広島大学学位請求論文

Rational curves on a smooth Hermitian surface

（非特異エルミート曲面上の有理曲線）

２０１９年

広島大学大学院理学研究科

数学専攻

尾白　典文

目　次

主論文

RATIONAL CURVES ON A SMOOTH HERMITIAN SURFACE

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Abstract. We study the set R of nonplanar rational curves of degree $d < q+2$ on a smooth Hermitian surface X of degree $q+1$ defined over an algebraically closed field of characteristic $p > 0$, where q is a power of p. We prove that R is the empty set when $d < q+1$. In the case where $d = q+1$, we count the number of elements of R by showing that the group of projective automorphisms of X acts transitively on R and by determining the stabilizer subgroup. In the special case where X is the Fermat surface, we present an element of R explicitly.

1. Introduction

Let q be a power of a prime p, and k an algebraic closure of the finite field $\mathbb{F}_q$. For a matrix m with entries in k, we denote by $m^{(q)}$ the matrix whose entries are the q-th power of those of m. We denote by a column vector $\boldsymbol{x} = {}^{\mathrm{t}}(x_0, x_1, x_2, x_3)$ a point in the k-projective space $\mathbb{P}^3$. Let A be a nonzero 4-by-4 matrix with entries in k. A k-Hermitian surface X_A is defined by

$$X_A := \{\boldsymbol{x} \in \mathbb{P}^3 \mid {}^{\mathrm{t}}\boldsymbol{x} A \boldsymbol{x}^{(q)} = 0\}.$$

If A is a Hermitian matrix, namely A has the entries in $\mathbb{F}_{q^2}$ and ${}^{\mathrm{t}}A = A^{(q)}$, the surface X_A is called a Hermitian surface. It is easily shown that X_A is smooth if and only if A is invertible.

The geometry of Hermitian varieties was systematically investigated by B. Segre in [8]. Especially, the number of linear spaces lying on a Hermitian variety and their configuration were considered. It was shown that the numbers of points and lines on a smooth Hermitian surface in $\mathbb{P}^3(\mathbb{F}_{q^2})$ are equal to $(q^3+1)(q^2+1)$ and $(q^3+1)(q+1)$ respectively, and no plane is contained. Further, the set of points and lines on a smooth Hermitian surface forms a block design, see also [3]. In recent years, the number of rational normal curves totally tangent to a smooth Hermitian variety X has been determined in [10] by considering the action of the automorphism group of X on the set of the curves. In [11], non-singular conics totally tangent to the smooth Hermitian curve of degree 6 in characteristic 5 were utilized for a geometric construction of strongly regular graphs. On the other hand, projective isomorphism classes of degenerate Hermitian varieties of

2010 *Mathematics Subject Classification.* 51E20, 14M99, 14N99.

Key words and phrases. rational curve, Hermitian surface, positive characteristic.

corank 1 and the automorphism group of each isomorphism class have been determined in [7].

Let A be an invertible 4-by-4 matrix with entries in k. We will be concerned with rational curves of degree > 1 on a smooth k-Hermitian surface X_A. Let d be a positive integer and F a 4-by-$(d+1)$ matrix of $\mathrm{rank}(F) \geq 2$ with entries in k. A rational curve C_F of degree d in $\mathbb{P}^3$ is the image of a rational map

$$\mathbb{P}^1 \ni {}^{\mathrm{t}}(s,t) \longmapsto F\ {}^{\mathrm{t}}(s^d, s^{d-1}t, \ldots, st^{d-1}, t^d) \in \mathbb{P}^3. \tag{1}$$

We call $\mathrm{rank}(F)$ the rank of the curve C_F. If $\mathrm{rank}(F) = 2$, then C_F degenerates to a line. If $\mathrm{rank}(F) = 3$, then C_F degenerates to a plane curve of degree ≥ 2. When $\mathrm{rank}(F) = 4$, the curve C_F is nondegenerate and is a space curve of degree ≥ 3. Then C_F is said to be nonplanar, namely C_F is not contained in any plane. Thus the study of rational curves of rank 2 on X_A is reduced to that of lines on X_A. Further, an algebraic curve of rank 3 on X_A is a smooth k-Hermitian curve of degree $q+1$, which is of genus $q(q-1)/2 > 0$. Hence we may restrict ourselves to the case of rank 4.

Our results are as follows:

Theorem 1.1. *There is no nonplanar rational curve of degree $\leq q$ on a smooth k-Hermitian surface.*

Let R be the set of nonplanar rational curves of degree $q+1$ on a smooth k-Hermitian surface X_A. As will be seen later, the set R is nonempty and each element is projectively isomorphic over k to the smooth curve

$$C_0 := \left\{ {}^{\mathrm{t}}(s^{q+1}, s^q t, st^q, t^{q+1}) \in \mathbb{P}^3 \mid {}^{\mathrm{t}}(s,t) \in \mathbb{P}^1 \right\}.$$

We denote by $\mathrm{Aut}(X_A)$ the group of projective automorphisms of X_A. Let n be a positive integer. We deal with the group $\mathrm{PGU}_n(\mathbb{F}_{q^2})$ defined by

$$\{Q \in \mathrm{GL}_n(\mathbb{F}_{q^2}) \mid {}^{\mathrm{t}}QQ^{(q)} = I\}/\boldsymbol{\mu}_{q+1}I,$$

where $\boldsymbol{\mu}_{q+1}$ denotes the group of $(q+1)$-th roots of unity and I denotes the unit matrix. As is well-known, the group $\mathrm{Aut}(X_A)$ is isomorphic to $\mathrm{PGU}_4(\mathbb{F}_{q^2})$. Then we shall prove the following theorem.

Theorem 1.2. *The group $\mathrm{Aut}(X_A)$ acts transitively on the set R, and the stabilizer subgroup is isomorphic to $\mathrm{PGU}_2(\mathbb{F}_{q^4})$.*

By Theorem 1.2, the cardinality of R is equal to $|\mathrm{PGU}_4(\mathbb{F}_{q^2})|/|\mathrm{PGU}_2(\mathbb{F}_{q^4})|$. We know by [6, pp.64-65] that

$$|\mathrm{PGU}_4(\mathbb{F}_{q^2})| = q^6(q^4-1)(q^3+1)(q^2-1) \text{ and } |\mathrm{PGU}_2(\mathbb{F}_{q^4})| = q^2(q^4-1).$$

Thus we have the following.

Corollary 1.3. $|R| = q^4(q^3+1)(q^2-1)$.

The number $|R|$ is 432, 18144, 249600, 1890000, 39645312, 383162400, … as $q = 2, 3, 4, 5, 7, 9, \ldots$.

In the special case where $A = I$, that is, where the surface X_A is the Fermat surface, we can explicitly give an element C_{F_J} of R such as

$$\left\{ {}^{\mathrm{t}}\!\left(\eta^{-q}\xi^q s^{q+1} - \eta^{-q}t^{q+1},\ s^q t,\ st^q,\ \omega\eta^{-1}\xi s^{q+1} + \omega\eta^{-1}t^{q+1}\right) \in \mathbb{P}^3 \,\middle|\, {}^{\mathrm{t}}(s,t) \in \mathbb{P}^1 \right\},$$

where ω, ξ, and η are elements of $\mathbb{F}_{q^2}$ satisfying $\omega^{q+1} = -1$, $\xi^{q+1} = 1$ with $\xi^2 \neq -1$, and $\eta^{q+1} = \xi^q + \xi$. Note that $\eta \neq 0$ because $\xi^2 \neq 0, -1$. The curve C_{F_J} is smooth since it is projectively isomorphic to the smooth curve C_0. On the other hand, a complete set of representatives for $\mathrm{Aut}(X_I)$ can be taken from $\mathrm{GL}_4(\mathbb{F}_{q^2})$ (see Lemma 4.1). Therefore we have the following.

Corollary 1.4. *All nonplanar rational curves of degree $q+1$ on X_I are projectively isomorphic over $\mathbb{F}_{q^2}$ to the smooth curve C_{F_J}.*

In the case where $q = 2$, we have $|X_I(\mathbb{F}_{q^2})| = 45$ where $X_I(\mathbb{F}_{q^2})$ denotes the set of $\mathbb{F}_{q^2}$-rational points of X_I, and $\mathrm{Aut}(X_I)$ is of order 25920. Then $|C_F(\mathbb{F}_{q^2})| = 5$ for each nonplanar cubic C_F on X_I. We can actually obtain by computation 432 nonplanar cubics on X_I and the stabilizer subgroup of $\mathrm{Aut}(X_I)$ fixing C_{F_J} of order 60. By restricting X_I to $X_I(\mathbb{F}_{q^2})$, we can verify that each cubic intersects 150 other cubics at a single point, 40 other cubics at two points and another cubic at five points. Here, when we say two cubics C_F, $C_{F'}$ intersect at n points we mean $|C_F(\mathbb{F}_{q^2}) \cap C_{F'}(\mathbb{F}_{q^2})| = n$. We can also verify that $\mathrm{Aut}(X_I)$ acts transitively on $X_I(\mathbb{F}_{q^2})$ and the stabilizer subgroup is of order 576, and furthermore, there are 48 cubics passing through each point of $X_I(\mathbb{F}_{q^2})$. These computational data files obtained by using `GAP` [4] are available upon request addressed to the author.

We give a brief outline of our paper. In the next section, we prove Theorem 1.1. By the same argument, we show directly that each irreducible conic, which is a rational curve of rank 3, is not contained in X_A. In section 3, we give a bijection between the set R and the quotient of certain sets consisting of invertible 4-by-4 matrices, by showing basic lemmas. In section 4, we first prove two lemmas which are necessary for our proof of Theorem 1.2. We prove Theorem 1.2 in the last of the section.

The author is grateful to Professor Ichiro Shimada for his encouragement during the course of the work and helpful suggestions on drafts.

2. Proof of Theorem 1.1

Proof of Theorem 1.1. Suppose that a nonplanar rational curve C_F defined by (1) is contained in a smooth k-Hermitian surface X_A. Denoting by $b_{i,j}$ the entries of the $(d+1)$-by-$(d+1)$ matrix ${}^{\mathrm{t}}FAF^{(q)}$, one has the identity

$$\sum_{i,j=0}^{d} b_{i,j} s^{d-i+q(d-j)} t^{i+qj} \equiv 0. \tag{2}$$

Therefore if $d < q$, all the coefficients $b_{i,j}$ must vanish because the exponents $(i+qj)$'s are all different. This implies that ${}^{\mathrm{t}}FAF^{(q)} = O$, but it is a contradiction. In fact, since $\mathrm{rank}(F) = 4$ by definition, we can take an invertible matrix F^* consisting of linearly independent 4 column vectors of F. Then, however, ${}^{\mathrm{t}}F^*AF^{*(q)}$ must be O. If $d = q$, the coefficients $b_{i,j}$ must vanish except for $b_{q,l-1} = -b_{0,l}$ with $1 \leq l \leq q$. This implies that $\mathrm{rank}({}^{\mathrm{t}}FAF^{(q)}) \leq 2$, but it is a contradiction by the argument above. Hence we conclude that $C_F \not\subset X_A$.

□

Remark 2.1. We can similarly give a proof for the case of irreducible conics. In fact, since an irreducible conic C_F is of rank 3, we can make an invertible matrix F^* consisting of linearly independent 3 column vectors of F and a vector linearly independent to those vectors. Suppose that $C_F \subset X_A$. Since $d = 2 \leq q$, one has $\mathrm{rank}({}^{\mathrm{t}}FAF^{(q)}) \leq 2$ in the same argument as the above proof. Therefore the 4-by-4 matrix ${}^{\mathrm{t}}F^*AF^{*(q)}$ must be of rank 3 at the most, but ${}^{\mathrm{t}}F^*AF^{*(q)}$ is of rank 4 by definition. This is a contradiction. As we have seen, this proof is valid for rational curves which are of rank ≥ 3 and degree $\leq q$.

3. Basic lemmas

In this section, we will prove some basic lemmas to prepare for our proof of Theorem 1.2. The following lemma gives a necessary and sufficient condition for a nonplanar rational curve of degree $q+1$ to be on a smooth k-Hermitian surface.

Lemma 3.1. *Let C_F be a nonplanar rational curve of degree $q+1$ defined by (1). The curve C_F is contained in a smooth k-Hermitian surface X_A if and only if the $(q+2)$-by-$(q+2)$ matrix ${}^{\mathrm{t}}FAF^{(q)}$ is of the form*

$$\begin{pmatrix} 0 & b_{0,1} & 0,\dots,0 & 0 & b_{0,q+1} \\ 0 & b_{1,1} & 0,\dots,0 & 0 & b_{1,q+1} \\ 0 & 0 & 0,\dots,0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0,\dots,0 & 0 & 0 \\ -b_{0,1} & 0 & 0,\dots,0 & -b_{0,q+1} & 0 \\ -b_{1,1} & 0 & 0,\dots,0 & -b_{1,q+1} & 0 \end{pmatrix}.$$

If the above condition is satisfied, the matrix F is of the form

$$(\boldsymbol{f}_0, \boldsymbol{f}_1, \boldsymbol{0}, \dots, \boldsymbol{0}, \boldsymbol{f}_q, \boldsymbol{f}_{q+1}).$$

Proof. As was seen above, the curve C_F is contained in X_A if and only if one has (2). In the present case where $d = q+1$, if $C_F \subset X_A$ then the coefficients $b_{i,j}$ must vanish except for $b_{q,l-1} = -b_{0,l}$, $b_{q+1,l-1} = -b_{1,l}$ with $1 \leq l \leq q+1$. Since $\mathrm{rank}(F) = 4$, there are 4 column vectors $\boldsymbol{f}_x, \boldsymbol{f}_y, \boldsymbol{f}_z, \boldsymbol{f}_w$ of F with $0 \leq x < y < z < w \leq q+1$ such that the matrix $F^* := (\boldsymbol{f}_x, \boldsymbol{f}_y, \boldsymbol{f}_z, \boldsymbol{f}_w)$ is invertible. Then none of x, y, z, w is from 2 to $q-1$ because ${}^{\mathrm{t}}F^*AF^{*(q)}$ is also invertible, and thus $x = 0, y = 1, z = q, w = q+1$. Let $\boldsymbol{f}_i$ be the i-th column vector with $2 \leq i \leq q-1$ of F. Then one has

$$ {}^{\mathrm{t}}\boldsymbol{f}_i A F^{*(q)} = (b_{i,0}, b_{i,1}, b_{i,q}, b_{i,q+1}) = (0,0,0,0), $$

and thus $\boldsymbol{f}_i = \mathbf{0}$. Hence F and ${}^{\mathrm{t}}FAF^{(q)}$ are of the form described above. The converse is obvious since (2) holds automatically.

□

A rational curve C_F defined by (1) is also obtained by replacing F by $\lambda F \varphi(g)$, where λ is an element of the multiplicative group $k^\times$ and φ is a homomorphism from $\mathrm{GL}_2(k)$ to $\mathrm{GL}_{d+1}(k)$ defined by the following: for each ${}^{\mathrm{t}}(s,t) \in k^2$ with ${}^{\mathrm{t}}(s,t) \neq {}^{\mathrm{t}}(0,0)$ and $g \in \mathrm{GL}_2(k)$, put ${}^{\mathrm{t}}(u,v) := g\ {}^{\mathrm{t}}(s,t)$, then

$$ \begin{array}{cccc} \varphi: & \mathrm{GL}_2(k) & \longrightarrow & \mathrm{GL}_{d+1}(k) \\ & \cup & & \cup \\ & \left(g : {}^{\mathrm{t}}(s,t) \mapsto {}^{\mathrm{t}}(u,v)\right) & \longmapsto & \left(\varphi(g) : {}^{\mathrm{t}}(s^d, s^{d-1}t, \dots, t^d) \mapsto {}^{\mathrm{t}}(u^d, u^{d-1}v, \dots, v^d)\right). \end{array} $$

Indeed, it is obvious by definition that $\varphi(I) = I$. Putting ${}^{\mathrm{t}}(x,y) := h\ {}^{\mathrm{t}}(u,v)$ for each $h \in \mathrm{GL}_2(k)$, one has

$$ \begin{aligned} \varphi(hg)\ {}^{\mathrm{t}}(s^d, s^{d-1}t, \dots, t^d) &= {}^{\mathrm{t}}(x^d, x^{d-1}y, \dots, y^d) \\ &= \varphi(h)\ {}^{\mathrm{t}}(u^d, u^{d-1}v, \dots, v^d) \\ &= \varphi(h)\varphi(g)\ {}^{\mathrm{t}}(s^d, s^{d-1}t, \dots, t^d). \end{aligned} $$

Hence $\varphi(hg) = \varphi(h)\varphi(g)$, and thus $\varphi(g) \in \mathrm{GL}_{d+1}(k)$.

Conversely if there is a matrix F' such that $C_F = C_{F'}$, then one has

$$ F\ {}^{\mathrm{t}}(s^d, s^{d-1}t, \dots, st^{d-1}, t^d) = F'\ {}^{\mathrm{t}}(u^d, u^{d-1}v, \dots, uv^{d-1}, v^d) \in \mathbb{P}^3. $$

This implies that there are homogeneous polynomials f, f' of degree d such that $f(s,t) = f'(u,v)$. Therefore there is an element g of $\mathrm{GL}_2(k)$ such that ${}^{\mathrm{t}}(s,t) = g\,{}^{\mathrm{t}}(u,v) \in \mathbb{P}^1$, and thus $F' = \lambda F \varphi(g)$ for some $\lambda \in k^\times$. Hence, denoting by $\mathrm{Im}(\varphi)$ the image of φ, we see that the set $k^\times F \mathrm{Im}(\varphi)$ corresponds one-to-one with C_F.

Let S be the set of matrices F such that ${}^{\mathrm{t}}FAF^{(q)}$ satisfies the condition of Lemma 3.1. Then by Lemma 3.1, for each $F \in S$ the set $k^\times F \mathrm{Im}(\varphi)$ corresponds one-to-one with the nonplanar rational curve C_F on X_A. Therefore one has the following bijection

$$ k^\times \backslash S / \mathrm{Im}(\varphi) \ni k^\times F \mathrm{Im}(\varphi) \longmapsto C_F \in R. \tag{3} $$

By Lemma 3.1, we define the map

$$ {}^* : S \ni F = (\boldsymbol{f}_0, \boldsymbol{f}_1, \mathbf{0}, \dots, \mathbf{0}, \boldsymbol{f}_q, \boldsymbol{f}_{q+1}) \longmapsto F^* = (\boldsymbol{f}_0, \boldsymbol{f}_1, \boldsymbol{f}_q, \boldsymbol{f}_{q+1}) \in S^*, $$

where S^* is written as

$$ S^* = \{F^* \in \mathrm{GL}_4(k) \mid {}^{\mathrm{t}}F^* A F^{*(q)} = D_B,\ B \in \mathrm{GL}_2(k)\}, $$

and D_B is a matrix defined by

$$ D_B := \begin{pmatrix} \mathbf{0} & \boldsymbol{b}_1 & \mathbf{0} & \boldsymbol{b}_2 \\ -\boldsymbol{b}_1 & \mathbf{0} & -\boldsymbol{b}_2 & \mathbf{0} \end{pmatrix} \in \mathrm{GL}_4(k) \text{ for } B = (\boldsymbol{b}_1, \boldsymbol{b}_2) \in \mathrm{GL}_2(k). $$

Further, we define the map $_*$ from $\mathrm{Im}(\varphi) \subset \mathrm{GL}_{q+2}(k)$ to $\mathrm{Im}(\varphi)_* \subset \mathrm{GL}_4(k)$ as follows:

for every $g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in \mathrm{GL}_2(k)$,

$$\varphi(g)=\begin{pmatrix} \alpha^{q+1} & \alpha^q\beta & ,\dots, & \alpha\beta^q & \beta^{q+1} \\ \alpha^q\gamma & \alpha^q\delta & ,\dots, & \gamma\beta^q & \delta\beta^q \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha\gamma^q & \beta\gamma^q & ,\dots, & \alpha\delta^q & \beta\delta^q \\ \gamma^{q+1} & \delta\gamma^q & ,\dots, & \gamma\delta^q & \delta^{q+1} \end{pmatrix} \mapsto \varphi(g)_*=\begin{pmatrix} \alpha^{q+1} & \alpha^q\beta & \alpha\beta^q & \beta^{q+1} \\ \alpha^q\gamma & \alpha^q\delta & \gamma\beta^q & \delta\beta^q \\ \alpha\gamma^q & \beta\gamma^q & \alpha\delta^q & \beta\delta^q \\ \gamma^{q+1} & \delta\gamma^q & \gamma\delta^q & \delta^{q+1} \end{pmatrix},$$

where $\mathrm{Im}(\varphi)_*$ is written as

$$\mathrm{Im}(\varphi)_* = \left\{ \begin{pmatrix} \alpha^q g & \beta^q g \\ \gamma^q g & \delta^q g \end{pmatrix} \in \mathrm{GL}_4(k) \;\middle|\; g \in \mathrm{GL}_2(k) \right\}.$$

Indeed, it is easy to see that $\det(\varphi(g)_*) = \det(g)^{2q+2}$ for every $g \in \mathrm{GL}_2(k)$, and thus $\varphi(g)_* \in \mathrm{GL}_4(k)$.

We denote by φ_* the composition of φ and $_*$, namely $\varphi_*(g) = \varphi(g)_*$ for every $g \in \mathrm{GL}_2(k)$.

Lemma 3.2. *The map φ_* is a homomorphism from $\mathrm{GL}_2(k)$ to $\mathrm{GL}_4(k)$. There is the following natural bijection*

$$k^\times \backslash S/\mathrm{Im}(\varphi) \longrightarrow k^\times \backslash S^*/\mathrm{Im}(\varphi)_*.$$

Proof. For each

$$g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},\ h = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \in \mathrm{GL}_2(k),$$

one has

$$gh = \begin{pmatrix} \alpha x + \beta z & \alpha y + \beta w \\ \gamma x + \delta z & \gamma y + \delta w \end{pmatrix}.$$

Therefore

$$\varphi_*(gh) = \begin{pmatrix} (\alpha x + \beta z)^q gh & (\alpha y + \beta w)^q gh \\ (\gamma x + \delta z)^q gh & (\gamma y + \delta w)^q gh \end{pmatrix}.$$

On the other hand,

$$\begin{aligned} \varphi_*(g)\varphi_*(h) &= \begin{pmatrix} \alpha^q g & \beta^q g \\ \gamma^q g & \delta^q g \end{pmatrix} \begin{pmatrix} x^q h & y^q h \\ z^q h & w^q h \end{pmatrix} \\ &= \begin{pmatrix} \alpha^q x^q gh + \beta^q z^q gh & \alpha^q y^q gh + \beta^q w^q gh \\ \gamma^q x^q gh + \delta^q z^q gh & \gamma^q y^q gh + \delta^q w^q gh \end{pmatrix} \\ &= \begin{pmatrix} (\alpha^q x^q + \beta^q z^q) gh & (\alpha^q y^q + \beta^q w^q) gh \\ (\gamma^q x^q + \delta^q z^q) gh & (\gamma^q y^q + \delta^q w^q) gh \end{pmatrix}. \end{aligned}$$

Since the q-th power is an automorphism of k, one has $\varphi_*(gh) = \varphi_*(g)\varphi_*(h)$ and thus φ_* is a homomorphism from $\mathrm{GL}_2(k)$ to $\mathrm{GL}_4(k)$.

For each $F \in S$, $g \in \mathrm{GL}_2(k)$, denoting by $a_{i,j}$ the entries of $\varphi(g)$, we can write the j-th column vector $\boldsymbol{g}_j$ with $j \in \{0, 1, q, q+1\}$ of $F\varphi(g)$ as

$$\boldsymbol{g}_j = \sum_{i \in \{0,1,q,q+1\}} a_{i,j} \boldsymbol{f}_i,$$

since $\boldsymbol{f}_i = \boldsymbol{0}$ for $2 \leq i \leq q-1$. Then it is immediate from definition that

$$F^* \varphi_*(g) = (\boldsymbol{g}_0, \boldsymbol{g}_1, \boldsymbol{g}_q, \boldsymbol{g}_{q+1}),$$

and thus $(F\varphi(g))^* = F^* \varphi_*(g)$. This implies that there is the natural map from $k^\times \backslash S / \mathrm{Im}(\varphi)$ to $k^\times \backslash S^* / \mathrm{Im}(\varphi)_*$. The bijectivity is obvious since by definition the map $S \to S^*$ is bijective.

□

By (3) and Lemma 3.2, one has the bijection

$$k^\times \backslash S^* / \mathrm{Im}(\varphi)_* \ni k^\times F^* \mathrm{Im}(\varphi)_* \longmapsto C_F \in R. \tag{4}$$

The following well-known proposition is useful. The readers may find a proof for example in [2] and [9, Proposition 2.5.].

Proposition 3.3. *For each element A of $\mathrm{GL}_n(k)$, there is an element B of $\mathrm{GL}_n(k)$ such that $A = {}^{\mathrm{t}}BB^{(q)}$. If A is a Hermitian matrix, then the matrix B can be taken from $\mathrm{GL}_n(\mathbb{F}_{q^2})$.*

By Proposition 3.3, it follows immediately that a smooth k-Hermitian (resp. Hermitian) surface is projectively isomorphic over k (resp. $\mathbb{F}_{q^2}$) to the Fermat surface X_I.

We define the set

$$M := \left\{ D_B := \begin{pmatrix} \boldsymbol{0} & \boldsymbol{b}_1 & \boldsymbol{0} & \boldsymbol{b}_2 \\ -\boldsymbol{b}_1 & \boldsymbol{0} & -\boldsymbol{b}_2 & \boldsymbol{0} \end{pmatrix} \in \mathrm{GL}_4(k) \,\middle|\, B = \begin{pmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 \end{pmatrix} \in \mathrm{GL}_2(k) \right\}.$$

Then the following map is surjective:

$$S^* \ni F^* \longmapsto {}^{\mathrm{t}}F^* A F^{*(q)} \in M. \tag{5}$$

In fact, by Proposition 3.3 there is an element D of $\mathrm{GL}_4(k)$ such that $D_B = {}^{\mathrm{t}}DD^{(q)}$ for each $D_B \in M$. Similarly there is an element A' of $\mathrm{GL}_4(k)$ such that $A = {}^{\mathrm{t}}A'A'^{(q)}$. Hence putting $F^* := A'^{-1}D$, one has ${}^{\mathrm{t}}F^* A F^{*(q)} = D_B$, and thus $F^* \in S^*$.

Lemma 3.4. *The set R is nonempty, and each element of R is projectively isomorphic over k to the smooth curve*

$$C_0 := \left\{ {}^{\mathrm{t}}(s^{q+1}, s^q t, st^q, t^{q+1}) \in \mathbb{P}^3 \mid {}^{\mathrm{t}}(s,t) \in \mathbb{P}^1 \right\}.$$

Proof. The set S^* is nonempty by the surjectivity of the map (5). Hence by (4) the set R is nonempty. For each element C_F of R, it is obvious by definition that

$$F^{*-1}F = (\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{0}, \ldots, \boldsymbol{0}, \boldsymbol{e}_3, \boldsymbol{e}_4) \text{ with } (\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4) = I.$$

This implies that C_F is projectively isomorphic over k to C_0. Then by definition, the curve C_0 is smooth clearly.

□

Remark 3.5. It is known that each nonplanar nonreflexive curve of degree $q+1$ is projectively isomorphic to the curve C_0 (cf. [1, Theorem 2]). For nonreflexive curves, see also [5]. Hence by Lemma 3.4, each element of R is projectively isomorphic to each nonplanar nonreflexive curve of degree $q+1$.

Remark 3.6. In the case where $A = I$, we can find an element of R. We put

$$J := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Then the matrix D_J is a Hermitian matrix. Hence by Proposition 3.3, there is an element ${F_J}^*$ of $\mathrm{GL}_4(\mathbb{F}_{q^2})$ such that ${}^{\mathrm{t}}{F_J}^*{F_J}^{*(q)} = D_J$. Actually taking ${F_J}^*$ such as

$$\begin{pmatrix} \eta^{-q}\xi^q & 0 & 0 & -\eta^{-q} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \omega\eta^{-1}\xi & 0 & 0 & \omega\eta^{-1} \end{pmatrix}$$

for ω, ξ and η as mentioned in Introduction, one has by (4) the corresponding curve C_{F_J} lying on X_I.

4. Proof of Theorem 1.2

The group $\mathrm{Aut}(X_A)$ of projective automorphisms of X_A is equal to

$$\{Q \in \mathrm{GL}_4(k) \mid {}^{\mathrm{t}}QAQ^{(q)} = \lambda A,\ \lambda \in k^\times\}/k^\times I.$$

By Proposition 3.3, the group $\mathrm{Aut}(X_A)$ is conjugate to $\mathrm{Aut}(X_I)$ in $\mathrm{PGL}_4(k)$.

We prove the following lemma on matrix groups of arbitrary rank because we need the lemma to our proof of Theorem 1.2.

Lemma 4.1. *Let n be a positive integer. The group $\mathrm{PGU}_n(\mathbb{F}_{q^2})$ is isomorphic to*

$$G := \{Q \in \mathrm{GL}_n(k) \mid {}^{\mathrm{t}}QQ^{(q)} = \lambda I,\ \lambda \in k^\times\}/k^\times I.$$

Proof. We consider the map

$$G \ni Qk^\times \longmapsto \xi_\lambda Q\boldsymbol{\mu}_{q+1} \in \mathrm{PGU}_n(\mathbb{F}_{q^2}),$$

where λ is the element of $k^\times$ satisfying ${}^{\mathrm{t}}QQ^{(q)} = \lambda I$ and ξ_λ is an element of $k^\times$ satisfying ${\xi_\lambda}^{q+1} = \lambda^{-1}$. Then the map is well-defined. In fact, it is obvious that ${}^{\mathrm{t}}(\xi_\lambda Q)(\xi_\lambda Q)^{(q)} = I$, and the matrix $\xi_\lambda Q$ has the entries in $\mathbb{F}_{q^2}$ because I is a Hermitian matrix. Hence $\xi_\lambda Q\boldsymbol{\mu}_{q+1}$ belongs to $\mathrm{PGU}_n(\mathbb{F}_{q^2})$. Further, putting $P := \alpha Q$ for each $\alpha \in k^\times$, one has ${}^{\mathrm{t}}PP^{(q)} = \alpha^{q+1}\lambda I$. It is easily shown by definition that

$$\xi_{\alpha^{q+1}\lambda}\boldsymbol{\mu}_{q+1} = \xi_{\alpha^{q+1}}\xi_\lambda\boldsymbol{\mu}_{q+1} \quad \text{and} \quad \alpha\xi_{\alpha^{q+1}}\boldsymbol{\mu}_{q+1} = \boldsymbol{\mu}_{q+1}.$$

Therefore we conclude that

$$\xi_{\alpha^{q+1}\lambda}P\boldsymbol{\mu}_{q+1} = \xi_\lambda Q\boldsymbol{\mu}_{q+1}.$$

Thus the map is independent of the choice of representatives for G.

Let $Q'k^\times$ be an element of G with ${}^{\mathrm{t}}Q'Q'^{(q)} = \eta I$ for some $\eta \in k^\times$. Then one has

$$(\xi_\eta Q' \boldsymbol{\mu}_{q+1})(\xi_\lambda Q \boldsymbol{\mu}_{q+1}) = \xi_{\eta\lambda} Q' Q \boldsymbol{\mu}_{q+1},$$

since $\xi_\eta \xi_\lambda \boldsymbol{\mu}_{q+1} = \xi_{\eta\lambda}\boldsymbol{\mu}_{q+1}$. Hence the map is a homomorphism from G to $\mathrm{PGU}_n(\mathbb{F}_{q^2})$. The injectivity and the surjectivity are immediate from definition.

□

By Lemma 4.1, the group $\mathrm{Aut}(X_A)$ isomorphic to $\mathrm{PGU}_4(\mathbb{F}_{q^2})$.

The following lemma is a key ingredient in our proof of Theorem 1.2.

Lemma 4.2. *For every g, $B \in \mathrm{GL}_2(k)$, one has*

$${}^{\mathrm{t}}\varphi_*(g) D_B \varphi_*(g)^{(q)} = \det(g)^q D_{{}^{\mathrm{t}}gBg^{(q^2)}}.$$

Proof. The proof is due to straightforward computation. We put

$$g := \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \ B := (\boldsymbol{b}_1, \boldsymbol{b}_2).$$

Then one has

$$\begin{aligned}
&{}^{\mathrm{t}}\varphi_*(g) D_B \varphi_*(g)^{(q)} \\
&= \begin{pmatrix} \alpha^q\ {}^{\mathrm{t}}g & \gamma^q\ {}^{\mathrm{t}}g \\ \beta^q\ {}^{\mathrm{t}}g & \delta^q\ {}^{\mathrm{t}}g \end{pmatrix} \begin{pmatrix} \mathbf{0} & \boldsymbol{b}_1 & \mathbf{0} & \boldsymbol{b}_2 \\ -\boldsymbol{b}_1 & \mathbf{0} & -\boldsymbol{b}_2 & \mathbf{0} \end{pmatrix} \begin{pmatrix} \alpha^{q^2} g^{(q)} & \beta^{q^2} g^{(q)} \\ \gamma^{q^2} g^{(q)} & \delta^{q^2} g^{(q)} \end{pmatrix} \\
&= \begin{pmatrix} -\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 & \alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 & -\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 & \alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\ -\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 & \beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 & -\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 & \beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \end{pmatrix} \begin{pmatrix} \alpha^{q^2+q} & \alpha^{q^2}\beta^q & \alpha^q\beta^{q^2} & \beta^{q^2+q} \\ \alpha^{q^2}\gamma^q & \alpha^{q^2}\delta^q & \gamma^q\beta^{q^2} & \delta^q\beta^{q^2} \\ \alpha^q\gamma^{q^2} & \beta^q\gamma^{q^2} & \alpha^q\delta^{q^2} & \beta^q\delta^{q^2} \\ \gamma^{q^2+q} & \delta^q\gamma^{q^2} & \gamma^q\delta^{q^2} & \delta^{q^2+q} \end{pmatrix}.
\end{aligned}$$

Putting

$${}^{\mathrm{t}}\varphi_*(g) D_B \varphi_*(g)^{(q)} := \begin{pmatrix} \boldsymbol{c}_1 & \boldsymbol{c}_2 & \boldsymbol{c}_3 & \boldsymbol{c}_4 \\ \boldsymbol{c}_5 & \boldsymbol{c}_6 & \boldsymbol{c}_7 & \boldsymbol{c}_8 \end{pmatrix},$$

one has

$$\begin{aligned}
\boldsymbol{c}_1 &= -\alpha^{q^2+q}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \alpha^{q^2}\gamma^q\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \alpha^q\gamma^{q^2}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \gamma^{q^2+q}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \boldsymbol{0}, \\
\boldsymbol{c}_2 &= -\alpha^{q^2}\beta^q\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \alpha^{q^2}\delta^q\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \beta^q\gamma^{q^2}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \delta^q\gamma^{q^2}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \det(g)^q(\alpha^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \gamma^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_2) \\
&= \det(g)^q\ {}^{\mathrm{t}}g(\boldsymbol{b}_1,\boldsymbol{b}_2)\ {}^{\mathrm{t}}(\alpha^{q^2},\gamma^{q^2}), \\
\boldsymbol{c}_3 &= -\alpha^q\beta^{q^2}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \gamma^q\beta^{q^2}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \alpha^q\delta^{q^2}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \gamma^q\delta^{q^2}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \boldsymbol{0}, \\
\boldsymbol{c}_4 &= -\beta^{q^2+q}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \delta^q\beta^{q^2}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \beta^q\delta^{q^2}\gamma^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \delta^{q^2+q}\alpha^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \det(g)^q(\beta^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \delta^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_2) \\
&= \det(g)^q\ {}^{\mathrm{t}}g(\boldsymbol{b}_1,\boldsymbol{b}_2)\ {}^{\mathrm{t}}(\beta^{q^2},\delta^{q^2}), \\
\boldsymbol{c}_5 &= -\alpha^{q^2+q}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \alpha^{q^2}\gamma^q\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \alpha^q\gamma^{q^2}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \gamma^{q^2+q}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= -\det(g)^q(\alpha^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \gamma^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_2) \\
&= -\det(g)^q\ {}^{\mathrm{t}}g(\boldsymbol{b}_1,\boldsymbol{b}_2)\ {}^{\mathrm{t}}(\alpha^{q^2},\gamma^{q^2}), \\
\boldsymbol{c}_6 &= -\alpha^{q^2}\beta^q\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \alpha^{q^2}\delta^q\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \beta^q\gamma^{q^2}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \delta^q\gamma^{q^2}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \boldsymbol{0}, \\
\boldsymbol{c}_7 &= -\alpha^q\beta^{q^2}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \gamma^q\beta^{q^2}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \alpha^q\delta^{q^2}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \gamma^q\delta^{q^2}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= -\det(g)^q(\beta^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \delta^{q^2}\ {}^{\mathrm{t}}g\boldsymbol{b}_2) \\
&= -\det(g)^q\ {}^{\mathrm{t}}g(\boldsymbol{b}_1,\boldsymbol{b}_2)\ {}^{\mathrm{t}}(\beta^{q^2},\delta^{q^2}), \\
\boldsymbol{c}_8 &= -\beta^{q^2+q}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 + \delta^q\beta^{q^2}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_1 - \beta^q\delta^{q^2}\delta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 + \delta^{q^2+q}\beta^q\ {}^{\mathrm{t}}g\boldsymbol{b}_2 \\
&= \boldsymbol{0}.
\end{aligned}$$

Hence one has

$$(\boldsymbol{c}_2,\boldsymbol{c}_4) = \det(g)^q\ {}^{\mathrm{t}}gBg^{(q^2)} = -(\boldsymbol{c}_5,\boldsymbol{c}_7), \quad \boldsymbol{c}_1 = \boldsymbol{c}_3 = \boldsymbol{c}_6 = \boldsymbol{c}_8 = \boldsymbol{0}.$$

This completes the proof.

□

Proof of Theorem 1.2. We define an equivalence relation $\sim$ on the set M as follows: $D_B \sim D_{B'}$ for D_B, $D_{B'} \in M$ if there is an element $g \in \mathrm{GL}_2(k)$ such that $D_{B'} = {}^{\mathrm{t}}\varphi_*(g)D_B\varphi_*(g)^{(q)}$. We denote by ${D_B}^{\varphi_*}$ an equivalence class containing D_B. On the other hand, the group $\mathrm{Aut}(X_A)$ acts on $k^\times\backslash S^*/\mathrm{Im}(\varphi)_*$ by multiplication from the left. Then the following map is bijective:

$$\begin{array}{ccc}
\mathrm{Aut}(X_A)k^\times\backslash S^*/\mathrm{Im}(\varphi)_* & \longrightarrow & k^\times\backslash M/\sim \\
\cup & & \cup \\
\mathrm{Aut}(X_A)k^\times F^*\mathrm{Im}(\varphi)_* & \longmapsto & k^\times({}^{\mathrm{t}}F^*AF^{*(q)})^{\varphi_*}.
\end{array}$$

Indeed, the surjectivity is obvious since the map (5) is surjective. If we assume that $k^\times({}^{\mathrm{t}}F^*AF^{*(q)})^{\varphi_*} = k^\times({}^{\mathrm{t}}{F_1}^*A{F_1}^{*(q)})^{\varphi_*}$ for some ${F_1}^* \in S^*$, then we have

$$ {}^{\mathrm{t}}(F_1{}^*\varphi_*(g)F^{*-1})A(F_1{}^*\varphi_*(g)F^{*-1})^{(q)} = \lambda A $$

for some $g \in \mathrm{GL}_2(k)$ and $\lambda \in k^\times$. Therefore $k^\times F_1{}^*\varphi_*(g)F^{*-1}$ belongs to $\mathrm{Aut}(X_A)$. This implies the injectivity, and thus bijectivity. By Proposition 3.3, there is an element B' of $\mathrm{GL}_2(k)$ such that $B = {}^{\mathrm{t}}B'B'^{(q^2)}$ for each $D_B \in M$. Then by Lemma 4.2, one has

$$ {}^{\mathrm{t}}\varphi_*(B'^{-1})D_B\varphi_*(B'^{-1})^{(q)} = \det(B'^{-1})^q D_I. $$

This implies that $k^\times D_B{}^{\varphi_*} = k^\times D_I{}^{\varphi_*}$. Hence $|k^\times\backslash M/\sim| = 1$ and thus $|\mathrm{Aut}(X_A)k^\times\backslash S^*/\mathrm{Im}(\varphi)_*| = 1$, and by (4) one has $|\mathrm{Aut}(X_A)\backslash R| = 1$. This proves half of our theorem.

Let $\Gamma/k^\times I$ be the stabilizer subgroup of $\mathrm{Aut}(X_A)$ fixing the element $k^\times F_I{}^*\mathrm{Im}(\varphi)_*$ of $k^\times\backslash S^*/\mathrm{Im}(\varphi)_*$ such that ${}^{\mathrm{t}}F_I{}^*AF_I{}^{*(q)} = D_I$. Then it follows immediately that

$$ \Gamma = F_I{}^*\mathrm{Im}(\varphi)_*F_I{}^{*-1} \cap \{Q \in \mathrm{GL}_4(k) \mid {}^{\mathrm{t}}QAQ^{(q)} = \lambda A,\ \lambda \in k^\times\}. $$

Hence each element of Γ can be written as $F_I{}^*\varphi_*(g)F_I{}^{*-1}$ for some element g of $\mathrm{GL}_2(k)$ satisfying

$$ {}^{\mathrm{t}}(F_I{}^*\varphi_*(g)F_I{}^{*-1})A(F_I{}^*\varphi_*(g)F_I{}^{*-1})^{(q)} = \lambda A \quad \text{for } \lambda \in k^\times, $$

or equivalently,

$$ {}^{\mathrm{t}}\varphi_*(g)D_I\varphi_*(g)^{(q)} = \lambda D_I \quad \text{for } \lambda \in k^\times. $$

By Lemma 4.2, this equality is equivalent to ${}^{\mathrm{t}}gg^{(q^2)} = \lambda I$ for $\lambda \in k^\times$. Consequently, one has the following isomorphism:

$$ \begin{array}{ccc} \{g \in \mathrm{GL}_2(k) \mid {}^{\mathrm{t}}gg^{(q^2)} = \lambda I,\ \lambda \in k^\times\}/k^\times I & \longrightarrow & \Gamma/k^\times I \\ \cup & & \cup \\ gk^\times & \longmapsto & F_I{}^*\varphi_*(g)F_I{}^{*-1}k^\times. \end{array} $$

By Lemma 4.1, we conclude that $\mathrm{PGU}_2(\mathbb{F}_{q^4}) \simeq \Gamma/k^\times I$.

□

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