Doctoral Dissertation

Research towards a Principle for the Statistics Curriculum in Japan from the Perspective of Context

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We hereby recommend that the dissertation by Mr. HIROTO FUKUDA entitled "Research towards a Principle for the Statistics Curriculum in Japan from the Perspective of Context" be accepted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN EDUCATION.

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Dedication

I extend my humble gratitude to my parents, Hiroaki and Teruyo,

for their unconditional love and for always supporting me,

and dedicate this dissertation to them.

Abstract

The world around us is rapidly transforming from determinism to indeterminism (Popper, 1982). ‘Information’ and ‘data’ symbolise the present and future of this society. Thus, the acquisition of statistical literacy is indispensable for students to discern and use essential information and data from the large amounts of information and data that they will encounter in day-to-day life. Under this situation, several studies on method knowledge, which refers to learning how to handle information and data, have been accumulated in statistics education research (e.g., Wild & Pfannkuch, 1999). Nevertheless, some issues remain unresolved in this research field, and highlight two features (‘the purpose is problem solving’ and ‘data are used in inquiry’) related to method knowledge.

Regarding the former (‘the purpose is problem solving’), the current society presents a complexity of information that extends far beyond the simplification or idealisation of problems in the real world (Hirabayashi, 2001). Therefore, it is important to clarify the problem to be solved before undertaking problem solving. A deeper study of the problem is still required, although research has been conducted for the Problem stage in the PPDAC cycle. PPDAC stands for the steps of Problem, Plan, Data, Analysis, and Conclusion (Wild & Pfannkuch, 1999). Regarding the latter (‘data are used in inquiry’), data are certainly necessary in inquiry process. Thus, determining the kind of data to be collected for inquiries has important educational significance. This is the Plan part of the PPDAC cycle. Based on the above, the issues of what is a problem and what kind of data must be collected via statistical inquiry have not been adequately clarified yet. This is the problem statement in this research.

Statistics education deals with this statistical inquiry and thus problem and data. However, statistics education in Japan is currently being planned to shift from content knowledge to method knowledge, but there are no consistent and common principles from elementary education to upper secondary education regarding the problem and data in statistical inquiry. Today, everyone can use the

Internet and obtain a variety of information and data by searching to find out solutions to the problems which they encounter in day-to-day life. Since the problems are always embedded in a certain ‘context’, it is essential to discuss the context to be treated even in statistics education. From these considerations, the objective of this research is to develop a principle for the statistics curriculum in Japan, paying attention to context. To attain the objective, the author formulates the following five research questions (RQs):

RQ1：What is context in statistics education? What is the current status of statistics education research on context?

RQ2：What are the characteristics and issues pertaining to context in current Japanese statistics education?

RQ3：What is the framework to conduct statistical inquiry that takes the context into account?

RQ4：Is the framework developed in RQ3 valid?

RQ5：What is a principle for the statistics curriculum in Japan focusing on context?

For RQ1, context in this research means the concrete situations of problems existing in the real world and is referred as the data-context: the real-world situation from which the problem arose (Pfannkuch, 2011, p. 28). The literature review revealed that statistics education research on context is insufficient and still needs to be studied.

For RQ2, the author conducted the textbook comparison between Japan and New Zealand. This is because the intention of statistics education in New Zealand is to teach and learn method knowledge for statistical inquiry, and this is consistent with the direction required for future statistics education in Japan. The comparison results revealed the characteristic of how context is treated in Japanese statistics education, namely, efforts are made to incorporate context in the problem itself. However, it cannot be said that context is included in the inquiry and answer when posing the problem. Moreover, there is a bias into types of contexts in Japanese statistics education: sports, weather, and tests. Comparatively, the observation of educational practices in New Zealand revealed that statistics education handles not only data but also contextual thinking. Contextual thinking here refers to emergence of another context when thinking about a specific context. To broaden an idea of contexts and acquire contextual thinking, students must acquire knowledge on various disciplines and use an interdisciplinary approach. This is one of the current issues in statistics education worldwide.

For RQ3, in order to handle these issues, this research established emergent hypothesis modelling as the framework to conduct statistical inquiry that takes the context into account. Emergent hypothesis modelling shows a series of processes, which requires two models: the hypothesis model-for (a hypothetical answer based on students' anticipation of the question) and the hypothesis model-of (a concrete premise of the problem-solving process based on search). In this modelling, the hypothesis model concretely emerges at the first stage of the statistical inquiry process. The core of emergent hypothesis modelling is the formation of hypotheses through contextual thinking. Therefore, emergent hypothesis modelling constructs the statistical inquiry cycle in the context world.

For RQ4, based on the above considerations, the author developed a teaching unit 'The Ecology of the Environment and Causal Relationships'. The author then conducted the teaching experiment based on this teaching unit and analysed it from the viewpoint of case study method. The causal inquiry resulted in both the beginnings of statistical literacy at the critical/mathematical level and the refinement of the hypothesis model-of based on the hypothesis model-for by emergent hypothesis modelling. Thus, the author could show empirically that emergent hypothesis modelling is valid.

For RQ5, the author developed a principle in order to not only pay attention to the PP stage but also conduct a survey of that stage within the PPDAC cycle based on the two theories, integrated modelling approach (IMA) (Manor Braham & Ben-Zvi, 2017) and the reasoning with informal statistical models and modelling (RISM) (Dvir & Ben-Zvi, 2018). The shuttling model among the five worlds is

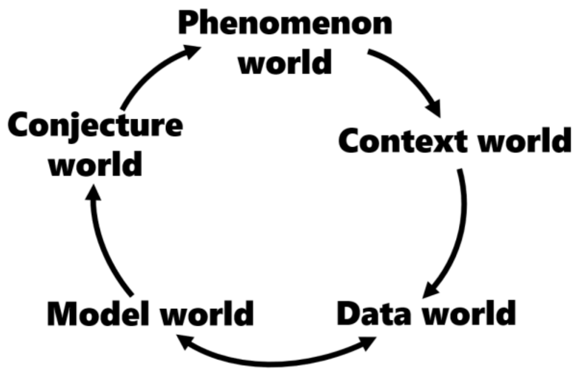

Figure. Cyclic shuttling between five worlds in statistical inquiry

proposed as shown in the figure. This model involves the emergent hypothesis modelling cycle, integrates IMA and RISM, and represents the whole statistical inquiry process. This model is common to all types of schools and can function as a normative model for the whole statistical inquiry process. Therefore, this model can serve as a principle for the statistics curriculum in Japan from the viewpoint of context. Finally, the author summarises the principle as the conclusion of this research: Statistical inquiry requires attention to the five worlds (phenomenon, context, data, model, and conjecture worlds) and the interconnections among them.

This research targeted at developing a principle for the statistics curriculum in Japan. The findings are relevant to statistics education in Japan since it seeks to convert the teaching and learning of content knowledge into that of method knowledge. However, this is not only a concern specific to Japan. Competency-based statistics education is required in all countries (cf. Wild, Pfannkuch, Regan, & Horton, 2011). Thus, the findings are also applicable to some extent to statistics education worldwide. This research can contribute to the positioning and treatment of context in competency-based statistics education as the international standard. These are the significances of this research and its implications for statistics education.

One of the potential research limitations is that although it considered method knowledge in statistics education in detail, the author was unable to take content knowledge into consideration to any great extent. This requires that the inquiry be conducted using technology based on big data. Since Japanese statistics education in the future will place greater emphasis on inferential statistics, a future statistical inquiry should use big data and technology in an integrated way. Thus, future research can explore how to realise statistical inquiry using big data and ICT (Information and Communication Technology) and thus how to equip teachers with such usage of technology.

Acknowledgements

My doctoral dissertation journey started when I had set foot in Hiroshima in 2008. Back then, I had planned for a four-year stint, but by the time I realised, I had already embarked on a 12-year long journey. Not to exaggerate, I assert that this dissertation is a condensed representation of these 12 years. Moreover, it is going to be the life and soul of my career as a researcher in the future. The story of an unskilled and inexperienced me eventually becoming a 30-year-old established Hiroto Fukuda after many metamorphoses remains incomplete without remembering the many people to whom I owe so much.

At the centre of these 12 years is my supervisor at Hiroshima University, Dr. Takuya Baba. He welcomed me with open arms as a research student after I completed the master's program and provided guidance for six years including the time I spent as a research student. Even after I became employed, he always looked after me, advising me not only on the contents of my dissertation but also on the next 40 years of my life as a researcher from various perspectives. In particular, as you may have guessed from the title of my dissertation 'in Japan', Dr. Baba taught me the importance of viewing Japan as one country among many others in the world and having the international perspective to be able to look at Japan objectively as a Japanese researcher. I truly appreciate this from the bottom of my heart. Although I still lack in many aspects as a disciple, I hope that I may continue to receive his guidance and encouragement.

Both Dr. Kinya Shimizu and Dr. Takayoshi Maki at Hiroshima University provided invaluable guidance as my deputy supervisors. Dr. Shimizu's critical teachings from the viewpoint of science education encouraged me to consider deeply the similarities and differences between statistics education, mathematics education, and science education. I have yet to provide a satisfactory result; however, I would like to apply what I have pondered to my future research. Regarding New Zealand as a comparative case in my dissertation, Dr. Maki offered me guidance from the perspective of comparative education and posed the simple, yet crucial question of 'Why New Zealand?' Moreover, as a senior researcher, he taught

me about the portrait of a researcher that will be indispensable for our society in the years to come. I express my deepest gratitude.

Even though Dr. Baba remains the focal point of my 12 years, tracing back to the time when Dr. Baba was not introduced in my life reminds me of a Professor Emeritus at Hiroshima University, Dr. Hideki Iwasaki, who served as the external examiner. Dr. Iwasaki was my supervisor during my master's program and an irreplaceable benefactor who provided me with the wonderful opportunity to meet Dr. Baba. He is my second mentor. I, a mathematics education novice who slows to grow, was guided by him to be able to get a glimpse of the depths of mathematics education as a discipline and become passionate about it. Now, I plan to make mathematics education research my profession. If I recall correctly, Dr. Iwasaki encouraged me on many occasions to 'conduct research that is daring and will take roots', so I hope that this dissertation can be counted as a move towards this challenge. However, I think there is still much to be done, and several challenges still remain. Dr. Iwasaki once told me that 'a researcher becomes mature on the way towards his or her first work'. This dissertation, therefore, is my first work of a long-term series, and I will orient the remainder of my life towards maturing in this direction. I sincerely thank Dr. Iwasaki for his tutelage.

After Dr. Baba came into my life, it is Dr. Maxine Jeanette Pfannkuch from the University of Auckland in New Zealand, an external examiner, that appeared in my life. While attending the statistics education session at the ICME 13 held in Hamburg, Germany in 2016, I met Dr. Stephanie Clare Budgett from the University of Auckland and Dr. Pip Arnold from the Cognition Education Group, who then introduced Dr. Pfannkuch to me. Furthermore, Dr. Pfannkuch introduced other teachers at a school in New Zealand to me, creating an opportunity that directly influenced this dissertation. Moreover, every time I visited New Zealand, Dr. Pfannkuch provided me with indispensable guidance based on cutting-edge findings in statistics education research. I cannot thank her enough.

Next, if we radially take a look at my past and future with reference to Dr. Baba's timeline, it is

filled with numerous seniors, juniors, and same cohort colleagues with Hiroshima University at the centre as well as those from across Japan and abroad. Naming all the individuals who have supported me might be time consuming; nevertheless, I extend my gratitude to all who shared their wonderful time with me during my four years at the School of Science, two years at the Graduate School of Education, and six years at the Graduate School for International Development and Cooperation. Your support has been vital in the completion of my dissertation. Thank you very much. I would also like to express my deepest gratitude towards Mr. Yuichi Kamimoto, Mr. Hiroki Otani, Mr. Daiki Urayama, and Mr. Masaaki Ishikawa who reviewed this dissertation in detail and gave me numerous valuable comments despite their busy schedule.

I would also like to take this opportunity to thank my parents and grandparents who have always supported me. I sincerely thank you and look forward to your continuous support.

Last but not least, including my intent to cautioning myself, I would like to close this acknowledgement with the words of Dr. Chikara Sasaki, a historian of science who earned his degree with the study of Descartes's mathematical thought, uttered to Mr. Tatsuo Yamauchi, a painter, as a reaction to his work 'Cloud - Aloofness': "Dear Yamauchi, I suggest you stop creating paintings that sell. I suggest you determine that sympathising with merchants is a corruption. I suggest you continue to stick only to the combination of shapes and colours that express the intrinsic beauty that satisfies you. I suggest you continue to maintain time of serenity to paint extraordinary work. I suggest you continue to reject words of appraisal with poor standards. While you are alive, I suggest you believe in only the high quality words of understanding from a few people. I suggest you believe in your high standards of beauty, keep practicing to elevate this standard even more, show only the paintings that make sense to you, and only to those who can understand. Then, … I suggest you continue to paint high standard works that exceed the standard like 'Cloud – Aloofness'. I suggest you aim for heights without compromise. I suggest you continue to create unapproachable beauty" (Sasaki, 2018, p. 294; translated by the author).

References

Sasaki, C. (2018). Unapproachable beauty. In Chubu University (Ed.), *Arena*, *21* (pp. 293-294). Aichi, Japan: Fubaisha.

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Hiroto Fukuda

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Chapter 1. Introduction

1.1 Background

Recently, the system of the world has been rapidly transformed from determinism to indeterminism (Popper, 1982). This is a paradigm shift in human history, and if there were a book called ‘human history’, the historical impact of this change would be such as to form a main part of the tale. Such a critical situation does not only affect a particular area, but unavoidably affects a wide swath of our world. The field of education is no exception. Thus, the main issue of this research is to implement this paradigm shift in education and to examine how education will be pursued from now on.

Every person in this world without exception passes through a period as a student. Even at this one point, the importance of education can never be reduced; it can only be increased. Moreover, character development is the greatest mission of education, so teaching literacy to enable students to survive in their society, become capable of transforming the society, and move with the times is also a task which only education can fulfil. Therefore, education can be said to be a fundamental factor which is indispensable for guiding students to adulthood.

In addition, people around the world have benefitted from innovations in information communication technologies such as personal computers. In our current society, a second world consisting of information, the Internet, has been born. The world of the Internet is completely different from the physical world. There are various physical limitations in the real world that are not present in this second world, which is growing without limit. As we live in the 21st century, we have to know how to survive in such an information-rich society. It can also be easily imagined that when our present students become adults they will be even more information-oriented. Therefore, ‘information’ and ‘data’ symbolise this society, including its future, and statistical literacy is indispensable because through it they can judge and use essential information and data correctly from the large amounts of information and data that they will encounter throughout their lives.

1.2 Historical transition of statistics education in Japan

Here, the author briefly summarises the history of statistics education in Japan and clarifies its features. At first, the current education system in Japan is briefly described. Japan has a 6-3-3 system, with 6 years for elementary education, 3 years for lower secondary education, and 3 years for upper secondary education; compulsory education includes elementary education and lower secondary education, nine years in total. More details are shown in Figure 1-1

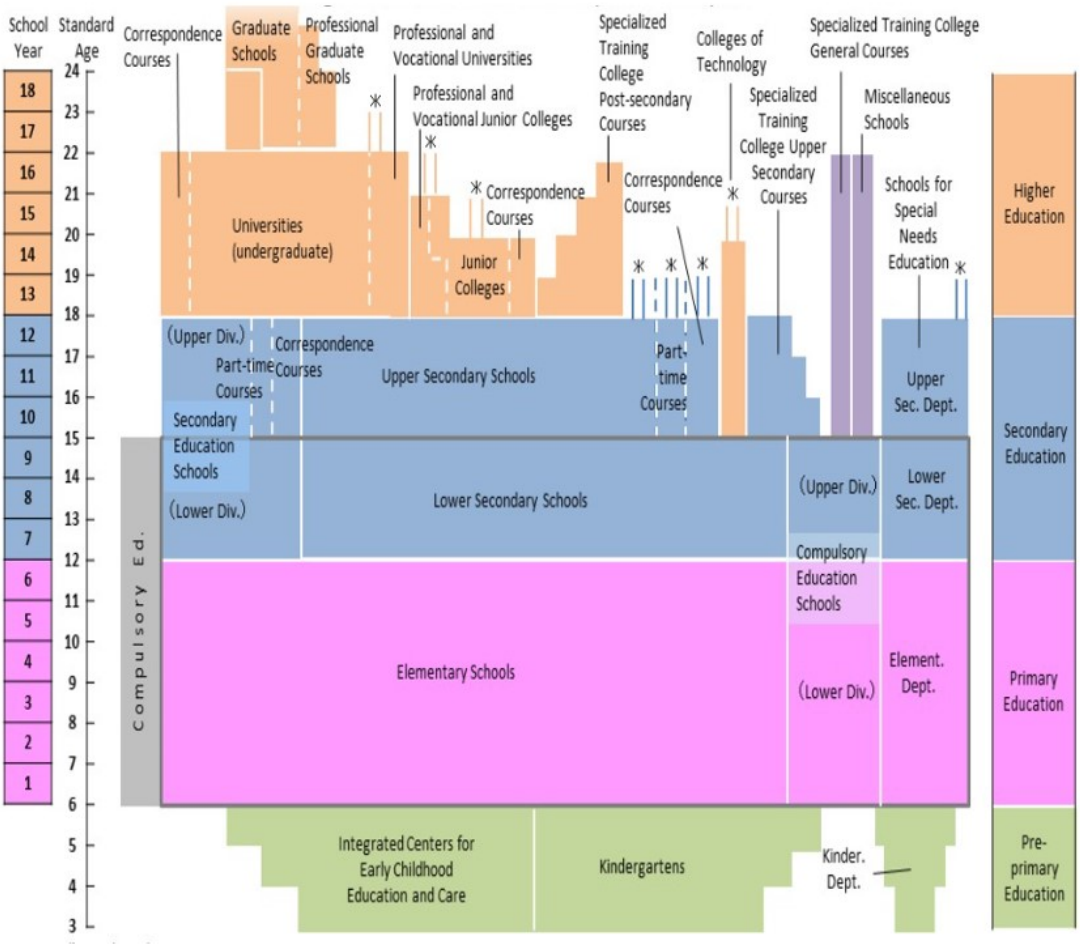


Figure 1-1. Organization of the school system in Japan

(Ministry of Education, Culture, Sports, Science and Technology (MEXT), n.d.)

Education in Japan aims to fully develop the individuals as defined in the Basic Act on Education. This entails the acquisition of many kinds of skills necessary to survive in society, which is in line with the global trend, such as literacy concepts whose origins are the liberal arts (e.g., Higuchi, 2010; Sato, 2001, 2003) and competency concepts as current literacy (e.g., Ishii, 2015; Matsushita, 2010; Organisation for Economic Co-operation and Development (OECD), 2000, 2019; Rychen & Salganik, 2001, 2003). These are also intended to inculcate the skills essential for people to participate fully in society. The Japanese enrolment rate in compulsory and upper secondary education has been almost 100% in recent years. Since almost all students receive education for 12 years from elementary school to upper secondary school, the Education for All has already been implemented. Thus, the educational content and teaching method implemented by each school should not differ, and standard education should be conducted at any school in Japan. This is why education is carried out based on a course of study wherein the educational content and teaching method in each school type and grade are specified. The preparation for the course of study was carried out only after the war. After the defeat of Japan in World War II, the United States Education Mission to Japan and the Civil Information and Educational Section (CI & E) of the General Headquarters of the Supreme Commander for the Allied Powers (GHQ / SCAP), conducted teaching for introducing educational reform in Japan (Ohya, 2002). After that, the Constitution of Japan was promulgated and the Basic Act on Education and the School Education Act, which constitute the basis of modern education in Japan, were enacted, and the course of study was based on the 'Course of Study' in the United States (Shinno, 2010; Ueda, 2010).

In Japan, statistics education is positioned as one area of mathematics education. It has also been conducted mostly based on the contents in the course of study. There is a detailed explanation that clarifies statistics education as a part of mathematics education. Table 1-1 summarises the educational content of statistics education for each grade, starting from elementary to upper secondary, in the past courses of study and those that are to be implemented, henceforth. Table 1-1 covers nine periods of time when the course of study changed.

Table 1-1. Educational content of statistics education in the Japanese courses of study (This is in addition to Ninomiya (2004) and Otani (2018). Translated by the author)

	1st period	2nd period	3rd period	4th period	5th period	6th period	7th period	8th period	9th period
	1947	1951	1958	1968	1977	1989	1998	2008	2018
Elementary school 1st grade		Using ○ and × to create simple tables and graphs						Representing by using pictures, diagrams, etc., and reading them	Representing by simple pictures and diagrams, and reading them
Elementary school 2nd grade	One-dimensional tables and figures	Creation of tables and graphs; using them effectively	Ordering and classifying materials and present them in a table	Summarizing matters into tables and graphs	Representing by tables and graphs, and reading them	Representing and reading matters in simple tables	Representing matters in simple tables and graphs, and reading them	Ordering and classifying quantities; representing in simple tables and graphs, and reading them	Ordering and classifying quantities; representing by using simple tables and graphs; reading and examining them
Elementary school 3rd grade	Picture graphs and bar graphs	Two-dimensional tables; picture graphs and bar graphs; using round numbers	Easy-to-understand classifications; creating tables, reading; bar graphs, line graphs	Making a correlation between two quantities, ordering and classifying materials; summarizing in tables; bar graphs and line graphs	Ordering and classifying materials; bar graphs	Simple ordering and classification of events; bar graphs	Simple ordering and classification of events; bar graphs	Ordering and classifying materials; representing by tables and graphs, and reading; bar graphs	Ordering and classifying data; representing in tables, and reading; bar graphs; examining and expressing using tables and graphs
Elementary school 4th grade	Line graphs; two-dimensional tables	Two-dimensional table; picture graphs and bar graphs; line graphs	Thinking and considering whether there are omissions or overlaps; line graphs	Ordering by using the concept of grouping, so that there are no omissions or unnecessary overlap	Line graphs; investigating the cases that occur with respect to the two events, thinking to avoid omissions or overlaps; investigating features and trends from graphs	Investigating the cases that occur with respect to the two events, and examining omissions or overlaps	Investigating the cases that occur with respect to the two events, and examining omissions or overlaps; line graphs	Ordering and classifying materials according to the purpose / ordering and classifying (from two viewpoints); representing in tables and graphs; investigating features; line graphs; examining omissions or overlaps	Data collection and analysis; ordering and classifying data according to the purpose / ordering and classifying (from two viewpoints); line graphs; considering, expressing and communicating the problem solving process and conclusion; examining omissions or overlaps
Elementary school 5th grade	Percentages	Line graphs, degree of increase/decrease and change; line graphs; practicing graphs on pre-defined paper spaces	Average, dispersion of the total data; pie graphs and band graphs; percentages	Percentages; dispersion of the data; average; tables and charts of frequency distribution; pie graphs and band graphs; percentage obtained from materials and tendency of the entire group	Ordering and classifying materials; pie graphs and band graphs; percentages	Ordering and classifying materials; pie graphs and band graphs; percentages	Ordering and classifying materials; pie graphs and band graphs; percentages	Collecting / ordering and classifying of materials according to the purpose, pie graphs and band graphs; investigating features; percentages;	Pie graphs and band graphs; understanding the method of statistical problem solving; collecting / ordering and classifying data according to the purpose; examining, expressing and communicating multifaceted process of problem solving and conclusions; averages; percentages;
Elementary school 6th grade	Reading pie graphs, square graphs and band graphs	Square graphs, belt graphs and pie graphs	Devising graphs using appropriate tables and graphs, and creating quick graphs for shopping and conversion graphs		Dispersion of materials; table of frequency distribution and histograms; understanding the overall tendency from some materials	Tables and graphs of frequency distribution; understanding the overall tendency from some materials	Average	Averages and dispersion of materials; tables and graphs showing frequency distribution	Representative values; tables and graphs showing frequency distribution; collecting / ordering and classifying data according to the purpose; critically considering, expressing and communicating the process of problem solving and conclusions

	1st period	2nd period	3rd period	4th period	5th period	6th period	7th period	8th period	9th period
	1947	1951	1958	1968	1977	1989	1998	2008	2018
Lower secondary school 1st grade	Bar graphs, line graphs, belt graphs, square graphs and pie graphs; organizing materials; selecting appropriate graphs; percentages	Bar graphs, line graphs, belt graphs, square graphs and pie graphs; organizing materials; selecting appropriate graphs; percentages	Collecting materials, organizing tables, graphs, representative value; frequency distribution, histograms; relative frequency / representative value; frequency, distribution, class, histogram, relative frequency, cumulative frequency, representative value					Collecting and organizing materials according to the purpose, reading trends of materials; histograms, representative value, average value, median value, mode, relative frequency, range, class; error and approximate value	Histogram, relative frequency; organizing into tables and graphs; collecting and analyzing data according to the purpose, reading, considering and judging trends of distribution; range, cumulative frequency
Lower secondary school 2nd grade	Percentages; finding the characteristics of graphs and the relationship between the two quantities; pie graphs and histograms	Percentages; pie graphs and histograms; finding the characteristics of graphs and the relationship between the two quantities			Collecting, ordering and classifying materials; representative values, dispersion; frequency distribution, histogram relative frequency, cumulative frequency, average value, range	Approximate value and error; collecting, ordering and classifying materials; representative value, dispersion, frequency distribution, histogram relative frequency, cumulative frequency, average value, range, correlation charts and correlation tables			Quartile range, boxplot; reading, critically considering and judging trends in data distribution
Lower secondary school 3nd grade	Predicting the relationship of varying amounts and expressing them in a table / graph; finding characteristics and regularity of changes	Percentages; predicting the relationship of varying amounts and finding the characteristics and regularity of changes shown in tables / graphs	Correlation table, correlation diagram; standard deviation; estimating the ratio in a population from the ratio in a sample	Understanding the trends in materials using tables, graphs, representative values, etc.; frequency distribution / representative value, histogram; correlation table and correlation diagram	Population and sample, sample survey; average value and ratio in a sample	Sample survey		Sampling, sample survey, description of population trends	Sample surveys, random sampling, critically considering and expressing the method and results of sample surveys; estimating and judging population trends; complete surveys
Upper secondary school 1st grade							Organizing materials; understanding trends of materials (Basic Mathematics)	Quartile deviation, dispersion, standard deviation; scatter diagram, correlation coefficient (Mathematics Ⅰ) (Mathematics application)	Dispersion, standard deviation, scatter diagram, correlation coefficient; organizing in tables and graphs; concept of hypothesis testing; data scattering condition and tendency; collection and analysis of multiple types of data according to the purpose; outlier (Mathematics I)
Upper secondary school 2nd grade		Features of various graphs; complete survey and partial survey; standard deviation; correlation; percentages; bookkeeping; understanding the significance of using probabilities and statistics	Standard deviation, concept of statistical estimation (Mathematics A) Descriptive statistics, binomial distribution, normal distribution, sample survey, t-test, χ^2-test, F-test (Applied Mathematics)	Statistical estimation (Mathematics A) (Integrated learning) Population and sample, probability distribution, statistical estimation (Applied Mathematics)	Binomial distribution, normal distribution; descriptive statistics, standard deviation; sample survey, sampling, population sample and sample distribution, estimation, examination of results (Mathematics II)	Probability distribution (random variables and probability distribution, binomial distribution) (Mathematics B)	Organization and classification using spreadsheet software, etc.; frequency distribution tables and correlation diagrams; representative value, dispersion, standard deviation, correlation coefficient (Mathematics B)	Random variable and probability distribution, binomial distribution; normal distribution; statistical estimation (population and sample, concept of statistical estimation) (Mathematics B)	Statistical estimation; concept of sample survey; random variables and probability distribution; binomial distribution and normal distribution; average, dispersion, standard deviation of random variables; critically considering the design of sample surveys according to the purpose, estimation and judgment of characteristics and trends of a population, method and results of sample surveys; confidence interval, significance level; scatter diagram (Mathematics B)
Upper secondary school 3rd grade			Distribution, average and dispersion, binomial distribution, normal distribution, and sample survey (Mathematics III)	Population and sample, probability distribution (distribution, dispersion, standard deviation of random variables), binomial distribution, normal distribution, statistical estimation, estimation, test (Mathematics III)	Distribution of variables, scatter diagram of representative values, random variables and probability distribution, binomial distribution, normal distribution, statistical estimation (population and sample, statistical estimation) (probability / statistics)	Representative value and scatter diagram, correlation statistical estimation (population and sample, normal distribution, concept of statistical estimation) (Mathematics C)	Random variables and probability distribution, binomial distribution; normal distribution (continuous random variables, normal distribution); statistical estimation (population and sample, concept of statistical estimation) (Mathematics C)		Mathematical representation of routine phenomena and social phenomena by using tables and statistical graphs (Mathematics C)

No major differences appeared from the 1st period to 6th period. Descriptive statistics, such as data shown in graphs and tables, is mainly taught from elementary education to the first half of lower secondary education, and inferential statistics, such as inferences from sample surveys, is mainly taught from the second half of lower secondary education to upper secondary education. A major change in statistics education in Japan occurred in the 7th period. In lower secondary education, statistics education was not a part of the curriculum. It appears that statistics education was implemented in upper secondary education in the 7th period. However, in upper secondary education before the 7th period, statistics education was positioned as a course of study, but in fact, was hardly put into practice. Therefore, it is only in elementary years that statistics education was implemented in the 7th period. In the course of study in the 8th period, another major change occurred. First, there was a return of statistics education, which was left out of lower secondary education in the 7th period; however, this was restored in the 6th period. Second, statistics education was positioned in 'Mathematics I', which is compulsory for upper secondary education. As a result, statistics education in upper secondary education, which thus far has been positioned only in the intended curriculum (courses of study) and has hardly been implemented, also functioned as the implemented curriculum. Further reforms are made in the course of study to be carried out in the 9th period. The key educational contents in statistics education in the course of study up to the 8th period are the different concepts (content knowledge) involved in inquiring statistically. On the other hand, the key educational content in the course of study in the 9th period does not only include content knowledge. For example, 'to consider the process and conclusion of problem solving (multidirectionally / critically)' in elementary education, 'to consider the method and result of sample survey critically' in lower secondary education, and 'to collect and analyse multiple types of data according to the purpose' in upper secondary education are also positioned as educational contents. In other words, statistical methods (method knowledge) such as utilising formed statistical concepts (content knowledge) and judging what kinds of statistical concepts (content knowledge) should be utilised in a certain statistical inquiry activity are also included as educational contents. In the 9th period, statistics education is included in all grades from elementary education to upper secondary education, which recognises as the increasing importance of statistics education within mathematics education.

As described above, there had been almost no major changes in the teaching contents even if the course of study had been revised. However, there were changes in the educational contents and their sequence in the very recent revision of the courses of study. It may also be seen that the importance of statistics education within mathematics education is increasing.

1.3 From determinism to indeterminism

In the previous section, the author mentioned that statistics education in Japan now takes a high position in mathematics education, but the reason was not clarified. The author thus considers this reason while providing an overview of changes from the perspective of statistics based on the studies of Ian Hacking, a Canadian philosopher of science.

After studying mathematics and physics, Hacking earned a Ph.D. in philosophy and wrote on the question, ‘Why have language and mathematics become the problem in philosophy?’ (Hacking, 1975, 2014) and ‘How are statistics and probability related to the society for development?’ (Hacking, 1990, 2006). While the former is a fully philosophical study of linguistics and mathematics, the latter is a study of probability and statistics, with respect to society. In other words, it can be said that his interest lay in exploring the connection between philosophy and society from both directions. More precisely, it would be better to regard the society mentioned here as humans, because it is only human beings who have developed society thus far, and he shows how probability and statistics were born and related to society in certain aspects as it developed. It is may be speculated that he focuses on probability and statistics because they emerge from society, and at the same time they influence the society and have brought about a paradigm shift in the view of science.

The following is a detailed explanation of what must be described here to summarise the contents of Hacking (1990). As a conventional view of science, it was thought that the proposition that all phenomena and events could be explained by some law was indisputable. Kant, Descartes, and Laplace also believed that this proposition was true. Laplace in particular, the founder of classical probability theory, made a clear reference to that point at the beginning of Laplace (1902): “All events, even those which on account of their insignificance do not seem to follow the great laws of nature, are a result of it just as necessarily as the revolutions of the sun” (p. 3). When stated this way, no famous scholars had any doubt of its validity, and this idea took root as a kind of common sense within society. Under such circumstances, it was Charles Sanders Peirce who denied this proposition forthrightly and all alone. Peirce, the famous scholar of semiotics, was the pioneer of indeterminism. Peirce (1892) stated that “I propose here to examine the common belief that every single fact in the universe is precisely determined by law” (p. 321), which he called ‘the doctrine of necessity’. At this moment, indeterminism was born, which takes ‘the doctrine of necessity’ as false as the result of his critique of the determinism based on this proposition.

The rule of Napoleon from 1820 to 1840 relates to the background of the replacement of determinism by indeterminism (Hacking called this ‘erosion of determinism’). Napoleon established institutions to collect data to provide information precisely describing the state of national power in several respects, including the Statistics Bureau as a national institution to support the compilation of data. It was considered that from the large amount of data compiled by the Statistics Bureau, it would be possible to infer rules which could not be possible from single data points. Therefore, the Statistics Bureau became an organisation which not only collected but also analysed data. Then, after Napoleon’s downfall, a large amount of data made by the Statistical Bureau and probabilistic inferences from them were spread all over the world, including more importantly the concomitant statistical know-how (Hacking called this ‘an avalanche of printed numbers’). Poisson’s publication of the law of large numbers in 1837 may not be irrelevant. In the analysis of the copious data compiled by the Statistics Bureau, including the law of large numbers, a single data point cannot be an object of statistical analysis, rather a large amount of data is assumed. One cannot make a definite inference from a single data but can make a probabilistic inference using a large amount of data. Therefore, it can be said that at this stage, the 'erosion of determinism' was progressing, and determinism was being implicitly replaced by indeterminism (Hacking called this 'the taming of chance') until it was finally announced in Peirce (1892).

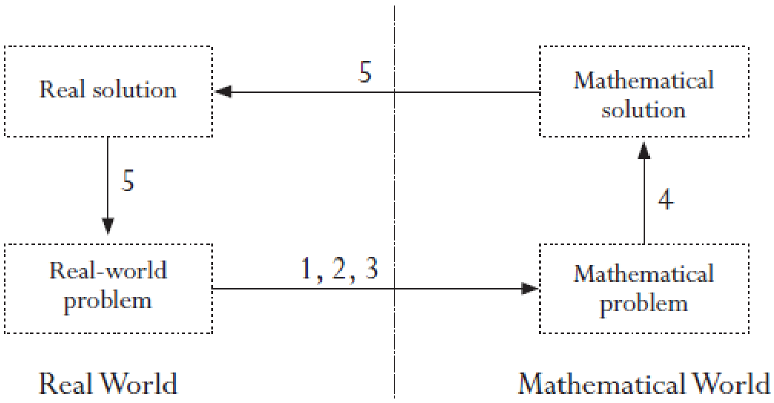
For the last 130 years since Peirce suggested a clear position within epistemology for chance, probability, and contingency, research on indeterminism has been conducted. For example, Monod (1972) (the French edition was written in 1970) discusses chance and necessity from the viewpoint of biology and the theory of evolution, Popper (1982) explains the protection of indeterminism and the lack of indeterminism, and Sober (2008) reports the relationship between three kinds of statistical philosophies, Bayesianism, likelihoodism, and frequentism, based on the theory of evolution. Research on indeterminism has recently gotten underway in Japan. For example, Miyakawa (2017) and Takeuchi (2018) considered the process of the acceptance of disciplinary statistics into Japan and the social and historical thought of disciplinary statistics.

As mentioned above, it can be said that statistics education has gained a high position in Japanese mathematics education because of the sea changes in the view of statistics and the immense social influence of statistics. In the next section, issues in this research will be clarified in view of this background.

1.4 Problem statement

The paradigm shift from determinism, which has dominated the society for a long time, to indeterminism has a great impact on the real world. Thus, today we see one personal computer per family and one mobile phone per person. Everyone carries around a large amount of information and data on his or her shoulder. Therefore, it is important to consider how education addresses the needs of the current age, particularly how statistics education handles information and data.

Methods of handling information and data are strongly related to mathematical modelling. According to OECD (2003), the mathematisation cycle is shown as Figure 1-2. This mathematisation cycle is based on Freudenthal's view of mathematics as an activity (cf. Freudenthal, 1968, 1973, 1991; Gravemeijer, 1997; van den Heuvel-Panhuizen, 2003) and progressive mathematisation consisting of horizontal mathematisation and vertical mathematisation (cf. Treffers, 1978).[1] The important point in this mathematisation cycle is that it starts from problems and situations in the real world, and problem solving is then conducted after formularisation into the world of mathematics, but these processes can be



(1) Starting with a problem situated in reality;

(2) Organising it according to mathematical concepts and identifying the relevant mathematics;

(3) Gradually trimming away the reality through processes such as making assumptions, generalising and formalising, which promote the mathematical features of the situation and transform the real-world problem into a mathematical problem that faithfully represents the situation;

(4) Solving the mathematical problem; and

(5) Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.

Figure 1-2. The mathematisation cycle (OECD, 2003, p. 38)

summarised as the modelling of reality. Thus, the mathematisation cycle can be a process of mathematical modelling. In particular, statistics education contains many elements of modelling. An example of this is the statistical investigative cycle (PPDAC cycle) (Figure 1-3), consisting of the steps of Problem, Plan, Data, Analysis, and Conclusion, as shown by Wild and Pfannkuch (1999).

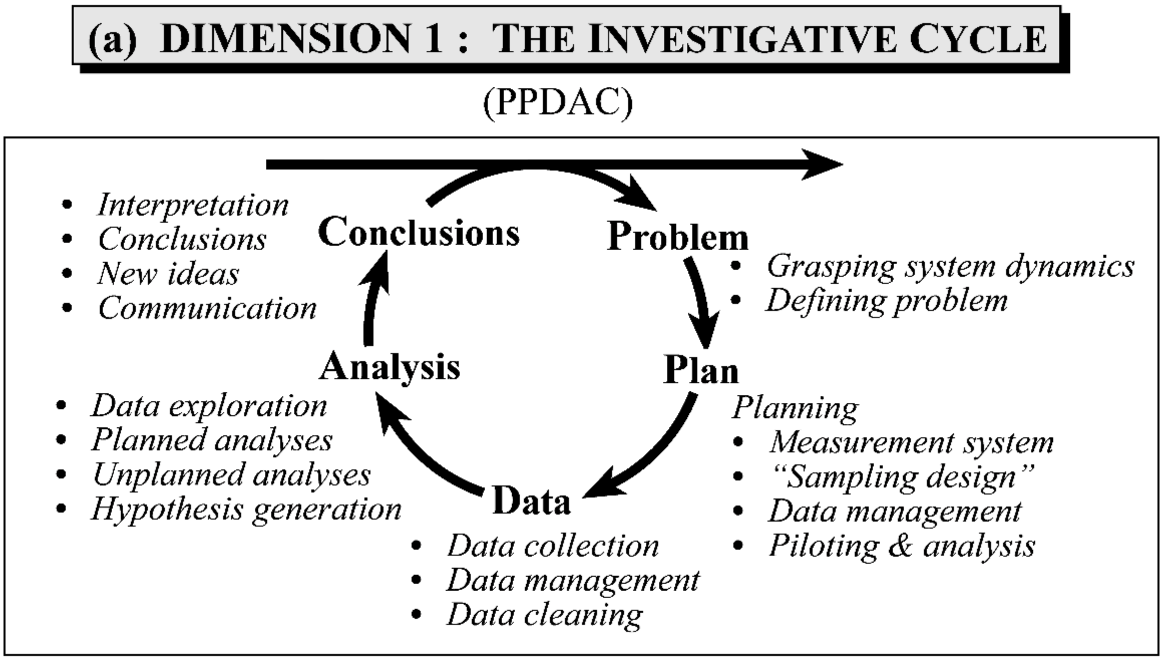


Figure 1-3. The investigative cycle (Wild & Pfannkuch, 1999, p. 226)

This model of statistical investigation is the result of an examination of the inquiry process of statisticians through interviews and the modelling of aspects in the complex inquiry. It is the repetition of a process of clarifying what the problem is, planning to solve the problem, collecting and analysing data, drawing a conclusion to the problem, and clarifying a new problem to improve the conclusion. Therefore, this PPDAC cycle can also be considered as the statistical modelling of the real world.

Since the beginning of the 21st century, a large number of studies has accumulated findings on data modelling, model-based statistical inference focusing on modelling, and Informal Inferential Reasoning (IIR). For example, components of data modelling in Lehrer and Schauble (2004), such as posing questions, sampling data, and making inferences, are shown as Figure 1-4 from Lehrer and English (2018), and model-based statistical inference by the interaction between data and models in statistical inquiry process is sketched out as in Prodromou (2017) in Figure 1-5.

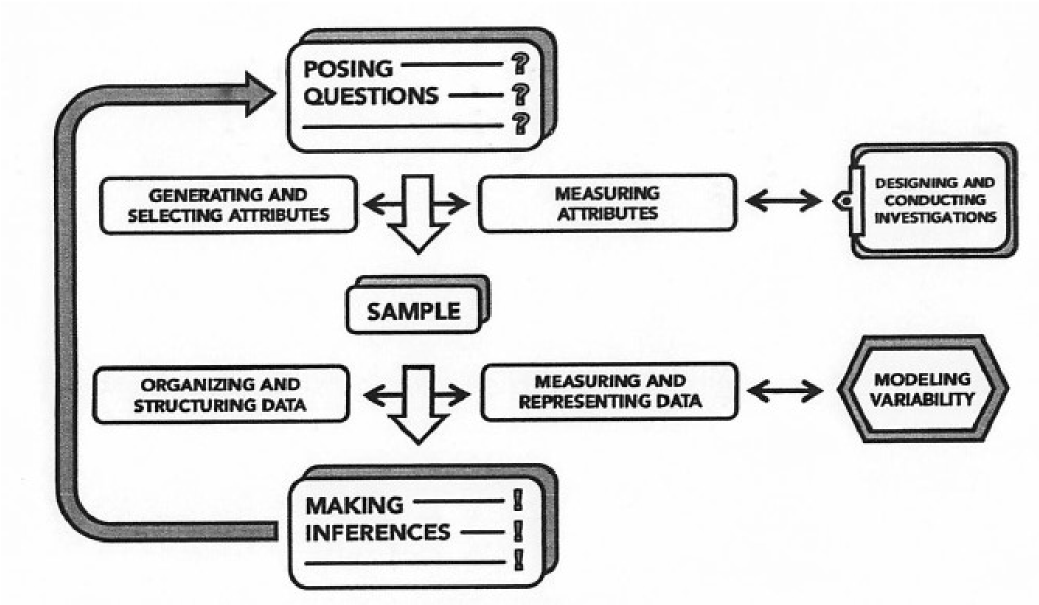


Figure 1-4. Components of data modelling (Lehrer & English, 2018, p. 232)

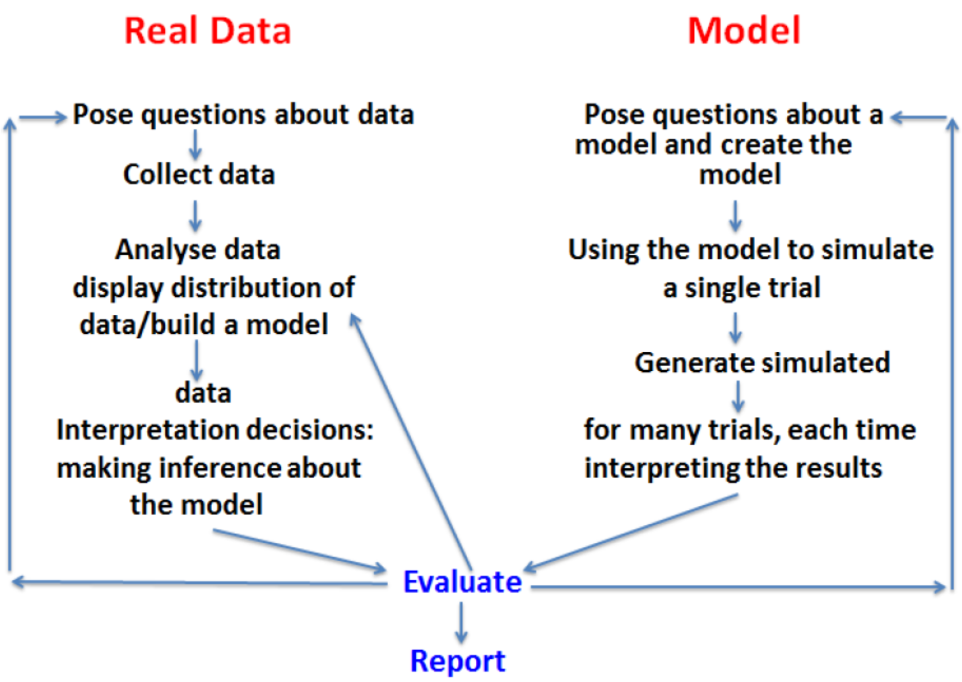


Figure 1-5. Bidirectional modelling diagram of model-based informal inference (Prodromou, 2017, p. 142)

In current statistics education research, the teaching is intended to focus more on the methods than on the contents of statistics by making students reason informally about statistics instead of teaching formal statistics in a formal way. Informal Statistical Inference (ISI) and Informal Inferential Reasoning (IIR) are proposed based on Tukey's (1977) Exploratory Data Analysis (EDA), which regarded statistics not as mathematics but as data analysis.[2] Makar and Rubin (2018) reviewed ISI and IIR, as defined by researchers in various ways, and found five points of commonality: Claim beyond the data, expressed with uncertainty, use of data as evidence, consideration of the aggregate, and integration of context knowledge (pp. 273–275).

In this way, many studies whose intention is to acquire method knowledge to learn how to handle information and data have been accumulated in statistics education research. For example, statistics is described as a methodological discipline and science dealing with data and uncertainty in Moore (1990) and Cobb and Moore (1997). What is common to varied method knowledge in statistics education (statistical inquiry cycle, data modelling, model-based statistical inference, ISI, IIR, and so on) is that 'the purpose is problem solving' and that 'data are used in inquiry'. As for 'the purpose is problem solving', for example, in data modelling and model-based statistical inference, the goal is to refine the model by repeating the modelling cycle. The model refinement here implies refining the solution for the original problem, so that the purpose of solving the problem is fulfilled. Regarding the point that a new problem is clarified by interpreting and evaluating the results and conclusions of the analysis, it is a problem whose solution still requires them to solve the original problem. Thus, it is a problem posing within the problem-solving process. As for the point that 'data are used in inquiry', this is characterised by inferring population characteristics from sample data and using statistical values or probabilities as evidence for inferences about the population. Therefore, data with numbers are always used in this inquiry. In summary, this is may be interpreted as indicating that statistics education research so far continues to discuss the 'method knowledge of inquiry using data for problem solving'.

The statistics curriculum in Japan will also be carried out in this direction in the future as the author explained in Section 1.2. As mentioned above, a salient characteristic of the statistics curriculum in the 9th period is the teaching and learning of statistical method knowledge to apply the formed statistical content knowledge and to judge what kind of statistical content knowledge should be used in a certain statistical inquiry activity. For example, in elementary education it is intended to introduce the PPDAC cycle explicitly, and in secondary education it is intended to treat the methods of the problem-solving process. In statistics education in Japan henceforth, the results of statistics education research thus far are to be reflected in the curriculum and incorporated into educational practices.

Nevertheless, there are issues that must be addressed in statistics education research. They underlie two features ('the purpose is problem solving' and 'data are used in inquiry') related to the method knowledge that are claimed and implemented in statistics education. It is closely related to the indeterminism mentioned above. Regarding the former, current society comprises a complex network of knowledge and information, and it presents a complexity of knowledge and information far beyond the simplification or idealisation of problems in the real world (Hirabayashi, 2001). Therefore, it is important to clarify the problem to be solved before undertaking problem solving. That is, although the research has been conducted for the Posing Question stage in data modelling or model-based statistical inference and the Problem stage in the PPDAC cycle, the accumulation of research is not sufficient.

For the latter, it is needless to say that inquiries based on data are necessary, but as with the former, there are too many variables in current society, which is too complicated for such an approach. Thus, inquiries into the kind of data to be collected itself has educational significance. In other words, inquiring at this stage does not necessarily require data. This is the Plan part of the PPDAC cycle. Thus, research focusing on the part where students do not use data in their inquiries is necessary.

From the above, we may see that the issues of what is a problem and what kind of data is necessary in statistical inquiry or statistical inference are not as sufficiently targeted for research as they are for the subsequent problem-solving situation. This is the problem statement in this research.

1.5 Research objective

In our excessively complex society that is facing an indeterministic world, defining problems and determining necessary data are important processes in which students do not necessarily use data. Instead, the investigator's own experience or common sense are used. Today, we are in an age when everyone can use the Internet, so we can obtain a variety of information by searching, and such information can also be a tool of inquiry. In such a situation, it is 'context' that is the key word for the problem statement. Since statistics has always developed within a context, it can be said that statistical inquiry also changes if the context changes. Therefore, it is essential to discuss the context to be treated even in statistics education.

According to Moore (1990), "data are not merely numbers, but *numbers with a context*" (p. 96), but this suggests that statistics education, which is a part of mathematics education in Japan, has different characteristics from mathematics education. Moore means that the object of mathematics is numbers, while the object of statistics is data. In this regard, Shaughnessy (2007) stated that "statistics is fraught with contextual issues, which is the nature of the discipline, whereas often mathematics strips off the context in order to abstract and generalize" (p. 1002). While mathematics aims at de-contextualisation, statistics cannot be separated from context and is context-dependent. From the above, the nature of mathematics education is different from statistics education. In statistics education, context is not data itself but an important element existing in the background of the data.

Additionally, statistics education in Japan is being planned to shift from content knowledge to method knowledge, but there are no consistent and common principles from elementary education to upper secondary education for what is a problem and what data are necessary in statistical inquiry. Therefore, principles about context are indispensable for the statistics curriculum. In this research, the objective is to develop a principle for the statistics curriculum in Japan, paying attention to context. To attain the objective, the author sets up the following five research questions (RQ):

RQ1：What is context in statistics education? What is the current status of statistics education research on context?

RQ2：What are the characteristics and issues pertaining to context in current Japanese statistics education?

RQ3：What is the framework to conduct statistical inquiry that takes the context into account?

RQ4：Is the framework developed in RQ3 valid?

RQ5：What is a principle for the statistics curriculum in Japan focusing on context?

1.6 Rationales for the research

The position of this research is explained by mathematics education as design science by Wittmann (1995). His view of mathematics is summarised as MATHEMATICS, regarding which he stated, "I suggest a use of capital letters to describe MATHEMATICS as mathematical work in the broadest sense; this includes mathematics developed and used in science, engineering, economics, computer science, statistics, industry, commerce, crafts, art, daily life, and so forth according to the customs and requirements specific to these contexts" (ibid., p. 359). Regarding mathematics education based on this view of mathematics, his statement that "they must see school mathematics as an extension of pre-mathematical human capabilities which develop within the broader societal context provided by MATHEMATICS" (ibid., p. 359) suggests that mathematics education is established in relation to the human activities that have created mathematics and the social infrastructure upon which mathematics has been constructed. In addition, he also referred to mathematics education as a discipline as follows: "mathematics educators need a lively interaction with MATHEMATICS and they must devote an essential part of their professional life to stimulating, observing and analysing genuine MATHEMATICAL activities of children, students and student teachers" (ibid., p. 359). Figure 1-6 shows the related areas of mathematics education and the core of mathematics education, that is, the identity of mathematics education.

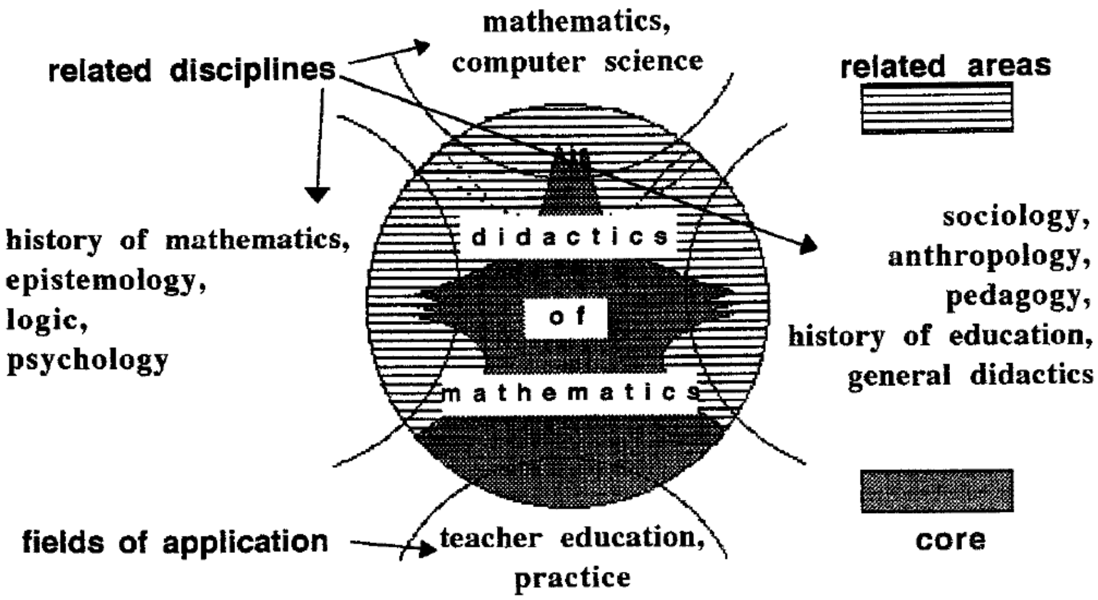


Figure 1-6. The core and the areas related to mathematics education, their links to the related disciplines and the fields of application (Wittmann, 1995, p. 357) [3]

It can be seen from the relationships with various related areas and related disciplines that the emphasis is placed in the end on the suggestions of the need for 'teacher education' and 'practice'. The core of mathematics education as design science consists of the following eight components (ibid., pp. 356-357).

1. analysis of mathematical activity and of mathematical ways of thinking,
2. development of local theories,
3. exploration of possible contents that focus on making them accessible to learners,
4. critical examination and justification of contents in view of the general goals of mathematics teaching,

5. research into the pre-requisites of learning and into the teaching/learning processes,
6. development and evaluation of substantial teaching units[4], classes of teaching units and curricula,
7. development of methods for planning, teaching, observing, and analysing lessons, and
8. inclusion of the history of mathematics education.

This means that mathematics education researchers must take this core as the object of their research. That is, the rationale of research must be discussed in relation to these cores.

The author considers this research in terms of these points. As mentioned above, mathematics is oriented toward de-contextualisation, while statistics is context-dependent, so they have different properties (Shaughnessy, 2007). However, mathematics education has to take current society into consideration, and the current trend is a transformation from determinism to indeterminism as described in Section 1.3 because mathematics education is a social-cultural practice (cf. Abe, 2010; Iwasaki, 2007). When we entered a highly information-oriented society and put data into a previously deterministic world, the role of chance appeared and determinism was eroded, so indeterminism can be said a fusion of determinism and chance. The 'reading, writing, and arithmetic' as literacy which has been required so far are equivalent to 'input, output, and processing' of information from the viewpoint of the highly information-oriented society (Iwasaki, 2007; Iwasaki, Nakamura, & Baba, 1999; Otani, 2018). 'Arithmetic' as the role of mathematics is replaced with 'processing', which means that statistics plays a significant role in 'processing.' Otani (2018) states "we cannot simply think that statistics is mathematics, and we cannot conclude that statistics is not mathematics at all" (p. 74; translated by the author)[5]. However, given these changes with the era, such as from determinism to indeterminism, from arithmetic to processing, or from traditional mathematics to MATHEMATICS, statistics in statistics education is clearly located in MATHEMATICS as a form of mathematical work in the broadest sense. In other words, statistics education is positioned in mathematics education whose view of mathematics is

MATHEMATICS, which is a position that this research takes.

In addition, with regard to the eight components forming the core of mathematics education, the objective of this research is to develop a principle for the Japanese statistics curriculum from the perspective of context. Therefore, this research deals predominantly with Component 6. At the same time, this research considers statistical inquiry and statistical inference (Component 1), aims to develop a local theory of the context for which sufficient accumulation has not been made in statistics education research (Component 2), and considers the types of context when referring to concrete teaching and learning processes (Components 3 and 5). However, this research does not deal with Component 4, 7, and 8. Finally, this research conducts educational practices in such a way that all the previous discussions are reflected. From the above, this research can serve as the core of mathematics education in Figure 1-6. Furthermore, mathematics education research in Japan based on the core of mathematics education has been accumulating (e.g., Iwasaki, 2007; Otaki, 2014; Otani, 2018; Suginomoto, 2015). In this way, the rationale of this research is also shown as a discipline of mathematics education.

1.7 Structure of the research

This section presents the overall structure of this study. In Chapter 2, the author analyses the term ‘context’, which is key to this research, through a literature review, then investigates quantitatively or through literature review how much context-related research exists in statistics education research thus far and what research has been conducted. Therefore, this chapter seeks to answer Research Question 1. Next, Chapter 3 focuses on Japan and presents a method for analysing the role of context in current statistics education and identifying issues in statistics education related to context, then shows the methodology necessary for developing the principle for the statistics curriculum. That is, the research method is explained. These involve a literature review, and then Chapter 4 and Chapter 6 apply the method. In Chapter 4, quantitative comparative research and qualitative analytical research are conducted to answer Research Question 2. Based on these results, a new framework of statistical inquiry with context is established in Chapter 5 (Research Question 3), and in Chapter 6, the statistics curriculum is developed based on the methodology designed in Chapter 3 and the teaching experiment is conducted to validate the framework (Research Question 4). This is conducted through a literature review and qualitative research. This result is interpreted in Chapter 7, whereupon the author aims to propose a principle for a future statistics curriculum in Japan from the perspective of context, and answers Research Question 5.

In summary, the structure of this research is shown in Figure 1-7.

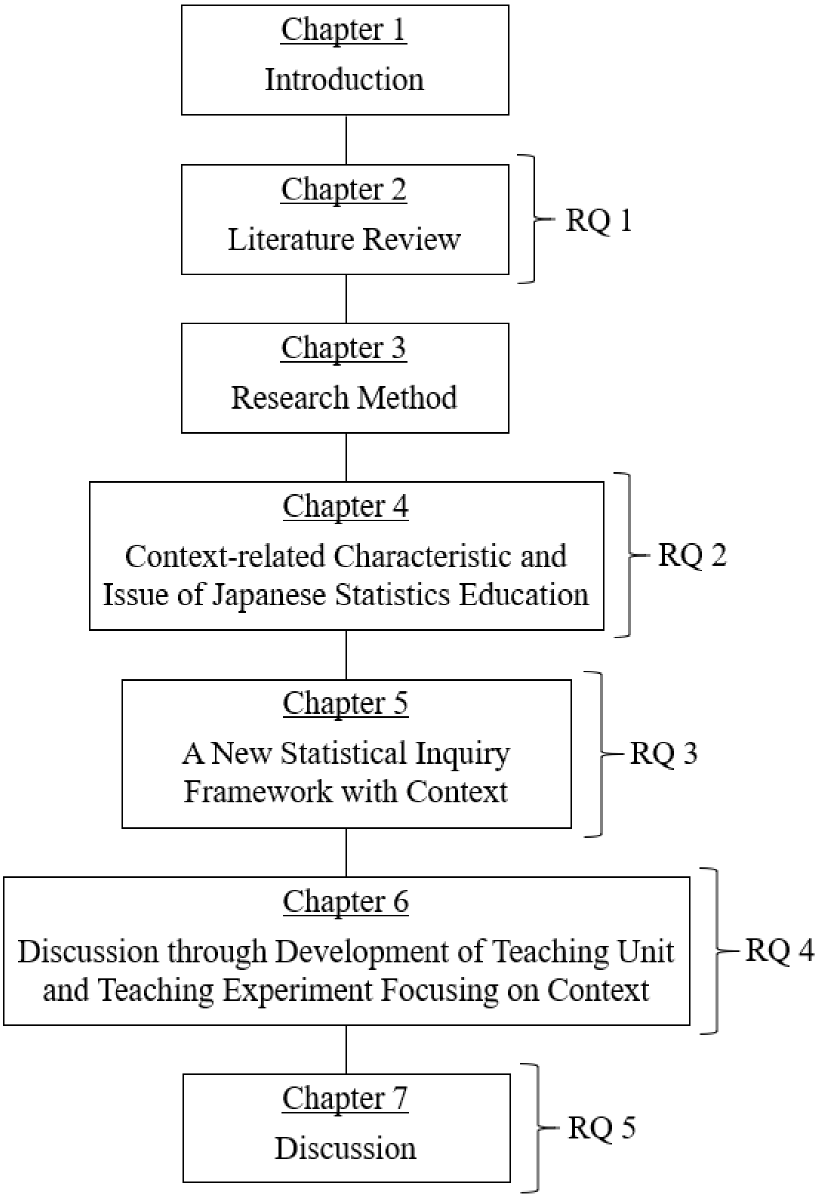


Figure 1-7. Structure of this study

Notes

1. Freudenthal suggests the significance of mathematisation: "What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible, even that of mathematizing mathematics" (Freudenthal, 1968, p. 7). In addition, Treffers describes the method of mathematisation based on the view of mathematisation of Freudenthal as follows: "The attempt to schematise the problem mathematically is indicated by the term 'horizontal' mathematisation. … The activities that follow and that are related to the mathematical process, the solution of the problem, the generalisation of the solution and the further formalisation, can be described as 'vertical' mathematisation." (Treffers, 1978, p. 71). It may be seen in Figure 1-8 that these two types of mathematisation are not independent but interact with each other, and the way of progress of mathematisation and the way the level rises are by steps and not linear.

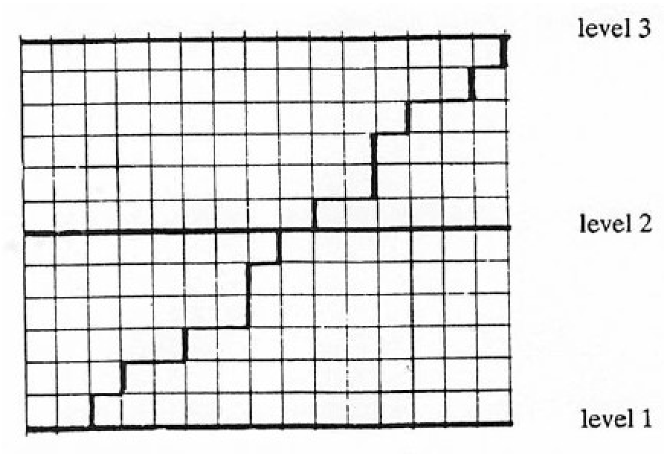


Figure 1-8. Progressive mathematisation (Treffers, 1978, p. 248)

2. Three key principles of ISI are defined, for example, by Makar and Rubin (2009) as follows: (1) *generalization*, including predictions, parameter estimates, and conclusions, that extend *beyond describing the given data*; (2) the use of data *as evidence* for those generalizations; and (3) employment of probabilistic language in describing the generalization, including informal reference

to levels of certainty about the conclusions drawn (p. 85). Additionally, ISI is defined as probabilistic generalisation from data as shown in Figure 1-9.

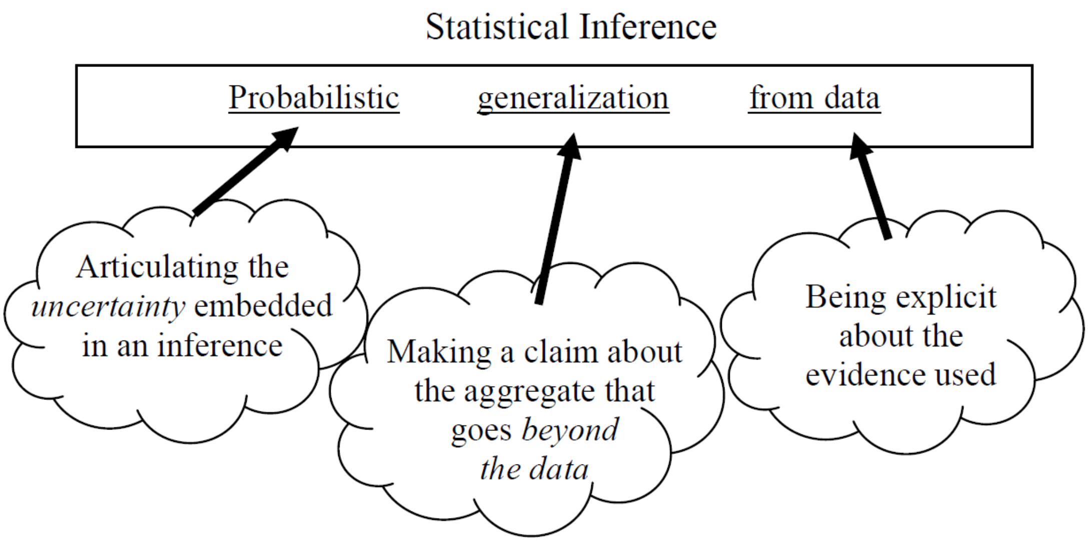


Figure 1-9. A framework for thinking about statistical inference (Makar & Rubin, 2009, p. 85)

Moreover, Pfannkuch (2007) defines informal reasoning as "the drawing of conclusions from data that is based mainly on looking at, comparing, and reasoning from distributions of data" (p. 149). Similarly, in Zieffler, Garfield, delMas, and Reading (2008), IIR is used to refer to "the way in which students use their informal statistical knowledge to make arguments to support inferences about unknown populations based on observed samples" (p. 44) and in Ben-Zvi, Gil, and Apel (2007), "the cognitive activities involved in informally drawing conclusions or making predictions about 'some wider universe' from patterns, representations, statistical measures and statistical models of random samples, while attending to the strength and limitations of the sampling and the drawn inferences" (p. 2). In addition, the difference between ISI and IIR is described such that ISI is not intended for formal statistical procedures, while IIR is intended to formalise ISI (Manor Braham & Ben-Zvi, 2017).

3. Regarding the core and related areas of mathematics education, Dr. Wittmann was invited to give a

plenary lecture on ‘Mathematics Education as Systemic-evolutionary Design Science: Revisiting’ at the 45th Meeting of the Japan Academic Society of Mathematics Education on 29 January 2017 (Sun). Here, the author would like to note that he used the related areas on the left side of Figure 1-6 with the addition of ‘semiotics’ in his plenary lecture. Please see the details in Wittmann (2019).

4. See Chapter 3 for more information on teaching unit.
5. Otani (2018) summarised the differences between statistics and mathematics based on the previous research as shown in Table 1-2:

Table 1-2. The differences between statistics and mathematics

(Otani, 2018, p. 74; translated by the author)

		Mathematics	Statistics
Nature		Structure-oriented and Application-oriented	Application-oriented
Object	Object of Inquiry	Number & Quantity, Shape & Space, Change & Relationship, and Data & Certainty	Data & Certainty
	Phenomenon	Deterministic Events and Indeterministic Events	Indeterministic Events
	Data	De-contextualised Numerical Values	Data with Context
Method	Problem Solving	Mathematical Modelling	Statistical Inquiry Cycle
	Justification	Demonstration	Argumentation

Here, the difference between statistics and mathematics as a discipline is described, but several previous studies regarding mathematics education and statistics education has been referenced. Thus, it can be interpreted that Table 1-2 shows the difference between statistics education and mathematics education. Iwasaki (2007) proposed " 'Object – Method' as the rule of didactical construction and 'Practice – Theory' as the law of didactical formation [in didactics of mathematics]" (p. 35; the parenthesis is by the author) with reference to Bigalke (1972, 1974). According to the rule 'Object – Method', both object and method in Table 1-2 are essential to construct mathematics education and statistics education. Thus, the similarity and difference between them can be confirmed. In particular, the major differences are data as object and the method of justification. About data as object, statistics education treats data with context while mathematics education treats de-contextualised numerical values. In addition, the method of justification in statistics education is argumentation to aim at persuasive claims while the method in mathematics education is demonstration following the deductive reasoning (Otani, 2018, pp. 73-74).

OECD (2016) defines and explains scientific literacy that is addressed in science education as the following (p. 20):

Scientific literacy is the ability to engage with science-related issues, and with the ideas of science, as a reflective citizen. A scientifically literate person is willing to engage in reasoned discourse about science and technology, which requires the competencies to:

- **Explain phenomena scientifically** – recognise, offer, and evaluate explanations for a range of natural and technological phenomena.
- **Evaluate and design scientific enquiry** – describe and appraise scientific investigations and propose ways of addressing questions scientifically.
- **Interpret data and evidence scientifically** – analyse and evaluate data, claims, and arguments in a variety of representations and draw appropriate scientific conclusions.

Additionally, there are three types of knowledge treated in scientific literacy as follows (p. 19):

・**Content knowledge** － knowledge of the content of science

・**Procedural knowledge** － knowledge of the procedures that scientists use to establish scientific knowledge

・**Epistemic knowledge** － an understanding of the role of specific constructs and defining features essential to the process of knowledge-building in science

Because statistics has a methodological nature, procedural knowledge and epistemic knowledge as the objects in science education are the objects in statistics education in a broad sense, and statistical inquiry cycle and argumentation as the methods in statistics education are the methods in science education (Otani, 2018, p. 81). In this way, there are many similarities between statistics education and science education. However, they are also different in terms of content knowledge which is the object in science education. Otani (2018) states the difference clearly: "In scientific literacy [science education], the inquiring object of natural science and knowledge peculiar to natural science are within content knowledge. Content knowledge is knowledge related to natural scientific phenomena such as chemical change or motion, so this viewpoint is treated as the variable for statistical literacy [statistics education]. In other words, content knowledge is related to natural science in the case that statistics is applied to the context of natural science, but needed knowledge is different in the case that statistics is applied to the context of social science." (p. 81; translated by the author; the parentheses are by the author). While content knowledge in science education is related to natural science, content knowledge in statistics education is related to data and certainty with its context in indeterministic events (Table 1-2). Therefore, content knowledge as the object in statistics education and content knowledge as the object in science education are different.

Chapter 2. Literature Review

In this chapter, the author defines ‘context’, which is polysemous, and summarises the roles of context in statistics education through reviewing previous studies in this field. These form the foundations of this research.

2.1 The meaning of the term ‘context’

In this section, the meaning of ‘context’ was discussed as the key word in this study. For example, common meanings of context are “the situation within something exists or happens, and that can help explain it” or “the text of speech that comes immediately before and after a particular phrase or piece of text and helps to explain its meaning” (Cambridge Dictionary, n.d.-a). The latter has a linguistic meaning referring to the relationship between pieces of text or between text and a phrase, so the former is related to this research. The meaning of the former is in line with the view of context espoused by Ichiei Hirabayashi, who obtained the first Ph.D. in mathematics education in Japan. Kanemoto (2012) states as follows: “I [Dr. Hirabayashi] think that context is ... an oriented situation or a direction of thinking. Furthermore, I can explain that the situation is a place where problems arise and a stage where their thinking begins when I am asked what the situation is” (p. 75; translated by the author; the parenthesis is by the author). With respect to this oriented situation, van den Heuvel-Panhuizen (2005) classifies it into two types (p. 2):

1. *a characteristic of a task presented to the students*

   : referring either to the words and pictures that help the students to understand the task, or concerning the situation or event in which the task is situated. The description of a context comes close to the interpretation of a context as a task characteristic.

2. *the learning environment*

: this includes both the different situations in which learning takes place and the interpersonal dimension of learning.

The second (learning environment) is the situation wherein learning takes place, for example, the characters of the students in the class, the relationships between the students, or the class form, such as individual activity, pair learning, group activity, and so on. On the other hand, the first (characteristic of a task presented to the students) is the situation which the object of learning has, for example, the problem sentences and figures treated in the educational practices, the characteristics of the tasks, and so on. According to Borasi (1986), context is “the situation in which the problem is embedded” (p. 128), which exactly matches the first definition of context.

The discussion thus far has concerned the lexical meaning of context and its meaning in mathematics education, but in statistics education, context is classified by Pfannkuch (2011) and Pfannkuch, Ben-Zvi, and Budgett (2018). In Pfannkuch, Ben-Zvi, and Budgett (2018), the meaning of context is summarised under the following three types: two types (data-context and learning-experience-context) by Pfannkuch (2011) and one type (designer-context) implicit in Wilkerson and Laina (2018). The meaning of each type is as follows:

1. *data-context*

: the real-world situation from which the problem arose (Pfannkuch, 2011, p. 28)

2. *learning-experience-context*

: the background students bring to a task and the physical and social learning environment in which they operate (Pfannkuch, 2011, p. 28)

3. *designer-context*

: [the] perspectives [of software designers for learning statistics and analytical purposes] on what structure, visualizations and elements to include, which may or may not assist student reasoning (Pfannkuch, Ben-Zvi, and Budgett, 2018, p. 1116; the parentheses are by the author)

The first context refers to the situation which the object of learning has, which is exactly the same as the first meaning of the two above-mentioned contexts of van den Heuvel-Panhuizen (2005). However, data-context is discussed within statistics education, so the object area is limited to the data. The second context is also almost the same as the second meaning of van den Heuvel-Panhuizen (2005), and the only difference between them is that the context in Pfannkuch (2011) includes learnt knowledge. Finally, the third context is related to software and technology, and means the background of how it works in the students' inquiries. Therefore, this can be regarded as the learning-experience-context about software and technology. There have been many previous studies about technology in statistics education (e.g., Ben-Zvi, Gravemeijer, & Ainley, 2018; Biehler, Ben-Zvi, Bakker, & Makar, 2013; Saldanha & McAllister, 2016), and it appears that the technology-specific context is shown as the third context to highlight its importance. Thus, Pfannkuch, Ben-Zvi, and Budgett (2018) classify context into three types, but there are these two types of data-context and learning-experience-context in substance. They are the same as the two types of context in mathematics education (characteristics of a task presented to the students and the learning environment).

Although the learning-experience-context, such as learnt knowledge, the class form, and statistical software, is definitely an important element when conducting statistical inquiries, this research can achieve its objective clearly through a focus on data-context. Therefore, context in this research means the concrete situations existing in the real world and refers to the data-context. In the following, this research treats context as data-context.

2.2 Statistics education research on context

The importance of context has been asserted and emphasised in the previous research on statistics education (e.g., Pfannkuch, Budgett, Fewster, Fitch, Pattenwise, Wild, & Ziedins, 2016; Pfannkuch & Wild, 2000; Wild & Pfannkuch, 1999). However, it was pointed out in 2011 that there were no further discussions on this point (Pfannkuch, 2011). It is possible that further research has been conducted since 2011, so the author would like to confirm the extent to which previous research on context has appeared in the important journals on statistics education. Focusing on *Statistics Education Research Journal* and *Journal of Statistics Education*, which are major international journals of statistics education, all 76 articles[1] from 2015 to 2017 were targeted for the analysis. Then, the author explains the method of analysis. Many of these 76 articles describe and analyse statistical inquiry and use some data with context. However, at this stage, they are not previous research on context, but previous research using context. Since the object of this research is context itself, it targets research whose object is how to use context or what kind of context to use. According to this, the author found only 2 out of all 76. They are Depaolo, Robinson, and Jacobs (2016) and Martonosi and Williams (2016). The focus of these papers is on higher education, and they conclude that the context should be set for the future occupations of the students. In summary, the importance of context is also mentioned in recent statistics education, but it was found that no further discussion of context at the elementary and secondary education levels has been conducted. Therefore, this literature review reveals that Pfannkuch's (2011) point that statistics education research on context is insufficient is still relevant.

However, there has been a little statistics education research on context in other international journals on statistics and mathematics education or the first handbook on statistics education research, published in 2018 (Ben-Zvi, Makar, & Garfield, 2018). The author uses these as clues to discuss the role of context in statistics education.

First, in Nilsson, Schindler, and Bakker (2018), the research questions necessary for the future of theory development in statistics education may be summarised as the following five (p. 374):

1. More explicit attention needs to be paid to how students can learn historically developed disciplinary, formal knowledge. More explicit attention on theorizing the relationship between formal and personal views of statistics will help to move the field forward.
2. In addition to static categorizations of student thinking, we need insights into the dynamics between categories or levels.
3. There is a need for more fundamental theories on the impact of digital technology on learning statistics but also on how to teach with digital technology. Reflection on how the nature of statistical knowledge itself changes due to such technology will also be necessary.
4. There is a need for a deeper theoretical conceptualization of context and contextualizing in statistics education.
5. Consider potential benefits of a semantic theory that has been proposed as underpinning research on statistical inference: inferentialism. We do not want to suggest this is the only or best way forward, but it is in our view an interesting candidate to shed a new light on long-standing issues.

As previously mentioned, Research Question 4 is the same as Pfannkuch's (2011) point, so it may be said that this is still a current issue. As the focus of this research is also context, it addresses Research Question 4.

As discussed in the previous section, there are several studies of context in mathematics education research, specifically Borasi (1986), Boaler (1993), van den Heuvel-Panhuizen (2005), Kastberg, D'Ambrosio, Mcdermott, and Saada (2005), and so on. Cobb and Moore (1997) mention the difference between the roles of context in mathematics and in statistics (data analysis) as follows: "In mathematics, context obscures structure. … In data analysis, context provides meaning" (ibid., p. 803).

In other words, context acts as a veil hiding the most essential structure in mathematics, and it is not an issue even in the case of a structure without context; indeed, the structure is clearer if there is no context. On the other hand, context gives meaning in statistics (data analysis), so no meaning is given without context, and statistics always involves a context (data analysis). That is, the structure is more clearly visible without context in mathematics. However, the meaning of context in mathematics education and that in statistics education differ slightly in quality. There *should* be context in mathematics education, while there *must* be context in statistics education. As Wild and Pfannkuch (1999) add, “one cannot indulge in statistical thinking without some context knowledge. The arid, context-free landscape on which so many examples used in statistics teaching are built ensures that large numbers of students never even see, let alone engage in, statistical thinking” (p. 228); this suggests that statistics education and context cannot be separated.

The results of a review of previous studies in Makar, Bakker, and Ben-Zvi (2011) may be summarised as follows: students use only statistical knowledge (e.g., Pfannkuch, Budgett, Parsonage, & Horring, 2004) or only contextual knowledge (e.g., Konold, Pollatsek, Well, & Gagnon, 1997; Makar & Confrey, 2007) in their statistical inquiries, but they find the integration of both kinds of knowledge challenging and worthwhile[2]. Therefore, students have not been able to perform such integration in their statistical inquiries, although they recognised that integrating contextual knowledge with statistical knowledge is meaningful. As per Makar, Bakker, and Ben-Zvi (2011)’s statement that “students need to learn to coordinate contextual and statistical knowledge to overcome their struggle to make sense of a perceived gap between what they know from experience and what they observe in data” (p. 156), statistics education research has to show how to harmonise both contextual and statistical knowledge.

It is Wild and Pfannkuch (1999) who most closely consider this harmonisation method, which is used when conducting practical research (Noll, Clement, Dolor, Kirin, & Petersen, 2018; Shaughnessy & Pfannkuch, 2002) and is applied to education on probability (e.g., Pfannkuch, Budgett, Fewster, Fitch, Pattenwise, Wild, & Ziedins, 2016). This approach has continued to make contributions until the present. Therefore, the following organises the consideration by Wild and Pfannkuch (1999) of the integration of context and statistics based on the concrete example in Shaughnessy and Pfannkuch (2002).

In short, their discussion clarified the nature of statistical thinking in statistical inquiry by statisticians, and this is summarised in Figure 2-1. Dimension 1 of statistical thinking is the same as Figure 1-3, and the PPDAC cycle shows the process of how to act and what to think in statistical inquiry. The

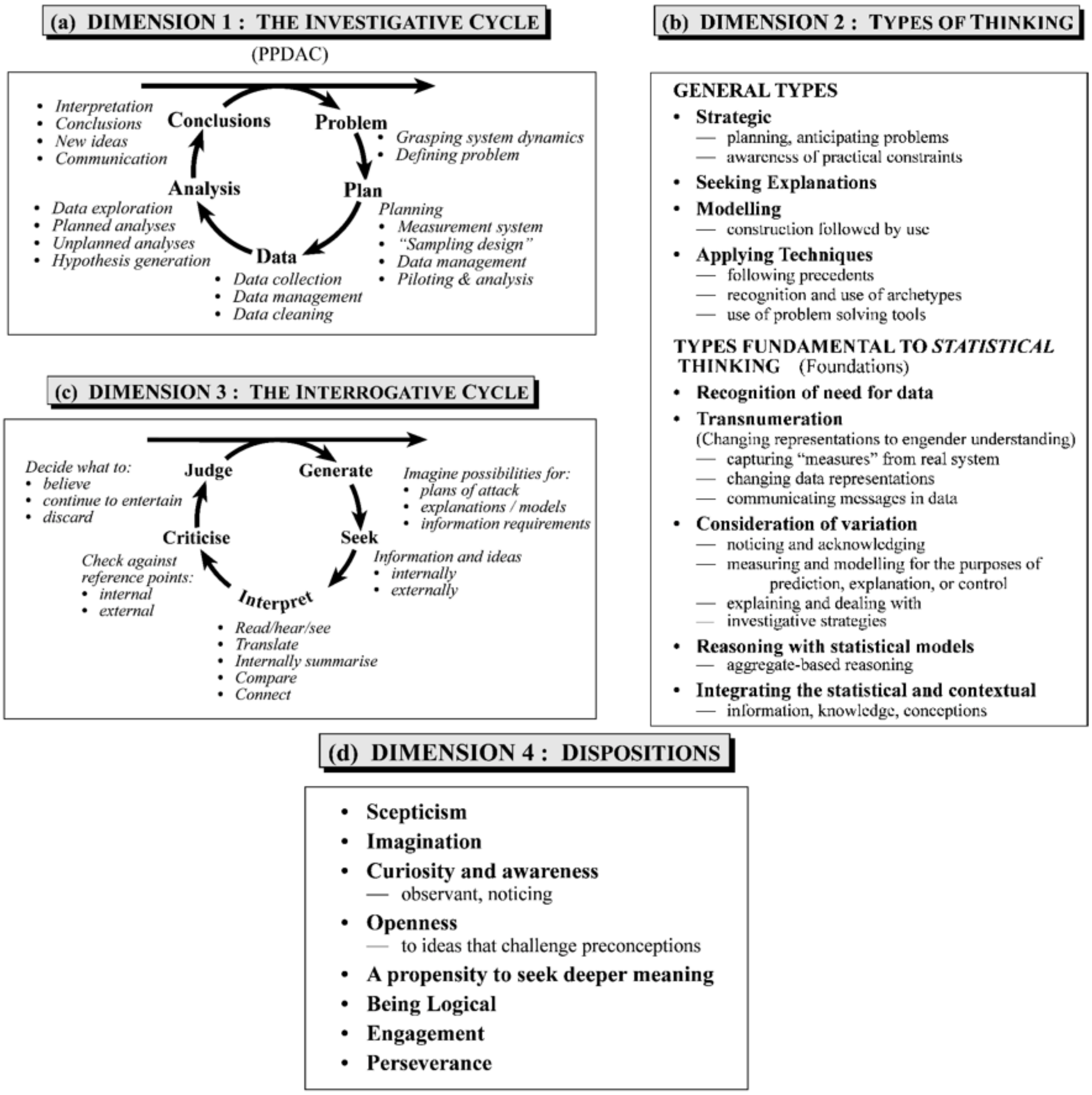


Figure 2-1. A 4-dimensional framework for statistical thinking in empirical enquiry

(Wild & Pfannkuch, 1999, p. 226)

types of thinking in dimension 2 are those used in statistical inquiry revealed by the interviews with statisticians, which may be classified into two: the general types of thinking common to all problem solving, and the types fundamental to statistical thinking, which are specific to statistical problem solving. Dimension 3 is the interrogative cycle and is described as "a generic thinking process in constant use in statistical problem solving" (ibid., p. 231). This interrogative cycle consists of five elements: Generate, Seek, Interpret, Criticise, and Judge. The PPDAC cycle of dimension 1 is a process of action in statistical inquiry, while the interrogative cycle of dimension 3 is a process of thinking in statistical inquiry. Finally, dimension 4 involves the emotions and personalities which influence the thinking revealed by the interviews of statisticians. Now let us further consider dimension 2 with a focus on integrating the statistical and contextual within the types of thinking fundamental to statistical thinking in Wild and Pfannkuch (1999) and the Judge stage in the interrogative cycle of dimension 3 with reference to context.

First, about the integrating the statistical and contextual within the types fundamental to statistical thinking of dimension 2, the interaction between context and statistics is shown in Figure 2-2.

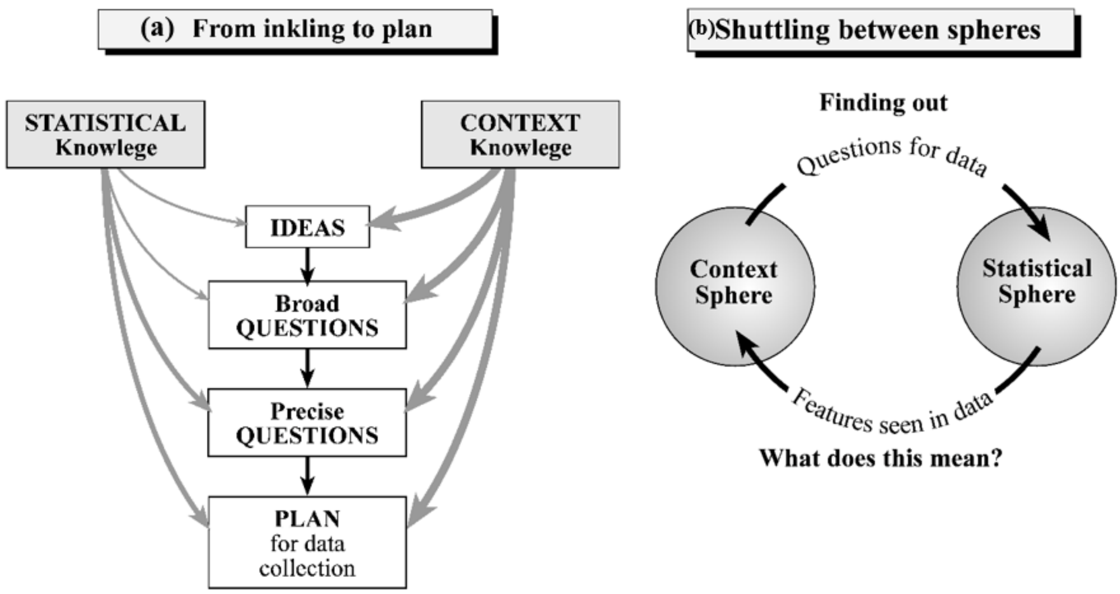


Figure 2-2. Interplay between context and statistics (Wild & Pfannkuch, 1999, p. 228)

In statistical problem solving, targeting real problems is fundamental, and, unlike equation and word problems in mathematics education, it is almost always the case that the structure of problems is not clear because of the complexity of the underlying reality. In other words, the structure of problems at the beginning of statistical inquiry is implicit. Figure 2-2 (a) shows the process of transforming problems with this implicit structure into explicit ones. In addition, such a transformation seems to clarify problems at Problem stage in the PPDAC cycle, but this figure shows more than just this. Of course, clarifying the problem is also an important aim, but the clarification of Plan is the real aim. Students cannot go to Plan stage without making an implicit problem explicit. Plan in this figure refers to designing a method of data collection including deciding the types of data, and it is exactly the clarification of the problem that transforms it into a problem which can be solved if data can be collected for the implicit problem. This is described by Wild and Pfannkuch (1999) as follows: "[Figure 2-2 (a)] traces the (usual) evolution of an idea from earliest inkling through to the formulation of a statistical question precise enough to be answered by the collection of data, and then on to a plan of action" (p. 228; the first parenthesis is by the author). Thus, Figure 2-2 (a) represents the stage of PP in the PPDAC cycle.

While Figure 2-2 (a) explains part of the PPDAC cycle in more detail, Figure 2-2 (b) shows what is intended by the PPDAC cycle. The intention is 'shuttling between spheres (context sphere and statistical sphere)', which is the caption of Figure 2-2 (b), and "the continuous shuttling backwards and forwards between thinking in the context sphere and the statistical sphere" (ibid., p. 228) is mentioned. As described, "[Figure 2-2 (b)] goes on all the time through PPDAC" (ibid., p. 228; the parenthesis is by the author), so that students continuously shuttle between the context sphere and the statistical sphere by repeating the PPDAC cycle. In other words, in Figure 2-2 (b), the features are found from the data collected in the statistical sphere, then they are interpreted and the collected data are critically reconsidered in the context sphere, and then the inquiring sphere is again returned to the statistical sphere.

In Shaughnessy and Pfannkuch (2002), practical research is conducted through a statistical inquiry with the context of the time interval of the hot water eruption at Old Faithful geyser in Yellowstone National Park, one of the most famous geysers in the world. The implicit problem is, 'How long do I have to wait until the next hot water eruption at Old Faithful?' For this problem, the teacher gave students the data showing the times of the eruptions for one day first, next the data for three days (that is, adding two more days), then the data for 16 days (adding 13 more days). The statistical inquiry proceeded by drawing stem-and-leaf displays, histograms, bar graphs, line graphs, and so on. Shaughnessy and Pfannkuch (2002) cite many problems which were not implemented in this practice but which can promote students' statistical thinking. For example, the problem of 'How much data do you need for the prediction? Are the data for one time, two times, one day, or one year?' arises if data are not given, and the problem of 'What kind of information can be obtained and lost by various graphical expressions? What type of relationship do these graphs have with each other?' occurs if the graphs are based on the data for 16 days, and if other presented problems are drawn from various situations (pp. 257-258). These problems are explicit ones, and this process is the clarification of Plan.

Let us now return to the students' specific inquiries. As a result of their inquiries, they have discovered the bimodal and oscillating patterns of the data for the time intervals of the eruption. This pattern is the result of the analysis of the data, so it is an inquiry in the statistical sphere. On the other hand, a new problem arises, 'Why does the time interval between eruptions change twice on average? What is the cause of this change? How does the system of this geyser work?' This stage constitutes inquiry in the context sphere because this inquiry concerns the interpretation of the found patterns. It is expected that the next PPDAC investigative cycle will seek to solve this new problem. In this way, the shuttling between the context sphere and the statistical sphere is performed as shown in Figure 2-2 (b).

Next, the process of distillation and encapsulation is shown as Figure 2-3 about the Judge stage of the interrogative cycle of dimension 3. Wild and Pfannkuch state regarding figure, "internal interrogative cycles help us extract essence from inputs, discarding distractions and detail along the way"

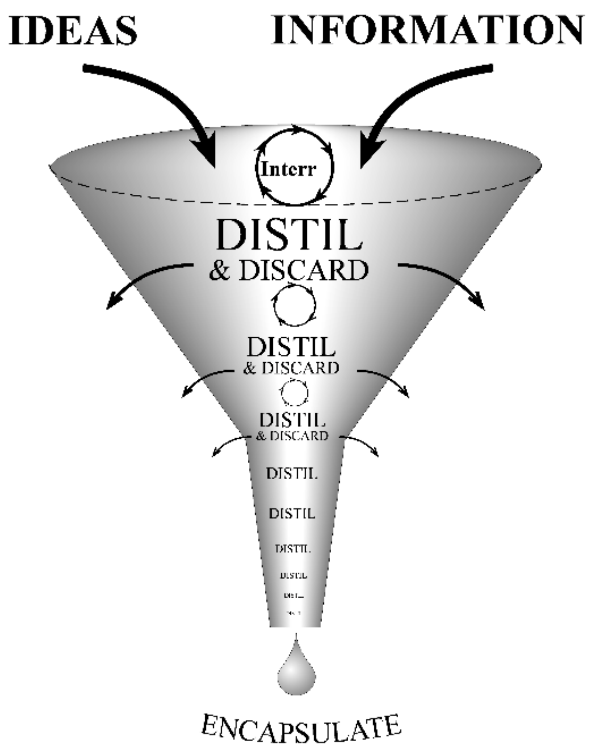


Figure 2-3. Distillation and encapsulation (Wild & Pfannkuch, 1999, p. 233)

(ibid., p. 233). By repeated application of the interrogative cycle to the ideas related to contextual knowledge and information related to statistical knowledge, ideas and information necessary for inquiries are left for the next interrogative cycle and unnecessary ideas and information are removed. Therefore, only the necessary ideas and information are left, allowing the essence to be encapsulated by repeating the interrogative cycle. As in Figure 2-2 (b), Figure 2-3 concerns the shuttling between the context sphere and the statistical sphere, and the PPDAC cycle always applies in Figure 2-2 (b), while Figure 2-3 implies that the interrogative cycle continues to apply.

As mentioned above, Figure 2-2 (a) shows the role of context in the PPDAC cycle, especially in the PP part, and Figure 2-2 (b) presents the role of context as the PPDAC cycle is repeated. The role of context in the repetition of the interrogative cycle is shown in Figure 2-3. In summary, the roles of context can be summarised under two points:

・The role of clarifying a problem and making a plan taking into account its relationship to statistical knowledge in the stage of PP within the PPDAC cycle in statistical inquiry; and

・The role of enabling shuttling to the statistical sphere by repeating the PPDAC cycle and the interrogative cycle.

Thus, the review of previous studies in Makar, Bakker, and Ben-Zvi (2011) pointed out that students used either contextual knowledge or statistical knowledge but not both when conducting statistical inquiry. Most of these previous studies relied on the findings of Wild and Pfannkuch (1999), and the roles of the summarised context were also taken into consideration. Therefore, the above two points are insufficient to completely explicate the role of context. In this study, the author considers this point from another viewpoint in Chapter 5.

Notes

1. The articles in special issues on specific themes were omitted, as the themes of these special issues did not include context.
2. The author summarises statistical knowledge and contextual knowledge including the relation to content knowledge and method knowledge. In Chapter 1, the author explained statistical content knowledge as the different kinds of concepts involved in inquiring statistically, and statistical method knowledge as knowledge about statistical method such as utilising formed statistical concepts (content knowledge) and judging what kinds of statistical concepts (content knowledge) should be utilised in a certain statistical inquiry activities. In this sense, statistical knowledge is both statistical content knowledge and statistical method knowledge. In some statistical inquiry, when knowledge related to context of the inquiry is used, the knowledge is called contextual knowledge in this research. Specifically, these knowledges are what follows in inquiring the cause of global warming based on

data:

・Contextual knowledge － Knowledge related to global warming

・Statistical content knowledge － Different kinds of graphs, Various regression analyses

・Statistical method knowledge － Knowledge such as judging what kind of graph and regression analysis should be utilised in analysing the collected data

・Statistical knowledge － Different kinds of graphs, Various regression analyses, Knowledge such as judging what kind of graph and regression analysis should be utilised in analysing the collected data

Chapter 3. Research Method

This chapter addresses the research questions set in Chapter 1 and designs the research process necessary to achieve the research objective. This process is a guide for conducting the research discussed after Chapter 4.

3.1 Comparative study with statistics education in New Zealand

The objective of this research is to develop a principle for the statistics curriculum in Japan which are to be consistent and common to all levels from elementary education to upper secondary education regarding what is a problem and what data are necessary in a statistical inquiry. Since both the problem and data are embedded in a context, this research then focuses on context as a key to achieve this objective. Therefore, identifying the context-related characteristics and issues of current statistics education in Japan provides us a perspective on developing a principle for the curriculum. In addition, to identify the characteristics and issues regarding the statistics education in Japan, it might be clearer to compare it with the statistics education in other countries. This research takes the case of New Zealand for the following reasons.

Statistics education in New Zealand has been developing since around 1950 (Forbes, 2014), and the 1992 version of the mathematics curriculum in New Zealand was particularly distinctive in the history. Statistics education was implemented in all school types within mathematics education. The mathematical process of problem solving, reasoning, and decision making was explicitly emphasised, and Tukey's (1977) idea of exploratory data analysis was introduced into statistics education (Begg, Pfannkuch, Camden, Hughes, Noble, & Wild, 2004). Additionally, Dr. Jane Watson, an Australian statistics education researcher, evaluated the statistics education highly in an assessment report of the mathematics curriculum in 1992 as follows: "Do not remove the emphasis that is in the 1992 NZ

Mathematics Curriculum. It is a leader in the world” (ibid., p. 12).

The current curriculum was then released in 2007, and statistics education held an even more important position. The main change was to rename the mathematics curriculum from ‘Mathematics’ to ‘Mathematics and Statistics’—that is, the statistical domain is treated not as one of domains in mathematics but rather as an independent subject like mathematics. This mathematics and statistics curriculum explicitly emphasising the formation of statistical literacy across all school types and grades is extremely rare worldwide (Watson, Fitzallen, Fielding-Wells, & Madden, 2018). As shown in Figure 3-1, the level corresponding to each grade is clearly indicated and divided into eight levels, so the mathematics curriculum also comprises eight levels.

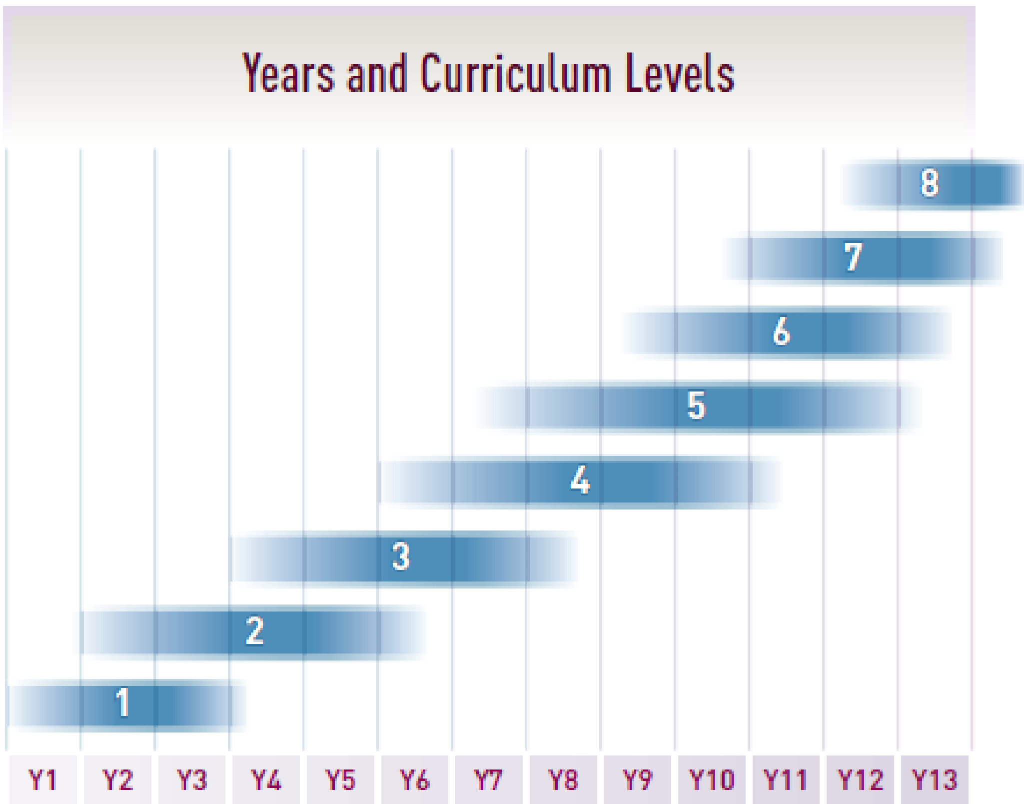
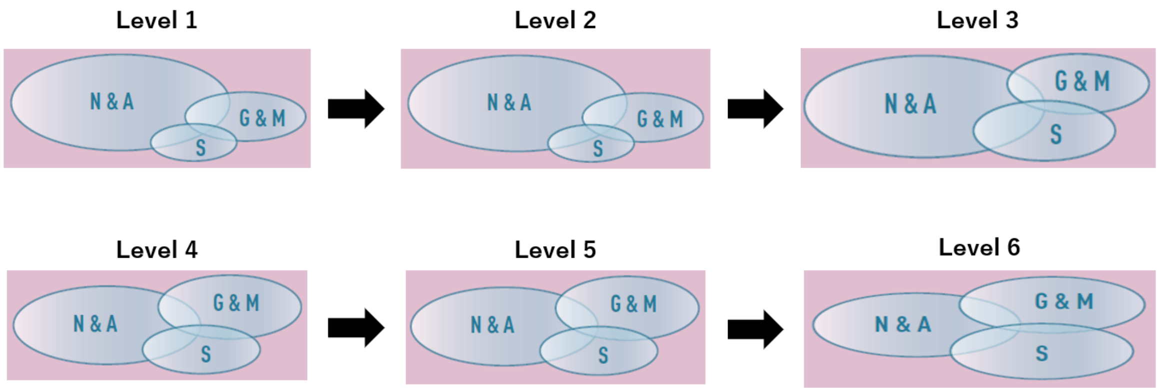


Figure 3-1. Years and curriculum levels (Ministry of Education, 2007a, p. 1)

Levels 1 to 6 consist of three areas: Number & algebra, geometry & measurement, and statistics. Levels 7 and 8 consist of two areas: Mathematics and statistics. While the teaching weight of number & algebra is high at Level 1, those of geometry & measurement and statistics increase as the level progresses, and the teaching weight of these three areas is set to be the same at Level 6 (Ministry of Education, 2007b, pp. 19-21). At Levels 7 and 8, with the two areas of mathematics and statistics, there is no description of teaching weight. Figure 3-2 shows the changes in the teaching weights of the three areas from Level 1 to Level 6.



Notes: N & A is 'Number & Algebra', G & M is 'Geometry & Measurement', and S is 'Statistics'.

Figure 3-2. The change in the teaching weight of areas in 'Mathematics and Statistics' (Ministry of Education, 2007b; extracted and edited by the author from pp. 19-21; Aoyama, 2013, p. 33)

The curriculum includes the teaching contents of statistics at each level, and it details are in Appendix A. The major changes in the statistics curriculum from 1992 to 2007 have occurred as follows: "from skills to immersing learning within the PPDAC (problem, plan, data, analysis, conclusion) statistical investigation cycle; from how to construct plots to how to reason from and interpret plots; from hand-drawn plots to automated plots; from reasoning from descriptive statistics to reasoning about populations from samples and underpinning inferential concepts including an appreciation and awareness of sampling variation; and from a non-assessed and hence non-taught interpretation and evaluation of media reports to an assessed statistical literacy component" (Pfannkuch & Wild, 2013, p. 609). Thus, it is necessary to form method knowledge for making decisions under uncertain situations using the PPDAC cycle. This is consistent with the international direction and also for future statistics education in Japan.

As mentioned above, the name of the subject in New Zealand has become 'Mathematics and Statistics'. There seems to be awareness of the difference between statistics and mathematics. As the previous studies stated, data with context is the object of statistics, while number without context is the object of mathematics. This is why statistics education in New Zealand may value context. Thus, the analysis of New Zealand may provide implications to consider the context-related characteristics and issues of current statistics education in Japan. Therefore, this research takes the case of New Zealand.

Furthermore, it would be important to focus on New Zealand from the perspective of the future of Japanese society, even though it is not directly related to this research. Since mathematics education is established as a social-historical practice (Abe, 2010; Iwasaki, 2007), it is to be expected that students' range of values to be dealt in mathematics education will expand not only to mathematical values but also to social ones[1] (Baba, 2007, 2009; Shimada, 2017; Shimada & Baba, 2012, 2014, 2015, 2016). Indeed, this should also apply to statistics education. Under this situation, education in New Zealand is informative for that in Japan. New Zealand is originally inhabited by Maori people, but has received many immigrants through English colonisation. It has a longer history as a multi-ethnic/multicultural nation than any other country, despite having a history of just over 70 years as a modern nation (Aoki & Sato, 2014). New Zealand has "the appearance of a high-quality welfare state supported by the spirit of mutual assistance in which everyone has equal rights and responsibilities" (Fukumoto, 2014, p. 93; translated by the author), and has led the world in implementing numerous pioneering social policies (i.e., the world's first women's suffrage law, the world's second elderly pension law, the world's second social security law, a community welfare law for disabled persons, anti-nuclear policy, and so on). This is because New

Zealand has a wealth of experience as a multi-ethnic/multicultural nation, utilises its direct experience with various conflicts between ethnic groups and cultures (i.e., tensions between the indigenous Maori and British immigrants), and has a method to implement policies before tension erupts. “The spirit of mutual assistance in which everyone has equal rights and responsibilities” (ibid., p. 93; translated by the author) also impacts the educational community, and this spirit is apparent throughout the curriculum (see Appendix A). It would be beneficial for Japan, which is expected to face the impending pressures of globalisation and internationalisation, to pay attention to education in New Zealand in such a sense that it has faced unification as a nation and diversification as multi-ethnic/multicultural one.

As mentioned in Chapter 1, it is not enough to mention the context in the course of study in Japan. Moreover, the New Zealand Curriculum (see Appendix A) also shows educational contents. The contents are not the specific contents to be taught but the abilities and skills to be required and acquired. Therefore, it would seem impossible to compare the Japanese course of study with the New Zealand curriculum.

Alternatively, this research focuses on textbooks and compares statistics education in Japan with that in New Zealand. The purpose as externalised values (Ernest, 2015) and the knowledge as the product of activities (Leontyev, 2009) are externalised things which are difficult to visualise, and “almost all issues in mathematics education are reduced to those in notation” (Hirabayashi, 1987/2013, p. 390; translated by the author). In this sense, it is textbooks as the central repository of mathematical notation that directly affects this purpose and knowledge. Furthermore, it is not an exaggeration to say that the purpose and knowledge mentioned above dominate all matters described in the textbooks since, under Article II of Act on Temporary Measures concerning Publication of Textbooks, it is obligatory for all schools from elementary to upper secondary to use textbooks in Japan. Therefore, this research focuses on textbooks for its comparison of Japan and New Zealand. This textbook comparison is conducted by quantitative analysis and case analysis.

In addition, the specific contents and methods for acquiring the abilities and skills shown in the New Zealand mathematics curriculum are entrusted to each teacher, and all matters, such as the contents of educational practices, their construction, and teaching materials, are up to the teacher because there is no compulsory system for textbooks (Ministry of Education, 2007c, pp. 37-44). Therefore, these may be unique characteristics related to context even in educational practices regarding statistics in New Zealand. For this reason, this research also analyses educational practices by a teacher qualitatively because the treatment is left up to each teacher in New Zealand unlike Japan. For this analysis of educational practices, a comparison with educational practices of statistics in Japan is not conducted because the educational practices for statistics in Japan are basically based on textbooks and reflect the same contextual characteristics as those of textbooks.

3.2 Reflection of previous studies on statistical inquiry with context

The comparison of textbooks in Japan and New Zealand and the analysis of educational practices in New Zealand clarify the characteristics and issues of the context in Japanese statistics education. This study then establishes a framework for method knowledge related to statistical inquiry. The method knowledge is the basic policy of statistics education in Japan, and is specifically concerned about the problem and data for statistical inquiry. This is the framework of statistical inquiry related to context. To establish it, the author uses the method of literature review and summarises the findings of the review of previous studies in Chapter 2 and of the current studies in statistics education research.

3.3 Teaching unit and teaching experiment focusing on context

The author develops a teaching unit (TU) based on the framework. The teaching unit was proposed by Dr. Wittmann, a mathematics education researcher in Germany, whose intention was to develop teaching materials or an educational practice which aims for students' independent inquiries, and it is also called the substantial learning environment (Akinwunmi, Höveler, & Schnell, 2014; Wittmann, 1984, 1995, 2001a, 2001b, 2005). The teaching unit has the following four properties (Wittmann, 2001a, p. 2):

1) It represents central objectives, contents, and principles of teaching mathematics at a certain level.
2) It is related to significant mathematical contents, processes, and procedures *beyond* this level, and is a rich source of mathematical activities.
3) It is flexible and can be adapted to the special conditions of a classroom.
4) It integrates mathematical, psychological, and pedagogical aspects of teaching mathematics, and so it forms a rich field for empirical research.

A teaching unit satisfying these four properties may become a resource in mathematics education from the elementary school to the upper secondary school, if the material contents and the teaching methodology are adjusted appropriately, as under properties (2) and (3). In fact, Wittmann has stated that "it is only from this perspective [MATHEMATICS] that the unity of mathematics teaching from the elementary through the upper secondary levels can be established" (Wittmann, 1995, pp. 359-360; the parenthesis is by the author). In other words, the teaching unit is defined as a model material which can be used at all levels, from elementary school to the upper secondary school. If the consistency at all levels is embedded in the teaching material, it implies a curriculum and can be said to be the teaching material capturing the essence of the curriculum. Since it is consistent with the intention of this research, the notion of teaching unit is adopted.

The teaching unit has the four properties expressed with the short terms 'objectives (O)', 'materials (M)', 'problems (mathematical problems arising from the context of the unit) (P)', and 'background (mostly mathematical, sometimes psychological background of the unit) (B)' (Wittmann, 1984). Additionally, Iwasaki (2007) mentions the relationship between the four terms and four properties as " 'objectives' is based on property (1), and 'materials' and 'problems' have to respond to properties (2) and (3). In addition, 'background' is needed to clarify property (4) precisely. Therefore, the teaching unit can be considered as a teaching material case consisting of these four items or an aggregation of them" (p. 185; translated by the author). Thus, this research also regards these four items as four properties of the teaching unit.

As mentioned in Chapter 1, Wittmann's view of mathematics was summarised as MATHEMATICS, and in fact he mentions that "the design of the teaching units, coherent sets of teaching units, and curricula has to be rooted in MATHEMATICS" (Wittmann, 1995, p. 359). Additionally, he takes another view of mathematics as the science of patterns. This is based on Sawyer (1955) and Devlin (1994). Therefore, a summary of his view of mathematics is MATHEMATICS as the science of patterns. This means that the teaching unit exists as teaching material whose mathematical objects are various patterns embedded in society. In fact, the example of teaching unit is the arithmogon (Figure 3-3), including the mathematical development from addition and subtraction, which are both used in science, commerce, and daily life, to simultaneous equations and then to matrixes:

TU *Arithmogons*

O: Adding, subtracting, operative investigation of these compositions, searching-discovering.

M: Trigonal and quadrilateral arithmogons (partly on working sheets).

P: Given numbers in some vertices and edges. Find the other numbers!

B: Linear independence of the numbers in vertices and edges, systematic solution by means of systems of linear equations, operative principle.

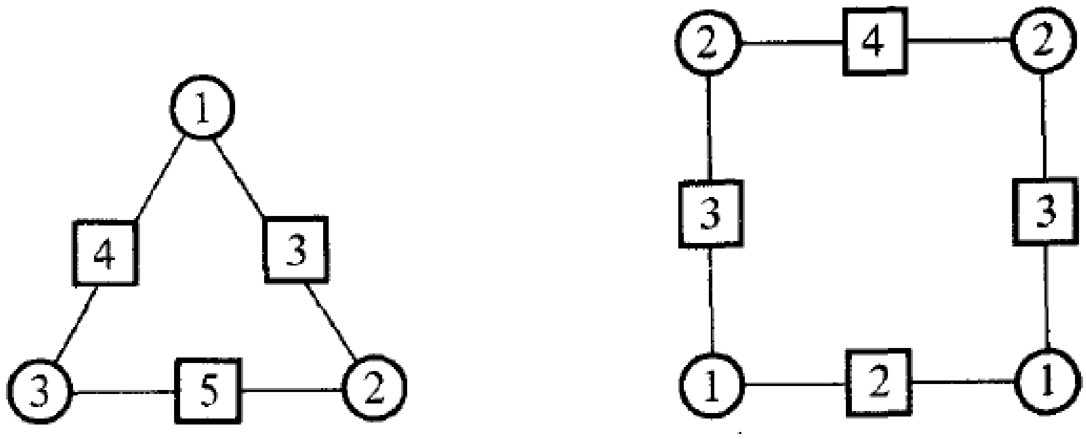


Figure 3-3. Teaching unit on 'Arithmogons' (Wittmann, 1984, p. 31)

In this way, it is usually mathematical sequences that guarantee consistency from elementary education to upper secondary education in teaching units. The reason that such mathematical sequences can be included is that mathematics itself is a patterned crystal substance. The laws, so to speak, which must be obeyed by mathematics are axioms. There are some exceptions to the standard axioms (e.g., non-Euclidean geometry), but it is doing mathematics that can construct any pattern within this system of axioms, and these constructs are mathematics. In doing mathematics, there is always an axiom to which one may always work back. Contrarily, in statistics, it can be said that the returning place is not an axiom but a context of the inquiry. Neither the 99% confidence interval nor the 95% confidence interval can be a place to go back when making the interval estimation. Rather, the choice depends on the decision of the investigator himself/herself based on the context. There are no reasons the confidence interval must be 99% or 95%. It may be possible to find a 95% confidence interval if you admit something may occur that does not fit within the confidence interval to a certain degree, but a 99% confidence interval is selected in medical statistics involving human life. In this way, it is not necessary for all people to obtain the same inquiry results. This is a characteristic of statistics such that different inquiry results are accepted depending on the person. Therefore, not only statistical sequences but also another point must be prepared in statistics education to offer consistency from elementary education to upper secondary education. This is a feature of a teaching unit.

Both sequence and consistency must be closely related to the goal of statistics education. In other words, this research needs to adopt an appropriate sequence for the development of method knowledge related to a statistical inquiry which accords with direction of statistics education in Japan. Therefore, this research adopts the structure of statistical literacy, as shown in Table 3-1.

Table 3-1. Statistical literacy construct (Watson & Callingham, 2003, p. 14)

Level	Brief characterization of step levels of tasks
6. Critical Mathematical	Task-steps at this level demand critical, questioning engagement with context, using proportional reasoning particularly in media or chance contexts, showing appreciation of the need for uncertainty in making predictions, and interpreting subtle aspects of language.
5. Critical	Task-steps require critical, questioning engagement in familiar and unfamiliar contexts that do not involve proportional reasoning, but which do involve appropriate use of terminology, qualitative interpretation of chance, and appreciation of variation.
4. Consistent Non-critical	Task-steps require appropriate but non-critical engagement with context, multiple aspects of terminology usage, appreciation of variation in chance settings only, and statistical skills associated with the mean, simple probabilities, and graph characteristics.
3. Inconsistent	Task-steps at this level, often in supportive formats, expect selective engagement with context, appropriate recognition of conclusions but without justification, and qualitative rather than quantitative use of statistical ideas.
2. Informal	Task-steps require only colloquial or informal engagement with context often reflecting intuitive non-statistical beliefs, single elements of complex terminology and settings, and basic one-step straightforward table, graph, and chance calculations.
1. Idiosyncratic	Task-steps at this level suggest idiosyncratic engagement with context, tautological use of terminology, and basic mathematical skills associated with one-to-one counting and reading cell values in tables.

Watson and Callingham (2003) derive a hierarchical structure of statistical literacy from records and surveys of more than 4000 students. This hierarchical structure comprises six levels (Idiosyncratic－Informal－Inconsistent－Consistent/Non-critical－Critical－Critical/Mathematical) as outlined in Table 3-1. Statistical literacy is shown sequentially from Level 1 to Level 6. Level 6, the last level, is the highest order of statistical literacy. Starting from the point where students can read the data, students are required to draw graphs and perform numerical calculations on which context has some effect, and finally must regard the context as the object of inquiries and explain it in words. These six levels are the deepening sequence of method knowledge. Thus, the author should be conscious of this sequence in developing a teaching unit.

Dr. Wittmann points out that the development of teaching units must be performed together with their evaluation by teaching experiments (Wittmann, 1995, 2001a). Wittmann (1995) stated that "the most important results of research in mathematics education are sets of carefully designed and empirically studied teaching units that are based on fundamental theoretical principles" (p. 369), and further indicated how to utilise data collected in teaching experiments so that "they [the data collected in teaching experiments] tell us something about the teaching/learning processes, individual and social outcomes of learning, children's productive thinking, and children's difficulties. They also help us to evaluate the unit and to revise it in order to make teaching and learning more efficient" (p. 368; the parenthesis is by the author). In other words, he emphasised the significance and role of teaching experiments in developing teaching units, which involves shuttling between theory and practice (Figure 3-4).

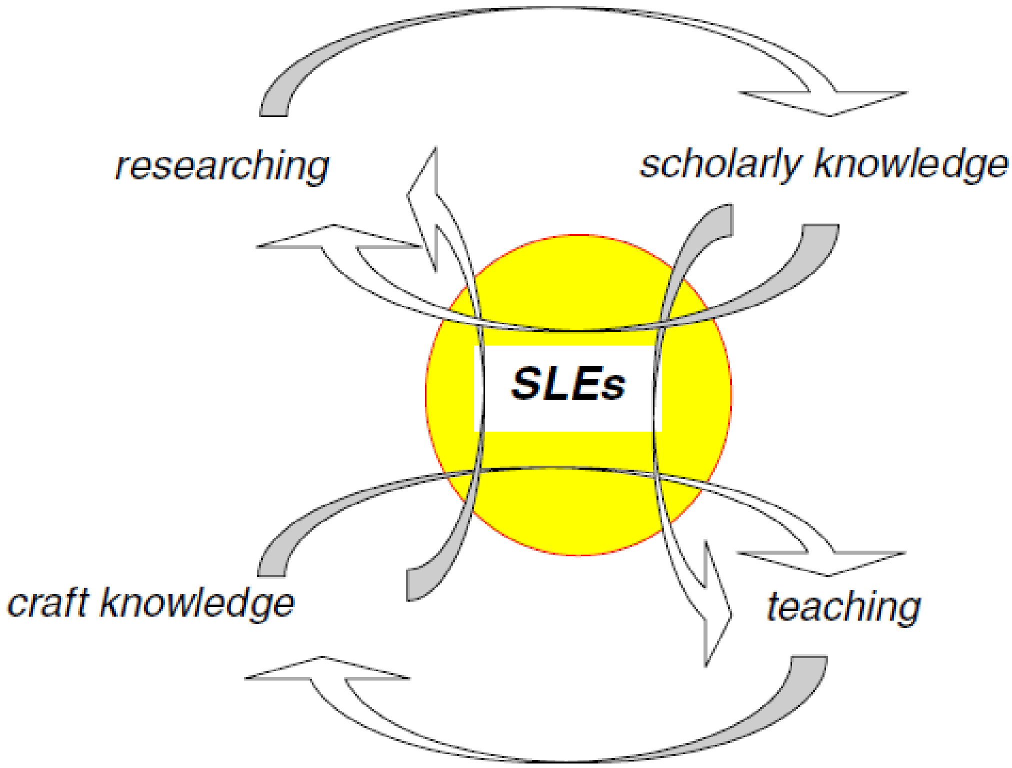


Figure 3-4. Shuttling between theory and practice (Wittmann, 2001a, p. 5)

With respect to theory, the theory used in the developmental stage of the teaching unit is evaluated by teaching experiments, and scholarly knowledge is thereby accumulated while further reinforcing the theory. As for practice, craft knowledge about students' thinking, the methods of teacher intervention, and so on are accumulated through teaching experiments of the developed teaching units. Therefore, this research aimed to develop a teaching unit through literature review, implement a teaching experiment of the developed teaching unit, and analyse the teaching experiment qualitatively to validate the framework.

3.4 Summary of the research method

Let us now summarise the research method of this study. By comparing Japanese and New Zealand textbooks quantitatively and qualitatively and analysing New Zealand's educational practices qualitatively, the author identifies the characteristics and issues of Japanese statistics education in terms of context. Then, based on these results, a framework is constructed for method knowledge related to statistical inquiry, which is the basic policy of statistics education in Japan. Then, the author validates this framework. To this end, a teaching unit is developed using the framework based on the literature review, and a teaching experiment of the developed teaching unit is conducted. Finally, this study develops a principle for the Japanese statistics curriculum focusing on context which are consistent and common to all levels from elementary education to upper secondary education, by analysing this teaching experiment qualitatively.

Notes

1. Mathematical values are criteria related to mathematics, while social values are criteria related to society.

Chapter 4. Context-related Characteristic and Issue of Japanese Statistics Education

In this chapter, the author compares Japanese and New Zealand textbooks and analyses New Zealand's educational practices from the perspective of context, then identifies the characteristics of and issues regarding context in the current statistics education in Japan.

4.1 Textbook comparison between Japan and New Zealand from the perspective of context

4.1.1 Method

According to Cooper and Harries (2002), with regard to the national test of mathematics conducted to students between ages 11 and 12 in the United Kingdom, the following two types of problem are given: A real problem presented in the national test and a modified problem wherein the reality is added to the original problem. As a result, it is reported that the students' motivation for the latter problem was higher and more realistic responses were seen. Similarly, several previous studies have suggested that more realistic and authentic problems should be applied in mathematics education (e.g., Sierpinska, 1995). It has also been pointed out that mathematics education in Japan has to better deal with more realistic and authentic problems and to pay attention to the fictitious property. The issue must be considered in light of the degree of reality and/or fictitiousness of the property, because the necessary elements for realistic mathematics materials are neither completely real nor completely unreal. (e.g., Hirabayashi, 1975, 1988; Koide, 2009).

As mentioned in Chapter 1, mathematics education is de-contextualisation-oriented, while statistics education is context-dependent because statistics is a discipline which faces contextual tasks (Moore, 1990; Shaughnessy, 2007). Therefore, it is important that the problems which students inquire in statistics education have some context. The reality and authenticity of the context are thereby confirmed. For example, the commentary on the course of study in Japan shows "to estimate the total number of headwords appearing in an English-Japanese dictionary by a sample survey" (MEXT, 2018, p. 156; translated by the author) for the unit on sample surveys to be taught in the 3rd grade of lower secondary school. Certainly, this is useless according to some criteria of usefulness in the real world. Therefore, a given context should be examined to see whether it bears reality or authenticity to a given problem.

The description thus far has consisted of an analysis of context for the problems, but it henceforth involves an analysis of the context in solving the problems. In Konold, Pollatsek, Well, and Gagnon (1997), it is pointed out that context within the problems in statistical classes may mislead the students, and the problems may be solved based on personal experience rather than collected data. Furthermore, Makar, Bakker, and Ben-Zvi (2011) stress the need to harmonise contextual knowledge with statistical knowledge, as described in Chapter 2. As mentioned above, mathematics education is de-contextualisation-oriented, while statistics education is context-dependent, but the current statistics education is treated in the same way as mathematics education in Japan. Thus, it is possible to set the hypothesis that 'the situation would be set in which students can solve even problems with context by ignoring the context'. Therefore, the author investigates whether the context is included within the (expected) inquiry or answer of the given problems. Finally, an investigation is conducted in terms of the kinds of context. The author classifies the contexts using six types of context, namely, Sports context, Scientific context, Social context, Artistic context, Economic context, and Political context (Zapata-Cardona & Escobar, 2016). Here, scientific context refers to not only contexts related to physics and chemistry but also any other contexts besides Sports contexts, Social contexts, Artistic contexts, Economic contexts, and Political contexts.

The following describes the object of analysis and the selection of the textbooks. First, the author focuses on the unit 'Analysis of Data' in Mathematics I as the object of analysis, which is implemented for 15- and 16-year-old students in the 1st year of upper secondary school in Japan. This is because this unit has received the closest attention in the current mathematics curriculum, and students must complete this unit to graduate from upper secondary school. For New Zealand, the author uses the Level 7 textbook, corresponding to the 1st year of upper secondary school in Japan. As for textbooks, most of New Zealand's textbook orders have been secured by Pearson, according to Aoyama (2013), and this publisher's textbook *GAMMA MATHEMATICS* (Barton, 2010) is used for the analysis. On the other hand, there are five textbook publishers in Japan, each of which has published several textbooks. Therefore, this research selects for analysis those which may be the most used from the tested textbooks[1]. Accordingly, the author contacted each publisher to ask: 'Which textbook is the most used?' or 'What is the most common textbook?' in case the publisher could not answer the first question. In this way, one textbook from each publisher was selected for analysis in this study (Fujita et al., 2016; Hasegawa et al., 2017; Matano et al., 2017; Okamoto et al., 2017; Oshima et al., 2017).

4.1.2 Quantitative analysis

The author conducted the analysis of the questions and exercises which students are expected to solve. The author then decided to apply the criteria of the presence or absence of context within the problem (A), the presence or absence of the reality/authenticity of context within the problem (B), and the presence or absence of context within the inquiry or answer required for the problem (C). Thus, each problem was classified in the form (A, B, C). When A is 'No', B and C are neither 'Yes' nor 'No'. This is because the problem itself does not have any context and we cannot classify it. In this case, it is expressed as (No, －, －). When there is a context within the problem, which means that A is 'Yes', but B and C are not mentioned, then the problem is expressed as (Yes, ○, ○). Therefore, there are five classification items: (No, －, －), (Yes, ○, ○), (Yes, No, No), (Yes, Yes, No), and (Yes, Yes, Yes).

The author would like to explain these items by using some concrete examples before the actual classification. First, (No, －, －) is considered in the following problem (Hasegawa et al., 2017, p. 126; translated by the author):

【Example Problem of (No, －, －)】

Find the quartiles Q1, Q2, and Q3 for the following data.

(1) 2, 3, 3, 4, 5, 6, 6, 8, 9

(2) 3, 4, 4, 5, 5, 7, 8, 10, 11, 13

In this problem, students find each quartile based on only given numerical values, so there is no context related to the real world. Thus, it is classified as (No, －, －).

Next, the problem of (Yes, No, No) is considered. If there is a problem such as “Estimate the total number of words in the headline listed in a certain English-Japanese dictionary by sample survey” (MEXT, 2018, p. 156; translated by the author), there is a context of the number of words in the English-Japanese dictionary. However, there is generally no opportunity to estimate the number of words in the dictionary. Even if there is, the numerical value is usually described in the dictionary, and it is not necessary to estimate it by sample survey. There is a context within this problem, but the context has no reality/authenticity. Thus, it is classified as (Yes, No, No).

In addition, the author gives an example of (Yes, Yes, No) as the following (Okamoto et al., 2017, p. 180; translated by the author).

【Example Problem of (Yes, Yes, No)】

The following data is a score record of repeated side jumps (points) for 12 high school basketball players. Answer the following questions.

54, 56, 60, 63, 66, 67, 53, 54, 56, 61, 64, 66

(1) Find the median.

(2) Draw a box and whisker plot.

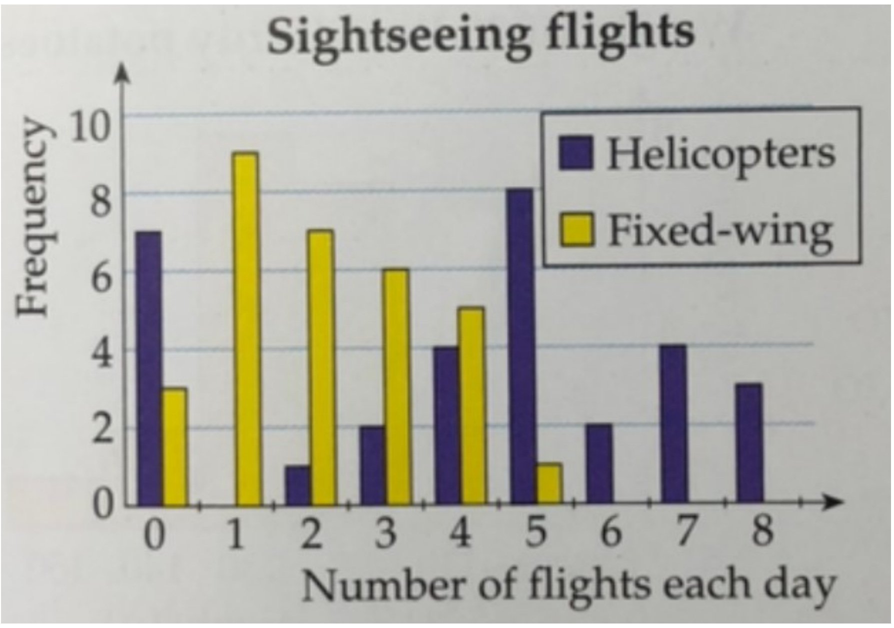
(3) Find the quartile range.

There is a context within this problem, which is the score record of basketball players' repeated side jumps. Furthermore, this is the context in which data can be collected practically unlike the previous example of (Yes, No, No). However, this context is not used at all in the problem solving, and the answer is obtained based on only numerical values. Therefore, this problem is classified as (Yes, Yes, No).

Finally, the author explains (Yes, Yes, Yes) using the following (Barton, 2010, p. 396).

【Example Problem of (Yes, Yes, Yes)】

A sightseeing company operates two kinds of aerial tours in the Mt Cook area – by helicopter and by fixed-wing aircraft. The bar graph below shows the number of flights each day over a period of one month.



Here are some conclusions from this data. Each one is wrong, or cannot be justified. Comment on each one, giving statistical or other reasons why it is incorrect.

(a) ~ (c) […]

(d) The column for "0" shows that helicopters are affected by bad weather more than fixed-wing aircraft are.

There is a context of the number of flights per day by helicopter and fixed-wing aircraft, and it can be said that this context has the reality/authenticity. In the problem solving, students cannot answer using only the numerical values, and this context needs to be used as an object of the inquiry. As this inquiry, students can answer, such as the following: 'This conclusion is incorrect because it could relate to passenger choice or helicopters needing more frequent maintenance'. Thus, this problem is classified as (Yes, Yes, Yes).

(Yes, ○, ○) means combination of all three types of (Yes, No, No), (Yes, Yes, No), and (Yes, Yes, Yes).

The problems are classified according to these categories. Since there are five textbooks, the averaged data were used for the comparison with New Zealand whose textbook orders have been secured mostly by one publisher. Japan, unlike New Zealand, has a textbook verification system whereby the textbooks were created under certain restrictions. Therefore, there are only minor differences among the five publishers (five textbooks), so the author judged that using the average for comparison with New Zealand presents no problem.

Table 4-1 shows the result of classifying the problems in Japan and New Zealand by context. As shown in Table 4-1, there is no significant difference between the two countries in terms of the proportion of context/non-context of problems. The fact that there is no context without reality/authenticity in New Zealand is unique. Compared with Japan, the proportion of problems with context within the inquiry or answer is high. It can also be said that there are many problems with no

Table 4-1. The result of classification of problems by context

(A, B, C)		Japan（80.8 problems）	New Zealand (322 problems）
(No, －, －)		19（23.5%）	60（18.6%）
(Yes, ○, ○)		61.8（76.5%）	262（81.4%）
	(Yes, No, No)	0.2（0.2%）	0（0%）
	(Yes, Yes, No)	60.2（74.5%）	173（53.7%）
	(Yes, Yes, Yes)	1.4（1.7%）	89（27.6%）

（The number in front of parentheses shows the number of problems and the number in parentheses shows their proportion amongst all problems for each country. To make better comparisons with New Zealand, the average for one textbook is used for each number of Japan. '－' means N/A and '○' means either.）

context within the inquiry or answer in Japan. However, the textbooks deal with problems with a degree of reality/authenticity, since there is only one problem that does not display reality/authenticity. In summary, the characteristic of how context is treated in Japanese statistics education is that there are efforts to incorporate context in the problem itself, but they do not include context in the inquiry and answer when posing the problem.

Next, the 309 (61.8×5) problems in the Japanese textbooks and the 262 problems in the New Zealand textbook containing context may be classified into six types of contexts that have been previously specified such as Sports context, Scientific context, Social context, Artistic context, Economic context, and Political context. The results are shown in Table 4-2 below.

If we consider the Japanese column, it can be said that most problems have sports or scientific contexts, in comparison with New Zealand. Furthermore, focusing on the scientific context, which accounts for 234 of the 309 problems (75.7% of the total), 97 problems had contexts related to the weather

Table 4-2. The result of classification of the contexts within the problems

Types of Context	Japan（309 problems）	New Zealand (262 problems）
Sports	69（22.3%）	13（5.0%）
Scientific	234（75.7%）	123（46.9%）
Social	3（1.0%）	75（28.6%）
Artistic	0（0%）	0（0%）
Economic	3（1.0%）	46（17.6%）
Political	0（0%）	5（1.9%）

and 80 problems had contexts related to tests. Therefore, about 80% of the total is occupied by the three contexts of sports, weather, and tests. In contrast, no problems in New Zealand have artistic context, but the remaining five kinds of contexts occur more evenly than in Japanese textbooks. In addition, although scientific contexts occur the most, a wide variety of subjects were included. Thus, in Table 4-1, the author found that problems have contexts at the roughly same proportion in Japan and New Zealand, but if we focus on the types of contexts, there is a difference in terms of types of context between two countries. In Japanese statistics education, there is large concentration of types of context such as sports, weather, and tests.

4.1.3. Case analysis

Thus far, this research has made a quantitative comparison of textbooks in Japan and New Zealand. Here, we shall analyse them qualitatively by examining specific problems in the textbook. It is possible to compare the textbooks in Japan and New Zealand by considering the problems with context within the inquiry and answer. Therefore, the author examined the (Yes, Yes, Yes) problems in both countries. Japanese textbooks have 7 problems for this type, while New Zealand textbook has 89 problems.

First, among the (Yes, Yes, Yes) problems, the only problem which requires decision making in Japan is the following (Matano et al., 2017, p. 184; translated by the author).

【Problem】

In a basketball club, you must select one among the three players A, B, and C to play the next game. You decided to analyse the characteristics of the three players using their performance data to appoint the player. The following table shows the scores of the three players in the last ten games, arranged in ascending order.

A	10	14	16	16	16	18	18	22	24	26
B	4	6	10	14	14	20	24	28	30	30
C	12	14	14	16	16	20	20	22	22	24

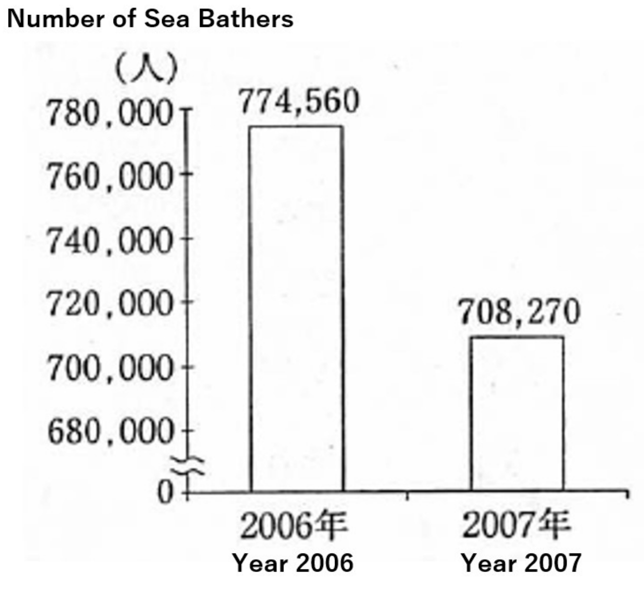
From these data, let's draw a boxplot … for each of the three players A, B, and C. In addition, let's consider what can be said about the characteristics of each player from the results. If you are the coach, which player will you select?

This problem is intended to elicit students' inquiries based on the following three characteristics: the data of the three players A, B, and C, which have the same mean and almost the same median, the boxplot of player B is longer than the boxplots of players A and C, and the boxplot of player C is shorter than the boxplots of players A and B. Expected answers include 'The average is the same for all players, but player C had the most stable score, so I will select player C' or 'The average is the same for all players, but player B's probability of scoring 30 points is highest from this data, so I select player B'. There is no one correct answer regarding which player to choose and the emphasis is placed on the explanation of why the player is selected. However, this problem contains the instruction that students calculate the mean, first quartile, median, third quartile, range, and interquartile range of each player's data, draw each boxplot, and judge the decision 'based on those calculation and figure'. Thus, at this stage, students' decision making is based not on contextual knowledge but on statistical knowledge. In this sense, knowledge to be used in this problem is limited.

In the past, a problem which contained contextual knowledge in the inquiry and answer could be seen in the unit 'Analysis of Data' (Okamoto et al., 2014, p. 202; translated by the author).

【Problem】

When upper secondary school student A was at a lower secondary school, he was investigating, as his research project over summer vacation, the number of sea bathers in July in Fujisawa City, Kanagawa Prefecture, where he lived. The following bar graph shows the records for 2006 and 2007 based on the results of his survey.



He analysed this graph and concluded that the number of sea bathers in Fujisawa City decreased sharply in 2007 over 2006. Let us discuss the correctness of his results based on the graph.

There are three main answers expected for this problem. The first is to make decisions after numerical processing. By calculating the difference between the numbers of sea bathers in 2006 and 2007 and dividing the result by 31, it is possible to estimate the difference per day. The second answer is 'There are only two data points for 2006 and 2007, and I cannot judge whether it is correct without checking the number of people in other years'. Finally, the third is an answer focusing on factors besides the number of people, such as 'There may have been a special event in 2006, so I cannot make a decision without looking at the number of people per day' or 'There is a possibility that there were a lot of rainy and cold days in 2007, so I cannot make a decision without looking at data about the weather'. While both the first and second answers focus on the number of people, the third answer is related to an inquiry into the reason for the difference in the numbers of people between 2006 and 2007. Under this perspective, the consideration has shifted from numerical points to various contexts (events, weather, and so on) in each year. In this way, the author presented a problem which includes not only statistical knowledge but also contextual knowledge.

Next, the following problem is one of the (Yes, Yes, Yes) problems in New Zealand which include contextual knowledge within the inquiry or answer (Barton, 2010, pp. 409-410).

【Problem】

Read the following media article about a poll on who should succeed the Queen.

Charles and William evens for throne

by Kara Segedin

As Prince William prepares to leave New Zealand, a poll of the *Herald* reader panel shows the 27-year old in a neck-and-neck race with his father as the popular choice to succeed the Queen.

The survey – taken before the Prince's recent three-day tour – found 33.3% wanted Prince Charles to be the next monarch, with 30.2% favouring William. But 29.4% of respondents preferred a republic in the event Queen Elizabeth II died or abdicated.

Women aged 18 to 44 and men aged 18 to 29 would prefer Prince William as the next head of state, while men aged 45 to 59 were particularly keen on New Zealand becoming a republic.

Almost half of the supporters of a republic – 49.1% – had no opinion on who should be the first head of state. Former Governor-General Dame Silvia Cartwright was the top choice, with 15.5% backing.

The online survey of the *Herald* Readers' Panel was conducted by the Nielsen Company between December 10 and 17.

(Source: *The New Zealand Herald*, 19 January 2010, p. A3)

Poll results

When Queen Elizabeth II dies or abdicates, who should replace her as New Zealand's head of state?

・ Prince Charles	33.3%
・ Prince William	30.2%
・ Another royal	1%
・ NZ to be a republic	29.4%
・ No opinion	6%

If New Zealand became an independent republic, who should be the first head of state?

・ No opinion	49.1%
・ Dame Silvia Cartwright	15.5%
・ Sir Paul Reeves	11.7%
・ Jim Bolger	3%
・ Jonah Lomu	1%

Give two reasons why this survey was *not* representative of all New Zealander's opinions.

There are two main answers expected to the above problem. The first is an answer using statistical knowledge, such as 'Only proportions are shown because of the survey, so there is the possibility that only a small amount of data was collected'. The second is an answer using contextual knowledge such as 'This is a survey of the readers of the *Herald*, so the data are likely to be biased'. As in '... the readers of the *Herald*, so ...', the second answer says that the criteria of the judgment are contextual. There are answers based on numerical data like the first one and answers relying on non-statistical knowledge such as readers of the media article in the second answer, which is the same as for the Japanese problem. However, the New Zealand problem requires both contextual knowledge and statistical knowledge in the second answer, unlike the Japanese problem. Even in the above-mentioned Japanese problem about the number of sea bathers, the statistical knowledge to be used is at most that of calculating the increase or decrease from 2006 to 2007. On the other hand, in this New Zealand problem, it is possible for students to answer in such a form that contextual knowledge and statistical knowledge are synthesised. In other words, answers may combine statistical knowledge of sample survey with contextual knowledge of opinion polls.

From this comparison of the two countries, it can be said that the New Zealand textbook posed more realistic/authentic and more complex problems. In addition, the context of opinions is on the next king in New Zealand, and is a social and cultural issue in this country. It uses a genuine media article, and moreover, places the statistical content of the sample survey properly. On the other hand, problems in Japan tend to emphasise the use of statistical knowledge but not contextual knowledge. Thus, it is found that there are efforts to include some context in the problem itself, but these efforts are not enough to include context in the inquiry and answer when posing the problem, and that there is a bias in the types of context within the problem itself. In addition, the synthesis of contextual knowledge and statistical knowledge is also an issue, as pointed out by Makar, Bakker, and Ben-Zvi (2011).

4.2 Educational practices in New Zealand[2]

4.2.1 Basic information

A school in Auckland, New Zealand, was chosen for this study because the Head of Mathematics and Statistics Department spoke Japanese. The period of the study is for a week (from 27 February to 3 March 2017). This research examines the educational practices given to the thirteenth-grade students (the final grade in upper secondary school). The students in this school are divided into different classrooms depending on their chosen route or degree of progress, and the author was allowed to observe the highest educational practices for those wanting to progress to the university.

The theme of the educational practices was time series. In the statistics' educational practices of the school, they spend from a month to a month and a half on each theme. In New Zealand, the new academic year starts between the end of January and the beginning of February. Thus, the observed educational practices were on the first theme for the thirteenth grade, and the observation was done during the final week in the five-week period dedicated to time series. The school followed the achievement levels set by the New Zealand Qualifications Authority (NZQA) as the learning objectives. The learning objectives of the observed educational practices were as follows: "Investigate time series data, with statistical insight involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle, and may include reflecting about the process; considering other relevant variables; evaluating the adequacy of any models; or showing a deeper understanding of models" (NZQA, 2017, p. 2).

The teacher performs two roles in the educational practices in New Zealand. The first is 'to facilitate students' exploration activity'. Most of the lesson time was allocated to students' exploration, and the teacher observed how the students were engaged with the activity and responded to students who requested the teacher's attention by raising their hands. The second is 'transmission of knowledge to the students'. Some of the questions that the students asked were not solvable without new statistical knowledge. In that case, the teacher not only responded to the student who raised the question, but also explained the new statistical knowledge to the whole class. For example, when making a future prediction by analysing time series data, a student was not sure of the method needed to express in numerical form the value that she could plot on the graph. In this instance, the teacher shared knowledge of regression lines with the whole class. In New Zealand, statistical exploration of data is performed using statistical knowledge. In other words, statistical knowledge is a tool for further statistical exploration. Consequently, the role of the teacher in New Zealand, 'to transmit knowledge to the students', is more accurately described as 'to transmit knowledge to the students unilaterally according to the students' needs'. Whether or not this is a general role of the teacher in educational practices in New Zealand is not clear, but at least this was evident by the observation. In addition, in the educational practices in New Zealand, the students were given a single task: 'To explore data and to write up a report'. The basic datasets were provided to the students, but if different information was required during the exploration, the students would look up it in the Internet by themselves. Since the students took an initiative in deciding which data to focus on from the given datasets, as well as what to explore, they engaged themselves with the exploration on their own volition. With reference to the other role of the teacher, 'to facilitate students' exploration activity', the teacher is required to have a depth of knowledge to respond to the students' needs at different levels of exploration.

4.2.2 Lesson analysis through episodes

Some episodes from a student's exploration (hereafter, Student A) in the educational practice that the author observed are discussed here. Student A focused on the time series data of wine consumption in New Zealand from the given datasets, and based on the time series graph (Figure 4-1), posed the question, 'Why did wine consumption, which was on the rise for several years, decrease in 2007 and 2008?'

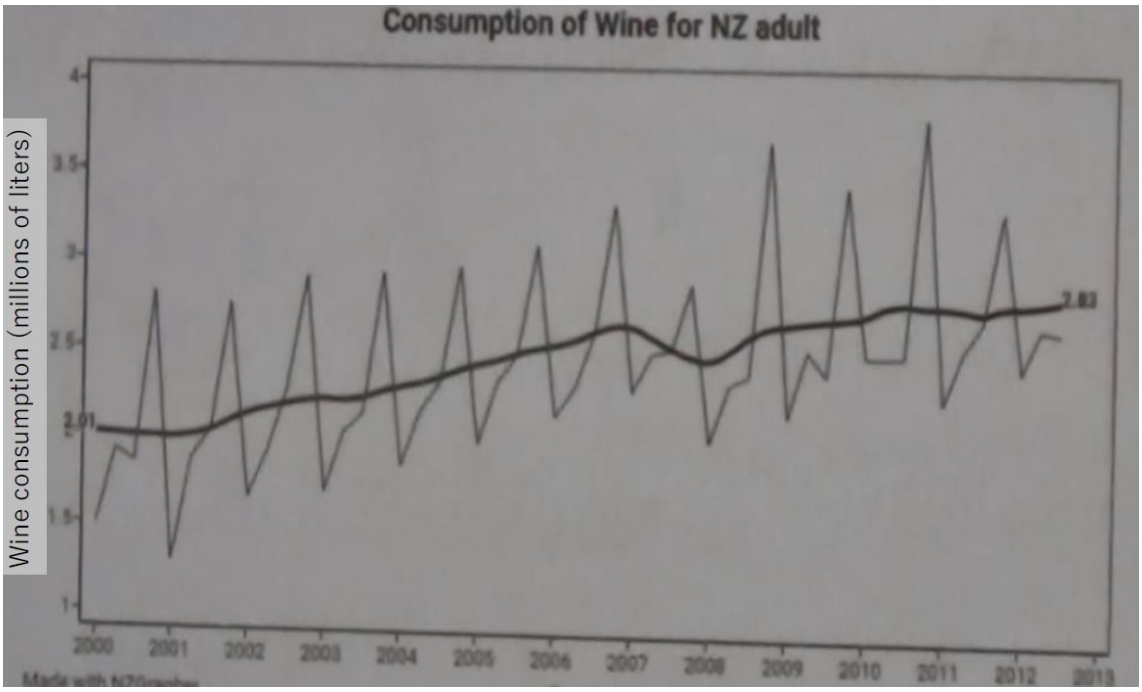


Figure 4-1. A screen shot of time series graph Student A made

(The letters on the left are rewritten by the author)

To solve this question, she investigated New Zealand's social situation around 2007 and 2008. After this investigation, she expected that the reason for the decrease of wine consumption in 2007 and 2008 was related to the financial crisis. Then, Student A focused on the next question, 'Why did wine consumption, which was on the rise for several years, decrease in 2011?' She then started the exploration. A hypothesis that the earthquake in Christchurch in 2011 could be a factor was formed. As shown in Figure 4-2, she performed an online search with 'wine', 'decreased', 'trend', '2011', and 'christchurch earthquake' to determine if the hypothesis would hold.

Figure 4-2. A screen shot of Student A's search 1

Next, Student A hypothesised that the climate might be a factor and performed another search on the Internet. Student A also noted that wine consumption in New Zealand was highest in 2010, and explored the question, 'Why was consumption highest in 2010?' In this exploration, the Internet was again used, and, as shown in Figure 4-3, a search for the question 'why are there highher wine consumption in 2010' was performed in Google Scholar, a search engine for academic articles. In other words, Student A searched for academic articles on wine consumption in New Zealand, continuing her exploration using a variety of academic articles.

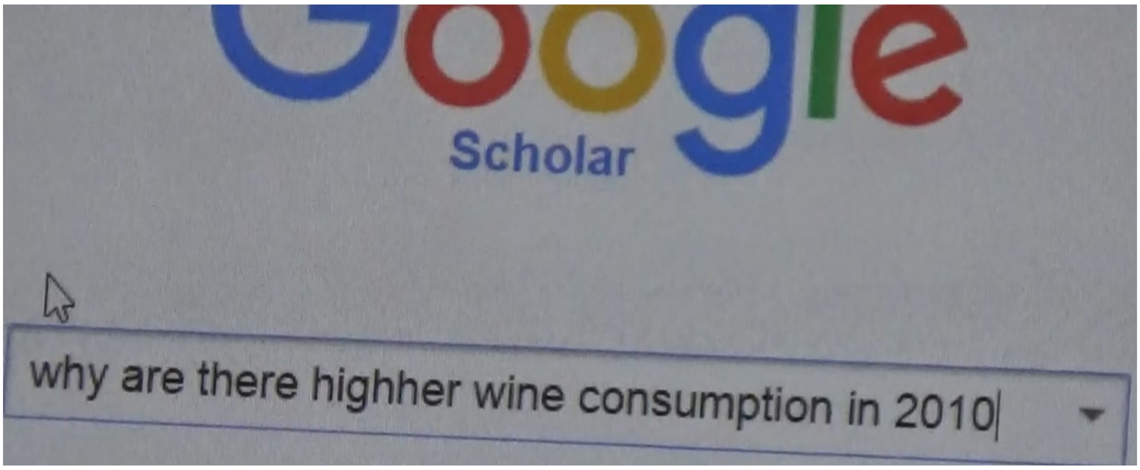


Figure 4-3. A screen shot of Student A's search 2

Student A's exploration has been described above. It may be said that activities for looking into the historical and geopolitical background of New Zealand using the Internet and Google Scholar are not very meaningful for statistics education. However, the author saw Student A's exploration in investigating the historical and geopolitical background of New Zealand's society as a process of analysing the time series data on wine consumption in New Zealand. It is important to combine contextual thinking and statistical knowledge for the statistical literacy within today's world. Therefore, it is very likely that such an approach to history and geopolitics will become a factor in the statistical inquiry cycle involving the use of the Internet. This is because the Internet is now the main method for acquiring information and knowledge in contemporary society. Research into 'questioning the world paradigm' (Chevallard, 2015), which investigates a problem in mathematics education by means of the Internet has begun to be pursued. Ben-Zvi (2007) has looked into exploration using Wikipedia, the Internet-based encyclopaedia, in statistics education.

Through the observation of educational practices in a New Zealand school and the author's analysis of the New Zealand curriculum, it can be suggested that statistics education should not only handle data but also involve contextual thinking. Contextual thinking means that students think about what kinds of data should be used, how to interpret the data processing, and so on. In other words, contextual thinking refers to the emergence of another context when thinking about a specific context. To capture the background of data, students must acquire knowledge on various disciplines and use an interdisciplinary approach. This is one of the current issues in statistics education worldwide.

4.3 Chapter summary

In this chapter, the author compared Japanese textbooks with New Zealand's textbooks, and analysed educational practices in New Zealand from the viewpoint of context. Consequently, it was found that statistical knowledge tends to be used more than contextual knowledge, which is characteristic of current statistics education in Japan. It was also pointed out as the issue on the current statistics curriculum in Japan that contextual thinking does not work in the inquiry and answer at the stage of posing the problem, and that there is some bias in the contexts which are the subject of the problem.

Notes

1. These textbooks are scheduled to be used in 2018.
2. This section is a modification of Fukuda (2018).

Chapter 5. A New Statistical Inquiry Framework with Context

In Chapter 2, the author described the roles of context in statistics education as follows: to clarify a problem (P) and its respective plan (P), while taking into account the relationship to statistical knowledge at the stage of PP within the PPDAC cycle of statistical inquiry, and enabling shuttling to the statistical sphere. Then, in Chapter 4, the lack of problems requiring contextual thinking within the inquiry and answer in statistics education in Japan was identified. In this chapter, based on these considerations, the author aims to establish a framework that is consistent across all school levels to define a problem and identify the necessary data in statistical inquiry, which is directly related to developing the principle concerning context in the statistics curriculum, which is the objective of this research.

5.1 Backwards emergent modelling

As briefly mentioned in Chapter 1, the rationale for this research can be traced to our current overcomplicated society. Therefore, this research focuses on the posing of problems, including the planning of problem solving, which is the starting point of statistical inquiry. The author explains the reasons based on Hirabayashi (2001), who focused on the systemic-evolutionary approach of Wittmann for the direction of mathematics education in such a society. The following quotation is the basis of the idea of a systemic-evolutionary approach:

> [The systemic-evolutionary approach's] basic paradigm is the spontaneous, self-generating ordering exemplified best by the living organism. Organisms are not constructed, they develop. … the only reasonable and feasible way of influencing and guiding a social system is to interact sensibly with the self-organising powers inside the

> system. … variety can only be absorbed by variety. (Wittmann, 2001a, p. 8; the first two sentences are a quotation from Malik (1986) translated by Wittmann into English; the parenthesis is by the author.)

This idea of a paradigm of the systemic-evolutionary approach can be reworded as "the idea of attempting to understand the substance of complex objects without artificially manipulating them" (Hirabayashi, 2001, p. 4; translated by the author). In other words, an attempt must be made not only to simplify and idealise complex phenomena but also to capture the substance of complex entities in statistical inquiry.

What is common to various forms of method knowledge (the statistical inquiry cycle, data modelling, model-based statistical inference, ISI, IIR, and so on) in previous statistics education research is that 'their purpose is problem solving', as stated in Chapter 1. In other words, all stages of method knowledge can be expressed in the form of '○○ in the problem-solving process', i.e. 'formulating the problem in the problem-solving process' and 'solving the equation in the problem-solving process'.

However, from the viewpoint of the systemic-evolutionary approach, the purpose of statistical inquiry is problem solving, but the educational object lies, instead, within the problem-posing process. This is because the essence of the problem may be lost if complex problems or phenomena are themselves not organised using the systemic-evolutionary approach before being solved. In other words, in the case of problems and phenomena with complexity, the process from the problem to the solution is like addition, while setting the premise of the process from problem to solution has educational value. In statistics education, in particular, defining the premise of the problem-solving process has educational value. Mathematics education, besides statistics education, is fundamentally based on axioms, so there cannot generally be any discussion beyond the world of axioms, though there are some exceptions such as the transition from Euclidean space to non-Euclidean space. In a sense, the world of axioms can be regarded as the criterion or foundation for problem solving. On the other hand, there are no axioms in statistics education, so it is essential to construct other criteria or foundations for statistical problem solving. In this case, it is impossible to construct strict criteria or foundations in a mathematical sense. Thus, the premise of the problem-solving process needs to be constructed with the context as key by aiming for a common understanding with others. This can be easily understood because statistics is supported not only by human logic but also by ethics, and that is why the systemic-evolutionary approach is consistent with the subject of establishing a framework for a problem and data in statistical inquiry.

To establish such a framework, this research focuses on emergent modelling, which proposes levels of mathematical activity in mathematical modelling. It is closely related to the issue of context and consistent with the systemic-evolutionary approach. Emergent modelling was proposed by Gravemeijer, a long-time member of the Freudenthal Institute. Additionally, within the mainstream of emergent modelling is realistic mathematics education (RME) theory (cf. Freudenthal, 1991). The tributaries of emergent modelling are mathematical modelling, a series of modelling cycles from which emerge informal methods for solving contextual problems, in which we construct a model-of which is limited to the context and a model-for which is a generalised model-of, de-contextualise the model-for, and reason with formal mathematics (cf. Gravemeijer, 1997, 2008). The important point to note here is the shift from the model-of to the model-for, which means a transition from a situation in which objects of students' thinking are modelled to a mathematical relationship (Gravemeijer, 2002). An example of model-of and model-for is the following problem addressed by Streefland (1991). In the problem of sharing a pizza, the division of the circle showing the pizza drawn by the students is the model-of, and a similar depiction to support the reasoning about the relation between the fractions (for example, '⅓', when dividing a pizza evenly among three people) is the model-for (Gravemeijer, 1999). This emergent modelling is classified into the following four levels (Gravemeijer, 2007a, p. 139) and shown as Figure 5-1. Furthermore, there is a sequence of increasingly higher levels from Level 1 to Level 4.

Level 1. *Activity in the task setting*

: in which interpretations and solutions depend on understanding of how to act in the setting

Level 2. *Referential activity*

: in which models-of refer to activity in the setting described in instructional activities

Level 3. *General activity*

: in which models-for derive their meaning from a framework of mathematical relations

Level 4. *Formal mathematical reasoning*

: which is no longer dependent on the support of models-for mathematical activity

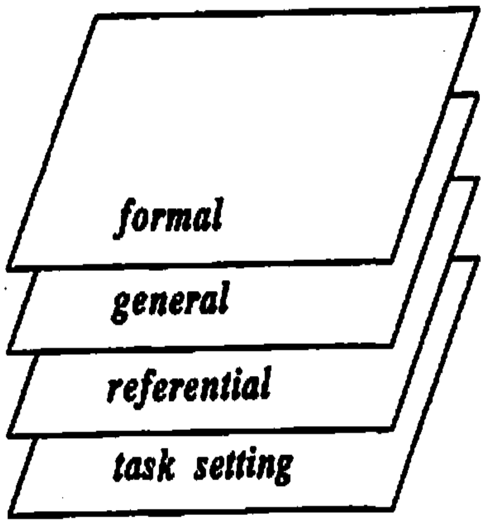


Figure 5-1. Levels of the activity (Gravemeijer, 1999, p. 163)

In this way, it is emphasised that emergent modelling constructs the model-of as a concrete activity, which is an operational activity depending on the context (Level 2), and then builds the model-for as an operational but abstract activity inquiring into mathematical relationships and mathematical constructions, in accordance with the context (Level 3). In other words, emergent modelling focuses on problem solving, in the sense that it shows the process from Level 1 to Level 4. However, considering the problem-posing process in statistics education, the author instead wishes to propose a system of backward modelling (backward emergent modelling) on the levels of mathematical activity. In other words, the problem-solving process itself is the activity from the context and situation at Level 1, or the model-of to the model-for, and ‘problem posing in the problem-solving process’ is the activity from the model-for, or mathematics at Level 4 to the context and situation at Level 1. On the other hand, the problem-posing process is from the model-for to the model-of. The context and situation at Level 1, that is, the posed problem, is updated from vague to definite by refining the model-of based on the model-for. The author does not mention Level 4 here because there is a context and situation for statistics education at the time of the problem-posing process.

Backward emergent modelling is explained by taking an environmental problem as an example. The reason for selecting an environmental problem as context is deliberated in the next chapter. When posing the problem, ‘What causes environmental destruction?’, a general answer is global warming. However, the general answers to the problem, ‘What causes global warming?’ are greenhouse gases and natural forces such as solar heating. Therefore, at this stage, greenhouse gases and natural forces such as solar heating are also applicable as answers to the first problem (‘What causes environmental destruction?’). As a matter of course, the inquiry into the causes of environmental destruction as an effect does not stop here but continues semi-permanently. There is also an ecology in which much knowledge is related in a complex organic manner. Therefore, the original problem of identifying the cause of environmental destruction is a vague problem, and it is necessary to refine it into a more definite problem. In other words, it is necessary to define the basis for stating that the environment is being destroyed. There can be a wide variety of reasons for this statement. Looking for data on where and how global warming has progressed, an inquiry into the reasons should be carried out. Therefore, contextual thinking, the emergence of another context when thinking about some context, is considered to occur spontaneously in this inquiry.

In such inquiry activities, the model-for is a model of the expected answer to a problem, and the model-of is a model of the premise of the problem-solving process. Students have the answers as knowledge from past learning and experiences to the question ‘What causes environmental destruction?’ (for example, global warming, greenhouse gases, and natural forces such as solar heating). They are general, highly subjective answers, since they do not have a quantitative basis, and, therefore, each answer is a model-for. Then, the statements such as ‘It is necessary to have data on where and how global warming has progressed’ and ‘It is necessary to have data on where and how greenhouse gases have increased’ serve to increase the objectivity of these highly subjective answers. They are referential premises of the problem-solving process for making models-for, which are subjective and general statements, to develop a common understanding with others. Here, each premise is a model-of. The refinement of this model-of reveals the substance of the first vague problem. In addition, the problem whose substance is clarified includes the mathematical and statistical premises (the premises of the problem-solving process) for solving the problem. The inquiry to reveal this premise is equivalent to refining a vague problem into a definite one, and the essential purpose of backward emergent modelling is the problematisation of the problem through a number of problem-solving processes. This is because the problem-solving process itself serves to verify the model-for as the general solution and to specify the model-of from it. As a result, there may be cases in which the model-for and model-of are not suitable as the solution to the problem or the premise of the problem-solving process. The trial and error in this problem-solving process refines the model-of, which directly affects the problematisation of the problem. Therefore, as backward emerging modelling involves multiple problem-solving processes, it is based not on a single educational practice but instead on multiple educational practices.

In the above discussion, backward emerging modelling is inspired by the emergent modelling of Gravemeijer, but the model-of and model-for in backward emerging modelling are different from those in emergent modelling. Regarding the basis of emergent modelling, there are two processes which influence mathematics and the learning process in mathematics education, as shown in Figure 5-2: "one must adapt ... abstract knowledge to solving problems set in reality. One has to translate real life problems into mathematical problems" (Gravemeijer, 1997, p. 330).

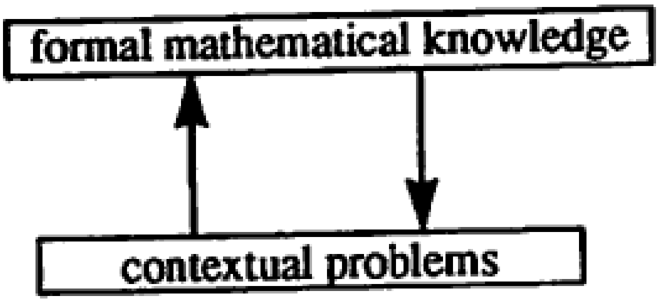


Figure 5-2. Application of formal mathematics (Gravemeijer, 1997, p. 330)

Additionally, taking mathematical problem solving into consideration, the following three stages are important: describing the contextual problem more formally, formally solving the problem, and translating the solution back into the contextual problem (Figure 5-3).

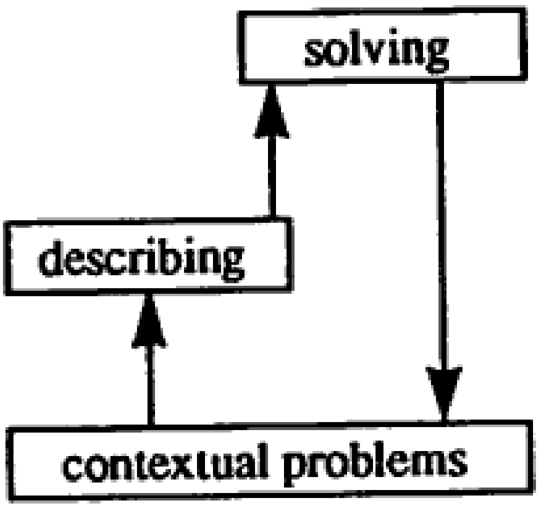


Figure 5-3. Realistic problem solving (Gravemeijer, 1997, p. 331)

Mathematical language is used to describe the contextual problem more formally, and vertical mathematisation, an algorithm in which the solution yields a certain pattern, represents the problem-solving process (Figure 5-4).

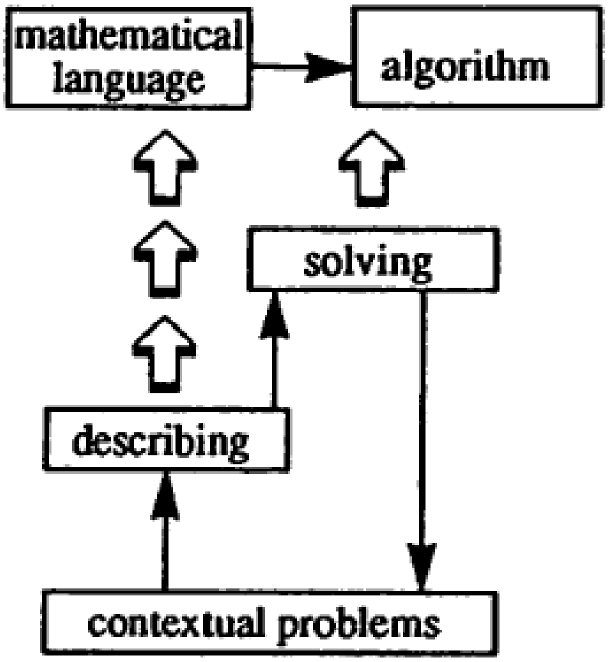


Figure 5-4. Vertical mathematisation (Gravemeijer, 1997, p. 332)

It is asserted as follows, in this respect: "Vertical mathematising is in the core of this process. Vertical progress is reflected in a sequence of gradually more and more formal symbolisations and solution procedures" (ibid., p. 333). In addition, Figure 5-5, which combines Figures 5-2, 5-3, and 5-4, shows the aspects of mathematisation of which formal mathematical knowledge is composed.

From the above, the final goal of emergent modelling, as per Gravemeijer, is to form formal mathematical knowledge, and it may be said that this modelling shows the process leading from the contextual problem to the formation of mathematical knowledge. Therefore, backward emergent modelling, intended for the problem-posing process of refining the premise of the problem-solving

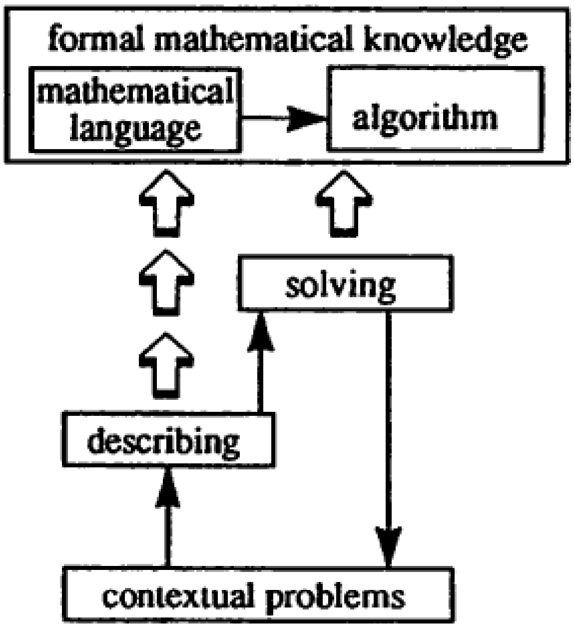


Figure 5-5. Reinvention (Gravemeijer, 1997, p. 332)

process, differs from emergent modelling in its final purpose. The models (model-of and model-for) emerging from the process in emergent modelling are different from those in backward emergent modelling. Previous studies (e.g. Gravemeijer, 2002, 2007b; Bakker, 2004; Büscher & Schnell, 2017) have shown that emergent modelling is consistent with statistics education, but, in those studies, the final purpose of emergent modelling is also regarded as the formation of statistical knowledge. While this statistical knowledge is not always formal and is often informal, this modelling indicates a process from contextual knowledge to (formal or informal) statistical knowledge. Thus, the purpose is the same as that of emergent modelling. Therefore, the terms model-of and model-for will be reconsidered in Section 5.2 and the framework in this study will then be established.

5.2 Emergent hypothesis modelling

The author will reconsider the terms model-of and model-for, and it is necessary to consider their positions in the statistical inquiry process as a whole when discussing the definition of terms.

Therefore, the author will position the model-of and model-for clearly based on the entire statistical inquiry process and construct a framework.

As mentioned in Chapter 1, the PPDAC cycle (Wild & Pfannkuch, 1999) is the root of statistical inquiry processes (e.g., data modelling, model-based statistical inference, ISI, and IIR), and a large number of previous studies on this topic have accumulated since the beginning of the 21st century (e.g., Lehrer & English, 2018; Lehrer & Schauble, 2004; Makar & Rubin, 2018; Prodromou, 2017). First, this research focuses on the integrated modelling approach (IMA) of Manor Braham and Ben-Zvi (2017), based on exploratory data analysis (EDA), and the probability-based approach for modelling activities. The EDA approach, in which data analysis is thorough, lacks probabilistic considerations, while the probability-based approach for modelling activities lacks data exploration. IMA is an approach which compensates for each of these deficiencies and integrates the two approaches, as shown in Figure 5-6.

The left-hand cycle is the statistical inquiry carried out through the EDA approach in the data world, and it indicates the process of deriving a conclusion by actually collecting and analysing real

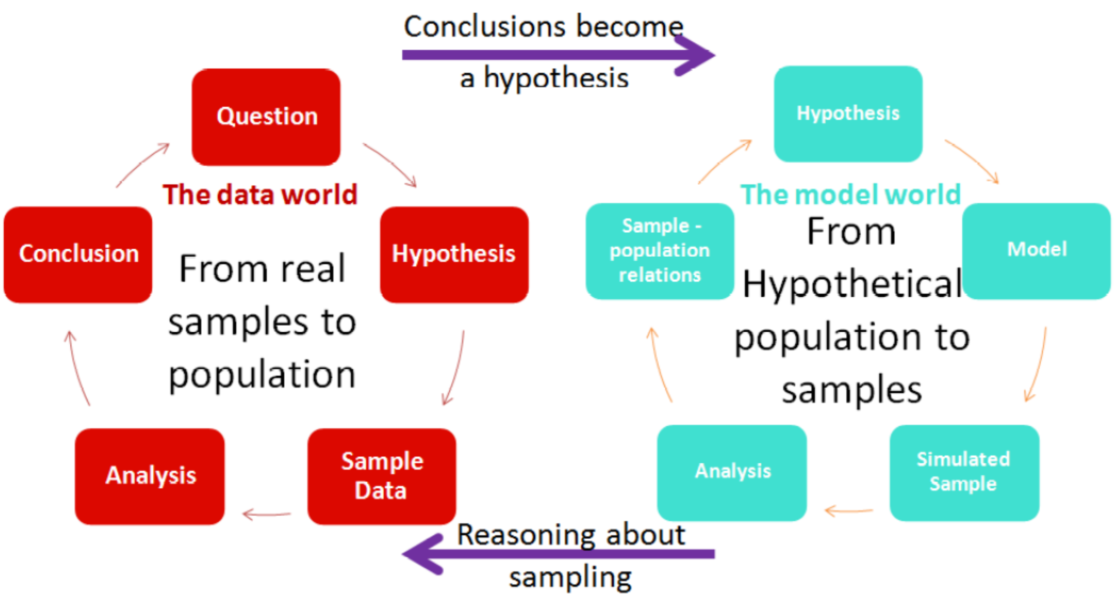


Figure 5-6. The rationale of the integrated modeling approach (IMA)

(Manor Braham & Ben-Zvi, 2017, p. 120)

sample data. A particular point to emphasise is that an informal inference is made about the characteristics of the population from sample data. The right-hand cycle, on the other hand, is the statistical inquiry carried out through a probability-based approach in the model world, and it indicates the process of determining sample data at the model level. In other words, it is the process of generating a sample through simulation and then inquiring about how to capture population characteristics at the stage of forming the basis of the relationship between the sample and the population, that is, inferential statistics as statistical knowledge. Additionally, the formation of statistical knowledge using probability such as the control of variability by the law of large numbers and the central limit theorem is also carried out in this cycle. Finally, the integration of the left-hand and right-hand cycles is explained. The conclusion obtained in the data world becomes the next hypothesis. When inferring population characteristics from the sample, it is impossible to obtain a 100% correct conclusion. In other words, the conclusion obtained from the actual sample data in the left-hand cycle can be regarded as a hypothesis. The right-hand cycle is then implemented to improve the accuracy of the obtained hypothesis. The hypothesis transferred to the right-hand cycle then transfers to the left-hand cycle again by reconsidering the sampling method in the model world to further improve accuracy. After being transferred to the left-hand cycle, the actual sample data are collected, according to the sampling method upon reconsideration, and then the hypothesis is updated. By repeating the shuttling between the left-hand and right-hand cycles, the sampling method is refined, and the conclusion of the original problem is also refined. As the process progresses, the accuracy of the probabilistic inference of the population distribution using the sample increases, and the effects of chance variation are controlled. The process is shown in Figure 5-7; the inquiries in the data world and model world gradually develop into an integrated form of inquiry, from which we take the name IMA.

While IMA emphasises integration of the data world and model world, the Reasoning with Informal Statistical models and Modelling (RISM) framework proposed by Dvir and Ben-Zvi (2018) emphasises integration of the conjecture world and phenomenon world. The RISM framework focuses on

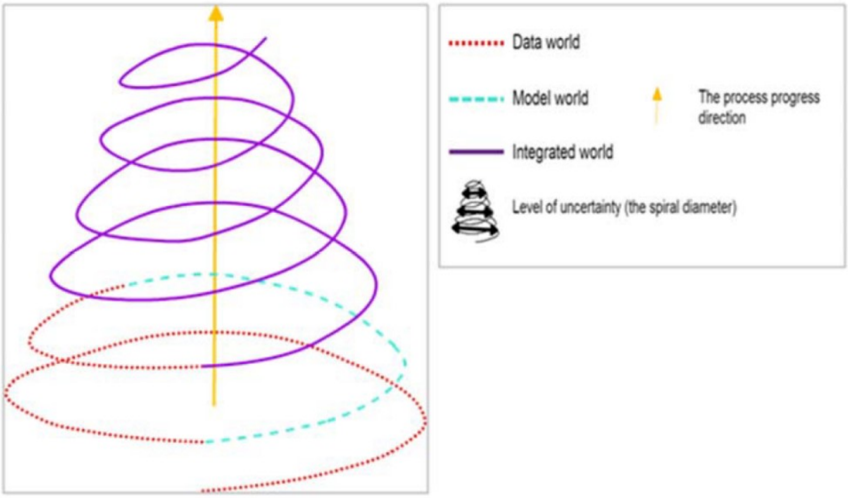


Figure 5-7. The integrated modeling approach (IMA) learning process (Manor Braham & Ben-Zvi, 2017, p. 121)

the comparison of models which are important elements of statistical modelling in the context of ISI. It is shown in Figure 5-8 (Pfannkuch, Ben-Zvi, and Budgett, 2018).[1] This represents the decision-making process of statistical inference. Thus, the conjecture model derived from the expected

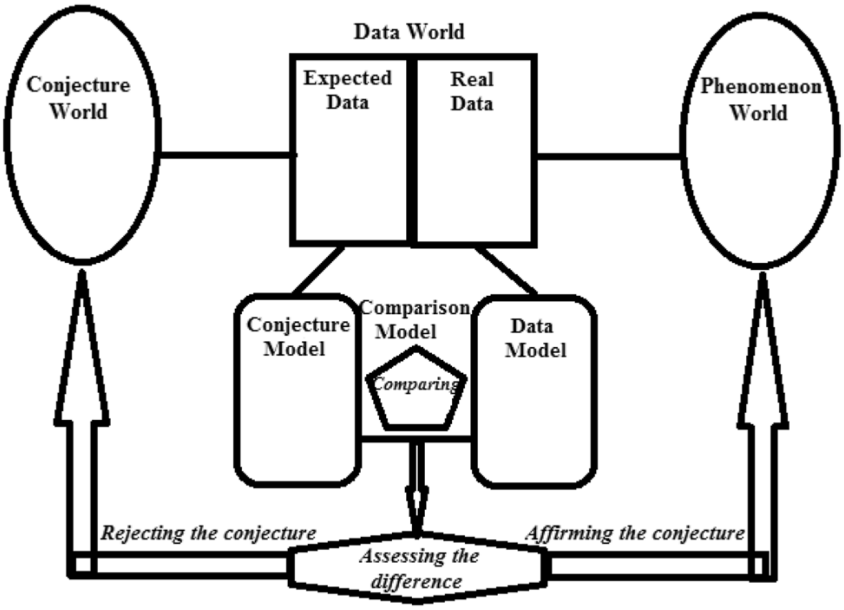


Figure 5-8. The model comparisons framework description of the informal statistical modeling process (Pfannkuch, Ben-Zvi, & Budgett, 2018, p. 1121)

data and the data model derived from the real data are compared, to judge whether a certain conjecture is correct or rejected. In case of the rejection, a new conjecture is created from the conjecture world. Otherwise, the conjecture is affirmed and applied to the phenomenon world. This process is also performed in the data world and model world, as in the IMA, although the process of shuttling between the context sphere and statistical sphere in Wild and Pfannkuch (1999) is more detailed. Furthermore, the particular emphasis in IMA is the relationship between sample and population, while the RISM framework focuses on the process of verifying the hypothesis. IMA also includes inquiries in both the data world and the model world, as shown in Figure 5-7, but the boundary between the two worlds disappears as the inquiries gradually become integrated. This is expressed in the RISM framework (Figure 5-8), which is the data world in the RISM framework, where it can be seen that the data and model are mixed.

However, the author has made minor changes to the research on RISM frameworks in Dvir and Ben-Zvi (2018) and Pfannkuch, Ben-Zvi, and Budgett (2018). First, it is better to distinguish between the data and the model in the data world because the conjecture model is created from the expected data, and the data model is derived from the real data. Further, the lexical meaning of conjecture is "a guess about something based on how it seems and not on proof", or "to guess, based on the appearance of a situation and not on proof" (Cambridge Dictionary, n.d.-b), while the lexical meaning of hypothesis is "an idea or explanation for something that is based on known facts but has not yet been proved" or "an idea or explanation for something that may be true but has not yet been completely proved" (Cambridge Dictionary, n.d.-c). Therefore, both words have almost the same meaning, but conjecture is more dynamic, while hypothesis is more static, because a hypothesis is an idea or explanation, and conjecture is a guess. Further, a hypothesis is based on known facts; therefore, the result of confirming validation of the created hypothesis is considered conjecture. From the above, the result of dividing the data world in Figure 5-8 into the data world and model world and modifying the conjecture model to yield a hypothesis model is

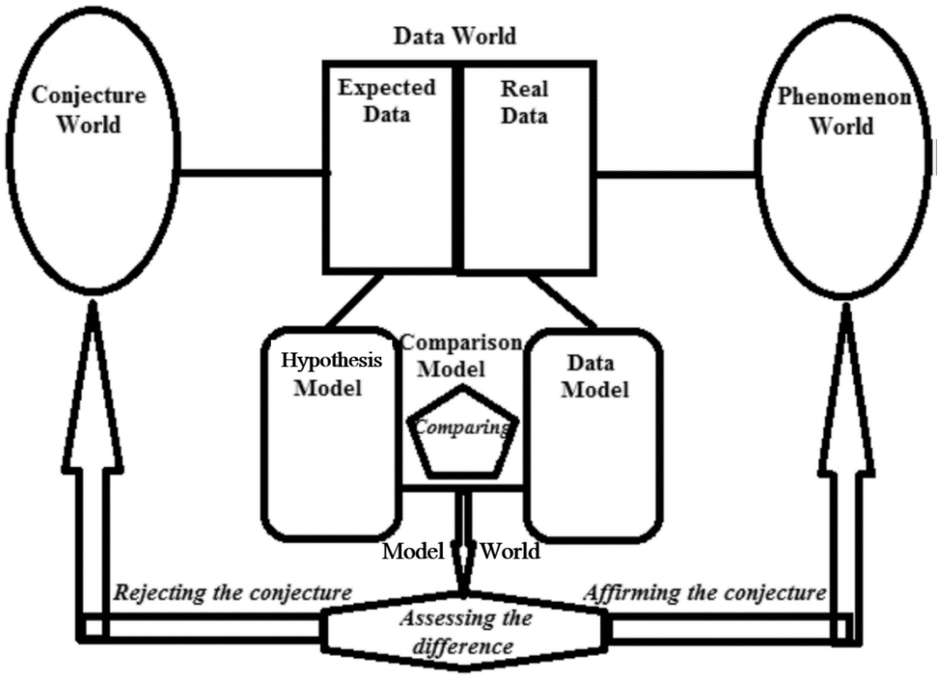


Figure 5-9. RISM framework

(Dvir & Ben-Zvi, 2018, Pfannkuch, Ben-Zvi, & Budgett, 2018; edited by the author from p. 1121)

shown in Figure 5-9.

The author considered IMA, the RISM framework, and their relationship, but have found that there is a problem common to both frameworks. Neither of them explains the first part of the statistical inquiry, which does not necessarily use data. In the IMA (Figure 5-6), the process from Question to Hypothesis in the data world has not been described. With the RISM framework (Figure 5-9), students must always pass through the data world to create a hypothesis model.

To solve those problems, backward emergent modelling is effective. The starting point of statistical inquiry is a vague problem, from which the hypothesis emerges, but there are two types of hypothesis, and students follow a slightly complicated process whereby they are related to the problem. These two types of hypothesis are exactly the same as the model-for and model-of in backward emergent modelling. In the case of the above-mentioned environmental problem, anticipated answers emerge, based on students' knowledge from past learning, experiences, common sense, and intuition to the question, ‘What is the cause of this environmental problem?’ The answers may be global warming, greenhouse gases, and natural forces such as solar heating. Each of them is a model-for in backward emergent modelling, but the author calls it ‘hypothesis model-for’ because it represents a hypothetical answer to the problem. Then, a concrete premise of the problem-solving process is formed, whereby objective factors are added to a highly subjective hypothesis model-for. For example, the statement, ‘It is necessary to obtain data on the proportions of greenhouse gases in the atmosphere’ can be made in the case of greenhouse gases as the hypothesis model-for. To form such a statement, contextual thinking is required by various searches, including the use of the Internet. This premise of the problem-solving process is the model-of in backward emergent modelling. It is the concrete problem related to the hypothesis model-for (e.g., the problem, ‘Is the proportion of greenhouse gases in the atmosphere rising year by year?’ and the hypothesis model-for is greenhouse gases). This also implicitly contains a concrete hypothetical answer for the hypothesis model-for. For example, a concrete hypothetical answer, ‘If the proportion of greenhouse gases in the atmosphere is rising year by year, then the cause of the environmental problem is greenhouse gases’ can be made in the case of greenhouse gases as the hypothesis model-for. It is called the ‘hypothesis model-of’.

In summary, the author has shown a series of processes for the emergence of two models: the hypothesis model-for by anticipation based on the question, and the hypothesis model-of based on search. It is called ‘emergent hypothesis modelling’ because the hypothesis model concretely emerges at the first stage of the statistical inquiry process. The core of emergent hypothesis modelling is the formation of hypotheses through contextual thinking, which basically constitutes a statistical inquiry process without the use of data. Of course, real data and a data model may appear when searching the Internet and in books, but this is not an essential element in emergent hypothesis modelling. Therefore, emergent hypothesis modelling constructs the statistical inquiry cycle in the context world, as illustrated in Figure 5-10.

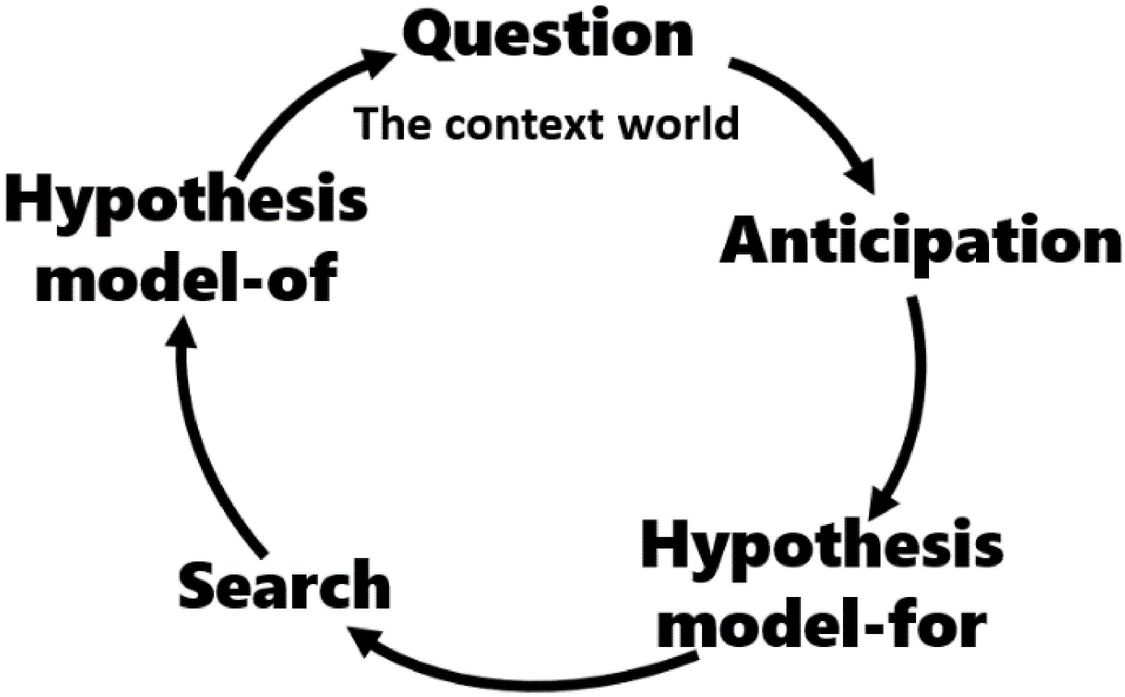


Figure 5-10. Statistical inquiry cycle through emergent hypothesis modelling

With respect to the issue of 'the inquiry and answer to the problem have no context, that is, no contextual thinking is accompanied them' regarding the context of statistics education in Japan, it seems that a solution will be possible if the problem is posed based on emergent hypothesis modelling as the framework. In the next chapter, the author develops a teaching unit on the causal inquiry into environmental problems presented in this chapter as an example, and then confirms whether emergent hypothesis modelling has been properly generated through a teaching experiment.

Notes

1. The original figure for the RISM framework is shown in Figure 5-11.

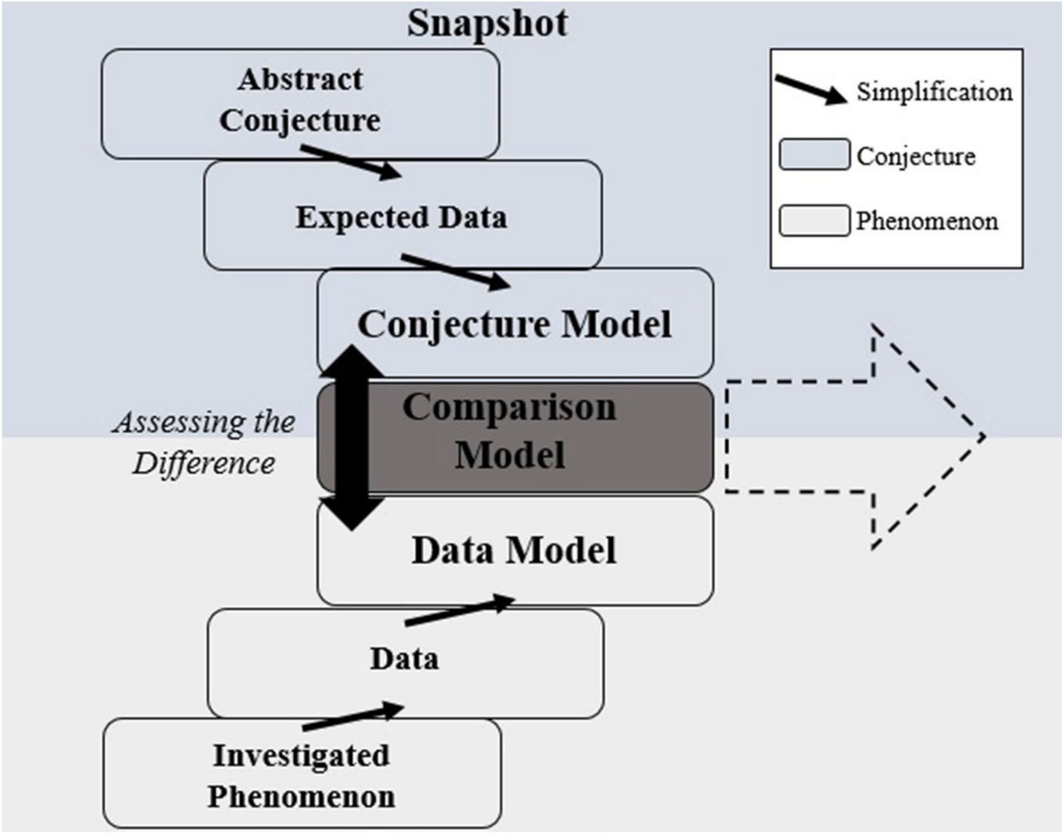


Figure 5-11. The RISM snapshot of an informal statistical modeling process

(Dvir & Ben-Zvi, 2018, p. 1185)

Chapter 6. Discussion through Development of Teaching Unit and Teaching Experiment Focusing on Context

As detailed in Chapter 3, a teaching unit is model material consistent with the whole school mathematics curriculum that consists of the following four properties: Objective (O), Material (M), Problem (P), and Background (B) (Wittmann, 1984). In this chapter, the author develops a teaching unit whose theme is a causal inquiry into environmental problems as a specific example of the emergent hypothesis modelling constructed in the previous chapter, then confirms whether the emergent hypothesis modelling properly occurs by means of a teaching experiment based on the developed teaching unit. First, the significance of treating the causal inquiry is discussed within statistics education.

6.1 Development of teaching unit

6.1.1 Focus on causal inquiry

Pearl (2000) describes the historical transitions in the view of causality in detail. The author discusses the significance of treating causal inquiry within statistics education through a summary of Pearl's discussion (2000). In science, causality was discussed by such figures as Galileo, Hume, and Russell. Galileo posited description as coming first and explanation second, that is, 'how' precedes 'why'. The use of descriptive expressions enables people to think of complex problems not only in terms of 'what if' but also 'how to'. Hume made this more robust, arguing that 'why' becomes completely unnecessary if 'why' is included in 'how'. Furthermore, Hume argued that causality is the product of observations, but Russell opposed this claim, arguing that the concept of causality is metaphysical and not a clearly definable concept, thereby showing that while science is full of abbreviations such as symbols, formats, and so on, causality cannot be abbreviated. On the other hand, in statistics, Galton claimed that it was necessary to clearly distinguish between causality and non-causal relations, and Pearson, a disciple of Galton, proposed the concept of correlation coefficients in the form of the contingency table as indicating non-causal relations. The invention of the correlation coefficient resulted in the development of a variety of disciplinary areas. Henceforth, causality was not handled in statistics. Pearl (2000) states that "his [Pearson's] crusade against animistic concepts such as 'will' and 'force' was so fierce and his rejection of determinism so absolute that he *exterminated* causation from statistics before it had a chance to take root" (p. 410; the parenthesis is by the author).

The above is a brief outline of the explication given by Pearl (2000), and mathematics and statistics have accordingly dealt with correlation, which is a necessary condition, and not with causality because of the impossibility of abbreviation and numerical interpretation. However, should statistics education treat correlation but not causality in the same way? Henceforth, the author considers this point by focusing on the significance for statistics education of causality and correlation. In Newton's equation of motion, $F = ma$, F (force) is the product of m (mass) and a (acceleration). Hence, this is the isomorphism of a correlative concept. Correlation is the relationship between two phenomena or variables, and the equation $F = ma$ shows the relationship between force, mass, and acceleration; that is, you can find one of these quantities if you know the other two. The meaning of $F = ma$, which is isomorphic to the correlative concept, is called the correlative meaning; $F = ma$ as a correlative statement does not show phenomena involving time, which entails the relationship between causes and effects. However, causal inquiry is part of human nature because humans live in the passage of time. In the equation of motion, the acceleration does not cause the force, but rather the force causes the acceleration, so it is necessary to inquire into the relationship between cause (force) and effect (acceleration). $F = ma$ does not involve time but rather the correlative meaning of the variables. On the other hand, the relationship between causes and effects exists although it is not clear whether direct causality is entailed by $F \rightleftarrows ma$ when including time. Thus, the meaning of $F \rightleftarrows ma$ that is isomorphic to the causal concept is called the causal meaning.

Further, Clement (2000) calls the correlative meaning the empirical law hypothesis and the causal meaning the explanatory model hypothesis. Clement (2000) and Lesh, Lovitts, and Kelly (2000) explain both hypotheses using the ideal gas law as an example.

Regarding the relationship between the correlative meaning and the causal meaning, there is correlation when there is causality, but the reverse is not necessarily true. In other words, correlation is a necessary condition for causality. Therefore, examining correlations is indispensable in causal inquiry, and such an examination is based on data. When judging whether a correlation is causal or not, it is difficult to conduct such a discussion based on data, so a discussion based primarily on context is performed. Thus, the causal inquiry requires a dialogue involving both data and context. Additionally, the context-based discussion which judges whether a correlation is causal or not involves individual experiences, so the discussion becomes subjective as a result. Therefore, an explanation to others is needed, whereby an individual's subjective evaluating process is transformed into an intersubjective one including other people. This can therefore be expected to foster statistical literacy at the critical/mathematical level (Watson & Callingham, 2003), in which students blend data-based mathematical elements with context-based critical elements and verbally express and explain them for the purpose of persuading others.

Current statistics research is approaching the original purpose of causal inquiry. With the development of Bayesian statistics, covariance structure analysis, statistical causal inference, and similar techniques, a methodology for treating causality statistically is being established. However, the causal inquiry required in statistics education does not aim at acquiring these statistical methodologies but rather at making decisions on one's own responsibility without presenting premature conclusions (cf. Wild & Pfannkuch, 1999). As Ishii (2014) argues, "As a living person, a worker, or a citizen, intellectual and social skills are required such as the abilities to respond to 'problems without a correct answer', to continue learning over a lifetime, and so on while working with others" (p. 6; translated by the author), so statistics education requires thinking directed towards making persuasive arguments in order to convince others of claims with strong subjectivity. Therefore, this argument is intended to include not only mathematical expressions but also verbal ones. This can be explained with reference to the concept of *Umwelt* proposed by Jakob von Uexküll, a German biologist. The environment seems to be an objective existent which surrounds everyone, and all living beings including human beings seem to exist in that same objective environment, but the concept of *Umwelt* suggests that this view is wrong. According to this concept, the environment is what all living beings make sense of subjectively, and there is thus a different environment for each living being (Uexküll, 1970). Therefore, critical thinking which facilitates self-reflection and the modification of one's *Umwelt* through a dialogue with others' *Umwelt* is situated at the core of statistical literacy as decision-making capabilities.

Thus far, the author has explained the significance of dealing with causal inquiry in statistics education. Although there is very little statistics education research on causal inquiry, whether domestically or internationally, there have been a few such studies, such as Rubin (2004) and Prodromou and Pratt (2013). Rubin (2004) expounds basic considerations of causal inquiry for university students and graduate students, and its characteristics are shown by comparing Fisher-style, Neyman-style, and Bayesian-style causal inquiries. Therefore, this study concerns formal causal inquiry using probability and does not involve context. In Prodromou and Pratt (2013), causal inquiry was implemented for 14- and 15-year-old students. More specifically, the cause of the success of a basketball shot was inquired into by a simulation with four variables: the angle of release, the speed of release, the height of release, and distance to the goal. This is not a formal causal inquiry like Rubin (2004) but involves numerically conducting an informal causal inquiry to explore the relationship between multiple causes for one effect, and the effect and causes are clarified at the beginning of the inquiry. However, as mentioned in Chapters 1 and 5, this thesis aims to foster statistical literacy to help survive our current overcomplicated information society, based on the paradigm of the systemic-evolutionary method, which seeks to capture a complex phenomenon as the substance of complex entities (Wittmann, 2001a). Therefore, under situations where the effect and the causes are not known at the beginning of the inquiry, students must set out to find the correlated variables, inquire which is the effect and which is the cause, and further search for the variables themselves.

In conducting a causal inquiry which cannot be expressed only by such mathematical expressions, a method is needed to express the results of inquiry. This research focuses on the cause and effect diagram, which is 'the relationship between the effect and causes' (Ishikawa, 1984, p. 89; translated by the author).[1] Figure 6-1 is an example of a cause and effect diagram about the causes of a traffic accident involving a car.

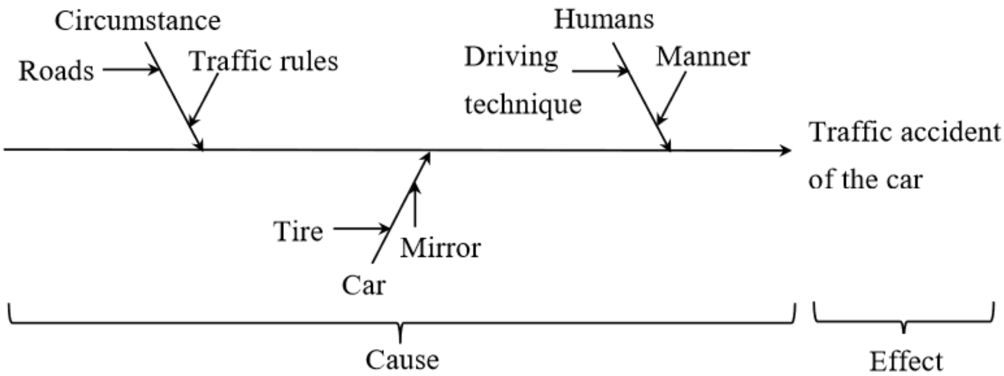


Figure 6-1. An example of a cause and effect diagram

First, the following viewpoints for finding causes are designed: Circumstance, Car, and Humans. Then, investigators try to find the causes from each viewpoint: Roads, Traffic rules, Tire, Mirror, Driving technique, and Manner. Finally, the cause and effect diagram shown in Figure 6-1 is made. What is important is that one story about the causal inquiry is made using both data-based explanations requiring mathematical expressions (mathematical) and context-based explanations using language (critical). By doing so, intersubjectivity in causal inquiry is ensured, and students can explain their own *Umwelt* persuasively to others. In summary, students can gain a consciousness of causal inquiry by preparing cause and effect diagrams and making stories about causal inquiries, and practices can be implemented in statistics education which aim at promoting statistical literacy at the critical/mathematical level by treating both mathematical elements and critical elements.

6.1.2 Focus on environmental problems

In developing a teaching unit for causal inquiry, it is also indispensable to consider the types of context. As there has been a need in recent years for mathematics education whose background is the social context (e.g., Atweh, Bleicher, & Cooper, 1998; Skovsmose, 1990, 1994), importance has been claimed for treating contexts related to crises of society in statistics education (e.g., Zapata-Cardona, 2018; Zapata-Cardona & Escobar, 2016).

In this social context of crises of society, this research focuses on environmental problems. In the United States, the world's leading country, environmental concerns have been loudly voiced recently. For example, Al Gore, the 45th Vice President, who received the Nobel Peace Prize in 2007, and Barack Obama, the 44th President, who received the Nobel Peace Prize in 2009, both have published books and articles on this subject (cf. Gore, 2006, 2017; Obama, 2017). Next, the author considers why human beings must pay attention to the environment today.

If the long human process of social and cultural formation is projected onto a timeline, the process may be divided into six stages in the following order: Human Revolution, Agricultural Revolution, Urban Revolution, Spiritual Revolution, Scientific Revolution, and Environmental Revolution (Ito, 2007, 2016). The Environmental Revolution is forecast to come to completion in the future and has already started. Thus, human beings will be forced to think about the base of their life, called the environment, because this base itself has started to show signs of strain from the processes by which human beings benefit from it. As a consequence, it is now necessary to look back on the societies and cultures which humankind has formed so far and reflect on them in our own society, which has reached the critical point of anthropocentricism. In fact, Rachel Carson expressed her concern over this same issue long before Ito considered environmental issues (e.g., Carson, 1962). The inventor of DDT, which demonstrated an unprecedented effectiveness in pest control, was awarded the Nobel Prize. However, since insects that ingested DDT were eaten by other insects and they in turn were eaten by still other animals, it was only a matter of time before human beings, who formed a part of the food chain, would be poisoned by DDT, but this was not considered at all at that time when human beings were seeking only immediate benefits. In other words, although they are pests to human beings, insects are indispensable to the planet-wide eco-system, which means human beings should abandon their anthropocentric perspective and adopt a more complex way of thinking.

Since then, many researchers have considered the relationship between environmental problems and humankind. For example, White (1968) stated that "Our ecologic crisis is the product of an emerging, entirely novel, democratic culture. The issue is whether a democratized world can survive its own implications [of causing ecological crisis because of the fusion of science and technology]. Presumably we cannot unless we rethink our axioms." (p. 79; the parenthesis is by the author). Furthermore, as shown in Figure 6-2 by Bateson (1972), it is argued that most of the problems concerning the ecosystem of the environment are caused not by scientific but by human factors.

As mentioned above, an inquiry into causality regarding the environment takes on educational significance through studying not only what is beneficial to human beings but also what is detrimental to them. D'Ambrosio (1994) has stated that what is necessary in developing future mathematics curriculum is not what is based on mathematical concepts but on the content and subjects, in particular environmental problems. Moreover, based on D'Ambrosio (1994), Abe (2006) argues the necessity of locating environmental problems in the methodology of developing current literacy. Even more recently, the special issue 'Mathematics Education and the Living World: Responses to Ecological Crisis' was

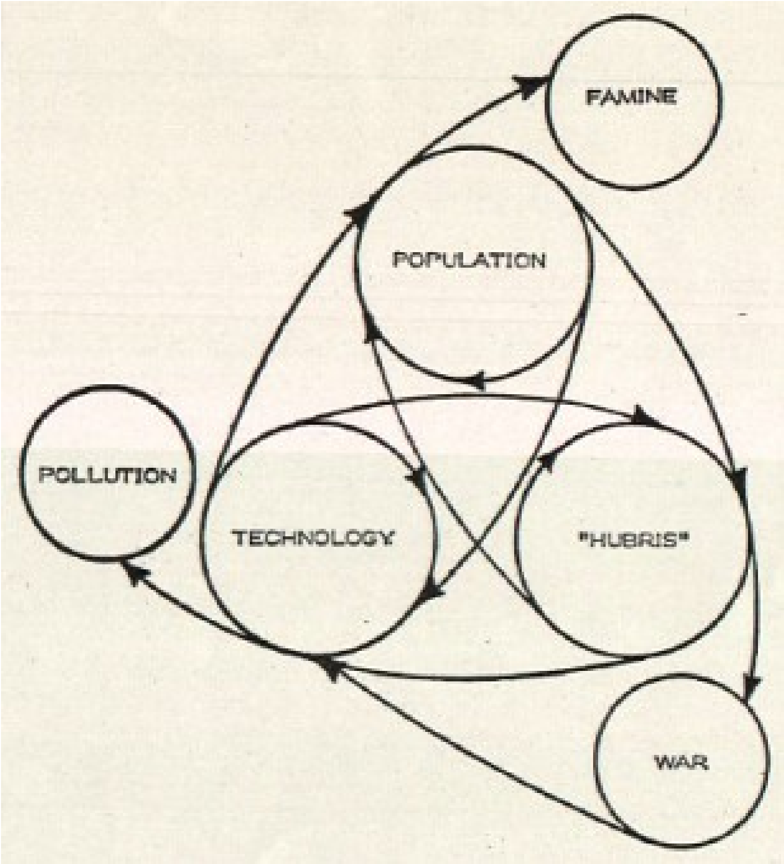


Figure 6-2. Dynamics of ecological crisis (Bateson, 1972, p. 651)

published in *Philosophy of Mathematics Education Journal*, and various mathematics education researchers explained the necessity of research in the context of environmental problems (e.g., Abtahi, Gøtze, Steffensen, Hauge, & Barwell, 2017; Boylan, 2017; Boylan & Coles, 2017; Coles, 2017; Gutiérrez, 2017; Karrow, Khan, & Fleener, 2017; Mikulan & Sinclair, 2017; Savard, 2017; Wolfmeyer & Lupinacci, 2017). Thus, selecting environmental problems as the context in statistics education also has significance for the above reasons. Overcomplicated environmental problems which cannot be simplified or idealised are consistent with the intention of this research. Therefore, this research focuses on environmental problems as a type of context and develops a teaching unit about them.

6.1.3 Teaching Unit 'The Ecology of the Environment and Causal Relationships'

Based on the above considerations, the author developed a teaching unit on causal inquiry into environmental problems. The following four properties constituting the teaching unit are shown in the development: Objective (O), Material (M), Problem (P), and Background (B) (Wittmann, 1984). The developed teaching unit on 'The Ecology of the Environment and Causal Relationships' is as follows.

Teaching Unit 'The Ecology of the Environment and Causal Relationships'

Objective: To develop decision-making capabilities by reflecting on one's *Umwelt* through a dialogue with others' *Umwelts*, and to modify one's *Umwelt*. To acquire statistical literacy at the critical/mathematical level.

Material: An investigation into causality in environmental problems

Problem: There are many relationship diagrams suggesting causes of environmental problems. Draw up your own diagram and describe it in words from the viewpoints of both data and context. In addition, you can use the Internet, calculators, and so on.

Background: An investigation into causality by way of data search and context search. Reflection on cultural/social historical changes of human society and awareness of the trade-off issue with regard to environmental problems. MATHEMATISATION of mathematics. Informal hypothesis testing.

With regard to the possibility of this teaching unit and the adjustment of teaching materials to specific types of schools, it is possible to carry out a causal inquiry on environmental problems locally, globally, or both. That is, there may be a local activity which tackles a specific task such as 'Let's think whether the cause of the environmental problem is …', and there may also be a more general global activity which tackles a task such as ‘Let’s think what the cause of the environmental problem is’. Also, there is an inquiry to conduct a global activity while performing a local activity which tackles ‘Complex, Unfamiliar and Non-routine problems (CUN)’ (Mevarech & Kramarski, 2014) such as ‘there are likely to be many causes of the environmental problem such as ⋯. Let’s think about the relationships between them and determine the strongest causal action for the problem after confirming them’. These can be changed in various ways depending on the school type, grade, and classroom situation.

In addition, it is conceivable that one may produce a new inference when the author discusses the potential for expansion and adjustment of the teaching materials from another point of view. In the case of causal inquiry into a result, the true result is obtained over time, and students can confirm the consistency of the prediction of the obtained result through causal inquiry. This entails a comparison between the hypothesis model created from the expected data and the data model created from the real data, which is the basic idea of the RISM framework described in Figure 5-9 of Chapter 5. However, in the case of causal inquiry into causes, people cannot judge the consistency of the prediction of the obtained causes through the causal inquiry without going back to the past, which is impossible. Thus, there cannot be any real data or data model in this case. That is, a case where the real data and the data model do not exist is considered, which means that people cannot compare them with the hypothesis model. Therefore, the reasoning in this case includes an expression of probability such as ‘probably’ or ‘would’. It can be said that this is the formation of a hypothesis. However, this research intends to form hypotheses in cases for which people may easily collect data, as this shows the process of conducting informal hypothesis testing. If it is generalised a little more, it can be considered an example of abduction.

Abduction is one of the methods of logic proposed by Peirce, and it has attracted much attention in science as a new method different from deduction and induction. For the definition of abduction, Peirce stated that “abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea” (CP 5.171), and characterised the difference between deduction and induction in the logic of inference as follows: "Deduction proves that something ***must*** be; Induction shows that something ***actually is*** operative; Abduction merely suggests that something ***may be***"(CP 5.171). As an example of abduction, Peirce writes: "Fossils are found; say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we suppose the sea once washed over this land" (CP 2.625). That is, abduction means forming a hypothesis about which facts can be true when a certain other fact holds true. If this fact is the data, abduction is the idea of inferring the characteristics of the population from samples, which is the essence of inferential statistics (cf. Ainley, Gould, & Pratt, 2015; Ben-Zvi, Bakker, & Makar, 2015; Dierdorp, Bakker, Eijkelhof, & van Maanen, 2011; Meletiou-Mavrotheris & Paparistodemou, 2015; Pfannkuch, Arnold, & Wild, 2015). Bakker, Kent, Derry, Noss, and Hoyles (2008) suggest that statistical inference includes not only deduction and induction but also abduction. In addition, the importance of the role of abductive reasoning in the development of informal inferential reasoning has continued to be asserted in statistics education research (e.g., Ben-Zvi, Aridor, Makar, & Bakker, 2012; Gil & Ben-Zvi, 2011; Kinnear, 2013).

Thus, another reason why statistics education focuses on abduction is that it is similar to the logic of hypothesis testing. Neyman and Pearson, the creators of frequentism, which constitutes the mainstream approach in current statistics, proposed hypothesis testing, which is one of the greatest inventions in statistics (Neyman & Pearson, 1933). Hypothesis testing is the method which "aims to state the evidence in a sample against a previously defined (null) hypothesis, minimizing certain risks" (Castro-Sotos, Vanhoof, van den Noortgate, & Onghena, 2007, p. 103). For the hypothesis (alternate hypothesis) to be argued, a hypothesis (null hypothesis) is set whose rejection provides support to the alternative hypothesis based on the inference that the observed sample can hardly be obtained under the assumption that the null hypothesis is correct. The example of abduction mentioned above can be stated differently from the standpoint of hypothesis testing: 'Under the assumption that there was not sea but land here in the distant past, it can be concluded that this area would have been covered by the sea in the past by rejecting the assumption based on the fact that fossils of fishes were found here.' In other words, abduction and hypothesis testing are different in the respect that hypothesis testing can start with the hypothesis one seeks to reject, while abduction can start from some fact, but the basic ideas are similar.

Statistics education research is also accumulating on hypothesis testing. Many previous studies in statistics education have found that it is difficult for students to understand formal hypothesis testing (e.g., Castro-Sotos, Vanhoof, Van den Noortgate, & Onghena, 2007; Cobb & Moore, 1997; Erickson, 2006; Falk & Greenbarm, 1995; Garfield & Ahlgren, 1988; Garfield, delMas, & Zieffler, 2012; Garfield, Le, Zieffler, & Ben-Zvi, 2015; Reaburn, 2014; Vallecillos, 1996). For example, Pfannkuch, Regan, Wild, and Horton (2010) propose the method of description of 'I notice', which means using descriptive statistics to find a tendency from samples, and 'I wonder', which means using inferential statistics to infer a characteristic of the population. Additionally, this method has recently been adopted in New Zealand (Aoyama & Matsumoto, 2015). However, as the discussion of how to cope with this difficulty remains at this level, it has thus far been inconclusive.

The emergent hypothesis modelling proposed in Chapter 5 contributes to informal hypothesis testing or informal abduction. The hypothesis model-of which is finally produced in emergent hypothesis modelling is a hypothetical concrete answer to the hypothesis model-for. For example, in the teaching unit developed for the causal inquiry into environmental problems, the hypothesis model-of is the hypothesis, 'If CO_2 is decreasing or paralleling as global warming progresses, the cause of the global warming is not CO_2', if the hypothesis model-for for the original problem 'What is the cause of the environmental problem?' is that the cause is not CO_2. In other words, it is implied that CO_2 also increases as global warming progresses if the cause of global warming is CO_2. This is the null hypothesis, and the hypothesis model-of shown above is the alternative hypothesis. In this way, emergent hypothesis modelling involves the formation of the alternative hypothesis and null hypothesis in hypothesis testing, and is intended to be part of posing the hypothesis in informal hypothesis testing. In other words, the developed teaching unit is intended to form the statistical concept of informal hypothesis testing by conducting a variety of searches in conjunction with contextual thinking. On this point it differs from science education. In science education, content knowledge is knowledge related to natural science, but this paper is not limited to that. As mentioned above, statistics education includes knowledge related to social science and the humanities as well. In other words, science education aims to consider environmental problems through abduction, while statistics education also considers environmental problems, but the purpose is the acquisition of method knowledge of abduction in order to evaluate uncertainty in the context of environmental problems.

6.2 Teaching experiment of the developed unit

6.2.1 Basic information

The teaching experiment in Japan based on the developed teaching unit 'The Ecology of the Environment and Causal Relationships' is implemented and analysed. The teaching unit and teaching experiment form one set, and the teaching unit has to be evaluated by the teaching experiment (Wittmann, 1995). One of the features of this teaching experiment is the following: "an orientation towards uncovering processes by which students learn school subject matter" (Thompson, 1979, p. 1). Therefore, the teaching experiment shows, not the effectiveness of the teaching unit but aspects of the students. The author then uses the case study as a qualitative research method for analysing the teaching experiment. Cranton and Merriam (2015) state that "the case study is an intensive description and analysis of a phenomenon or social unit such as an individual, group, institution, or community. … The goal of a case study is holistic description and interpretation. … case studies are concerned with describing a phenomenon as it exists at a particular time" (p. 57). Therefore, the case study is consistent with the intention of this teaching experiment to show aspects of the students. The results of the author's analysis shown later reflect opinions from a few practice researchers including the teacher in this teaching experiment, so efforts were made to guarantee validity and reliability through peer/colleague examination (cf. Cranton & Merriam, 2015).

The procedure of the teaching experiment is as follows:

・Date: 5 days from 12 March 2018 (Mon) to 18 March 2018 (Fri)

・Number of lessons: 6 lessons[2]

・Students: 25 second-year lower secondary school (13–14-year-old) students in Niigata Prefecture, Japan[3]

・Content: Inquiry into the problem of the teaching unit 'The Ecology of the Environment and Causal Relationships'

・Development: See below, but see Appendix B for details

As a background to this class, usual mathematics lessons rarely use computers, but the 2nd, 4th, and 5th lessons were conducted in the computer room, and the rest of the lessons were held in the classroom. Although knowledge about environmental problems is imparted in science education, the 1st lesson takes up opinions against the generally held views of CO_2 as a cause of global warming. As a result, a scene involving questioning the knowledge that the students had learned was set up. The teacher is a fifth-year male teacher with a master's degree in research on mathematical literacy in the context of the statistical domain. This was the first time the teacher conducted lessons using a cause and effect diagram. Since the teacher and the author did not have any specialised knowledge on environmental problems, they searched on the Internet and other sources to prepare the lessons and made the lesson plan in Appendix B in advance. In addition, the author conducted non-participating observations, shared the aspects of the students' inquiries with the teacher after each lesson, and revised the plan for the next time based on the information.

Under these situations, the teaching experiment was conducted over six lessons. An outline of the teaching experiment with six lessons is shown in Table 6-1.

Table 6-1. Outline of development of the teaching experiment

Lesson	Development
1	The teacher shares information on global warming through video images, and students confirm opinions for and against views of CO_2 as a cause.
2	Students divide into groups between pros, cons, and unknown for the view that 'the cause of global warming is CO_2',[4] and each group inquiries into the reasons with an Internet search.
3	Students make presentations about the results of their inquiries and exchange their opinions in the whole classroom.
4	The pros inquire into the additional causes of CO_2 increase as the cause of global warming, and the cons inquire into other causes of global warming and their causes. In addition, the teacher introduces the cause and effect diagram to the students and asks them to indicate the results of their inquiries in the diagram.
5	Since there were some students who could not understand how to draw the cause and effect diagram which the teacher introduced in the previous lesson, the teacher explains it again while paying attention to the relationship between the cause and the effect. Then, the pros inquire into the additional causes of CO_2 increase as the cause of global warming, and the cons inquire into other causes of global warming and their causes, just as in the previous lesson. However, the teacher asks them to focus on the influence of the sun and to inquire into the relationship between it and global warming in the con group.

6	Students make presentations about the results of their inquiries (including the cause and effect diagram) in the 4th and 5th lessons to the whole classroom to share their opinions. Finally, each student writes a strategy to prevent global warming in words while they make a cause and effect diagram about it with each other.

The cause and effect diagram treated in the teaching experiment is slightly different from that described in Section 6.1. In Section 6.1, several viewpoints are set first, and then the causes of the effect are inquired into based on each viewpoint in making the cause and effect diagram. In this teaching experiment, the viewpoints are not set, but the causes of the effect are, and then the further causes of each cause are searched into. This is because the intention is not for the students to make the cause and effect diagram itself in this teaching experiment, but the method of expressing the cause and effect diagram can be used visually when conducting the causal inquiry. In addition, the teacher presented the local issue, 'Is CO_2 the cause of global warming or not?', to the students at the beginning of this teaching experiment, and then let the con groups freely inquire into other possible causes of global warming besides CO_2. Here, the author would like to mention that the teacher let the con groups inquire into the influence of the sun after sharing that it can be a cause besides CO_2 in the 5th lesson. However, it was possible to describe other causes freely besides CO_2 and the sun when each student finally constructed a cause and effect diagram.

6.2.2 Perspectives of analysis

Henceforth the author analyses the teaching experiment. The objects of this analysis are the cause and effect diagrams and worded expressions by the students or groups in the 6th lesson teaching experiment and the transcript developed throughout the whole classroom (see Appendix C).[5] Additionally, the author analyses them from the following three perspectives: 'causal inquiry', 'emergent hypothesis modelling', and 'the renewal of one's own *Umwelt* through the dialogue with others' *Umwelts*'. The causal inquiry includes not only explanations based on data but also explanations based on words. The causal inquiry including explanations based on words is based on the context as shown in such previous studies as Moore (1990), Wild and Pfannkuch (1999), and Shaughnessy (2007). The author analyses them in terms of whether students can explain the timewise relationship[6] between one context and another context. The causal inquiry is a common theme throughout the six lessons, so the object of this analysis is all six lessons. Regarding emergent hypothesis modelling, the author examined whether there was a scene where they constructed the premise of the problem-solving process (the refinement of the hypothesis model-of based on the hypothesis model-for). Since the author set the environmental problem, which was the context from which the emergent hypothesis modelling emerged, the emergent hypothesis modelling should be conducted by the students during causal inquiry. Thus, all six lessons are the object of this analysis. Finally, concerning the renewal of one's own *Umwelt* through the dialogue with others' *Umwelts*, the objects of this analysis are the cause and effect diagrams and the worded expressions by which they conveyed their own opinion at the end of the 6th lesson after conducting an inquiry with their own group and listening to the opinions of the other groups. An analysis is conducted of the interaction between the individual and the other.

6.2.3 Analysis of teaching experiment from the perspective of 'causal inquiry'

First of all, the author analyses it from the viewpoint of 'causal inquiry'. The author would like to show how the beginnings of statistical literacy at the critical/mathematical level (Watson & Callingham, 2003) emerge by giving two examples of explanations, one data-based and the other word-based. The data-based explanation was given in the group presentation in the 3rd lesson. The following statement was made using the portion enclosed in the handwritten boxes in Figure 6-3 by Group A.

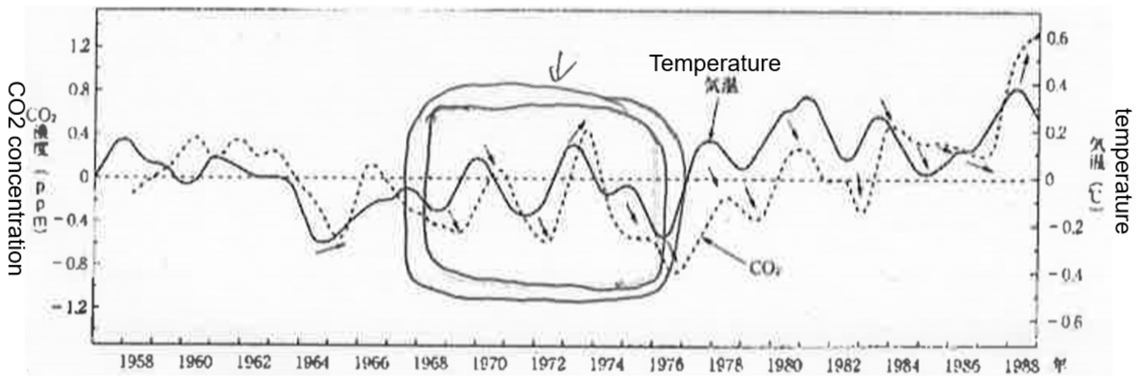


Figure 6-3. The graph used in Group A's presentation in the 3rd lesson

(with handwritten parts by group A; translated by the author)

Group A: … there are other researchers studying the correlation between the global temperature and carbon dioxide. These indicate at a glance a strong correlation between these two. If we examine them carefully, we find that there is indeed a correlation, but the causal relationship is completely the opposite. Instead of the temperature rising after carbon dioxide increased, we found that the temperature increased first, and after a little while, carbon dioxide increased. …

Group A found Figure 6-3 using the Internet,[7] and argued using the graph that CO_2 is not the cause of global warming. Regarding this, there are many overlapping parts of the analysis from the viewpoint of emergent hypothesis modelling, but it is hard to imagine that Group A focused on the temporal relationship between CO_2 and temperature spontaneously after finding Figure 6-3 on the Internet. In other words, it is more likely that they had a purpose of finding data showing an increase in CO_2 after the temperature rise before they found Figure 6-3. Therefore, it can be said that this explanation using Figure 6-3 by Group A is a causal inquiry based on data.

Next, the word-based explanation emerged in the presentation by Groups B and C in the 6th lesson. Both groups were opposed to the hypothesis that CO_2 is the cause of global warming, and gave presentations about the results of inquiring into the causes of global warming besides CO_2 from which the following passages are quoted:

Group B: … According to the latest research, when oceans become oxidized, the production of dimethyl sulfate declines. … Plankton in the sea produces dimethyl sulfate. It is said that dimethyl sulfate … becomes the source of clouds. … The resulting large amount of clouds blocks sunlight and decreases the Earth's temperature. … A certain amount of CO_2 in the atmosphere dissolves into the oceans, lowering the pH level of oceans, that is, oceans become acidic. Therefore, the more CO_2 levels in the atmosphere increase, the more oxidized the oceans will become. …

Group C: … if the sunlight is pouring over the Earth with its full strength, then clouds reflect back 75% of it, and the remaining 25% hits the Earth and contributes to the Earth's temperature. As the other group (Group B) said, the amount of clouds above oceans is decreasing, which means that the reflective power of the clouds is also decreasing. Consequently, the heat of the sun directly hits the Earth. …

The result of the inquiry by these groups is shown as follows in the form of '(cause) → (effect)': CO_2 increase → acidification of the sea → dimethyl sulfate decrease → cloud decrease → amount of sunlight (the influence of the solar energy) increase → temperature rise (global warming). In this scheme, no data from the causal inquiry have been presented, so it can be considered a word-based causal inquiry. It is contextual thinking that is always raised in causal inquiries based on words. As explained in Chapter 4, this refers to the emergence of another context in thinking about a particular context. In watching the video image in the 1st lesson, those who argued against the opinion that CO_2 is the cause of global warming listed clouds as the cause of global warming, so Groups B and C were expected to conduct research on clouds. However, the cause of cloud reduction is decreases in dimethyl sulfate, whose cause is the acidification of the sea, and the cause of this is an increase in CO_2. Contextual thinking was conducted such that research into the context 'cloud' produces the context 'dimethyl sulfate', as well as the context 'sea', then the context 'CO_2' in a continuous chain. This also coincides with the analysis from the viewpoint of emergent hypothesis modelling, but the search by Group B in particular formed only a hypothesis. That is, the increase of CO_2 is listed as the cause of global warming, but Group B did not provide data on the increase of CO_2. Therefore, as a result of Group B's inquiry, the hypothesis 'If CO_2 increases, global warming is caused by the amount of sunlight (the influence of the sun)' is formed. In addition, if the increase or decrease of CO_2 is examined in the future, the truth or falsehood of this hypothesis is verifiable, and it can thus be said that the premise of the problem-solving process is constructed.

6.2.4 Analysis of teaching experiment from the perspective of 'emergent hypothesis modelling'

Next, the author analyses the teaching experiment from the viewpoint of 'emergent hypothesis modelling'. As mentioned above, this viewpoint emerged in conducting the causal inquiry, so the analysis had already been conducted when analysed from the viewpoint of 'causal inquiry', but there was a scene where emergent hypothesis modelling appeared prominently. In the presentation by Group D which inquired into the reason argued in the 3rd lesson that CO_2 is not the cause of global warming, the following explanation was made using Figure 6-4.

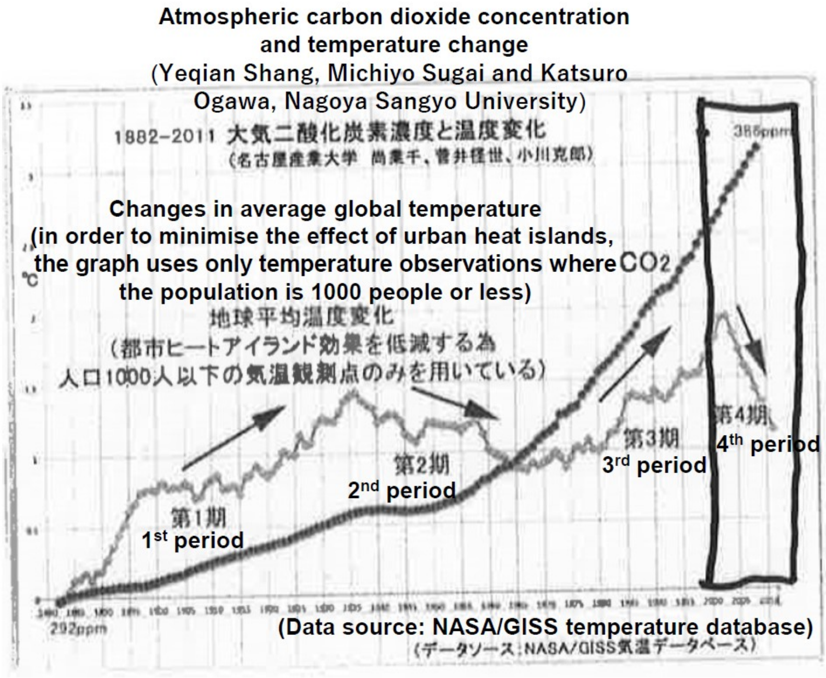


Figure 6-4. The graph used in Group D's presentation in the 3rd lesson

(with handwritten parts by group D; translated by the author)

Group D: … If carbon dioxide was indeed related to global warming, we suppose that the global temperature would keep rising without lowering. This is why we thought carbon dioxide was not related to global warming.

Figure 6-4 is the supporting figure which Group D found from the Internet.[8] Group D regards the fact that the global average temperature is decreasing although CO_2 is increasing as evidence, based on which Group D showed that a direct correlation between CO_2 and global warming does not exist, so a causal relationship cannot exist. The hypothesis was clearly indicated in the explanation of Group D. It is the hypothesis 'If carbon dioxide is involved in global warming, then the temperature of the world would continue to rise without falling'. In order to judge whether this hypothesis is correct, Figure 6-4 was used in the discussion. As a result, it was argued that CO_2 is not the cause of global warming. Therefore, if it is explained in terms of emergent hypothesis modelling, an anticipation was performed for the original problem ‘Is CO_2 the cause of global warming or not?’, then the hypothesis model-for ‘The cause of global warming is not CO_2’ for Group D was determined, and the search was carried out by contextual thinking on CO_2 based on this, and finally, the hypothesis model-of ‘If carbon dioxide is involved in global warming, then the temperature of the world would continue to rise without falling’ emerged. That is, it can be said that the premise of the problem-solving process was established, and then the problem solving itself was also conducted of verifying the hypothesis using Figure 6-4.

6.2.5 Analysis of teaching experiment from the perspective of ‘the renewal of one’s own *Umwelt* through the dialogue with others’ *Umwelts*’

Next, the author analyses the teaching experiment based on the third viewpoint, ‘the updating of one’s own *Umwelt* through dialogue with others’ *Umwelts*. In the final scene of the 6th lesson, the students constructed their final cause and effect diagrams individually, and student Sachi made the diagram shown in Figure 6-5.

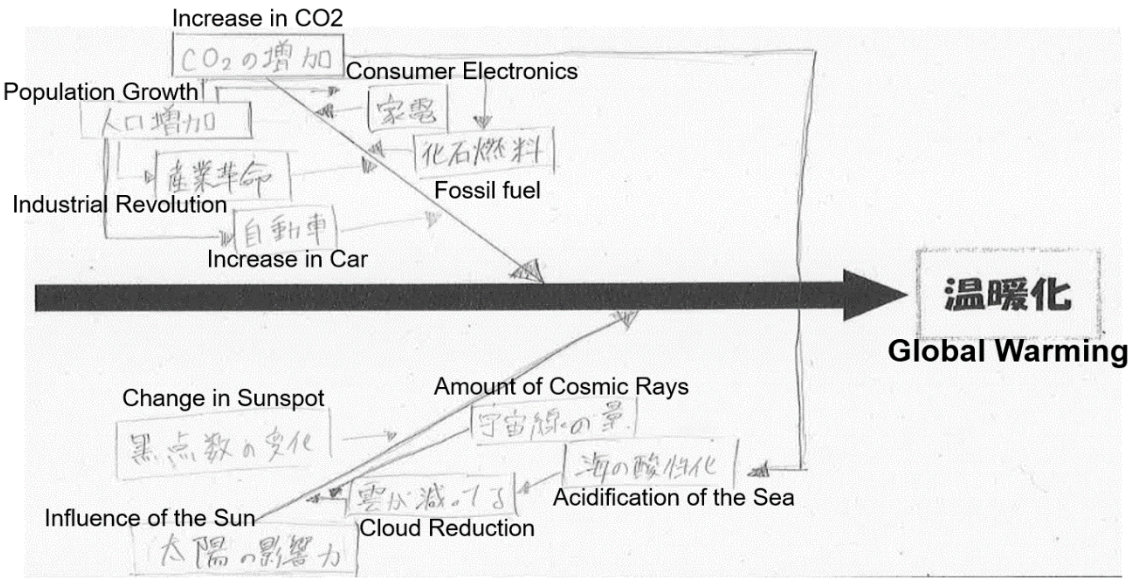


Figure 6-5. The cause and effect diagram finally created by student Sachi (translated by the author)

Student Sachi is a member of Group B, which presented '(cause) → (effect)': CO_2 increase → acidification of the sea → dimethyl sulfate decrease → cloud decrease → amount of sunlight (the influence of solar energy) increase → temperature rise (global warming) in the 6th lesson. The lower part of Figure 6-5 is almost exactly the same as that presented by Group B. As mentioned above, the inquiry by Group B formed only the hypothesis 'If CO_2 increases, global warming is caused by it', so the verification of the hypothesis concerning the increase or decrease in CO_2 remained to be performed. However, the increase in CO_2 was listed as a cause of global warming in the cause and effect diagram in Figure 6-5 by student Sachi. It is thought that this is because the results of the inquiry by Group E, which agreed that the cause of global warming is CO_2, affected Sachi's inquiry. As a result of the inquiry by Group E, 'population growth', 'industrial revolution and people's lives', 'increase of car penetration rate', 'use of fossil fuel', and 'evaporation of the sea' were listed as causes for the increase of CO_2, which results in a diagram almost identical to the upper part of Figure 6-5. In this way, it can be said that student Sachi decided the cause and effect diagram of global warming by her own intention through the dialogue with the result of Group E's inquiry.

As mentioned above, the author analysed the teaching experiment based on the developed teaching unit, 'The Ecology of the Environment and Causal Relationships'. As a result of the causal inquiry which was the intention of the teaching unit, the beginnings of statistical literacy at the critical/mathematical level and the refinement of the hypothesis model-of based on the hypothesis model-for by emergent hypothesis modelling both emerged.

6.3 Findings of the teaching experiment to improve the teaching unit

In Section 6.2, the author showed that there were some scenes where the intention of the teaching unit appeared. In the following, the author considers issues with which to improve the teaching unit that were identified by the teaching experiment.

The first is the issue of the students' orientation toward causal inquiry. In the group presentations in the 3rd lesson, four groups explained that CO_2 is not the cause of global warming, but three of the groups argued that there is no causality because there is no correlation. Of course, it is sufficient to show that there is no correlation in order to explain that there is no causality, and it would seem to be a natural idea to confirm whether there is a correlation or not. However, the intention of this research was also an inquiry to judge whether two variables showed only a correlation or causality with a rational explanation. The discussion of whether the cause of global warming is CO_2 or not has not been scientifically settled and opinions are divided. Therefore, as much data shows no correlation between the amount of CO_2 and the temperature, the teaching unit needs a description, such as establishing one stage with a focus on the data with correlation as the target of the classroom discussion and conducting an elaboration to inquire whether there is causality or only correlation between these data.

The second is the issue of emergent hypothesis modelling. The intention was to set up a hypothesis, that is, the construction of the premise of the problem-solving process, and there were groups which could set up the hypothesis as described above. However, it was only the above-mentioned Group D that set their hypothesis explicitly; the rest of the groups did so only implicitly. Thus, the issue is to clarify a method to make implicit hypothesis explicit and conduct teaching intervention on hypothesis setting before, after, or during the inquiry. By doing so, it is possible to have the students inquire at the stage of verifying the constructed hypothesis, which can form the foundation of hypothesis testing in an informal way. As a concrete measure to this end, it is considered an effective measure to incorporate a question for the purpose of directing informal hypothesis testing within the 'problem' of the teaching unit before the question on environmental problems. Therefore, the author refers to the teaching unit 'Galton Board' developed by Wittmann (1984). The subject of this teaching unit 'Galton Board' is probability and probability distributions using the Galton Board as shown in Figure 6-6, and the teaching unit comprises the following four properties: Objective (O), Material (M), Problem (P), and Background (B) (ibid., p.

32).

TU *Galton Board*
O: Mathematizing a stochastic situation.
M: Galton boards of various sizes, balls.[9]
P: Where will the first ball fall?
Where will the second one fall?, etc.
Why?
What path can a ball take?
How many balls exist?
Which paths lead to the same goal?
Compare the probabilities of the paths, etc.
B: Bernoulli chain, binomial distribution.

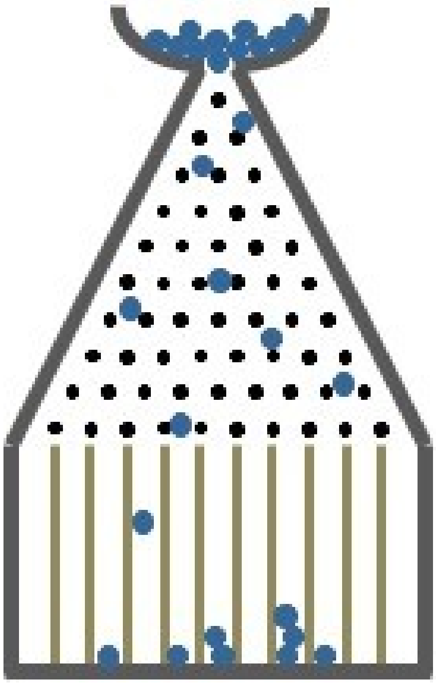


Figure 6-6. Galton board

Since the subject of this teaching unit ‘Galton Board’ is not hypothesis testing but probability and probability distributions, it is necessary to convert it to a teaching unit whose subject is hypothesis testing. As mentioned above, formal hypothesis testing is a statistical concept which is difficult for students to understand, so the author developed a teaching unit which includes connections with proof by contradiction, a similar idea to this concept, and informal hypothesis testing. First, based on Otani (2019) about the connection of hypothesis testing with proof by contradiction, the similarities and differences between statistical testing and proof by contradiction are as follows:

＜Similarity＞

・Start the inference with some (negated) hypothesis to claim the validity of a hypothesis

・Lead with the ‘contradiction’ under the posed hypothesis, and reject the posed hypothesis, and support the original claim

＜Difference＞

・The contradiction in hypothesis testing is not identical to the strict contradiction in proof by contradiction but is only regarded as a state just like a contradiction

・The hypothesis testing cannot lead to a state just like a contradiction, and it may suspend the assertive judgement.

Then, based on these similarities and differences, the author developed the teaching unit ‘Pinball Board’ using the pinball board shown in Figure 6-7.

TU *Pinball Board*

O: Consider the basic idea of hypothesis testing.

M: Pinball board.

P: ① When dropping a ball from the centre, let's predict where the ball will fall.

② Let's think about where the ball is likely to fall by calculation.

③ Let's compare the experiment and the theoretical value.

④ When observing that one ball is at the left end, let's consider whether the ball was dropped from right of centre.

⑤ When observing that one ball is at the left end, let's consider whether the ball was dropped from the centre.

⑥ When one ball fell to a certain place in the left side, let's consider whether the ball was dropped from left of centre.

B: Hypothesis testing, proof by contradiction, Bernoulli process, binomial distribution

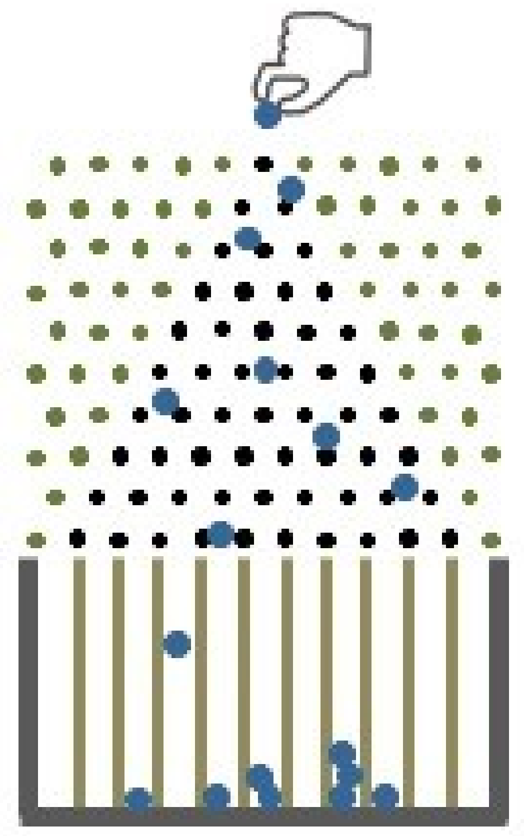


Figure 6-7. Pinball board

As a series of ‘Problems’, the problem is first to compare the experimental results with theoretical values using a 4-divider pinball board (from Problem ① to Problem ③), and then the pinball board is changed to have 10 dividers. At that time, the activity of calculating the theoretical value for the 10-divider pinball board is not set. The approximation of the experiment and the theoretical values found in the 4-divider pinball board are only to be used to confirm that the result does not change even if the number of dividers increases. In addition, in the 10-divider pinball board shown in Figure 6-8, the dividers into which the ball can drop when falling from the centre are, in order from the top of the figure, numbered ①, ②, ③, ..., ⑪, and information on the theoretical probability of dropping the ball in each divider is given to the students.

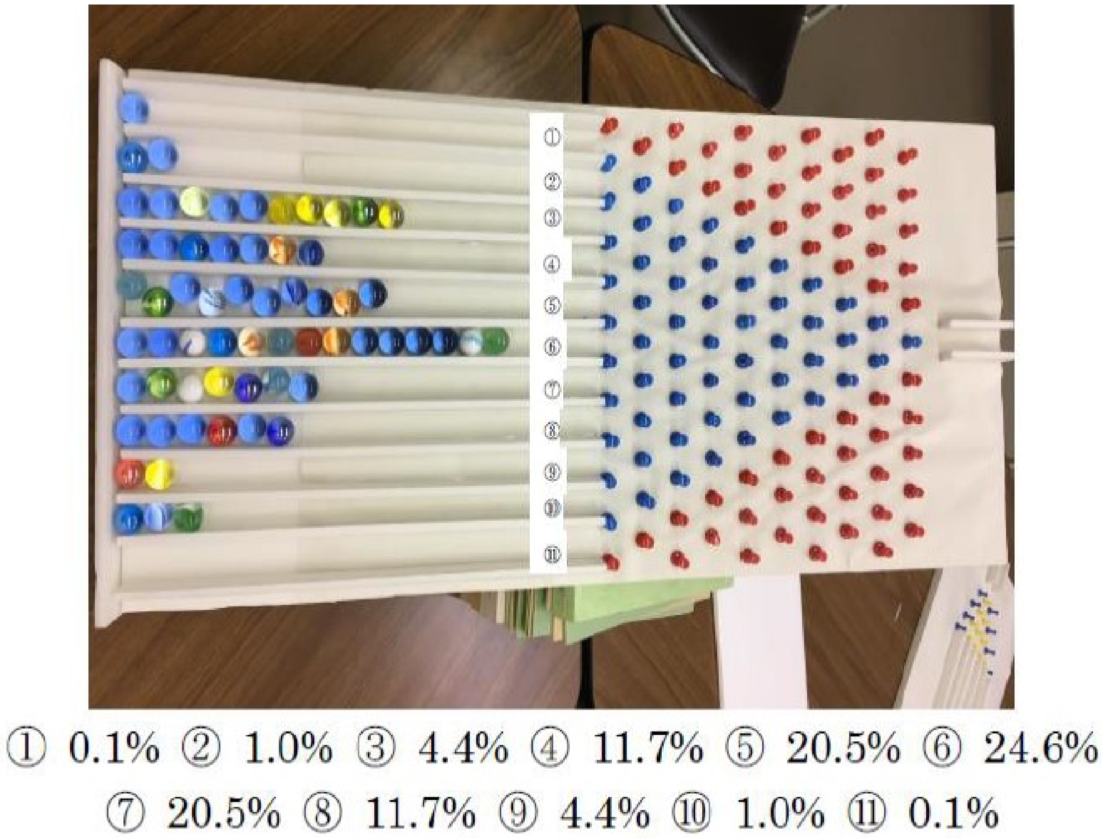


Figure 6-8. Probability for dropping balls in each divider after falling from the centre of the pinball board

The steps so far are a simplified version of the teaching unit 'Galton Board'. The author considers the position where the ball is likely to fall through a numerical calculation based on prediction and experiment. In the same way, it is intended to be able to yield the theoretical values for a given situation, such as when the number of dividers is 10. If the subject is the probability distribution, the object of inquiry thenceforth is the binomial distribution, and the inquiry will be conducted to determine its characteristics, such as its mean and variance, but in the case of hypothesis testing as the subject, the binomial distribution remains the method of consideration. In order to treat this material as hypothesis testing, the author focuses on where the ball was dropped and fell to introduce the perspective of assumptions and results.

Problem ④ verifies the validity of the hypothesis that the ball was dropped from one space to the right of the centre for the result that the ball fell into the leftmost divider (①) (Figure 6-9). Under this assumption, it can be inferred that the assumption is not correct because the ball cannot fall into the leftmost divider (①) (it may fall into the second-leftmost divider (②)). The logic of proof by contradiction is used here, and it is possible for it to emerge for those who have not yet studied it, and, for those who

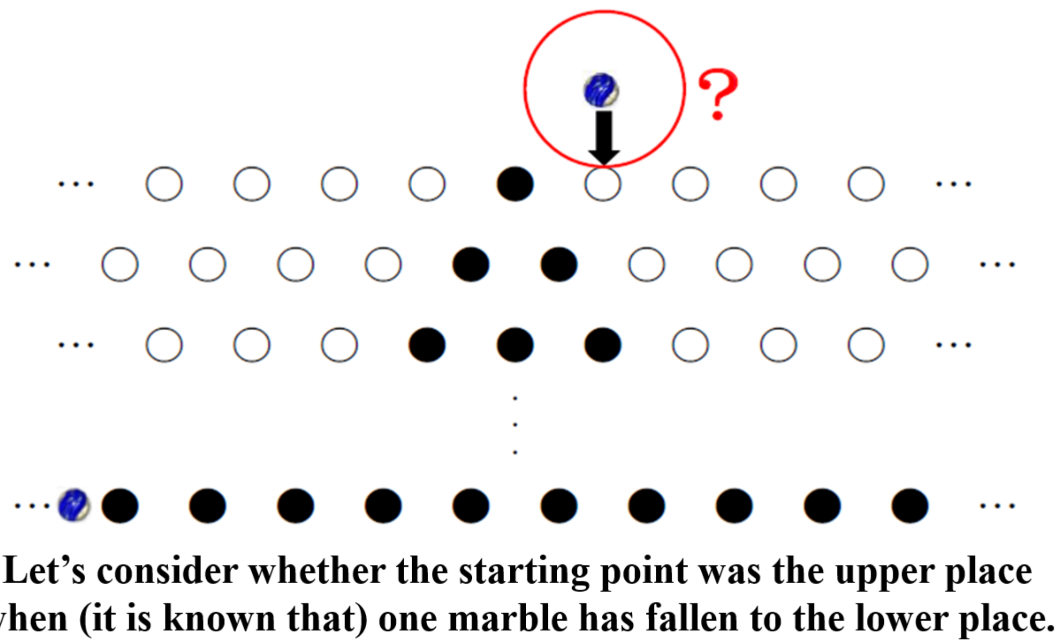


Figure 6-9. The situation of Problem ④ in the Teaching Unit 'Pinball Board'

have already learnt it, to utilise it.

Problem ⑤ verifies the validity of the hypothesis that the ball was dropped from the centre with the same result as Problem ④ (Figure 6-10). Unlike Problem ④, it cannot be said where it definitely cannot fall, so it is impossible to exactly deny this assumption using the logic of proof by contradiction. However, under this assumption, the provisional conclusion is reached that theoretically this event would occur in only 0.1% according to the theoretical value, so it is likely that the ball was not dropped from the centre but rather from left of centre.

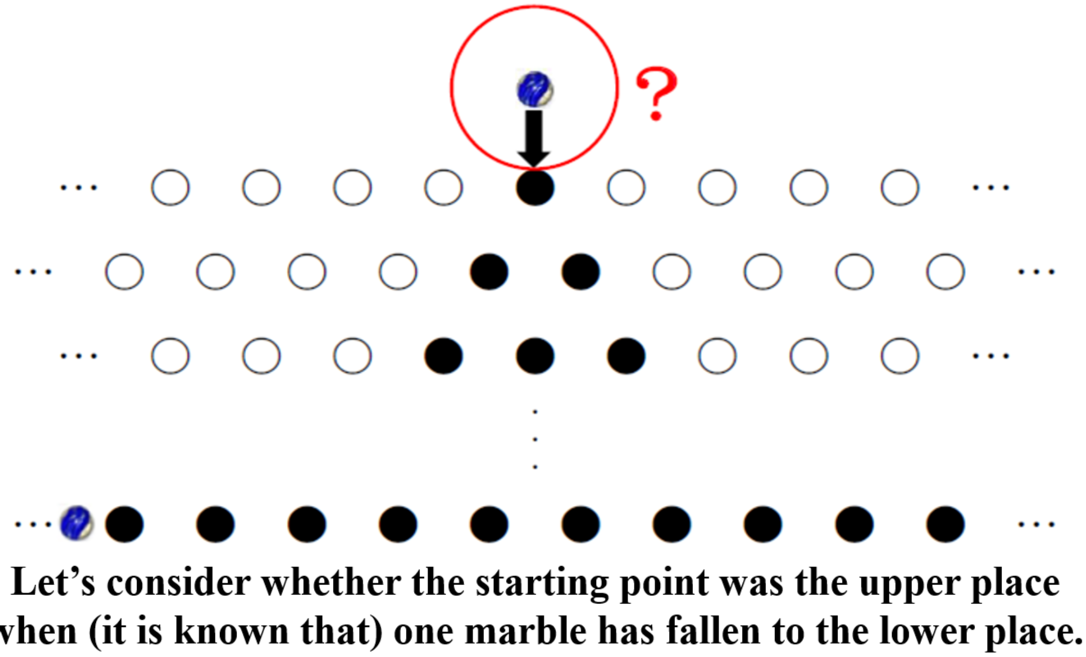


Figure 6-10. The situation of Problem ⑤ in the Teaching Unit ‘Pinball Board’

Problem ⑥ is a case where the null hypothesis cannot be rejected (Figure 6-11). By setting events with not very low probabilities, a situation is made where they cannot make a judgement about the assumption that the ball was dropped from the centre. Unlike proof by contradiction, there may be cases where they cannot judge what kind of assumption is reasonable.

As mentioned above, the teaching unit ‘Pinball board’ is intended to cause the idea of hypothesis testing to emerge mathematically using probabilistic ideas and to form the statistical concept of informal

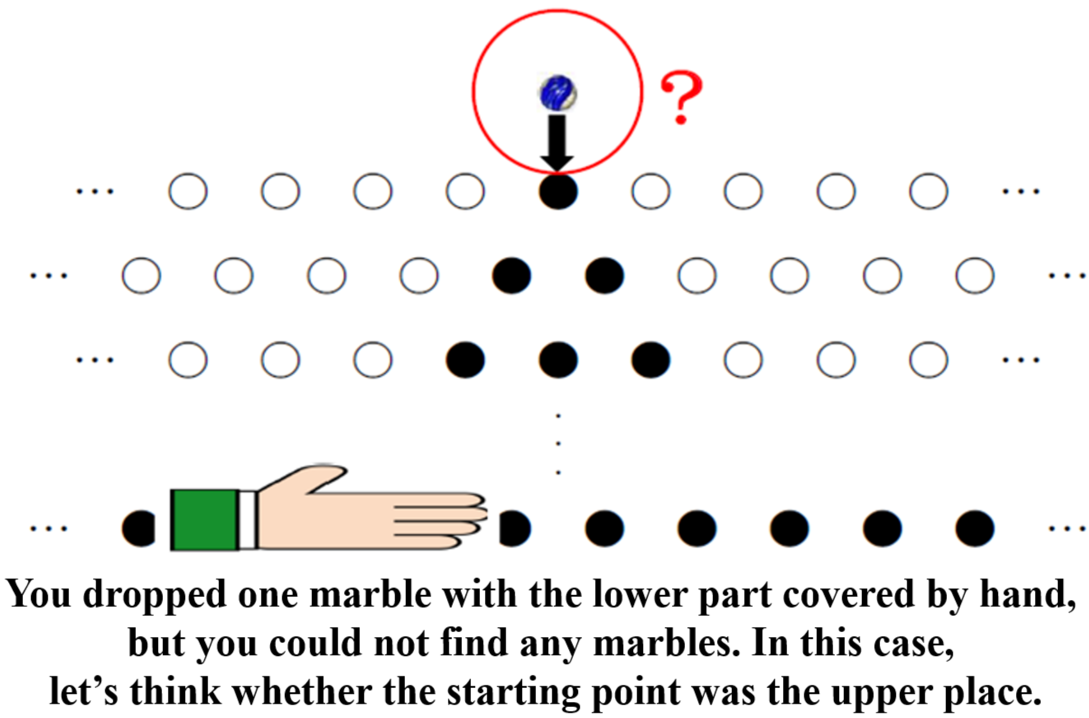


Figure 6-11. The situation of Problem ⑥ in the Teaching Unit ‘Pinball Board’

hypothesis testing. In this way, conducting causal inquiry into environmental problems after recognising the idea of hypothesis testing mathematically without taking the context into consideration may enable the idea of explicit hypothesis testing to emerge for the students, and at the very least it facilitates the explicit setting of hypotheses.

6.4 Chapter summary

In this chapter, the author aimed to develop a teaching unit in statistics intended for emergent hypothesis modelling. In order to do so, this research focused on causal inquiry as statistical knowledge and environmental problems as contextual knowledge. Next, the author developed the teaching unit ‘The Ecology of the Environment and Causal Relationships’ intended to generate contextual thinking in causal inquiries into environmental problems. The teaching experiment was then implemented based on this teaching unit, and the author could show empirically that the students performed emergent hypothesis modelling through contextual thinking.

Notes

1. The cause and effect diagram is sometimes called an ‘Ishikawa diagram’, after the name of its proponent, or ‘fish bone diagram’, because of the shape of the diagram.
2. Each lesson of the teaching experiment lasted 50 minutes.
3. The object of analysis is the students who participated in all six lessons of the teaching experiment. In addition, the students of the subject of the teaching experiment were not divided into lessons according to their degree of proficiency or other factors, but constituted a mixture of students with high and low achievement in mathematics.
4. The students who did not know whether CO_2 is the cause of global warming were made to inquire as opponents.
5. Regarding the inquiry in groups, the aspect of discussions in groups should have been retained in some form originally, but this was not possible due to such limitations as the number of videos. Therefore, in the inquiry in groups, the objects of the analysis were descriptions in the worksheets obtained as the result of inquiries and the statements in the group presentation.
6. Time in causal inquiry may be reworded as the order of things.
7. It is unclear from which Internet page the group cited this, but the following URL contains the same graph: http://www.env01.net/main_subjects/global_warming/contents/s003/kaisuionn_kionn_co2/kaisuionn_kionn_co2.html (Final confirmation on February 20, 2019).
8. It is unclear from which Internet page the group cited this, but the following URL contains the same graph: http://www.env01.net/fromadmin/contents/2015/2015_ 07.html (Final confirmation on February 20, 2020).
9. Since there is no further explanation in Wittmann (1984), it was interpreted as ‘Galton boards of

various sizes and balls used in them' in this research.

Chapter 7. Discussion

In this chapter, the author summarises the discussion up to Chapter 6 and develops a principle for the statistics curriculum in Japan with special attention to context. The author then discusses the significance of this research for statistics education, the limitations of this research, and future directions of research.

7.1 A Principle for the statistics curriculum in Japan from the perspective of context

Chapter 2 showed that context works at the stage of PP within the PPDAC cycle. The purpose of PP is to clarify what the problem is in the statistical inquiry and what kinds of data are necessary. Additionally, this research also focused on theorising this stage. Next, Chapter 4 made a comparison of Japanese and New Zealand textbooks and an analysis of educational practices in New Zealand. A need for an awareness of contextual thinking was suggested. Then, Chapter 5 proposed a statistical inquiry cycle through emergent hypothesis modelling in the context world as in Figure 7-1. Then, a teaching unit was developed for causal inquiry. Implementing and analysing the teaching experiment in Chapter 6 showed that the theory of Figure 7-1 functions effectively. Thus, the statistical inquiry through emergent hypothesis modelling (Figure 7-1) aims to refine the hypothesis model-of based on the hypothesis model-for. As a result, it can be found that this inquiry works normatively in the development of the curriculum and the development of educational practices.

Another point of the role of context shown in Chapter 2 is to enable shuttling to the statistical sphere by repeating the PPDAC cycle and the interrogative cycle. Thus, it is necessary to develop a theory in order not only to highlight the PP stage within the PPDAC cycle as in Figure 7-1, but also to survey the PP stage within the PPDAC cycle. In order to develop the principle, previous studies that were reviewed regarding the whole statistical inquiry process in Chapter 5 are considered and referenced in the

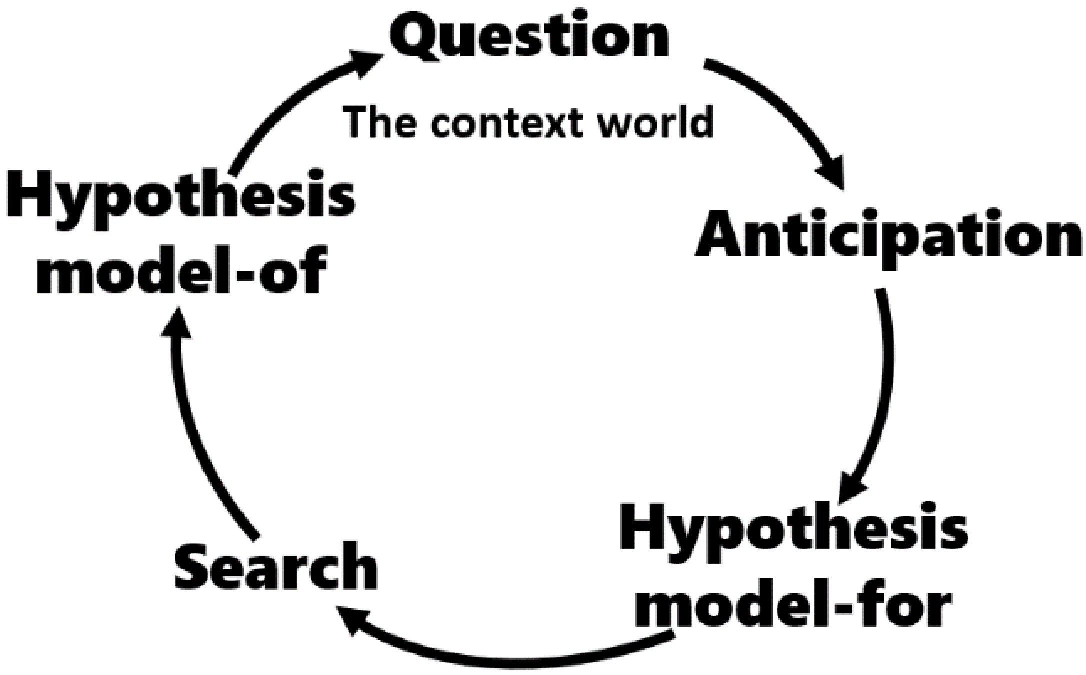


Figure 7-1. Statistical inquiry cycle through emergent hypothesis modelling（Reprint）

following discussion.

First, the author introduces the RISM (reasoning with informal statistical models and modelling) framework based on previous research on the statistical inquiry process. The RISM framework is intended to integrate the phenomenon world, in which specific problems in the real world exist, and the conjecture world, in which predictions are made regarding specific problems in the real world. For a specific problem extracted from the phenomenon world, in the conjecture world, the expected data are predicted in the data world under the given conjecture, which serve in turn as the basis for forming the hypothesis model in the model world. On the other hand, in the phenomenon world, real data are collected in the data world and the data model is made in the model world on that basis. Then, if this statistically inferred hypothesis model is consistent with the data model based on the data actually collected, then the initially predicted conjecture is transferred to the phenomenon world and the plausibility of the conjecture regarding the solution of the problem is shown. On the other hand, if the hypothesis model is not consistent with the data model, then the initially predicted conjecture is rejected, so it is not transferred to the phenomenon world and a new conjecture is made in the conjecture world. Figure 7-2 shows the steps of the verification of the conjecture.

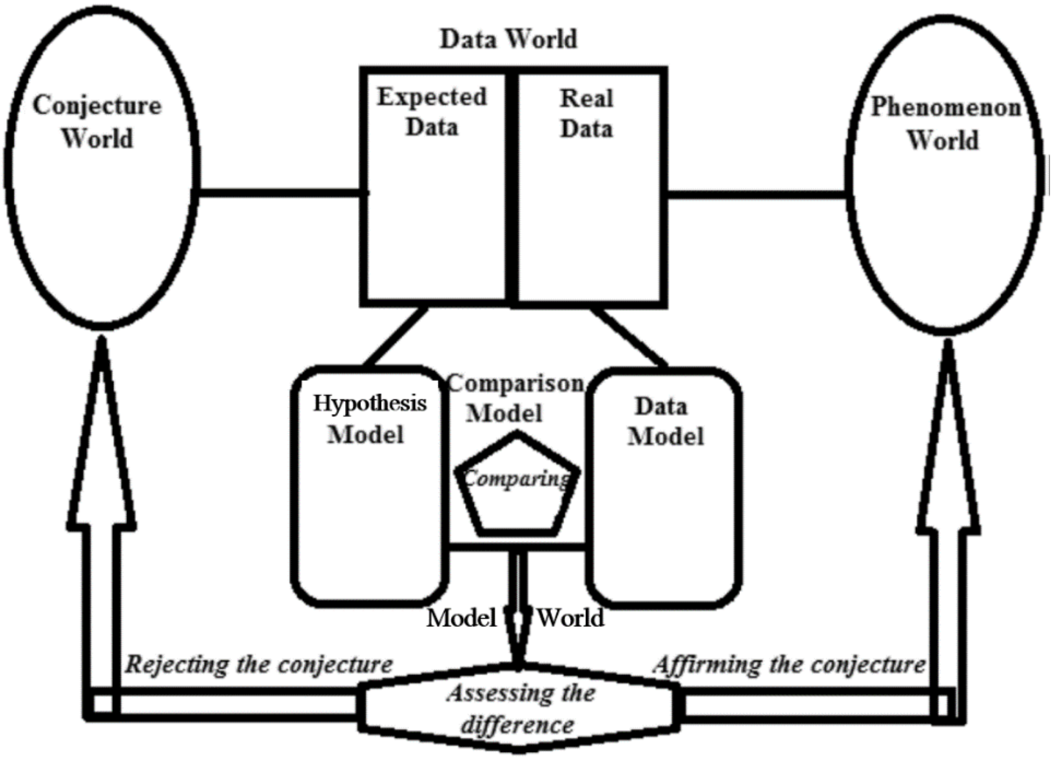


Figure 7-2. RISM framework (Reprint)

(Dvir & Ben-Zvi, 2018; Pfannkuch, Ben-Zvi, & Budgett, 2018; edited by the author from p. 1121)

Next, the author introduces the IME (integrated modelling approach) on the basis of previous research on the statistical inquiry process. IME is an approach which integrates an EDA (exploratory data analysis) approach emphasising the data search and a probability-based approach for modelling activities. IME aims to build relationships between samples and a population. For a particular problem, the RISM framework strives to integrate the EDA approach in the data world, which is used to draw a conclusion to the problem through the collection of actual sample data and an analysis to estimate the characteristics of the population, with a probability-based approach to modelling activities in the model world, in which the conclusion obtained in the data world is regarded as the hypothesis; assume the samples in a virtual population after making the hypothesis model; and on that basis form the statistical concept of inferential statistics to derive the characteristic of the virtual population. In other words, this process has an annular structure, which means that the conclusion presented in the data world is transferred to the model world as a hypothesis, and the method knowledge on sampling formed in the model world is used in the data world again to form a more refined conclusion. Figure 7-3 shows the steps of these processes.

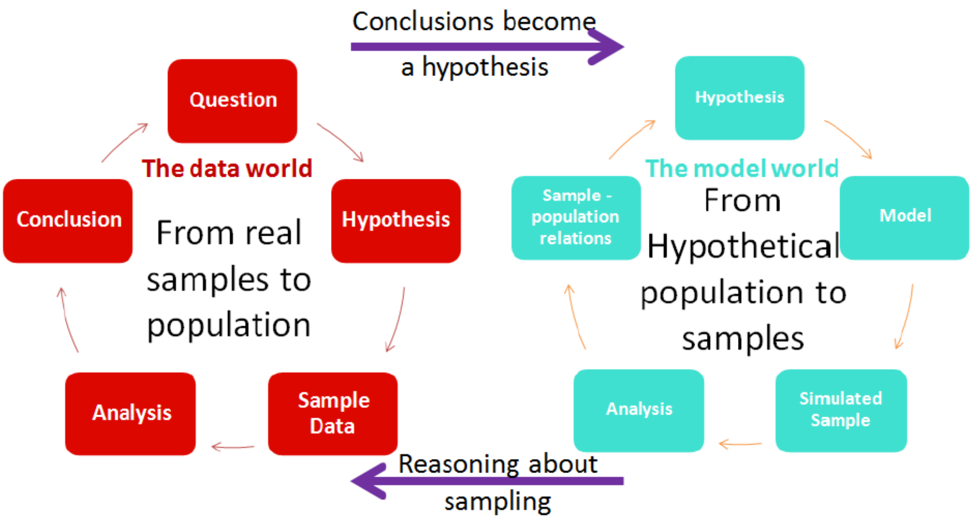


Figure 7-3. The rationale of the integrated modeling approach (IMA) (Reprint)

(Manor Braham & Ben-Zvi, 2017, p. 120)

There is a relationship between IMA and the RISM framework discussed above. The shuttling between the data world and the model world in the RISM framework aims to form method knowledge on sampling by identifying the relationship between the samples and the population, which is precisely IMA itself. That is, the shuttling between the data world and the model world in IMA is a framework which explains this shuttling within the RISM framework in more detail. Therefore, IMA details a part of the RISM framework.

In addition, the framework for the emergent hypothesis modelling cycle in Figure 7-1 was constructed for an issue common to both IMA and the RISM framework, which is that the first part of the statistical inquiry, which does not necessarily use data, is not explained. The shuttling model between the five worlds can then be proposed as shown in Figure 7-4 to survey the emergent hypothesis modelling cycle showing the statistical inquiry cycle in this context world, retrieve the idea of IME and the RISM framework, and comprehend the whole statistical inquiry process.

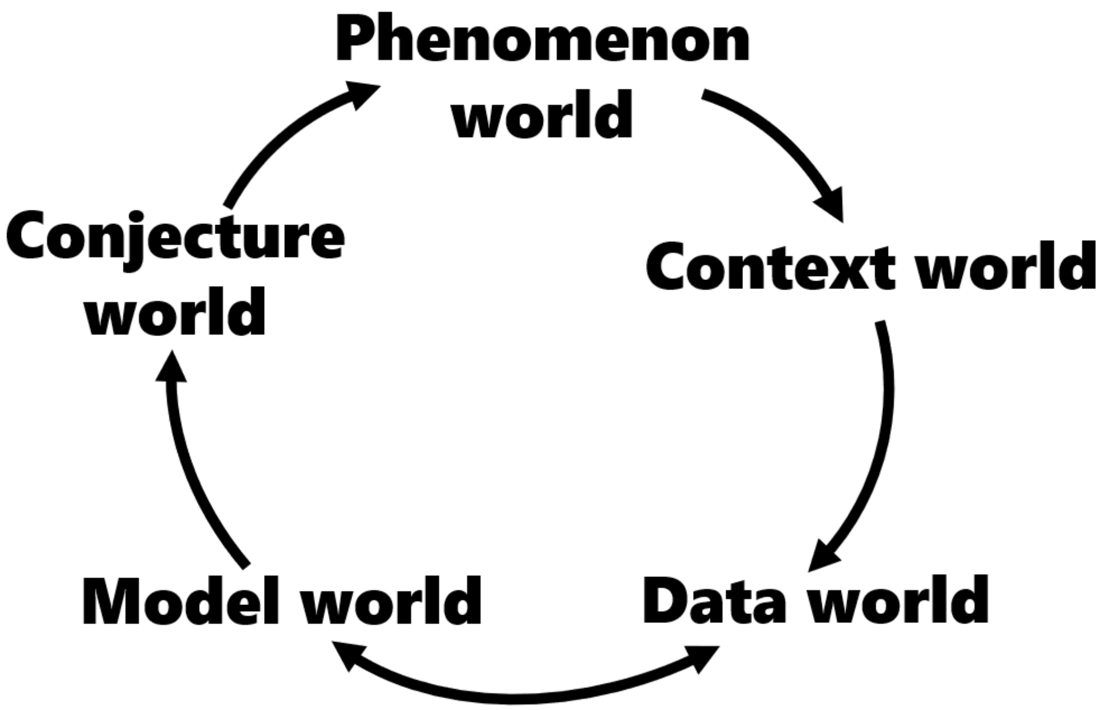


Figure 7-4. Cyclic shuttling between five worlds in statistical inquiry

First, emergent hypothesis modelling (Figure 7-1) in the context world is conducted in order to transform the vague problem which occurred in the phenomenon world into a definite problem and clarify what kinds of data are necessary. After that, the formation of method knowledge on sampling and actual sampling are implemented by shuttling between the data world and the model world (IMA). However, the data world and the model world here are separated because the formation of method knowledge on sampling is performed from the beginning, but the distinction between the two worlds gradually disappears as they are integrated into one world (Manor Braham & Ben-Zvi, 2017). The solution to the original problem is then conjectured as a decision by verifying the hypothesis model-of that emerged in the context world through informal statistical inference in the conjecture world (RISM framework). If the hypothesis model-of is rejected by this verification, then the second cycle towards the emergence of a new hypothesis model-of is started. If the hypothesis model-of is not rejected, then the second cycle is developed towards a more sophisticated one. The author would like to add only one comment. The shuttling model between five worlds in the statistical inquiry shown in Figure 7-4 does not necessarily proceed in sequence between all the worlds with this arrow, and each world is not completely distinct from the others, so they overlap. Therefore, it should be noted that this shuttling model between five worlds is just one model.

Taking the teaching experiment in Chapter 6 as an example, the author would like to explain the shuttling model between five worlds in this statistical inquiry. The vague problem ‘Is the cause of global warming CO_2 or not?’ was raised in the phenomenon world. In order to clarify this problem and determine the data necessary for problem solving, emergent hypothesis modelling is performed in the context world. Then, in the inquiry by Group D taken up in the analysis, the hypothesis model-for ‘The cause of global warming is not CO_2’ was set, and finally the hypothesis model-of ‘If carbon dioxide is involved in global warming, then the temperature of the world would continue to rise without falling’ emerged through contextual thinking on CO_2. A comparison between the experimental group and the control group cannot be made because this teaching experiment is an inquiry into the causes of the result, and the global context of global warming is treated. Therefore, it is extremely difficult to actually collect data, and it is only possible to use data already collected. If it were possible to collect data, it could be said to be an inquiry in the data world. However, such an inquiry was not found in all groups of this teaching experiment, so it was basically an inquiry in the model world. Then, as explained in Chapter 5, the hypothesis model in the RISM framework is positioned in the model world, so the hypothesis model-for and the hypothesis model-of in the context world of the shuttling model between the five worlds are also positioned in the model world. Therefore, the hypothesis model-for ‘The cause of global warming is not CO_2’ and hypothesis model-of ‘If carbon dioxide is involved in global warming, then the temperature of the world would continue to rise without falling’ which emerged through the efforts of Group D are the hypothesis models. Additionally, as shown in Chapter 6, Group D verified the hypothesis model using Figure 7-5, selected by searching the Internet.[1] Because Figure 7-5 is a model using certain data, it is a data model. In this case, the hypothesis model was not rejected after comparing the hypothesis model and the data model, so this hypothesis model in the conjecture world (in particular, the hypothesis model-for)

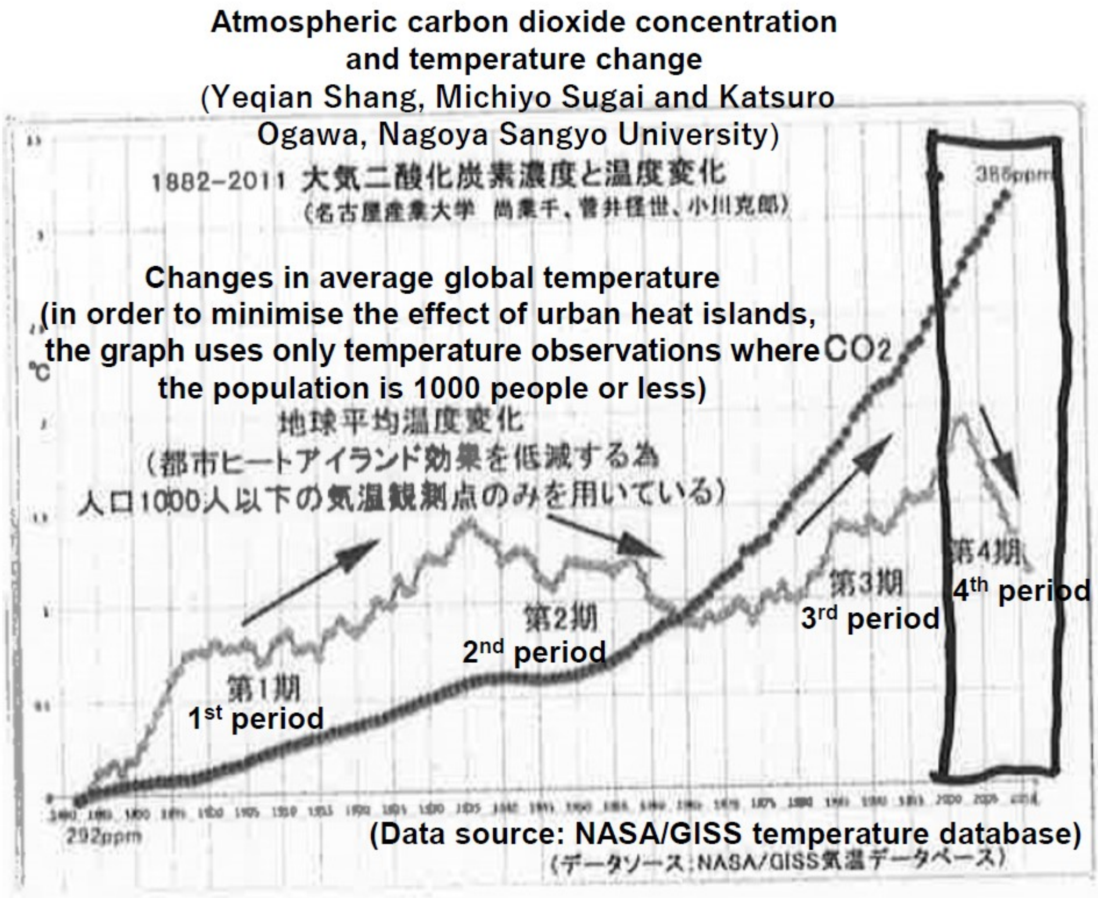


Figure 7-5. The graph used in Group D’s presentation in the 3rd lesson (Reprint)

(with handwritten parts by group D; translated by the author)

is regarded as a valid conjecture and is then returned to the phenomenon world to become the solution to the original problem. Although there was no further inquiry in Group D, it would be ideal for a second shuttling between the five worlds to be conducted to further refine this conjecture.

As described above, the shuttling model between the five worlds in this statistical inquiry is common to all types of schools and can function as a normative model for the whole statistical inquiry process, and is the model surveying the emergent hypothesis modelling in the context world resulting from the whole statistical inquiry process. Therefore, this model can serve as a principle for the statistics curriculum in Japan from the viewpoint of context. Finally, the author summarises the principle of Figure 7-4 as the conclusion of this research: Statistical inquiry requires attention to the five worlds (phenomenon, context, data, model, and conjecture worlds) and the interconnections among them.

7.2 Significance of the research and implications for statistics education

The author explains the significance and implications for statistics education of this research. The problem statement in this research is that it is difficult to identify the problem itself, which is the object of statistical inquiry in our current society where knowledge and information are too complicated to allow the simplification and idealisation of the problem in the real world. In order to address this problem statement, this research focused on the systemic-evolutionary method (cf. Hirabayashi, 2001; Wittmann, 2001a), which is an attempt not only to simplify or idealise a complex phenomenon but also to try to capture the essence of a complex substance, and constructed the problematising process of the problem so as to update a vague problem into a definite one. The author proposed emergent hypothesis modelling as in Figure 7-1, whose core is contextual thinking, from the viewpoint of context. Finally, the author proposed a shuttling model between the five worlds in statistical inquiry as in Figure 7-4, consisting of the phenomenon world, the context world, the data world, the model world, and the conjecture world as a principle for the Japanese statistics curriculum in the form of surveying contextual thinking. Therefore, the significant point is that this research could represent the process which any student from elementary school to upper secondary school should follow in an inquiry in order to enhance the feasibility of such statistical inquiry in the current situation where it is difficult to make a complex phenomenon the object of statistical inquiry. Furthermore, it can also be appreciated that this research could reveal the students' specific process of statistical inquiry through the teaching experiment concerning the complex context of environmental problems. Therefore, the success of this research suggests that it can be used as one guideline to develop a statistics curriculum and establish the teaching and learning process in devising the units and educational practices for a future society where extensive informatisation is expected to continue. In summary, for one item on a list of research questions necessary for the future of theory development in statistics education presented in Nilsson, Schindler, and Bakker (2018), "There is a need for a deeper theoretical conceptualization of context and contextualizing in statistics education" (p. 374), this research was able to theorise about a proper principle of the statistics curriculum from the viewpoint of context.

Moreover, it is suggested that the success of this research is applicable not only to Japan but also to some extent to statistics education around the world. Statistics education in Japan seeks to convert the formation of content knowledge into method knowledge, so this research intended to examine the implications of this current situation. However, such a situation is not limited to Japan, as competency-based statistics education is required in each country around the world (cf. Garfield, Le, Zieffler, & Ben-Zvi, 2015; Moore, 1998; Wild, Pfannkuch, Regan, & Horton, 2011). For example, students are required to conduct statistical inquiry in all school types and grades (see Appendix A), and competency-based statistics education is intended to be conducted in New Zealand, on which this research focused. This research obtained some findings for statistics education in Japan because contextual thinking has arisen in the educational practices in New Zealand. However, these practices were observed for one week in only one school in New Zealand, so it is not always the case that educational practices are conducted in all schools in New Zealand where the contextual thinking occurs. Furthermore, there are some descriptions related to contextual thinking in the curriculum. However, attention has not been paid to the issue of how to promote and structure the use of contextual knowledge at the beginning of an investigation. Therefore, the theoretical results in this research can be useful as a guideline for reflecting on and developing the statistics curriculum in New Zealand. These results can serve to refine statistics education in not only New Zealand but also many other countries/regions. Therefore, this research can contribute to the positioning and treatment of context in competency-based statistics education as the international standard.

7.3 Limitations of the research

In Section 7.2, the author showed the significance and implications of this research, but there are also limitations to this research. First, this research proposed a principle for a statistics curriculum consistent from elementary school to upper secondary school, but the teaching experiment was only carried out in one second-year lesson in lower secondary school. Of course, the intention of the teaching experiment was not to show the effectiveness of the developed principle but to evaluate the teaching unit while analysing what kind of inquiry the students performed, so it would be possible to implement comparative analysis by conducting the teaching experiments in multiple lessons; the validity of the evaluation of the teaching unit could then be enhanced by conducting comparative analysis. Also, the context in the teaching unit and teaching experiment applied in this research was environmental problems, and other authentic complex contexts required socially (e.g., contexts of decision-making related to equity and fairness) could not be considered. Therefore, it is one of the limitations of this research that it was not possible to conduct multiple teaching experiments dealing with various authentic contexts in a variety of school types and at various grade levels.

In addition, as the intention of this research was to examine how to conduct statistics education in Japan so as to convert the formation of content knowledge into method knowledge, this research focused on the context, after which the author considered the statistical inquiry process as method knowledge throughout this research. That is, the object of the discussion was always method knowledge in this research. However, it is also necessary to handle content knowledge in statistics education. Traditional statistics education in Japan has focused on content knowledge, so it is necessary to introduce method knowledge as a change of direction, but it is not intended for statistics education to aim at the formation of only method knowledge. Therefore, the issues of the content knowledge to be acquired in statistics education and the relationship between content knowledge and method knowledge are also urgent research questions. One of the limitations of this study is that this research could not fully consider this research question.

7.4 Future directions of research

Finally, the author will discuss the future directions of research. As mentioned in Section 7.3, this research always considered method knowledge in statistics, but the author could not take content knowledge into consideration to any great extent. Content knowledge in statistics education has been researched in terms of 'big ideas' (cf. Watson, Fitzallen, Fielding-Wells, & Madden, 2018). Garfield and Ben-Zvi (2004) proposed eight big ideas in statistics (data, distribution, trend, variability, models, association, samples and sampling, and inference), which Garfield and Ben-Zvi (2008) reorganised into nine (data, statistical models and modelling, distribution, centre, variability, comparing groups, samples and sampling distributions, statistical inference, and covariation). In this way, these studies took up the central concepts of teaching and learning in statistics education. Recently, Watson, Fitzallen, and Carter (2013) summarised these as five big ideas (informal inference, distribution, variation, expectation, and randomness) for statistics in the classroom, which are illustrated in Figure 7-6.

This research considered the statistical inquiry process intended for informal hypothesis testing through the formation of hypotheses, so it can be said that this research forms a part of informal inference

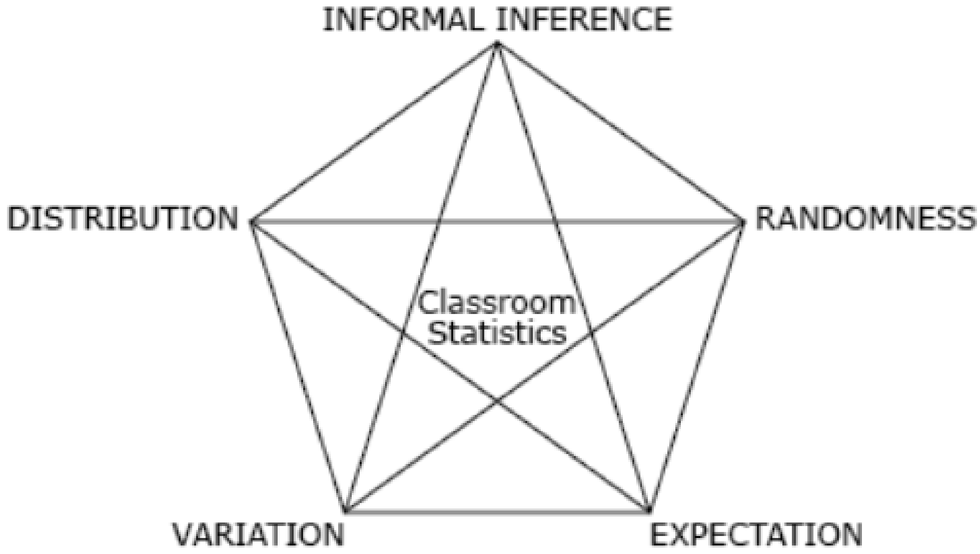


Figure 7-6. Interrelated big ideas underlying statistics in the classroom

(Watson, Fitzallen, & Carter, 2013)

within the big ideas in Figure 7-6. However, controversial points remain concerning the concept of sampling aimed at constructing the relationship between samples and the population located at the core of informal inference, and the relationship with distribution, randomness, variation, and expectation. Studies focusing on each big idea have accumulated; for example, Reading and Shaughnessy (2004) and Garfield and Ben-Zvi (2007) on distribution, Batanero and Serrano (1999), Paparistodemou, Noss, and Pratt (2002), and Batanero (2015) on randomness, Watson (2002), Garfield and Ben-Zvi (2007), and English and Watson (2016) on variation, and Watson and English (2015) and English and Watson (2016) on expectation may be mentioned. Therefore, it is thought that it is possible to refer to content knowledge in statistics education by considering how, within the shuttling between five worlds in statistical inquiry shown in Figure 7-4, shuttling between the data world and the model world relates to hypothesis testing based on the findings of these studies.

Furthermore, this requires that the inquiry be conducted using technology based on big data (e.g., Ben-Zvi, Gravemeijer, & Ainley, 2018; Biehler, Ben-Zvi, Bakker, & Makar, 2013; Saldanha &

McAllister, 2016). The role of technology is listed as one of the research questions necessary for the future of theory development in statistics education presented in Nilsson, Schindler, and Bakker (2018), which is why it is considered a current research question. In particular, NZGrapher and iNZight, which statistics education research groups in New Zealand are researching, and TinkerPlots, which has now spread all the world over after research by statistics education research groups in Australia, are expected to be informative as technology specialised for statistics education.

In addition, Japanese teachers rarely use these technologies in statistics education, partly because there are few findings regarding the effects of using technologies on teaching and learning statistics in Japanese statistics education research. However, since statistics education in Japan as implemented henceforth will place greater emphasis on inferential statistics, a statistical inquiry based on big data using technology is necessary. Additionally, it can be said also to be required in the case of teacher education by Figure 7-7, repeated from Chapter 1, which illustrates the identity of mathematics education in Wittmann (1995).

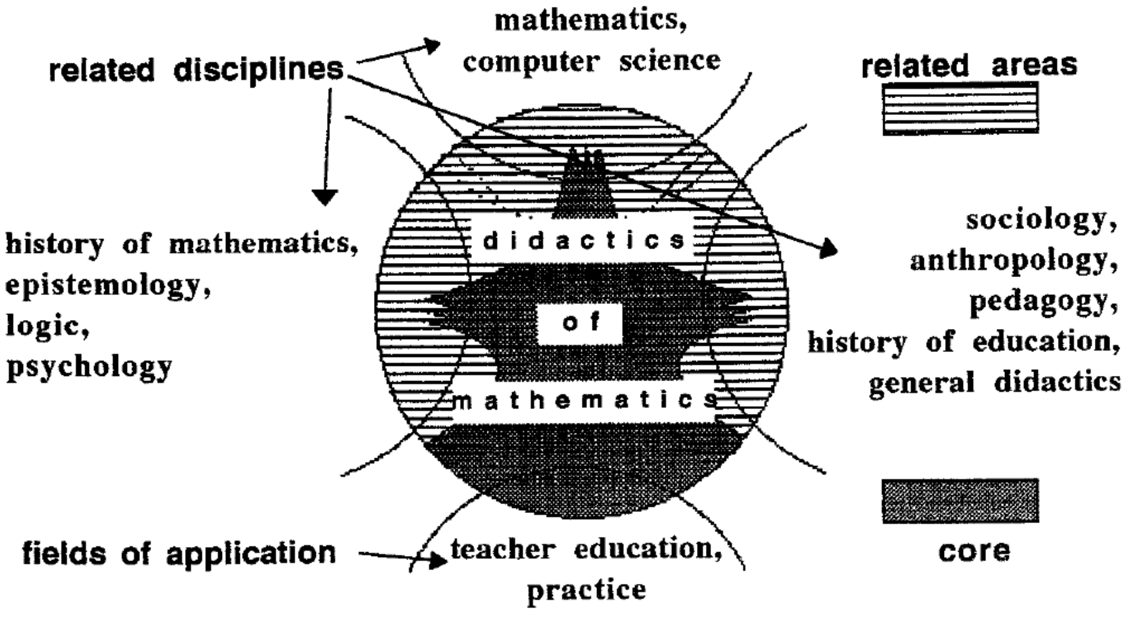


Figure 7-7. The core and the areas related to mathematics education, their links to the related disciplines and the fields of application[2] (Reprint) (Wittmann, 1995, p. 357)

The final form in which various core and related areas are retrieved is practice and teacher education. Regarding practice, this research developed a teaching unit and conducted a teaching experiment based on considerations from related areas such as hypothesis testing, environmental problems, and the history of human civilizations. On the other hand, teacher education is left as a future direction of research.

Finally, about the context which is the theme in this research, not only statistical knowledge and mathematical knowledge but also scientific knowledge, social knowledge, and similar realms of knowledge are required when practising contextual thinking in the statistical inquiry cycle, which is why an interdisciplinary approach must be adopted in order to complete an authentic statistical inquiry cycle (cf. Gal & Garfield, 1997; Rao, 1975; Zieffler, Garfield, & Fry, 2018). Therefore, as mentioned at the beginning of Chapter 1, the critical issue in the future is to provide a solution to the main issue in this research, that is, examining the world system during its ongoing conversion from determinism to indeterminism from the aspect of education and deciding how to pursue education from now on, through a consideration of its connections with other subjects and seeking a comprehensive form of school education whose centre is statistics education.

Notes

1. It is unclear from which Internet page Group D cited this, but the following URL contains the same graph: http://www.env01.net/fromadmin/contents/2015/2015_07.html (Final confirmation on February 20, 2020).
2. Regarding the core and related areas of mathematics education, Dr Wittmann was invited to give a plenary lecture on ‘Mathematics Education as Systemic-evolutionary Design Science: Revisiting’ at the 45th Meeting of the Japan Academic Society of Mathematics Education on January 29, 2017 (Sun). Here, the author would like to note that he used the related areas on the left side of Figure 7-7

with the addition of 'semiotics' in his plenary lecture. Please see the details in Wittmann (2019).

Publications associated with this research

Fukuda, H. (2014a). Present issues and perspectives on statistics education. *Journal of Japan Society of Education*, *68*(1), 68-71. (in Japanese)

Fukuda, H. (2014b). A development research on teaching unit about statistics education: Presentation of teaching unit raising critical thinking for a decision-making skill. *Journal of JASME Research in Mathematics Education*, *20*(2), 169-182. (in Japanese)

Fukuda, H. (2014c). The significance of statistics education as mathematics education. *Journal of Japan Society of Mathematical Education: Research Journal of Mathematical Education*, *96(Fall Special Issue)*, 153-160. (in Japanese)

Fukuda, H. (2015). A study on statistical inquiry process involving mathematics educational significance. In K. Beswick, T. Muir, & J. Wells (Eds.). *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 160). Hobart, Australia: PME.

Fukuda, H. (2016). Future of statistics education for realizing modeling with systemic-evolutional perspective. *Journal of JASME Research in Mathematics Education*, *22*(2), 153-162. (in Japanese)

Fukuda, H. (2017a). A comparative analysis of statistical problems between Japan and New Zealand. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, p. 26). Singapore: PME.

Fukuda, H. (2017b). The characteristic of statistics education in Japan by comparison with New Zealand. *Journal of JASME Research in Mathematics Education*, *23*(2), 151-158. (in Japanese)

Fukuda, H. (2018a). Development of teaching unit on causality focusing on environment. In F.-J. Hsieh

(Ed.). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education* (Vol. 1, p. 371). Taipei, Taiwan: EARCOME.

Fukuda, H. (2018b). The future of statistics education from the perspective of educational practices in New Zealand. In M. A. Sorto, A. White, & L. Guyot (Eds.). *Proceedings of the Tenth International Conference on Teaching Statistics*. Voorburg. The Netherlands: International Statistical Institute. Retrieved from https://icots.info/10/proceedings/pdfs/ICOTS10_1D3.pdf?1531364186

Fukuda, H., Otani, H., & Iwasaki, H. (2018). Development research of statistical hypothesis testing: Focusing on expansion and distinction from reduction to the absurd. *Journal of Science Education in Japan*, *42*(4), 335-349. (in Japanese)

Fukuda, H. & Kamimoto, Y. (2019). Comparison between school types by problem analysis in textbooks of Japanese statistics education: From the perspective of context. *Journal of Science Education in Japan*, *43*(4), 362-372. (in Japanese)

Appendix A: The New Zealand Curriculum and the Teaching Content of 'Mathematics and Statistics'

The current school system and structure of the entire curriculum is shown in Fighre A-1 and Figure A-2.

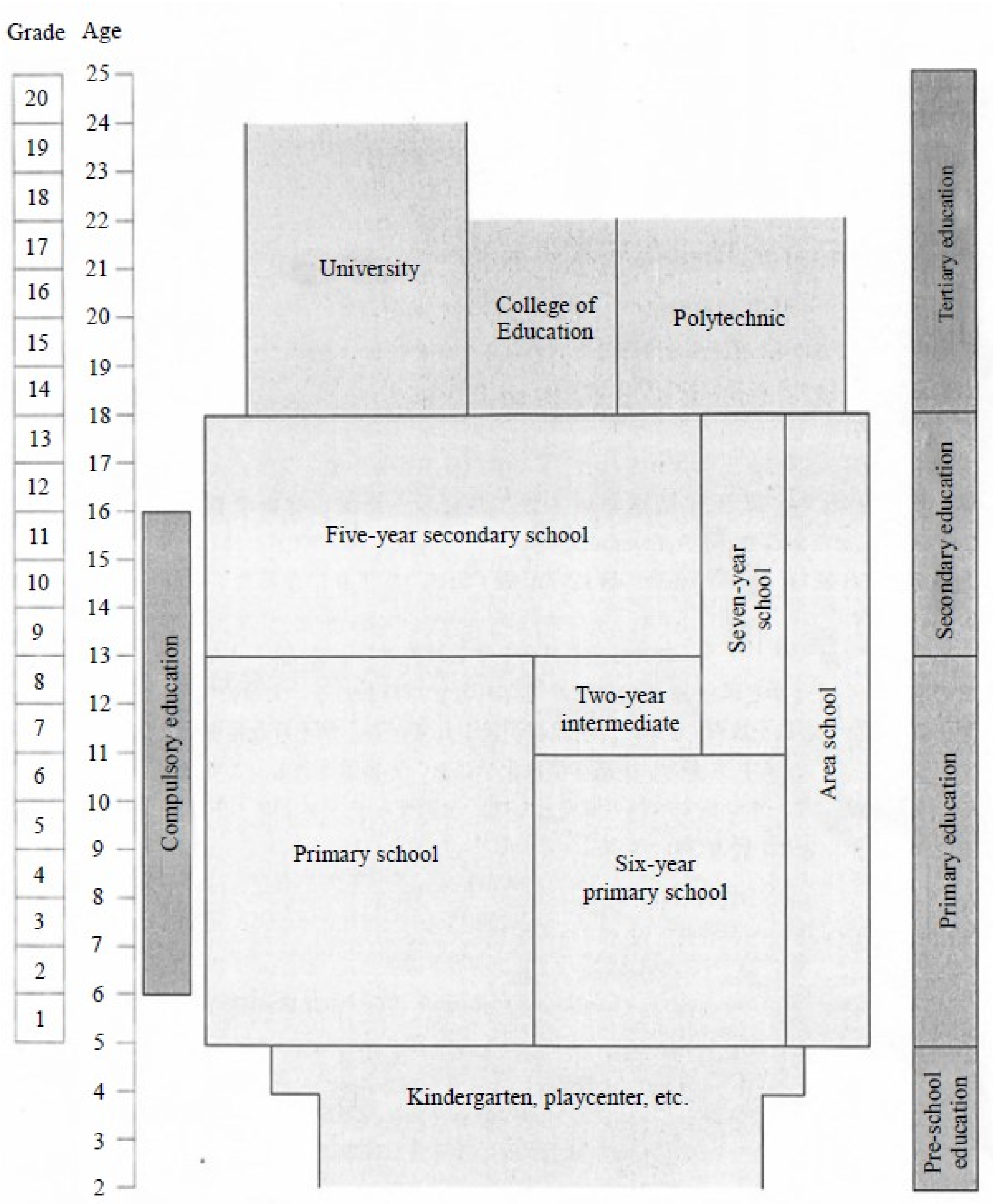


Figure A-1. The school system chart in New Zealand (MEXT, 2017, p. 86)

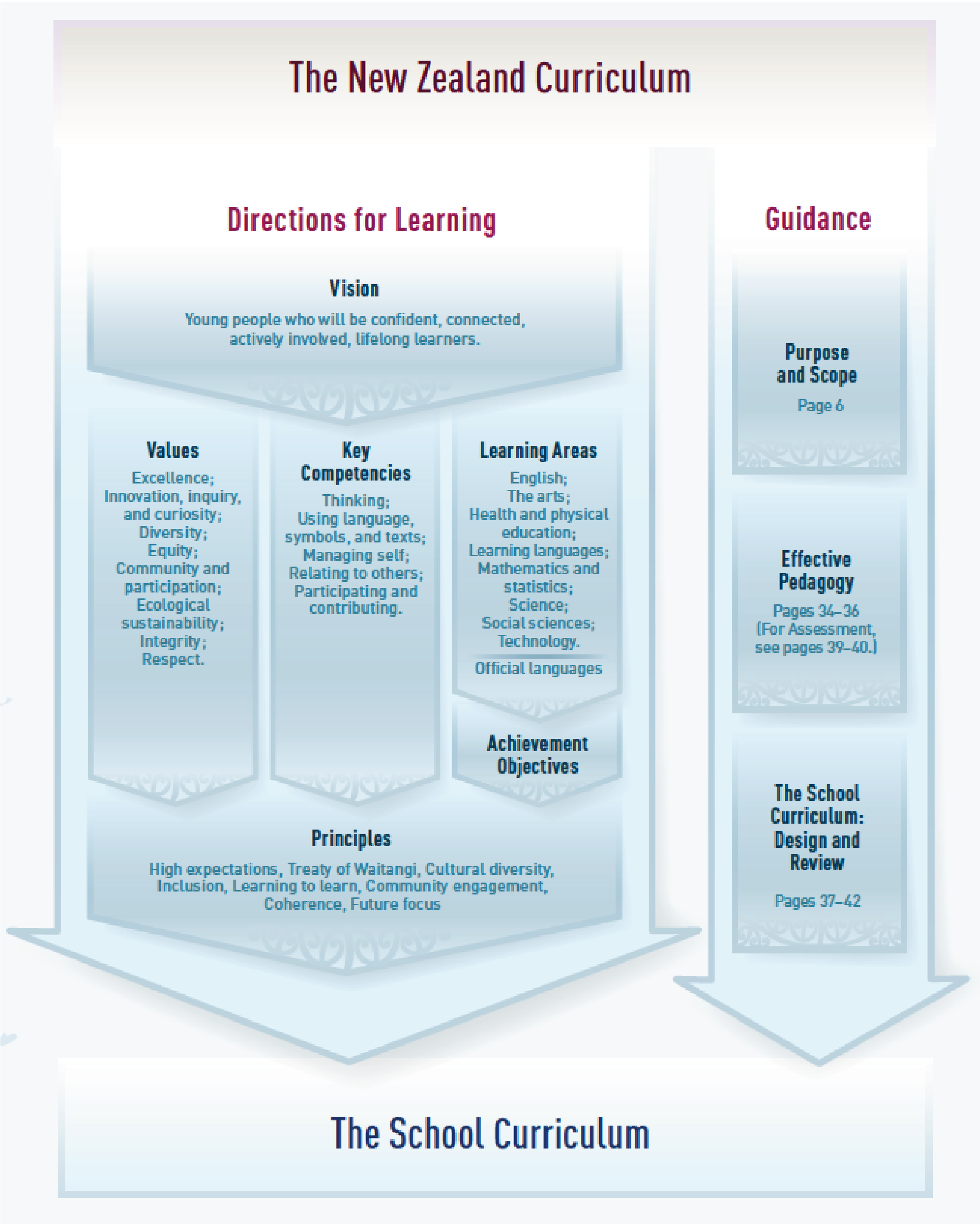


Figure A-2. Overview of a schematic view of curriculum in New Zealand (Ministry of Education, 2007c, p.7)

This curriculum is based on the vision of what New Zealand society expects students to do in their school years and what they want them to acquire before leaving school (Ministry of Education, 2007c). In addition, there are three pillars: 'values' as beliefs to continue to be explored, modelled, and encouraged about what is important or desirable, 'key competencies' as capabilities for living and lifelong learning, and 'learning areas' in each subject for the achievement of the vision. Moreover, there exist 'principles' which embody beliefs and underpin all school decision making. The learning area points to each subject (e.g., Mathematics and Statistics, English, Science, and so on), so it is easy to understand its content. Therefore, the other four elements (vision, values, key competencies, and principles) are explained below (Ministry of Education, 2007c, pp. 8–13).

<Vision>

Our vision is for young people:

・who will be creative, energetic, and enterprising

・who will seize the opportunities offered by new knowledge and technologies to secure a sustainable social, cultural, economic, and environmental future for our country

・who will work to create an Aotearoa New Zealand in which Māori and Pākehā (those whose ancestors were not Māori) recognise each other as full Treaty partners, and in which all cultures are valued for the contributions they bring

・who, in their school years, will continue to develop the values, knowledge, and competencies that will enable them to live full and satisfying lives

・who will be confident, connected, actively involved, and lifelong learners

<Values>

Values are deeply held beliefs about what is important or desirable. They are expressed through the ways in which people think and act. Every decision relating to curriculum and every interaction that takes place in a school reflects the values of the individuals involved and the collective values of the institution. Students will be encouraged to value: excellence, innovation/inquiry/curiosity, diversity, equity, community/participation, ecological sustainability, integrity, and to respect themselves, others, and human rights. Through their learning experiences, students will learn about:

・their own values and those of others

・different kinds of values, such as moral, social, cultural, aesthetic, and economic values

・the values on which New Zealand's cultural and institutional traditions are based

・the values of other groups and cultures

Through their learning experiences, students will develop their ability to:

・express their own values

・explore, with empathy, the values of others

・critically analyse values and actions based on them

・discuss disagreements that arise from differences in values and negotiate solutions

・make ethical decisions and act on them

<Key Competencies>

The New Zealand Curriculum identifies five key competencies: thinking, using language, symbols, & texts, managing self, relating to others, and participating & contributing. People use these competencies to live, learn, work, and contribute as active members of their communities. More complex than skills, the competencies draw also on knowledge, attitudes, and values in ways that lead to action. They are not separate or stand-alone. They are the key to learning in every learning area. The development of the competencies is both an end in itself (a goal) and the means by which other ends are achieved. Successful learners make use of the competencies in combination with all the other resources available to them. These include personal goals, other people, community knowledge and values, cultural tools (language, symbols, and texts), and the knowledge and skills found in different learning areas. As they develop the competencies, successful learners are also motivated to use them, recognising when and how to do so and why.

<Principles>

The principles embody beliefs about what is important and desirable in the school curriculum—both nationally and locally. They should underpin all school decision making. These principles put students at the centre of teaching and learning, asserting that they should experience a curriculum that engages and challenges them, is forward-looking and inclusive, and affirms New Zealand's unique identity. The principles relate to how the curriculum is formalised in a school; they are particularly relevant to the processes of planning, prioritising, and review. All curricula should be consistent with these eight statements: high expectations, the Treaty of Waitangi [the treaty concluded between the Māori and the British royal authorities], cultural diversity, inclusion, learning to learn, community engagement, coherence, and future focus.

Next, the author explains the learning area. In Figure 3-1, the levels corresponding to each grade are shown, and it is explained that it is composed of eight levels. The educational content of the subject "Mathematics and Statistics" is shown below, but what is presented at each level is not the specific contents to be taught but the skills and techniques to be required and acquired. Therefore, they are only the abilities and skills which the students at the relevant grade and level are required to acquire, and the specific contents and methods by which the teachers teach them these abilities and skills are left up to the individual teacher; in addition, teachers have a free hands in all such matters as the class content, construction, and teaching materials since the textbooks do not have a certification system (Ministry of Education, 2007c, pp. 37–44).

Finally, the educational content of the statistical area in the subject "Mathematics and Statistics" is shown below (Ministry of Education, 2007b, pp. 18–21).

<Level One>

Statistical investigation

・Conduct investigation using the statistical enquiry cycle:

- posing and answering questions;
- gathering, sorting and counting, and displaying category data;
- discussing the results.

Statistical literacy

・Interpret statements made by others from statistical investigations and probability activities.

Probability

・Investigate situations that involve elements of chance, acknowledging and anticipating possible outcomes.

<Level Two>

Statistical investigation

・Conduct investigation using the statistical enquiry cycle:

- posing and answering questions;
- gathering, sorting, and displaying category and whole-number data;
- communicating findings based on the data.

Statistical literacy

・Compare statements with the features of simple data displays from statistical investigation or probability activities undertaken by others.

Probability

・Investigate simple situations that involve elements of chance, recognizing equal and different likelihoods and acknowledging uncertainty.

<Level Three>

Statistical investigation

・Conduct investigation using the statistical enquiry cycle:

  － gathering, sorting, and displaying multivariate category and whole-number data and simple time-series data to answer questions;

  － identifying patterns and trends in context, within and between data sets;

  － communicating findings, using data displays.

Statistical literacy

・Evaluate the effectiveness of different displays in representing the findings of a statistical investigation or probability activity undertaken by others.

Probability

・Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary.

<Level Four>

Statistical investigation

・Plan and conduct investigations using the statistical enquiry cycle:

  － determining appropriate variables and data collection methods;

  － gathering, sorting, and displaying multivariate category, measurement, and time-series data to detect patterns, variations, relationships, and trends;

  － comparing distributions visually;

  － communicating findings, using appropriate displays.

Statistical literacy

・Evaluate statements made by others about the findings of statistical investigations and probability activities.

Probability

・Investigate situations that involve elements of chance by comparing experimental distributions with expectations from models of the possible outcomes, acknowledging variation and independence.

・Use simple fractions and percentages to describe probabilities.

<Level Five>

Statistical investigation

・Plan and conduct surveys and experiments using the statistical enquiry cycle:

− determining appropriate variables and measures;

− considering sources of variation;

− gathering and cleaning data;

− using multiple displays, and re-categorising data to find patterns, variations, relationships, and trends in multivariate data sets;

− comparing sample distributions visually, using measures of centre, spread, and proportion;

− presenting a report of findings.

Statistical literacy

・Evaluate statistical investigations or probability activities undertaken by others, including data collection methods, choice of measures, and validity of findings.

Probability

・Compare and describe the variation between theoretical and experimental distributions in situations that involve elements of chance.

・Calculate probabilities, using fractions, percentages, and ratios.

<Level Six>

Statistical investigation

・Plan and conduct investigations using the statistical enquiry cycle:

− justifying the variables and measures used;

− managing sources of variation, including through the use of random sampling;

− identifying and communicating features in context (trends, relationships between variables, and differences within and between distributions), using multiple displays;

− making informal inferences about populations from sample data;

− justifying findings, using displays and measures.

Statistical literacy

・Evaluate statistical reports in the media by relating the displays, statistics, processes, and probabilities used to claims made.

Probability

・Investigate situations that involve elements of chance:

− comparing discrete theoretical distributions and experimental distributions, appreciating the role of sampling size;

− calculating probabilities in discrete situations.

<Level Seven>

Statistical investigation

・Carry out investigations of phenomena, using the statistical enquiry cycle:

− conducting surveys that require random sampling techniques, conducting experiments, and using existing data sets;

− evaluating the choice of measures for variables and the sampling and data collection methods used;

– using relevant contextual knowledge, exploratory data analysis, and statistical inference.

・Make inferences from surveys and experiments:

– making informal predictions, interpolations, and extrapolations;

– using sample statistics to make point estimates of population parameters;

– recognising the effect of sample size on the variability of an estimate.

Statistical literacy

・Evaluate statistically based reports:

– interpreting risk and relative risk;

– identifying sampling and possible non-sampling errors in surveys, including polls.

Probability

・Investigate situations that involve elements of chance:

– comparing theoretical continuous distributions, such as the normal distribution, with experimental distributions;

– calculating probabilities, using such tools as two-way tables, tree diagrams, simulations, and technology.

<Level Eight>

Statistical investigation

・Carry out investigations of phenomena, using the statistical enquiry cycle:

– conducting experiments using experimental design principles, conducting surveys, and using existing data sets;

– finding, using, and assessing appropriate models (including linear regression for bivariate data and additive models for time-series data), seeking explanations, and making predictions;

– using informed contextual knowledge, exploratory data analysis, and statistical inference;

− communicating findings and evaluating all stage of the cycle.

・Make inferences from surveys and experiments:

− determining estimates and confidence intervals for means, proportions, and differences, recognising the relevance of the central limit theorem;

− using methods such as resampling or randomisation to assess the strength of evidence.

Statistical literacy

・Evaluate a wide range of statistically based reports, including surveys and polls, experiments, and observational studies:

− critiquing causal-relationship claims;

− interpreting margins of error.

Probability

・Investigate situations that involve elements of chance:

− calculating probabilities of independent, combined, and conditional events;

− calculating and interpreting expected values and standard deviations of discrete random variables;

− applying distributions such as the Poisson, binomial, and normal.

Appendix B: Lesson Plan for 6 Hours of Teaching Experiment

Second Year Mathematics Teaching Plan

March 12–16, 2018

Supervisor: Name abbreviation (licensed teacher)

(0) Sub-unit objective and outline (total six hours)

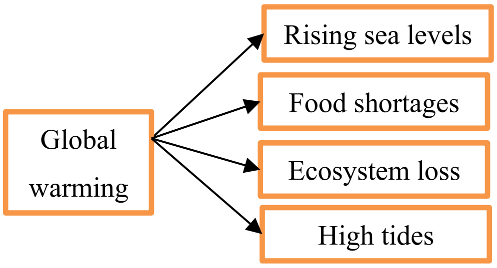
To acquire statistical literacy on the critical mathematical level, through deciding to update ones own world-view on the basis of reflection on ones own experience of the world through dialogue with others about their experiences and views of the world.

Main Question (P)	Time	Main learning activity (PPDAC)	Main assistance (・)
P0 What is happening to the Earth? P1 What is causing global warming?	First Hour	Problem1 Share information about causes of global warming and global environmental problems. Plan1 Division into groups relating to causes of global warming (study topics)	・Watch video providing an outline explanation of global environmental problems. ・Watch video with arguments from proponents and opponents of the CO_2 Theory of Climatic Change.
P1-① Why could it be said that CO_2 is causing global warming? P1-② Why can it not be said that CO_2 is causing global warming?	Second Hour	Data1 & Analysis1 Examine the basis (data) relating to causes of global warming.	・ Have students put the data examined on their worksheets, and describe which data led to Conclusion1.
	Third Hour	Conclusion1 Summarise what is causing global warming on the basis of the data. Each group will present in front of the class, confirming that a range of factors cam be thought of as causing behind global warming.	・Confirm which parts of the data investigated led students to Conclusion1.

P2-① What should we do to reduce CO_2? (What is causing the cause of global warming?) P2-② What else is causing global warming, other than CO_2?	Fourth Hour	Problem2 & Plan2 Confirm what would be worth further investigation, in order to prevent global warming. Data2&Analysis2 Investigate the basis (data) for thinking of increased CO_2 as a cause, and for thinking of other factors as causes of global warming.	・Ask proponents of the CO_2 Theory of Climatic Change think about what can be done to reduce CO_2, and opponents to investigate other causes of global warming.
	Fifth Hour	Conclusion2 Create a cause and effect diagram summarising CO_2 as a cause, and other causes of global warming.	・ Introduce cause and effect diagrams. ・Put the data examined into a cause and effect diagram.
P3 What can be done to prevent global warming?	Sixth Hour	Conclusion3 Each group will present in front of the class; based on the summary of causes of global warming, individuals will write reports on preventing global warming.	・Investigate man-made and natural causes of global warming, and create a cause and effect diagram summarising these as a class. ・Focus on man-made causes from the perspective of positive solutions to global warming, and have the students consider measures to prevent global warming.

(1) Objectives and outline for the first hour

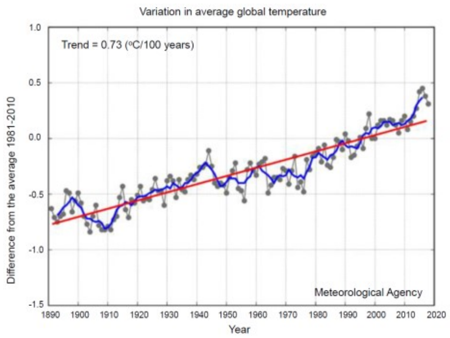
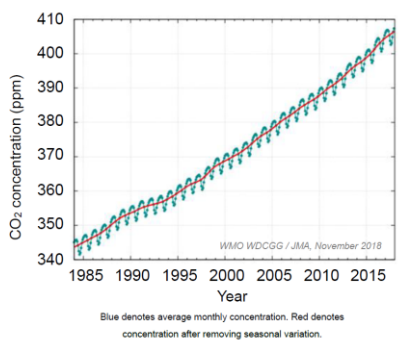
To learn about the existence of proponents and opponents of the CO_2 Theory of Climatic Change by watching videos about what is causing global warming, and for individuals to identify what they would like to investigate.

Learning activity	Teacher encouragement and student response	Key points (・) & evaluation (■)
Task Presentation 10 mins	【Question】 What is happening to the Earth? S: The average temperature and sea level are rising year on year. S: Glaciers are retreating. S: Food is becoming scarce, and ecological systems are being destroyed. S: High tides and super typhoons are increasing. T: What is the cause behind the occurrence of these environmental problems? S: The fact that temperature is increasing is the cause.  【Study project】 What is causing global warming?	・Watch a video summarising environmental problems (https://www.youtube.com/watch?v=URSj7PA_ZwY). ・ Set out environmental problems in terms of “cause” and “effect” relationships.
Independent Problem Solving 15 mins Summary 20 mins	S: Increasing CO_2 is the cause. S: It seems that there may be causes other than CO_2. T: Do you think CO_2 is the cause? Do you think that it is not the cause? 【Summary】 CO_2 might be the cause; there might be other causes T: Would each group please summarise what you would	・ Watch videos relating to causes of global warming (https://www.youtube.com/watch?v=URSj7PA_ZwY and https://www.youtube.com/watch?v=CTZqFvXUwlQ&t=69). ・Divide the class into groups that support and oppose the

	like to investigate in the next period? S: In the video people stated reasons in support and opposition of the CO_2 Theory of Climatic Change, but we didn't fully understand them. S: We would like to look into the possible reasons for CO_2 causing global warming. S: We would like to investigate the relationship between the fact that CO_2 levels are increasing and the fact that temperatures are rising. S: We would like to look into how much CO_2 levels are increasing. S: We would like to investigate why it could be said that CO_2 is not causing global warming. S: We would like to investigate what the causes of global warming other than CO_2 might be.	CO_s Theory of Climatic Change (groups of approximately six people each to form the basis of class activities). ■ Understanding that a basis is needed in order to argue either for or against the CO_2 Theory of Climatic Change.

(2) Objectives and outline for the second and third hours

By researching why CO_2 can (or can not) be said to be causing global warming in books and online, and presenting to the class in groups, and using (as the basis for such presentations) data that indicates a relationship between increasing CO_2 density and rising temperatures, students will be able to explain that "CO_2 would seem to be a cause of global warming, but it would seem that there are also other causes."

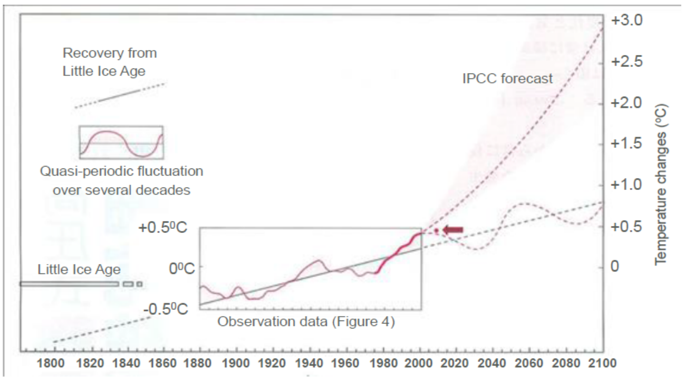
Learning activity	Teacher encouragement and student response	Key points (・) & evaluation (■)
Task Presentation 5 mins	T: Let us confirm what we will be investigating today. S: We will be investigating why it could be said that CO_2 is causing of global warming. S: We will be investigating why it could be said that CO_2 is not causing global warming. 【Study project】 Why could it be said that CO_2 is (or is not) causing global warming?	・Confirm what was planned in the previous hour, and set the study project.
Independent Problem Solving 40 mins	【Reasons why aaaaCO2 could be said to be causing global warming】 ・ CO_2 has the effect of raising temperatures, so it must be causing global warming. 【Reasons why CO_2 could be said to be causing global warming】 ・Changes in temperature and CO_2 density have been consistent from 1990 to 2015.  (Ministry of Land, Infrastructure, Transport and Tourism website)  (Japan Meteorological Agency website)	・ When students state "Based on this data, it would appear that there is a relationship between CO_2 and global increasing temperatures," ask them, "which is the cause, and which is the effect?" ・Ask "why are you using that data?" and "what can be understood about global warming from that data?" and make students aware of

Refining

40 mins

【Reasons why it could not be said that CO_2 is causing global warming】

・When one considers the fact that temperature increases in waves every few decades, the recent temperatures are consistent with that fact, and so CO_2 is not a primary cause for global warming.

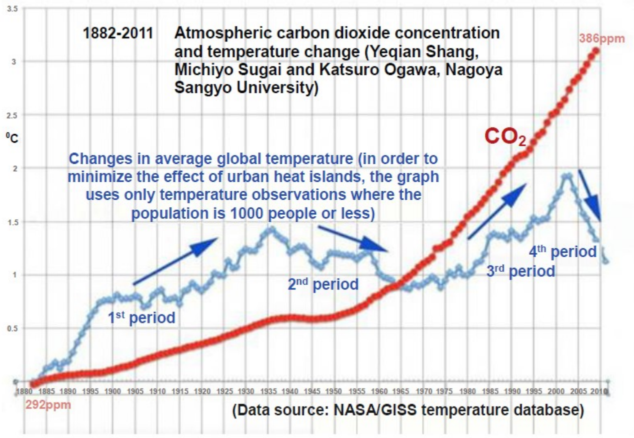


(AKASOFU Syun-Ichi)

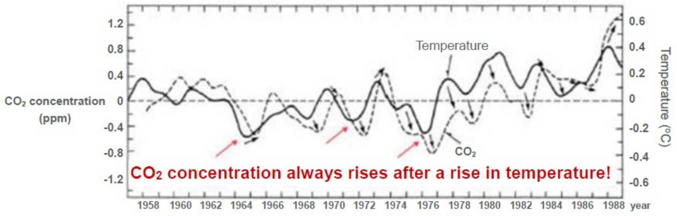
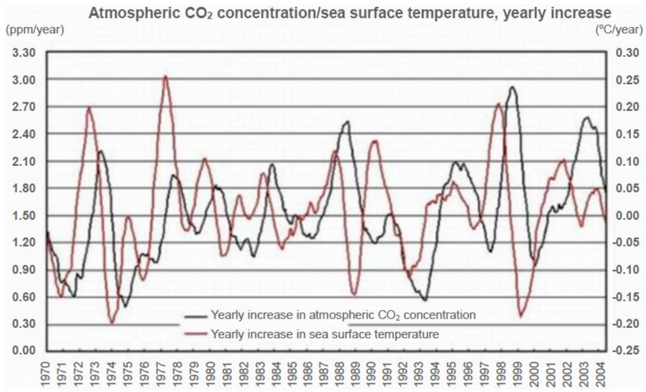
【Reasons why it could not be said that CO_2 is causing global warming】

・Temperatures rose from 1890 to 1900 despite only marginal increases in CO_2.

・Temperatures decreased from 1955 to 1975, despite rapid increaes in CO_2.

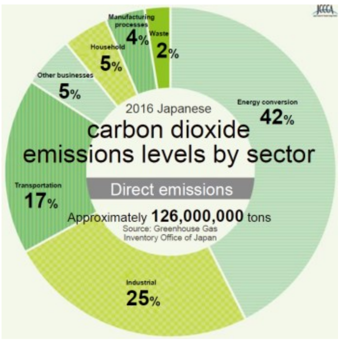


the relationship between data and conclusion (study project).

	【Reasons why it could not be said that CO_2 is causing global warming】 ・CO_2 density is increasint following increases to atmospheric and sea temperatures. If one thinks that CO_2 density is increasing due to industry and the like becoming active due to warmer temperatures, then CO_2 density would not be a cause of global warming.  (Keeling, 1989)  (Kondo, 2006) T: I would like each group to present on why it could be said, or could not be said, that CO_2 is causing global warming. S: CO_2 is causing global warming, because trends in temperatures and CO_2 density have been consistent from 1990 to 2015. S: It cannot be said that CO_2 is causing global warming, because temperatures decreased from 1955 to 1975 despite rapid CO_2 increases.	・ During presentations, ask the following questions: "What kind of data did you use?"; "What part of the data were you looking at?"; and "What could be said, as a result?"
Summary 5 mins	T: Can it be said (or not said) that CO_2 is causing global warming? 【Summary】 There are consistencies and inconsistencies between increases to CO_2 and rising temperatures, so it would seem that CO_2 is a cause of global warming, but that there are also other causes.	・ Summarise as a class.

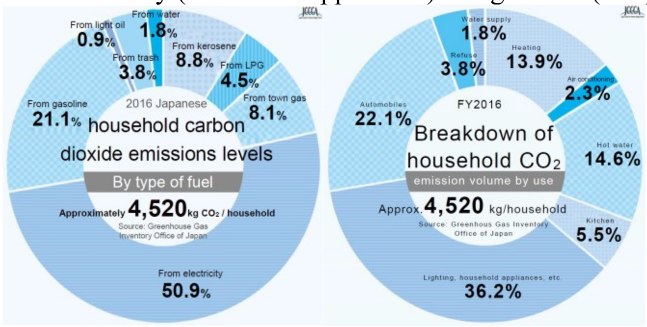
(3) Objectives and outline for the fourth and fifth hours

As each group uses books and the Internet to research what can be done to reduce CO_2 levels and what other causes there may be for global warming, students will become able to summarise in a cause and effect diagram how human life causes increases in CO_2, and how other causes (such as natural environmental cycles) can be thought to be causing global warming.

Learning activity	Teacher encouragement and student response	Key points (・) & evaluation (■)
Task Presentation 5 mins	T: It seems like there were thought to be several causes for global warming. So, what would you like to investigate further, with a view to the prevention of global warming? S: We will research what can be done to reduce CO_2 levels. S: We will investigate else is causing global warming, other than CO_2. 【Study project】 What can be done to reduce CO_2 levels? What else is causings of global warming, other than CO_2?	・ Confirm what each group will be investigating, and set the study project.
Independent Problem Solving 40 mins	【Causes of increasing CO_2 levels】 ・CO_2 emissions are caused mainly be power plants, factories, and transportation.  (Japan Center for Climate Change Actions website)	

【Causes of increasing CO_2 levels】

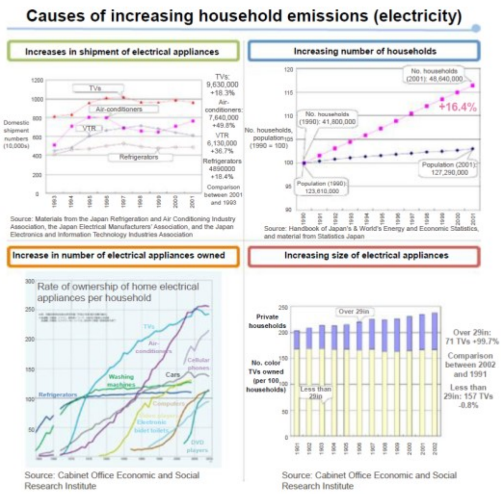
・The main causes household causes of CO_2 emissions are the use of electricity (household appliances) and gasoline (cars).



(Same as above)

【Causes of increasing CO_2 levels】

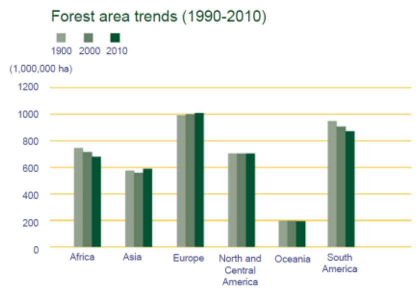
・Increases to the number of households, to the variety of electrical appliances, and electricity consumption, are causing increased household CO_2 emissions.



(Same as above)

【Causes of increasing CO_2 levels】

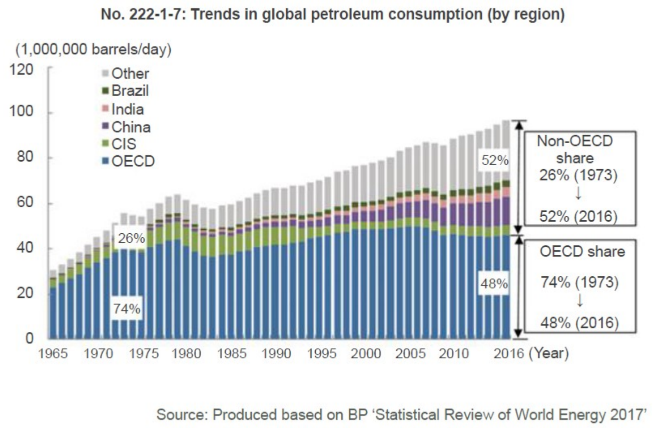
・Decreases to the area of the earth's forests (which absorb CO_2) are also causing increased levels of CO_2.



(Forestry and Forest Products Research Institute website)

【Causes of increasing CO_2 levels】

・Increased consumption of oil is also causing increased levels of CO_2.



(Ministry of Economy, Trade and Industry website)

・Introduce cause and effect diagrams.

・Distribute enlarged cause and effect diagrams to each group.

・Have students

Refining

40 mins

【Causes of global warming other than CO_2】

・Because sunspot cycles and temperature fluctuations are consistent, solar activity is causing global warming.

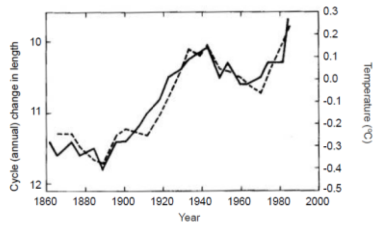
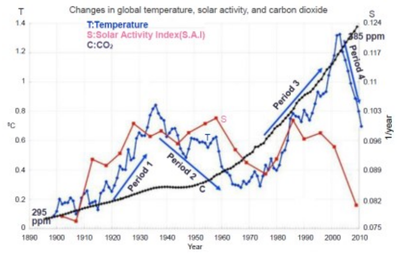


Figure 2-3: Relationship between change in length of sunspot cycle (solid line) and Northern Hemisphere temperature variation (dotted line), 1861-1989 (from Friis-Christensen and Lassen, 1991)

(Blog from general population)

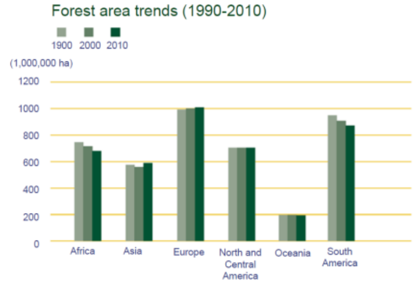
・Temperatures are falling despite CO_2 density rising. Solar activity is similarly declining. Therefore, it is not CO_2 but rather changes in solar activity that is causing global warming.



(OGAWA Katsuro)

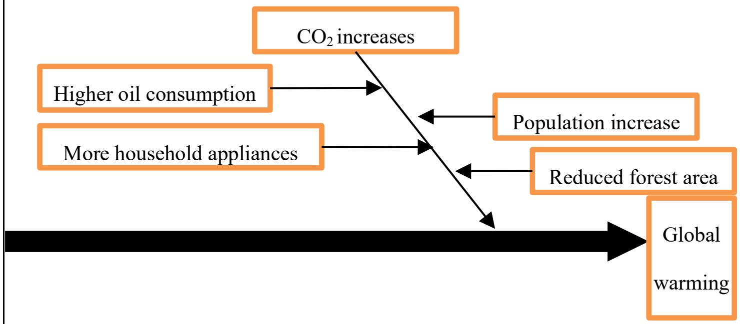
【Causes of global warming other than CO_2】

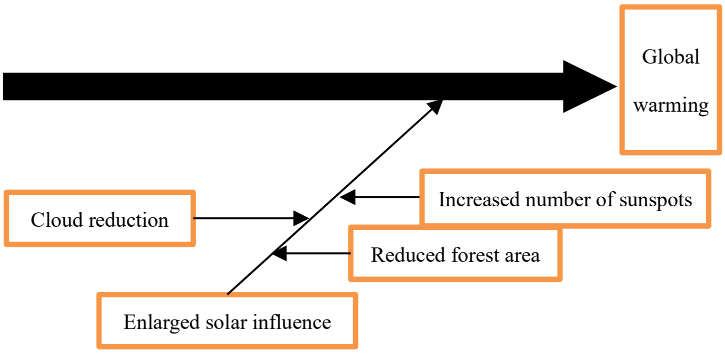
・Deforestation and changes to surface reflectance are causing global warming.



(Forestry and Forest Products Research Institute website)

attach data used as the basis of their conclusions to the enlarged cause and effect diagrams.

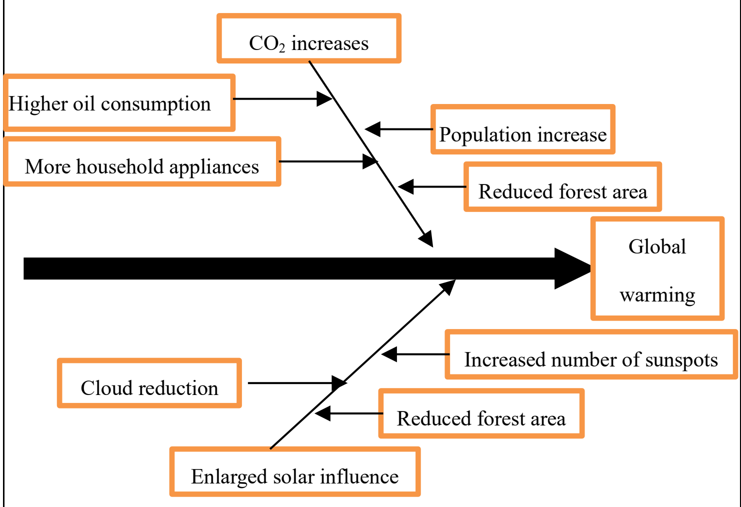
<table>
<tr>
<td></td>
<td>T: Having investigated the causes of global warming, there would seem to be a range of causes; let us try to summarise them in an easily understandable cause and effect diagram.


S: CO_2 causing global warming


S: Causes of global warming other than CO_2</td>
<td></td>
</tr>
<tr>
<td>Summary
5 mins</td>
<td>T: What can we do to reduce CO_2 levels? What else is causing global warming?
【Summary】・CO_2 is released when humans create and use energy, so it is necessary to review how we humans live.
・Since heat from the sun causes increases to air temperature, it is thought that changes to solar temperature and cloud coverage, and, changes to reflectance due to deforestation are causing global warming.</td>
<td>・Have each group give a summary.
・Have students confirm which parts of the data they looked at.</td>
</tr>
</table>

(4) Objectives and outline for the sixth hour

Through interactions with other groups concerning what they have researched about what can be done to prevent global warming, students will be able to think about how human life causes increased CO_2 levels, and how natural environmental cycles cause global warming.

Learning activity	Teacher encouragement and student response	Key points (・) & evaluation (■)
Task Presentation 5 mins	T: There seemed to be a range of causes for global warming. What can we do to prevent global warming? Let us share what we have researched, and think about this. Please listen to presentations from other groups, and if you are convinced by a point, add it to your diagram. 【Study project】 What can we do to prevent global warming?	
Independent Problem Solving (Presentations) 15 mins	S: We thought it was likely that there are other causes of global warming as well as CO_2, such as the influence of the sun. S: We learned that CO_2 levels have been increasing because people use a lot of energy from oil. T: Let's listen to the presentations, and complete our cause and effect diagrams.	・ Print each group's cause and effect diagram, and distribute them.
Refining 15	S: We understand a range of factors as coming together to cause global warming.  S: Global warming might be caused by CO_2 released by humans, or by natural phenomena. In either case, we see that "foreast area" is a factor.	・ Post the enlarged cause and effect diagram on the blackboard, and have each team write it out. ・ Have students write down what they can understand from the cause and effect diagram in their

		worksheets.
Summary 10 mins	T: Based on the fact that human life is causing CO_2 increases, and that natural phenomena are also causing global warming, please summarise what we can do (as humans living on this planet). 【Summary】 There is not a single fixed cause of global warming, but rather a range of factors effect each other in complex ways. Even so, that is not to say that we cannot prevent global warming. There are things that we can all do as individuals. For example, if we were to increase foreast area, this would lead to the absorption of CO_2 and reflection of sunlight, so I think it is important to undertake tree-planting activities. I also think that it is possible to prevent increased oil consumption by being cnoscious of not using more energy needlessly.	・Have students summarise individually in the form of written compositions(?). ・Have students think about whether there isn't anything that they can do in terms of their commonplace activities.

Appendix C: Transcript of Teaching Experiment

<The first session: Monday, March 12, 2018>

Teacher: Starting today, we are going to think about the global environment for a few class hours. Does anyone think the Earth has recently been changing? Anyone think that the Earth is drastically changing? I suppose we do not feel so. How old are you now?

Student: 22 years old.

Teacher: Liar. You have only lived just over a dozen years. I am sure it is difficult to feel changes in the Earth. Now, has anyone heard from someone that the Earth has been changing?

Student: [Raising hands]

Teacher: Okay. Now, we are first going to check how the Earth has been changing. What is happening on Earth now? I am now going to distribute the worksheet for today. You may only place the worksheet on your desk. Take your pens or pencils out. [Distributing the handout] Now, let us watch a video and check what is happening to the Earth now. It is a three-minute video. While watching, please take notes on the memo section. Only keywords are fine.

Student: [Watching the video]

Teacher: Up to the section we have just watched, please write down what is happening to the Earth.

Student: [Writing]

Teacher: Now please compare your notes with the people near you. What is happening? Everyone had the same answer?

Student: [Checking with others]

Student: Let's check now. What is happening?

Student: Global warming.

Teacher: Global warming. Warming. Anything else?

Student: Sea level rise.

Teacher: Sea level rise. The sea level is becoming higher. Anything else?

Student: Loss of coral reefs.

Teacher: Loss of coral reefs. I guess this means the ecosystem is being damaged. Anything else?

Student: The ice is melting.

Teacher: The ice is melting. Anything else?

Student: Intensive heat in summer.

Teacher: Intensive heat in summer. It is like the summer is becoming scorching hot. The temperature and climate are becoming abnormal. You just said intensive heat in summer. Do you remember the heavy rain in the area of the Kinu River and flooding? Abnormal means the temperature becomes really hot, and there's also really heavy rain. Snowfall was also quite heavy this year. Anything else? At the back.

Student: Carbon dioxide is increasing.

Teacher: Carbon dioxide is increasing. Anything else?

Student: Greenhouse gases.

Teacher: Greenhouse gases. Does anyone know what the greenhouse gases are?

Student: Carbon dioxide.

Teacher: Yes, carbon dioxide. It is called one of the greenhouse gases. Anything else? Nothing? Is that all?

Student: Are these all abnormal climate?

Teacher: Yes, I guess these are all leading to abnormal climate. Anything else besides abnormal climate?

Student: Drought.

Teacher: Drought. Do you know what drought is?

Student: Desertification.

Teacher: Yes, desertification. Drought means that the ground is becoming really dry. Anything else?

Student: No.

Teacher: Nothing. Do you know what the high tide in the video is? In the video, the wave was getting really high. Changes in the Earth are appearing in many ways like this. Other changes are damage to forests and deforestation. A lot of changes are happening, but do you know what is causing them?

Student: People.

Teacher: People. Why are these changes happening? What do you think? What did the video say? What is the background of the cause? Do you know what I mean? Okay? What is the background of all these changes? Talk to people around you and check.

Students: [Checking with others]

Teacher: My question is, why are these changes happening? What are people doing?

Student: People are ... gas.

Teacher: Gas. People are ... gas? What happens when people release greenhouse gases?

Student: The Earth becomes hot.

Teacher: Does this mean that it might be the cause of global warming? I guess global warming is causing many problems such as sea level rise, destruction of ecosystems, abnormal climate that melts the ice, droughts, and high tides. This is quite a serious problem, don't you think? What do you think we need to do to prevent the Earth from becoming hot? If this continues, it will be a big problem. Then, what do you think we should do to prevent global warming?

Student: Reduce carbon dioxide.

Teacher: Yes, reduce carbon dioxide. Does anyone agree? Do you think we should reduce carbon dioxide?

Student: [Raising hands]

Teacher: Many of you agree. Why do you think we should reduce carbon dioxide?

Student: The ice will stop melting.

Teacher: The ice will stop melting. Do you mean the ice melts when there is a lot of carbon dioxide? Why?

Student: Because it gets hot.

Teacher: Ah, it gets hot. This means the increase of CO_2. Like everyone said, we should reduce CO_2 because it is the cause of global warming. Then, let's watch a video to check why the increase in CO_2 will result in global warming. Video, are you getting it?

Student: No.

Teacher: Then, for now, can you write this down [what is written on the blackboard] in the memo section? Just write the main points. Write this [what is written on the blackboard] in the memo section. Write what the cause of the changes is.

Student: [Watching the video]

Teacher: You have probably learned things from videos in elementary school and quickly listed the causes. Meanwhile, do you know there is a discussion like this?

Student: [Watching the video]

Teacher: Now, you just said using common sense that CO_2 was probably the cause, and we should reduce it. Meanwhile, some people are saying that CO_2 is not the cause. Has anyone heard a discussion like this? Oh, you have, great. Two of you have heard of it, but basically, I suppose that you thought CO_2 was the cause. Now, open the handouts. We just heard a little story, and we are not sure whether CO_2 is the cause or not. There may be another cause. Based on your current opinion, please draw a circle there.

Student: [Watching the video]

Teacher: Surprisingly, some people say that snow is the cause of global warming instead of CO_2. Based on the discussions so far, please select whether the cause of global warming is CO_2 or anything else, or that you simply do not know. Did you make your choice? Are you okay with it? Did anyone change their opinion? Now, please raise your hand once. We are going to check for today. Who of you thinks CO_2 is the cause?

Student: [Four students raised hands]

Teacher: Good, only four? Who of you thinks CO2 is not the cause?

Student: [Three students raised hands.]

Teacher: Three? Who is not sure?

Student: [Many students raised hands.]

Teacher: Wow, yes, okay. In the end, most of you said you were unsure. We have to do research to find out whether CO_2 is actually related or not. We are going to clarify this in the next session. I am going to assign groups to study this. Once again, which of you thinks CO_2 is the cause? Then, can you look up reasons that CO_2 is the cause? Okay? Who thought CO_2 was not the cause? Here, change here. You, can you switch seats for now? Did you think CO_2 was not the cause? Can you raise your hand again if you were unsure? Okay, here is the idea. I am sorry, but for those of you who were unsure, can you study the reasons that CO_2 is the cause with this group? And can you study the reasons that CO_2 is not the cause with this group? Do you know what I mean? Okay? Now, these groups, you must decide which you want to study. Groups 1 and 2 here and Groups 5 and 6 need to discuss in groups which viewpoint you want to study. Okay? Which are you going to study?

Student: [Talking in groups]

Teacher: [Talking to individual groups as they decide which they will study] You are going to go to the Computer Room and use the computers to check data in individual groups tomorrow. I want these groups to explain why CO_2 is not related to global warming using data to convince us. I want the groups who claim that CO_2 is the cause to explain why it is the case using data. Then, I want you to check in individual groups what kind of data your group will investigate in the next class and write it down at the bottom of the handout. We are going to switch seats, but only for the math class, we are going to use this group formation. Now, form groups and discuss what kind of data you are going to examine in the next class.

Student: [Talking in groups]

Teacher: Good, now, let's check what you are going to investigate tomorrow. I want you to investigate this tomorrow. Look at the blackboard. Whether CO_2 is related to global warming or not. What is this group going to investigate?

Student: Greenhouse gases, functions of clouds, and graphs of CO_2.

Teacher: What did you say? CO_2 graph and what?

Student: Greenhouse gases and the functions of clouds.

Teacher: Functions of clouds. Won't you examine temperature? Take a look. In the next class, I want you to search data such as CO_2 and temperature and decide whether CO_2 is related to global warming or not. Now, after the end of the class, please bring the handout to me. Class is dismissed.

<The second session: Tuesday, March 13, 2018>

Teacher: We watched a video yesterday, but we were unsure whether CO_2 was related to global warming or not. Today, you are going to investigate this in groups. So, today's goal is to explain why CO_2 is the cause of global warming, or not the cause of it. I want you to search for data and explain your position. Groups 2 and 3 are going to investigate reasons that CO_2 is the cause. Groups 1, 4, 5, and 6 are going to investigate reasons that CO_2 is not the cause. Yesterday, did you determine what kind of data you would be looking for? Do you remember? What kind of data?

Student: Graphs.

Teacher: Yes, graphs. What kind of graphs? What kind of graphs? What kind of data?

Student: CO_2.

Teacher: Yes, CO_2. Good, tell me more. CO_2, CO_2 data, and what kind of data are you going to look for? Anyone?

Student: Temperature.

Teacher: Yes, temperature. Temperature, because we need to check changes in the temperature. Individual groups will keep looking for data like this and explain why CO_2 is related or not related to warming based on the data. Each of you has the worksheet. In the end, individual groups will summarize their findings in the worksheet. You are going to complete this within class. I will make copies of the worksheet in the fifth class. You will give presentations based on the worksheet, and the entire class will reach a conclusion. Do you understand what you are going to do? Okay? The desk there is for you to have a meeting and discuss what you find out. If you need scissors, they are at the front. Please come and take them. Now, start your investigation.

Student: [Group activities using computers]

Teacher: Now, return to your seats for a moment. Hey, have a seat. I know you are in the middle of your investigation. Now, turn your face to me. The class hour is short today, and you are doing a great job. But do you mean you have not finished writing your report? Report or a summary of your group. Okay, please complete it and submit it by the recess of the third hour. I can print them out if you can submit your reports by then. So please help each other in your groups and complete the report during the recess. Are you okay? So please submit them by the recess of the third hour. Thank you. Class is dismissed.

<The third session: Tuesday, March 13, 2018>

Teacher: Now, you did a great job writing reports in the last class. I printed them out and will hand them to you later. What you are going to do first is to perhaps prepare for the presentation. Each group has an enlarged version of the handout. Please use pens or other tools and prepare for explanations to make it easy to understand. You can ask questions after the presentations, if there are any. In the end, you are going to summarize why CO_2 is related or not related and review your opinions. Any questions so far? Now, gather in groups, and one member of each group needs to come get this handout. Gather in groups.

Student: [Preparation for presentation in individual groups]

Teacher: Okay, now, you are going to give presentations. But before that, I suppose you want to know what the other groups are thinking. Please skim through it and find any questions or uncertainties. If you find any, write all of them down here. I will give you two to three minutes. First, I want you to skim through it and write comments.

Student: [Individual activity]

Teacher: Good. Anyone want more time to read the worksheets of other groups? Fine? Did you skim through mostly all of them? Are you ready? Now we will proceed to the presentations. The first up is Group 2. Go ahead. Give your presentation.

Students in Group 2: We of Group 2 are now going to start our presentation. This is the annual amount of global CO_2 emissions in 2006. The bottom is the graph from 2014. This is the average temperature indicating how much the global average temperature has risen. These data indicate that the CO_2 emissions increased by about 5.7 billion tons from 2006 to 2014. Also, the rates of temperature rise in these years indicate that the difference in standard values has increased from 0.1 to 0.2 degrees. Based on these data, we conclude that the cause of global warming is carbon dioxide. This is why we think CO_2 is causing global warming.

Everyone: [Applause]

Teacher: Why did you think CO_2 was related?

Student: We found that out.

Teacher: You found out. From which years did you check the data?

Student: 2006.

Teacher: From 2006 to 2014. Up to 2014, how many billion tons?

Student: 5.7.

Teacher: 5.7 billion tons of CO_2 have increased, you said. You checked this period here. You then concluded that the CO_2 emissions had increased and explained so in the presentation. Any questions? Nothing? Next, Group 3.

Students in Group 3: Yes, we will start now. First, our group looked for data that would support that CO_2 was the cause of global warming. Then we found this pie chart in Number 3. It shows the types and ratios of greenhouse gases and indicates that carbon dioxide has been increasing a lot. Next, I want you to take a look at these two graphs. This one shows the global annual mean temperature, and this one indicates that the amount of CO_2 has been increasing. The comparison between 1970 and 2000 indicates that both are increasing. These data imply that when the concentration of carbon dioxide increases, the annual mean temperature also increases. This is why our group thinks that CO_2 is the cause of global warming.

Everyone: [Applause]

Teacher: Here, from which year to which year did you make a comparison?

Student: From 1970 to 2000.

Teacher: You compared from 1970 and 2000 and thought that CO_2 had increased. You also added references like this. It says it is the majority. I mean, CO_2 is the majority. Any questions? Nothing? From here, groups that think CO_2 is not the cause will give presentations. Group 1.

Students in Group 1: We of Group 1 are now going to start our presentation. First, we researched whether the carbon dioxide concentration and carbon dioxide were related to global warming and found a graph. Please take a look at this graph. In this graph, the blue line indicates the carbon dioxide concentration and red line the global temperature. We found out that here, although the blue, the carbon dioxide concentration, had increased, the red had decreased. If carbon dioxide was indeed related to global warming, we suppose that the global temperature would keep rising without lowering. This is why we thought carbon dioxide was not related to global warming.

Everyone: [Applause]

Students in Group 1: We also found another website saying that global warming was a lie to begin with. According to the website, when the global population increases, and people start to gather [in cities], the temperature increases and triggers a so-called heat island phenomenon, which leads to the warming of the Earth.

Teacher: Has anyone heard of the heat island phenomenon? No one? Thank you. Next, Group 6.

Students in Group 6: We of Group 6 are now going to start our presentation. Group 6 researched reasons that CO_2 was not related to global warming and instead clouds were probably related to it. Please take a look at this chart. It indicates that even when carbon dioxide was gradually increasing, the temperature was gradually lowering. Based on this, we concluded that carbon dioxide was not related. Next, please take a look at this. This shows that from around 1990, the intensity of sunlight kept increasing. This one here indicates that the amount of clouds became less. This is why we thought that instead of CO_2, the temperature rose as the amount of clouds decreased, and the intensity of sunlight also increased. We therefore thought that CO_2 was not the cause of global warming.

Everyone: [Applause]

Teacher: You used the same data as the previous group. The difference from the other group is that you added the explanation about clouds. Any questions? Any questions about this explanation section? Then, can I ask a question? You said that the sun was becoming hotter. Do you have any data indicating how hot the sun is getting?

Students in Group 6: No.

Teacher: No data. Did you find any data? You found them, but you did not put them here? Also, the clouds have been decreasing? Or are they increasing? You said the amount of clouds seemed to be decreasing. Do you know how much they are decreasing? They are decreasing by 0.5 points every year. I suppose you have detailed data for this. I see. Your explanation will be easier to understand with data for this. It will show how much the clouds have been decreasing. Okay. Next, Group 5, please.

Students in Group 5: We of Group 5 are now going to start our presentation. We researched changes in the temperature of Antarctica and the carbon dioxide concentration. Zero here is our current data. Hundred here is the data from a million years ago. The carbon dioxide concentration a million years ago was as high as today. Yet, the temperature was low. Although the carbon dioxide concentration today is about the same, the temperature back then did not become high but instead became low. This is why we concluded that CO_2 could not be the cause of global warming.

Everyone: [Applause]

Teacher: You presented a slightly different graph. From which year to which year the previous groups examined? From 1960 to 2000. They examined recent data. How many years did this group say? A million years ago. You checked the data from a long time ago. As a result, you discussed that CO_2 was high here while the temperature was low, and CO_2 increased here while the temperature remained the same here and decreased here. Any questions? The last group. Next, Group 4, please.

Students in Group 4: We of Group 4 are now going to start our presentation. Please. Our group looked for data showing the relationship between carbon dioxide and temperature to support the position that CO_2 is not the cause of global warming. We often hear that when carbon dioxide increases, global warming is exacerbated. Yet, there are other researchers studying the correlation between the global temperature and carbon dioxide. These indicate at a glance a strong correlation between these two. If we examine them carefully, we find that there is indeed a correlation, but the causal relationship is completely the opposite. Instead of the temperature rising after carbon dioxide increased, we found that the temperature increased first, and after a little while, carbon dioxide increased. This one shows that after the pink increased, CO_2 increased after a little time lag. These data imply that carbon dioxide is irrelevant to global warming; rather, global warming was causing carbon dioxide to increase. This is why we concluded that carbon dioxide was not the cause of global warming.

Student: [Applause]

Teacher: Did you notice that this group was a little different from the others? Do you understand what this circle in the graph means? Take a look on your handout. What do you think Group 4 is trying to claim? The red in this graph is the temperature, the blue below it is CO_2. Did you understand that Group 4 was saying that there is a gap there? Which one is changing first? Which one is the first to change?

Student: Temperature.

Teacher: The temperature started to rise first, and carbon dioxide started to change a little later. What does this mean? CO_2 changes after the temperature changes. What is the cause then? Talk to people around you a little while. Do you know what I mean? Check with others what this graph implies.

Student: [Checking with others]

Teacher: Good, now look at the front. So, after all, which one is the cause, and which one is the consequence? What is the cause, and what is happening? You found data indicating that the temperature was the cause and the CO_2 level was the consequence. This is the opposite of what we have heard before. Before, people said CO_2 was the cause. Yet, you presented an interesting data analysis saying that global warming was the cause to begin with. Then, based on this and the other presentations so far, please take the time to think about whether CO_2 is related to global warming as a cause. I am going to ask questions now. Who thought CO_2 was related? I want an honest opinion. But, did you understand and agree with the explanations by Groups 2 and 3? Now, who thought CO_2 was irrelevant? Who thought that CO_2 seemed to be related, but that there may be another cause? Who thought CO_2 was related along with other factors? Okay. I am going to first summarize the target for today. What were the grounds for the discussion of people who claimed that CO_2 was the cause? What kind of change shown in the graphs did they use as the reason? What was the basis of the discussion that a CO_2 increase results in a temperature rise as well as the opposite discussion? It is because the temperature would not change even with a CO_2 increase in some cases. Now, from here on, you are going to write your own diagnosis. Okay? You raised your hands a little earlier. You found out that the temperature increased in some cases, and in other cases the temperature remained the same. So based on the findings, you must decide whether CO_2 is related or not. You don't have the handout I distributed this morning, do you? For now, please write on the back of this handout.

Student: [Writing]

Teacher: CO_2 is related. CO_2 is not related. I thought many of you had thought CO_2 was related. Anyone thinking otherwise? No one? You must raise your hand some time. Who is thinking CO_2 is related? [Zero students] Who is thinking CO_2 is completely irrelevant? [Four students] Who is thinking CO_2 is also related? [Many students] This means that many of you think that CO_2 seems to be relevant, and there many other factors. Okay, so in the next class, we need to look for other factors. Also, if CO_2 is really related, we must reduce CO_2 then. Next class, we are going to search for ways to reduce CO_2. Write your name on the handout and please submit it at the front.

<The fourth session: Wednesday, March 14, 2018>

Teacher: I am going to check what we discussed yesterday. Can you form into your groups? What did you research yesterday?

Student: CO_2.

Teacher: Yes, we researched whether CO_2 was related. Then we concluded yesterday that CO_2 seemed to be related, but factors other than CO_2 may also be related. We can also draw a diagram like this because there are various factors. Do you know what I mean? It seems like a demonstration in math. There is global warming, and the increase in CO_2 is the cause of it. Also, there are other uncertainties in '?' We can draw a diagram indicating that there are other uncertain factors. Okay? So today, based on this diagram, I want you to come up with other aspects that you should look into to prevent global warming. What do you think you should research?

Student: Causes.

Teacher: Causes of what?

Student: Causes besides CO_2.

Teacher: 'Besides' means that we want to clarify what this '?' is, right? Anything else? Can we leave this CO_2 unchecked? It is better if we reduce it, right? This means that I want you to research ways to reduce CO_2 and why CO_2 increases to begin with. So, today's topic is, what are the causes of global warming besides CO_2? Also, why is CO_2 increasing? If we find causes, we may be able to see ways to reduce them. So now, I want you to do two things today. I want you to do it all at once by assigning you to different tasks. Can Groups 2 and 3 who think CO_2 is related work on researching why CO_2 is increasing? The other groups, Groups 1, 4, 5, and 6, are assigned to researching causes other than CO_2. Then, please take a look at the handout on the table. Like this, I want you to add arrows to indicate the causes of the increase in CO_2 and why it is increasing. Do you know what I mean? So, I want you to research what is right to the point using data. For other sections, please research this '?' section using the yellow piece of paper there. Make a guess on the causes of global warming and write them down on the yellow piece of paper. Guess what the causes of the CO_2 increase are. Okay? Do you know what I mean? First, I want you to talk in groups and make guesses. Go ahead and start.

Student: [Talking in groups]

Teacher: When you finish writing, select one person from each group to paste your diagram on the whiteboard. I want to explain more about how to interpret the diagram. Can you take a look at the back of the handout? It shows a diagram that describes the causes of a traffic accident. For example, people may be the cause. In particular, their driving skills are inadequate. Or, their driving manners are bad. Other causes may be speeding too much. Others include defects in cars, such as the mirrors being too small, or the tires were defective. Other factors include situations, such as where the traffic accident occurred. Other cars may be hard to see on the road, or traffic rules may be involved. I want to create something like this together in the end. Do you get the picture? In the end, we are going to combine what you found out through research and create a diagram showing what is related to global warming. Are you ready? Now, two groups, please go ahead and paste the papers.

Student: [Pasting papers on the whiteboard]

Teacher: I want you to calm down because we are in a different room from usual. I think it is too small to see. I want you to check what other groups are researching whenever needed. The cause of global warming in ① means the causes of global warming other than CO_2. Someone wrote 'greenhouse gases' over there, but CO_2 is one of the greenhouse gases. Now, please go ahead and start your research on other causes. Start researching, please.

Student: [Group activities using computers]

Teacher: Now, please listen. The class is ending in a few minutes. I want to collect the small worksheets that you have. You don't have to have data there as long as you have written your guesses on what is related. Okay? I want to collect the worksheets with your guesses based on your online research on factors other than CO_2 which seem to be related and factors which seem to be related to the reason for the increase of CO_2. If you have data, please write down that you have data. Please finish your work by printing out whatever data or information you want to print out. I am going to collect the worksheets at the beginning of the fifth session. Please briefly describe your guesses by then. A greeting is not needed. Please finish on your own.

<The fifth Session: Thursday, March 15, 2018>

Teacher: Yesterday, I asked you to investigate the causes of global warming besides CO_2 so as to prevent global warming. However, since CO_2 is also one of the causes of global warming, I asked each group to investigate the causes of the increase in atmospheric CO_2 once again. I also asked you to use this chart with arrows when summarizing findings. However, some of you told me that you didn't understand how to draw this chart with arrows. So, today, I will first explain how to draw this chart. Look at the slide on how to draw a chart. Groups 2 and 3 investigated the causes of the increase in CO_2. Someone in a certain group made this easy-to-understand chart. Do you understand what it represents or what this chart shows? Can you read it? First, it says that an increase in atmospheric CO_2 is the cause of global warming. The causes of global warming are written at the upper end of the chart. Do you know what this middle arrow represents?

Student: The reason.

Teacher: Yes, right. It indicates the reason or the cause. Therefore, it says in the middle that the increases in CO_2 levels due to industrial development, the evaporation of seawater, population growth, and thermal power generation are the causes of the increase in atmospheric CO_2. These causes are written in this middle section, and I want you to use this same format when you summarize your findings. In the cases of Groups 1, 4, 5, 6, this person also summarized the findings in an easy-to-understand manner. What does this person think causes global warming?

Student: Population growth.

Teacher: Right. This person thinks that an increase in population causes global warming. This person writes the breakdown … things that are caused by population growth, which then leads to global warming. According to the aforementioned format, what should be written in this middle section? What are you supposed to write here? What was written below this middle arrow a while ago? What was written?

Student: An arrow.

Teacher: Yes, an arrow, a middle arrow. What are these?

Student: Reasons for the increase.

Teacher: Right, reasons for the increase are written. Then, what should be written in the middle section in this case?

Student: Reasons why the number of people increases.

Teacher: Right, you are supposed to write the reasons why the number of people increases. Then, how about this one? Is this a cause of population growth?

Student: No, it is not.

Teacher: Look at this. For example, if the amount of exhaust gas increases, does the number of people increase? No, it is not people that increase. If the trees in forests are cut down, then the number of people increases? Then, people are like Pinocchio. So, unfortunately, these are incorrectly positioned. If you correct it, how does it look? If you correct it, how does it look? If it is corrected, then it looks like this, and exhaust gas becomes the cause of global warming. Then, what causes the increase in exhaust gas?

Student: Cars.

Teacher: What about cars?

Student: Gas from cars.

Teacher: Gas from cars. It is called exhaust gas. What do you think increases the amount of exhaust emissions?

Student: People.

Teacher: Because of population growth. Then, this arrow, the direction of this arrow is the other way around. Do you see? The location of the arrow … Similarly, the number of home electric appliances seems to be one of the related causes. Then, why does the number of home electric appliances increase?

Student: Population.

Teacher: Right, because of population growth, and maybe an increase in the number of nuclear families. These may be the reasons. So, I want you to write the causes of global warming at the edge of the chart like this and add the causes to the middle arrow. Okay so far? Look at this. There is no mention of CO_2. They are investigating the causes of global warming besides CO_2, but is this okay? What do you think of that? The members of Group 6 are talking about this now. Can you tell that this is eventually an investigation of CO_2, a cause of global warming? Although the word CO_2 is not mentioned, these are the causes of the increase in atmospheric CO_2. Groups 1, 4, 5, and 6 do not seem to have investigated the causes of the increase in atmospheric CO_2, but actually, their investigation results lead to them. So, you have to be careful. Then, in the past two-hour session, what did we think were potential causes of global warming besides CO_2?

Student: Vapor something, water vapor.

Teacher: Water vapor and what else?

Student: Gas, heat islands.

Teacher: Heat islands, what else? Group 6, what did you think the causes were other CO_2 and population growth?

Student: The sun.

Teacher: Oh, the sun, what else? The clouds, in the second and third periods, you talked about the sun and clouds as causes. Then which one do you think goes into this square, sun or clouds?

Student: Water vapor.

Teacher: Oops, water vapor, would you put it aside for a while? Why don't you talk about which one, the sun or clouds, goes into this square with people around you?

Students: [Talking with people around them]

Teacher: Now, let me check your answers. Please raise your hand to the statement with which you agree. The cause of global warming, how many of you think that changes in the impact of clouds cause global warming? How many of you think that the declining impact of the clouds causes global warming? Okay, put your hands down. How about here? How many of you think that changes in the impact of the sun cause global warming? All right, thank you. Suppose that the impact of clouds is placed here, then why does the decline in the impact of clouds cause global warming?

Student: Sunlight.

Teacher: A lot of sunlight comes in. Then, putting the impact of clouds here is not right. You know what

I mean? The declining impact of clouds should lead to a lot of sunlight. Otherwise, it would not lead to global warming. Do you get it?

Student: No, I don’t get it.

Teacher: You don’t get it. Do you understand? Okay? So, the impact of the sun on the Earth is changing. The cause of the change in the impact of the sun on the Earth is clouds. That is, the impact of clouds, which are related to the impact of the sun on the Earth, is declining, which accelerates global warming. Okay so far? All right? For global warming, what I want Groups 2 and 3 to investigate, based on the understanding that global warming is caused by atmospheric CO_2, is the causes of the increase in atmospheric CO_2. I want these groups to enter the results or the data here. For Groups 1, 4, 5, and 6, since we don’t have enough time, I want you to focus on the impact of the sun and investigate how it is related to global warming and the causes of the change in the impact of the sun. Add the findings, the data, into the chart. Okay so far? Then, summarize the findings in each group in the remaining time. While confirming the findings you got from yesterday’s investigation, enter the summary into the worksheet. Share the work of obtaining required data among members of each group. Let’s start working.

Students: [Group activity using personal computers]

Teacher: Okay, stop. Please summarize the findings of your group during recess time and submit them to me before the end of the day. I will print them out so that we can discuss the summary of each group in the next session together. Each group, please complete your summary.

<The sixth session: Friday, March 16, 2018>

Teacher: So, this is the last session on global warming. You guys did a great job, way beyond my expectations. Today, we are going to talk about what we can do to prevent global warming, for which each group investigated many things, such as why atmospheric CO_2 is increasing and what the causes of global warming other than CO_2 are. After hearing the presentation of each group, I will ask each person to write a summary of what you have learned through these sessions. Now, split into groups and one person from each group, please come and get a big sheet of paper for your presentation and start preparing for your presentation.

Students: [Preparations for presentation in each group]

Teacher: Okay, thank you for making the preparations in such a short time. I printed out the summary of each group. I am going to distribute them. After you get one set of summaries, write your name down on it, quickly look through them, and put a question mark where you don't understand. It sometimes happens that some groups end up investigating items that are different from those of other groups. Maybe you have not finished reading the summaries, but let's start the presentations. I want you to hear the presentations of the other groups and think what causes global warming and what the causes of the causes are. Group 2, would you start?

Student in Group 2: What caused the increase in atmospheric CO_2 is the massive consumption of petroleum, coal, and natural gas after the Industrial Revolution. Next is that atmospheric CO_2 increases due to deforestation. For oceans, while they are originally alkaline, they have become acidic because they absorb CO_2. Since oceans themselves do not emit CO_2 in large quantities, we thought that excess CO_2 beyond the amount that oceans can absorb remains in the atmosphere. This chart shows that the causes of the increase in atmospheric CO_2 are the Industrial Revolution, deforestation, oceans, home electric appliances, and the exploitation of fossil resources. Therefore, we conclude that in order to prevent global warming, we need to prevent increases in atmospheric CO_2.

All students: [Clapping hands]

Teacher: Do you have any questions for Group 2? Any questions? No? Sure? Okay, then I have one question just to check. Did you understand this part? Did you understand why atmospheric CO_2 increases due to oceans? Did Group 2 explain it well? Why?

Student: Oceans cannot absorb that much.

Teacher: Yes, since oceans cannot absorb that much, some CO_2 remains in the atmosphere. What did they say about the current state of the oceans?

Student: They have become alkaline.

Teacher: Have they become alkaline?

Student: They are acidic.

Teacher: Yes, they have become acidic. It has become so acidic that it cannot absorb any more CO_2. I see. Thank you. Next presentation is by Group 3. Group 3, please start.

Student in Group 3: I will start the presentation on behalf of Group 3. First of all, we in Group 3 investigated the causes of the increase of atmospheric CO_2. We thought that one of the causes must be population growth. It is expected that the population will increase by half in the coming several years. We thought that population growth will result in active industrial activities and an increase in the penetration rate of automobiles, which in turn will lead to global warming. For industrial activities, this pie chart shows the breakdown of the consumption of fossil fuel that emits CO_2: Coal and petroleum account for the majority of consumption; coal is 68.7% and petroleum is 10.6%.

Teacher: The largest part is petroleum?

Student in Group 3: No, this pink part is coal.

Teacher: Okay, it seems the printer did not do a good job. This is not easy to see, but the largest part is coal.

Student in Group 3: In the breakdown of petroleum that accounts for the second largest part in the consumption of fossil fuel, it is the transportation sector that accounts for the largest part (35%). The transportation sector leads to the increased penetration rate of automobiles. The vehicle that makes up the greatest part in the transportation sector is the passenger car that we use for traveling, and its penetration rate has been increasing over the past several years. You know that these cars emit CO_2, but recently cars with few CO_2 emissions, such as electric cars and hybrid cars, have become available. However, this section says that the penetration rate of hybrid cars is 9.8% and that of electric cars is 0.2%. Their penetration rates remain very low. Therefore, it seems appropriate to think that CO_2 will still continue to be emitted by cars. Third, we thought that evaporation of seawater has something to do with the increase of atmospheric CO_2. This bar chart shows that while some parts of the oceans absorb CO_2, some other parts emit CO_2 when seawater evaporates. Therefore, we concluded that CO_2 is also emitted when seawater evaporates. In summary, the increase in atmospheric CO_2 results from the increased penetration rate of passenger cars, the large consumption of coal and petroleum, and the evaporation of seawater. Based on the results, we think that it is important to reduce atmospheric CO_2 in order to prevent global warming. Thank you for listening.

All students: [Clapping hands]

Teacher: Thank you. This group explained a lot of things. Let’s see, the word ‘industry’ was mentioned before. What was it related to? What was the keyword?

Student: Revolution

Teacher: Yes, it was the Industrial Revolution. In short, people started using a large amount of fossil fuel during the Industrial Revolution. The group said that industry has since been a major cause of the increase of atmospheric CO_2. Do you have any questions? Why does CO_2 increase when the number of people increases?

Student: If the number of people increases, industrial activities and the penetration rate of automobiles also increase.

Teacher: Then, it does not mean that CO_2 increases because of breathing of the increased number of people, does it?

Student: No, it's not because of breathing.

Teacher: No, it's not. It is not because a large number of people breathe out CO_2. Then where should this arrow be placed? What increases when the number of people increases?

Student: CO_2.

Teacher: You mean breathing causes the CO_2 increase?

Student: No. Cars.

Teacher: I see. It is the number of cars that increases. As the number of people increases, the number of cars also increases. When the number of cars increases, atmospheric CO_2 increases, leading to global warming. Are there any other places where the arrow can be placed?

Student: Industry.

Teacher: Oh, in the industry part. If the number of people increases, then industrial activities become more active. Where else?

Student: Use of fossil fuels.

Teacher: Yeah, it is people who use fossil fuels. Do animals use fossil fuels? Imagine that a dog is burning coal. It's kind of cute, isn't it? I would like to see that. Then how about the evaporation of seawater? Are people involved in it? Maybe not. Maybe these are the places where the arrow can be placed. Okay, thank you. Now I want the groups that investigated the causes of global warming besides CO_2 to make a presentation. The first group is Group 6.

Student in Group 6: I will start the presentation of the investigation results by Group 6. We thought that the impact of the sun, let's see, changes in the number of sunspots could be one of the causes other than CO_2 of global warming and investigated it. Please look at this chart. This line at the bottom shows the number of sunspots and the line above it represents global temperature. Greenhouse gases, including CO_2, are placed in this white part between the number of sunspots and the temperature. We thought all of these factors caused a rise in global temperature. CO_2 in the white part is relatively low, and the difference between the temperature and the number of sunspots is not great from 1850 to 1950. On the other hand, in the 2000s, the difference between the number of sunspots and the temperature becomes greater, showing that greenhouse gases, including CO_2, have been increasing. Then, we thought that a decrease in clouds must be related to global warming and investigated it. According to the latest climate science research, a decrease in the amount of clouds over oceans, which more than one climate model predicts, will increase global warming by over 3 degrees Celsius in this century. It is said that this increase is rather high among conventional global warming estimates. We thought that the oxidation of oceans was the cause of the decrease of clouds above oceans. According to the latest research, when oceans become oxidized, the production of dimethyl sulfate declines. If the amounts of dimethyl sulfate decline, fewer clouds are formed, leading to global warming. Plankton in the sea produces dimethyl sulfate. It is said that dimethyl sulfate is released out of the sea into the atmosphere and becomes the source of clouds. When the Earth's temperature rises, plankton produces more dimethyl sulfate, facilitating the formation of clouds. The resulting large amount of clouds blocks sunlight and decreases the Earth's temperature. This is the normal temperature stabilization mechanism. However, if atmospheric CO_2 increases, it will disturb this mechanism. A certain amount of CO_2 in the atmosphere dissolves into the oceans, lowering the pH level of oceans, that is, oceans become acidic. Therefore, the more the CO_2 levels in the atmosphere increase, the more oxidized the oceans will become. We first thought that a change in the number of sunspots and a decrease in the amount of clouds were the causes of global warming besides CO_2, but it turned out that CO_2 seems to be involved also in the decrease of the amount of clouds. So, we conclude that for the prevention of global warming, it is essential to reduce CO_2 in the atmosphere. Thank you.

All students: [Clapping hands]

Teacher: Thank you. There were many keywords. Let me check one by one. The first is sunspots. Do you know what sunspots are? They are black spots that appear on the surface of the sun. These are places where the temperature is low. Can you see the white part around each spot? This is the place where the temperature is high. These high-temperature areas increase, and consequently the number of sunspots increases. An increase in the number of sunspots means that there should be times when the number of sunspots decreases, but it keeps increasing. There are places where the temperature increases. So the number of sunspots and temperature are changing in a similar manner. But then, a gap is formed between them. What did Group 6 say fills this gap?

Student: CO_2.

Teacher: Right, they think it might be CO_2. It was a plausible explanation. CO_2 might be involved here. And … the amount of clouds is decreasing. Where did the summary say clouds were decreasing?

Student: In the sky.

Teacher: Oh, well, clouds exist only in the sky. Where did it say the clouds were?

Student: Above the oceans.

Teacher: Yes, above oceans. The amount of clouds above oceans is decreasing, and if the clouds above oceans decrease in quantity, global temperatures rise … clouds above oceans absorb.... In relation to the cause of the decrease in the amount of clouds, they mentioned acidic oceans. The other group also mentioned something about acidic oceans. Which one was it? It was the first group, wasn't it? Oxidation seems to be involved here. They said that the amount of clouds might be decreasing due to oxidation. Thank you. Now, next is Group 1.

Student in Group 1: I will make a presentation on behalf of Group 1. At first, we in Group 1 investigated the relationship between global warming and the sun. While studying the impact of the sun, we came across the keyword sunspots. So, we looked into sunspots. Look at this graph. The figures surrounded by the blue line are the number of sunspots and those in red are the Earth's temperature. The graph shows that from 1950 to 2000, while the number of sunspots did not change significantly, the Earth's temperatures increased over the 50 years. So, we first thought that the number of sunspots and Earth's temperature are not closely related to each other. We then investigated the impact of the sun on the Earth. Briefly speaking, the red line is the impact of the sun and the one above it is the Earth's temperature. From 1910 to the 1950s, as the impact of the sun increased, the Earth's temperature also increased. However, from 1980 to 2010, the temperature increased despite a decline in the impact of the sun. Therefore, we thought that something different is working here. Next, we investigated clouds and found that there is a certain relationship between clouds and the sun. Look at this illustration, although it is a little messy. For example, this graph shows that if the sunlight is pouring over the Earth with its full strength, then clouds reflect back 75% of it, and the remaining 25% hits the Earth and contributes to the Earth's temperature. As the other group said, the amount of clouds above oceans is decreasing, which means that the reflective power of the clouds is also decreasing. Consequently, the heat of the sun directly hits the Earth. Therefore, our group concluded that in order to prevent global warming, clouds should be produced somehow. In the process of cloud production, care should be taken not to emit any CO_2. Since it is said that too many clouds also lead to a temperature increase because heat is trapped in the atmosphere, the thickness of the clouds to be produced should be appropriate. Thank you for listening.

All students: [Clapping hands]

Teacher: The conclusion of this group is interesting, isn't it? Produce clouds. Shall we make cotton candy? No. However, we should somehow produce clouds. By the way, where did they say the clouds should be produced?

Student: Above the oceans.

Teacher: Above the oceans. Since the amount of clouds above oceans is decreasing, they suggest that clouds should be produced above oceans. Thank you. Next is Group 4.

Student in Group 4: I will make a presentation on behalf of Group 4. Our group investigated whether or not the impact of the sun accelerates global warming and found the first keyword, cosmic rays. We then started investigating cosmic rays. However, since there were no appropriate data available, we could not go into the details. We found, however, that if the amount of cosmic rays increases, then the amount of clouds around the Earth increases. Then we looked into sunspots, and we found that if the number of sunspots decreases, solar activities go down, and that if the number of sunspots increases, solar activities become more active. As the teacher said, the temperatures of sunspots are lower than those of the surrounding areas, but still they are no less than 4000 degrees. One sunspot is as big as the Earth. Next, look at this graph. The green line shows the Earth's temperature, and the red line shows the number of sunspots. As the number of sunspots increases, the Earth's temperature also increases. So we think that the number of sunspots is related to the Earth's temperature. From this graph, we found that the sun has a negative impact on the Earth. Therefore, in order to prevent global warming, the quantities of cosmic rays, clouds, and sunspots need to be increased. However, it is impossible to increase them since they are all natural phenomena. Therefore, all we can do is to reduce the amount of atmospheric CO_2. Thank you for listening.

All students: [Clapping hands]

Teacher: By the way, what is this negative impact?

Student in Group 4: Global warming.

Teacher: Global warming. Global warming occurs due to the impact of the sun. Okay, thank you. The next group is the last one. Group 5, please start.

Student in Group 5: I will make a presentation on behalf of Group 5. Our group thought that the sun affects global warming. First, changes in the number of sunspots, the number of sunspots … Graph A shows changes in the number of sunspots. Graph B shows the average temperature. When a peak appears on Graph A, a similar peak also appears on Graph B. Therefore, we thought that the number of sunspots is related to the average temperature. Next, we suspected that the amount of cosmic rays is also related to global warming. Graph C shows that there is a proportional relationship between the amount of clouds and that of cosmic rays. Moreover, the two peaks ① and ② on Graph C correspond to the peaks on Graphs A and B, which means that if the cosmic rays increase, clouds also increase, leading to heat trapping. This is why we thought that cosmic rays are also related to global warming. We think that the causes of changes in the impact of the sun include at least the four factors mentioned above. In order to prevent global warming, CO_2 needs to be reduced.

All students: [Clapping hands]

Teacher: Look at this, is the amount of clouds increasing or decreasing?

Student: Increasing.

Teacher: Right, this shows changes in the amount of clouds. If the amount of clouds increases, what happens to the average temperature?

Student: Same peak.

Teacher: Right, a similar peak is formed. When the amount of clouds increases, the temperature also rises sharply. So, this point, 'when the amount of clouds increases, the temperature also rises,' is opposite from what the other groups mentioned before. Do you get it? Then, when the temperature increases, where do you think clouds are increasing?

Student: Above the ground.

Teacher: Yes, not above oceans, but above the ground … so, what is important is the balance between them. I created one more presentation by Student MS (pseudonym) as an extra edition. Look at the last material by Student MS. What does he/she think are the causes of global warming? Can you tell from the chart? What do you think they are?

Student: Temperature of the oceans.

Teacher: The temperature of the oceans, do you mean that the temperature of the oceans is the cause of global warming? For the causes, where are we supposed to look for?

Student: Greenhouse gases.

Teacher: Right, greenhouse gases. What is written in the circle?

Student: H_2O.

Teacher: Do you know what H_2O is?

Student: Water.

Teacher: Yes, water, it is water vapor. Actually, besides CO_2, water vapor also has a greenhouse effect. According to some theories, the greenhouse effect of water vapor is stronger than that of CO_2, and this is what Student MS investigated. Student MS described the reason why water vapor increases on the chart. Please read the chart. I know that you don’t have enough time, but I have one more thing that I want you to do. I want each one of you to write a report at the beginning of the fifth period. I want you to add the causes of global warming and what should be done to prevent it to the chart based on these summaries, including the last one. Did you get it? Since you don’t have much time left, use your free time and add them to the chart before today’s assembly. I’m sorry to rush you, but please finish it before the assembly. Okay, I will finish off this session after distributing these. Based on the results of investigations and what you heard in presentations, add the causes of global warming and the causes of the causes to the worksheet, the chart. Then, based on the chart, write also what we should do to prevent global warming. Okay, class dismissed.

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