論 文 の 要 旨

題目 H_∞-Constrained Dynamic Games for Linear Stochastic Systems with Multiple Decision Makers
（複数の意思決定者を伴う線形確率システムにおける H_∞制約付き動的ゲーム）

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In this thesis, an H_∞-constrained incentive Stackelberg games for stochastic systems with deterministic external disturbances are investigated. As part of the preliminary study, some results of the deterministic system are also presented. Although we focus on continuous-time, we have studied dynamic games for both discrete- and continuous-time systems. In the case of discrete-time, both deterministic and stochastic systems are investigated. One leader and multiple followers are considered for both finite- and infinite-time cases. On the other hand, multiple leaders and multiple followers are investigated for a continuous-time stochastic system. To simplify the calculation, only the infinite-time horizon of continuous-time is emphasized.

In the incentive Stackelberg game, players are divided into two categories; the leader group and the follower group. For a single leader game, incentive Stackelberg strategy is an extensive idea in which the leader can achieve his team-optimal solution in a Stackelberg game. Multiple leaders and multiple followers have made the game more complex and challenging. In the leaders' group and followers' group, players are supposed to be non-cooperative, and Nash equilibrium is investigated. Several theorems and lemmas are designed to study the incentive Stackelberg game problems. For multiple leader games, an incentive structure is developed in such a way that leaders achieve Nash equilibrium attenuating the disturbance under H_∞-constrained. Simultaneously, followers achieve their Nash equilibrium ensuring the incentive Stackelberg strategies of the leaders while the worst-case disturbance is considered. The deterministic disturbances and their attenuation to stochastic systems under the H_∞-constrained is one of the main attractions of this thesis. Problems involving deterministic disturbance must be attenuated at a given target called disturbance attenuation level $\gamma>0$. Surprisingly, the concept of solving the disturbance reduction problem under the H_∞-constrained seems like a Nash equilibrium between the disturbance input and the control input.

In this research, a very general and simple linear stochastic system governed by Ito differential equation has been studied. This thesis studies the most common linear quadratic (LQ) optimal control in the game problems. In order to solve the LQ problem, stochastic dynamic programming (SDP) and stochastic maximum principle are deeply studied. Cooperative game problems and non-cooperative game problems are solved based on the concepts of Pareto optimality and Nash equilibrium solutions, respectively. Several basic problems are completely solved and useful for current research. The main task to solve the LQ problem is to find a matrix solution of algebraic Riccati equations (AREs). Newton's method and Lyapunov iterative method are used to solve such AREs.

However, the main objective of this research is to investigate the incentives Stackelberg strategy, preliminary research and synthesis of LPV systems for multiple players have been done to implement our current idea for LPV system in future. H_∞-constrained Pareto optimal strategy for stochastic linear parameter varying (LPV) systems with multiple decision makers is investigated. The modified stochastic bounded real lemma and linear quadratic control (LQC) for the stochastic LPV systems are reformulated by means of linear matrix inequalities (LMIs). In order to decide the strategy set of multiple decision makers, Pareto optimal strategy is considered for each player and the H_∞-constrained is imposed. The solvability conditions of the problem are established from cross-coupled matrix inequalities (CCMIs). Several academic and real-life numerical examples have also been resolved to demonstrate the usefulness of our proposed schemes.

This thesis consists of seven chapters. In **Chapter 1**, the research background, motivation, research survey, objectives and outlines of the thesis are descried. Some basic definitions and preliminary results are also introduced in this chapter. **Chapter 2** of the thesis summarizes some of the preliminary mathematical problems based on discrete-time and continuous-time stochastic optimal control. In **Chapter 3**, the incentive Stackelberg game for a discrete-time deterministic system is considered. It explains two levels of hierarchy with one leader and multiple followers. Followers are supposed to act non-cooperatively. Exogenous disturbance also exists in the system and is attenuated under the H_∞-constrained. **Chapter 4** investigates the incentive Stackelberg game for discrete-time stochastic systems. The structure of the game is very similar to the previous Chapter 3. It is a single leader and multiple non-cooperative followers with exogenous disturbance which is attenuated under the H_∞-constrained in the 2-level hierarchy. Therefore, Chapter 4 can be viewed as the stochastic version of the deterministic game described in Chapter 3. In **Chapter 5**, the continuous-time incentive Stackelberg games for multiple leader and multiple followers are investigated. The external disturbance is included with the system, as usual. The information pattern of the game is more complex than before. Each leader must achieve Nash equilibrium and use the H_∞-constrained to reduce the external disturbance. Each leader separately announces the declares incentive Stackelberg strategies for each follower. Each follower employs a leader incentive mechanism that follows the Nash equilibrium in a follower group. **Chapter 6** discusses the Pareto optimal strategy for stochastic LPV system with multiple players. In the dynamic game of uncertain stochastic systems, multiple participants can be used for more realistic plants. The system includes disturbances that are attenuated under the H_∞-constrained. Finally, in **Chapter 7**, the thesis is concluded with some motivating guidelines for future research.

In this thesis, two appendices are included. The **Appendix A** discusses how to solve convex optimization problems using linear matrix inequalities (LMIs) and special cases to solve systems and control theory problems. Some preliminary results on static output feedback optimal control are given in **Appendix B**. Here we consider the linear quadratic optimal cost control problem for stochastic Ito differential equations. Several definitions, theorems, and lemmas are studied for future research. To solve the output feedback control problem, Newton's algorithm and corresponding MATLAB codes are already developed. Numerical examples of a very basic problem have been solved. The problem is already formulated for future investigation.