quad B_1Y & B_2 & 0 & Q & \end{matrix} \qquad 0 \tag{6.5}$$

$$\begin{matrix} Q & & \\ ^{(l)}_x Q & ^{(l)}_u Y & \frac{^{(l)2}}{} \end{matrix} \qquad 0 \quad l \quad I[1 \ \ n_s] \tag{6.6}$$

where the symbol stands for symmetric block in matrix inequalities. Further, we suppose that $x(0)$ $(P\ )$*, where* $P :\ Q^{1}$*. Then, by applying the feedback control law*

$$u(t) \quad Fx(t) \tag{6.7}$$

where $F \quad YQ^{1}$ *to the system (6.1), (6.2), the relations* $x(t) \quad (P \quad) \ t \quad 0$ *and* $_{x}x(t) \quad _{u}u(t) \quad t \quad 0$ *hold. In addition, the following inequality holds.*

$$z_{\ l_2} \qquad _{l_2} \quad \overline{} \tag{6.8}$$

where $:\ \hat{}^{1\ 2}$.

Proof) The closed-loop system (6.1), (6.2) and (6.7) can be rewritten as

$$x(t \quad 1) \qquad x(t) \quad B_2 \ (t) \tag{6.9}$$

$$z(t) \qquad x(t) \quad D_2 \ (t) \tag{6.10}$$

where $:\ A \quad B_1F, \quad :\ C \quad D_1F.$

By performing a congruence transformation with block-diag $[Q^{1}\ I\ I\ I]$ on (6.5), and substituting $Q^{1} \quad P$ for the resulting inequality, we obtain

$$\begin{bmatrix} P & & & \\ 0 & I & & \\ & D_2 & \hat{}I & \\ & B_2 & 0 & P^{1} \end{bmatrix} \quad 0 \tag{6.11}$$

Then, by applying Schur complement to (6.11), we have

$$B_2 \ P \quad B_2 \quad \begin{matrix} P\,0 \\ 0\,I \end{matrix} \quad \frac{1}{\hat{}} \ D_2 \qquad D_2 \quad 0 \tag{6.12}$$

By multiplying (6.12) from the left by $[x(t) \quad (t)\ ]$ and from the right by $[x(t) \quad (t)\ ]$ and using (6.9), (6.10), we obtain

$$V(x(t \quad 1)) \quad V(x(t)) \qquad (t)\ _2^2 \quad \frac{1}{\hat{}}\ z(t)\ _2^2 \tag{6.13}$$

where $V(x) :\ \ x\ \ Px$. From (6.13) and $x(0) \quad (P\ )$, we have

$$V(x(t)) \quad V(x(0)) \quad \sum_{i\ 0}^{t\ 1} \quad (i)\ _2^2 \qquad t \quad 0 \tag{6.14}$$

Eq.(6.14) implies that $x(t) \quad (P \quad) \quad t \quad 0$.

Furthermore, from Eq.(6.13), we obtain

$$\sum_{i\ 0}^{t} z(i) {}_2^2 \quad \hat{} \quad \sum_{i\ 0}^{t} (i) {}_2^2 \quad V(x(0)) \tag{6.15}$$

Therefore, we can conclude that Eq.(6.8) holds.

By performing a congruence transformation with block-diag$[Q \;\; 1]$ ^{1} on (6.6), we have

$$\begin{matrix} & P & \\ {}_{x}^{(l)} & {}_{u}^{(l)}F & \frac{(l)2}{} \end{matrix} \quad 0 \quad l \quad I[1 \;\; n_s] \tag{6.16}$$

By applying Schur complement [3] to (6.16), we have

$$(\;{}_{x}^{(l)} \quad {}_{u}^{(l)}F) \; (\;{}_{x}^{(l)} \quad {}_{u}^{(l)}F) \quad \frac{(l)2}{}P \quad l \quad I[1 \;\; n_s] \tag{6.17}$$

Hence, ${}_{x}x(t) \quad {}_{u}u(t) \quad t \quad 0$ holds since the relation $x(t) \quad (P \quad) \quad t \quad 0$ holds.
Q.E.D.

6.3 Main Results

In this section, we show a control algorithm in which the feedback gain F is updated online so that the l_2 gain is minimized. In the proposed control algorithm, the controller state x_c is also used as a tuning parameter to improve control performance.

Algorithm 4

Step 0: Set $t \quad 0$.

Step 1: Measure $x_p(t)$. At time $t \quad 0$, go to Step 2. Otherwise, go to Step 3

Step 2: Solve $\min_{Y Q \tilde{x}_c} \hat{}$, s.t. (6.5), (6.6) and

$$\begin{matrix} x_p(t) & \\ \tilde{x}_c & Q \end{matrix} \quad 0 \tag{6.18}$$

Set $P(0) \quad Q$ ^{1}, $Y(0) \quad Y$, $x_c(0) \quad \tilde{x}_c$, $\tilde{x}(0) \quad [x_p(0) \quad \tilde{x}_c]$, $p(0) \quad \tilde{x}(0) \;\; P(0)\tilde{x}(0)$ and go to Step 5.

Step 3: Solve $\min_{Y Q \tilde{x}_c} \hat{}$, s.t. (6.5), (6.6), (6.18) and

$$\begin{matrix} (t) \\ x_p(t) \\ \tilde{x}_c \end{matrix} \quad Q \qquad 0 \tag{6.19}$$

where (t) : $p(0)$ $p(t$ $1)$ $x(t)$ $P(t$ $1)x(t)$. Set $P(t)$ Q ^{1}, $Y(t)$ Y, $x_c(t)$ $\tilde{x}_c$ and $\tilde{x}(t)$ $[x_p(t)$ $\tilde{x}_c]$. If the above optimization problem is infeasible, set $P(t)$ $P(t$ $1)$, $Y(t)$ $Y(t$ $1)$ and $\tilde{x}(t)$ $x(t)$.

Step 4: Update $p(t)$ by $p(t)$ $p(t$ $1)$ $[x(t)$ $P(t$ $1)x(t)$ $\tilde{x}(t)$ $P(t)\tilde{x}(t)]$.

Step 5: Apply $u(t)$ $F(t)\tilde{x}(t)$ with $F(t)$ $Y(t)P(t)$ to the plant (6.1), (6.2).

Step 6: t t 1 and go to Step 1.

It should be noted that the controller state x_c is reset at Step 2 and Step 3 so that $\hat{}$ is minimized when the optimization problem is feasible. The optimization problems at Step 2 and Step 3 are LMI optimization problems with respect to the variables Q Y and $\tilde{x}_c$. Hence, the problems are convex optimization problems.

As for Algorithm 4, the following results hold.

Theorem 7 *Consider the system (6.1), (6.2). Assume that there exist Q Y $\tilde{x}_c$ which satisfy the constraints at Step 2 at time t 0. Then, by applying Algorithm 4 to the system (6.1), (6.2), the feasibility of the algorithm holds for all time. Further, the following inequality holds.*

$$z_{\ l_2} \qquad (\quad {}_{l_2} \qquad \bar{}) \tag{6.20}$$

where : max (1) ( )

Proof of Theorem 7) Firstly, we show that the feasibility holds for all time. From the assumption, $x(0)$ $(P(0)$ $)$ holds. Hence, at time t 0, the parameters $P(0)$ and $Y(0)$ are determined by solving the optimization problem at Step 2. Then, the control signal is calculated at Step 5 and applied to the plant. At time t 1, if the optimization problem at Step 3 is feasible, $P(1)$, $Y(1)$ and $\tilde{x}_c$ are obtained by solving the optimization problem. On the other hand, if the optimization problem is infeasible, $P(1)$, $Y(1)$ and $\tilde{x}_c$ are chosen as $P(1)$ $P(0)$, $Y(1)$ $Y(0)$ and $\tilde{x}_c$ $x_c(t)$. Hence, it is possible to determine the parameters at time $t \quad 1$. The same arguments hold for $t \quad 2\ 3$. Hence, we can conclude that Algorithm 4 is feasible for all time.

Then we show that the inequality (6.20) holds. It follows from (6.13) that the following inequality holds at each sampling time.

$$
\begin{aligned}
x(t \quad 1)\ P(t)x(t \quad 1) \quad \tilde{x}(t)\ P(t)\tilde{x}(t) \\
(t)\ {}^{2}_{2} \quad \frac{1}{\hat{}(t)}\ z(t)\ {}^{2}_{2}
\end{aligned}
\tag{6.21}
$$

Hence, from eq. (6.21), we have

$$
\begin{aligned}
x(t \quad 1)\ P(t)x(t \quad 1) \\
\overset{t}{\underset{i\ 1}{}}[x(i)\ P(i \quad 1)x(i) \quad \tilde{x}(i)\ P(i)\tilde{x}(i)] \\
x(0)\ P(0)x(0) \\
\overset{t}{\underset{i\ 0}{}} \frac{1}{\hat{}(i)}\ z(i)\ {}^{2}_{2} \qquad (i)\ {}^{2}_{2}
\end{aligned}
\tag{6.22}
$$

Let us consider the second term of the left-hand side of the inequality (6.22). From the equality $p(t) \quad p(t \quad 1) \quad [x(t)\ P(t \quad 1)x(t) \quad \tilde{x}(t)\ P(t)\tilde{x}(t)]$ at Step 4, the following relation holds.

$$
\begin{aligned}
p(0) \quad p(t \quad 1) \\
\overset{t\ 1}{\underset{i\ 1}{}}[x(i)\ P(i \quad 1)x(i) \quad \tilde{x}(i)\ P(i)\tilde{x}(i)]
\end{aligned}
\tag{6.23}
$$

Firstly, we suppose that the optimization problem at Step 3 is feasible at time t. Then, from (6.19), we have

$$
\begin{aligned}
p(0) \quad p(t \quad 1) \quad x(t)\ P(t \quad 1)x(t) \\
\tilde{x}(t)\ P(t)\tilde{x}(t) \quad 0
\end{aligned}
\tag{6.24}
$$

From (6.23) and (6.24), the following inequality holds.

$$
\overset{t}{\underset{i\ 1}{}}[x(i)\ P(i \quad 1)x(i) \quad \tilde{x}(i)\ P(i)\tilde{x}(i)] \quad 0
\tag{6.25}
$$

It should be noted that the above inequality holds at all times when the optimization problem at Step 3 is feasible. Then, we suppose that the optimization problem at Step 3 is infeasible at time t. In this case, the equality

$$x(t)\ P(t\quad 1)x(t)\quad \tilde{x}(t)\ P(t)\tilde{x}(t)\quad 0 \tag{6.26}$$

holds, since $P(t)$ and $\tilde{x}(t)$ are chosen as $P(t)\quad P(t\quad 1)$ and $\tilde{x}(t)\quad x(t)$ at that time. Hence, from the above discussion, we can conclude that the inequality (6.25) holds for all time.

Hence, from (6.22), (6.25) and $x(t\quad 1)\ P(t)x(t\quad 1)\quad 0$, we have

$$\overset{t}{\underset{i\ 1}{}}\ \frac{1}{\hat{}(i)}\, z(i)\ ^{2}_{2}\qquad (i)\ ^{2}_{2}\qquad x(0)\ P(0)x(0) \tag{6.27}$$

The above inequality holds for t . Hence, we can conclude that the inequality (6.20) holds. **Q.E.D.**

Remark 17 *In this remark, we provide comments on the convergence property of the state x when Algorithm 4 is carried out. It has been shown in Theorem 7 that the inequality (6.20) holds. This implies that $z(t)\quad 0, (t\quad)$ holds. Hence, if the matrix $C\quad D_1F$ is full column rank, the state x also converges to zero since the equality $z\quad (C\quad D_1F)x\quad D_2$ holds. It would be di cult to make $C\quad D_1F$ be a full column rank matrix in general since the matrix F is time-varying. However, in the case of $D_1\quad 0$, it would be possible to make the matrix $C\quad D_1F$ be a constant full column rank matrix by adding extra outputs to z.*

Remark 18 *A moving horizon H control problem for constrained linear systems has been studied in Reference [5]. In the method of Reference [5], the state-feedback gain is updated online by solving an LMI optimization problem. It has been shown in Reference [5] that the dissipation inequality holds when the dissipation constraint introduced in Reference [5] is added to the optimization problem. Algorithm 4 has been developed based on the idea of Reference [5]. The relationship between the proposed method and the method in Reference [5] is explained as follows.*

> *The inequality (6.19) corresponds to the dissipation constraint of Reference [5]. More specifically, the inequality (6.19) is equivalent to the dissipation constraint of Reference [5] when $\tilde{x}$ is replaced with $x(t)$. It should be noted that, by this modification, the dissipation inequality still holds even when the resets of the controller state occur.*

In the method of Reference [5], the Lyapunov matrix Q and the feedback gain matrix Y (which correspond to Q( ) and Y( ) in this chapter) are updated online by solving an LMI optimization problem. In contrast, in the proposed method, the decision variables for online computation are ˜ and $\tilde{x}_c$ *only. This enables us to solve the optimization very e ciently.*

Remark 19 *In Reference [20], we have proposed a gain-scheduled control law with state resets for attenuating* l_2 *disturbances. In this approach, a control law with a scheduling parameter is designed o ine based on a parameter dependent Lyapunov function, and the scheduling parameter and the controller state are updated online so that the disturbance performance is improved. The control method of Reference [20] requires smaller computation time as compared with the proposed method. However, the control performance achieved by Reference [20] depends on the choice of the feedback gain designed o ine. Hence, careful consideration is required in designing the feedback gain. Also, the structure of the feedback gain is restrictive as compared with that of the proposed control law.*

Remark 20 *The proposed control method can be extended to observer-based output feedback control by applying a similar procedure as in Reference [37].*

Remark 21 *The proposed control method could be applied to systems with a hierarchical structure[33, 15]. For example, in Reference [33], a disturbance attenuation control problem for a ship by means of an active mass damper has been studied. In the literature, a position tracking controller for a movable mass is designed at the first stage. At the second stage, a disturbance attenuation controller is designed in which the reference signal for the position control system of the movable mass is used as the control input. This control problem could be handled within the proposed control framework by regarding the state of the position tracking controller for a movable mass and the position reference signal as* x_c *and* u*, respectively. Also, the constraints on the movable area of the mass and the actual control signal calculated by the position servo controller could be rewritten in the form of (6.3). It should be noted that the actual control signal is not always equivalent to the signal* u*.*

Remark 22 *In this remark, we explain the relationship between the proposed method and the existing MPC methods that can be used in the presence of disturbances. In References*

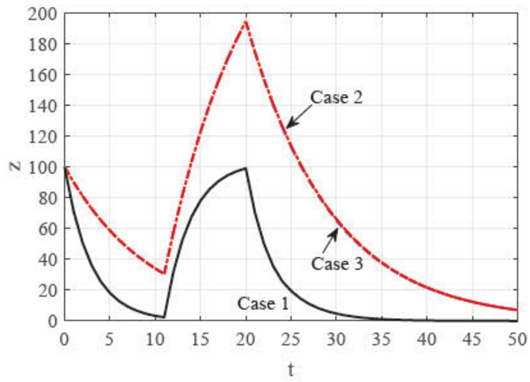

Figure 6.1: Controlled output z (solid: Case 1, dash-dot: Case 2, dashed: Case 3)

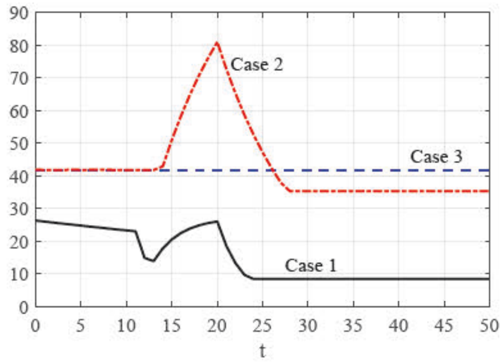

Figure 6.2: l_2 gain (solid: Case 1, dash-dot: Case 2, dashed: Case 3)

[36, 26], MPC algorithms that ensure feasibility and stability in the presence of persistent bounded disturbances have been developed. Also, in References [21], an MPC algorithm that rejects the e ects of deterministic disturbances has been proposed. In these methods, an open-loop optimal control sequence is determined at each sampling time so as to optimize disturbance attenuation performance. On the other hand, in References [31, 5, 14, 7], several MPC algorithms that optimize the l_2 *performance in the presence of* l_2 *bounded disturbances. Unlike the MPC methods in References [36, 26, 21], the variable-gain feedback control law described by* $u(t)$ $F(t)x(t)$ *is used in these literatures. As far as we know, the MPC methods for attenuating* l_2 *bounded disturbances adopt the control law of this structure. This structural constraint on the controller might limit the control performance. In this chapter, though the control law with the same structure is basically adopted, the controller state* x_c *is utilized as the extra degree of freedom to improve control performance. The control performance could be further improved by incorporating the open-loop optimal control*

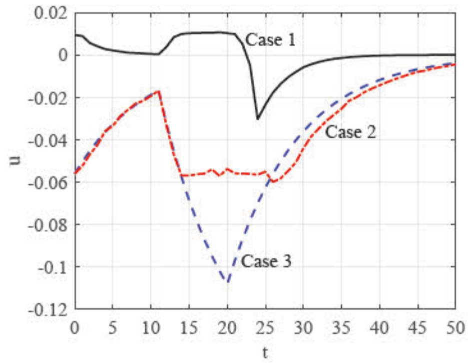

Figure 6.3: Control input u (solid: Case 1, dash-dot: Case 2, dashed: Case 3)

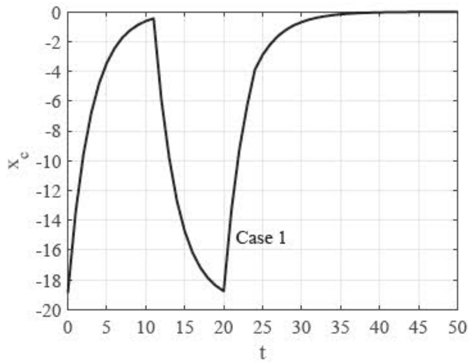

Figure 6.4: State x_c (Case 1)

sequence into the control algorithm as in References [36, 26, 21]. The development of such control method is a future research topic.

Remark 23 *When the value of the disturbance can be measured or estimated at each sampling time, it would possible to reduce the value of   online. In this case, since the constraint (6.6) is relaxed when the value of   decreases, the control performance could be further improved.*

6.4 Numerical Example

Consider the system (6.1) with the matrices

$$A \quad \begin{matrix} 0\,9 & 1 \\ 0 & 0\,8 \end{matrix} \qquad B_1 \quad \begin{matrix} 0 \\ 1 \end{matrix} \qquad B_2 \quad \begin{matrix} 1 \\ 0 \end{matrix}$$

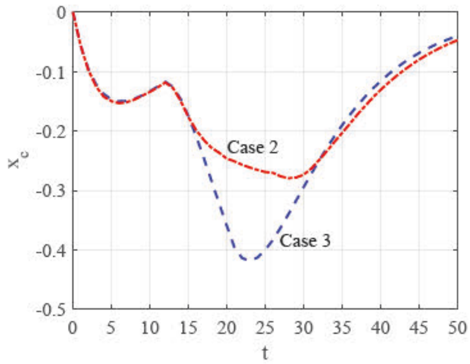

Figure 6.5: State x_c (dash-dot: Case 2, dashed: Case 3)

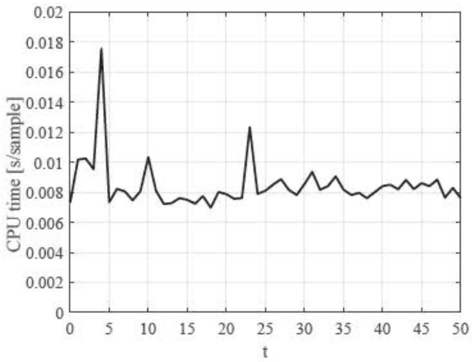

Figure 6.6: CPU time in Case 1 [s sample]

and C [1 0] D_1 0 D_2 0. We assume that x $[x_p\ x_c]$, where $x_p\ x_c$. Also, we assume that the constraint u 1 is imposed on the control signal. In the following numerical simulations, we apply the disturbance given by

$$(t) \quad \begin{matrix} 30 & 11 \quad t \quad 19 \\ 0 & \text{otherwise} \end{matrix} \tag{6.28}$$

to the system. Also, the initial state of the plant is $x_p(0)$ 100. We have applied the following three types of control laws to the above system.

Case 1: Algorithm 4

Case 2: Algorithm 4 without state resets

Case 3: State-feedback control with a constant feedback gain

The control law in Case 1 is the proposed control law. The control law in Case 2 corresponds to that in Reference [5]. The control law in Case 3 is designed by solving $\min_{YQ}$ subject to (6.5), (6.6) and $x(0)$ $(P \quad)$ with $x_c(0)$ 0. In all cases, we have chosen 30 and 8100.

Figs. 6.1–6.5 show the simulation results. The solid line shows the simulation result in Case 1. The dash-dot line shows the simulation result in Case 2. The dashed line shows the simulation result in Case 3. Fig. 6.1 shows that the magnitude of the controlled output z in Case 1 is e ectively suppressed as compared with the other cases. Fig. 6.2 shows the value of the l_2 gain in Case 1 is smaller as compared with the other cases. Figs. 6.4 and 6.5 show that the response of x_c in Case 1 is significantly di erent from those of the other cases as the result of the state resets.

The control algorithms were implemented on the digital computer (Intel Xeon 3.3GHz, 8GB RAM), using MATLAB. The optimization problem in Algorithm 4 was solved by the interior point method implemented on Robust Control Toolbox. The maximum computation time required to solve the optimization problem was 18 ms (see Fig. 6.6).

6.5 Conclusions

In this chapter, we have proposed a control law for constrained linear systems in the presence l_2 disturbance. In the proposed approach, the feedback gain and the controller state are updated online so that the l_2 gain is minimized under the input constraint. The problem of determining the feedback gain and the controller state is formulated as an LMI optimization problem. It has been shown that the system with the proposed control algorithm is finite-gain l_2 stable. To implement the proposed control algorithm, the LMI optimization problem needs to be solved at each sampling time. Hence, the development of an e cient numerical optimization algorithm to solve the optimization problem is a future research topic. Also, the application of the proposed method to practical control problems is a future research topic.

Chapter 7

Online Optimization of Disturbance Attenuation Performance of Input Constrained Systems by Gain-scheduled Control with State Resets

In Chapter 6, we have proposed a l_2-disturbance attenuation control method for constrained control systems. To carry out the control algorithm, we need to solve an LMI optimization problem at each sampling time. In this chapter, we show a method for reducing the computation time required to solve the optimization problem. In this chapter, a control law with a scheduling parameter is designed o -line, and the scheduling parameter and the controller state is updated on-line so that the l_2-gain of the system is minimized. The problem of determining the scheduling parameter and the controller state is formulated as an optimization problem with constraints described by linear matrix inequalities. Then the optimization problem is reduced to a convex optimization problem with respect to a scalar variable. We show that both feasibility of the control algorithm and dissipation inequality are guaranteed for all times.

7.1 Introduction

In Chapter 6, we have proposed a l_2-disturbance attenuation control method for constrained control systems. The control method is based on the mode predictive control. In the control method in Chapter 6, the feedback gain and the value of the controller state are updated on-line so that the l_2-disturbance attenuation performance is improved. To carry out the control algorithm, we need to solve an LMI optimization problem at each sampling time. Since the LMI optimization problem is a convex optimization problem, it can be solved e ciently by an interior point method. However, as shown in the numerical example in Chapter 6, it takes around 10ms to compute the control signal even when the control algorithm is applied to the simple second order example.

In this chapter, we show a method for reducing the computation time required to solve the optimization problem in Chapter 6. In this chapter, a control law with a scheduling parameter is designed o -line, and the scheduling parameter and the controller state is updated on-line so that the l_2-gain of the system is minimized. The problem of determining the scheduling parameter and the controller state is formulated as an optimization problem with constraints described by linear matrix inequalities. Then the optimization problem is reduced to a convex optimization problem with respect to a scalar variable. Further, we show a method for solving the optimization problem based on the bisection method. We show that both feasibility of the control algorithm and dissipation inequality are guaranteed for all times. The e ectiveness of the method is shown through a numerical example.

Notations: For a vector u ^{m}, we define the multivariable saturation function as (u) : $(\ (u_1)\quad (u_m))^T$, where

$$(u_i) : \quad \begin{matrix} \mathrm{sgn}(u_i) & u_i & 1 \\ u_i & u_i & 1 \end{matrix}$$

For a vector ^{n}, we denote its Euclidean norm as $_2$: $(\ ^T\ )^{1\ 2}$. For a signal (k) defined on $[0\quad)$, we define its l_2 norm as $_{l_2}$: $(\ _{k\ 0}\ (k)^T\ (k))^{1\ 2}$. For a positive definite matrix P $^{n\ n}$, we denote $(P\ )$: x $^n : x^T P x$. For a matrix H $^{m\ n}$, we denote the ith row of H as $H^{(i)}$. Furthermore, we define (H) : x ^{n} : $H^{(l)}x$ $1\ l\ 1$ m . For integers k_1 and k_2 such that k_1 k_2, we define $I[k_1\ k_2]$: $[k_1\ k_1\ 1\ \ k_2]$. Let be the set of m m diagonal matrices whose diagonal elements are either 1 or 0. We suppose that each element of is labeled as $\mathbf{E}_j$ j 1 2 2^m, and denote $\mathbf{E}_j$: I $\mathbf{E}_j$.

7.2 Problem Formulation and Preliminaries

Let us consider the system described by

$$x(t\quad 1)\qquad Ax(t)\quad B_1\ (u(t))\quad B_2\ (t) \tag{7.1}$$

$$z(t)\qquad Cx(t)\quad D_1\ (u(t))\quad D_2\ (t) \tag{7.2}$$

where x ^{n} is a state vector, u ^{m} is an input vector and z ^{q} is an output vector. ^{p} is a disturbance that satisfies : ^{p} : $_{l_2}$. We assume that the above system consists of a plant and dynamics of a controller. Further, we assume that the structure of the state x is x $[x_p^T \; x_c^T]^T$, where x_p ^{n_p} is the plant state and x_c ^{n_c} is the controller state.

The purpose of this chapter is to design a feedback control law

$$u(t) \quad F(t)x(t) \tag{7.3}$$

that minimizes the l_2-gain from to z of the system (7.1)-(7.3) at each sampling time and achieves large region of attraction.

In the following, we show a design method of a control law which has a structure that a high-gain control law and a low-gain control law are interpolated by a single scheduling parameter. Then, in Section 7.3, we propose a control algorithm for the obtained control law and show that the system is finite l_2-gain stable.

We introduce the following lemma.

Lemma 5 *[11] Let u ^{m}. Suppose that $_j$ 1 j $[1 \; m]$, then (u) can be represented as (u) $^{2^m}_{j\;1}$ $_j(\mathbf{E}_j u \quad \mathbf{E}_j^-)$, where 0 $_j$ 1 $^{2^m}_{j\;1}$ $_j$ 1.*

We initially introduce the following theorem which is used to design a feedback controller with a scheduling parameter.

Theorem 8 *Consider the system (7.1), (7.2). For given positive scalars , $\hat{}_i$ $(i \quad 0 \; 1)$ such that $\hat{}_0$ $\hat{}_1$, assume that there exist matrices $Q_i \; Y_i \; Z_i$ $(i \quad 0 \; 1)$ that satisfy*

$$\begin{matrix} Q_i & & & \\ 0 & I & & \\ CQ_i \quad D_1(\mathbf{E}_j Y_i \quad \mathbf{E}_j^- Z_i) & D_2 & \hat{}_i I & \\ AQ_i \quad B_1(\mathbf{E}_j Y_i \quad \mathbf{E}_j^- Z_i) & B_2 & 0 & Q_i \end{matrix} \quad 0$$

$$i \quad I[0 \; 1] \quad j \quad I[1 \; 2^m] \tag{7.4}$$

$$\begin{matrix} Q_i & \\ Z_i^{(l)} & \frac{1}{} \end{matrix} \quad 0 \quad i \quad I[0 \; 1] \quad l \quad I[1 \; m] \tag{7.5}$$

$$Q_0 \quad Q_1 \tag{7.6}$$

where the symbol stands for symmetric block in matrix inequalities. Further, for some constant $[0 \; 1]$, we suppose that $x(0)$ $(P(\;) \quad)$ where $P(\;)$: $Q(\;)^{1}$ $Q(\;)$:

$(1 \quad)Q_0 \quad Q_1$. *Then, by applying the feedback control law*

$$u(t) \quad F(\ )x(t) \tag{7.7}$$

where $F(\ ) \quad Y(\ )Q(\ )^{\ 1}$ *and* $Y(\ ) : \ (1 \quad)Y_0 \quad Y_1$ *to the system (7.1), (7.2), the relations* $x(t) \quad (P(\ ) \qquad) \ t \quad 0$ *and the following inequality holds.*

$$z_{\ l_2} \quad (\ ) \qquad _{l_2} \tag{7.8}$$

where $(\ ) : \ \hat{}(\ )^{1 \ 2} \ \hat{}(\ ) : \ (1 \quad)\hat{}_0 \quad \hat{}_1$.

Proof) From Lemma 5, while $x \qquad (H(\ ))$, the saturation nonlinearity $(F(\ )x)$ can be represented as $(F(\ )x) \quad \sum_{j \ 1}^{2^m} \ _j \ \mathbf{E}_j F(\ ) \quad \mathbf{E}_j^{-} H(\ ) \ x$, where $H(\ ) \quad Z(\ )Q(\ )^{\ 1} \ Z(\ ) :$ $(1 \quad)Z_0 \quad Z_1$. Hence, while $x(t) \qquad (H(\ ))$, the closed-loop system (7.1), (7.2) and (7.7) can be rewritten as

$$x(t \quad 1) \qquad (\ (t))x(t) \quad B_2 \ \ (t) \tag{7.9}$$

$$z(t) \qquad (\ (t))x(t) \quad D_2 \ \ (t) \tag{7.10}$$

where $(\ (t)) : \ \sum_{j \ 1}^{2^m} \ _j(t) \ _j \quad _j : \ A \ \ B_1 \ \mathbf{E}_j F(\ ) \ \ \mathbf{E}_j^{-} H(\ )$, $(\ (t)) : \ \sum_{j \ 1}^{2^m} \ _j(t) \ _j \quad _j :$ $C \quad D_1 \ \mathbf{E}_j F(\ ) \quad \mathbf{E}_j^{-} H(\ )$.

From (7.5), we can show that $H(\ )^{(l)T} H(\ )^{(l)} \quad \frac{1}{\ } P(\ ) \quad l \quad I[1 \ m]$. This implies that $(P(\ ) \qquad) \qquad (H(\ ))$.

Also, from (7.4), we can show that

$$V(x(t \quad 1)) \quad V(x(t)) \qquad (t) \ _2^2 \quad \frac{1}{\hat{}(\ )} \ z(t) \ _2^2 \tag{7.11}$$

where $V(x) : \quad x^T P(\ )x$. From (7.11) and $x(0) \qquad (P(\ ) \quad)$, we have $V(x(t)) \qquad V(x(0))$ $\ _{i \ 0}^{t \ 1} \quad (i) \ _2^2 \qquad\qquad t \quad 0$. This implies that $x(t) \qquad (P(\ ) \qquad) \quad t \quad 0$. Hence, we can conclude that the relation $x(t) \qquad (H(\ )) \quad t \quad 0$ holds. Furthermore, from Eq.(7.11), we obtain $\ _{i \ 0}^{t} \ z(i) \ _2^2 \quad \hat{}(\ ) \quad _{i \ 0}^{t} \quad (i) \ _2^2 \quad V(x(0))$. Therefore, we can conclude that Eq.(7.8) holds. **Q.E.D.**

In this chapter, based on Theorem 8, we design a gain $F(1) \qquad Y_1 Q_1^{\ 1}$ which makes $(P(1) \qquad)$ large and a gain $F(0) \quad Y_0 Q_0^{\ 1}$ which achieves small l_2-gain $\ (0)$ in $(P(0) \qquad)$. Then we construct the control law (7.7) by interpolating the obtained gains.

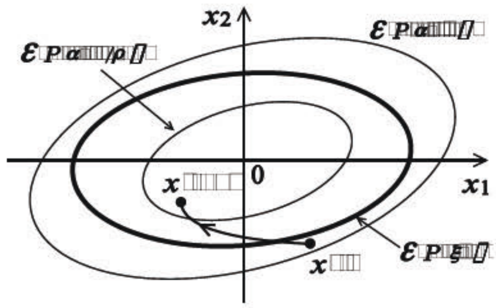

Figure 7.1: Invariant set

7.3 Main Results

In this section, we show a control algorithm in which the parameter of the control law is actively changed online so that the disturbance attenuation performance is enhanced. In addition to , the controller state x_c can be used as a tuning parameter to improve control performance. Hence, in this section, we show a control algorithm that enhance disturbance attenuation performance by suitably determining and x_c online. The function ( ) takes a smaller value when a smaller value is chosen as . Hence, it can be expected that the disturbance attenuation performance is improved when the smaller value is chosen as at each sampling time. Also, from Theorem 8, for a constant , a set of initial state in which the l_2-gain performance is guaranteed under the control law (7.7) is given by $(P( )\ )$. With these in mind, we propose the following gain-scheduling control algorithm.

Algorithm 5

Step 0: Set t 0.

Step 1: Measure $x_p(t)$. If t 0, go to Step 2. Otherwise, go to Step 3.

Step 2: Solve min$_{\tilde{}\ [0\ 1]\ \tilde{x}_c}$ $\tilde{}$, s.t.

$$\begin{matrix} x_p(t) \\ \tilde{x}_c \end{matrix} \quad Q(\tilde{}) \qquad 0 \qquad (7.12)$$

Set (t) $\tilde{}$, $x_c(t)$ $\tilde{x}_c$ and $\tilde{x}(t)$ $[x_p(t)^T\ \tilde{x}_c^T]^T$, $p(0)$ $\tilde{x}(0)^T P(\ (0))\tilde{x}(0)$ and go to Step 5.

Step 3: Solve $\min_{\tilde{\ } [0\ 1]\ \tilde{x}_c} \tilde{\ }$, s.t. (7.12) and

$$\begin{matrix} (t) \\ x_p(t) \\ \tilde{x}_c \end{matrix} \quad Q(\tilde{\ }) \qquad 0 \tag{7.13}$$

where $(t) :\ \ p(0) \ \ p(t \ \ 1) \ \ x(t)^T P(\ (t \ \ 1))x(t)$. Set $(t) \ \ \tilde{\ }$, $x_c(t) \ \ \tilde{x}_c$ and $\tilde{x}(t) \ \ [x_p(t)^T \ \tilde{x}_c^T]^T$. If the above optimization problem is infeasible, set $(t) \ \ (t \ \ 1)$ and $\tilde{x}(t) \ \ x(t)$.

Step 4: Update $p(t)$ by $p(t) \ \ p(t \ \ 1) \ \ [x(t)^T P(\ (t \ \ 1))x(t) \ \ \tilde{x}(t)^T P(\ (t))\tilde{x}(t)]$.

Step 5: Apply $u(t) \ \ F(\ (t))\tilde{x}(t)$ to the plant (7.1), (7.2).

Step 6: $t \ \ t \ \ 1$ and go to Step 1.

In the above algorithm, the constraint (7.13) is omitted at time $t \ \ 0$. Also, Step 3 is skipped at time $t \ \ 0$. It should be noted that the controller state x_c is reset at Step 2 so that the scheduling parameter is minimized when the optimization problem is feasible. The optimization problem at Step 2 is an LMI optimization problem with respect to the variables $\tilde{\ }$ and $\tilde{x}_c$. Hence, the problem is a convex optimization problem. A method for solving the optimization problem is explained in Remark 25.

As for Algorithm 5, the following results hold.

Theorem 9 *Consider the system (7.1), (7.2). Assume that there exist matrices Q_i Y_i Z_i which satisfy the matrix inequality conditions (7.4)–(7.6). Further, assume that $x(0)$ $(P(1) \ )$. Then, by applying Algorithm 5 to the system (7.1), (7.2), the feasibility of the algorithm holds for all time. Further, the following inequality holds.*

$$z_{\ l_2} \qquad (1)(\qquad _{l_2} \qquad \bar{\ }) \tag{7.14}$$

Proof of Theorem 9) Firstly, we show that the feasibility holds for all time. From the assumption, $x(0) \qquad (P(\ (0)) \ \ )$ holds. At time $t \ \ 0$, the parameters (0) and $\tilde{x}_c$ are determined by solving the optimization problem at Step 2, and the control signal is calculated at Step 4 and applied to the plant. At time $t \ \ 1$, if the optimization problem at Step 2 is feasible, the parameters (1) and $\tilde{x}_c$ are obtained by solving the optimization problem. On the other hand, if the optimization problem is infeasible, (1) and $\tilde{x}_c$ are chosen as $(1) \qquad (0)$

and $\tilde{x}_c \quad x_c(1)$. Hence, it is possible to determine the parameters at time $t \quad 1$. The same arguments hold for $t \quad 2\ 3$. Hence, we can conclude that Algorithm 5 is feasible for all time.

Then we show that the inequality (7.14) holds. It follows from (7.11) that the following inequality holds at each sampling time.

$$x(t \quad 1)^T P(\ (t))x(t \quad 1) \quad \tilde{x}(t)^T P(\ (t))\tilde{x}(t) \quad (t) \ {}^2_2 \quad \frac{1}{\hat{}(\ (t))} \ z(t) \ {}^2_2 \tag{7.15}$$

Hence, from eq. (7.15), we have

$$x(t \quad 1)^T P(\ (t))x(t \quad 1) \quad {}^t_{i\ 1} [x(i)^T P(\ (i \quad 1))x(i) \quad \tilde{x}(i)^T P(\ (i))\tilde{x}(i)] \quad \tilde{x}(0)^T P(\ (0))\tilde{x}(0) \quad {}^t_{i\ 0} \frac{1}{\hat{}(\ (i))} \ z(i) \ {}^2_2 \quad (i) \ {}^2_2 \tag{7.16}$$

Let us consider the second term of the left-hand side of the inequality (7.16). From the equality $p(t) \quad p(t \quad 1) \quad [x(t)^T P(\ (t \quad 1))x(t) \quad \tilde{x}(t)^T P(\ (t))\tilde{x}(t)]$ at Step 3, the relation $p(0) \quad p(t \quad 1) \quad {}^{t\ 1}_{i\ 1}[x(i)^T P(\ (i \quad 1))x(i) \quad \tilde{x}(i)^T P(\ (i))\tilde{x}(i)]$ holds. Firstly, we suppose that the optimization problem at Step 2 is feasible at time t. Then, from (7.13), we have $p(0) \quad p(t \quad 1) \quad x(t)^T P(\ (t \quad 1))x(t) \quad \tilde{x}(t)^T P(\ (t))\tilde{x}(t) \quad 0$. Hence, the following inequality holds.

$${}^t_{i\ 1} [x(i)^T P(\ (i \quad 1))x(i) \quad \tilde{x}(i)^T P(\ (i))\tilde{x}(i)] \quad 0 \tag{7.17}$$

It should be noted that the above inequality holds at all times when the optimization problem at Step 3 is feasible. Then, we suppose that the optimization problem at Step 2 is infeasible at time t. In this case, the equality $x(t)^T P(\ (t \quad 1))x(t) \quad \tilde{x}(t)^T P(\ (t))\tilde{x}(t) \quad 0$ holds, since (t) is chosen as $(t) \quad (t \quad 1)$ at that time. Hence, from the above discussion, we can conclude that the inequality (7.17) holds for all time.

Hence, from (7.16), (7.17) and $x(t \ \ 1)^T P(\ (t))x(t \ \ 1) \quad 0$, we have ${}^t_{i\ 1} \frac{1}{\hat{}(\ (i))} \ z(i) \ {}^2_2 \quad (i) \ {}^2_2$ $\tilde{x}(0)^T P(\ (0))\tilde{x}(0)$. This inequality holds for t . Also, the relation $\hat{}(\ ) \quad \hat{}_1 \quad [0\ 1]$ holds. Hence, we can conclude that the inequality (7.14) holds. **Q.E.D.**

Remark 24 *A moving horizon H control problem for constrained linear systems has been studied in [5]. In the method of [5], the state-feedback gain is updated on-line by solving* *an LMI optimization problem. It has been shown in [5] that the dissipation inequality holds when the dissipation constraint introduced in [5] is added to the optimization problem. Algorithm 5 has been developed based on the idea of [5]. The relationship between the proposed method and the method in [5] is explained as follows.*

The inequality (7.13) corresponds to the dissipation constraint of [5]. More specifically, the inequality (7.13) is equivalent to the dissipation constraint of [5] when $\tilde{x}$ is replaced with $x(t)$. It should be noted that, by this modification, the dissipation inequality still holds even when the resets of the controller state occur.

In the method of [5], the Lyapunov matrix Q and the feedback-gain matrix Y (which correspond to $Q(\ )$ and $Y(\ )$ in this chapter) are updated on-line by solving an LMI optimization problem. In contrast, in the proposed method, the decision variables for on-line computation are $\tilde{}$ and $\tilde{x}_c$ only. This enables us to solve the optimization very e ciently.

Remark 25 *In this remark, we explain a method for solving the optimization problems at Step 2 in Algorithm 5. It is clear that the optimization problem at Step 2 is equivalent to*

$$\min_{\tilde{}\ \tilde{x}_c} \tilde{} \quad \text{s t} \quad \begin{matrix} & \tilde{x}(t)^T \\ & Q(\tilde{}) \end{matrix} \quad 0 \tag{7.18}$$

where $\ :\ \min\ \ (t)$. *A method for solving the above optimization problem has been studied in [49]. The above optimization problem can be reduced to a convex optimization problem with respect to a scalar parameter. A simple method for solving the optimization problem based on the bisection method will be shown in Appendix.*

7.4 Numerical Example

Consider the system (7.1) with the matrices

$$A \quad \begin{matrix} 0\,9 & 1 \\ 0 & 0\,8 \end{matrix} \quad B_1 \quad \begin{matrix} 0 \\ 1 \end{matrix} \quad B_2 \quad \begin{matrix} 1 \\ 0 \end{matrix}$$

and $C \quad [1\ 0]\ \ D_1 \quad 0\ \ D_2 \quad 0$. For this system, we solved a feasibility problem with LMI constraints in Theorem 8 with $\hat{}_1 \quad 2000 \quad \hat{}_0 \quad 72 \quad\quad 30 \quad\quad 8100$ and obtained $Q_0\ Q_1\ Y_0$ and Y_1.

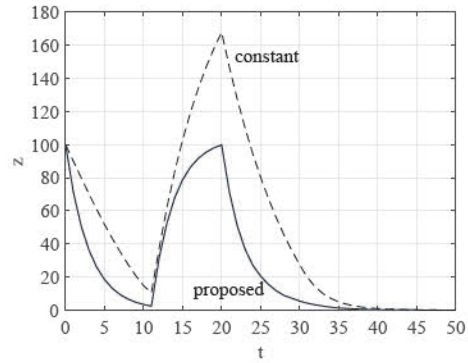

Figure 7.2: Controlled output z (solid: proposed, dashed: constant gain feedback)

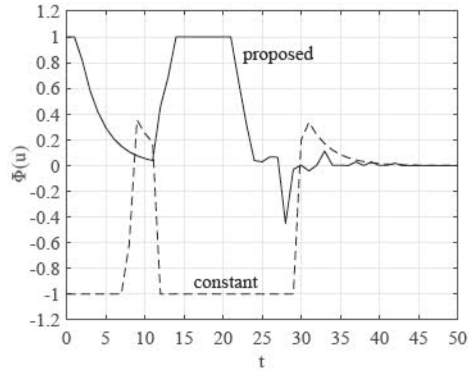

Figure 7.3: Control input (u) (solid: proposed, dashed: constant gain feedback)

Figs. 7.2–7.5 show the simulation results for $x_p(0)$ 100 and

$$(t) \quad \begin{array}{ll} 30 & 11 \quad t \quad 19 \\ 0 & \text{otherwise} \end{array} \tag{7.19}$$

The solid line shows the simulation result with the proposed control law. The dashed line shows the simulation result with the constant gain feedback control law $u(t)$ $F(1)x(t)$. It can be seen from these figures that the controlled output z converges to zero in both cases. However, it can be seen that, when the proposed control law is utilized, $z(t)$ converges to zero quickly as compared to the case of the constant gain feedback control law.

The control algorithms were implemented on the digital computer (Intel Xeon 3.3GHz, 8GB RAM), using MATLAB. The optimization problem in Algorithm 5 was solved by the bisection method explained in Remark 25. The maximum computation time required to solve the optimization problem was 0.35 ms (see Fig. 7.6).

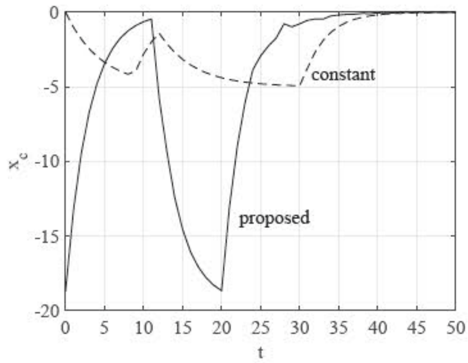

Figure 7.4: Controller state x_c (solid: proposed, dashed: constant gain feedback)

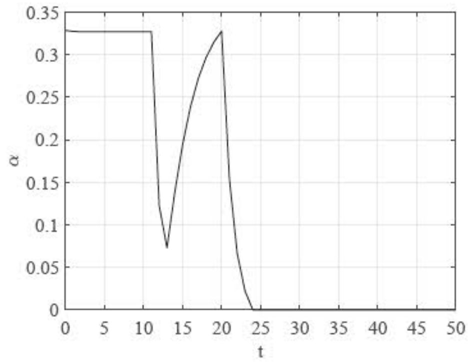

Figure 7.5: Scheduling parameter

7.5 Conclusions

In this chapter, we have proposed a gain-scheduled control law for input constrained discrete-time systems in the presence l_2-disturbance. In the proposed approach, the scheduling parameter and the controller state are updated online so that the l_2-gain is minimized. The problem of determining the scheduling parameter and the controller state is formulated as an LMI optimization problem and is reduced to a convex optimization problem with a single variable. It has been shown that the system with the proposed control algorithm is finite-gain l_2 stable.

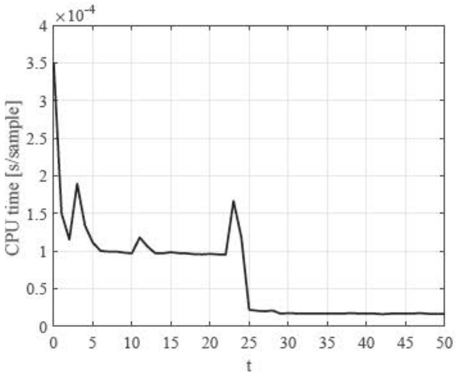

Figure 7.6: CPU time [s sample]

Chapter 8

Conclusions

In this dissertation, we have studied online optimization-based control methods for enhancing tracking and disturbance attenuation performance of constrained systems. The conclusions are summarized as follows.

In Chapter 3, a tracking control for a class of linear parameter varying systems with input constraints has been proposed. The proposed controller includes a single scheduling parameter, and the control performance and the size of the region of attraction can be tuned by the parameter. In the proposed control algorithm, the scheduling parameter and the state of the controller are determined on-line so that the tracking control performance is improved. It has been shown that the problem of computing the scheduling parameter and the state of the controller is reduced to a convex optimization problem with an LMI constraint. Further, it has been shown that both feasibility of the control algorithm and convergence of the tracking error are guaranteed when the reference signal is constant.

In Chapter 4, we have applied the control method in Chapter 3 to a torque control problem for a permanent magnet synchronous motor under input voltage limitation. The value of the system matrix of the PMSM changes depending on the variation of the rotor speed. Hence, it was required to design a control law that robustly achieves zero tracking error under the variation of the rotor speed and the input voltage limitation. It has been shown that, by using the proposed control method, the setpoint tracking is achievable under the variation of the rotor speed and the input voltage limitation.

In Chapter 5, the control method in Chapter 4 has been extended so that the output torque of the PMSM tracks time-varying reference signal under input voltage limitation. To this end, we have combined the controller in Chapter 4 with a reference governor. In the proposed control algorithm, when the tracking error is large, the reference governor generates a modified reference signal that can be tracked under the input voltage limitation. The main feature of the proposed method is that the controller state is reset at each sampling time so that the tracking control performance is improved. The e ectiveness of the method has been shown by a numerical example and an experimental result.

In Chapter 6, we have studied a disturbance attenuation problem for constrained control systems. In the proposed approach, the feedback gain and the controller state are updated online so that the l_2 gain is minimized under input and state constraints. The problem of determining the feedback gain and the controller state is formulated as an LMI optimization problem. It has been shown that the system with the proposed control algorithm is finite gain l_2 stable. To carry out the control algorithm, the LMI optimization problem needs to be solved at each sampling time.

In Chapter 7, we have shown a method for reducing computation time required to carry out the control algorithm in Chapter 6. The control law of this chapter has been designed so that the scheduling parameter and the controller state are updated online so that the l_2-gain is minimized. The problem of determining the scheduling parameter and the controller state has been formulated as an LMI optimization problem and has been reduced to a convex optimization problem with a single variable. It has been shown that the system with the proposed control algorithm is finite-gain l_2 stable.

Bibliography

[1] Balas, G., Chiang, R., Packard, A. and Safonov, M., Robust control toolbox 3, user's guide; The MathWorks, Inc., Natick, MA., 2008.

[2] Beker, O., Hollot, C. V. and Chait, Y., Plant with integrator: An example of reset control overcoming limitations of linear feedback, *IEEE Transactions on Automat. Control*, Vol. 46, No. 11, pp. 1797-1799, 2001.

[3] Boyd, S., Ghaoui, L. E., Feron, E. and Balakrishnan, V., Linear Matrix Inequalities in System and Control Theory, SIAM: Philadelphia, PA, 1994.

[4] Chai, S., Wang, L. and Rogers, E., Model predictive control of a permanent magnet synchronous motor with experimental validation, *Control Engineering Practice*, Vol. 21, No. 11, pp. 1584-1593, 2013.

[5] Hong, C. and Scherer, C. W., Moving horizon H control with performance adaptation for constrained linear systems, *Automatica*, Vol. 42, No. 6, pp. 1033-1040, 2006.

[6] Clegg, J. C., A nonlinear integrator for servomechanisms, *Trans. AIEE, Part II, Appl. Ind.*, Vol. 77, pp. 41-42, 1958.

[7] Sindareh, Esfahani P. and Pieper, J. K., H model predictive control for constrained discrete-time piecewise a ne systems, *International Journal of Robust and Nonlinear Control*, Vol. 28, pp. 1973-1995, 2018.

[8] Francis, B. A., The linear multivariable regulator problem, *SIAM Journal on Control and Optimization*, Vol. 15, pp.80-505, 1975.

[9] Gilbert, E. G., Kolmanovsky, I. and Tan, K. T., Nonlinear control of discrete-time linear systems with state and control constraints: a reference governor with global

convergence properties, *Proceedings of the IEEE Conference on Decision and Control*, pp.144-149, 1994.

[10] Hirata, K. and M. Fujita, M., A switching state feedback control of input constrained linear discrete-time systems, *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 11, No. 6, pp. 316-323, 1998.

[11] Hu, T. and Lin, Z., Control Systems with Actuator Saturation: Analysis and Design, Springer, 2001

[12] Hu, T., Lin, Z. and Chen, B.M., An analysis and design method for linear systems subject to actuator saturation and disturbance, *Automatica*, Vol. 38, pp. 351-359, 2002.

[13] Kadota, M., Doki, S. and Okuma, S., Application of model predictive control for current control system of permanent magnet synchronous motor, *Trans. of the Institute of Electrical Engineers of Japan. C*, Vol. 131, No. 4, pp. 860-869, 2011.

[14] Kim, K. B., Yoon, T.W. and Kwon, W. H., Stabilizing receding horizon H controls for linear discrete time-varying systems, *International Journal of Control*, Vol. 75, No. 18, pp. 1449-1456, 2002.

[15] Kinoshita, D., Yoshida, K. and Matsumoto, I., Stabilizing control for an inverted pendulum with restricted travel, *Transactions of the Society of Instrument and Control Engineers*, Vol. 53, No. 7, pp. 408-415, 2017.

[16] Kothare, M. V., Balakrishnan, V. and Morari, M., Robust constrained model predictive control using linear matrix inequalities, *Automatica*, Vol. 32, No. 10, pp. 1361-1379, 1996.

[17] Kothare, M. V., Balakrishnan, V. and Morari, M., Robust constrained model predictive control using linear matrix inequalities, *Automatica* , Vol. 39, pp. 837-846, 2013.

[18] Feng, L., Wang, J. and Poh, E., O -line formulation of robust model predictive control based on several Lyapunov functions, *IEEE Trans. on Control Systems Technology*, Vol. 3, pp. 1705-1710, 2004.

[19] Lin, Z., Global control of linear systems with saturating actuators, *Automatica*, Vol.34, No.7, pp.897-905, 1998.

[20] Khonjun, S. and Wada, N., Online optimization of disturbance attenuation performance of input constrained systems by gain-scheduled control with state resets, *Proceedings of SICE Annual Conference*, pp. 709-712, 2018.

[21] Maeder, U., Borrelli, F. and Morari, M., Linear o set-free model predictive control, *Automatica*, Vol. 45, No. 10, pp. 2214-2222, 2009.

[22] March, P. and Turner, M. C., Anti-windup compensator designs for nonsalient permanent-magnet synchronous motor speed regulators, *IEEE Transactions on Industry Applications*, Vol. 45, No. 5, pp. 1598–1609, 2009.

[23] Matsui, N., Kameda, K. and Takeshita, T., DSP-based software de-coupling current control of brushless motor, *IEEJ Transactions on Industry Applications*, Vol. 107-D, No. 2, pp. 215–222, 1987.

[24] Mantri, R., Saberi, A., Lin, Z. and Stoorvogel, A. A., Output regulation for linear discrete-time systems subject to input saturation, *International Journal of Robust and Nonlinear Control*, Vol. 7, pp. 1003-1021, 1997.

[25] Mayne, D. Q., Rawlings, J. B., Rao, C. V. and Scokaert, P. O. M., Constrained model predictive control: Stability and optimality, *Automatica*, Vol. 36, pp. 789-814, 2000.

[26] Mayne, D. Q., Raković, S. V., Findeisen, R. and Allgöwer, F., Robust output feedback model predictive control of constrained linear systems, *Automatica*, Vol. 42, No. 7, pp. 1217-1222, 2006.

[27] Megretski, A., L_2 BIBO output feedback stabilization with saturated control, *Proc. IFAC World Congress*, pp.435-440, 1996.

[28] Nakagawa, S., Yamaguchi, T., Numasato, H., Hosokawa, H. and Hirai, H., Improving the disturbance resistance of magnetic disk drives by using anti-windup and model following controls with initial value compensation, *JSME International Journal*, Series C, Vol. 43, No. 3, pp.618-624, 2000.

[29] Wagener, N., Cheng, C. Sacks,J. and Boots, B., An Online Learning Approach to Model Predictive Control, *Robotics: Science and Systems 2019 Freiburg im Breisgau 2019*, pp. 22-26, 2019.

[30] Okuyama, A. and Yamaguchi, T., Controller switching strategy for constrained systems and its application to hard disk drives, *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, Vol.1, No.4, pp.453-461, 2007.

[31] Orukpe, P. E., Jaimoukha, I. M. and El-Zobaidi, H. M., Model predictive control based on mixed H_2 H control approach, *Proceedings of American Control Conference*, pp.6147-6150, 2007.

[32] Pillay, P. and Krishnan, R., Modeling of permanent magnet motor drives, *IEEE Trans. Industrial Electronics*, Vol. 35, No. 4, pp. 537–541, 1988.

[33] Sasaki, H. and Watanabe, R., Active rolling control of ships-design of control system in consideration of movable range of mass–. *IEEJ Transactions on Industry Applications*, Vol. 120, No. 11, pp. 1382-1390, 2000.

[34] Satoh, A., State feedback synthesis of linear reset control with L_2 performance bound via lmi approach, *IFAC Proceedings Volumes*, Vol. 44, No. 1, pp. 5860-5865, 2011.

[35] Schuster, E., Walker, M. L., Humphreys, D. A., and Krstic, M., Plasma vertical stabilization with actuation constraints in the DIII-D tokamak, *Automatica*, Vol. 41, No. 7, pp. 1173-1179, 2005.

[36] Scokaert, P. O. M. and Mayne, D. Q., Min-max feedback model predictive control for constrained linear systems, *IEEE Transactions on Automatic Control*, Vol. 43, No. 8, pp. 1136-1142, 1998.

[37] Shamma, J. S., Anti-windup via constrained regulation with observers, *Systems and Control Letters*, Vol. 40, No. 4, pp. 261-268, 2000.

[38] Stoorvogel, A. A. and Saberi, A. (Editors), Special issues: Control problems with constraints, *International Journal of Robust and Nonlinear control*, Vol.9, No.10, 1999.

[39] Teel, A. A., Linear systems with input nonlinearities: Global stabilization by scheduling a family of -type controllers, *International Journal of Robust and Nonlinear control*, Vol.5, pp.399-411, 1995.

[40] Teel, A. R. and Bu ngton, J. M., Anti-windup for an F-16's daisy chain control allocator, *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, pp. 748-754, 1997.

[41] Umemura, Y. and Sakamoto, N., Nonlinear optimal servo control design for PMSM with inverter voltage norm constraints, *Trans. of the Institute of Systems, Control and Information Engineers*, Vol. 26, No. 7, pp.252–260, 2013.

[42] Wang, Y. and Boyd, S., Fast Model Predictive Control Using Online Optimization, *IEEE Trans. on Control Systems Technology*, Vol. 12, No. 2, pp. 267-278, 2010.

[43] Ding, Y., Xu, Z., Zhao, J. and Shao, Z., Fast Model Predictive Control Combining O ine Method and Online Optimization with K-D Tree, *Mathematical Problems in Engineering*, Article ID 982041, 2015.

[44] Gao, Y. Wang, Y. and Xie, L., Robust stability of reset control systems with uncertain output matrix, *Automatica*, Vol. 48, pp. 1879-1884, 2013.

[45] Wada, N. and Saeki, M., An LMI based scheduling algorithm for constrained stabilization problems, *Systems and Control Letters*, Vol.57, pp.255-261, 2008.

[46] Wada, N., Minami, M., Matsuo, Y. and Saeki, M., Tracking control of a twin-rotor helicopter model with actuator constrains, *Journal of the Robotics Society of Japan*, Vol.27, No.2, pp.199-208, 2009.

[47] Wada, N., Constrained tracking control by continuous resets of the state of a gain-scheduled controller, *Mechanical Engineering Journal*, **1**(3), DOI: 10.1299 mej.2014dr0012, 2014.

[48] Wada, N., Model predictive tracking control for constrained linear systems using integrator resets, *IEEE Transactions on Automatic Control*, Vol. 60, No. 11, pp. 3113-3118, 2015.

[49] Wada, N., Miyahara, H. and Saeki, M., Constrained tracking control by gain-scheduled feedback with optimal state resets: A general servo problem and an online optimization method, *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 138, No. 12, 121008, 2016.

[50] Wada, N., Li, Y., iyake, D. and Khonjun, S., Permanent magnet synchronous motor torque control by gain-scheduled feedback with state resets, *IEEJ Transactions on Electrical and Electronic Engineering*, Vol. 12, No. 5, pp. 744-752, 2017.

[51] Wada, N., Kawaoka, N. and Saeki, M., A gain-scheduled control algorithm for input constrained systems to track time-varying references using controller state resets, *IEEJ Transactions on Electrical and Electronic Engineering*, Vol. 12, No. 1, pp.87-95, 2017.

[52] Walsh, M. J., Mahdi Alavi, S. M. and Hayes, M. J., Practical assessment of hardware limitations on power aware wireless sensor networks - An anti-windup approach, *International Journal of Robust and Nonlinear Control*, Vol. 20, No. 2, pp.194-208, 2009.

[53] Yamaguchi, T., Shishida, K., Tohyama, S., Soyama, Y., Hosokawa, H., Ohsawa, H., Numasato, H., Arai, T., Tsuneta, K. and Hirai, H., A mode-switching controller with initial value compensation for hard disk drive servo control, *Control Engineering Practice*, Vol. 5, No. 11, pp. 1501-1646, 1997.

[54] Zanma, T., Kawasaki, M., Liu, K., Hagino, M. and Imura, A., Model predictive direct torque control for PMSM with discrete voltage vectors, *IEEJ Journal of Industry Applications*, Vol. 3, No. 2, pp.121-130, 2014.

Publications

Journal Articles and Refereed Conference Articles:

1. S. Khonjun, N. Wada and K. Inoue: Online Optimization of l_2 Gain Performance for Constrained Linear Systems by Model Predictive Control with State Resets, IEEJ Transactions on Electrical and Electronic Engineering, Vol.14, No.9, pp.1359-1363, 2019.

2. N. Wada, Yi Li, D. Miyake and S. Khonjun: Permanent magnet synchronous motor torque control by gain-scheduled feedback with state resets, IEEJ Transactions on Electrical and Electronic Engineering, Vol.12, No.5, pp.744-752, 2017.

3. S. Khonjun and N. Wada: Online optimization of disturbance attenuation performance of input constrained systems by gain-scheduled control with state resets, Proceedings of SICE Annual Conference, pp.709-712, 2018.

4. S. Khonjun, D. Miyake and N. Wada: Torque control of a PMSM using a reference governor with integrator resets, Proceedings of SICE Annual Conference, pp.1494-1497, 2019.

Acknowledgment

I would like to express my sincere gratitude to Professor Nobutaka Wada for his guidance, encouragement and insight throughout this study. He gave me full support not only with my research, but also with my life and career, and has played an important role in both my academic and personal development.

I would like to thank Emeritus Professor Masami Saeki. He gave me so much positive help and instructive discussion in my study, and I have learned so much from him. Additionally, I extend many thanks to Professor Keiji Yamada, Professor Soichi Ibaraki and Professor Hiroaki Mukaidani for our precious discussions and their advice. I would like to express my heartfelt thanks to Associate Professor Yu Kawano for helping my study.

I would like to thank the past and present members of my research group: Mr. Daichi Miyake, Mr. Kotaro Inoue, Mr. Makoto Kawachi.

I would also like to thank all the past and present members of the Control Engineering Laboratory, in the Department of Mechanical System Engineering, Hiroshima University, for their enthusiastic help to both my life and study.

I would like thank my parents for their support and encouragement from my youth to my time as a doctoral student. Their teaching is gratefully appreciated and has been invaluable in my life.

Appendix A

A Method for Solving the Optimization Problem

In this section, we show an algorithm for solving the optimization problem at Step 3 in Algorithm 1 e ciently. In the following, firstly, we transform the optimization problem at Step 3 to a convex optimization problem with a scalar decision variable. Then, we propose an algorithm for solving the transformed optimization problem.

By applying the Schur complement to (3.29), we obtain

$$(\tilde{x} \quad r)^{\mathrm{T}} P(\tilde{\ })(\tilde{x} \quad r) \quad 0 \tag{A.1}$$

where $\tilde{x} : \ [x_p^{\mathrm{T}} \ \ \tilde{x}_c^{\mathrm{T}}]^{\mathrm{T}}$. The condition (A.1) can be rewritten as

$$\tilde{x}^{\mathrm{T}} P(\tilde{\ })\tilde{x} \quad 2r^{\mathrm{T}} \quad {}^{\mathrm{T}} P(\tilde{\ })\tilde{x} \quad r^{\mathrm{T}} \quad {}^{\mathrm{T}} P(\tilde{\ }) \quad r \qquad 0 \tag{A.2}$$

We partition the matrices $P(\tilde{\ })$ and as follows.

$$P(\tilde{\ }) \quad \begin{matrix} P_1(\tilde{\ }) & P_2(\tilde{\ }) \\ P_2(\tilde{\ })^{\mathrm{T}} & P_3(\tilde{\ }) \end{matrix} \qquad \begin{matrix} _1 \\ _2 \end{matrix}$$

where $P_1(\tilde{\ }) \quad {}^{n_p \ n_p} \ P_2(\tilde{\ }) \quad {}^{n_p \ n_c} \ P_3(\tilde{\ }) \quad {}^{n_c \ n_c} \quad {}_1 \quad {}^{n_p \ n_r}$ and $_2 \quad {}^{n_c \ n_r}$.

Then the inequality (A.2) can be rewritten as

$$(\tilde{x}_c \ \tilde{\ }) : \ \tilde{x}_c^{\mathrm{T}} P_3(\tilde{\ })\tilde{x}_c \quad N(\tilde{\ })\tilde{x}_c \quad L(\tilde{\ }) \quad 0 \tag{A.3}$$

where

$$\begin{aligned} N(\tilde{\ }) \ &: \quad 2[x_p^{\mathrm{T}} P_2(\tilde{\ }) \quad r^{\mathrm{T}}(\ {}_1^{\mathrm{T}} P_2(\tilde{\ }) \qquad {}_2^{\mathrm{T}} P_3(\tilde{\ }))] \\ L(\tilde{\ }) \ &: \quad x_p^{\mathrm{T}} P_1(\tilde{\ })x_p \quad 2r^{\mathrm{T}}(\ {}_1^{\mathrm{T}} P_1(\tilde{\ }) \\ &\qquad {}_2^{\mathrm{T}} P_2(\tilde{\ })^{\mathrm{T}})x_p \quad r^{\mathrm{T}} \quad {}^{\mathrm{T}} P(\tilde{\ }) \quad r \end{aligned}$$

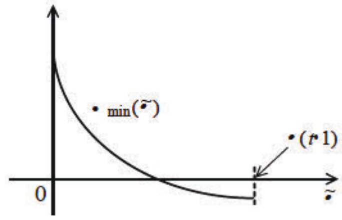

Figure A.1: $\Lambda_{\min}(\tilde{\alpha})$

It is clear that there exist $\tilde{\alpha}$ and $\tilde{x}_c$ that satisfy (3.29) if and only if there exist $\tilde{\alpha}$ and $\tilde{x}_c$ that satisfy (A.3). Also, it should be noted that the set of $\tilde{\alpha}$ and $\tilde{x}_c$ that satisfy (3.29) is convex since the condition (3.29) is an LMI.

For any fixed $\tilde{\alpha}$, there exists $\tilde{x}_c$ such that $\Lambda(\tilde{x}_c, \tilde{\alpha}) < 0$ if and only if the inequality $\Lambda(\tilde{x}_c^*, \tilde{\alpha}) < 0$ holds, where $\tilde{x}_c^* := \arg\min \Lambda(\tilde{x}_c, \tilde{\alpha})$. In the following, based on this fact, we show a method to compute the minimum value of $\tilde{\alpha}$ that satisfies $\Lambda(\tilde{x}_c^*, \tilde{\alpha}) < 0$. By solving the equality $\partial\Lambda/\partial\tilde{x}_c = 2P_3(\tilde{\alpha})\tilde{x}_c + N(\tilde{\alpha})^{\mathrm{T}} = 0$, $\tilde{x}_c^*$ can be obtained as

$$\tilde{x}_c^* = -\frac{1}{2}P_3(\tilde{\alpha})^{-1}N(\tilde{\alpha})^{\mathrm{T}}. \tag{A.4}$$

It should be noted that $\tilde{x}_c^*$ is a function of $\tilde{\alpha}$.

Then we define the following function.

$$\Lambda_{\min}(\tilde{\alpha}) := \Lambda(\tilde{x}_c^*, \tilde{\alpha}) = \Lambda\left(-\frac{1}{2}P_3(\tilde{\alpha})^{-1}N(\tilde{\alpha})^{\mathrm{T}}, \tilde{\alpha}\right) \tag{A.5}$$

Hence, the optimal solution to the optimization problem at Step 3 in Algorithm 1 can be obtained by solving the following equality.

$$\Lambda_{\min}(\tilde{\alpha}) = 0 \tag{A.6}$$

Further, by substituting $\tilde{\alpha}$ computed by solving (A.6) for (A.4), $\tilde{x}_c$ can be obtained as $\tilde{x}_c = -1/2P_3(\tilde{\alpha})^{-1}N(\tilde{\alpha})^{\mathrm{T}}$.

From the definition of the function $\Lambda_{\min}(\tilde{\alpha})$, the equality $\Lambda_{\min}(\tilde{\alpha}) = 0$ has a unique solution on the interval $[0, \alpha(t-1)]$. Moreover, $\Lambda_{\min}(\tilde{\alpha})$ takes a negative value at $\tilde{\alpha} = \alpha(t-1)$ (see Fig. A.1). Based on these facts, we show an algorithm for solving the equality (A.6) as follows.

Algorithm 6

Step 1: *Set* $_{\min}$ $0,$ $_{\max}$ $(t \quad 1),$ i 1 *and* $i_{\max}$.

Step 2: *If* i $i_{\max},$ *set* $\tilde{}$ $_{\max}$ *and break.*

Step 3: *Compute* $\tilde{}$ $(_{\min} \quad _{\max})$ 2.

Step 4: *If* $_{\min}(\tilde{})$ $0,$ *then set* $_{\min}$ $\tilde{}$. *Otherwise, set* $_{\max}$ $\tilde{}$. *Then set* i i 1 *and go to Step 2.*

$i_{\max}$ 1 is an integer. The relative error of the above algorithm is $2^{\ i_{\max}}$. Hence, we can obtain $\tilde{}$ with high accuracy by choosing a su ciently large value as $i_{\max}$. When the optimization problem at Step 3 in Algorithm 1 is solved by Algorithm 6, the closed-loop stability is assured in the case where the parameter $i_{\max}$ is chosen so that the solution that satisfies (t) $(t \quad 1)$ for all time. It can be expected that such a solution is obtained by choosing a su ciently large value as $i_{\max}$. In the above algorithm, it is not required to compute the gradient and or the hessian. Hence, it is quite easy to implement as a computer program as compared with the interior point method.