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Non summability of formal solutions of certain partial differential equations
(ある偏微分方程式の形式解の非総和可能性)

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主論文

Non summability of formal solutions of certain partial differential equations

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1 Introduction

In this paper, we shall study the non-summability for partial differential equations. There are many results for the summability. Lutz-Miyake-Schäfke [3] studied the Borel summability of divergent formal solutions of the heat equation. Ōuchi [6], [7] studied the multisummability of formal solutions of nonlinear partial differential equations. Tahara-Yamazawa [8] studied the multisummability of a formal solution of a certain class of partial differential equations. Malek [4] studied the summability of formal solutions of linear partial differential equations. Hibino [1] studied the Borel summability of divergent solutions for singular first order linear partial differential equations with polynomial coefficients. In [2], the author gave an example such that a divergent formal power series solution is not 1-summable in any direction. The object of this paper is to give the examples such that a divergent formal power series solution is not 1-summable in any direction.

In order to state our result we introduce some notation. Let $(t, x) \in \mathbb{C}\times\mathbb{C}$. For $r > 0$ we write $D_r = \{t \in \mathbb{C} |\, |t| < r\}$.

Let $\{C_m\}_{m=1}^{\infty}$ be a sequence of numbers such that $\sum_{m=1}^{\infty} |C_m| < \infty$ and $\{h_m\}_{m=1}^{\infty}$ be a sequence of bounded complex numbers. We define $a(x)$

$$a(x) = \sum_{m=1}^{\infty} C_m e^{h_m x}. \tag{1.1}$$

Let $n, k \in \mathbb{N}^*(\mathbb{N}^* = \{1, 2, 3, \dots\})$. For $j = 1, 2, \dots, n$ we define the polynomial of ξ, $P_j(\xi)$ by

$$P_j(\xi) = \sum_{i=1}^{k} \alpha_{i,j}\xi^i, \tag{1.2}$$

where $\alpha_{i,j} \in \mathbb{C}$. We consider the following Cauchy problem

$$\begin{cases} \dfrac{\partial}{\partial t}u = a(x)t + \displaystyle\sum_{j=1}^{n}\left(t^2\frac{\partial}{\partial t}\right)^j \frac{\partial}{\partial t}P_j(\partial_x)u \\ u(0, x) = 0. \end{cases} \tag{1.3}$$

One can easy see that (1.3) has a formal power series solution in t of the form

$$u(t, x) = \sum_{n=1}^{\infty} u_n(x)t^n, \tag{1.4}$$

where $u_n(x)$ is analytic on D_r for some $r > 0$ and continuous up to the boundary for any $n \in \mathbb{N}^*$. Then we have

Theorem 1.1. Assume that $\alpha_{k,1} \neq 0$. Then there exists a bounded sequence $\{h_m\}_{m=1}^{\infty}$ such that, for $a(x)$ in (1.1) any formal solution (1.4) of (1.3) is not 1-summable in any direction.

Next we study the non-summability of formal solution of the next equation. Consider the Cauchy problem

$$\begin{cases} \dfrac{\partial}{\partial t} u(t,x) = a(x) + \left(t\dfrac{\partial}{\partial t}\right)^2 u(t,x) + \left(t\dfrac{\partial}{\partial t}\right)^2 \partial_x^{\alpha} u(t,x) \\ u(0,x) = \varphi_0(x), \end{cases} \tag{1.5}$$

where $\alpha \in \mathbb{N}^*$ and $a(x)$ and $\varphi_0(x)$ are entire functions. Equation (1.5) has a formal solution (1.4). Then we have

Theorem 1.2. There exists a bounded sequence $\{h_n\}_{n=1}^{\infty}$ such that every formal solution (1.4) of (1.5) is not 1-summable in any direction.

Theorem 1.2 means that we give a simple example of a non-summable partial differential equation which does not satisfy the sufficient condition for the 1-summability given in [8].

This paper is organized as follows. In Section 2, we give some definitions and notation. In Section 3, we show the Gevrey estimate of formal solution of equation 1.3. In Section 4, we prove Theorem 1.1. In Section 5, we show the Gevrey estimate of formal solution of (1.5). The proof of Theorem 1.2 is given in Section 6.

2 Definitions and notation

We give some definitions and introduce symbols necessary for the proof of main theorem.

Let $\mathbb{C}[[t]]$ be the ring of formal power series in t with coefficients in $\mathbb{C}$. We say that formal power series $\hat{f}(t) = \sum_{n=0}^{\infty} f_n t^n \in \mathbb{C}[[t]]$ has Gevrey order $1/k > 0$ if there exists $C > 0$ such that, for all $n \geq 0$

$$|f_n| \leq C^{n+1} \Gamma\left(1 + \frac{n}{k}\right),$$

where $\Gamma(x)$ is a Gamma function. We denote by $\mathbb{C}[[t]]_{1/k}$ the set of all formal power series with Gevrey order $1/k$.

A sector on the Riemann surface of the logarithm is the set of the form

$$S \equiv S(d,\alpha,\rho) := \left\{ re^{i\theta} \middle| |\theta - d| < \frac{\alpha}{2}, 0 < r < \rho \right\},$$

where d is an arbitrary real number, α is a positive real and ρ is positive real or ∞. In case $\rho = \infty$, we mostly write $S(d, \alpha)$:

$$S(d,\alpha) \equiv S(d,\alpha,\infty) := \left\{ re^{i\theta} \middle| |\theta - d| < \frac{\alpha}{2}, 0 < r \right\}.$$

A closed subsector $\bar{S}_1$ of S is the set of the form

$$\bar{S}_1 \equiv \bar{S}_1(d', \alpha', \rho') := \left\{ re^{i\theta} \middle| |\theta - d'| \leq \frac{\alpha'}{2}, 0 < r \leq \rho' \right\},$$

where $0 < \alpha' < \alpha$, $0 < \rho' < \rho$, $|d - d'| < \alpha/2 - \alpha'/2$.
Given $k > 0$, we say that a function f being holomorphic in a sector S asymptotically equals $\hat{f}(t) \in \mathbb{C}[[t]]_{1/k}$ or $\hat{f}$ is the asymptotic expansion of Gevrey order $1/k$ of f if, for every closed subsector $\bar{S}_1$ of S there exists $C > 0$ such that for every $N \geq 0$ and every $t \in \bar{S}_1$

$$|r_f(t, N)| \leq C^{N+1} \Gamma\left(1 + \frac{N}{k}\right),$$

where $r_f(t, N) = t^{-N} \left(f(t) - \sum_{n=0}^{N-1} f_n t^n \right)$. In such a case, we write, for short, $f(t) \cong_k \hat{f}(t)$. We denote by $A_k(S)$ the set of all holomorphic functions on S having an asymptotic expansion of Gevrey order $1/k$.

Definition 2.1. For a formal power series $\hat{f}(t) \in \mathbb{C}[[t]]_1$ without constant term, the formal Borel transform $\hat{B}_1 \hat{f}$ is defined by

$$\hat{B}_1 : \hat{f}(t) = \sum_{n=1}^{\infty} f_n t^n \mapsto f(\xi) = \sum_{n=1}^{\infty} f_n \frac{\xi^{n-1}}{\Gamma(n)}. \tag{2.1}$$

Definition 2.2. For $\hat{f}(t) \in \mathbb{C}[[t]]_1$, we say that $\hat{f}(t)$ is 1-summable in the d-direction if there exists an $\epsilon > 0$ such that $\hat{B}_1 \hat{f} \in A_1(S(d, \epsilon))$ with exponential size at most 1. We denote by $\mathbb{C}\{t\}_{1,d}$ the set of all formal power series that are 1-summable in the d-direction. We say that a function $f(t)$ on $S(d, \epsilon)$ has exponential size at most 1 on $S(d, \epsilon)$ if, for every subsector $\overline{S}_1$ in $S(d, \epsilon)$ there exist $C > 0$ and $h > 0$ such that

$$|f(t)| \leq Ce^{h|t|} \quad (t \in \overline{S}_1).$$

3 Estimate of formal solution

In this section, we show the Gevrey estimate of formal solution of (1.3).

Proposition 3.1. Let $a(x)$ be given by (1.1), and let $h > 0$ satisfy that $|h_m| \leq h$ for any $m \in \mathbb{N}^*$. Then, for every $\ell \in \mathbb{N}^*$ we have $u_\ell(x)$ uniquely. Moreover, we have the estimate: there exist $K > 0$ and $M > 0$ such that, for any $\ell \in \mathbb{N}^*$ and $x \in \mathbb{C}$

$$|u_\ell(x)| \leq \frac{(\ell-1)!}{\ell} M^\ell K e^{h|x|}. \tag{3.1}$$

Proof Substituting (1.4) into (1.3) we get

$$\sum_{m=1}^{\infty} u_m(x) m t^{m-1} = a(x)t + \sum_{j=1}^{n} \left(\sum_{m=2}^{\infty} P_j(\partial_x) u_m(x) \frac{m(m+j-2)!}{(m-2)!} t^{m+j-1} \right).$$

Compare the coefficients of both sides about the power of t, to obtain

$$\begin{aligned}
u_1(x) &= 0, \quad u_2(x) = a(x), \\
u_3(x) &= \frac{1}{3} P_1(\partial_x) u_2 = \frac{1}{3} P_1(\partial_x) a(x), \\
u_4(x) &= \frac{1}{4} \{6P_1(\partial_x) u_3(x) + 4P_2(\partial_x) u_2\} \\
&= \frac{1}{4} \left\{2P_1(\partial_x)^2 a(x) + 4P_2(\partial_x) a(x)\right\}, \\
&\vdots \\
u_\ell(x) &= \frac{1}{\ell} \left\{ \sum_{m+j=\ell, j\geq 1} P_j(\partial_x) u_m(x) \frac{m(\ell-2)!}{(m-2)!} \right\}.
\end{aligned} \tag{3.2}$$

Therefore, we have the formal solution uniquely.
By assumption, there exists $K > 0$ such that for every $j = 1, 2, \ldots, n$ and any $x \in \mathbb{C}$

$$|P_j(\partial_x) a(x)| \leq K e^{h|x|}.$$

We prove (3.1) by induction on l. The case $l = 1$ is trivial. Suppose now that (3.1) holds up to $l - 1$. Because the sum with respect to j in (3.2) is a finite sum and u_l is a function of $a(x)$, (3.1) is obtained. □

4 Proof of Theorem 1.1.

Set $(\partial u/\partial t) =: v$. Then, (1.3) is written as follows

$$v = a(x)t + \sum_{j=1}^{n} \left(t^2 \frac{\partial}{\partial t} \right)^j P_j(\partial_x) v.$$

Applying the formal Borel transform to both sides and by setting $\hat{B}_1(v) = \hat{w}$ we have

$$\hat{w} = a(x) + \sum_{j=1}^{n} \tau^j P_j(\partial_x)\hat{w}, \tag{4.1}$$

where we used

$$\hat{B}_1 \left\{ \left(t^2 \frac{\partial}{\partial t} \right)^j P_j(\partial_x) v \right\} = \tau^j P_j(\partial_x) \hat{B}_1 (v) .$$

If we set

$$\hat{w} = \sum_{m=1}^{\infty} \hat{w}_m(\tau) e^{h_m x},$$

then we have

$$P_j(\partial_x)\hat{w} = \sum_{m \geq 1} \hat{w}_m(\tau) P_j(h_m) e^{h_m x}. \tag{4.2}$$

Substituting (4.2) and the expansion of $a(x)$ into (4.1) we have

$$\begin{aligned} \hat{w}_m(\tau) &= C_m + \sum_{j=1}^{n} \tau^j P_j(h_m)\hat{w}_m(\tau) \\ &= C_m + \sum_{j=1}^{n} \tau^j \sum_{i=1}^{k} \alpha_{i,j} h_m^i \hat{w}_m(\tau) \end{aligned}$$

for every $m \in \mathbb{N}$. Therefore, we have

$$\hat{w}_m(\tau) = \frac{C_m}{1 - \sum_{j=1}^{n} \tau^j \sum_{i=1}^{k} \alpha_{i,j} h_m^i} \tag{4.3}$$

for every $m \in \mathbb{N}$.

Because $\alpha_{k,1} \neq 0$, we take $\epsilon > 0$ sufficiently small such that

$$\left| \sum_{j=1}^{n} \tau^j \alpha_{k,j} \right| > |\alpha_{k,1}|\epsilon/2 \tag{4.4}$$

on $|\tau| = \epsilon$. Define

$$F(\tau, h) := \sum_{j=1}^{n} \tau^j \sum_{i=1}^{k} \alpha_{i,j} h^i - 1. \tag{4.5}$$

Take a countable infinite set $T = \{T_m\}_{m=1}^{\infty}$ which is dense on $|\tau| = \epsilon$. For each $\tau_m \in T$, consider the equation $F(\tau_m, h) = 0$. Take one root arbitrarily and put it by h_m. Then $H := \{h_m\}_{m=1}^{\infty}$ is a bounded set. Indeed, every coefficient of h^i in $F(\tau_m, h)$ is uniformly bounded when $\tau_m \in T$. Moreover, the coefficient of the highest power, h^k, $\sum_{j=1}^{n} \tau_m^j \alpha_{k,j}$ is uniformly bounded from the below by $|\alpha_{k,1}|\epsilon/2$ by (4.4). Because the solution h of the algebraic equation $F(\tau_m, h) = 0$ is a continuous function of the coefficients of the equation, it follows that H is a bounded set.

We shall show that there exist constants $\delta_1 > 0$ and $0 < \epsilon_1 < \epsilon$ such that, for every $h_m \in H$

$$|F(\tau, h_m)| \geq \delta_1 \quad \text{on } |\tau| < \epsilon_1.$$

Suppose that this is not true. Then, for every $\nu = 1, 2, \ldots$ there exist a positive integer $m(\nu)$ and τ_ν such that

$$|F(\tau_\nu, h_{m(\nu)})| < 1/\nu, \quad |\tau_\nu| < 1/\nu.$$

On the other hand, by (6.4) we have

$$F(\tau_\nu, h_{m(\nu)}) = \sum_{j=1}^{n} {\tau_\nu}^j \sum_{i=1}^{k} \alpha_{i,j} (h_{m(\nu)})^i - 1.$$

Because τ_ν tends to 0 as ν tends to infinity and H is a bounded set, the right-hand side does not tends to zero. This is a contradiction. By the above definition of $T = \{\tau_m\}$ and $H = \{h_m\}$, the formal Borel transform $\hat{w}(\tau, x)$ has pole singularities at each point in $T = \{\tau_m\}$ which is dense on $|\tau| = \epsilon$. Therefore u is not 1-summable in any direction. This completes the proof.

5 Estimate of formal solution

In this section, we show the Gevrey estimate of formal solution of (1.5).

Proposition 5.1. Assume that $a(x)$ is of exponential size at most 1. For $m \in \mathbb{N}^*$, let $u_m(x)$ be a coefficient function of formal power series solution in t of (1.5). Then, for any $m \in \mathbb{N}^*$ we have

$$u_m(x) = \frac{(m-1)!}{m} \left(\sum_{j=0}^{m-1} \binom{m-1}{j} \partial_x^{\alpha j} a(x) \right).$$

Moreover, we have the estimate: there exist $K > 0$ and $h > 0$ such that, for any $m \in \mathbb{N}^*$ and $x \in \mathbb{C}$

$$|u_m(x)| \leq \frac{(m-1)!}{m}\,(1+e^{h\alpha})^{m-1}\,K\,e^{h|x|}.$$

Proof Substituting (1.4) into (1.5) we have

$$\sum_{j=1}^{\infty} u_j(x)\,j\,t^{j-1} = a(x) + \sum_{j=1}^{\infty} u_j(x)\,j^2\,t^j + \sum_{j=1}^{\infty} \partial_x^\alpha u_j(x)\,j^2\,t^j.$$

Compare the coefficients of both sides about the power of t, to obtain

$$\begin{aligned}
u_1(x) &= a(x)\\
u_2(x) &= \frac{1}{2}\,(a(x) + \partial_x^\alpha a(x))\\
&\vdots\\
u_m(x) &= \frac{(m-1)^2}{m}(u_{m-1}(x) + \partial_x^\alpha\,u_{m-1}(x))\\
&= \frac{(m-1)^2}{m}(1+\partial_x^\alpha)u_{m-1}(x).
\end{aligned}$$

Solving the recurrence relation we have

$$u_m(x) = \frac{(m-1)!}{m}\left(\sum_{j=0}^{m-1}\binom{m-1}{j}\,\partial_x^{\alpha j} a(x)\right), \qquad (m \geq 1). \tag{5.1}$$

By assumption, there exist $K > 0$ and $h > 0$ such that for any $x \in \mathbb{C}$

$$|a(x)| \leq K\,e^{h|x|}.$$

Hence, we have

$$\begin{aligned}
|\partial_x^{\alpha i} a(x)| &\leq \frac{(\alpha i)!}{2\pi}\int_{|x'-x|=R}\left|\frac{a(x')}{(x'-x)^{\alpha i+1}}\right| |dx'|\\
&= \frac{(\alpha i)!}{2\pi}\int_{|x'-x|=R}\frac{K\,e^{h|x'|}}{R^{\alpha i+1}}|dx'|\\
&= \frac{(\alpha i)!}{R^{\alpha i}}\,K\,e^{h(|x|+R)}.
\end{aligned}$$

By setting $R = \alpha i$ we see that the right-hand side is estimated by $K\, e^{h(|x|+\alpha i)}$. By (5.1), we have

$$\begin{aligned} |u_m(x)| &\le \frac{(m-1)!}{m} \left(\sum_{j=0}^{m-1} \binom{m-1}{j} |\partial_x^{\alpha j} a(x)| \right) \\ &\le \frac{(m-1)!}{m} \left(\sum_{j=0}^{m-1} \binom{m-1}{j} K\, e^{h\alpha j}\, e^{h|x|} \right) \\ &\le \frac{(m-1)!}{m} (1+e^{h\alpha})^{m-1} K\, e^{h|x|}. \end{aligned}$$

□

6 Proof of Theorem 1.2.

Let $n \in \mathbb{N}^*$ and $q \in \mathbb{N}^*$ satisfy $q(q-1)/2 < n \le q(q+1)/2$. Set $\nu = n - q(q-1)/2$ and define h_n by

$$\frac{1}{1+h_n^\alpha} = \Big(1 - \frac{1}{q+1}\Big) e^{\frac{2\pi i \nu}{q+1}}, \tag{6.1}$$

where α is given in (1.5). Then we have

Lemma 6.1. Let h_n be given by (6.1). Then we have

$$\left| \frac{1}{1+h_n^\alpha} \right| = 1 - \frac{1}{q+1} \tag{6.2}$$

and

$$\arg\left(\frac{1}{1+h_n^\alpha} \right) = 2\pi \frac{\nu}{q+1}, \tag{6.3}$$

for $\nu = n - q(q-1)/2$, $q(q-1)/2 < n \le q(q+1)/2$. The function $a(x)$ in (1.1) is well defined.

Proof (6.2) and (6.3) are immediate consequences of the definition. By (6.2) we have $1/2 \le |1/(1+h_n^\alpha)| < 1$. Hence, we have $|h_n| \le 3^{\alpha^{-1}}$, and

$$|a(x)| \le \sum_{n=1}^{\infty} |C_n||e^{h_n x}| \le \sum_{n=1}^{\infty} |C_n| \exp(3^{\alpha^{-1}}|x|).$$

Hence, $a(x)$ converges in $|x| < R$ for every $R > 0$. □

By taking $u_1 := u - \varphi_0(x)$ as a new unknown function, (1.5) is equivalent to

$$t\frac{\partial}{\partial t}u_1 = a(x)t + t\left(t\frac{\partial}{\partial t}\right)^2 u_1 + t\left(t\frac{\partial}{\partial t}\right)^2 \partial_x^\alpha u_1. \tag{6.4}$$

We prove Theorem1.2 by using (6.4).

Proof of Theorem1.2 Set $\partial u_1/\partial t =: v$. Then, (6.4) is written as follows

$$tv = a(x)t + t^2\frac{\partial}{\partial t}tv + t^2\frac{\partial}{\partial t}t\partial_x^\alpha v.$$

Applying the Borel transform to both sides and by setting $\hat{B}_1(tv) = \hat{w}$ we have

$$\hat{w} = a(x) + \tau\hat{w} + \tau\partial_x^\alpha\hat{w}, \tag{6.5}$$

where we used

$$\begin{aligned}\hat{B}_1\left(t^2\frac{\partial}{\partial t}tv\right) &= \hat{B}_1\left\{\sum_{j=1}^{\infty} u_j(x)j^2t^{j+1}\right\} \\ &= \sum_{j=1}^{\infty} u_j(x)\frac{j}{(j-1)!}\tau^j = \tau\hat{B}_1(tv).\end{aligned}$$

If we set

$$\hat{w} = \sum_{n=1}^{\infty}\hat{w}_n(\tau)e^{h_n x},$$

then we have

$$\partial_x^\alpha\hat{w} = \sum_{n=1}^{\infty}\hat{w}_n(\tau)h_n^\alpha e^{h_n x}.$$

Substituting $\hat{w}$ and (1.1) into (6.5) we have

$$\hat{w}_n(\tau) = \frac{C_n}{1-\tau(1+h_n^\alpha)}$$

for every $n \in \mathbb{N}^*$.

Take $R \in (0,1)$ and let $|\tau| < R$. Choose $N \in \mathbb{N}^*$ such that $|1/(h_n^\alpha + 1)| \geq R$ for every $n \geq N+1$. We consider

$$\sum_{n=1}^{\infty} \hat{w}_n(\tau)e^{h_n x} = \sum_{n=1}^{N} \hat{w}_n(\tau)e^{h_n x} + \sum_{n=N+1}^{\infty} \hat{w}_n(\tau)e^{h_n x}.$$

Singular points of the first term in the right-hand side are given by $\{\tau = 1/(h_n^\alpha + 1) \mid n = 1, \ldots, N\}$. According to the assumption on h_n^α, we have

$$R \leq |1 + h_{N+1}^\alpha|^{-1} \leq |1 + h_{N+2}^\alpha|^{-1} \leq \cdots < 1.$$

So, there exists $A_0 \in \mathbb{R}$ such that $\sup_{n \geq N+1, |\tau|<R} \{1/(1 - |\tau||1 + h_n^\alpha|)\} = A_0$. It follows that

$$\begin{aligned} \left| \sum_{n=N+1}^{\infty} \frac{C_n}{1 - \tau(1 + h_n^\alpha)} e^{h_n x} \right| &\leq \sum_{n=N+1}^{\infty} \frac{1}{1 - |\tau||1 + h_n^\alpha|} |C_n||e^{h_n x}| \\ &\leq A_0 \exp(3^{\alpha^{-1}}|x|) \sum_{n=N+1}^{\infty} |C_n| < \infty \end{aligned}$$

for $|\tau| < R$. Since R is arbitrary, $\sum_{n=1}^{\infty} \hat{w}_n(\tau)e^{h_n x}$ is a meromorphic function on $|\tau| < 1$.

By Lemma 6.1, we have (6.2) and (6.3) for $\nu = n - q(q-1)/2$ and $q(q-1)/2 < n \leq q(q+1)/2$. It follows that $1 \leq \nu \leq q$. We note that q tends to infinity as $n \to \infty$. Hence, the set $\{1/(1 + h_n^\alpha)\}_n$ accumulates to any points on the unit circle. Therefore, v is not 1-summable in any direction. □

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公表論文

An example of a non 1-summable partial differential equation.
Masafumi Yoshino, Kenji Kurogi
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Counterexample to the 1-Summability of a Divergent Formal Solution to Some Linear Partial Differential Equations.
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