An impact of inferentialism on mathematics lessons

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1. Introduction

Inferentialism, a philosophy proposed by neo-pragmatist R. Brandom (cf. Brandom, 2000), has received attention in educational research. Many educational implications that differ from traditional views are drawn from this philosophy. In traditional educational philosophy, for example, it is argued that the rationality of spontaneous concepts is comparable to that of scientific concepts (Derry, 2008). Standard epistemologies tend to consider knowledge acquisition individualistic and not to pay attention to the social nature of knowledge as social norms (Derry, 2013). The two famous metaphors, *acquisition* and *participation*, proposed by Sfard (1998) are integrated and the third metaphor of *mastering* has been proposed (Taylor, Noorloos, & Bakker, 2017). In mathematics education research, it is argued that mathematical texts and contexts cannot be separated, and there is a need to question whether we can teach concepts by providing their definitions (Bakker & Derry, 2011). An inferentialist view of knowledge, i.e., a normative pragmatist view of knowledge, does not lapse into relativism (Noorloos, Taylor, Bakker, & Derry, 2017), unlike socio-constructivism proposed by Cobb and his colleagues (Cobb, 1994; Cobb, Stephan, McClain, & Gravemeijer, 2001; Yackel & Cobb, 1996). Authorities emerging in students' mathematical activities play an important role in constructing mathematical meanings (Seidouvy, Helenius, & Schindler, 2019); teaching notetaking risks strengthening teachers' control and limiting opportunities of students' creative reasoning (Nilsson, 2018). We acknowledge that inferentialism sheds light on overlooked aspects of educational phenomena and has many implications for education and educational research.

Some researchers also refer to an inferentialist framework in research on mathematical task design (Schindler & Joklitschke, 2016; Seidouvy & Eckert, 2017). However, the role of inferentialism in such research tends to be to describe and analyze classroom interactions between students engaging in designed mathematical tasks. The design principles of the mathematical tasks do not necessarily depend on inferentialism itself. The direct impacts of inferentialism on the ways of designing mathematics tasks and lessons, thus, have only been implicit. Considering the abovementioned implications from inferentialism, we can expect that fruitful implications are drawn from inferentialism for mathematical task and lesson designs.

The purpose of this paper is to elaborate on an impact of inferentialism on mathematics lessons.

2. Theoretical frameworks

Inferentialists consider themselves to be conceptual pragmatists rather than conceptual platonists.

An account of the conceptual might explain the *use* of concepts in terms of a prior understanding of conceptual *content*. Or it might pursue a complementary explanatory strategy, beginning with a story about the practice or activity of

applying concepts, and elaborating on that basis an understanding of conceptual content. The first can be called a *platonist* strategy, and the second a *pragmatist* (in this usage, a species of functionalist) strategy. […] [Inferentialism] is a kind of conceptual pragmatism (broadly, a form of functionalism) in this sense. It offers an account of knowing (or believing, or saying) *that* such and such is the case in terms of knowing *how* (being able) to *do* something. (Brandom 2000, p. 4, italics in the original)

Inferentialists think that explicitation produces understanding rather than that understanding produces explicitation.

[W]e might think of the process of expression in the more complex and interesting cases as a matter not of transforming what is inner into what is outer but of making *explicit* what is *implicit*. This can be understood in a pragmatist sense of turning something we can initially only *do* into something we can *say*: codifying some sort of knowing *how* in the form of a knowing *that*. (Brandom 2000, p. 8, italics in the original)

Reversing the traditional order of explanation about conceptual understanding, inferentialists find that human thoughts can be vague before making them explicit (Uegatani & Otani, in press).

The process of explicitation is to be the process of applying concepts: conceptualizing some subject matter. (Brandom 2000, p. 8)

Human thoughts, thus, become solid through making them explicit.

The conceptual pragmatist stance necessarily implies a holistic stance in terms of human concept use. Explicitation is considered a foreground process of conceptualization, while many applied concepts are still implicit in the background of the explicitation process.

[I]nferentialist semantics is resolutely *holist*. On an inferentialist account of conceptual content, one cannot have *any* concepts unless one has *many* concepts. (Brandom 2000, p. 15, italics in the original)

Inferentialists, hence, think that conceptualizing a particular concept never stops:

[U]nderstanding a concept […] (in relation to other concepts) is always a matter of degree: Users of a concept may have practical mastery of many inferences it is involved in but not all" (Bakker, 2018, p. 178)

Based on these philosophical stances, inferentialists have an interest in what people talk about:

The context within which concern with what is thought and talked *about* arises is the assessment of how the judgments of one individual can serve as reasons for another. The representational content of claims and the beliefs they express reflect the social dimension of the game of giving and asking for reasons. (Brandom 2000, p. 159, italics in the original)

In this game, people judge if the assertions are true and also judge what the true assertions are about. Our locutions "make the words 'of' and 'about' express the intentional directedness of thought and talk" (Brandom 2000, p. 169). Each participant in a discourse has their own intentionality and mutually assesses what each participant intends to talk about. For this reason, we should distinguish between different discourses when we focus on intentionality. In mathematics education research, we should distinguish between the observer's and the observed leaner's

intentionality since they participate in different discourses (Uegatani & Otani, 2019).

3. Avoiding confusion between an observer's and an observed learner's intentionality

The two inferentialist ideas of conceptualization as explicitation and the distinction between the observer's and the observed learner's intentionality imply that a mathematics lesson should be designed on the basis of consideration on *what the lesson is about from the perspective of the learner's intentionality*. For example, suppose that the teacher asks a learner to factor $x^2 + 5x + 6$ and that the learner answers $(x + 2)(x + 3)$. Then, the learner uses the concepts of factoring, exponents, multiplication, addition, and so on in the process of factoring. From an observer's perspective, this scene may be characterized as a situation where the observed learner learns factoring. However, from our inferentialist perspective, this scene is not such a situation. We as inferentialists think that the learner does not learn *about* factoring, though she uses the concept of factoring. In this situation, *what she actually talks about* is the expression $x^2 + 5x + 6$. She learns about the expression $x^2 + 5x + 6$ but does not learn about factoring itself. She can learn about factoring only if she intends to talk about factoring itself. For example, if the teacher asks her what factoring is, then she has an opportunity to learn about factoring.

This confusion between learning about the expression $x^2 + 5x + 6$ and about factoring itself stems from a confusion between the two perspectives of the observer's and the observed learner's intentionality. When the observer witnesses the learner factoring many quadratic expressions in the lesson, the observer may want to describe the lesson as about factoring. However, this description is an over-abstraction by the observer. Since the observer wants to summarize her observation, she tends to report her observation in an abstract manner. As Mason (2011) cautions in his discussion about roles of theories, "[t]o express is […] to over-stress" (p. 2490). The learner does not necessarily recognize the significance of the exercises she practices nor describe her own experiences in the same vein as the observer does.

Taking a holistic stance in concept use, inferentialists think that a learner always and implicitly uses many concepts. Nevertheless, she does not learn about most of them even when she uses them. She only learns about what she intends to talk about. For this reason, a lesson about a mathematical concept should provide students opportunities to intentionally talk about the concept. Without this discussion, the lesson is not about the concept from the perspective of the learner's intentionality.

One of the easiest ways of making students form their own intentions may be questioning what the day's lesson is about. For example, if they characterize it as a lesson about factoring, then we can think that they learn something about factoring. However, of course, what they learn about factoring is not trivial. In addition, they do not necessarily characterize the lesson as a lesson about factoring. They may characterize it as a lesson about the fact that math is boring. It is not easy for us to make students identify an expected intention. Reflection on the day's lesson does not always succeed in making them view the lesson from the observer's perspective.

Note that we do not intend to deny taking an observer's perspective when analyzing what a lesson is about. We would like, rather, to argue the necessity to avoid confusing the observer's and the observed learner's perspectives. Indeed, as radical constructivists argue (cf. von Glasersfeld, 1990, 1995), the observer's thought of what a learner thinks from a learner's perspective is constructed from a kind of observer's perspective. The observer cannot avoid taking her own perspective. Rather, we must understand the existence of the observer's intentionality (Uegatani &

Otani, 2019). We should not deny the observer's intention itself to summarize the essence of the observed lesson. We can only criticize a gap between her original intention and her actual expressions. If her original intention is not to summarize what the students learn about in the lesson, then it can be valid that she characterizes a lesson including factoring many quadratic expressions as a lesson about factoring.

We should always distinguish between the observer's and the observed learner's intentionality and try to make the learner have her own intention. However, it is not easy to make her have our intended intention. In the next section, we will discuss a promising way of attempting to do this from an ethical perspective.

4. Encouragement and forbiddance
4.1. From an ethical point of view

Ernest (2012) discusses our first philosophy in mathematics education and argues the importance of ethics. In his argument, he emphasizes the ethical consideration of the relationship between the self and the other:

> The other is not a phenomenon but an enigma, *something ultimately refractory to intentionality* and opaque to understanding. If we could fully comprehend the other, we could reduce it to an object. The other has a right to be herself, unlimited by our expectations and understanding.
>
> (p. 13, italics added)

Ethics is our first philosophy. All of our educational practices should be done with ethics. From this point of view, we as mathematics teachers cannot control and should not try to control students' intentionality. There is, hence, an inferentialist dilemma: students must have their own intention to talk about what they are expected to learn about, on the one hand; the teacher cannot control their intentions from an ethical point of view. This dilemma should not be discussed negatively, however. We adopt a positive tone. If the teacher respects students' intentionality, then she can grasp what they learn in lessons at a moment.

To discuss how we can respect students' intentionality, it is worth mentioning the classic distinction between instrumental and relational understanding by Skemp (1976). Superficial understanding of how to execute mathematical procedures without their essential meanings is considered instrumental, and deep understanding of why such procedures are correct is considered relational. Following this distinction, we had traditionally thought that superficially correct students' behaviors were meaningless and insufficient. If a student only obeys mathematical rules, then she seemingly does not intend to behave in such a way. However, adopting a positive tone, we can interpret it in a different way. From an inferentialist perspective, the meanings of words are determined in terms of its inferential role. Completely meaningless expressions cannot exist in principle. Thus, even a result of uncritically obeying given rules has its own meaning, that is, its inferential role, even though it is less meaningful than rules with relational understanding.

It is noteworthy that words do not have their literal meanings. Suppose that we sing a song written in an unfamiliar foreign language in a music lesson. Then, we might be unable to sufficiently fulfill our own intention to sing the song with its literal meaning because of the lack of appropriate understanding of the language. Instead, we can intend to sing in tune, for example. Our music teacher judges whether or not we hit the right tones. Our teacher instructs how to sing better if we are not in tune. Her instruction is not a mechanical response but a result of conceptual

judgement on how she instructs better. Thus, our songs as our expressions do not have their literal meanings, but inferential meanings. In fact, they cause our teacher's conceptual reasonings and her next intentional instruction. After listening to her advice, we reflect on our previous ways of singing the song and try to sing it better the next time. Our conceptual judgement influences our future singing, to a greater or lesser extent. Our voices also have inferential meanings for us. From our inferentialist perspective, there is no expression corresponding to platonic existence. As long as expressions cause the next inferences of the participants in a discourse, they always have their inferential meanings.

4.2. Inferential freedom

A practical implication can be drawn from the two abovementioned claims that students' intentionality is ethically uncontrollable and that there is no inferentially meaningless expression: *increasing students' degrees of inferential freedom, first; decreasing them, second*. We will validate this derivation in this subsection.

There is a delicate relationship between inferential freedom and inferential meaningfulness. Let us raise three cases so that we may feel that expressions are less meaningful. First, if a student can derive any claim from a given expression, then there is virtually no conceptual contribution of the expression to her inference. We can see such an expression as less meaningful. A contradiction is a typical example of this case. Second, if a student always derives the same claim from any number of given expressions, then those expressions do not conceptually contribute to her inference well. We can also see such expressions as less meaningful. For example, suppose that whenever a student sees mathematical terms, she claims that she hates math. Then, those terms are less meaningful for her. They play a too-strong inferential role of recalling her bad experiences to her. Third, suppose that a student encounters a particular type of expression. Suppose also that she can only obey some rigid rule and cannot flexibly cope well with unexpected affairs in the situation; then this type of expressions also has a too strong inferential role. We can also see such types of expressions less meaningful. This is a case of instrumental understanding. That is, if the teacher suddenly and unexpectedly asks the student why such procedures are applicable, then the student cannot cope with the question. As these examples show, the inferential role of an expression should be neither too weak nor too strong. There should be an appropriate degree, and such a degree provides meaningfulness.

In fact, emphasizing the degree of inferential freedom is consistent with value scholars in mathematics education has placed on flexibility. For example, flexible operational understanding is considered an indicator of better mathematical performances (Gray & Tall, 1994; Tall, 2011). Professional mathematicians' interchangeable uses of the two terms *concepts* and *objects* are also considered flexible rather than ambiguous (see a debate between M. Inglis and D. Tall; cf. Inglis, 2003; Tall, 2004). Recent reconceptualization and reappraisal of flexible procedural knowledge in problem solving by J. Star and his colleagues has been supported our claims (Star, 2005, 2007, 2018; Star & Stylianides, 2013). From our inferentialist perspective, procedural knowledge is considered a kind of conceptual knowledge, and the distinction between these two types of knowledge disappears.

Based on this reconceptualization of the distinction between procedural and conceptual knowledge, we can argue that the degree of inferential freedom reflects on social acceptability of the consequences of inferences, that is, social norms. For this reason, we should emphasize that establishing appropriate classroom norms is one of mathematics teachers' roles. Respecting students' intentionality, a mathematics teacher should temporarily accept that her students infer from an expression presented in the lesson to any expression except when they behave in a

remarkably unethical manner. It seems to be difficult for students to freely infer in an environment that strongly encourages them to behave only in a normative way. The teacher should, thus, try to increase her students' degrees of inferential freedom, first. This teacher's attitude is also expected to encourage her students' creativity since it prevents them from responding in a stereotypical way. However, keeping an attitude toward accepting any expression by the students is liable to lead to the establishment of a too-weak social norm. As discussed above, the inferential role of an expression should be neither too weak nor too strong. Therefore, the teacher should try to decrease her students' degrees of inferential freedom, second.

This practical implication requires mathematics teachers to behave in their classrooms in a delicate manner. Social acceptability of the consequences of inferences is socially constructed by the classroom members, that is, not only the teacher but also all the students in the classroom. We can never introduce such social norms from outside of the classroom (Yackel & Cobb, 1996). If the teacher neglects this fact and forcibly introduces explicit social norms, then her students' degrees of inferential freedom will seriously decrease. For example, a mathematics teacher should not ask her students not to be afraid to be wrong during the classroom discussion. This requirement contradicts the fact that she actually places value on correct mathematical answers. Since her students usually want to be correct, they necessarily are afraid to be wrong. Thus, if the teacher wants to ask so, then she must stop putting value on mathematically correct answers. However, it is not a realistic solution. In this paper, we recommend that the teacher interprets her students' apparently wrong answers as potentially correct ones. As constructivists have strongly asserted (cf. Confrey, 1991), students often behave correctly from their limited perspectives. Therefore, we would like to argue that both quickly identifying how limited their perspectives are from their mathematically wrong responses during classroom discussion and providing a special and different value to their wrong responses from the typical value of mathematical correctness are the teacher's very important roles. The teacher should make explicit what her students should do, by illustrating an actual and concrete student's behavior, rather than make explicit what her students should not do. This explicitation helps her students to intend to behave in a normative way.

5. Conclusion

To elaborate an impact of inferentialism on mathematics lessons, we argued that a mathematics teacher should try 1) to identify what her students actually talk about in order to grasp what they conceptualize, 2) to realize a classroom environment to accept appropriate degrees of inferential freedom, and 3) to find her students' mathematically wrong answers valuable from a different perspective and to orient them toward aiming at socially established norms and intentionally behaving in such normative ways. However, these are still only potential recommendations. The following questions are still open, for example: What degree of inferential freedom is appropriate? What is the value of mathematically wrong responses? How are classroom norms established through explicitation from an inferentialist perspective? Further theoretical discussions and further empirical research are needed to better understand how inferentialism impacts mathematics lessons.

Acknowledgements

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数学の授業への推論主義の影響

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1. 序論

　教育研究において, 推論主義が注目を集めてきている. 推論主義は, タスクデザインの研究においても参照されている (Schindler & Joklitschke, 2016; Seidouvy & Eckert, 2017). しかしながら, そうした研究における推論主義の役割は, 主に教室での相互作用の記述や分析にあり, タスクそれ自体のデザイン原理として推論主義が機能しているわけではない. そこで本稿では, 数学のタスクや授業をデザインする上で, 推論主義がどのような直接的影響を有するかを示す. 本稿の目的は, 数学の授業への推論主義の影響の精緻化にある.

2. 理論的枠組

　推論主義は, 概念的プラトニストを否定し, 概念的プラグマティストの立場に立つ. それは, 理解によって思考の明示化が可能になると考える立場ではなく, 思考の明示化によって理解が生み出されるとする立場である. 人間の思考は, 明示化されるまでは曖昧であり得るのである (Uegatani & Otani, in press). 概念的プラトニズムは, 不可避的に概念使用の全体論を採用する. 明示化されていなくとも, 人は推論において様々な概念を使用している. それゆえ, 推論主義者にとって, 人が何について話しているかは重要である. ある表現が何についての表現であるか, すなわち, 何を志向しているかは, 志向性 (intentionality) と呼ばれる. 表現者の立場から見れば, 意図 (intention) を持つということである.

3. 観察者の志向性と観察される学習者の志向性との混同を避ける

　推論主義のアイディアは, その授業が学習者の志向性という観点から見て, 何についての授業であるかを考慮して, 授業をデザインすべきであると示唆する. 例えば, ある学習者が, $x^2 + 5x + 6$ を因数分解せよ, と問われて, $(x + 2)(x + 3)$ と答えたとしよう. このとき, その学習者は, 因数分解, 指数, 乗法, 加法などを利用している. 観察者の視点からは, この場面は, 学習者が因数分解を学習している状況として特徴付けられ得る. しかし, 推論主義の視点からは, この場面はそのような状況ではない. 推論主義者は, たとえ因数分解の概念を使用していたとしても, 学習者は因数分解について学んでいないと考える. この状況において, その学習者が実際に何について話しているかと言えば, $x^2 + 5x + 6$ という数式についてである. 彼女は, この数式について学んだが, 因数分解それ自身については学んでいない. 彼女は, 因数分解について話す意図を持つときに限り, 因数分解について学ぶことができる. 例えば, 教師が因数分解とは何かとその学習者に問うならば, その学習者は因数分解について学ぶ機会を得る.

　これは, 観察者視点で授業を要約することを否定するものではない. むしろ, 観察者は, 観察者視点で観察した授業について話をしているということを, 意識すべきである (Uegatani & Otani, 2019).

4．奨励と禁止

4.1．倫理的視座から

Ernest (2012) の議論に基づけば，倫理的観点からは，我々は他者をコントロール可能だと思うべきではない．他者は，各々志向性を有している．学習者の志向性も，コントロールしようとしてはならない．

また，あらゆる表現は，意味を持つ．完全に無意味な表現は存在しない．例えば，馴染みのない言語の歌を歌うときでさえ，我々は決まった音程を出したいという意図を持つことができる．表現が，字義通りの意味を有するかどうかはともかくとして，表現は，それが誰かの次の概念的推論を誘発する限り，わずかながらでも意味を有する．

4.2．推論的自由度

生徒達の志向性は倫理的にコントロール不可能であり，推論的に無意味な表現は存在しない，という上述の主張から導かれる実践的示唆は，第一に生徒達の推論的自由度を増大させ，第二にそれらを減少させる，というものである．

推論的自由度と推論的有意味性の間には，デリケートな関係がある．ある表現がより有意味であるためには，その表現から次に導き得ることが，適度に豊かである必要がある．何でも導かれ得るようであれば，それは矛盾であるし，決まった推論しか生み出さないのであれば，それはそれで理解の仕方が貧しい (例えば，道具的理解は，how にしか答えられず why には答えられないような，決まった推論しか生み出さない硬直的な理解の仕方である).

推論的自由度の強調は，手続き的知識と概念的知識の間の再概念化を唱導する近年の Star らの研究と整合的である (Star, 2005, 2007, 2018; Star & Stylianides, 2013)．推論主義の視座からは，手続き的知識と概念的知識の区別は不要である．

こうした考察に基づくと，社会的規範の強さが，推論的自由度を低下させる可能性を指摘することができる．すなわち，禁止の指導は，生徒達の振る舞いの自由度を低下させる．つまり，推論的自由度を低下させる．まずは何をやっても大丈夫だという自由度を与えなければならない．しかし，その一方で，常に何でもありでは，学習にならない．一旦自由度を増大させておき，次に，数学的な正しさを指導していくことが重要である．このように，禁止の指導が推論的自由度を低下させると危惧するならば，数学の教師は，「間違いを恐れるな」と生徒に指導すべきでないかもしれない．正解したいと思う以上，必然的に間違いは恐れられる．正解することを褒めるのをやめない限り，それは一貫性を欠いている．しかし，それは現実的ではない．そこで本稿では，教師が，生徒の間違いが潜在的な正解である可能性を取り上げていくべきであると推奨したい．生徒の間違いは，数学的に間違いであるが，数学的な正解以外の価値観から価値付けることができるかもしれない．何かを禁止するのではなく，何かを具体的に奨励することが重要である．

結論

数学の授業への推論主義の影響を精緻化すべく，我々は次の 3 点を主張した．1) 数学の教師は，ある生徒が概念化していることを把握するために，その生徒が実際に何について話しているのかを同定する努力をすべきである．2) 数学の教師は，適切な推論的自由度を許容する教室環境を実現するよう努力すべきである．3) 数学の教師は，生徒の数学的に誤った答えを，異なる視座から価値付け，生徒達を社会的に確立された規範を目指し，そうした規範的方法で意図的に振る舞おうとするよう仕向けていくべきである．しかしながら，これらはまだ潜在的に有用な推奨に留まっている．推論主義が授業にどのような影響を及ぼすかについて，経験的研究が必要である．