

## 論文の要旨

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論文題目 Essays on the Whittaker-Henderson Method of Graduation  
(ウィットテイカー・ヘンダーソン平滑化法に関する諸論)

論文の要旨

The Whittaker-Henderson (WH) method of graduation is a smoothing tool in econometric time series analysis. This dissertation consists of three research papers on this method. It was first introduced by a German scholar George Bohlman (1899) where he used the first order differences for graduating data. Later, the method was well developed by Whittaker (1923) and Henderson (1924) separately and now it is popularly known as the WH method of graduation. There are two popular cases of WH method of graduation, first one is called exponential smoothing (ES) filter (King and Rebelo, 1993) or WH method of order 1 and the second one is popularly known as Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) or WH method of order 2. This dissertation contributes to the literature by providing explicit formulas for both the WH graduation of order 1 and 2. The dissertation consists of four chapters.

In chapter 1, the introductory survey of research, some preliminary definitions, examples and relevance methods are discussed.

In Chapter 2, to derive the explicit formula for the smoother weights of the WH graduation of order 1, we apply a different approach to that of

Cornea-Madeira (2017) and our approach leads to a simpler formula. Several theorems and lemmas are designed for the richer understanding of the filter.

In Chapter 3, we provide an alternative simpler formula for the HP filter and explains the reason why our approach leads to a simpler formula. By applying the Sherman-Morrison-Woodbury (SMW) formula and a discrete cosine transformation matrix, De Jong and Sakarya (2016) recently derived an explicit formula for the smoother weights of the HP filter. More recently, by applying the SMW formula and the spectral decomposition of a symmetric tridiagonal Toeplitz matrix, Cornea-Madeira (2017) provided a simpler formula. Here, we provide a simpler alternative formula for the smoother weights of the HP filter. A MATLAB code to find the smoother weights of the popular HP filter is included which guaranteed the efficiency of the proposed method. Wang et al. (2015), developed a method for deriving the explicit inverse of a pentadiagonal ( five-diagonal) Toeplitz matrix. Our approach may be regarded as an application of Wang et al. (2015).

In Chapter 4, based on the result of Yamada (2019), simple formulas for calculating the smoother matrix of the WH method is also provided. In addition, we show some results, which include that two other smoother matrices related with the WH graduation are also bisymmetric. The main purpose of this dissertation is to establish alternative methods to gain simpler formulas.