Fundamentals and Bubbles in Stock Prices: Theory and Evidence

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1 INTRODUCTION

The existence of excess volatility in stock prices has been a known fact for a long time and, together with traditional asset pricing models, some hypotheses were developed attempting to explain it. One of these, the efficient market hypothesis, besides explaining stock price variations through the effect of the arrival of new information on the market, assumes the possibility of small disturbances, known as white noise, that can bring prices momentarily out of line with their true value. Another, the rational expectations hypothesis, postulates that economic agents react to events rationally both by not committing past errors again and by adjusting their behavior to new expectations about the future, and thus offers a basis for other explanations to the stock price fluctuations.

More recently, a possible explanation to stock price volatility compatible with the rational expectations hypothesis has appeared: rational bubbles. A rational bubble is a self-confirming belief that stock prices depend on a variable, or a combination of variables, that is intrinsically irrelevant to the determination of the fundamental price (or true value of the stock). A bubble, or a succession of bubbles, plausibly can explain why stock prices seem to soar and then nose-dive.

The object of this paper is to examine the admissibility of rational bubbles explaning the volatility of stock prices in Japan. After Japanese stock prices began falling in 1990, it has become habitual in Japan both in the press and in academia, to call the period of stock price increase that ended in 1989 the "bubble period". The starting point of this possible bubble is not clearly defined but 1986 (after the "high-yen recession period") and

1982 are two popular possibilities. Although during the 1980s Japan exhibited scant consumer-price inflation, it suffered accentuated asset-price inflation with sharp rises not only in stock prices, but also in bonds, land and art prices.

The volatility tests performed by LeRoy and Porter [26] and Shiller [33] were the first to suggest that stock prices were too volatile to be compatible with the existing stock price formation models. These results sparked controversy and originated a large body of literature concerning rational bubbles and their testing. A classic reference is Blanchard and Watson [2] where bubble theory is presented in its actual form together with some tests. The original tests were revised and criticised by Flavin [15], Kleidon [25] and West [40], among others. In an attempt to overcome those criticisms Campbell and Shiller [5] and others tested whether or not stock prices were cointegrated and found they were not, adding evidence of excess volatility in stock prices. However, at the same time theoretical arguments against rational bubbles were presented by Diba and Grossman [9], empirical evidence against them was also found, among others, by Diba and Grossman [8] using unit root tests, and by Dweyer and Hafer [13] using cointegration and other tests. Two review essays, by Cochrane [7] and by LeRoy [27], seem to suggest that the debate may be approaching conclusion with the relinquishment of rational bubbles as an explanation for volatility and a reinterpretation of the meaning of excess volatility. We, too, did not find concluding evidence in favor of the presence of a rational bubble in Japanese stock prices.

We will start section 2 by presenting a model of stock prices' formation based on an arbitrage equation. This equation has two kinds of solutions: one is called fundamental, the other bubble. After presenting these solutions, we discuss briefly the processes of bubble formation, bubble bursting and the cases when bubbles can be ruled out. We will then proceed, in section 3, to present the results of several tests performed to determine whether Japanese stock prices movements are better described by the fundamental or by the bubble solution to the arbitrage equation. Finally, in section 4 we present the concluding remarks. Whether there is a bubble in stock prices or not can be tested through an examination of the characteristics of stock prices' time series. In principle, if a rational bubble is present, stock prices should not follow a random walk process insofar as a rational bubble should be increasing at a constant exponential rate. Inversely, if stock prices follow a random walk process we should be able to conclude that no rational bubble is present.

Depending on the different specifications of the models for fundamentals and bubbles the corresponding appropriate statistical tests should be used. Slightly different hypothesis can call for different testing procedures as we will see later.

Evidence in support of the presence of a bubble in a certain data set is, in a way, evidence against the fundamental hypothesis, and vice-versa. But for sake of clarity we will classify some as tests for fundamentals and others as tests for bubbles.

In testing if a fundamental model holds, that is, if a fundamental model adequately describes the behavior of a given time series of stock prices, we start by using the classical empirical tests of weak form of market efficiency (Fama [14]), and then we proceed by using the more recently developed unit root tests. Then we test if stock prices are cointegrated with profits (that we use as a proxy for dividends). Failure to reject the fundamental model by these tests is an indication that probably a rational bubble is not present in the data tested.

As we will demonstrate later, if a rational bubble is present the rate of increase of stock prices should be increasing through time. This characteristic that bubbles imprint in stock prices' time series should provide us with a rationale for testing their possible existence. We will do this with two tests: time trend regression tests and autoregression tests. Should these tests indicate the presence of a bubble and then the fundamental model is being rejected at the same time. Conversely, if these tests do not indicate the presence of a bubble then our confidence in the fundamental hypothesis becomes reinforced.

Another hypothesis tested is whether or not Japanese and American stock prices have moved together. If they indeed did move together and if a bubble is found to exist in the stock prices of one of the countries then a bubble can be expected to exist in the stock prices of the other country: this is called a contagious bubble. If the stock prices in both countries moved together and it is found that no bubble was present in one of them, then we can also expect to find no bubble in the other. But in the case that stock prices in both countries did not move together, what can be infered is only that no contagious bubble was present.

2 THEORY OF FUNDAMENTALS AND BUBBLES

After presenting some basic assumptions in 2.1, we will introduce a simple expectational difference equation in 2.2, its fundamental solution in 2.3, and its possible bubble component in 2.4. In this last sub-section, after defining bubbles we present some possible explanations concerning their formation and bursting. Lastly we discuss some arguments on the possibility of ruling out bubbles.

2.1 ASSUMPTIONS OF A STOCK MARKET MODEL

Let us assume a stock market where shares can be traded in a continuous time sequence at (discrete) dates $t = 1, 2, ..., t, ...$. During period or time t the share of company i is traded at price ${}_ip_t$ (for convenience, whenever i is a representative or typical company, we drop the i subscript if no confusion results). The nonnegative dividend d_t is declared during period t, to be paid during period t+1 to individuals who held the share at time t. We assume furthermore that the sequence of dividends $\{d_1,d_2,...,d_t,...\}$ follows an exogenously given stochastic process (that can, for example, be operationally dependent on the company's output or on the demand in the market for its output, etc; in other words, dividends are determined by the real side of the company).

There is a finite set of individuals $j = 1, 2,..., J$, with rational expectations and finite lives, who trade or transact the existing shares. We assume also that individual, investor or trader j is risk neutral and that he is also able to borrow and lend at the riskless interest rate r_t. The individual's risk neutrality implies that they would not speculate (a speculative trade is a trade where the average gain is expected to be zero and where insurance plays no role) at time t if the market were not to reopen after t. The individual j holding of shares of company i at time t is ${}_{ij}q_t$ (subscripts i and j are also droped whenever the meaning is clear from the context and no confusion results) and, given an aggregate quantity of shares of company i, ${}_iq$, the market clearing conditions are $\Sigma_j\, {}_{ij}q_t = {}_i\bar{q}$ $(=\Sigma_j\, {}_{ij}q_{t-1})$ and $\Sigma_i \Sigma_j\, {}_{ij}q_t = \bar{q}$ $(=\Sigma_i \Sigma_j\, {}_{ij}q_{t-1})$[2]. Short sales are allowed so that ${}_{ij}q_t$ can have both positive and negative values, but in situations where they are not allowed ${}_{ij}q_t \geqq 0$.

2.2 ARBITRAGE EQUATION

We can use the following arbitrage equation between the return on a share and the interest paid by a riskless asset as a behavioral specification for the stock market model:

$$\frac{E[\, p_{t+1} \mid t\,] - p_t}{p_t} + \frac{d_t}{p_t} = r_t \tag{1}$$

where p_t is the price of a share or of an index or portfolio of shares at period t, d_t is the dividend of period t to be paid during period t+1 (because it is announced at t the dividend is known with certainty at that time), and r_t is the rate of interest paid by the riskless asset[3] on period t, that for simplicity we assume to be constant over time. For the dividend of t the share becomes ex-dividend at t+1, that is, a share purchased at t+1 doesn't entitle its new holder to the dividend of t but only to the dividend of t+1 (that will be paid at t+2). The riskless interest rate r can also be viewed as the opportunity cost of capital for the buyer of shares and the riskless asset can be thought of consisting of government bonds. Finally, $E[\, p_{t+1} \mid t\,]$ denotes the expectation held at t (today) of the price of the share at t+1 (tomorrow).

If risk neutral individuals can arbitrage between shares and the riskless asset, then the expected return on the share, which is equal to the expected rate of capital gain plus the dividend-price ratio on the left hand side of equation (1), must equal the riskless rate on the right hand side. By simple reorganization of terms of equation (1) we can get:

$$p_t = a\,E[\,p_{t+1} \mid t\,] + a\,d_t \tag{2}$$

where $a \equiv (1 + r)^{-1} < 1$.

Equation (2) is an expectation difference equation where the current price depends on the current expectation of its value next period plus this period's dividend. Because the coefficient a is equal to the one period discount factor and is less than one as long as the interest rate is positive, this implies that that dependence is less than one for one. For reasons that will be apparent later, we assume that the dividends do not grow explosively over time, a reasonable assumption considering the life cycle every company seems subjected to. This is expressed by the following transversality condition:

$$\lim_{m\to\infty} a^m\,E[\,d_{t+m} \mid t\,] = 0 \tag{3}$$

To solve equation (2) for the behavior of p_t, we need to state explicitly how individuals form their expectations. Several theories on formation of expectations exist, but because of several advantages and due to the structure of our model, we will assume the rational expectations hypothesis. Adaptive expectations for instance, although it assumes that individuals change or adapt their expectations to take account of past forecast errors, still accepts that the future will be pretty much like the past. Rational expectations, in contrast, make no such assumption that the past will be projected into the future, but simply that individuals make use of all available information and in doing so they are on average correct (they make no sistematic prediction errors). So, in analysing this kind of phenomena where the present is more a function of the future than the future a function of the past, rational expectations seem to be the most appropriate model to use because of the exact mathematical relationship between actual and expected prices that it provides.

To make it operative we define rational expectations of p_{t+m} as equal to the mathematical expectation of p_{t+m} based on information available during period t + n for n < m. We assume furtherthat the individuals know the model (ie., equation (2)) and its specification (ie., parameter a) and also that all individuals receive the same information at the same time. These are strong assumptions that, although not strictly necessary for the rationality of formation of expectations, are useful to keep the model simple and allow us to speak of a typical individual's expectation as the mathematical expectation based on the given information set. In practice, however, different people will be using different models and acquiring different information, and thus will be forming different expectations.

Let us define:

$$E[\, p_{t+1} \mid t \,] = E[\, p_{t+1} \mid I_t \,] \tag{4}$$

with $I_t = \{\, p_{t-m}, d_{t-m}, x_{t-m};\ m = 0,\ldots,\infty \,\}$.

$E[\, p_{t+1} \mid t \,]$ is equal to the mathematical expectation of p_{t+1} based on the information set I_t. The information set contains current and lagged values of p, d, and x. x is a variable summarizing other factors that though not present in equation (2) may help predict future values of p and d. Since $\ldots \subset I_{t-1} \subset I_t \subset I_{t+1} \subset \ldots$ no memory is lost because what was known during period t it is still known during period t+1.

2.3 FUNDAMENTALS

Linear rational expectations equations can be solved through the use of analytical methods (such as the undeterminated coefficients and factorization methods). But the most convenient method for simpler cases and the one that we use here is repeated substitution.

We can write equation (2) at time t+1 and take expectations of both sides conditional on the information set at time t:

$$E[\, p_{t+1} \mid I_t \,] = a\, E[\, E[\, p_{t+2} \mid I_{t+1}] \mid I_t \,] + a\, E[\, d_{t+1} \mid I_t \,]$$

Using the law of iterated expectations[4] this equation becomes:

$$E[\,p_{t+1} \mid I_t\,] = a\,E[\,p_{t+2} \mid I_t\,] + a\,E[\,d_{t+1} \mid I_t\,]$$

Replacing this result in equation (2) gives:

$$p_t = a^2\,E[\,p_{t+2} \mid I_t\,] + a^2\,E[\,d_{t+1} \mid I_t\,] + a\,d_t$$

Repeating these steps once again for t+2, and then again up to time T we get:

$$p_t \;=\; a_{T+1}\,E[\,p_{t+T+1} \mid I_t\,] \;+\; \sum_{m=o}^{T} a^{m+1}\,E[\,d_{t+m} \mid I_t\,] \qquad (5)$$

For the second term on the right hand side of equation (5) to converge, the expected dividends average growth rate must be lower than the riskless interest rate, that is, the expected value of d should grow at a rate smaller than (1/a) - 1, ie., the interest rate. This is assured by the transversality condition in equation (3). Then, if:

$$\lim_{T\to\infty} a^{T+1}\,E[\,p_{t+T+1} \mid I_t\,] = 0 \qquad (6)$$

equation (5) reduces to:

$$p_t^* = \sum_{m=o}^{+\infty} a^{m+1}\,E[\,d_{t+m} \mid I_t\,] \qquad (7)$$

Equation (7) is a solution to equation (2). It gives p_t as the discounted sum of future d_{t+m}'s, what means that the price of a share or portfolio of shares is equal to the present value of the perpetual stream of cash dividends of that share or portfolio of shares. This solution is usually called the fundamental solution because it reflects solely the basic or fundamental component of a share value: dividends.

Imposing the transversality condition (6) on equation (5) implies that (7) is a unique solution to equation (2). This is so as long as $a < 1$, that is, as long as the interest rate is positive. The effect on equation (2) of expected future prices on current prices has to be less than unity, what is a plausible restriction.

Another question is whether imposing condition (6) is justified or not. Since the implications of explosion of the endogenous variable p are not inconsistent with the assumptions of our stock market behavioral model its imposition is arbitrary. Usually the reason for its imposition or not lies in whether the objective is to study phenomena other than bubbles or not. Actually in most models of the stock market this kind of condition is only implicit.

So although this fundamental solution is the only one traditionally considered in financial economics, it is not the only solution to equation (2) as we will shortly see in the next sub-section.

2.4 BUBBLES

2.4.1 Definition

Because in equation (6) we imposed the condition that the expectation of the price does not increase too fast, equation (7) was the only solution to the simple expectational difference equation (2). But once we lift this arbitrary condition, many other solutions to equation (2) are also possible. A possible solution is given by:

$$p_t = p^*_t + b_t \tag{8}$$

provided that

$$E[\, b_{t+1} \mid I_t \,] = a^{-1}\, b_t \tag{9}$$

Thus a possible solution to equation (2) may differ from the equillibrium p_t* of equation (7) by a term b_t provided that b_t satisfies the condition of equation (9). If b_t is not identically zero, p_t may depend on extraneous variables, that is, on variables not affecting the fundamental value p_t*. A most important characteristic of b_t is that as $a > 1$, if b_t is not equal to zero the value of b_{t+m} increases to infinity as m increases; to $+\infty$ if $b_t > 0$ or to $-\infty$ if $b_t < 0$ as will be apparent from the next two examples. Then b_t can become an important component of p_t and because of these characteristics it is referred to as a bubble.

For better understanding of what a rational bubble is, the above equation (8) can be rewritten as: $b_t = p_t - p_t^*$. A bubble is thus simply the difference between the actual price of a stock and the fundamental price of that stock (defined in the above section as the present value of the future flow of dividends); furthermore, it should be increasing in accordance with the expression given in equation (9).

We can consider two families of bubbles: deterministic and stochastic. A deterministic of the type:

$$b_t = b_0\, a^{-t} \tag{10}$$

for arbitrary b_0, simply follows a time trend and is ever-expanding.

A stochastic bubble with the following process:

$$\begin{aligned} b^1_{t+1} &= (aq)^{-1}\, b_t + e_{t+1} \qquad & P[\, b^1_{t+1} \,] &= q \\ b^0_{t+1} &= e_{t+1} \qquad & P[\, b^0_{t+1} \,] &= 1 - q \end{aligned} \tag{11}$$

with $E[\, e_{t+1} \mid I_t \,] = 0$ and arbitrary b_0, and has each period the probability 1-q of bursting (q is larger than zero but smaller than unity).

Let us assume for simplicity that dividends, and thus p^*_t are constant. If b_0 is positive, under the deterministic bubble the price of the share will increase exponentially to infinity although the dividends remain constant; under the stochastic bubble, as long as it does not burst, the price of the share will also increase exponentially to infinity but at a faster pace than the deterministic bubble to compensate for the probability of a crash. The price in both cases will increase independently of any increase in dividends because, if individuals antecipate the possibility of a higher price for the share next period, they will buy it notwithstanding the price already being higher than the present value of the expected future dividends. This antecipation of ever-increasing prices satisfies the arbitrage condition and is self-fulfilling as long as the bubble does not burst. If b_0 were negative the price of the share would become negative in finite time. Although negative bubbles can exist in some models, we will not deal (for reasons given in sub-section 2.4.4 Rulling out bubbles) with them and will limit ourselves to the case of positive bubbles. If the stochastic bubble bursts, the disturbance e_t allows for the formation of a new bubble. And although e_t can have an independent distribution, it can also be correlated with any variable in vector $x_t \in I_t$ and still satisfy the condition that its conditional expectation be zero. So if individuals believe that unexpected sunspots affect the price, they will indeed affect it.

2.4.2 Bubble formation

For a probabilistic model of bubbles like the one in equation (11), the disturbance e_t allows for the stochastic formation of bubbles. But for a deterministic model like (10) it is difficult to specify a formation mechanism.

Several reasons exist that can explain the emergence of bubbles. The first is profit motive: anybody that initiates a bubble (and then those who hold it immediately after him before it bursts) can expect to receive large returns without engaging in any productive activity. This reason is also known as pyramiding or Ponzi game and to it was attributed the rise and fall of shares prices in Wall Street in 1928-29)[5].

The second, usually known as the peso problem, is the response of market participants to inaccurate news. Even if their reaction to those news were rational, the fact that the news were inaccurate (or false) leads them to follow a course of action that they would not follow had they had accurate information. For example, the announcement by an oil company of the discovery of a new large field can lead to an increase in the price of the shares of that company (a bubble), even if it turns out that the field was overestimated and the present discounted value of its production (its fundamentals) was much lower. Because of the profits that market manipulation through the spreading of inaccurate or false information can give rise and the harmful consequences to other market participants as well as to the market as a whole, stock market regulations usually exist that try to curb it .

A third reason for the formation of bubbles are sunspots, that is, variables that affect prices only because individuals believe it does[6]. If, for example, the market believes that companies that do more research should be valued more, the price may reflect the size of the investigation budget even if that investigation does not translate in practice into any product improvement or whatever.

A fourth reason may be cross share-holdings. To see this assume there are two companies i = 1,2. Consider also that, by definition, a share's discounted future flow of dividends (it's fundamental value) multiplied by the number of shares outstanding must be equal to the net worth of that company[7]. As a consequence the fundamental value of a share should be equal to the net worth divided by the total number of shares outstanding, that is

$$p_1 q_1 = W_1 \quad ; \quad p_2 q_2 = W_2 \tag{12}$$

where p, q and W stand, respectively, for the price of the share, the number of shares outstanding and the total net worth of a company. For simplicity let us assume that no company has debt or financial assets so that total net worth is equal to total physical assets: W finances only and completly the company's productive activities. We can now introduce financial assets in the form of a cross share-holding between the two companies. Equation (12) becomes then:

$$p_1 q_1 = PW_1 + FW_1 \quad ; \quad p_2 q_2 = PW_2 + FW_2 \tag{13}$$

where PW and FW represent the part of net worth corresponding to productive (non-financial) and finacial assets (or shares). If v represents the proportion of the other company's shares bought we have that $FW_1 = v\, p_2 q_2$ and $FW_2 = v\, p_1 q_1$. Substituting FW_1 and FW_2 into equation (13) and rearranging the terms we get (for company 1):

$$p_1 q_1 = \frac{PW_1 + v\, PW_2}{1 - v^2}$$

Differentiating we obtain:

$$\frac{d(p_1 q_1)}{dv} = \frac{PW_2(1 - v^2) - (-2v)(PW_1 + v\, PW_2)}{(1 - v^2)^2} > 0 \tag{14}$$

As can be seen from equation (14), an increase in the value of the cross share-holding implies an increase in the market value of both companies, even though the productive net worth of both companies remains unchanged. Cross share-holdings can then be one of the mechanisms of bubble formation.

A fifth reason that can lead to the formation of a bubble can be an institutional arrangement. This is the case of fiat money that has no intrinsic value at all. Nevertheless people accept it as having value because they have confidence that everybody else does the same due to some institutional arrangement (in most cases, a law that constrains the residents in a country to use the paper issued by some designated institution). Although fiat money is the best example of an institutional arrangement that leads to the formation and maintenance of a bubble, Malkiel [28] describes the formation of the "South Sea Bubble" in the stock prices of The South Sea Company, 300 years ago in England, as due also to an institutional arrangement given force by the government.

2.4.3 Bubble bursting

Bubble bursting is probably the least understood phase of a rational bubble life. To begin with, the point in time when the bubble will burst and the bursting itself are uncertain. If investors knew with certainty that the bubble would burst at a certain point in time they would not want to hold it during the period before, neither during the period before that, nor during the previous one, ... all the way back to the present point in time. The same reasoning, known as backward deductions, applies also when the bursting is certain but the bursting point in time is uncertain: if investors knew with certainty that the bubble would burst at a certain unspecified point in the future, they would not want to hold the bubble the period before, . . . nor at the present moment. The bursting of the bubble must thus remain incertain if a rational bubble is ever to appear.

One possible cause of the (possible) bursting of a rational bubble, is new information. Because the information set I_t is a subset of the information set I_{t+m} $(m > 0)$, we can have that $E[\, p_{t+n} \mid I_t \,] \neq E[\, p_{t+n} \mid I_{t+m} \,]$ for $n > m$, where p denotes the price of a stock including a bubble as given by equation (8). That is, the expected value of a stock price at a certain future time period $t + n$ may differ from period to period due to new information. This new information most certainly is not information concerning fundamentals or changes in fundamentals, because rational bubbles are independent of fundamentals by definition. But it can perhaps be information related to the bubble itself or to a sunspot.

Another possible mechanism that can possibly trigger the bursting of a rational stochastic bubble (as it also can start the process of formation) is the the random term e_{t+1} of equation (11). If in a certain period the difference $b_{t+1} - (aq)^{-1}b_t = e_t$ becomes a large negative value investors may conclude that the return they received did not match their required return. If e_t is unacceptably large negative value this may start a process of revision of expectations, and lead to the conclusion that the expected return of a bubble will not match anymore the required return; and if they start selling, this will depress further the price and the prospective returns and start a vicious cicle of selling and price decreases that becomes, like the bubble itself, a self-fulfilling prophecy.

A related cause that can lead to the bursting of a stochastic bubble is a decrease in the expected probability of continuation of the bubble, q, that for some reason is not accompanied buy the corresponding required increase in the rate of growth of the bubble. This would also start a vicious cycle of lower than expected returns, sales and lower prices.

2.4.4 Ruling out bubbles

One aspect that was skipped in the previous subsections should be considered now. It refers to the situations when bubbles can be ruled out from occurring in the stock market.

We can start, for example, by ruling out negative bubbles. Assets like shares of common stock were the liability is restricted to the value of the share itself can not have a negative price[(8)]. Because a bubble must always be increasing in any dynamic model a negative bubble would imply a negative price in finite time, which is impossible. In a more general way, bubbles can be ruled out from occuring in assets that can be freely disposed when they become liabilities.

It has beem argued that deterministic ever-expanding bubbles can be ruled out because they would become too large to be compatible with the finiteness of the economy. This argument can be disposed of when the economy itself is growing at a pace equal or larger that the bubble: in this case the relative size of the bubble to the economy would be constant or decreasing posing no such problem. But even if the bubble were increasing at a faster pace than the economy no such reason allows us to rule out deterministic bubbles. This can be seen using equation (14) and supposing that the economy is composed by z companies all similar to company i. v represents now the sum of the percentage of the total number of shares of every company, that are held by company i ($v = \Sigma^z v_i$). As we assume that all companies are similar, every company holds an equal percentage of all other companies so that $v = zv_i$ if the companies hold 100v% of their own shares, or $v = (z - 1)v_i$

if they do not hold any of their own shares (0 < v < 1). Aggregating all companies equation (14) would become:

$$\sum_{i=1}^{Z} p_i = \sum_{i=1}^{Z}\left(\frac{{}_pW_i}{q_i} + \frac{v}{1-v} \cdot \frac{{}_pW_i}{q_i}\right)$$

or

$$p = zp = z\left(\frac{{}_pW}{q_i} + \frac{v}{1-v} \cdot \frac{{}_pW}{q}\right) \qquad (15)$$

because i = k for all i, and where P stands for the aggregate value of the economy (fundamentals and bubbles). This equation shows that for given ${}_pW$, q, and z and thus for a given aggregate fundamental value of the economy equal to $z({}_pW/q)$, the bubble itself will be larger than that fundamental value for 1/2 < v < 1. In fact as the value of v approaches unity the value of the bubble will explode to infinity even though the fundamental value remains constant at a small fraction of the value of the bubble. The size of the economy then, poses no theoretical limit to the size of a positive bubble.

Another argument advanced to eliminate positive bubbles in physical assets is that if a substitute exists in infinitely large supply, possibly at a very high price, then there cannot exist positive bubbles. If a positive bubble would appear, then the expected price would go to infinity and consequently exceed the price at which the substitute was available, what is impossible in a world with rational agents.

One other argument, this time specific of the stock market, argues that as soon as a bubble would appear in a company's shares it would be in the interests of the initial shareholders to issue more shares and invest the proceeds as long as the bubble would exist. As more and more shares were issued at an increasing rate it is doubtful that the market would continue to absorb that ever increasing supply of shares unless their price decreased.

However, positive bubbles can not be ruled out when individuals have finite lives (exhibit myopic rational expectations), but they can be ruled out when individuals have fully dynamic rational expectations (what would happen with infinitely lived individuals).

We can also rule out bubbles in assets whose prices are subject to some terminal condition in the future, such as bonds. This is because their price at maturity is a fixed value (equal to their fundamental value) and no bubble can exist then. If no bubble exists at the terminal period it will also not exist the period before. Working bakward in time we can see that a bubble can never exist in this kind of assets were there is a terminal condition on their price.

Concluding, bubbles can be ruled out from occuring in securities markets when: (1) they are negative, (2) a terminal price exists for the asset, and (3) individuals have fully dynamic rational expectations. If any one of these conditions are present bubbles cannot exist. Bubbles possibly may exist (cannot be ruled out) when all the following conditions are present: (1) fundamentals are difficult to evaluate with precision and certainty, and (2) individuals exhibit short run behavior (not fully dynamic, that is, without infinite horizons rational expectations).

3 TESTING FOR FUNDAMENTALS AND BUBBLES IN JAPANESE STOCK PRICES

After describing in 3.1 the data used in performing the various tests, we present the weak form efficiency tests in 3.2, an excess volatility test in 3.3, tests for fundamentals in 3.4, and tests for bubbles in 3.5.

3.1 THE DATA USED

Three sets of data were used in our query of whether bubbles existed or not in Japanese stock prices. We will present them briefly here, but more detailed information can be found in Dos Santos [12] and will be furnished to anybody interested.

The first data set is composed of the daily series of the Nikkei 225 index from

September 29, 1987, to January 22, 1990, but the tests are performed for the period between November 12, 1987 and December 29, 1989, respectively the minimum after the October 1987 crash and the maximum before the 1990 crash. The Nikkei 225 is used as a measure of Japanese stock prices and is chosen because of its popularity and wide usage in financial circles and popular and academic press, and because of its easy availability. The reported values of the index are their values at the end of a trading day and all trading days are included (including those Saturdays when trading also took place).

The second data set is composed of the daily Nikkei 225 index (for Japanese stock prices), the Dow 30 (for US stock prices) and the yen-dollar exchange rate time series for a period ranging from September 29, 1987, to February 5, 1990. The reported values of the Nikkei 225 and the yen-dollar exchange rate are their values at the end of the trading day in Japan and the reported values of the Dow 30 is its values at the end of the trading day in New York. All days in which trading occured in all three markets (Tokyo stock and foreign exchanges and New York stock exchange) are included. But when no trading occured in one market no trading is reported for all markets. This was done to keep a correspondence between the values of the three series.

The third data set consists of the quarterly averaged TOPIX, call rate, GNP deflator and business profits (seasonally adjusted) series, from the first quarter of 1968 to the last quarter of 1989. As it will be explained later, the business profits series is intended to be used as substitute for dividends, an use justified by the institutional characteristics of Japanese economic life that do not make dividends a good measure of the performance and future prospects of Japanese companies. Because business profits includes the profits of all companies, the more broad measure of Japanese stock prices, TOPIX, is used instead of the Nikkei 225.

3.2 WEAK FORM EFFICIENCY TESTS

We know already that a bubble, either of the deterministic type of equation (10) or of the stochastic type of equation (11), should increase at an exponential rate. Consequently, from either of these equations and from equation (8) we should expect that if a bubble exists in stock prices then those stock prices should also increase exponentially. Moreover, if stock prices are increasing exponentially, present price increases should be explained, at least in part, by past price increases. Or putting it in another way, past price increases should be able to predict future price increses, with certainty in the case of a deterministic bubble, and with a certain probability in the case of a stochastic bubble.

But the ability to predict future price increases based on the examination of the sequence of past prices contradicts the efficient markets theory in its weak form(9). Then, one of the consequences of the existence of a rational bubble is that the weak form of the efficient markets theory should not hold. This has strong implications for all modern financial theory and it is one of the reasons that attracted so much attention to rational bubbles during the past ten years.

The hypothesis that past returns cannot be used to predict future returns cannot be proven since there are an infinite number of ways that the sequence of past prices can be used to forecast future prices. All that can be done is to test particular ways of combining past price data to predict future returns. However, a very large number of tests of alternative price patterns have already been made using data for different countries and different periods of time (Fama [14], Granger[18]). And the conclusion of these studies is that information in the past price series is already incorporated in the present stock price.

One of the most used process to test the weak form efficient markets hypothesis is to see if stock prices follow a random walk. If in fact they follow a random walk then the weak form of efficient markets hypothesis can not be rejected, and it follows that a rational bubble should not be present. This random walk test can be done, among other ways, by examining the correlation between past price changes and future price changes as in the following equation:

$$p_t - p_{t-1} = \mu + \beta(p_{t-1-T} - p_{t-2-T}) + \varepsilon_t$$

where the term μ , also called drift, measures the expected change in price unrelated to the previous price change. As long as stocks have a positive capital gain return, μ should be positive. The term β measures the relationship between two prices changes, between the one occuring at t and the one occuring at t-1-T. If T=0 then it mesures the relationship between two consecutive price changes; if T=1 it measures the rélationship between a price change and the price change that occured two periods previously; and so on. ε_t is a random variable assumed to be normally independent and identically distributed with mean value of zero. It incorporates the variability of the current price change not related to the previous price change. Denoting a price change during period t by Δp_t the above equation can be written as

$$\Delta p_t = \mu + \beta \Delta p_{t-1-T} + \varepsilon_t \tag{16}$$

Estimation of equation (16) for T=0 using the Nikkei data for the period from November 12, 1987 to December 29, 1989, gives:

$$\Delta p_t = \underset{(3.4571)}{29.609} + \underset{(1.0278)}{0.042993} \Delta p_{t-1}$$

$R^2 = 0.0019$, DW = 2.0239

where the values in parenthesis are the t-values of the estimated parameters. Both in the case when T=0 as well in the cases when T=1,2,3,4 (reported in Dos Santos [12]), β is not significatively different from zero. From this test the random walk hypothesis cannot be rejected.

Not all random walk tests have been done by utilizing equation (16). Another popular approach has been to test the logarithm of price relatives as in the following equation:

$$\ln(p_t/p_{t-1}) = \beta \ln(p_{t-1-T}/p_{t-2-T}) + \varepsilon_t \qquad (17)$$

The logarithm of the ending price divided by the previous price is the continuously compounded rate of return.

Estimating equation (17) for T=0 using the Nikkei daily series between November 12, 1987 and December 29, 1989 yields:

$$\ln(p_t/p_{t-1}) = 0.086568 \ln(p_{t-1}/p_{t-2})$$
$$(2.0880)$$

R^2 = Adjusted R^2 = 0.0050 , DW = 2.0282

These results seem also to support the random walk hypothesis: the correlation coefficient is so small that one can not expect past return rates to explain meaningfully future return rates.

3.3 AN EXCESS VOLATILITY TEST

By the end of the 1970s the efficient market hypothesis was a well established theory, considered to be useful and well supported by evidence. The two twin ideas behind it were very simple: prices reflect all available and relevant information and are optimal forecasts of the future flow of real dividends. Not only this, but the efficient markets hypothesis was the basis of most of the modern financial theory.

But in 1981 two papers, one by LeRoy and Porter [26] and another by Shiller [33], argued that stock prices were actually too volatile to be determined by the present value of the optimal forcast of the future flow of real dividends. Their argument is as follows.

Let p_t denote the actual price of an asset at time t, and p_t^* denote the discounted value of the expected future flow of dividends at time t. This p_t^* corresponds to the theoretical price given by the fundamental solution in equation (7) and Shiller calls it the "ex-post value". If, as the fundamental value solution of equation (7) asserts, price p_t is the rational expectation of p_t^*, the present value of actual future dividends, then the data must satisfy the variance inequality var(p_t^*)≧var(p_t). This is because since p_t is known at time t we have that $p_t^* = p_t + u_t$, where u_t is a forecast error. And because a forecast error must be uncorrelated with the corresponding forecast, so u_t must be uncorrelated with p_t. Therefore it follows that var(p_t^*) = var(p_t) + var(u_t) . Since variances are nonnegative the above variance inequality follows. However, Shiller and others found that the above inequality was violated for the U.S. data tested and this originated the subsequent excess volatility debate. One of the explanations offered for the excess volatility found by Shiller is the possible existence of a rational bubble.

Although we could not do an exact replica of the inequality test performed by Shiller due to the unability to construct the series p_t^*, we tried a substitute test that consists in comparing the coefficient of variation $(\sum(X_i - \overline{X})^2/\sum X_i^2)$ for the series of the actual price (for what we used the Topix index) and of the ratio earnings/nominal interest rate. This ratio earnings/nominal interest rate is used as a proxy of present value of the expected future flow of dividends (see Funaoka [18]). We used for earnings a seasonally adjusted series of business profits, and for the nominal interest rate a series of average call rates. This comparison was performed for the period from 1968 to 1989 using quarterly data and the results were:

	Stock Prices	Earnings/interest rate
Coefficient of variation	0.4778	0.3835

These results show that in fact stock prices seem to be more volatile than a measure of their fundamental value, what calls for an explanation. One advanced hypothesis is the existence of a rational bubble. Other explanations for crashes and excess volatility that do not assume rationality also exist (for example, Shiller [34]). But a very simple example can show that even minimal changes in expectations about interest rates can have a massive effect on prices. Let us assume that investors expect a dividend of a certain stock to be 5 yen at the end of this year and that it will increase thereafter at the rate of 3% per year. If they expect the interest rate to remain constant at 5%, then the value of this stock will be 250 yen. However, if the expectations about the interest rate change, let's say to 4%, while the expectations about the company future prospects remain unchanged, then the value of the stock will become 500 yen, a 100% change! Because small changes in interest rates can have large repercussions in stock prices it is surprising that stock prices are not even more volatile.

3.4 TESTS FOR FUNDAMENTALS

3.4.1 Preliminaries

Let us now remember the stock price determination model presented in equation (2):

$$p_t = a\,E[\,p_{t+1} \mid I_t\,] + a\,d_t \tag{2}$$

where $a \equiv (1 + r)^{-1} < 1$, with the further assumption that the riskless interest rate remains constant ($r_t = r$, $\forall t$). As assumed before, investors are risk neutral and have rational expectations. If the transversality condition in equation (6) holds then a solution to equation (2) is given by

$$p_t^* = \sum_{m=0}^{+\infty} a^{m+1}\,E[\,d_{t+m} \mid I_t\,] \tag{7}$$

the fundamental solution. Equation (7) gives the fundamental price as the present value of the future stream of dividends discounted at the constant riskless interest rate.

To see if the fundamental solution is a reasonable representation of the actual process of price formation we can test it using unit root tests. But before we can do this we need to specify a model for dividend behavior. Different models for dividends have different implications and require the use of different testing procedures. Once having specified a model for dividends we can test joinly that dividends' model and the above fundamental model by using the appropriate unit root tests.

3.4.2 Deterministic model for dividends

We can have two classes of dividends' models: deterministic and stochastic. In the deterministic category we can have a relation of the following type:

$$d_t = (1 + g^d)^t \, d_0 \tag{18}$$

for an arbitrary d_0, and where g^d is a constant growth rate. Substitution of equation (18) into equation (7) gives

$$p_t^* = \sum_{m=0}^{+\infty} a^{m+1} d_{t+m}$$

and calculation of the proportional change in the fundamental price brings:

$$\frac{\Delta p_{t+1}^*}{p_t^*} = g^d \tag{19}$$

Equation (19) implies that if prices are determined according to the fundamental solution and dividends are determined as in equation (18) then the rate of change in stock prices should equal the rate of growth of dividends. To test if this hypothesis[(10)] holds, two tests (in every aspect equivalent to each other) can be performed. Noticing that $\Delta p_{t+1}^*/p_t$ is approximately equal to $\ln(p_{t+1}^*/p_t^*)$ we can transform equation (19) into

$$\ln(p_{t+1}^*/p_t^*) = g^d \tag{20}$$

We can test equation (20) using the following regression:

$$\ln p_{t+1} = g^d + \beta \ln p_t + e_t \tag{21}$$

Estimation of equation (21) using the Nikkei 225 daily series between November 12, 1987 and December 29, 1989 gives:

$$\ln p_{t+1} = \underset{(1.0430)}{0.022764} + \underset{(470.70)}{0.99789} \ln p_t$$

$R^2 = 0.9975$, Adjusted $R^2 = 0.9975$, DW = 1.8944

The regression corresponding to equation (21) has an estimated β coefficient that is approximately one and significant.

This result allows us not to reject (accept) the hypothesis tested, namely that stock prices are determined according to the fundamental model of equation (7) with dividends following a deterministic relation of the type of equation (18) as summarized by equation (20), and under the assumption that the approximation used is reasonable.

3.4.3 Stochastic model for dividends

Another class of dividends' models are stochastic models. One specification for this kind of models can be

$$d_t = (1 + g^d)\, d_t + \eta_{t+1} \tag{22}$$

where η_t is the unexpected part of divideds growth in period t and it is distributed with mean of zero. The difference between the deterministic model of equation (18) and this model resides in that this one follows a stochastic process due to the inclusion of the random term η_t.

In a simple stochastic dividend model as the one of equation (22) we can add further assumptions about the term η_t. As we said above η_t is distributed with mean zero. But besides this basic assumption we can assume that η_t is either (i) normal independent identically distributed with mean zero and variance one ($\eta_t \sim$ iid $N(0, \sigma^2)$), or (ii) it follows a stochastic process of the type: $\eta_{t+1} = \eta_t + \varepsilon_t$, with ε_t normal independent identically distributed with mean zero and variance one ($\varepsilon_t \sim$ iid N(0,1)). However, choice of either (i) or (ii) will be irrelevant in what follows.

Substitution of equation (22) into equation (7) and then calculation of the proportional change in the fundamental price brings:

$$\frac{\Delta p^*_{t+1}}{p^*_t} = g^d + \frac{\eta_{t+1}}{p^*_t} \sum_{m=0}^{+\infty} a^{m+1}(1+g^d)^m \tag{23}$$

Denoting the second term of the right hand side of equation (23) by

$$\eta'_{t+1} = \frac{\eta_{t+1}}{p^*_t} \sum_{m=0}^{+\infty} a^{m+1}(1+g^d)^m$$

we can write equation (23) as

$$\frac{\Delta p^*_{t+1}}{p^*_t} = g^d + \eta'_{t+1} \tag{24}$$

Independently of the assumptions relative to η_{t+1} made above, the term η_{t+1} will always be serially correlated(11).

Using the approximation $\Delta p_{t+1} / p_t = \ln(p_{t+1} / p_t)$, equation (24) can be transformed to

$$\ln p_{t+1} = g^d + \rho \ln p_t + \eta'_{t+1} \tag{25}$$

Because of the serial correlation of η'_{t+1}, we cannot regress equation (25) directly to test the possible existence of an unit root $\rho=1$. But we can make some simplifying assumptions about the behaviour of η'_{t+1}. One possible assumption is that it follows a stochastic process of the type:

$$\eta'_{t+1} = \delta \ \eta'_t + \varepsilon_t \tag{26}$$

with $0 < \delta < 1$ (this ensures that the process is a stable, non-explosive process) and ε_t normal independent identically distributed with mean zero and variance one ($\varepsilon_t \sim$ iid N(0,1)).

Then if we subtract from equation (25) written at t+1 the same equation written at t multiplied by the factor δ of equation (26) we can get

$$\Delta \ln p_{t+1} = (1-\delta)g^d - (1-\delta)(1-\rho)\ln p_t - \delta\ \rho\ \Delta \ln p_t + \varepsilon_t$$

where $\varepsilon_t \sim$ iid N(0,1) is not serially correlated. Making $\alpha_0 = (1-\delta)g^d$, $\alpha_1 = -(1-\delta)(1-\rho)$, and $\alpha_2 = -\delta\rho$ we can write the above equation as:

$$\Delta \ln p_{t+1} = \alpha_0 + \alpha_1 \ln p_t + \alpha_2 \Delta \ln p_t + \varepsilon_t \tag{27}$$

To see whether the model behind equation (24) (that is, the fundamental price determination model of equation (7) and the dividend formation process of equations (22) and (26)) holds or not, we want to test whether the value of ρ in equation (25) is one or not. Because we cannot test this directly in equation (25) we need to test whether α_1 in equation (27) is zero or not. As we assume in equation (26) that $\delta < 1$ we can accept without any further ado that if $\alpha_1 = 0$ then $\rho = 1$. (If we do not impose on equation (26) the stability condition that $\delta < 1$ then even if $\alpha_1 = 0$ we cannot be sure that $\rho = 1$: we can have in this case that either $\delta = 1$ and $\rho \neq 1$, or $\delta \neq 1$ and $\rho = 1$, or $\delta = 1$ and $\rho = 1$. One possible solution to distiguish between these three cases would involve first to test whether the constant term is $\alpha_0 = 0$ or not. According to the result of this test we would have that either a) $\delta \neq 1$ or b) $\delta = 1$. If a) were the case then we could proceed to test whether $\alpha_1 = 0$, and if this hypothesis could not be rejected then we would be able to conclude that $\rho = 1$. But if b) were the case then any test whether $\alpha_1 = 0$ would not allow us to infer that $\rho = 1$. Another possible solution would be to test directly whether the estimated values of α_0 and α_1, a_0 and a_1, are such that $a_1/a_0 = 0$ or not. This test would be equivalent to the test that the estimated $-(1 - \rho) = 0$ or $\rho = 1$. However, there is not yet a statistical distribution available to this test).

Estimation of equation (27) using the Nikkei 225 daily series between November 12, 1987 and December 29, 1989 gives

$$\Delta \ln p_t = \underset{(1.0842)}{0.023629} - \underset{(-1.0394)}{0.0022005} \ln p_{t-1} + \underset{(1.6979)}{0.070468} \Delta \ln p_{t-1}$$

$R^2 = 0.0069$, Adjusted $R^2 = 0.0034$, DW = 2.0318

We cannot reject the null hypothesis H_0: $\alpha_1 = 0$ (using the t-distribution the critical values of the 95% confidence interval are $\pm$ 1. 96). Assuming that $\delta < 1$ we cannot reject the hypothesis that $\rho = 1$ and should be able to infer that Japanese stock prices between November 12, 1987 and December 29, 1987 followed the fundamental price determination model and that no bubble was present.

If we were to relax the assumption that $\delta < 1$ we still would be able to find that the estimated $a_1/a_0 = -(1-\delta)(1-\rho)/(1-\delta) = -(1-\rho) = -0.09$ or that the estimated $\rho = 1.09$, but we have no distribution to construct a confidence interval, and thus have no measure of the goodness of the $\rho = 1.09$ estimate.

The results of this test, that is, the values of the t-ratios obtained for the estimated values of α_1, as well as those of the tests peformed by Dwyer and Fuller [13] for levels and changes(12) for other two periods also with daily data are summarized in the following

Table 1.

TABLE 1 UNIT ROOT TESTS

		t-ratio		Durbin-Watson	
Author	Sample	Levels	Changes	Levels	Changes
Dwyer & Hafer	Aug 4 1986 Jun 11 1987	0.208	-7.952	2.02	1.99
Dwyer & Hafer	Jan 2 1987 Jun 11 1987	-0.637	-6.258	2.01	1.98
Dos Santos	Nov 12 1987 Dec 29 1989	-1.039	-17.306	2.03	2.05

The test statistics reported above for the levels indicate that we cannot reject the hypothesis (H_0: $\alpha_1 = 0$ or $\rho = 1$) of an unit root in the in the series for the three sample periods. The test statistics for the changes, however, rejects the the hypothesis of a second unit root. The test statistics are compared with thet-ratio critical value (for a 95% confidence interval) of ±1. 98.

3.4.4 More tests for fundamentals: stock prices and profits

We will now analyse the joint behavior of stock prices and profits under the assumptions of (1) constant and (2) variable real interest rates.

3.4.4.1 Constant expected real interest rate

Let us continue to assume that stock prices are determined according to the fundamental solution given by equation (7), with a constant real interest rate r. Let us also assume that dividends follow a stochastic process such as represented by the general equation (28),

$$d_{t+1} = \mu + (1 + g^{d})\, d_t + \eta_{t+1} \qquad (28)$$

with $\mu = 0$ and η_{t+1} itself following a stochastic process of the type:

$$\eta_{t+1} = \delta\, \eta_t + \varepsilon_{t+1} \qquad (29)$$

with $0 \leqq \delta \leqq 1$ and $\varepsilon_{t+1} \sim$ iid $N(0, \sigma_{\varepsilon}^{2})$. Notice that when $\beta = 1 + g^{d} = 1$ and $\delta = 0$ the process has a unit root $\beta = 1$ and dividends follow a random walk. The restrition imposed on equation (28) that there is no drift ($\mu = 0$) is not essencial and can be lifted if it is convenient.

Let us now introduce the concept of cointegration. If both of two[(14)] variables each has a unit root, then we know that they are stationary. And if the residuals of the simple linear equation characterizing the relationship of the two variables do not have an unit root, then the two variables are said to be cointegrated. For example, if both stock prices and dividends have an unit root, and if they are regressed one in another and the resulting residuals do not have an unit root then stock prices and dividends are said to be cointegrated. The meaning of cointegration is that even if two variables follow a random walk there is a significant relationship between them, that is, the two move together. In the previous example of stock prices and dividends, even if both stock prices and dividends follow a random walk, if they are cointegrated they move together.

If stock prices and dividends each follows a random walk process and each has an unit root, then, according to the fundamental stock price determination model, they should be cointegrated. That is, a regression of stock prices on dividends of the form of:

$$p_t = \alpha_0 + \alpha_1 d_t + \varepsilon_t \qquad (30)$$

should have a residual that does not have a unit root[15]. This can be demonstrated as follows. Equation (7) for the fundamental price can be rewritten as

$$p_t^* = d_t \sum_{m=0}^{+\infty} a^{m+1} + \sum_{m=0}^{+\infty} a^{m+1}(E[d_{t+m} | I_t] - d_t) \qquad (31)$$

The expression inside parenthesis is equal to:

$$E[d_{t+m} | I_t] - d_t = \sum_{k=1}^{m} (E[d_{t+k} | I_t] - E[d_{t+k-1} | I_t])$$

Substitution of this equation into equation (31) gives:

$$p_t^* = r^{-1}E[d_{t+1} | I_t] + r^{-1} \sum_{m=1}^{\infty} a^m (E[d_{t+m+1} | I_t] - E[d_{t+m} | I_t]) \qquad (32)$$

If dividends follow a random walk process and have one unit root as we are assuming, the difference $E[d_{t+m+1} | I_t] - E[d_{t+m} | I_t]$ does not have one unit root because its expected value $E[\varepsilon_{t+m+1}] = 0$ for all m. Because, in addition, the discount factors a^{m+1} form a set of geometrically declining coefficients on these already stationary values this implies that the second term of the right hand part of equation (32) does not have an unit root. And because this term of equation (32) corresponds to the residuals of equation (30), these residuals should not have an unit root if the expected real interest rate is constant and fundamentals determine stock prices[16].

The above conclusions do not apply for the case when a bubble is present on stock prices. This is because when a bubble is on, the bubble part of the stock prices increases over time, and since the bubble component is independent of the dividends by definition, it should appear in the residuals ε_k of equation (30). And if it appears in those residuals their estimated value should increase with time and have a root greater than one $(1 + r/q)$. Thus, if a rational bubble exists and is an important component of the observed stock prices then the stock prices will not be cointegrated with dividends.

To perform the cointegration test between stock prices and dividends we will have to make three unit root tests: one for stock prices, another for dividends and a last one for the residuals of the regression between stock prices and dividends. In these unit root tests we will always be testing the null hypothesis H_0: $\beta = 1$ against H_a: $\beta \neq 1$. We can perform the cointegration test having two different basic assumptions about the process followed by dividends. These are that:

(A) dividends do indeed follow a simple random walk process ($\delta = 0$ in the above equation (26));

(B) dividends follow the stochastic process described by the above equation (26) with $0 < \delta < 1$.

Let Y_t denote the variable being tested for an unit root (stock prices, dividends or residuals), ϕ (L) denote a lag polonomial with lag operator L, and μ denote a constant term (the rate of growth of dividends). β, δ, ε_t, and η_t are as defined before.

Let us then consider (A). In this case we have the general expression:

$$\phi(L)\,[(1 - \beta L)Y_t - \mu] = \varepsilon_t \tag{33}$$

Let us take the simple case where the lag polonomial $\phi(L) = 1 - \phi_1 L$. Making the appropriate transformations we get:

$$\Delta Y = (1 - \phi_1)\mu - (1 - \phi_1)(1 - \beta)\,Y_{t-1} - \phi_1 \beta\ \Delta Y_{t-1} + \varepsilon_t$$

or, by making $\alpha_0 = (1 - \phi_1)$, $\alpha_1 = -(1 - \phi_1)(1 - \beta)$, and $\alpha_2 = -\phi_1 \beta$ this becomes

$$\Delta Y = \alpha_0 \mu + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-1} + \varepsilon_t \tag{34}$$

We can now consider two cases:

(A1) $\phi_1 = 0$, and

(A2) $0 < \phi_1 < 1$.

In case (A1) with $\phi_1 = 0$ we have that $\alpha_0 = 1$, $\alpha_1 = \beta - 1$, and $\alpha_2 = 0$, and equation (34) becomes

$$\Delta Y_t = \mu + (\beta - 1) Y_{t-1} + \varepsilon_t \qquad (35)$$

Equation (35) corresponds to equation (21) we saw in 3.4.2. We test, in this equation, the existence of an unit root using the null hypothesis H_0: $\alpha_1 = \beta - 1 = 0$. The t-ratios for α_1 are given below in Table 2 for stock prices, dividends and residuals of the regression between the two (for the later we made $\alpha_0 = 0$, since there is no reason to include an intercept term in the case of residuals).

In case (A2), with $0 < \phi_1 < 1$, we test in equation (34) the existence of an unit root with the null hypothesis H_0: $\alpha_1 = -(1-\phi_1)(1-\beta)=0$. Because $\phi_1 < 1$ this is equivalent to the hypothesis $\beta - 1 = 0$. As we are making the same basic assumptions in cases (A1) and (A2), the two are equivalent and the results we will get from the two should be approximately similar (provided that ϕ_1 is in fact less than one, a question we already discussed at some length in 3.4.3. The t-ratios for α_1 are also given in the following Table 2 for stock prices, dividends and the residuals of the regression between the two.

Let us now consider (B). In this case we have the general expression:

$$\phi(L)\,[(1 - \beta L)Y_t - \mu - \eta_t] = 0 \qquad (36)$$

Equation (36) differs from equation (33) in that the residuals η_t are considered to be autocorrelated in opposition to the normal independent identical distribution assumed for ε_t. Taking the case where the lag polonomial is $\phi(L) = 1 - \phi_1 L$ and making $\phi_1 = \delta$ and after doing the appropriate manipulations we get:

$$\Delta Y = (1 - \phi_1)\mu - (1 - \phi_1)(1 - \beta)\, Y_{t-1} - \phi_1 \beta\, \Delta Y_{t-1} + \varepsilon_t$$

and from this equation the above equation (34) follows.

In performing the cointegration test for Japanese stock prices and dividends, we used quarterly data between the first quarter of 1968 and the last quarter of 1989 for the Topix, a stock price index including all companies listed in the first section of the Tokyo stock exchange based in January 4, 1968, and a seasonally adjusted series for the business profits of all industries. The use of business profits as a proxy for dividends calls for a few words of explanation.

The model behind equation (7) is based on the hypothesis that individuals are concerned with dividends plus capital gains. Also, it assumes that capital gains are just a reflection of information about future dividends. Profits or earnings, on the other hand, are statistics that can provide an indicator of how well a company is doing but, theoretically, there is no reason why price per share should be the discounted value of expected future profits per share if some profits are retained to finance the future expansion of the company. Such a present value formula would entail a sort of double counting. It is incorrect to include in the present value formula both profits at time t and the later profits that accrue to the reinvested time t earnings. However when firms pay no dividends (or when these dividends are such a small amount of profits that they can be considered negligible) then the price of a share of stock cannot be determined by the present value of dividends given by equation (7). And, as it has been shown recently by Campbell and Shiller [6], profits, when averaged over the years, are a good substitute for dividends. In conclusion, our use of business profits instead of dividends is justified because dividends in Japan are, as a rule, of very small value and are not a good indicator of profitability - dividends in Japan remain unchanged for long periods of time irrespective of profitability, that is, they do not reflect fundamental changes in the prospects of companies when those changes happen.

Another problem with the series of business profits used is that it includes the profits of all companies in Japan (not only the companies listed in the first section of Tokyo's stock exchange whose price Topix measures) and is not in per share values. This is a problem that could not be avoided due to the difficulty of obtaining data to construct a series only for those companies listed in the first section of Tokyo's stock exchange. This series should then be taken only as an approximation of the real profits per share series. Also, this series is composed only by the operating profits not including profits arising from speculative investments in the stock market and land.

Performing the unit root test for the logarithms of stock prices, the logarithms of business profits and the residuals of the regression between the two for the period between the first quarter of 1968 and the fourth quarter of 1989 yields the results presented in Table 2.

TABLE 2 COINTEGRATION BETWEEN STOCK PRICES AND PROFITS

Variable	t-ratio on lag variable		Durbin-Watson	
	Levels	Changes	Levels	Changes
(A1)				
Stock Prices	0.6073		1.5058	
Profits	-1.2213		1.9361	
Residuals	-1.5170		1.8852	
(A2) and (B)				
Stock Prices	0.3513	-6.5311	1.9142	1.9305
Profits	-1.2200	-4.3718	2.0180	1.9990
Residuals	-1.5667	-5.3444	2.0083	1.9928

The results for cases (A1) and (A2) and (B) are similar as we expected. They show that we cannot reject the null hypothesis that stock prices, dividends (profits) and the residuals of the regression between the two had a unit root. The conclusion is then that with constant interest rates, stock prices and dividends (profits) are not cointegrated variables. This is evidence against the fundamental model and in favor of the possible existence of a rational bubble.

3.4.4.2 Variable expected interest rate

Let us remember that the arbitrage between the return on a share of stock and the interest paid by a riskless asset can be expressed by the above equation (2):

$$p_t = a_t\, E[\, p_{t+1} \mid I_t\,] + a_t\, d_t \tag{2a}$$

where $a_t = (1 + r_t)^{-1} < 1$.

The interest rate as well as the discount factor have now a time subscript to indicate their nonconstancy. Because we will have to deal with rather complicated equations and to avoid cumbersomeness we will from now on use a simpler notation for expectations: $E_t y_{t+k}$ will from now on represent the rational expectations at time t of variable y at time t + k . The above equation (2a) will accordingly be written as

$$p_t = a_t\, E_t p_{t+1} + a_t\, d_t \tag{2a}$$

The fundamental solution when the interest rate is variable is found, as before, by writing equation (2a) at time t + 1 and taking expectations of both sides conditional on the information set at time t:

$$E_t p_{t+1} = E_t(a_{t+1}\, E_{t+1} p_{t+2} + a_{t+1}\, d_{t+1})$$

Replacing this result in equation (2a) gives:

$$p_t = a_t E_t(a_{t+1} E_{t+1} p_{t+2} + a_{t+1} d_{t+1}) + a_t d_t$$

Repeating these steps again for t + 2 , and so forth, we get:

$$p_t = a_t d_t + a_t E_t(a_{t+1} d_{t+1}) + a_t E_t(a_{t+1} E_{t+1}(a_{t+2} d_{t+2})) + \ldots$$

Assuming that the discount factors and dividends have independent distributions, and that the discount factors are not serially correlated, if we use the law of iterated expectations the resulting expression can be written as:

$$p_t = \sum_{m=0}^{\infty} \gamma_{t+m} d_{t+m} \tag{37}$$

with $\gamma_{t+m} = \prod_{h=0}^{m} E_t a_{t+h}$, where $E_t a_t = a_t$.

Equation (37) can be rewritten as:

$$p_t = d_t \sum_{m=0}^{\infty} \gamma_{t+m} + \sum_{m=0}^{\infty} \gamma_{t+m} (d_{t+m} - d_t) \tag{38}$$

Assuming that the expected real interest rate is constant overtime (that is, assuming that the term structure is flat), then $\gamma_{t+m} = a_t^m$. This simplifies equation (38) to:

$$p_t = \frac{d_t}{r_t} + \sum_{m=0}^{\infty} a_t^m (E_t d_{t+m} - E_t d_t) \tag{39}$$

Substitution into equation (39) of $E_t d_{t+k} - E_t d_t = \sum_{k=1}^{m} (E_t d_{t+k} - E_t d_{t+k-1})$ gives, after simplification:

$$p_t = \frac{d_t}{r_t} + \frac{1}{r_t} \sum_{m=1}^{\infty} a_t^k (E_t d_{t+m+1} - E_t d_{t+m})$$

Assuming that the expected one-period change in dividends is a constant, c, we have:

$$p_t = \frac{d_t}{r_t} + \frac{c}{r_t^2} \tag{40}$$

Equation (40) is the basis for the cointegration tests between stock prices and dividends when interest rates are not assumed constant. The t-ratios of the unit root tests for the residuals of equation (40) when the interest rate used is first in real and then in nominal terms are presented below in Table 3.

TABLE 3 COINTEGRATION TESTS BETWEEN STOCK PRICES AND PROFITS WHEN INTEREST RATE IS VARIABLETABLE

Variable : Residuals	t-ratio on lag variable		Durbin-Watson	
	Levels	Changes	Levels	Changes
Real interest rates	-2.7249	-9.8011	1.9407	2.1373
Nominal interest rates	-2.1679	-4.6803	2.0310	1.9449

The critical value for the size of the sample used and for a 10% significance level is -2.57 according the table given by Fuller (1976) and reproduced by Yamamoto [41] (pp. 330). This means that, when real interest rates are allowed to fluctuate, the hypothesis that stock prices and profits were cointegrated cannot be rejected for that significance level for the period considered. However, the same does not happen when the interest rate used is in nominal terms: in this case the hypothesis that stock prices and profits were cointegrated can be rejected. The evidence seems thus somewhat inconclusive to whether stock prices followed fundamentals or included a bubble component.

3.5 TESTS FOR BUBBLES

3.5.1 Preliminaries

To test for the existence of rational bubbles, besides assuming the previous models for fundamentals and dividends as given by equations (7) and (22), let us remember the definition of rational bubbles given in equation (8):

$$p_t^b = p_t^* + b_t \tag{8}$$

with

$$E[b_{t+1} | I_t] = a^{-1} b_t \tag{9}$$

Also, the process for a stochastic bubble is given by:

$$b_{t+1} \begin{cases} = (aq)^{-1} b_t + e_{t+1} & p[b_{t+1}] = q \\ = 0 & p[b_{t+1}] = q-1 \end{cases} \tag{11}$$

As long as the bubble is on, the bubble part of the price grows at a rate:

$$g^t = (aq)^{-1} - 1 = \frac{1+r}{q} - 1 = r + (1+r)\frac{(1-q)}{q} > r \tag{41}$$

Because q is larger than zero and smaller than one the bubble part of the price grows at a rate larger than the riskless interest rate r. When the bubble bursts its value becomes zero. A simplifying assumption used here is that when the bursting occurs the bubble value immediately becomes zero implying at that point of time a negative infinite growth rate.

This obviously is probably not true of a real bubble whose value perhaps becomes zero after a certain number of trading sessions take place. When the bubble component of the stock price becomes zero the stock price is solely determined by the fundamentals and its growth rate by their growth rate because the expected growth rate of the bubble component becomes zero.

But as long as there is a bubble in stock prices the proportional changes in stock prices should be predictable for any given finite period. If the expected dividend grows at a constant rate as either of the above equations (18) and (22), then the proportional change in the stock price that, as in equation (8), includes both a fundamental and a bubble component should be:

$$\frac{\Delta p^{b}_{t+1}}{p^{b}_{t}} = g^{*}_{t}\frac{p^{*}_{t}}{p^{b}_{t}} + g^{b}\frac{b_{t}}{p^{b}_{t}} \tag{42}$$

Rearranging this equation,

$$\frac{\Delta p^{*}_{t+1}}{p^{b}_{t}} = g^{*}_{t}\frac{\left(p^{b}_{t} - b_{t}\right)}{p^{b}_{t}} + g^{*}\frac{b_{t}}{p^{b}_{t}} = g^{*}_{t} + \left(g^{t} - g^{*}_{t}\right)\frac{1}{1 + p^{*}_{t}/b_{t}} \tag{43}$$

According to equation (43) the rate of change of the stock price when it includes a bubble can be divided into two different parts. The first part is the rate of growth of fundamentals g_t (that should be equal to the rate of growth of dividends as we saw above). The second part is due to the bubble and itself has two parts: the first is the difference of the growth rates of the bubble component and fundamental component of price, g^b - g^*_t, and should have a positive expected value since $g^b > r$ (from equation (41))and the expected value of g_t is $g^d < r$ (due to the imposition of the transversality condition). The second part of the term due to the bubble is an increasing function of time because the ratio p^*_t/b_t is a decreasing function of time. The ratio of the fundamental price to the bubble part of the price is given by:

$$\frac{p_t^*}{b_t} = \frac{\left(1+g_t^*\right)^t}{\left(1+g^b\right)^t} \cdot \frac{p_0^*}{b_0} \tag{44}$$

with period zero being the first period when the bubble is on and p_0^*/b_0 being constant for all $t > 0$. As time goes to infinity, the ratio in equation (44) goes to zero because $g^d < g^b$ and the bubble part of the price eventually dominates the fundamental component in the stock price.

Equation (43) implies that, for any finite period when the bubble is on the proportional change in price is an increasing function of time. Consequently the proportional change in observed stock prices should be predictable from its own past values. This is not the case for the Nikkei 225 daily series between November 12,1987, and December 29, 1989 as we saw in the weak form efficiency tests in 3.2. But two more tests can be done to try to detect the existence of a possible bubble. They are time trend regressions and autoregressions.

3.5.2 Time trend regressions

From equation (43) we can see that if a bubble is on, the proportional increase in stock prices is an increasing function of time. The reason for this is that the growth rate of the bubble component of price is larger than the growth rate of the fundamental component of price, and the bubble component increasingly dominates the stock price. Whether a bubble is present or not in a series of stock prices can then be tested by regressing the proportional changes in stock prices on time to see if there is, say, a linear or quadratic relationship between those proportional changes and time. The existence of such a relationship would provide evidence in support of the existence of a rational bubble.

The following Table 4 provides the results of regressions of proportional changes in stock prices on time for three sample periods, the first two performed by Dwyer and Hafer [10] and the third by us. For each period two regression results are reported: one in which time enters only as a linear term and another that adds a quadratic term.

TABLE 4 TIME TREND REGRESSIONS

Author	Sample(13)	Const	Time	Time^2	R^2	DW	χ^2
Dwyer & Hafer	Jul 30 1986 Jun 6 1987	0.001 (0.52)	1.176×10^{-5} (0.80)		0.003	1.82	0.635
		0.001 (0.40)	9.648×10^{-6} (0.19)	9.406×10^{-9} (0.04)	0.003	1.82	0.636
Dwyer & Hafer	Jan 2 1986 Jun 6 1987	0.004 (1.80)	-7.80×10^{-6} (-0.22)		0.000	1.89	0.051
		0.004 (1.14)	-5.01×10^{-6} (-0.04)	-2.42×10^{-8} (-0.02)	0.000	1.89	0.052
Dos Santos	Nov 12 1987 Dec 29 1989	0.001 (1.86)	-6.67×10^{-7} (-0.35)		0.000	1.90	
		0.002 (2.05)	-1.07×10^{-5} (-1.25)	1.60×10^{-8} (1.20)	0.003	1.90	1.564

In all the three cases the regressions with time entered only as a linear function do not indicate the proportional change in stock prices to have any significant trend (for a 95% confidence interval the appropriate t-statistic is approximately 1.98). And adding a quadratic term does not alter this finding. In conclusion, for these three time periods, according to these time regressions tests, the hypotesis that a bubble was present can not be accepted.

3.5.3 Autoregressions

The presence of bubbles can also be tested by examining the results of autoregressions of the proportional change in stock prices. If bubbles are present then observed changes in stock prices should exhibit significant serial correlation. To test this Dwyer and Hafer [13] regressed, for two sample periods, the proportional change in stock prices on a constant and twenty five lagged values of that change. In addition we regressed for a third sample period the proportional change in stock prices on aconstant and seven lagged values of that same change. If the lag coefficients are not positive and significant we can reject the hypothesis that a bubble is present in the data tested.

The test statistics are reported in the following Table 5.

TABLE 5 AUTOREGRESSIONS

Author	Sample	Sum of Coefficients	χ^2
Dwyer & Hafer	Sep 3 1986 Jun 11 1987	$\sum_{i=1}^{25} \beta_i = -0.140$	24.001
Dwyer & Hafer	Jan 2 1987 Jun 11 1987	$\sum_{i=1}^{25} \beta_i = -1.108$	*41.819
Dos Santos	Nov 12 1987 Dec 29 1989	$\sum_{i=1}^{7} \beta_i = -1.07 \times 10^{-3}$	—

To test the hypothesis that autoregressive parameters are significant as a group χ^2 -statistics were calculated for the first two sample periods. Significance is denoted by the asterisk * . The autoregressive parameters are found to be significant as a group for the second sample period but the sum of coefficients is negative, contrary to the hypothesis being tested. For the other two samples, besides the sum of coefficients being negative they are not significant either individually and/or as a group (for more detailed results concerning the first two samples see Dwyer and Hafer [13], pp. 42 - 46, and for the third one see Dos Santos [12]).

In conclusion, the autoregressive tests realized are largely inconsistent with the hypothesis of presence of a rational bubble in the data samples tested.

3.5.4 Contagious bubbles

"All major stock markets began an impressive period of growth in 1982.(...) Foreign stock exchanges enjoyed bull markets similar to the U.S. during this period (1982-1987).(...) Increased by significantly improved computer and communications technology, cross-border equity investment increased rapidly during this period.(...) As cross-border investment grew, so did U.S. investors' sensitivity to foreign common stock performance. Investors made comparisons of valuations in different countries often using higher valuations in other countries as justification for investing in lower valued markets. Consequently, a process of ratcheting up among worldwide stock markets began to develop.(...) Trading on U.S. markets tended to lead other markets around the world. " (Brady Report [4], pp 9-10). This excerpt from the Brady Report on the crash in October 1987 argues that stock markets around the world moved together, a hypothesis that can be called "contagious bubble".

We can investigate the validity of this hypothesis by examining the joint behavior of stock prices in Japan and the U.S. To do this we start by assuming that all transaction costs are zero, that investors are risk neutral and that expectations are rational. Under these assumptions the expected return from holding common stock in any common currency must be the same everywhere. This implies the following "stock-return parity":

$$E\left[h_t^{us} | I_t\right] = E\left[h_t^j | I_t\right] + E\left[\Delta e_t | I_t\right] \qquad (45)$$

where h_t^{US} and h_t^J are respectively the rate of return from holding stock in the U.S. and in Japan in their own currency in period t, and Δe_t is the rate of change in the exchange rate between the dollar and the yen(17). Equation (45) can be written in expost terms as:

$$h_t^{us} = h_t^j + \Delta e_t + \varepsilon_t^{us} - \varepsilon_t^j - \varepsilon_t^e \qquad (46)$$

where the ε 's are the unexpected difference between the expected and actual values of h_t^{US}, h_t^J and e_t , and are normal independently distributed with $\varepsilon_t \sim$ iid $N(0, \sigma^2)$. Assuming further that dividends are zero we can write equation (46) in terms of the levels of stock prices as:

$$p_t^{us} - p_{t-1}^{us} = p_t^j - p_{t-1}^j + e_t - e_{t-1} + \varepsilon_t^{us} - \varepsilon_t^j - \varepsilon_t^e \qquad (47)$$

were the p's are the logarithms of stock prices, and e is the logarithm of the exchange rate. If we define a stock price relative as:

$$x_t = p_t^{us} - p_t^j - e_t \qquad (48)$$

equation (47) can be rewritten as:

$$x_t = x_{t-1} + \varepsilon_t^{us} - \varepsilon_t^j - \varepsilon_t^t \qquad (49)$$

Equation (49) states that the relative stock prices this period equal their value last period plus the unexpected part of the U.S. stock return (ε_t^{US}) minus the unexpected part of the Japanese stock return (ε_t^J) and minus the unexpected part of the change in the exchange rate (ε_t^e).

Equation (56) can be tested for the existence of a unit root using the following regression:

$$\Delta X_t = \alpha x_{t-1} + \varepsilon_t \tag{50}$$

were the ε's are assumed to be normal independently distributed. If the t-ratio for the estimated α is less in absolute value than the critical value of the appropriate t-statistic (1.98 for an interval of confidence of 95%), then we cannot reject the null hypothesis H_0: $\alpha = 0$. The existence of a unit root implies that returns from U. S. stocks and returns from Japanese stocks were roughly in line with each other, that is, relative gains in one market against the other in a trading day would imply losses in a following day so that the average of relative gains and relative losses would be zero and the relative stock prices would be approximately constant over time. If, for example, U. S. stocks were increasing at a faster rate than Japanese stocks, at a given constant exchange rate, x_t would increase over time and α would certainly be significantely different from zero[18]. Then, the assumption that dividends are zero for stocks in both countries and the existence of a unit root imply that U. S. stocks prices moved in line with Japanese stocks (and/or vice-versa). This means that if there was a bubble in Japanese stock prices then a bubble of the same magnitude should have been present in U. S. stock prices and vice-versa, and if there was not a bubble in Japanese stock prices a bubble should not have been present in U. S. stock prices (and vice-versa). On the other hand, from the no existence of a unit root we can only conclude that Japanese and U. S. stock prices did not move together.

Results of the t-ratio obtained by regressing the above equation (50) are given for three periods (the first, for all available values of the series, includes both the October 1987 crash and the first five weeks of the 1990 crash, the second goes from the minimum value for Japanese stock prices after the October 1987 crash to their maximum value before the 1990 crash, and the third is for the period around the 1990 crash) in the following Table 6.

TABLE 6 CONTAGIOUS BUBBLES

Sample	Estimated Coefficient	t-ratio	Significance
Sep 29 1987 Feb 5 1990	8.071×10^{-5}	0.7043	No
Nov 12 1987 Dec 29 1989	6.962×10^{-5}	0.8203	No
Oct 20 1989 Feb 5 1990	2.112×10^{-4}	1.2769	No

The above t-ratios were compared with the critical values of the t-distribution of 1.96 (the first two) and 2.00 (the third) (95% confidendence for ∞ and 60 degrees of freedom). These results show that we cannot reject the hypothesis that the stock prices in Japan and the U. S. moved together during the three periods tested. If there was a bubble in Japan then also there was one in the U.S. and vice-versa, and if there there wasn't any bubble in one country's stock prices then also there wasn't in the other country's.

4 CONCLUDING REMARKS

Although the rational bubble hypothesis presented in section 2 seemed to be able to offer an explanation to the volatility of stock prices in Japan, the tests performed in section 3 do not seem to offer it much support. Excess volatility was indeed found but so was the cointegration between stock prices and earnings. Besides, the time series of stock prices did not present the behavior that would be expected to be found if there was indeed a bubble. Instead, the excess volatility found can perhaps be explained by causes other than rational bubbles (for example, small changes in interest rates can explain large variations in stock prices; fads are also an alternative explanation).

NOTES

(1) I am grateful to the advice given by the Professors Takayoshi Kitaoka, Shuichi Komura and Koichi Maekawa. Needless to say that any mistakes are all of my making only.

(2) $\Sigma_j {}_{ij}q_t = \Sigma_j {}_{ij}q_{t-1}$ is a condition that ensures that the company i is not issuing new shares.

$\Sigma_i \Sigma_j {}_{ij}q_t = \Sigma_i \Sigma_j {}_{ij}q_{t-1}$ ensures that no new company is issuing new shares, that is, the total number of shares is constant.

(3) If individuals were not risk neutral as we assumed in sub-section "2.1 Assumptions of a Stock Market Model" the interest rate of the riskless asset would have to be adjusted with a risk premium. For a presentation along this line see Keizai Hakusho [23].

(4) If Ω is an information set and ω a subset of this information set, then for any variable y: $E[\, E[\, y \mid \Omega \,] \mid \omega \,] = E[\, y \mid \omega \,]$.

(5) For example, by John Kenneth Galbraith (in "The 1929 Parallel", "The Atlantic Monthly", January 1987). He writes: "In the months and years to the 1929 crash there was a wondrous proliferation of holding companies and investment trusts. The common feature of both the holding companies and the trusts was that they conducted no pratical operations; they existed to hold stock in other companies, and these companies frequently existed to hold stock in yet other companies. Pyramiding, it was called. The investment trust and the utility pyramid were the greatly admired marvels of the time. (...) [This leveraged process] meant that any increase in the earnings of the ultimate companies would flow back with geometric force to the originating company. (...) It was a grave problem, however, that in the event of failing earnings and values, leverage would work fully as powerfully in reverse."

(6) Sunspots were believed by the english economist S. Jevons to influence agricultural output. However, their meaning in economics nowadays is that of a variable

that affects equilibrium only because individuals believe it does.

(7) Common stock holders have residual claims on the assets of a company after all creditors (employees, suppliers, banks bond holders, etc) have been paid. These residual claims are expressed by the concept of net worth and their fundamental value is equal to the present value of expected future dividends.

(8) This does not happen with proprietorships and partnerships that have unlimited liability. "The owners' of proprietorships and partnerships typically have unlimited liability, which means that business creditors can look for repayment beyond the business entity's assets to the owners' personal assets. " (from Horngren and Sunders (1987), "Introduction to Financial Accounting").

(9) The efficient markets theory states that stock prices fully reflect all vailable information. Based on the type of information the efficient market theory has been subdivided into three categories. The first, called weak form, states that current prices include all the information that might be in past prices. Thus, weak form efficiency tests are tests of whether all information contained in historical prices is fully reflected in current prices. The second, called semi-strong form, states that current prices include all publicly available information. Finally, the called stong form states that all information, whether public or private is reflected in current stock prices. A market is said to be weak form, semi-strong form or strong form efficient if no excess profit can made by using past prices (technical analysis), publicly available information (fundamental analysis) or any kind of information, public or private (any kind of analysis).

(10) We had better say these hypothesis since, as said previously, we are making two hypothesis: (1) that stock prices are determined according to the fundamental solution (equation 7), and (2) that dividends follow the deterministic model of equation (18).

(11) This can be seen by substitution of the fundamental price p_t by its definition as given by equation (7) and the dividend's process specification of equation (22):

$$\eta^1_{t+1} = \frac{\eta_{t+1}}{p_t} \sum_{m=0}^{\infty} a^{m+1}\left(1+g^d\right)^m$$

$$= \frac{\eta_{t+1}}{\sum_{m=0}^{\infty} a^{m+1}\left(1+g^d\right)^m d_t} \sum_{m=0}^{\infty} a^{m+1}\left(1+g^t\right)^m = \frac{\eta_{t+1}}{d_t}$$

(12) The test for unit roots was performed for levels and changes: for the former, as explained, the first differences in the logarithms of stock prices (changes) are regressed on a constant, the lagged value of the logarithms of stock prices (levels) and the lagged value of the first difference in logarithms of stock prices; for the later, the second differences in logarithms of stock prices are regressed on a constant, the lagged first difference of the logarithms of stock prices and the lagged value of the second difference in the logarithms of stock prices, or:

$$\Delta^2 \ln_{t+1} = \alpha_0 + \alpha_1 \ln p_t + \alpha_2 \Delta^2 \ln p_t .$$

The test statistic is in the first case the reported t-ratio for the lagged level, and in the second case the reported t-ratio for the lagged first difference. For reference, if in the test for changes we omit the constant term α_0 the t-ratio would be -16.851 instead of -17.306.

(13) Sample period endpoints represent peaks in stock price indexes.

(14) Two or more, but for simplicity and because the case at hand involves only two variables, we will refer only to the two variables case.

(15) Equation (30) is the correct specification when $\mu \neq 0$, in equation (28). When there is no drift and $\mu = 0$ then $\alpha_0 = 0$ and the correct specification of equation (30) becomes: $p_t = \alpha_1 d_t + \varepsilon_t$.

(16) If $\beta = 1$ and $\delta = 0$ and dividends have a unit root, then equation (28) is $d_{t+1} = \mu + d_t + \varepsilon_{t+1}$. If we take expectations at time t we have that $E[d_{t+1} \mid I_t] = \mu + d_t$. Also if we take expectations on $d_{t+m+1} = \mu + d_{t+m} + \varepsilon_{t+m+1}$, we can have that $E[d_{t+m+1} \mid I_t] - E[d_{t+m} \mid I_t] = \mu$.

Substitution of these two results into equation (32) gives that

$$P_t^* = r^{-1}m + r^{-1}d_t + r^{-1}m \sum_{m=o}^{\infty} a^m$$

or

$$P_t^* = r^{-1}\mu + r^{-1}d_t + r^{-2}\mu$$

Making $\alpha_0 = r^{1}\ \mu + r^{2}\ \mu$ and $\alpha_1 = r^{1}$ we arrive at

$$p_t = \alpha_0 + \alpha_1 d_t + \varepsilon_t$$

If there no drift in equation (28), that is $\mu = 0$, then we have that

$$E[\, d_{t+m+1} \mid I_t\,] - E[\, d_{t+m} \mid I_t\,] = 0$$

and

$$p_t = \alpha_1 d_t + \varepsilon_t$$

as already said in note (15).

(17) We ignore the second-order term $h \cdot \Delta e$.

(18) If, for example, U.S. stock prices (P^{US}) were increasingat a faster pace than Japanese stock prices (P^J), we would have that (assuming a constant exchange rate):

$$\frac{P_t^{us} - P_{t-1}^{us}}{P_{t-1}^{us}} > \frac{P_t^{j} - P_{t-1}^{j}}{P_{t-1}^{j}}$$

and because $P_t/P_{t-1} = \ln (P_t/P_{t-1}) = \ln P_t - \ln P_{t-1} = p_t - p_{t-1}$, this inequality becomes:

$$p_t^{us} - p_{t-1}^{us} > p_t^{j} - p_{t-1}^{j}$$

or

$$p_t^{us} - p_t^{j} > p_{t-1}^{us} - p_{t-1}^{j}$$

or

$$x_t > x_{t-1}$$

what implies that $\alpha > 0$.

Inversely if Japanese stock prices were increasing at a faster rate than U.S. stock prices we would have that $\alpha < 0$.

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