Doctoral Dissertation

The Open Approach and its Impact on Jamaican Elementary Students' Understanding of Mathematical Concepts in the Number Strand: A Gender and Class Setting Comparison

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The Open Approach and its Impact on Jamaican Elementary Students' Understanding of Mathematical Concepts in the Number Strand: A Gender and Class Setting Comparison

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DEDICATION

To my mother Ms. Sheral Gouldbourne and the rest of my family.

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ABSTRACT

Using Jamaican grade four students as participants, the purpose of this research project was to examine the impact the open approach has on the understanding of mathematical concepts in the Number Strand. The study was conducted in order to provide insight into teaching male and female students in different class settings using the open approach. The participants were from two public schools in Jamaica and were organised into co-ed, all-boys and all-girls classes respectively. Both quantitative and qualitative data were used to compare students' responses to an open-ended problem and its impact on their understanding of mathematical concepts. A pre-test was administered to all participants, followed by six months of teaching with the open approach method and the administering of a post-test. Data were gathered from observation of lessons and from assessment of written tests done by the students. The Statistical Package for the Social Sciences (SPSS) was used to assist in descriptive and inferential statistical analyses of the quantitative data. One Way of Analysis Variance (ANOVA) tests were used for comparing mean values of test items. The qualitative analysis was based on the observation notes which were collected throughout the process (Bogan & Biklen, 1998; Creswell, 2009; Marshall & Rossman, 2006). Results from quantitative data were used to answer research question one, two and three while results from qualitative data were used to answer research questions four and five.

Results show that the use of the open approach with open-ended problems had a positive impact on students' understanding of mathematical concepts regardless of gender or class setting. This was evidenced by the fact that all groups had an increase in performance on the post-test when compared with the pre-test, and all were able to produce more solutions at the end of the intervention than they did at the beginning.

Boys showed higher averages and displayed a greater understanding of concepts than the girls did. Girls showed a greater tendency towards using traditional methods but had little understanding of the method they used. Girls obtained higher scores than boys on closed items, but boys obtained higher scores than girls on open-ended items.

With regards to class setting, boys in the co-ed group displayed greater understanding and had more solution methods than boys in the single-sex class, but this was not significant for most items. The girls in the single-sex class showed greater understanding of mathematical concepts than girls in the co-ed class, but this too was varied and had no significance on most items.

It was concluded that the classroom environment created by the open approach that resulted in a positive impact on students' understanding is characterised by student-autonomy, discussion of a multiplicity of ideas, inter-connectedness of concepts, thoughtful reflection and relevance to students' everyday life. The researcher contends that through the synthesis of the findings of this study, teacher educators and educational policy makers can revisit and revise instructional practices so that teachers can better assist students to develop greater understanding of mathematical concepts.

CHAPTER ONE:

INTRODUCTION

1.1 Background to the Problem

Research into mathematics education has typically divided the understanding of mathematics into two types: procedural knowledge and conceptual understanding (Crooks & Alibali, 2014). Procedural knowledge is knowing *how* or recalling from memory the steps required to solve a particular problem. It is mainly seen as correctly applying a predetermined algorithm to achieve an answer. Conceptual understanding is seen "as an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know *why* a mathematical idea is important and they know the "contexts" in which is it useful." (Kilpatrick, Swafford, & Findell 2001, p.118). These students are able to identify operations, and relations between mathematical concepts, and are able to strategically apply them to solve problems while avoiding common misunderstandings (Nohda, 1991, 1995). It is generally agreed that solely having procedural knowledge is not as valuable as having conceptual understanding of mathematics (Boaler, 1998; Crooks & Alibali, 2014; Lin, Becker, Ko & Byun, 2013).

There has been a shift in emphasis from procedural knowledge to conceptual understanding since the beginning of the 21st century (van Ores, 2002). This shift was influenced by changes in contemporary society such as globalisation, increase in technology and increased access to information (MOE, 2013). The increase in the use of computers, calculators, smart-phones and other forms of technology has caused employers to seek school leavers who are able to go beyond the mere memorization of information to the application of their knowledge in a variety of unfamiliar situations. An understanding of the underlying meaning behind a calculation, the knowledge of why a mathematical concept is important and in what context it is useful, are necessary for one to be able to apply that knowledge in an unfamiliar situation (Hoosain, 2001). The teaching for conceptual understanding is therefore important. Moreover, mathematics is seen as an essential part of everyday life which is why it occupies a central place in the education system of most countries and why it is a compulsory subject in most schools. Mathematical skill is not just about the ability to compute accurately, it also requires a proper understanding of the reason for the calculation (Benjamin, 2011; van Ores, 2002). Careers, especially in the fields of Science, Technology, Engineering, and Mathematics (STEM), require personnel to explain, interpret and analyse information. They are expected to evaluate arguments and situations based on evidence and to provide logical reasoning for their decisions. An understanding of mathematical concept is needed to carry out skills such as explaining, creating, analysing and evaluating (Lin, Becker, Ko & Byun, 2013). The teaching of mathematics should therefore go beyond procedural knowledge to include an understanding of mathematical concepts.

The Ministry of Education (MOE) of Jamaica, concerned about the learning outcomes of students, has developed a number of initiatives to facilitate the improvement of students' understanding of mathematical concepts. These include a new mathematics curriculum and new National Mathematics Policy Guidelines. The new mathematics curriculum places more emphasis on teaching for conceptual understanding than its predecessor. The new National Mathematics Policy calls for teachers to use a flexible approach to the teaching of mathematics "so that learners will be encouraged to develop their own strategies for calculating and for problem-solving, which they are able to explain to others" (MOE, 2013, p.10). These changes show Jamaica's intent to align itself with world trend in mathematics education. However, the learning environment in Jamaica is different from that which exists in other countries and therefore has its own unique challenges. Among them is the existing disparity in achievement between the genders (Clarke, 2005; Evan, 1999; MOE Statistics Report, 2012, 2014, 2015; Stewart, 2015; Williams-Raynor, 2011). Any pedagogical approach introduced in Jamaican schools should also be able to manage this phenomenon.

Instructional techniques that develop conceptual understanding in students have been extensively studied in the field of mathematics education, and many suggestions have been given regarding how to develop this skill in students. The main suggestions include Realistic Mathematics Education (RME); Hands-on Approach; Problem Solving Approach and Open Approach. RME which was developed in the Netherlands in the 1970's, focuses on connecting mathematical concepts to the student's daily life (Dickinson & Hough, 2012; Van den Heuvel-Panhuizen, 2014). In the Hands-on Approach, tangible materials are used as aids in bringing the concept across to students (Korn, 2014). Students think about the properties of the object, then deduce connections and relationships as they manipulate the object. The Problem Solving Approach, being promoted in America, encourages students to think about patterns and identify relationships during the process of solving the problem (Kilpatrick et al., 2001; National Council of Teachers of Mathematics [NCTM], 2000; Polya, 2004). The Open Approach or Open-Ended Approach, was developed in Japan and encourages students to create their own strategies for solving open-ended problems (Becker & Shimada, 1997; Nohda, 2000). The situation in Jamaica should be considered before selecting an approach.

1.2 Problem Statement

In 2004, Jamaica's Ministry of Education (MOE) reported that poor performance rates and the increasing gender disparities favouring girls are among the main challenges facing Jamaica's education system (Stewart, 2015).

In 2008, the MOE of Jamaica dispatched 63 mathematics specialists to 413 elementary schools across the island. Some were placed as teachers of mathematics in select schools while others were required to offer support to existing teachers within a cluster of schools. Before the intervention, we observed that lessons were "teacher – centred" and characterised by extensive rote learning. This was later confirmed by reports of the National Education Inspectorate (NEI[1]). In lessons aimed at developing conceptual understanding, teachers gave students both the problem and the heuristic strategy such as "use a table", "work backwards" and "find a pattern", which was to be applied in order to arrive at a solution. The teacher then guided students to carry out the heuristic procedure. These problem solving lessons primarily consisted of a closed question which required all students to apply the same method and to arrive at the same answer. There were no in-depth discussions about the mathematical concept or the reason for applying the given procedure. Additionally, problem solving lessons were normally unplanned, ad hoc activities with questions which had little or no connection to the topic being taught or to the students' everyday experiences. While it was uncertain whether rote teaching was the main cause of the poor performance among Jamaican students, its overuse has certainly contributed to student's lack of conceptual understanding. Stewart (2015) agreed and argued that the prevalence of rote and

[1] National Education Inspectorate (NEI) reports can be retrieved at http://www.moey.gov.jm/moe-publications or at http://www.nei.org.jm/Inspection-Findings/School-Reports

lecture-style teaching in many schools increase issues of inequity in teaching and learning practices. The main types of assessments in elementary school are summative tests with multiple choice and short answer questions (NEI, 2015). Also, the mathematics specialists saw that the teaching of mathematics was driven by assessment. That is, teachers teach for recall and memorization because they see this method as a fast way of preparing students to pass multiple choice tests; however, this form of teaching mainly develops procedural understanding and students tend to quickly forget the algorithms they learn in the rote teaching classroom (Cobb, 1988; Miller, 1997). The mathematics specialists thought that an understanding of mathematical concepts would help students perform better in the subject as well as relieve issues of inequality. As part of the intervention, teachers were encouraged to use open-ended problems relating to the objectives of the lesson. The expectation was that these problems would elicit a variety of responses from students and initiate meaningful class discourse. Discussions could then be used to expand students' understanding of mathematical concepts. The dispatch of specialists to various schools was expected to improve the performance of students in mathematics but was not sustainable due to financial constraints.

In Jamaica, on average, girls do better than boys in schools. This problem has existed for many years (Miller, 1997; Moyston, 2014; Teape, 2015) and has been observed at the primary, secondary and tertiary levels according to the Grade Six Achievement Test (GSAT), the Caribbean Secondary Education Certificate (CSEC) exams, and that more females than males graduate from Jamaican universities each year. This suggests that there are or will be more educated females than males in the society.

In order to bridge the academic gap between boys and girls, some school principals proposed the practice of single-sex classes in co-educational schools. In one such school, the Allman Town Primary, boys in selected grades are separated from girls and are taught by male teachers while their female counterparts are taught by female teachers (Hibbert, 2015). Due to the fact that the phenomenon of girls outperforming boys in mathematics is rare, not many researches into its causes, effects and mitigating strategies have been carried out. However, the limited research into this subject indicates that boys do not benefit from single-sex classes in co-educational elementary school as much as girls do (Jackson, 2002; McFarland et al., 2011; Mulholland et al., 2004). Tyson (2012) argued that most elementary school teachers in Jamaica are not equipped with the skills to facilitate learning among students at varying levels of cognitive development. Tyson examined 23 reports published by the National Education Inspectorate (NEI) and concluded that most teachers used ineffective teaching strategies which are not able to stimulate students to learn. Separating students by gender and applying the same traditional teaching strategies, may not yield any improvement in students' conceptual understanding or reduce the gender gap in mathematics. It is a known fact that students learning styles are different (Matalon, 1994) and this difference transcends gender. All boys do not learn in the same way, neither do all girls. A change to single-sex classes may not be effective without a change in teaching methodology. This study proposes the implementation of the open approach and seeks to examine its impact on the understanding of mathematical concepts among students when they are placed in different class settings.

The open approach was selected from the four recommendations given in the "Background to the Problem" based on three reasons. First, the open approach uses open-ended problems. Open-ended problems can be structured so that students can form connections between the concept they are learning and their daily lives (Meyer, 2010; Pehkonen, 1997). Additionally, tangible materials can be used to help concretize the concept in students' minds while they solve open-ended problem. Not only do these problems aid students to think about patterns and identify relations, but they also allow flexibility in procedures and variety in outcomes. This forces students to think about the relationship among concepts and thereby develop their understanding of them.

The second reason is that open-ended problems give students the freedom to solve the problem in their own unique ways. This feature is important as it alludes to the idea that boys can solve the problem using their own understanding and experience; likewise, girls can solve the problem using their own understanding and experience. This ability has led some researchers to argue that open-ended problems are able to remove bias and create fairness or equity in the classroom (Boaler, 2008; Strong, 2009).

Finally, the unique ability of open-ended problems to promote students' understanding by allowing various possible solutions has made them attractive to teachers in Jamaica and the Ministry of Education has begun promoting the use of these problems in schools (MOE, 2013). It is best to look at the impact this pedagogical approach will have on Jamaican students' understanding of mathematical concepts.

The MOE recommends a change to methodology that focuses on developing conceptual understanding and that this methodology should begin in primary schools (MOE, 2011, 2013). Grade four was chosen as the intervention grade for three reasons.

First, in Jamaica, mathematics is introduced as a discrete subject in grade four. An integrated curriculum, designed to offer a holistic education, is used in grades one, two and three; this makes it difficult to measure the influence of mathematics only, on students' understanding. On the other hand, students in grades five and six are preparing for their school leaving exams, and this would likely have an impact on the outcomes of a research among these students. It was therefore decided that a research conducted among grade four students may give more accurate results.

Second, grade four is seen as a pivotal year for students to grasp fundamental concepts in mathematics. It is at this grade that concepts from the integrated curriculum are reviewed and the foundation is laid for concepts that will be done in grades 5 and 6. The importance of grade four is reflected in it being chosen as the target grade for initiating the implementation of the new mathematics curriculum. While all grades will use it in the near future, students in grades 2 and 4 began learning mathematics with this new curriculum in September 2016.

The third reason for choosing grade four is that only a few studies on open approach have been conducted with participants at this grade level. Chapter three of this dissertation will show that participants in previous researches are mainly from grades one, two, five and six. A research at the grade four level is expected to fill this gap in literature.

The Grade Four Achievement in Numeracy (GAIN) test is given towards the end of grade four. Table 1 shows data from the GAIN test over a six-year period which reveals the two issues mentioned above, (1) girls are outperforming boys in mathematics and (2) approximately 50% of the cohort each year, fail mathematics.

Table 1. Students' Performance on GAIN Test from 2009 to 2014

Year	Number in Cohort	National Average	Boys' Average	Girls' Average	Gender Difference
2009	46,588	45%	36%	55%	14%
2010	46,278	41%	33%	50%	17%
2011	45,769	49%	40%	59%	19%
2012	43,447	54%	47%	62%	15%
2013	42,436	58%	50%	67%	17%
2014	40,981	58%	50%	65%	15%

Source: Ministry of Education: Assessment Division (modified)

The National Mathematical Policy Guidelines highlighted the 2011 GAIN results in strands, as shown in Table 2.

Table 2. Mastery Levels per Strand on the GAIN Test in 2011

STRAND	PERCENTAGE MASTERY
Number Representation	58.2
Number Operation	45.3
Measurement	69.3
Geometry	75.3
Algebra	54.5
Statistics	47.0

Source: New National Mathematics Policy Guidelines 2013, Pg.7.

Table 2 shows that on the GAIN test in 2011, less than half of the students were able to master Number Operation and Statistics. Two years later, in 2013, for both genders, there was a twenty-four percent (24%) increase in the mastery rate on the Algebra and Statistics components, but a significant decline of twelve percent (12%) in the mastery of the Number Representation and Number Operation components from the 2012 scores. In an attempt to explore and address students' weaknesses in these areas, this research will target topics in the Number Strand in grade four. Another rationale for choosing the Number Strand and is that it is the foundation upon which the other strands are built (MOE, 2011, 2013). Number concepts and operations form the foundational understanding in elementary grades which are vital to the further development of Measurement, Algebra, Geometry and Statistics. The overall weakness in mathematics may stem from a weakness in the Number Strand. The topics of focus for this research include types of numbers, counting, the four basic operations of numbers, factors, multiples, equivalency and fractions. In mathematics education, concept image and concept name are two ways in which one can understand a concept. Concept image can be visual representations, collections of impressions or experiences that are evoked on hearing or seeing the concept name. A deeper understanding can be had by relating a concept name to its image. In the open approach, students are exposed to multiple images and expressions of a concept which allows them to form multiple connections with the concept name. The more connections or images of a concept that one has, the greater the potential for understanding such a concept.

1.3 Purpose Statement

The general purpose of this study was to determine if the open approach applied, has a different impact on the understanding of mathematical concepts for boys and girls in grade four classes in rural Jamaica. The specific purposes can be described as follows:

1. To determine if rural elementary school students showed increased understanding of mathematical concepts after being exposed to teaching in the open approach.
2. To compare the responses of students in single-sex class setting with those in co-educational setting and between the genders when they are exposed to teaching via the open approach.
3. To determine the impact of the learning environment created by the open approach on students' understanding of mathematical concepts.

General Question

What impact does the open approach have on Jamaican grade four students' understanding of mathematical concepts in the Number Strand with respect to gender and class settings?

1.4 Research Questions

1. What is the difference between the mean score of open-ended items on the pre-test and post-test for each group of students?
2. What is the difference between genders or class settings in the mean score of closed items compared with open-ended items on the pre-test and post-test?
3. When comparing students' responses for each item on the post-test, is there a difference between the genders or class settings?
4. What gender-specific differences and similarities reflecting conceptual understanding of mathematics, are displayed among students as they respond to open-ended problems in the open approach?
5. How do teachers create a learning environment that supports students' understanding of mathematical concepts in the open approach?

1.5 Significance of the Study

The significance of the study can be discussed in relation to the local and scholar levels. At the local level, the study can be used to enhance the teaching of mathematics and the way Jamaican teachers prepare students for further studies and future jobs. The study is based on a teaching method that is fairly new to Jamaican teachers; therefore, the information provided by the study will provide teachers with enhanced knowledge of the teaching method and how they can apply it during lessons. The study also provides teachers with insight into teaching for understanding and factors that may enhance students' understanding of mathematical concepts. It is also significant to policy makers and stakeholders within the Jamaican education sector because its result can be used to influence future policies. Countries faced with similar problems of students' poor understanding of mathematical concepts may also find this information useful. The data can be used to make informed decisions about the teaching and learning of mathematics.

At the scholar level, the study compares gender responses to open-ended problems by using single-sex class setting comparing with co-educational class setting. To the knowledge of the author, this comparison has not yet been done in the field of mathematics education. This knowledge is worthy of attention as it may impact teaching and learning with open-ended problems.

The research examines the unusual case of gender disparity where females are out-performing males in mathematics, and considers which class setting may be better at reducing the gap in gender performance when the open approach is used in instruction.

1.6 Conceptual Framework

In the present study, students were actively engaged in creating their own solutions to mathematical open-ended problems, thereby reflecting the theory of constructivism. The theory of constructivism argues that humans construct meaning from current knowledge structures and this leads to an improvement in the individual's schema (Brooks & Brooks, 1999; Piaget, 1977).

The conditions required for the establishment of a classroom environment using the open approach were adapted from Nohda's Problem Solving Activities Model (Nohda, 1991, 2000). Nohda explains that lessons in the open approach learning environment are student-centred and that they evolve according to students' intuition, solutions and the nature of class discussions. The background of classroom interactions is formed by the experiences and beliefs of both the teacher and the students and these are in turn influenced by social and cultural factors. The open approach allows for freedom of interaction between the teacher and students, among students, and between a student and mathematics. These interactions facilitate development of conceptual understanding. The connection between the open-ended problems and conceptual understanding is made possible by linking the rubric of open-ended problems with the definition of conceptual understanding (Davis, 2006; Hoosain, 2001).

In the case of Jamaica, conceptual understanding is defined as the comprehension of mathematical ideas, relationships, and meaning for operations and procedures which are revealed in application (MOE, 2011, 2013). The teacher can support students' understanding of mathematical concept by allowing them to (1) create their own strategies for solving a problem, (2) share and explain their strategy with others, and (3) apply learnt concepts both in the world of mathematics and the world of reality. The variations in student-presented solutions triggers discussions. These discussions cause students to interact with each other, with mathematics and with the teacher. Greater understanding of a concept is gained from discussions and interactions. The increase in understanding, in turn, produces an increase in fluency, flexibility, elegance, originality and strategies (Davis, 2006; Hoosain, 2001; Munroe, 2015b, 2016a).

1.7 Definition of Terms

Classroom environment: Refers to the psycho-social surroundings in which students

Interact with mathematics as they solve open-ended problems.

Co-education: Having both male and female students receiving instruction simultaneously. In this study, the abbreviation *co-ed* also refers to co-education.

Conceptual understanding: Conceptual understanding is defined as connection understanding, principles underlying procedures and application. The ability to identify relationships between two concepts and to organise one's knowledge as a coherent whole as well as knowing why a mathematical idea is important and the kinds of contexts in which it can be applied; both in the mathematics world and the real world.

Elegance of solution: The quality of the explanation given for a solution.

Equity: Fairness and inclusiveness; where the possibility for a child to achieve his or her educational potential is not obstructed because of personal or social circumstances such as gender, socio-economic status, ethnicity, or academic ability.

Flexibility of solution: Measures how many different mathematical ideas, principles or properties were discovered by the student. If two solutions or methods have the same mathematical idea, they are included in the same group (Becker & Shimada, 1997).

Fluency of solution: The number of correct responses to a problem that a student produces (Nohda, 1999).

Gender: In this study, gender is used to distinguish between male and female. It is used synonymously with the term “sex”.

Open approach:	A student-centred method that employs an open-ended problem. Students are allowed to choose their own way to solve the problem and to produce various solutions to the problem.
Open-ended problem:	A question with more than one correct solution and/or more than one method for obtaining a solution (Nohda, 1991).
Originality of solution:	The degree to which the students' ideas are unique, or Uncommon (Becker & Shimada, 1997).
Reflection:	The act of self-assessment, self-correcting and self-monitoring. It can be vocalised or applied internally.
Single-sex class:	A class in which students of one gender are educated.
Solution:	The process employed by a student to resolve a problem. The solution process ends with an answer which may or may not be correct.
Strategy:	A method of solution. This may be created by the student or adapted from an existing source.

CHAPTER TWO:

PRIMARY MATHEMATICS EDUCATION IN JAMAICA

2.1 An Overview of the Education System in Jamaica

The Ministry of Education (MOE) is responsible for the education system in Jamaica and is guided by the *philosophy* that "Every Child Can Learn, Every Child Must Learn***".*** The *vision* of the MOE is *"*a customer- centred, performance oriented education system producing globally competitive, socially conscious Jamaican citizens*,"* and the *mission***:** "to provide strategic leadership and policy direction for quality education for all Jamaicans to maximize their potential, contribute to national development and compete effectively in the global economy, as it pursues its developmental goals for the nation*"*. All acts by the MOE, from building new schools to creating new policies, are guided by its mission and vision statements and philosophy.

Currently, there are approximately 792 public and 237 private primary schools in Jamaica (MOE, 2015). Ten of these public schools offer special education for the physically and mentally challenged students. The formal education system has four levels: early childhood, primary, secondary and tertiary. Except for St. Georges Girls Primary and Infant, all primary schools offer co-gender education. It is compulsory for all children to attend school up to the ninth grade, but free education is provided to the end of secondary school. According to the Education Act (1980), a school year has 190 days beginning in early September and ending in June of the following year. There are three terms in the school year, each consisting of approximately 12 weeks. A mid-term break is given approximately every six weeks and lasts for two or three days. End of term breaks are marked by longer periods of two weeks or two months - Christmas and Easter Holidays last for two weeks; summer holidays last for approximately two months. Schools are required to conduct assessments at the end of each term. Some schools also carry out monthly or 6-week periodical assessments in respective subject areas. The medium of instruction is English. The number of instructional hours per school day as stipulated by the regulations should be no less than four and a half hours at the primary, all-age and secondary schools on a shift system, and five hours for whole-day schools. ‘Instructional hours’ refer to the hours that teachers and students are present together in a teaching-learning process. Public schools have lessons on Mondays to Fridays from 8 a.m. to 3:30 p.m. with minor variations based on the idiosyncrasies of each institution. Some schools operate on a shift system to accommodate their large student population. At these schools students are placed in two sets, one set attends school in the morning and the other set in the afternoon. Morning shift runs from 7 a.m. to 12 noon and afternoon shift, from 12 noon to 5 p.m. According to statistics document MOE (2015), the national average of teacher-to-student ratio is 1:32, but 14% of schools have a ratio of 1:42 or more. A child begins early childhood (0 - Kindergarten, Basic or Infant school) at age 3 and may take different paths to tertiary education (6) - see Figure 1. Approximately 81% of teachers in the primary school system are qualified, but rural and remote schools generally have a higher proportion of inadequately trained teachers. Approximately 52% of the schools are in "good" to "satisfactory" condition, and 86% of the students have satisfactory seating arrangements (MOE, 2011, 2015). According to the MOE (2015), the number of children reading at their grade level has increased from 13.5% for boys and 21.9% for girls in 2013 to 37% for boys and 55% for girls in 2015. However, boys are still lagging behind girls.

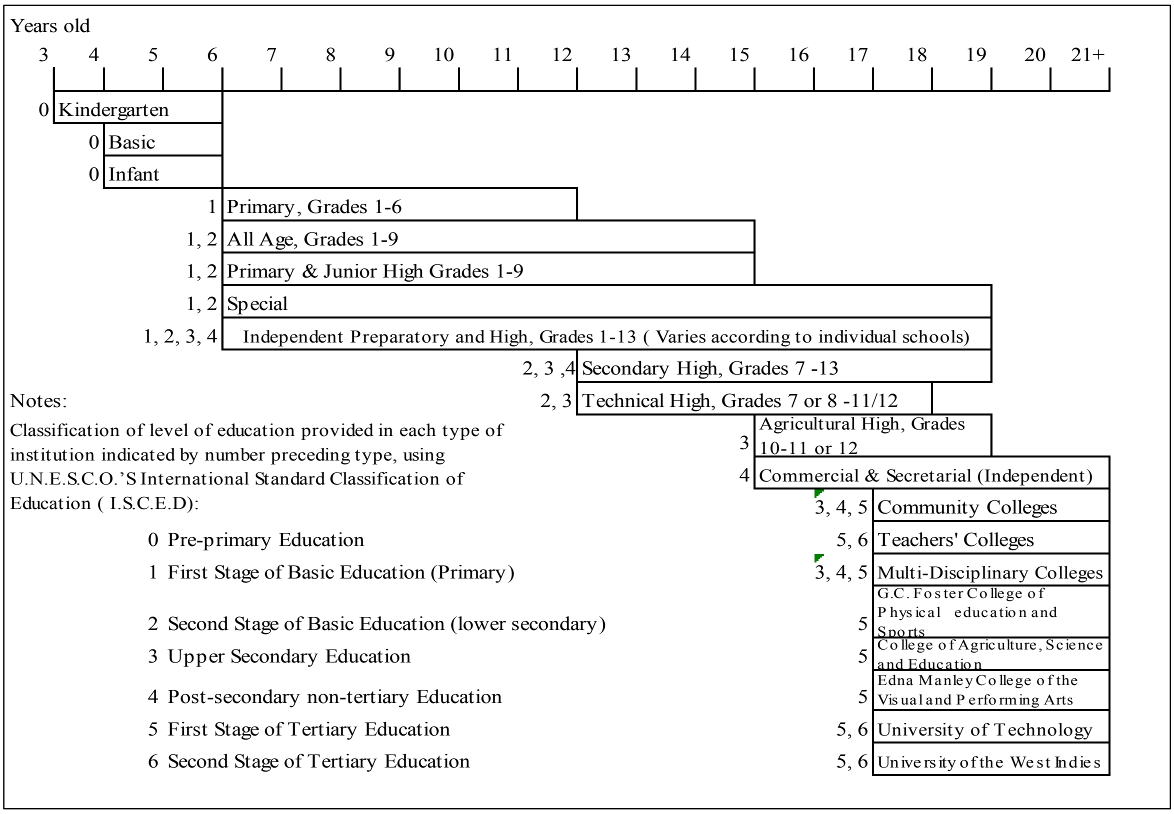


Figure 1. Jamaica's Formal Education System
Source: Ministry of Education Statistics Report 2014/2015 (p.6)

Primary Education

A child begins primary school at age six. In the school year 2014/2015, the enrolment rate at primary school was 99.6% and attendance rate was 86.3% (85.4 % boys and 87.3 % girls). Reports have shown that the attendance rates for girls have been consistently three to four percentage points higher than those for boys over the years (MOE, 2012, 2015). Attendance rates also tend to be higher in urban areas. The basic curriculum in primary schools includes Reading, Language Arts (English), Mathematics, Social Studies, Science, Art, Music, Religious Education and Physical Education. Generally, students are guided in all subjects by a single teacher and they occupy the same classroom for that school year. In the 2014/2015 school year, approximately 27 % of schools had 20 students or less within a class and 10 % had 35 students or more. This shows a decrease in the number of over-populated schools from previous years. On average, approximately 96% of enrolled students complete primary schooling each year (MOE, 2012, 2015).

2.2 Mathematics Education in Primary Schools

The content of mathematics to be taught in Jamaica is laid out in four national curricula at the primary level. These include: the integrated curriculum for lower primary of grades one to three, and discrete curriculum for grade four, grade five and grade six respectively. This section gives a brief description of the integrated curriculum followed by in-depth analysis of the grade four curriculum.

2.2.1 The Integrated Curriculum at Grades One to Three

The integrated curriculum spans grades one, two and three. Designed for teaching the child as a holistic individual, the curriculum is organised into themes rather than under subjects. The general theme of the integrated curriculum is “Me and My Environment.” The theme is separated into sub-themes (e.g. my body, my family, and my community) which are further divided into teaching units. Content and objectives for sub-theme and teaching units are organised for each term in the school year. Students learn mathematics content as it emerges from the integrated lesson. For example, when grade one students are learning about “The Body” they are required to count different parts of their body, example their eyes, ears and fingers. This type of teaching clearly shows the connection of mathematics to other subjects’ areas and its use in daily life. Topics and objectives in mathematics to be covered in the integrated curriculum are listed, in strands, at the end of the curriculum guide under grade level headings. Topics in mathematics that do not readily emerge from integrated lessons are taught separately in the "Mathematics Window". The Mathematics Window is a teaching session which concentrates on numeracy only. It lasts for 60 minutes and is used for:

- Introducing concepts necessary for learning in a particular area of the integrated content.
- Reinforcing concepts previously introduced through an integrated lesson.
- Introducing areas not included in the integrated content.

The integrated curriculum guide provides objectives, activities and suggestions for assessment of learning, but the methodology is left to the individual teacher. Students sit a diagnostic test on completing the integrated curriculum in grade three. The test consists of multiple-choice items designed to evaluate knowledge of, and competence in using number related concepts. An analysis of test results over a number of years reveals low-level achievement in students' scores which can be interpreted as a lack of understanding of mathematical concepts (MOE, 2013).

2.2.2 The Grade Four Mathematics Curriculum

The curriculum reform of 1999 for grade four, places emphasis on literacy and numeracy of all students. It was designed to be delivered in such a way that empowers students to make connections between what they are learning in school and activities in the outside world. Organisation of the content and objectives in the grade four curriculum guide are similar to that of the grades five and grade six curriculum guides.

Content in the Grade Four Curriculum

The content in the grade four curriculum is organised into two sections. The first section presents information in columns under the headings of "*focus questions*", "*attainment* *targets", "objectives"* and *"vocabulary/concepts"*. Focus questions define the scope and sequence of the topics in a unit so that essential concepts in each unit are covered. Attainment targets are what students of different abilities should know and understand at the end of the unit. Objectives are obtained from attainment targets. Vocabulary is the list of mathematical terms that each student is expected to know at the end of the unit. The vocabulary list may also contain concepts to be covered within the unit.

The second section is called "*activity plan*"; it gives suggestions for achieving objectives of each focus question in five categories: "a*ctivities", "skills", "assessment", "evaluation" and "materials/resources"*. The content is organised under the teaching units for each term and the topics to be covered within a teaching unit over a specific time frame are listed under five strands: Number, Measurement, Geometry, Algebra and Statistics with Probability. A comparison of content in each strand, based on attainment targets to be covered at each grade level, is shown in Figure 2**.**

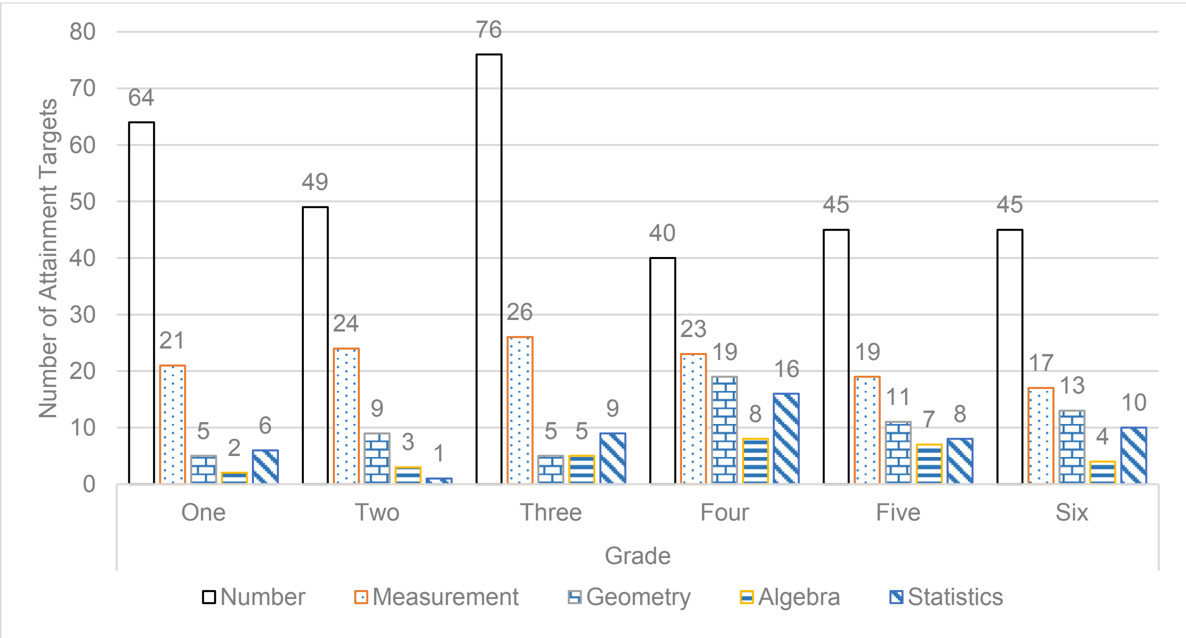


Figure 2. Distribution of Attainment Targets in Mathematics for Grades One to Six

An analysis of Figure 2 shows that more emphasis is placed on the Number Strand at each grade level. This is because it is the foundation upon which the other stands are built. At the grade four level, the number of attainment targets for the Number Strand is about two times or more of the other strands. The Number Strand also dominates more than 50% of the contact time in a school year, followed by Measurement, Geometry, Algebra and Statistics. An analysis of the Number Strand in the grade four curriculum reveals six major topics that are broken down into attainment targets: Place value (3-attainment targets); Addition and Subtraction ideas-2; using the calculator-3; Multiplication and Division ideas-12; Problem solving-6; and Fractions including decimal forms-14. Topics in the curriculum are ordered in sequence with each objective being a prerequisite for the next. Students are expected to master all the content of a topic before moving to the next topic. Topics in the Number Strand spiral across the curricula, so the contents are revisited each school year at increasing depth and complexity. Content not learnt in previous grades are re-taught, often resulting in the teacher not being able to complete the curriculum for the current year and students move on to the next grade without learning its full content. The section of the curriculum guide that focuses on the Number Strand for term one is provided in Appendix H.

Objectives in the Grade Four Curriculum

The grade four curriculum defines objectives as the measurable indicators of what students should be able to do after completing a specific lesson or set of lessons. It provides four relational contexts for writing objectives to teach a new concept: (1) relate the new concept to previously learnt ones; (2) relate the new concept to visual models; (3) relate the new concept to students' real life experiences; (4) relate the new concept to other subject areas. However, no suggestion is given on how a teacher may link objectives to these areas.

The verb used in an objective is important because one can determine the degree of cognitive rigour of a lesson by examining the verb used in the lesson's objective(s). Benjamin Bloom gives a theoretically-based taxonomy framework in the cognitive domain for verbs used in writing objectives. Stemming from the idea that the verb indicates the level of thinking expected of students, Bloom listed six hierarchical thinking levels under the headings of: knowledge, comprehension, application, analysis, synthesis, and evaluation (Krathwohl, 2002). Teachers in Jamaica refer to Bloom's list of verbs when writing lesson objectives. The verbs used to describe the cognitive demands in the Number Strand along with an analysis of such verbs using Bloom's taxonomy as assessment framework are shown in Table 3. The list of objectives in the Number Strand for term one in the grade four curriculum, is given in Appendix H.

Table 3. Verbs used in the Number Strand in the Grade Four Curriculum

Bloom's Taxonomy Level	Number of Verbs (One verb per objective)	Example of Verbs
Knowledge	7	Identify, name, define
Comprehension	2	Distinguish, recognize
Application	4	Apply, discover, compute
Analysis	3	Investigate, solve
Synthesis	2	Write, compose
Evaluation	3	Estimate, select

Table 3 shows that except for knowledge (33.3%) and application (19.5%), verbs used in writing objects are approximately equally distributed across Bloom's taxonomy levels; comprehension, 9.5%; analysis, 14.3%; synthesis, 9.5 %; and evaluation, 14.3%. These skills should allow students to think in a convergent and divergent manner, to investigate challenges and create solutions to new problems as well as to think in complex and creative ways (O'Tuel & Bullard, 1995). However, the implemented mathematics curriculum in Jamaica is different from the intended one (Buddo, 2015).

Jamaican teachers are bound by the curriculum. They are expected to complete all curriculum units before national tests are administered. Results of national tests show that some content in the curriculum are not adequately covered which may indicate that teachers are not implementing it as suggested. This argument was confirmed by Buddo (2015) who used a questionnaire to capture the reflections of 37 teachers from 15 schools on six areas of implementing the primary mathematics curriculum: Personal Attributes, Teaching Methods, Classroom Learning Environment, Classroom Management and Control, and Assessment and Feedback. The teachers' reflections indicate that they are aware of their own abilities and inadequacies to effectively deliver the mathematics curriculum. Teachers consider themselves to have good class control but continue to employ traditional methods. Buddo stated that "there is room for improvement in the methods for teaching and the techniques for assessing students' understanding" (p.199). Teachers also listed school administration, the curricular content, and students' disposition towards mathematics as other factors that negatively impact the implementation of the grade four mathematics curriculum. The following were the main issues raised by the teachers about the grade four mathematics curriculum:

- "The curriculum guide does not cater to the students who are performing below the grade level.
- Ways of teaching mathematics are not suggested in the curriculum document.
- Resources stated in the curriculum document are not provided at the school.
- Activities are limited and often unclear.
- The lessons and objectives are not set out in a systemic way e.g. geometry is scattered over the three terms.

- Suggested instructional materials are not provided by the Ministry of Education." (p.207).

2.2.3 Curriculum Reform of 2016

This study was conducted between 2014 and 2015. Since then, a new curriculum has been developed. Implementation of the newly revised curriculum began in September 2016 starting with students in grades two and four. The outline of the revised curriculum is different from its predecessor in that objectives and standards for each strand are shown in parallel columns for grades 4, 5 and 6. Also the heading called *evaluation* is replaced by *learning outcomes* and *objectives* replaced by *benchmark* in the new revised curriculum. Suggestions given for activities in the revised curriculum are more focused on developing conceptual understanding. The new curriculum provides suggestions and guidelines on how the teacher can (1) relate new concept to previous learnt ones; (2) relate new concept to visual models; (3) relate new concept to students' real life experiences; (4) relate new concept to other subject areas. A comparative analysis highlighting changes in curriculum reform of 2016 from the curriculum reform of 1999 was carried out by the researcher and is shown in Table 4.

Table 4. Comparison of Curriculum Reform of 1999 with Curriculum Reform of 2016

Category	1999	2016[2]	Change
Aim	To produce citizens who are numerate and guided by moral and social principles.	To produce successful lifelong learners who are proud citizens of Jamaica.	More emphasis is placed on teaching for conceptual understanding.
Topics	The word "topic" is not used. Teachers are required to deduce topics from attainment targets and objectives.	• Sets • Number value • Fraction ideas • Estimation Mental calculation.	"*Sets*" is introduced and placed as the first topic in the first term. The significance of this was not given.
Objectives	Verbs are stated at all levels of Bloom's Taxonomy.	More verbs included at the higher cognitive level in Bloom's taxonomy.	Direct link between objectives (Benchmarks) and activity is given.
Content	Covers all strands in mathematics.	Increase in content for all strands.	Detailed information on how content is related to other subject areas and real life are included.
Method	Suggest the use of various methods for teaching.	Suggest methods that emphasized concept building and show the relevance of mathematics.	Cautions teachers about the danger in using traditional method only.
Number Strand	No separation of sections in the number strand.	Two sections shown 1. Number representation 2. Number operation and application.	Provides information on websites that support teaching.
Assessment	Test, portfolios and projects.	Increase in the variety of ways to assess students.	Encourages varying assessment methods including formative and summative.

Table 4 shows that many changes have been made to the 1999 curriculum reform. These modifications are reflective of the changes in the society over the past 17 years; therefore, most of these changes are geared towards a shift in teaching from procedural to conceptual understanding. However, the newly revised curriculum did not address all comments given by teachers in Buddo (2015); for example, the curriculum still does not cater to students who are performing below the grade level.

[2] This is a draft version of the 2016 curriculum reform.

2.2.4 Teaching Method

According to the NEI's Chief Inspector Report (2015) and as stated by teachers in Buddo (2015), the expository method also known as the traditional approach is the most common method used for teaching mathematics in Jamaica. Other studies that have described this form of teacher-centred teaching are Buddo (2013), Davis (2004) and Williams (2008). Using information from these studies, the pattern of a typical lesson in Jamaica can be described as follows: The teacher introduces the lesson with a story, demonstration, recap of previous lessons or some other preferred method. The teacher outlines the objectives of the lesson, gives definition(s) and shows one or two examples of the procedure to be followed in solving the given closed problems. Different props, including hands-on materials may be used during explanation of the algorithm. The teacher lists similar problems on the board for students to solve. The teacher sits at his/her desk and waits for students to solve the problems. The teacher may occasionally move around the room to view pupils' work. Students follow the given algorithm to solve the problems listed on the board. On completing the list of problems, students carry their books to the teacher to be marked. Correct answers are marked by with a tick (✓) and incorrect answers are marked by an ex (×). The teacher points out the error and the child returns to his or her seat to re-do the problem. After the majority of students attempted the problems, the teacher shows the solutions on the board and these are copied by students. The teacher answers students' questions and in some cases re-explains steps in the algorithm. The board is erased and a new list of problems is given. The teacher returns to his or her seat and students solve the problems. This cycle of listing problems on the board for students to solve using a set algorithm continues for the duration of the lesson.

Buddo (2013) shows results of a surveyed carried out with 899 grade four students from 15 schools in urban Jamaica. The purpose was to obtain information on students' views about the teaching of mathematics in school. Students' perspectives were sought based on six indicators of teaching: Personal Attributes of the teacher; Planning and Preparation; Teaching Methods; Classroom Learning Environment; Classroom Management and Control; and Assessment and Feedback. The students had positive perspectives for all six indicators; however, they scored the teachers lowest in the areas of *Teaching Methods* and *Assessment and Feedback,* and highest on *Personal Attributes* and *Classroom Management and Control*. This result shows that the students think that there is a need to improve or modify the method used for teaching mathematics.

2.2.5 The Grade Four Achievement in Numeracy test.

The Grade Four Achievement in Numeracy (GAIN) was first implemented in 2009. It is administered towards the end of the third term in the school year. All students in grade four are expected to sit the test at their respective school. All grade four students across the nation sit the test on the same day. The test consists of two papers; paper one has multiple choice items and paper two has short answer items. The test is designed to assess three strands; Number Representation and Operations, Geometry and Measurement, and Algebra and Statistics. The test ranks students in three groups: mastery (a score of 50% or above in all strands); near master (a score of 50% or above in 1 or 2 strands); and non-mastery (less than 50% in all three strands). Data provided by the Ministry of Education shows that each year, girls outperformed boys on the GAIN test and approximately 50% of students who sit the test are able to achieve mastery (see Table 1).

2.3 The New Mathematics Education Policy

The new National Mathematics Policy Guideline (2013) outlines a framework for implementing, monitoring and evaluating mathematics teaching and learning in the Jamaican schools. As its rationale, the policy states that the "lack of mathematical understanding has been, too often, reflected in the unsatisfactory performance of students of mathematics at all levels of the education system. Poor attitudes to the subject are also very evident among many students, and some view mathematics as being of little use to them outside of school" (p.5). In order to move towards a positive change, the document provides *guidelines* and *standards* for the teaching and learning process, teacher education and the respective roles that each group of stakeholders should play in mathematics education. *Guidelines* are statements suggesting how teaching should be carried out in schools. *Standards* are expectations that must be met in order to fulfil the guidelines. Stakeholders are the individuals who have roles to play in maintaining the standards and carrying out the guidelines. Listing the headings and sub-headings from the policy's Table of Contents produces the summary of areas which the policy addresses. This summary is shown in Table 5.

Table 5. Sections and Sub-sections of the New National Mathematics Policy Guidelines

Guidelines	**Standards**	**Stakeholders**
Teaching approach	Qualifications to teach mathematics in Jamaica	The Government of Jamaica
Planning for instruction	Components of the mathematics curriculum	The Ministry of Education
Assessment	*The teaching approach*	The Jamaica teaching council
Reporting	*Teacher education*	Teacher training institutions
Teaching time		School leadership
Teacher education programmes		Parents and the community
		Learners

Many of the guidelines and standards given in the mathematics policy are reinforcements of what already exists in other policies on general education. For example, teaching time for a lesson remains at one hour per day. However, the policy provides more detailed guidelines and standards on *Teaching approach* and *Teacher education* for mathematics education in Jamaica. Policy guidelines and standards for teacher education are discussed in the section entitled "Teacher Education" of this chapter. With regards to the teaching approach, the policy recommends that each mathematics lesson be guided by its own lesson plan; that the teaching of mathematics be student-centred, and that teaching be focused on understanding, applying and communicating mathematical ideas. It promotes three principles that should guide each lesson: conceptual understanding, computational fluency and problem solving skills. Conceptual understanding will be discussed since it is the focus of this dissertation.

Conceptual Understanding

Conceptual understanding as defined in the policy and other ministerial documents such as The National Comprehensive Numeracy Programme (2011), is the comprehension of relationships, underlying structure and meaning for operations and procedures which are revealed in application, both in the world of reality and in different domains in mathematics. In order to develop conceptual understanding in students, the policy recommends that teachers use "appropriate teaching methodologies which are underpinned by the notion of constructivism and which focus on understanding and the development of skills and processes rather than number crunching and the memorization of facts and formulae" (p.11). It further states that classroom activities should focus on the development of mathematical processes, and that learners should be equipped to represent mathematical ideas in ways that make sense to them. This is best done in an interactive classroom setting and which gives students the opportunity to solve problems using their own strategies. For example, in adding 45 and 27, a student may arrange the numbers vertically and add corresponding digits.

This results in the need to rename 12 (the result of adding 7 ones and 5 ones) as one group of 10 and 2 ones. Another student, being aware that 45 needs 5 more "ones" to make 50, may remove 5 ones from the 27, add them to the 45 and solve for "50 + 22". Both approaches show the underlying concept of place value. Discussing these two strategies helps students to see the similarities between them and to better understand the place value concept. Students with conceptual understanding are able to appreciate the fact that 7 + 5 can be represented as 5 + 2 + 5 and as 10 + 2. More advanced students may be able to see that 45 + 27 gives the same result as 50 + 22. This not only develops conceptual understanding, but computational fluency as well.

The policy states that understanding of a mathematical concept is revealed in its application both in everyday life and in mathematics domains. To explain this point, the policy gives the following example: "in teaching number, students will be involved in activities that reflect everyday use of numbers so that they become not only aware of *how* numbers are used but also *why* they are used. This will aid their *understanding* of numbers. Further, they would be encouraged to demonstrate this understanding by *applying* this knowledge in a variety of situations and to *communicate* their understanding in different ways, both orally to their peers and to the teacher as well as in written form" (p.11).

The information given in this policy can have far-reaching effects on the teaching of mathematics in Jamaica. The main concern, however, is that teachers and other stakeholders may not be aware of this policy and those who are aware of it may not see it as important. Case in point, one of the three teachers in this study (conducted in 2014/2015) was unaware of the existence of the policy, even though it was received by the school in 2013.

Welsh (2012) argues that the poor implementation of policy in Jamaica negatively affects the development of its education system and by extension its economy. Welsh compared the education systems of Jamaica and Singapore from 1960 to 2010. He explored critical factors in education development between the two countries and identified three significant factors that impacted on the development of their education system. First, the timing of reform is important, not just the content of the reform. Second, having a vocational strategy is significant. Third, having a balanced, forward-looking education development strategy that closely ties education, economic development and national development. He argued that these factors greatly contributed to Singapore's GDP (Gross Domestic Product) per capita moving from $4,383 to $55,862 and the lack of them contributed to Jamaica's GDP moving from $6,417 to $8,539 during the same period. It can also be said that the lack of awareness of developed policies has had damaging effects on the education system in Jamaica. It is important for the government to develop new policies for improving the lives of its citizens, it is even more important for such policies to be implemented.

2.4 Teacher Education

In general, a minimum of 5 Caribbean Secondary School Certificate (CSEC) subjects including Mathematics and English are required for an individual to be accepted in a teacher training institution. The policy recommends an increase in qualification by stating that applicants to be trained as teachers of mathematics should obtain scores in the upper percentile range of the CSEC Mathematics examination. The policy also stipulates that all candidates must sit a diagnostic test with open-ended problems so as to identify gaps and misconceptions in their understanding of mathematical concepts. These gaps and misconceptions must be filled and corrected respectively before the candidate is allowed to graduate from the programme.

According to the MOE statistic report 2014/2015, there are 13 public institutions that offer degree courses in education. Ten of these institutions offered a four-year degree in primary education and eleven offer a degree in Mathematics Education for secondary schools. Table 6 shows the institutions that offer degrees in Primary Education and the number of candidates enrolled for the year 2014/2015. Most of these institutions also offer training in other areas including non-education fields.

Table 6. Enrolment at Teacher Training Institutions for the School Year 2014/2015

Name of Institution	Male	Female	Total	Type of Degree
Bethlehem Teachers' College	22	184	206	Primary Education
Church Teachers' College	11	67	78	Primary Education
Knox Community College	7	44	51	Primary Education
Mico Teachers' College	25	370	395	Primary Education
Sam Sharp Teachers' College	37	236	273	Primary Education
St. Joseph Teachers' College	24	373	397	Primary Education
Moneague Community College	27	213	240	Primary Education
Edna Manley College of Arts	156	185	341	Primary Education
C.A.S.E.*	16	129	145	Primary Education
UWI**	440	1395	1835	Primary Education

- ** C.A.S.E. means College of Agriculture and Science Education, **UWI means University of the West Indies.*

The information in Table 6 reflects the typical enrolment ratio of male and female candidates each year. As can be seen, there is a low registration of male candidates. Most males shy away from teaching due to its low salary and the stigma of it being a female profession (Campbell, 2015).

Shulman (1987) argues that in producing an effective lesson, a teacher must have at least three types of professional knowledge: content knowledge, pedagogical knowledge and knowledge of students. All three can be seen within each teacher education programme in Jamaica. The programme of study for a degree in Mathematics Education contains main courses in mathematics, education and pedagogy. Mathematics for primary school teachers in training covers content up to the grade nine level in all strands. The policy recommends that teachers re-learn these topics from a conceptual perspective so as to improve their knowledge of the mathematics content. Effective Pedagogical Content Knowledge (PCK) of teachers is necessary for developing students' conceptual understanding in mathematics (Mansor, Halim Osan, 2010). In Jamaica, teachers in training are taught a wide range of teaching approaches and methodologies including hands-on approach, expository approach, and corporative grouping method and so on. Teachers are also trained to carry out student-centred lessons and are required to use such knowledge in their teaching practice.

Teachers in training participate in a practicum course where they are assigned to a class in a public school to teach on a daily basis for three consecutive months. Primary school teachers are required to have at least 60 hours in the teaching of mathematics. Secondary school teachers are given more than one class and should have at least 180 contact hours. The teacher in training is guided by an in-house lecturer; and are assessed towards the end of their teaching practice by at least two external lecturers. The policy guidelines state that all primary school teachers in training must be observed at least three times by a mathematics education specialist towards the end of their practicum experience and the grade given must be used in calculating the final grade for the practicum course. Pre-service teachers must receive a pass in their practicum course before they receive their degree in education.

There is no licensing of teachers in Jamaica. Teachers apply for jobs based on the requirements of the institution and the qualification they have been awarded by the Joint Board of Teacher Education (JBTE). Due to insufficient staff, some schools accept individuals without a degree in education. These individuals are referred to as untrained teachers. At the primary level they may be individuals with 5 or more passes in CSEC examinations. At the secondary level, they may have a degree in another field. For example, a school may hire an accountant to teach accounts or mathematics. Trained instructors are teachers who have acquired professional training in teaching a skill such as electrical installation. Figure 3 shows the distribution of teachers for the school year 2014/2015.

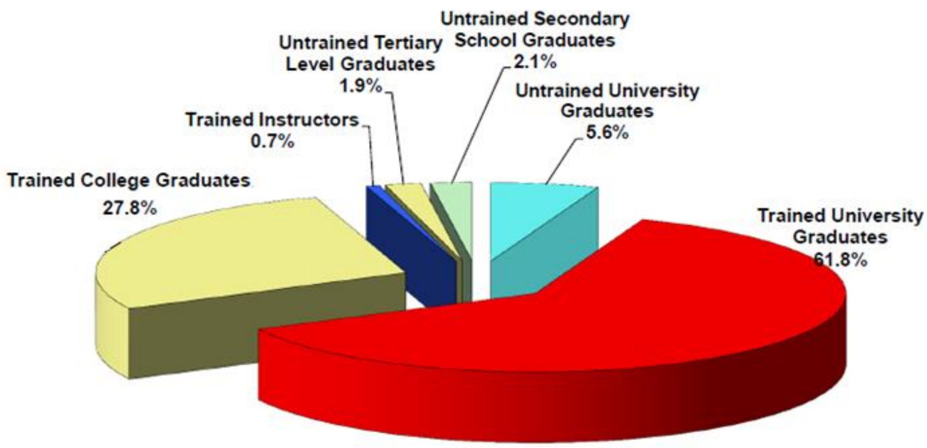


Figure 3. Distribution of Teachers by Qualification
Source: Ministry of Education Statistical Report 2014/2015 (p.23).

Figure 3 shows that the 90.3 % of teachers in the public sector are trained. The ratio of trained to untrained teachers increased from 7 to 1 in 2013/2014 to more than 9 to 1 in 2014/2015. While this is true for the overall number of teachers, a different situation exists for mathematics teachers. There is an acute shortage of graduates trained in the field of mathematics as there has been a constant brain drain of teachers of mathematics from the education system. Between January and October 2015, more than 70 teachers of mathematics and science left Jamaica to work in other countries (Campbell, 2016).

CHAPTER THREE:

LITERATURE REVIEW

This study proposes a consideration of the impact of the open approach on students' conceptual understanding with regard to gender and class setting. The related literature, presented in this chapter, is organised into four sections.

- Section one looks at the open approach and open-ended problems.
- Section two focuses on conceptual understanding and how it is developed through the open approach
- Section three examines gender specific responses to open-ended problems and the influence of Jamaica's culture on mathematics education.
- Section four discusses the implementation of the open approach in the regular classroom and the environment which is to be created in the open approach classroom.

3.1 The Open Approach and Open-ended Problems

3.1.1 The Open Approach

The open approach is a student-centred pedagogical method which was first implemented in Japan in the 1970s (Becker & Shimada, 1997). At that time it was referred to as the open-ended approach. Ikeda (2010) explains that research in the open-ended approach began in 1971 with the main purpose of evaluating students' higher order thinking skills. In addition to evaluating students, the open-ended approach also had the potential for enhancing higher order thinking skills. Between 1974 and 1976, research into the open-ended approach was simultaneously carried out in six different areas across the country and the focus changed from using it as a tool for evaluation to a strategy for the development of higher order thinking skills and conceptual understanding in students.

Nobuhiko Nohda, a member of the original group of researchers, extended the open-ended approach by describing openness in the classroom in terms of student activity, mathematical activity and both student and mathematical activity (Nohda, 1995). He called this extended open-ended approach the *Open Approach* (Nohda, 1991). According to Nohda (1995), the open approach focuses on three main situations: formulating a mathematical problem, investigating various approaches to a formulated problem, and posing more advanced problems relating to the first. These three situations can be considered independently or collectively depending on the teacher's intention or objective for giving the problem. For example, in formulating a problem, the teacher may show students a picture of various items, differently priced, and ask them to formulate mathematical problems using the information in the picture. Alternatively, students may be presented with a problem and asked to solve it using different approaches, or they may be asked to provide more than one solution to the problem. Finally, having solved a given problem, students could be asked to create extensions to the problem or to develop similar problems.

The term "Open Approach" became known in the western world in the 1980s when the Cockcroft Report of England and National Council of Teachers for Mathematics (NCTM) under the banner of "An Agenda for Action" in America began promoting problem solving strategies in the classroom (Pehkonen, 1997). Since then, much research has been conducted into the open approach, and researchers have debated whether the term "Open Approach" or "Open-Ended Approach" should be used. Lin, Becker and Byun (2013) state that in the western world, the terms open approach and open-ended approach have become synonymous in the field of mathematics education. Since 1995, most papers published on the subject have used the term “open approach” to describe the teaching strategy and the term “open-ended problem” to describe the type of questions used in the teaching strategy (Chan, 2007; Lin et al., 2013; Nohda, 1995; Silver, 1995) and these terms are similarly employed in this study. Researchers such as Nohda (1991), Silver (1995) and Pehkonen (1997) agree that the open approach develops an understanding of mathematical concepts in students and have called for more research to be carried out in this area.

3.1.2 Open-Ended Problems

Open-ended problems in mathematics, are questions or statements formulated to have multiple solutions and one or more methods for obtaining a solution (Becker & Shimada, 1997; Nohda, 1995). Nohda (1991) explained that open-ended problems allow many possible avenues for different solutions and discovery of new approaches by combining previously learnt knowledge. Nohda (1995) described two characteristics of open-ended problems. Firstly, they should evoke in all students, an interest in solving the problem. The open-ended problem should be flexible and should take into account, students’ different mathematical abilities and allow various solutions at diverse levels. Students of varying abilities, believe that they have the prerequisite knowledge necessary for arriving at a solution and they experience a sense of achievement after solving the problem. Secondly, open-ended problems should be suitable for generating mathematical understanding and should also be easy to generalise into new problems. That is, in solving the problem, students gain more understanding of the intended concept within the problem, and how it is connected with other concepts. In most cases, this growth in understanding ignites the curiosity in students to explore the problem more deeply. Further exploration of the problem also leads to more discoveries and increased understanding. The focus of open-ended problems is not necessarily about arriving at a final answer, but on what can be learnt during the solution process. In this study, two types of problems are considered to be open-ended; (1) problems with multiple solutions (Becker & Shimada, 1997), and (2) problems with one correct answer but multiple methods of obtaining the answer (Nohda, 1995; Pehkonen, 2014). Examples of open-ended math problems promoting number sense at the grade four level are:

1. *Which number does not belong to the group and why? 2, 8, 9, 18* (Focusing on multiple solutions).
2. *Add 195 and 27 in at least two different ways.* (Focusing on method).

Open-ended problems foster understanding of mathematical concepts as students cannot rely on pre-determined rules or on memorization to provide answers (Silver, 1995). In solving open-ended problems, students are engaged in multiple intellectual processes such as exploring, testing, discussing, connecting, criticising, communicating, investigating, generalising , Hypothesising , and using reasoning to explain different ideas (Kwon et al., 2006; Lin et al., 2013; Sullivan, 2009). Pehkonen and Ahtee (2005) discussed the use of the open-ended problems on a world-wide scale while Zimmermann (2010) described the development of the open approach in Germany over the past twenty years. Both papers deliver a broad picture on the use of open-ended problems and suggest that further research be carried out on the subject.

3.1.3 Description of the Open Approach Lesson: The Case of Japan

In Japan, an open approach lesson usually begins with the teacher posing an open-ended problem. Students work individually, then in groups to discuss and solve the problem. This is followed by presentation and reflection on the various solutions presented (Becker & Shimada, 1997). Students are asked to integrate previously learnt knowledge, individual skills, and their own learning styles, in order to solve the problem. Different features of the regular Japanese lesson, *Hatsumon, Kikan-shido*, *Bansyo*, *Neriage,* and *Matome*, usually appeared in the open approach lesson (Munroe, 2015; Stigler & Hiebert, 1999; Ueda, 2011). *Hastumon* means questioning in Japanese and mainly refers to the manner in which the open-ended problem is introduced to the class and the types of questions that the teacher uses to elicit information from students. That is done to stimulate conjectures and understanding in students as they solve the problem. After posing the question, the teacher listens to conjectures from students before allowing them to proceed in solving the problem. *Kikanshido* means instructions at student's desk. There are two types of observations: observation of students working on the problem while walking around the room, and observation made during discussions with students. The teacher walks among the desks, carefully observing and providing guidance to students where necessary. Guidance is given to help students proceed in their own way of understanding and chosen method. Errors and misconceptions are also corrected during this time. The teacher mentally records the different types of solutions observed and purposefully calls on students to write varying solutions on the board. *Bansyo* is the word used for "writing on the board". Ueda (2011) states that, in *Bansyo*, the teacher arranges students' ideas so that students can clearly see the mathematical relationships among them. Therefore, the teacher writes or guides students to write on particular sections of the board. Students are then asked to discuss the different solutions given on the board. Discussion of multiple solutions is a prominent characteristic of the open approach. Discussing the connections between these solutions provides students with more insights into the relationships between the mathematical concepts involved. *Neriage* is the term used to describe the organisation of students' ideas from discussions.

The lesson concludes with *Matome,* where the teacher and students summarise the main points of the lesson and link them to the objective of the lesson.

In Japan, open-ended problems help in the development of students' understanding within the world of mathematics. Becker and Shimada (1997) describe using open-ended problems in Japanese classroom with the aid of a diagram, see Figure 4, and explained that in solving open-ended problems, one takes "the path from (f) to (g) or from (n) to (o) without considering the real world situation (a)" (p.8).

In Jamaica, the definition of conceptual understanding includes seeing the connections among concepts in the mathematics world and being able to apply a concept in the real world. The solution process should therefore include consideration of real-life situations. In order to reflect this aspect of the research, the figure used by Becker and Shimada (1997) to explain the practice in Japan was modified to show the research's proposal for the Jamaican context - see Figure 4.

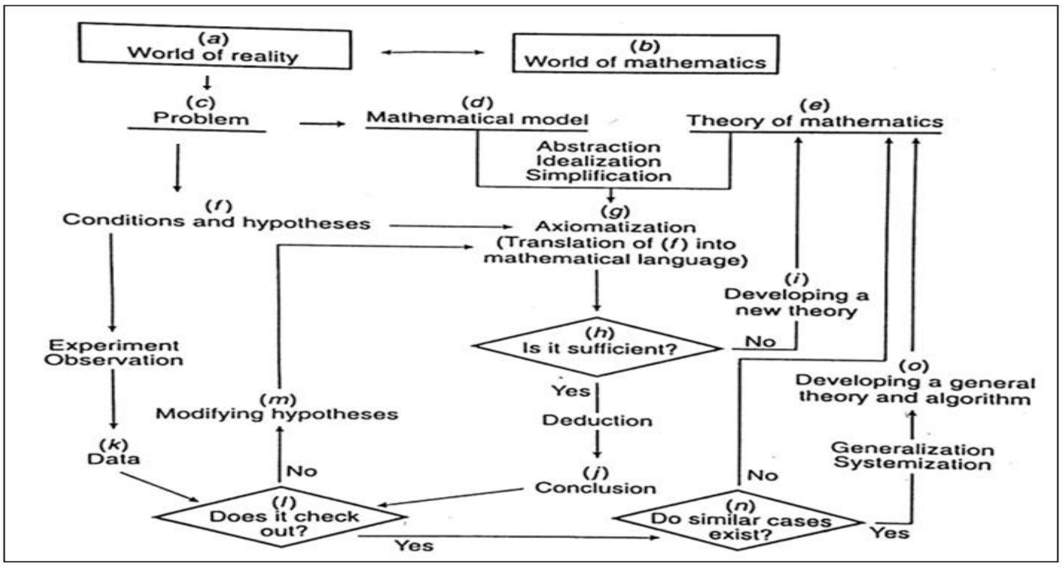


Figure 4. Modified Model of Mathematical Activities (Becker & Shimada, 1997, p.4.)

Modifications include creating a direct link from the world of reality (a) to the world of mathematics (b); from problems in the real world (c) to mathematical model (d); and from problems in real world (c) to conditions and hypothesis (f). In this new model, the problem emerged from the relationship between the world of reality and the world of mathematics. Also, (a) may be an imaginary world which is less abstract than the world of mathematics (b). In solving the problem, students may create a mathematical model and use that model to solve the problem. The link from (c) to (f) also shows that students may create their own conditions and hypotheses which are mainly formulated from their experiences. Students begin with conditions in the problem that they are aware of, and search for the conditions in the problem that they are less familiar with. In doing so, they may create their own model or formulate their own hypothesis based on their own experiences. The problem is therefore interpreted at an individual level. In solving the problem, each student tries to connect his or her cumulative knowledge and experiences to the problem presented in order to determine a solution path. As students seek to solve the problem, they search for familiar features within the problem and consider how the problem may be applied to their own daily lives. The process of contextualizing the problem is called axiomatization (g). The problem solver follows the path as laid out in the diagram. This contextualizing makes it possible to modify the problem to various situations including conditions seen among students in Jamaican schools.

3.1.4 Assessment Using an Open-Ended Problem

An open-ended problem is a legitimate assessment tool which may be used by the teacher to determine students' understanding of mathematical concepts (Becker & Shimada, 1997; Goetz, 2005; Hancock, 1995; Lin et al., 2013; Munroe, 2016a**)**. It is here that the assessment rubric as proposed by Becker and Shimada (1997) may be applied. In this model, assessment seeks to determine the level of *fluency, flexibility, originality* and *elegance,* which is reflected in the solution presented by the student. *Fluency* and *Flexibility* are measurements of quantity, requiring the teacher to record, "how many…"..., whereas *Originality and Elegance* are qualitative assessment which asks "how innovative" and "how well?" respectively. Further explanation of the rubric is given in section 3.2 of this literature review.

Fluency

Fluency is a measure of the number of different correct responses to a problem that a student presents. According to Nohda (1999), the stronger student may give more correct responses while a weaker student though presenting fewer responses is still accredited with some level of accomplishment. The nature of student centeredness demands that each child is allowed to develop from his individual starting point and work at his own pace to complete a given task. In doing so, students with greater understanding of the mathematical concept (s) are able to produce more correct solutions to the problem. Examples of papers that have used this category in assessment are Lin et al. (2013) and Laine et al. (2014).

Lin et al. (2013) conducted a study to explore whether using instruction in the open approach enhances students' knowledge of fractions. Participants were 125 undergraduate students (106 female, 19 male) aged 18–23 from a university in America. Participants were given a test consisting of 32 items that were designed to examine their procedural and conceptual understanding of fractions, before and after receiving instruction. The findings show that most of the participants achieved improved learning outcomes through the open approach instruction. All participants had increased fluency in their procedural knowledge showing that the instructions in the open approach not only developed their conceptual understanding but also had positive effect on their computational skills. However, while participants increased in their conceptual understanding of other categories tested, they did not show improvement in the multiplication and the division of fractions categories. The researchers explained that these categories require models and informal scenarios for conceptual sense making. This suggests the need for models, preferable from students' daily life, to be used in the classroom as aids for increasing students' understanding on some concepts.

Laine et al. (2014) conducted a study using open-ended problems to determine how Finnish pupils' understanding of mathematical structure develop as they progressed from the 3rd to 5th grade. The participants in the study were 348 third-graders and 356 fifth-graders from Finland and Chile. Students were given the pre-test in 3rd grade and a post-test in 5th grade. Both tests consisted of four questions relating to each other. The purpose of the formulation of the problem was to help the pupils to find how many solutions for a certain problem existed. They measured how many solutions students could provide for each test item. Pupils' fluency, was found to correlate with their ability to solve the problem. Their result also showed that fluency correlated with students' conceptual understanding of numbers. However, most students were not able to provide justification for their solutions.

Flexibility

Flexibility measures how many different mathematical ideas were discovered by the student. In carrying out the assessment, solutions with the same mathematical concept are grouped together and points are awarded based on the number of groups. It is deemed important for students to demonstrate the ability to think flexibly and measurement of this ability has been documented in a number of studies that focus on students' development of concepts related to mathematical constructs. (Hembree, 1992; Klavir, & Hershkovitz, 2008; Nohda, 1991).

Nohda (1991) shows different solutions presented by 35 grade two students on an open-ended problem. Students were asked to find a way to get the total number of apples in a figure without counting any apple twice. The grade two students devised unique ways of counting the apples and grouped the apples differently to count them including, counting by 1, 2, 5, 10 and using symmetry. Many students discovered that there was more than one pattern presented for counting by each number. The communication in the class surrounding students' presentations helped students to see flexibility in counting and better understanding of the concept of counting. This shows that students can become more flexible in their approaches in solving problems and this can be used to enhance their understanding of a concept.

For flexibility, Hembree (1992) postulates that better problem solvers are able to see different perspectives and to produce different approaches to solving a problem, substantiating the expectation that students with higher conceptual understanding will be able to produce more solution groups. This idea is reflected in students' solutions to a problem discussed by Klavir, and Hershkovitz (2008), where the responses of 164 fifth grade students in Israel, were examined. Using flexibility as one of the main categories for assessing students' understanding on number concepts, the researchers were able to verify that students' solutions to open-ended problems could be used as an indicator of their levels of mathematical knowledge. They explained that student's flexibility tells the number concept with which they are more familiar. Information gathered from such assessment measures, is useful for lesson planning, and helps the teacher to determine what concepts students need more practice with.

Elegance

Elegance examines the child's explanation about his or her solution and how well he/she defends an opinion or a position taken. Explanations with a higher degree of clarity, are awarded more marks for elegance. Here, both written and verbal statements are considered by the teacher and accorded equal importance. Moskal (2000) concentrated on the elegance category as a way of finding out more about students' understanding of concepts in geometry. Moskal argues that the 12-year-old students in the study were capable of providing detailed written explanations that reflected their mathematical reasoning; she went on to suggest that teachers should take the time to read and make sense of students' written statements as well as their verbal expressions. A similar assessment of students' abilities in geometry is seen in Sanchez (2013). The solutions to open geometric problems, presented by high school students in America, were examined to determine their levels of fluency, flexibility, and elegance. Sanchez also stated that this approach to assessment is useful for exposing students' misconceptions as it requires students to give reasons for, and to mount a defence of their choices. With this, the teacher will know if the student's reasoning is mathematically sound. She went on to endorse open-ended problems as being able to prepare students for their role in the society by showing them relevance of mathematics in their daily lives. With regard to students written and verbal statements, Klavir, and Hershkovitz, (2008) added that the teacher should listen carefully to what students say in the open approach classroom as students tend to reveal more in what they say than in what they write. This level of attention to students' responses becomes even more important in situations where students experience academic challenges due to poor penmanship, poor grammar or other intellectual deficiencies as seen in Jamaica and other Caribbean countries (Davis, 2002; MOE, 2013).

Kosyvas (2016) produced a qualitative study that assessed the elegance of students' responses. Observations were used as the main method to collect data. Twenty-six 12-year-olds from medium socio-economic background took part in the study. They were given an open-ended task on simultaneous equations without any prior exposure to the algorithmic processes of solving such problems. Students worked in corporative groups of four to discuss the problem. They were then required to present their answers to the whole class. Results showed rich communication within the groups as students participated actively, making their ideas public and presenting arguments that were convincing to their classmates. The authors stated that as students worked together and questioned each other's ideas, they gained new insights about different ways to connect the concept which advanced their conceptual development.

Originality

Originality refers to uniqueness or insightfulness of ideas generated by the student. Originality measures the quality of the mathematical thought. Examples of papers that have used this category in assessment are Kwon, Park and Park (2006); Silver and Cai (2005). Kwon et al. (2006) used fluency, flexibility and originality to measure and develop students' divergent thinking skills in mathematics. The participants were seventh-grade students in 13 classes at five middle schools in Seoul. Most of the students were from families of lower middle to middle socioeconomic status and were of average academic ability. Eight classes from public middle schools served as treatment groups while five classes from two of the middle schools were selected as comparison group. The five schools were located in the same school district and had approximately the same level with regard to students' academic ability. Instruction utilizing open-ended problems was held for eight months from April to November of 2003 and three teachers participated in the treatment groups. Students in the control group continued to receive instruction with the traditional approach. Results showed a statistically significant difference in fluency, flexibility and originality in favour of the treatment group. The researchers added that the "originality" displayed by students in the treatment group was particularly outstanding where they responded to questions which required them to draw a figure. Their result proved beneficial for this current research as it was expected that students might rely on drawing figures rather than on writing sentences due to their age and low reading ability.

The recognition and reward of students "originality" in the classroom, creates students with increased self-esteem, confidence, and an enthusiasm for life. This can lead them into becoming more creative and self-actualized individuals.

3.2 Conceptual Understanding and the Open Approach

A concept is an abstract or generic idea generalised from a particular set of instances (Skemp, 1987). The ability to learn new mathematical concepts depends on what one already knows because new concepts are linked to old ones (Crooks & Alibali, 2014; Kilpatrick et al., 2001). It is therefore reasonable to assume that partial understanding of the basic concept will lead to partial understanding of other related concepts.

3.2.1 Understanding

There is a variety of models for defining understanding in mathematics education. For example, Skemp (1976) proposed a two-part scale model of *instrumental* and *relational* understanding. Instrumental understanding refers to the child's ability to apply rules or carry out algorithms without knowing the reason behind the calculation. Relational understanding is deeper than instrumental understanding and includes knowing why an operation is carried out and, being able to deduce rules from general patterns and relationships. Byers and Herscovics (1977) extended Skemp's (1976) classification system and added two other levels of understanding, namely, *intuitive understanding* and *formal understanding*. Intuitive understanding is the ability to solve the problem without prior analysis. Formal understanding is the ability to combine mathematical ideas into chains of reasoning. They explained that a student might at any one time, have a combination of these different types of understanding in relation to a particular topic.

Kieran (1994) pointed out that mathematical understanding has been equated with "knowing, applying, and analyzing" (p. 593) since the 70's. She opined that understanding is a continuum that spans the learning of mathematics and that some form of understanding is required regardless of the level at which one is operating.

Perkins (1998) adopts the concept of understanding as a continuum and goes further by listing some activities which indicate understanding. These are: explaining, proving, generalising, applying, analysing, creating, representing ideas in a new way and using more than one way to represent an idea. Here "understanding" encompasses both lower and higher order thinking skills as opposed to the view of understanding presented in Bloom's taxonomy. Kilpatrick (2014), also seeks to question Bloom's Taxonomy and warns against interpreting or applying understanding in mathematics as cognition at the lower level. In addition, Pehkonen (2014) states that mathematical understanding entails, among other factors, the skills to analyse mathematical statements. This author shares the view that understanding in mathematics is a continuum and adds that conceptual understanding is at the higher end of the continuum as it requires more cognitive rigour than procedural understanding.

Conceptual Understanding

The book, **"***Adding it Up: Helping Children Learn Mathematics"* by Kilpatrick et al. (2001) places conceptual understanding as the first component of mathematical proficiency and defines it as "an integrated and functional grasp of mathematical ideas" (p.118). Students with conceptual understanding "know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organised their knowledge into a coherent whole, and this enables them to learn new ideas by connecting those ideas to what they already know" (p.118).

Crooks and Alibali (2014) reviewed 85 different articles on conceptual understanding in mathematics and listed six definitions from this literature. The definitions, listed in descending order according to the number of articles that used them are:

- *Connection understanding.* Having knowledge of relationships and connections between two concepts and within a domain. It is seen as a web where discrete information is connected through shared knowledge.
- *General principle understanding.* This is seen as non-verbalized, abstract principle that one uses in calculation. Being able to apply a general principle to a specific situation.
- *Understanding of principles underlying procedures.* This can be defined as the knowledge basis for procedures, or knowing why a procedure works.
- *Category understanding.* This is being able to put information into sets or groups based on a commonality.
- *Symbol understanding*. This is the awareness of what mathematical symbols mean
- *Domain structure understanding.* Can be defined as having knowledge of the conceptual structure of a domain (Crooks & Alibali, 2014, p.348).

The article suggests that the definition of conceptual understanding used, is tied to how it is assessed.

The view of conceptual understanding in Jamaica's mathematics education is the comprehension of relationships, underlying structures and meaning for operations and procedures which are revealed in application both in the world of reality and different domains in mathematics (MOE, 2013**).** Benjamin (2011), argues that not enough time has been given in Jamaican classrooms for the development of conceptual understanding. Students, it is believed, spend more time gaining procedural knowledge rather than learning concepts. It is recommended that more lessons be focused on developing mathematical concept (Benjamin, 2011; MOE, 2013).

The working definition of conceptual understanding for this study is broken down into *connection understanding*, *principles underlying procedures* and *application*. Connection understanding is seen as the ability to identify relationships between two concepts and to organise one's knowledge as a coherent whole. Understanding principles of underlying procedure is perceiving why a mathematical idea is important and the kinds of contexts in which it can be applied. Application of mathematics concepts can be both in the world of mathematics and the real world. This study proposes that using open-ended problems in the classroom can foster understanding of mathematical concepts in students.

3.2.2 Teaching for Conceptual Understanding

In a discussion about what it means to teach for understanding, Alkin (1992) suggests that teachers need to do more than just present information. He points out that teaching for understanding means teaching for conceptual change in students. This process is tedious and often means navigating through obstacles and resolving conflicts and confusion, but the end results are long lasting. In order to teach for conceptual understanding, Burns (2000) proposed that the teacher must be willing to play a less active role in the instruction and allow students enough time in the class to grapple with problems, search for strategies and solutions on their own, and learn to evaluate their own results. Conceptual understanding is different from procedural knowledge. Knowledge means that one can assimilate and maybe replicate what he or she has read or heard, but understanding requires that one is able to interpret and put new meaning into what is read or heard. Teaching for understanding means giving students the opportunity to provide their own interpretation of what they see and hear.

A comparison of the outcomes of students taught with the open approach and the outcomes of those taught with traditional methods, showed results in favour of using the open approach for the development of conceptual understanding (Boaler 1998, 2002, 2008; Cobb, 1988; Lin et al., 2013). Boaler (1998) conducted a three-year case study where she compared procedural and conceptual understanding of students from two high schools in London. In her study, Amber High used a traditional, textbook approach to teaching mathematics while Phoenix Park High used open approach with open-ended activities. Boaler observed 100 lessons in each school and collected both qualitative and quantitative data. She stated that students at Amber High found mathematics lessons in Years 9, 10, and 11 extremely boring and tedious and that they often demonstrated a noticeable lack of interest and lack of involvement with their work. Lessons at Phoenix Park High were noisy and unstructured, students spent more time exploring mathematics rather than practicing procedural skills. Triangulating the results using observations, interviews and questionnaires, Boaler found that students in both groups had similar knowledge of mathematics but "students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations" (p.41). Boaler (2008) also showed how Railside High School in California was able to improve students' performance on national tests after it adopted the principles of the open approach teaching.

Hitz and Scanlon (2001) used open-ended problems in their study which compared traditional teaching methods with teaching which used the open approach. The objective of their three-month study was mainly to assess students' achievement, their attitude toward instruction, and the retention of knowledge and skills that they displayed. Seven classes with 95 students at a private high school in Pennsylvania were given instruction in surface area, volume and coordinate geometry. The results showed that students exposed to the open-ended problems had higher retention and more positive attitude towards learning. Other researchers have recommended using open approach when teaching problem solving in mathematics and for the deeper conceptual understanding (Ellerton & Clements, 1997; Lin et al., 2013; Pehkonen, 1997, 2014; Silver, 1995).

Open-ended items induce more cognitive strategies and invite a wider range of solutions and methods than do traditional assessment items such as multiple choice questions. They are also more effective at revealing and promoting students' understanding of mathematical concepts (Sanchez & Ice, 2004). When given the opportunity to create multiple representations and to discuss these with their peers, students' conceptual understanding is enhanced. The teaching process (planning and implementation) should therefore facilitate this. Powell, Maher, and Alston (2004) explored this idea by observing students' working on the mathematical investigations and by reviewing students' writings. Participants were 24 sixth grade students from an impoverished community in America. Students were encouraged to share their ideas, to develop and justify their ideas with convincing arguments, and to pose extensions to the given problem as well as to create new related problems. The authors claim that these actions allowed students to build a foundational understanding of concepts and ideas about fractions and their operations.

Choppin (2007) suggests that patterns of discourse that are dialogic provide opportunities for students to generate ideas; this then becomes the focal point for collective reflection. Students in this study were encouraged to participate in social interactions which allowed them not only to create multiple solutions but also to share their ideas about these solutions. Students actions during the solutions process include; describing connections with prior knowledge, giving reasons, finding underlying similarities or differences, working on extended tasks, creating and sharing their own methods, making comparisons, changing one's mind and posing questions. The teacher conducts formative assessment of students' understanding based on their explanation of a concept and provides opportunities for them to refine their thought processes. Students' explanation of their thought process is a construct of mathematical elegance as proposed by Becker and Shimada (1997).

Mathematics education that is centred on problems with a single representation of an answer, retards students' understanding of related concepts as well as renders them unable to transfer such concepts to new and unfamiliar situations (Santos-Trigo, 1996). It must be admitted however, that multiple representations alone is insufficient for developing conceptual understanding. Evidence of this is shown in a research conducted with 170 high school students from Germany. This research, conducted by Berthold and Renkl (2009), used open-ended problems to measure conceptual understanding. The main goals of this study were to test whether multiple representations, such as diagrams and equations, helped students to acquire conceptual understanding in probability, and to investigate whether pupils needed instructional support in order for them to make multiple representations of a given problem. The authors found that multiple representations alone did not develop conceptual understanding but that the added dimension of social interactions among students was also required as a part of the teacher's attempt to enhance understanding.

These views of the research on teaching for understanding are consistent with recommendations and perspectives of documents published by Jamaica's Ministry of Education. These include the National Comprehensive Numeracy Programme (2011) and the National Mathematics Policy Guidelines (2013). These views are also consistent with this researcher's view on what it means to teach for understanding. The National Mathematics Policy Guidelines (2013) encourages teaching for understanding and higher order applications. It advocates a format of instruction that emphasises students "doing" mathematics. The guidelines recommend that teachers provide opportunities for students to use their own strategies to explore concepts and ideas, and to construct their own understanding of mathematics.

3.2.3 Assessing Conceptual Understanding

Among the proposed approaches for assessing conceptual understanding presented in literature, two suggestions are worth noting in relation to the current study. Firstly, an instrument may be developed to examine students' understanding of a particular content domain such as fractions (Niemi, 1996) or of a particular relation such as equivalence (Rittle-Johnson, Matthews, Taylor & McEldoon, 2011). Here, separate instruments are developed and administered independently to examine each concept being considered. This current study focuses on a group of concepts in the Number Strand, administering several individuals and independent instruments would have proven impractical, hence this approach was not adopted.

The second approach involves creating a teaching-learning environment, observing the activities within this environment and comparing students' responses to a predetermined rubric established to "operationalize understanding" (Davis, 2006; Lampert, 1986; Perkin 1998). This approach was deemed more appropriate for the current research, but the established rubrics were designed to examine both procedural and conceptual understanding and would therefore have gathered information not relevant to this study. In adopting this approach, it was therefore left for this researcher to adopt or establish a similar rubric designed to examine conceptual understanding only. Since the focus of this research is on the open approach with open-ended problems, any rubric so created would be required to centre on students' responses in an "open environment" as they interact with open-ended problems. A further review of existing literature was necessary to gather information on how such a rubric may be created.

The design and implementation of the selected rubric was influenced by suggestion presented in the book "*The Open-Ended Approach: A New Proposal for Teaching Mathematics"* (Becker & Shimada, 1997). Along with developing higher order thinking skills, it was hypothesised in the book that the rubric of fluency, flexibility, original and elegance could also be used to assess students' understanding of mathematical concepts. The hypothesis was mentioned as a possible task for future research and this served as a springboard for the current research, hence the attempt herein to use a rubric relating to open-ended problems as an instrument for assessing conceptual understanding. Some modification to the initial rubric used by Becker & Shimada (1997) was however required as their rubric was created in relation to higher order thinking skills. Indicators specific to "conceptual understanding" would have to be coined to satisfy the requirements of the current work.

A connection between Becker and Shimada's (1997) rubric and indicators of conceptual understanding can be established by using the definitions of the rubric's criteria. This connection can be explained using the mathematical transitive property of equality; i.e., if 'A' = 'B' and 'B' = 'C', then 'A'= 'C'. In this case, 'A' is considered to be the rubric for assessing open-ended problems (fluency, flexibility, originality and elegance), 'B' is the definition of each criterion in the rubric and 'C' is indicators of conceptual understanding. To establish this connection, the discussion will provide a review of the rubric for open-ended problems and the definition of each criterion ('A' and 'B' respectively), followed by definitions and indicators of conceptual understanding ('B' and 'C' respectively), and finally, showing the connections between open-ended problem and conceptual understanding (‘A’ and ‘C’ respectively) in a table.

The rubric for assessing students’ responses to open-ended problems, as presented by Becker and Shimada (1997), defines each criterion as follows:

“Fluency - how many solutions can each student produce?” (p.35)

“Flexibility - how many different mathematical ideas (properties or principles) can each student discover?” (p.35)

“Originality - to what degree is student's idea original?” (p.35)

“Elegance - how well can students explain their idea?” (p.35)

In seeking to justify the use of this rubric, it was necessary to conduct a review of existing literature to determine if such an approach to assessing conceptual understanding may be validated. Two models, Hoosain (2001) and Davis (2006), were discovered to have similar properties and were worthy of inclusion in this discussion. These researchers also had an interest in establishing a replicable system for assessing conceptual understanding in students in the mathematics classroom. Though the terms “fluency”, “flexibility”, “originality” and “elegance” were not used in their discourse, a parallel could be drawn between their descriptions of the behaviours that reflected conceptual understanding and the definitions of these criteria as presented by Becker and Shimada (1997).

Hoosain (2001) offered eight “*indicators of understanding*” that a learner should be able to do. These are:

1. Recognize relationships among concepts and within a concept.

2. Represent a concept in different ways and identify the connections among these representations.
3. Recognize the underlying structure of the mathematics embedded in a situation.
4. Communicate mathematics orally and in writing.
5. Apply mathematics to real-life and other situations
6. Generate examples and non-examples of the concepts
7. Monitor and control his/her thought process so that he/she recognizes when something is not correct and takes the appropriate corrective measures
8. Recognize that a result is meaningful and make sense (p.20-21).

Davis' model is based on the "moves" that students make as they solve mathematical problems. These moves, he suggested, may be in the form of gestures, body language, oral expression, written expressions, tone of voice and the quality of the actual solutions that they offer. Any such response which emanates from a student, he explains, may be an indication of what the student knows and how well he or she knows it. The model describes four states of cognition; (a) understanding mathematical concepts, (b) understanding mathematical generalisations, (c) understanding mathematical procedures and (d) understanding number facts. The section on understanding mathematical concepts is divided into two levels as shown below.

"Level 1:
Students understand a concept to the extent that they can make the following moves

1. Give or identify examples of the concept
2. Defend choices of examples of the concept
3. Give or identify non examples of the concept
4. Defend choices of non-examples of the concept

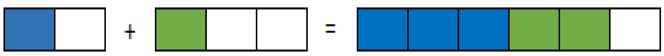
Level 2:

Characteristics of the concept

5. Identify things that are necessarily true about examples of the concept
6. Determine properties sufficient to make something an example of the concept
7. Tell how one concept is like (or unlike) another concepts
8. Define the concept
9. Tell how we use the concept" (p.14). Recognize its applicability in unfamiliar context. (p.15)

In the work of Becker and Shimada (1997), the term "fluency" relates to the total number of solutions to a problem that the student produces. This bears some correlation to Hoosain (2001) and Davis (2006) who both suggest that the number of examples and non-examples produced by a student, is an indication of the degree to which they understand a given concept. For example, students may be asked to list numbers which equal 50 when rounded. A student may give the numbers 48, 49, 51, 53 as solutions to the problem. Fluency, according to Becker and Shimada (1997), is determined by how many "*solutions*" the student produces; in this case- four solutions. Likewise, for Hoosain (2001) and Davis (2006), if students can give "*examples*" of numbers that when rounded equal 50 (in this case, 48, 49, 51, 53), then they have an understanding of the concept of rounding. The solution to the fluency criterion is the same in both cases. This shows the similarity in the meaning of fluency in the different explanations. The term "*example*" in the explanation given by Hoosain's (2001) and Davis (2006) can be replaced with the term "*solution*" in Becker and Shimada's (1997) explanation or vice versa. The consensus is that students with greater understanding of the mathematical concept (s) are able to produce more correct solutions to the problem or, they are able to give more examples of situations in which the concept is applicable.

Flexibility is a measure of the amount of different ideas, principles or properties that are embedded in the student's solution (Becker & Shimada, 1997). To this idea, Hoosain (2001, p. 20) offers a parallel, that is, can students represent a concept in "*different ways"*? And can they identify the connections among these representations? The "*different ways*" (Hoosain, 2001) was interpreted to mean the same as *different methods* (Becker & Shimada, 1997), where each method represents a different mathematical idea, property or principle. The more one understands a concept, the more he/she is able to manipulate it and reproduce it in different forms. For example, asking students to add the fractions $\frac{1}{2}$ and $\frac{1}{3}$ may produce the following four ideas.

Idea 1.  (Using diagram)

Idea 2. $\frac{1}{2}+\frac{1}{3}=\frac{3+2}{6}=\frac{5}{6}$ Using least common multiple

Idea 3. $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ Using equivalence

Idea 4. $0.5 + 0.33 = 0.83$ Using decimals

These are seen as four "methods" (Becker and Shimada, 1997) which is the same as saying four "different ways" (Hoosain, 2001) of producing the solution to the problem. However, idea 4 is not an expected solution of Jamaican grade four students. Davis also asks, can students tell how one solution is like another? Or, can they tell how two solutions differ? Davis' argument can be interpreted as saying that from a list of presented solutions, a student should be able to say which solutions utilise the same idea or principle. This would imply that the student is able to put solutions together based on their similarities and thereby obtain different groups where each group has solutions that are common to that particular group. For example, from an open-ended problem a student may produce eight solutions; 4 using addition, 2 using subtraction and 2 with multiplication. According to Becker and Shimada's rubric, this student would have a score of three in the flexibility category. Davis' argument aligns with this in saying that a student should be able to place the three solutions that utilized addition together, likewise the two solutions with subtraction together and the two solutions about multiplication together, thereby making three groups. Davis argument also suggests that in listing similarities, the student is inadvertently saying which solutions do not utilise the same idea. Students with a higher level of flexibility are more adept at performing such abstract mental negotiation and are deemed to have a more in-depth understanding to the concept.

From Becker and Shimada's perspective, an original (unique, innovative, or creative) solution is one that requires students to think "out of the box". Hoosain (2001) argued that a student's ability to "recognize the underlying structure of the mathematics" (p.20) and to apply such knowledge can lead to original solutions. It should be noted here that a student may have a knowledge of the underlying structure of the concept within the problem and still not be able to produce an original solution. The weakness in the indicator of originality from Hoosain's model rectified by using Davis' model. In Davis' model, originality is seen as the student being able to "recognize the applicability of a concept in an unfamiliar context" (Davis, 2006, p. 14). This "unfamiliarity" from the student's perspective can be seen as an innovative solution created by the student's creative way of thinking. For example, using decimals to add the fractions $\frac{1}{2}$ and $\frac{1}{3}$ (idea 4 given above) would be categorized as an original solution because it would be innovative for a Jamaican grade four student.

Elegance, which is a reflection of how well students can *explain* their ideas (Becker & Shimada, 1997) was deemed to require a similar thought process as does the ability of students to *communicate* mathematics *orally and in writing* (Hoosain, 2001) and the ability of students to "*defend choices* of examples of the concept" (Davis, 2006, p.14). On one level students may be able to solve the problem but the ability to effectively communicate, orally or in writing, and to so defend a particular choice of action is of equal importance in the mathematics classroom. It is through the sharing of ideas and in the understanding of ideas shared that mankind has been able to perpetuate itself and to ensure the evolution from one civilization to the next. Knowledge hoarded is of little value in the grand scheme of human existence, hence the value accorded to the skills of communication that students are required to display. Explanation of one's thoughts often requires giving reasons for making certain decisions or taking certain actions, this can be interpreted as defending choices. In recognizing the value of communication, all three works opined that in explaining, or defending, students should be encouraged to use mathematical jargon as this helps to advance their own intellectual development as well as that of their peers.

Hoosain (2001) and Davis (2006) also recommended using problems which require multiple representations in lessons geared towards developing conceptual understanding. Additionally, they suggest that teachers give special attention to students' "ways of solving the problem" and that students should be allowed to connect the problem to situations in their daily lives. These thoughts correspond with the focus of this current research and were included in the rubric under the heading "strategy". The term "strategy" in the rubric is an indication of how well students are able to apply their knowledge of the concept both in the world of mathematics and in the society in which they live. Here, reference must be made to the importance of the students' ability to apply a concept, as stated in the working definition of conceptual understanding presented in this research. The relationship between the rubric presented by Becker and Shimada (1997) and the ideas of Hoosain (2001) and

Davis (2006) regarding the indicators of conceptual understanding, is outlined in Table 7 below.

Table 7. Using Open-Ended Problem to Assess Conceptual Understanding

Assessment Criteria for Open-Ended Problem (OEP)	**Definition of OEP Criteria**	**Indicators of Conceptual Understanding by Definition.**	
	Becker & Shimada (1997)	**Hoosain (2001)**	**Davis (2006)**
Fluency	How many solutions can each student produce?	Generate examples and non-examples of the concept.	Give or identify examples of the concept
		Recognize relationships among concept and within a concept.	Give or identify non examples of the concept.
Flexibility	How many different mathematical ideas, (properties or principles) are discovered by the students?	Represent a concept in different ways and identify the connections among these representations.	Determine properties sufficient to make something an example of the concept.
			Tell how one concept is like another concept.
Originality	To what degree is student's idea original?	Recognize the underlying structure of the mathematics embedded in a situation.	Recognize its (the concept) applicability in unfamiliar contexts.
Elegance	How well can students explain their idea?	Communicate mathematics orally and in writing.	Define the concept.
			Defend choices of examples of the concept.
Strategy	Is the student's strategy based on real-world context?	Apply mathematics to real-life and other situations.	Tell how we use the concept…to help us.
	Is the student's strategy based on abstraction?	Recognize that a result is meaningful and make sense.	Show understanding of the concept in generalisation.

It is reasoned that in applying this rubric, one may accurately assess a student's level of conceptual understanding based on the nature of the responses to a given problem that they present. Each criterion is defined in such a way that the teacher or the researcher can observe students as they respond to open ended problems in the open approach teaching method.

It must be reiterated here that one key element in the open approach is discourse and the assessment of discourse during the problem solving process is also important. The work of Pirie and Schwarzenberger (1988) focused on the discourse between students within the classroom. They are of the view that the quality of mathematical talk may be an indication of mathematical understanding. In a year-long study, he observed students engaged in the learning of mathematics and determined that there were four different types of "talk" which emanate from them during discourse. These he describes as:

- *Purposeful talk*. Those with well-defined goals even if not every group member is aware of them.
- *On a mathematical subject talk. Content Talk* - Relating to either the goals themselves, or a subsidiary goal which emerges during discussion.
- *Genuine contribution talk*. Input from at least some of the students which enriches and propels the discourse.
- *Interactions*. Indications that the line of discourse has been picked up by other participants.

Pirie and Schwarzenberger (1988) state that a distinction must be made between reflective statements and operational statements. Reflective statements which describe concepts and the relationships between them are characterized as relational understanding (Skemp, 1976). Operational statements which describe actions, are characterized as instrumental understanding (Skemp, 1976). Given the nature of this dissertation, more attention was placed on reflective statements.

3.3 Open Approach and Gender

The open approach has been considered by some researchers as one of the learning approaches able to create equity in the mathematics classroom (Boaler, 2002; Hancock, 1995). Some studies have been carried out to this effect (Lubienski, 2000; Strong, 2009; Sullivan & Siemon, 2003); however, the issue of gender disparity in favour of girls is not a common phenomenon and as such, it has not been thoroughly investigated. Before looking at these researches, it is necessary to define the term "equity" as used in this research.

In this study, equity in the classroom is defined as fairness. On one level, this means that the possibility of a child achieving his or her educational potential is not obstructed because of personal and social circumstances such as gender, socio-economic status, ethnicity, or academic ability. Teaching therefore, should not be biased towards personal or social circumstances, but be inclusive of all. On another level, equity implies that each student is given the opportunity to explore mathematics in his or her own way and at his or her own pace. A student is not treated differently because of academic ability. As a result of the level of cognitive development or academic challenges, a particular student may be inclined to think differently and progress through a solution at a slower or faster pace than his peers. The teacher should ensure that this child is accommodated within the lesson. Open-ended problems in the open approach have the potential to create equity in the classroom (Boaler, 2002; Nohda, 1991; Sullivan & Siemon, 2003; Strong, 2009).

In Japan, investigations were carried out among high and low performing students in elementary, junior high, and high schools (Nohda, 1991, 1995, 2000; Shimada, 1977). It was found that open-ended problems were able to challenge and engage equally, students at all levels of academic ability. Studies conducted in elementary schools in the western hemisphere confirm these results, suggesting that open-ended problems can meet the needs of diverse learners in the same classroom (Hancock, 1995; Kabiri & Smith, 2003).

Lubienski (2000) looked at students' reactions to learning mathematics through problem solving. She compared students from low Socio-Economic Status (SES) groups with those from high SES groups. Performing the role of a teacher-researcher, she conducted the research with her grade seven class. The class was a pilot site for the reform-oriented, problem-based Connected Mathematics Project (CMP) in America. The project used open-ended problems and required teachers to facilitate students' exploration, discussion, and sense making of important mathematical ideas. Data were collected through observations, written assignments, surveys and interviews. Lubienski concluded that students from both low and high SES groups showed improvement in understanding mathematical concepts, but added that students from high SES groups may benefit more. Similar to Lubienski, Strong (2009) was the teacher and researcher of her study. She also used the students in her grade seven class as participants. Strong selected six students to represent the different SES groups, gender and ethnicity of the class. She observed how these students interacted with open-ended problems and with each other. From her year-long study, she concluded that open-ended problems developed students' conceptual understanding and their confidence in doing mathematics regardless of their SES group, gender or ethnicity. At the elementary level, Sullivan and Simon (2003) examined the impact of open approach on students from lower socio-economic backgrounds. They guided the teachers in using the open approach with open-ended problems to teach mathematics to elementary school children. The teachers incorporated open approach teaching in their daily lessons. Data were gathered from observations, interviews and survey. The pair found that the open approach did address the needs of students from low socio economic status as students in their study showed improvement in knowledge and application of mathematical concepts.

These are among the researchers who agree that the open approach can meet the needs of diverse learners in the same classroom; however, more information is needed on the issue, especially on its impact on how the different genders understand mathematical concept when learning in the open approach. It has been argued that the open approach promotes individual learning where each student solves the problem based on his/her own way of understanding. From a research perspective, the researcher considered it interesting to know whether solutions to an open-ended problem provided by boys, among themselves, would be similar and likewise, whether girls too, would produce similar solutions. The argument here is that, if boys and girls think and learn differently, then this difference should be revealed in their solutions to problems where they can use their own way of thinking to solve the given problem.

The idea of assessing the responses of the respective genders within the open approach environment stems from the openness of the approach. Nohda (1995) explained openness in terms of students' activity and mathematical activity. When student activity is open it means that both boys and girls can explore and deepen their understanding in their own way and at their own pace. This creates diversity in students' responses that are based on individual students' beliefs, preferences and experiences. Openness is examined with the perspective that students from the same gender may share similar preferences which are reflected in their chosen solution strategy. Mathematical activity in the open approach classroom is the process of abstraction from the concrete experience of real life to the world of mathematics and vice versa. The openness of mathematical activity is similar to the scenario of a story teller beginning a story and leaving the listeners to determine how the story ends. The teacher may begin the story and students can create their own suitable "ending". Each student explores the problem in depth based on his or her own preferences, self-determination and mathematical ability. The teacher assists students in understanding and elaborating their mathematical ideas in response to students' solution path and disposition. The research, therefore, looks at how students' self-exploration and disposition are affected by their class setting or gender.

In the open approach, more room is given to active construction of meaningful structure by the student. Boys and girls solve the problem according to their own interest and ability and this makes them favour doing mathematics more. Nohda (2000) described this as opening students' hearts towards mathematics. Often, students become dismayed when doing mathematics and tend to dislike the subject. One of the main reasons is the constant memorization of facts and mathematical rules. Openness allows students to discover these facts for themselves thus mathematics become more personal, memorable and understandable.

3.3.1 Response of the Genders in Solving Mathematical Problems

The consensus in early literature is that, in middle school and beyond, boys usually perform better in mathematics than girls do (Leahey & Guo, 2001). This view has changed, as current analysis of international tests has shown that the gap between the performances of the genders is usually very narrow or it is in favour of girls. Else-Quest, Hyde, and Linn (2010) meta-analysed the 2003 Trends in Mathematics and Science Studies (TIMSS) and Program for International Student Assessment (PISA) studies and concluded that there were gender similarities in students' scores. Mullis et al. (2012) stated that of the 50 countries in which fourth-grade students participated in the 2011 TIMSS study, "26 had no significant gender difference in mathematics achievement…, 20 had small differences favouring boys, and 4 had relatively larger differences favouring girls" (p.79). Studies comparing gender performance in mathematics with qualitative data and those that examined gender performance on open-ended problems using quantitative data are discussed next.

From studies with qualitative data, three types of cognitive abilities; quantitative, spatial and verbal, are believed to have significant impact on the type of strategies students use in solving problems. An understanding of these can help to explain gender-related differences in the way students solve mathematical problems (Tartre &Fennema 1995). It is generally accepted that males possess higher quantitative ability than females do (Maccoby & Jacklin, 1974; Benbow & Stanley, 1980); however, this does not necessarily mean better performance in mathematics. It can be interpreted simply that males may reason differently from females (Moreno & Mayer, 1999). According to Nuttall, Casey, and Pezaris (2005), spatial ability is a cognitive process based on the use of mental pictures, instead of words. Spatial ability is believed to be an important component of mathematical thought during mathematical problem solving (Halpern, 2000). Students with higher mathematical ability are more inclined to solve problems by using more spatial processes while students with lower mathematical ability solve problems using more verbal processes (Krutetskii, 1976). Studies investigating gender differences in spatial ability have reported inconsistent results. Ben-Chaim, Lappen, and Houang (1988) found that there were differences in spatial visualization favouring boys. However, from a study with 10 and 11-year-old students, Seng and Chan (2000) found that while there was a significant positive relationship between spatial ability and mathematical performance, boys did not perform significantly better than girls on spatial tasks. They concluded that there were no significant gender-related differences in the relationship between spatial ability and mathematical performance. Verbal ability is using words to clearly express one's understanding. Evidence has shown that there were gender differences in verbal skills with females outperforming males on many verbal tasks (Halpern, 2000; Maccoby & Jacklin, 1974); but in their study, Linn and Hyde (1989) showed that gender differences in verbal abilities are declining. In 1995, Tartre and Fennema conducted a study which examined the spatial and verbal skills of 60 students as they progressed through 6th, 8th, 10th and 12th grades. Results showed no consistent significant gender difference between means for spatial skills, verbal skills and achievement in mathematics. Spatial skills were consistently found to be significant predictors of mathematical achievement among females, but not for males, and verbal skill was a consistent significant predictor of mathematical achievement for males, but not for females. Gurain (2006) states that males are more spatial and that they use non-verbal planning tools such as pictures and symbols, to communicate their thought processes. McNeil (2008) states that girls are generally more verbal and tend to perform better in group situations with guidance and encouragement. Regardless of which skill is more prominent in each gender, researchers have agreed that both spatial and verbal skills can influence the different strategies students use in solving problems (Sax, 2005). In order to enhance mathematical understanding in both genders, an approach that requires the application of spatial and verbal abilities is necessary.

Some studies have attempted to compare gender-related performance on open-ended problems with such performance on traditional and multiple choice items by using quantitative data. Ben-Shakhar and Sinai (1991) examined the difference in the performance on tests, by conducting a study with students in ninth-grade and applicants to Israeli universities. The results revealed that males did better than females on multiple-choice items and females did better than males on open-ended items. Hellekant (1994) suggests that because of personality traits, males are more prone to guessing and consequently favour multiple-choice items, but this does not explain why they perform better. On the other hand, Bellar and Gafni (1996) compared the respective achievement of females and males from an international perspective. The authors focused on seven countries - Hungary, Ireland, Israel, Korea, Scotland, Spain and the USA - and tested 9-year-olds and 13-year-olds in mathematics and science. They concluded that the format of the item did not influence the difference in the performance between boys and girls. In their study, females did relatively better on multiple-choice items than on open-ended items. Additionally, Wester and Henriksson (2000) converted selected mathematics items from the Third International Mathematics and Science Study (TIMSS) into open-ended problems and gave both sets of problems to students in grade 6 and grade 8. Their results contradict Hellekant's, (1994) argument as no correlation between item format, the personality of males and the performance of students was found. The contradicting results from these researches on gender performance on open-ended items is probably due to shortcomings with quantitative measures. These contradictions do make it necessary for further research to be done and have served as a catalyst for the current research.

Identifying the classroom conditions and pedagogical approaches that influence gender differences in learning has been difficult. Factors previously thought to be significant, have now been found to be of little impact on the teaching-learning process. Zhu (2007) completed a review of the literature regarding gender differences in mathematical problem solving and noted that there were "many complex variables including biological, psychological, and environmental" (p. 187) that may contribute to the strategies boys and girls used and on their learning. An overall analysis revealed that it is a combination of factors that affect students' performance and that this combination is unique to each classroom. This leads to the conclusion that above anything else, each case needs to be examined based on the unique way the factors are combined for that particular environment. Therefore, it is significant to look at the particular case of Jamaica.

3.3.2 Socialization of Boys and Girls in Jamaica.

Boy and girls are socialized differently in Jamaica. It is believed that negative socialization, learned gender roles and the society's perception of the value of education are among the main reasons for the under-performance of boys in schools (Clarke, 2005; Evans, 1999). Socialization in the home and community teaches girls obedience, cooperation and other skills that help them to cope with the demands and routines of school. Girls are kept under the strict supervision of parents and constantly learn from them. Girls develop discipline and focus by assisting their mothers with daily chores (Moyston, 2011). In most homes, boys receive less supervision and are allowed more free time away from home. In the community, boys are pressured more aggressively by their peers into negative behaviours and those who do not fit into the group are considered to be effeminate. Boys are expected to be "macho" which is translated to being defiant of authority and existing rules (Clarke, 2005). Brown and Chevannes (1998) explained that boys are socialized into perceiving school and education as effeminate and as such have little interest in attending school and in learning. This negative attitude towards school is further compounded by the "get rich quick" mentally among young men, which is the main catalyst for the country's high crime rate. In most communities, success is measured in terms of material possessions and young men are bombarded with images of "success" displayed by individuals with little academic achievement and of questionable character. On the other hand, it is the educated who are deemed "unsuccessful". These gender-based socialization practices within the home and community have shaped an identity for girls that is in congruence with achieving and maintaining a high academic standard; but for boys, an identity that rebels against the school system. Evans (1999) state that in high school, "roughly 40% of boys think that if a boy wants to be popular and respected by peers he cannot be serious about school work, and that 24% of boys think that boys who are studious, are strange"(p.34). It is uncertain whether or not boys in elementary schools share this perception, nevertheless, the general thought is that social factors have negative effects on boys' participation in school (Clarke, 2005; Moyston, 2011) and more specifically, in the mathematics class.

3.3.3 The Cultural View of Mathematics in Jamaica

Bishop (1991) states that the teaching and learning of mathematics is embedded in culture. The cultural view of mathematics in Jamaica is that not everyone can do mathematics. An individual's ability to do well in mathematics is seen more as a function of innate ability rather than a function of effort (Benjamin, 2011). Benjamin went on to say that, in Jamaica, while no one will admit to being illiterate, most people will admit that they cannot do mathematics. The culture of innumeracy pride in the wider community has impacted negatively on how students see mathematics.

Most Jamaicans fear mathematics and believe it has little use outside of school. Hofman, Hofman & Guldemond (2001) state that the larger community provides an environment encompassing informal interactions in which beliefs, fears, values and norms are shared and where in turn, students' knowledge and behaviour are shaped. The child carries these beliefs, fears, values, norms and expectations into the mathematics classroom. Reports on mathematics education in Jamaica show that most Jamaican students view mathematics as being of little use to them outside of school, and as such they display a poor attitude towards learning the subject (Francis, 2006). Few students take responsibility for their own learning and this is reflected in the number of them who do assignments and in the poor results on tests. Parents defend their children's poor performance in mathematics by explaining that they too were poor in math at school. Students, in turn, use this defence as a reason for not exerting much effort in learning the subject. The onus is therefore on the teacher to get students interested in learning and succeeding in mathematics. Tragically, the fear of mathematics is also reported among teachers. Benjamin (2015) revealed that 19 % of 4000 teachers who were surveyed indicated a fear and dislike for the subject. Teachers with a fear of mathematics may pass on this fear to their students. Hence retarding their progress in learning the subject.

3.3.4 Mathematics Classroom Culture in Jamaica

The culture of teaching mathematics in Jamaican classroom as described by Chevannes (2003) is one in which "students sitting quietly, listening to the teacher and repeating after the teacher" (p. 1). This style of teaching is supported by the description of the teaching method given in chapter two of this dissertation. An analysis of this description shows that the culture of most mathematics classrooms in Jamaica has taught students that getting the answer to what is usually a "closed question", is what is important, and that the "bright' child is the one who can get the correct answer quickly. Therefore, students do not spend time to think about the problem or to provide logical reasons for their calculations. Instead, they try to memorize the given algorithm and reproduce it as quickly as possible. The pattern of the mathematics class, as was discussed in Chapter 2, is that the teacher provides an algorithm for solving a problem then lists similar problems on the board. Students follow the given algorithm to provide the answer to each question. The teacher checks to see if "the answer" is correct, then presents a new algorithm along with its list of similar problems. This situation leaves only a few students experiencing a feeling of satisfaction and accomplishment at the end of the mathematics class. For this study to be successful, students' mind-set must be changed from this method of learning to focus more on the solution process and not a final answer. It is expected that the teacher will need to create new social norms in the classroom in order to achieve this goal.

3.4 Implementing the Open Approach in the Regular Classroom

This section looks at: (1) the implementation of the open approach in the regular classroom; (2) the intended classroom environment relevant to this study.

The open approach, as described by Nohda, (2000) is one that opens students' hearts and minds to learn mathematics. He explains that teaching anchored in the logics of the teacher cannot open students' minds and as such students should formulate their own logics and their own way of learning. The role of the teacher is to assist students in understanding and exploring their mathematical ideas according to each student's ability. Therefore openness in the classroom means the solution process is open; the solution is open and the way to develop the problem is open (Nohda, 1995). From Nohda's perspective, in the open approach, students take an active lead in class discussion and express their ideas freely. They have a chance to feel the fulfilment of discovery as each student can answer the problem in his or her own meaningful way. Implementing the open approach in the regular classroom has its challenges. Two important issues that have been researched are:

- Lessons took two or more sessions to be completed and
- Insufficient amount of open-ended problems available to teachers.

On the issue of open-ended problems that can be used in the regular classroom time; Charles and Lester (1984) conducted a state-wide instructional program in the open approach known as Mathematical Problem Solving (MPS) for fifth-grade and seventh-grade students in America. Twelve fifth-grade and ten seventh-grade teachers of mathematics, implemented MPS over 23 weeks while a similar number of teachers conducted their usual instruction with similar age groups. Teachers implementing the MPS, gave students 10-25 minutes each day to solve open-ended problems. They employed the open approach instructional format beginning with them giving a problem and orchestrating a discussion focused on understanding the problem, followed by students sharing possible ways to solve the problem. Next, students worked independently or in small groups and finally, they shared their representations, procedures, and solutions with the class. Students gave each other feedback, reflected on the problem-solving process, and shared these reflections during the whole-class discussions. The pair of researchers evaluated students' achievement based on standardized tests and classroom observations during the intervention and found that both students and teachers in the MPS program exhibited improved attitudes toward mathematics. The conclusion drawn was that open approach instruction enhances students' problem-solving behaviour and performance; however, the limited time allotted for investigation and discussions may not have been sufficient for slower students to explore the problem properly.

Foong (2002) conducted workshops on creating short open-ended problems with several elementary school teachers in Singapore. The teachers converted textbook questions into open-ended problems. The features of these types of open-ended problems include:

- No fixed method of solution; problems yielding many possible solutions
- Problems being accessible to all students regardless of their level of cognition
- They facilitate students' idiosyncrasies, creativity and imagination

- They relate to students' real-life context and experiences. (P. 138)

Foong pointed out that teachers were able to determine students' level of understanding by the solutions that they presented. It was found that these open-ended problems did not require several class periods, but that they could be completed within the normal 45 - 55 minutes lessons. Thus they were called "short" open-ended problems. Since short open-ended problems develop students' understanding of mathematical concepts as much as other open-ended problems, they were seen as more favourable for daily class lessons. Since then, many researchers have used "short open-ended problems" in their study (Kabiri & Smith, 2003; Chan, 2007). Short open-ended problems were implemented in this current study because participants were young children with ages ranging from 9 to 11 years.

The practice of creating open-ended problems from closed textbook questions has become widespread among both teachers and researchers. As a result, teachers now have access to a vast reservoir of open-ended problems as there are numerous websites, books and articles with well-crafted open-ended problems and information regarding their development and uses; (example, Sullivan & Lilburn, 1997; Kabiri & Smith, 2003). While it cannot be said that this issue is completely solved, as there are many topics for which open-ended problems have yet to be created, the present question bank is a step in the right direction. A teacher can obtain a problem from one of these sources and modify it to meet the needs of his or her students.

3.4.1 Open Approach and Lesson Study

The open approach teaching method was refined through "Lesson Study" in Japan. Lesson study involves a group of teachers collaborating to plan, implement and evaluate a lesson with the intention of improving their didactical techniques and content quality. During the implementation phase, a member of the group teaches the lesson while the others observe and take notes. The teachers then meet to evaluate the lesson and progressively re-design it to better promote students' grasp of content and concepts. This continuous cycle of professional development aims at improving instruction and hence, students learning. Catherine Lewis of America and Maitree Inprasitha of Thailand have dedicated their research to developing instruction in the open approach through lesson study.

Lewis rallied teachers in America to create a lesson study community. By 2005, the lesson study community in North America consisted of at least 900 members from 125 school districts in 32 states (Lewis, 2006). The open approach is used in most of these lesson study researches (Lewis, 2002, 2004). Lewis argued that using open problems in lessons helps teachers to deepen student's understanding of connections between mathematical concepts. In addition, this approach enhances teachers' skill in assessment and so they are better able to meet the needs of their students.

Maitree Inprasitha has written many articles about using open approach in lesson study in Thailand (see Inprasitha, 2002, 2003, 2006, 2007, and 2012) with student teachers as participants. The common format for instruction on open approach seen in his articles is; 1) posing an open-ended problem, 2) students' self-learning, 3) whole class discussion and comparison, and 4) summing-up by connecting students' emergent mathematical ideas (Inprasitha, 2002, 2007, 2012).

The drawback with combining lesson study and open approach is that studies tend to focus on developing "lesson study", not on "the open approach". That is, the focus is on the teacher and creating "good lesson plans" (Lewis 2006, p. 5) rather than on the students. Also, with lesson study, no provision is made for the lone teacher who seeks to use open approach in teaching. It may not be convenient for every teacher who wants to implement open approach teaching method to be a part of a lesson study group. There is therefore a need to consider creating a model that an individual teacher can use when implementing the open approach lesson.

An individual teacher attempting to implement the open approach without proper knowledge of how to do so, can confuse students. This may lead to incorrect understanding of mathematical concepts. Wu (1994) documented three examples of questions that were too difficult for the students at the level at which the questions were applied. The students did not understand the problem and hence could not arrive at a solution. With no intent to discredit the use of open problems in the classroom, Wu argued that some questions may be too open and teachers should put boundaries on the problems in order not to overwhelm and confuse students.

Regarding instruction, teachers should be cautioned against the possibility of instrumental understanding of content being replaced by an instrumental understanding of process (van ores,2002). This means that students may shift from memorizing formulas to memorizing problem solving strategies. This approach was observed in some Jamaican classrooms where teachers gave students a specific problem solving strategy with which to solve the problem. The problems used by teachers in Jamaica were closed; it is unlikely that the teacher will be able to give all the possible strategies for solving an open-ended problem.

3.4.2 The Proposed Open Approach Classroom Environment

This study is built on the theory of constructivism. According to Piaget (1977), the goal of education is for students to construct their own ideas through inquiry and discovery and not just to memorize facts. Von Glasersfeld (1990) argued that finding more methods to solve a problem reflects the core of Piaget's idea of constructivism. Constructivists support the idea that student's gain greater understanding when they create their own solutions path. "When children are actively engaged in understanding why things work the way they do, transfer follows naturally and without great effort" (Devens-Seligman, 2007).

The classroom environment refers to the psychosocial surroundings in which students interact with mathematics as they solve open-ended problems. The term psychosocial refers to the psychological influences and effects that the social surroundings have on an individual. The social surroundings include gender, interactions with mathematics and with resources available in the mathematics classroom (Pirie & Schwarzenberger, 1988). It is believed that students work better in a surrounding where they feel safe, free, accepted and encouraged (Boaler, 2008; Clarke, 2005; Nohda, 2000).

Teaching for conceptual understanding means helping the learner to make connections between what they already know and the new information they are receiving. "The more connections the learner can make between the new experience and previous experiences, the greater and consequently the more useful the understanding" will be (Haylock, 1982, p. 54). Each concept is part of a network of concepts, procedures and facts (Davis, 2006). For example, the concept of fraction is part of a network with decimals, percentages, ratios, proportions and so on. Teaching for conceptual understanding means helping students to understand the intricacies of this network. The more connections that can be identified for a concept, the greater the understanding of that concept. Students may already have intuitive understanding about the connections between two concepts. The teacher can therefore build on these intuitions to help the child to move from intuitive understanding to formal understanding (Byers & Herscovics, 1977). A teachers' recognition and interpretation of students' strategies, statements, body language and idiosyncrasies can help in this process of teaching for conceptual understanding. Ball (1991) suggests that "teaching for understanding entails keeping a wide range of considerations in mind. This includes the substance of the content, the ways in which the nature and discourse of mathematics are represented, and the social and cultural aspects of mathematics" (p.81). Open-ended mathematics problems enable students to develop conceptual understanding (Nohda, 2000; Pehkonen, 2014).

Teaching for conceptual understanding means teaching the *why* for each step in a calculation and the reason for carrying out the calculation as a whole. For example, students should be able to see the reason for using addition in a particular calculation and not subtraction. Knowing "*why"* helps students to better understand *how*. Students who know why a calculation is done are better at using an appropriate procedures in a given situation and at effectively monitoring their progress and making sense of their solutions (Boaler, 1998; Strong, 2009). In order to develop conceptual understanding, students should be given an opportunity to defend their chosen method and also to defend not choosing a particular method (Davis, 2006; Hoosain, 2001).

Pertaining to the world of mathematics, students are encouraged to think like mathematicians as they solve open-ended problems. A study related to this view is Lampert (1990). Lampert conducted a study with fifth-grade students with the aim of helping them to understand the relationships among mathematical concepts. She planned and enacted lessons around tasks that required students to write various representations of solutions and mathematically justify their presented solutions. Lampert encouraged students to think like a mathematician by using mathematical language and logical arguments in their discussion. Students shared different solutions and revised their ideas in the face of contradicting evidence. Lampert stated that eliciting multiple solutions, and facilitating collaboration and discourse, can provide opportunities for students to identify the connection between two concepts. Another example is seen in Strong (2009) where students not only displayed a positive view of themselves as mathematicians, but showed an increase in their problem-solving abilities and understanding of mathematical concepts.

A rationale for learning mathematics is its applicability to everyday situations such as using money, reading a clock or designing a building. Open-ended problems can be linked to real-world situations as this makes it easier for students to see and understand the practical use of the mathematics that they are learning. Hitz and Scanlon (2001) conducted a study where they compared the effectiveness of project-based experiential learning method (open problems with real life applications) and the traditional classroom method in the teaching of mathematics. They concluded that when students can develop a connectedness through real-life experiences, they can apply that learning to other situations. This helps them to better understand how mathematical ideas are used in the real world. Another example is seen in Technology, Entertainment and Design (TED) conference with Dan Meyer's (2010) presentation. The students in Dan Meyer's class were able to understand volume, capacity and speed after answering a question about how long it would take water flowing from a tap to fill a tank. Students in mathematics classroom are always asking "Why do I need mathematics?" and "Why is this concept relevant?" Creating open-ended problems related to a real-world context will help them see the relevance of mathematics in their daily lives, and this will further encourage them to learn the subject.

3.5 Gaps in Previous Studies

This study seeks to provide new data regarding the use of the open approach on students' conceptual understanding in mathematics by making comparisons between the genders and between single-sex and co-educational classes. These relationships have not been considered in previous researches. Other limitations of previous studies include:

1. Most researches looked at the ability of open-ended problems to meet the needs of diverse learners without considering gender differences. This research focusses on the ability of open-ended problems to meet the needs of both genders.
2. Most researches that examined the gender gap in mathematics have focused on situations in which the disparity favoured male students. This research was conducted in a setting where the disparity was in favour of female students.

3.6 Summary

This chapter addresses the relationship between the open approach, open-ended problem, conceptual understanding and gender. Solving an open-ended problem goes beyond simply arriving at a right or wrong answer but encompasses students' thought processes, ideas and general understanding of the inter-connectedness of the concepts in the problem. The approach provides a learning environment where various ideas and alternative ways of solving the problem are presented. Students gain clarity on a concept by sharing ideas about the concept, discussing its relationships with other concepts and discussing the rationale behind each solution.

The open approach has been used to gain insight into students' understanding of mathematics. Among the documented studies on open approach, the focus has been on developing students' creativity, mathematical thinking and higher order thinking skills. However, this research looks at how the open approach with open-ended problems supports students' conceptual understanding. As a result of the lack of information regarding gender specific responses to open-ended problems, interest has been sparked in this area, and this interest has led to the focus of this study.

The open approach has been advocated as a form of differentiated instruction which allows students to work at their pace, use their own learning styles, investigate their own interest and use their own methods to solve the problem. Literature suggests that with a given open-ended problem, it should be possible for a teacher to improve the learning outcomes of students of different academic ability in the same classroom. Along with this, a student's understanding of a mathematical concept can be diagnosed from his/her solutions to an open-ended problem. This diagnosis may also be useful in determining the respective impact that a learning experience has on students.

CHAPTER FOUR:

METHODOLOGY

4.1 Overview

The goal of this study was to examine the impact of instruction in the open approach on fourth-grade students' understanding of mathematical concepts. The study compares this impact on boys as opposed to girls, and on students in co-ed class as opposed to those in single-sex class. This chapter presents the methodology used for the pilot study and the main study. The schedule of activities is outlined in Table 8 below. A description of both studies follows.

Table 8. Outline of the Study

	Month, Year	Activities	Remarks
Pilot Study	September, 2014	Teacher training - 2^{nd} week of the month	Establishing the open approach environment, creating social and sociomathematical norms
		Implementation - 3^{rd} and 4^{th} weeks	
		Pilot test- 4^{th} week	
Main Study	October, 2014	Pre-test- 1^{st} week	To obtain data for comparison
		Intervention, observations, Target lesson 1	Types of Numbers, Comparing
	November, 2014	Intervention, observations, Target lesson 2	Estimating, Place Value
	December, 2014	Intervention, observations, Target lesson 3	Addition, Subtraction, Multiplication, Division
	January, 2015	Intervention, observations, Target lesson 4	Operations, Fractions
	February, 2015	Intervention, observations, Target lesson 5	Fractions
	March, 2015	Intervention, observations	To obtain data for comparison with pre-test
		Post test	

Topics in decimal and measurement were also done during the research period, but these lessons were not included in the research. Most topics in decimal were not taught at Bath Primary due to school events, therefore, these lessons could not be compared. Topics in measurement were not included because the research focused on the Number Strand.

4.2 Pilot Study

The pilot study was designed to verify if any modifications to the open approach, which was adopted from a Japanese context, would have been required before any attempt was made at implementation in the context of primary schools in Jamaica. The main concern was how teachers would manage the class, facilitate discoveries and organise students' solutions to increase their understanding of mathematical concepts.

4.2.1 Participating Teachers

Three teachers, two females and one male, volunteered to participate in the study. The female teacher who taught the co-educational class was in her late 40s. She has a Bachelor's Degree in Primary Education and had been teaching for 25 years. She spent 17 consecutive years teaching grade two students, this was her second year teaching at grade four. She spoke in an even pleasant tone. She had excellent class control skills as students respected and obeyed her whenever she spoke. She volunteered for the study because she thought that in order for students to improve their academic performance, teachers must improve their teaching methods.

The other female teacher taught the all-girls' class. She was in her late 30's and had been teaching for 12 years. She had been teacher grade four for eight consecutive years prior to the implementation of the study. She has a Bachelor's Degree in Primary Education and a Master's Degree in Education. Her senior teacher duties kept her busy throughout the day. She had an excellent command of the class as students obeyed her when she spoke. She commented that even though she knew of the open approach, she was afraid of trying it. She anticipated that she would benefit from participating in the research, and this motivated her to volunteer for the project.

The male teacher who taught the all-boys' class was in his early 30's. He has a Bachelor's Degree in Primary Education and had been teaching for five years. He had a business-like attitude towards his work. His tone of voice changed depending on what he required from the students. A harsh tone was used if students became disruptive. He had excellent class control as all students responded appropriately whenever he spoke to them. His reason for volunteering was that he thought that the open approach could help students to take more responsibility for their learning.

The teachers claimed that they knew of open-ended problems from MOE workshops, but had not implemented them in their lessons. While they have used other forms of student-centred methods, they thought that open-ended problems posed unique challenges that were more difficult to manage. The common reason given for not teaching with open-ended problems was the fear that students may ask questions or produce solutions which they (the teachers) could not answer or explain. Teachers were also uncertain of how to manage multiple responses to a problem and how to support students learning in the open approach. This attitude may be described as common among teachers in Jamaica and may be the reason the approach is not widely used.

4.2.2 Teacher Training

The teachers were trained to use the open approach method in the second week of September 2014. The main purpose was to acquaint them with using the method and to develop a common format for lessons. The three teachers participated in a four-day workshop. In days one and two, the teachers watched video recordings of lessons conducted in the open approach and discussed the main sections of the lessons, introduction, between desk instructions, organising students' solutions, summing up and reflection. Day three was spent modifying textbook questions to create open-ended problems (Foong, 2002) and creating assessment rubrics for selected problems (Becker & Shimada, 1997). The focus was on changing the question to highlight a pre-determined concept and to ensure that they allowed for multiple solutions. Table 9 shows examples of questions that were modified from students' regular textbooks. On day four, the teachers conducted mock lessons and continued discussions on using the open approach and open-ended problems. The teachers then participated in a lesson study practicum exercise, conducted as implementation of the pilot study.

Table 9. Modified Problems from Student's Textbooks

Textbook closed-ended questions	Concept (s)	Modified to open-ended problem
Show that 5 + 3 = 10 - _____	Equivalency	What pair of numbers can go in the spaces to make the statement true? _______ + ______ = 15
Round 53.6 to the nearest tens.	Rounding off. Place value. Decimals	Find **eleven** numbers that can be rounded off to 50.
What is the place value of 3 in the number 238?	Place value	Using the digits 4, 5, 2 and 7, write different numbers with 5 in the tens place.

4.2.3 Implementation

The 97 students from the three classes participated in the study. All parents gave their consent to students' participation by signing a permission slip, and students were informed of their new roles in the classroom. Students were told not to rely solely on the teacher, but to try and solve the problems themselves, create their own solutions and give their own explanations. Social and socio-mathematical norms to govern each class were created. Among the social rules generated were:

(a) Actively listen to each person's contribution,

(b) Do not talk while someone else is talking

(c) Only talk about things that are related to the topic of discussion (Lampert, 1990).

Some examples of sociomathematical norms included:

(a) Giving explanations for your answer,

(b) Disagreeing politely,

(c) Commenting on individual's ideas; not on the individual (Yackel & Cobb, 1996).

Implementation of the teaching intervention was done using a lesson study format where the researcher worked with each teacher individually, to plan, teach and evaluate each lesson. In the lesson study practicum exercise, lessons were conducted with the open approach pedagogical method for three days each week. Modifications of lessons were done based on feedback from students. The researcher gradually reduced his participation in the process until the teacher was confident enough to implement the process alone. By the end of the second week, (fourth week in the month), all teachers were able to plan and implement their lessons without assistance from the researcher.

2.4.1 Data Collection and Analysis

The methods employed for data collection included participant observation, student journals, student' feedback, students' questionnaire, teacher interviews and student interviews. Information collected was used to determine whether or not it would be feasible to conduct the main study.

Analysis of information gathered from observations was carried out simultaneously with data collection and was used to set in motion corrective measures for revealed discrepancies. Observations focused on teacher's competence in using the open approach and on students' aptitude for operating in the open approach. Data from students' questionnaires, interviews and journals were analysed in terms of students' academic ability, their willingness to participate in the open approach lesson, and on their opinion of learning with open-ended problems. The selection and placement of students was done in such a way that participants from the different classes were of similar academic ability and socio-economic status.

4.2.5 Pilot Test

The pilot test was conducted in September 2014 to determine how well the created open-ended problems might measure Jamaican fourth grade students' understanding of mathematics. The pilot test also served to confirm each measure's dimensionality, explore item parameters, and investigate measure reliability.

Test items were created from topics outlined in the grade four curriculum as stated in chapter two of this study. The twelve open-ended items on the test covered topics in types of numbers, number operations, place value, equivalent fractions, adding fractions and formulating problems. There were two items for each topic on the test. These items were modified from students' textbook. The test was given to 42 grade four students in a neighbouring school. The researcher selected these students because their results on the grade three test were similar to those of the prospective participants in the study.

Solutions were assessed using the criteria of fluency, flexibility and originality as previously described. The reliability of each item was judged based on the number of students who were able to produce correct solutions. Two items were removed as more than 80% of the participant's scored maximum marks on these items. One item was removed as more than 80% of the participants omitted it or attempted it with very little success. It was at times difficult for the researcher to ascertain whether an error on the part of a student was due to misunderstanding or miscalculation. Closed items for testing students' computational ability were added to clarify this difficulty. For verification, the final test instrument consisted of 17 items; 8 closed and 9 open-ended problems. Four of the open-ended problems were paired with closed questions and the other five remained "stand alone" problems (see Appendix A for test items).

4.2.6 Results of the Pilot Study

Before implementing the pilot study, teachers gave suggestions regarding expected challenges for both teachers and students. The major challenge pertaining to the conceptual framework was on establishing and maintaining a supportive environment. That is, teachers were worried about moving around the room to observe and give feedback to individual students. Previously, teachers used the student-centred method once or twice a month, but the research required them to do this for three days each week. It was thought that this action would be tiring and difficult. It was also difficult to walk within the small spaces between the desks that the layout of the classroom allowed. From the students' standpoint, it was expected that some students would have difficulty adjusting to their new role in the open approach classroom. Students were accustomed to listening to the teacher and applying given strategies. The research however, required that they contributed more to class discourse and created their own strategies for solving a given problem. The teachers thought that students would not readily adjust to such unfamiliar activities. Corrective measures were planned to overcome these and other challenges, however, some challenges faced were different from the expected challenges previously stated. Please see Table 10.

Table 10. Foreseen Challenges and Actual Events

	Foreseen Challenges	Corrective Measures	Actual Events (Results)
Teachers	Co-ordinating students' responses so that they learn something from the "chaos".	Emphasise the underlying concept behind each strategy and the connection between two strategies. Teach the "why".	Corrective measure worked well. Each lesson focused on one or two main concept (s). Teachers guided students to compare two strategies at a time and to defend their choice.
	Being mobile for the duration of the class to support students.	Rest while standing and observing students at work.	Teachers occasionally sat at their desks during the lesson.
	Class control (daydreaming, noise, distractions, lack of pencil or book).	Create stimulating activities that nine-year-olds could relate to. Have extra pencils and books available.	Students did not show many signs of distractions. Actions reflecting a lack of interest gradually reduced during the intervention.
Students	Willingness to share ideas and participate in classroom discourse	Offer incentives for participation.	Students were willing to show their solutions and often created extensions to the problem.
	Working collaboratively.	Structure the lesson to encourage group work.	Students' willingness to work in groups increased over time.
	Writing in Journal.	Provide reflection time in the lesson. Teachers to ask students to verbalised their thoughts and then to list them on the board.	Slower students copied all the sentences from the board instead of only the ones relevant to what they had learnt.

4.2.7 Summary of Pilot Study

Results from the pilot study showed the potential for implementing the open approach in Jamaican classroom. The teachers were capable of facilitating students learning in the classroom. Students were eager to learn with this approach. They explored math at their own pace and showed signs of increased understanding of mathematical concepts. Both boys and girls seemed to welcome the idea of using their own strategies to solve the problem. It was however, necessary to make three changes to the structure of the lessons based on the results of the pilot study.

Change 1. Relate a concept to students' experience

Teachers created problems relating to daily life to which the students could relate. It was observed that students tended to create more diverse and unique solutions when such problems were stated, as opposed to when problems were from abstract or unfamiliar contexts. Due to this observation, most lessons or problems were given names that reflected daily life activities, for example "Sharing Sweets," or "Shopping at the Market."

Change 2: The Open Environment

Exploring extensions of the problem in the same class period was allowed. Nohda (1991) spoke of the possibility of extending a problem in class; however, most lessons observed in Japan consisted of one open-ended problem. During the pilot study, the teachers realised that some students often tried to create their own extension to the problem. For example, in asking students to use five digits to create different numbers, some students also tried to find the largest number and smallest number the four digits could produce. Extension of the problem was assigned as additional activities. One rationale for this was that, apart from further exploration with the problem, students were familiar with having more than one activity in a lesson. The extra or extended activity kept students interested and on task. Extensions to the problem often allowed students to solve a wide range of related examples and non-examples (Davis, 2006; Hoosain, 2001). As such, extensions to the problem were welcomed and encouraged and were included in the lesson. The teachers also began to include expected extensions to the problem in their lesson plans.

Change 3: Brief Small Group Discussion

Brief small group discussion was used when students had difficulty producing a response to a comment or question made by a peer or by the teacher. In these cases, along with allowing students to think about the comment or question before responding, students would be asked to share their opinions with others in the class. Students were at this point, allowed to move about the class to listen to each other or share their opinions as they saw fit. Students were then called back to their seats and responses were solicited from different members of the class. These short group discussions lasted from about 30 seconds to about 2 minutes and were carried out during general class discussions.

4.3 Main Study

4.3.1 Research Design

The study used a concurrent mixed method approach. This means that both quantitative and qualitative data were combined to provide a comprehensive analysis of the research problem (Creswell, 2009). However, more emphasis was placed on qualitative data. There were three stages to the process: pre-test, intervention and post-test. Observations were conducted during the intervention period.

Creswell (2009) described the concurrent mixed method as one in which the researcher collects both types of data at the same time. These are subsequently collate in order to provide a basis for interpretation of the overall results.

4.3.2 Rationale for Research Design

The mixed method was chosen after conducting a literature review and considering the purpose of the study. The literature review focused on the keywords: open approach, open-ended problems, gender, and conceptual understanding in mathematics. It shows that most studies on the open approach used observation because they sought to gather information on students' solutions to open-ended problems (Nohda, 1991; Chan, 2007; Strong, 2009; Lin et al., 2013; Laine, 2014). Summative tests were used as the main source of data for studies that compare gender performance in mathematics (Leahey & Guo, 2001; Bessudnov & Makarov, 2015). This study sought to compare the respective approaches to solving open-ended problems presented by boys as opposed to girls, therefore combining both qualitative and qualitative data was most appropriate.

The second rationale for choosing the mixed method was the purpose of the study. Creswell (2009) identified treatment integrity as one of the main rationale for using the mixed method. The study looks at the integrity of whether the proposed open approach with open-ended problems would have an impact on students' understanding of mathematical concepts. In addition, the intention was to compare the respective approaches to solving open-ended problems presented by boys as opposed to girls, therefore combining both quantitative and qualitative data was most appropriate. With these aims in mind, it was more appropriate to use the mixed method than it was to use quantitative or qualitative method only. Pre-eminence was given to the collection of qualitative data because information from this source was deemed to have a greater value due to the nature of the research.

4.3.2 Participants

Ninety seven grade four students from two rural schools in Westmoreland, Jamaica, participated in the study. Bath Primary and West Primary were used as pseudonyms for the schools. These schools were purposely selected due to the similarities in their students' abilities and close proximity to each other. The researcher was able to conveniently commute between the schools to observe classes on a given day. One school had average and below average students while the other school had average, below average and advanced students. In an effort to ensure homogeneity among the groups, students deemed to be "average" or "below average" were selected from among their peers. At Bath Primary, these students were further divided into single-sex groups with 28 students in the all-girls class and 31 students in the all-boys class. At West Primary, a co-educational arrangement with 38 students was maintained. The identity of each student in this study remained confidential with pseudonyms being used instead. Permission for students' participation was obtained from teachers, parents, and students before intervention. In addition, permission to conduct the research at the schools was obtained from each school board through the principal.

All participating classes consisted of Jamaican students of African descent. Students attending these schools were from communities in rural Jamaica and were from household within the range of low socio-economic status. This information was gathered from teachers and the school records. The average age of the participants was ten years. The students, now in grade four, had each received a score of less than 80% on the Grade Three Diagnostic Test administered at the end of the previous school year. Forty-four students, (28 boys and 16 girls) were on government support. This means they were assisted with daily transportation cost and were given free lunches at school for three days each week. Forty-one students had only one notebook in which all subjects were done, the other 58 students used one notebook per subject, as recommended by the respective schools. Students carried one or two pencils to school. Teachers reported that 46 students (30 boys and 16 girls) were reading below the grade four level; of these, 13 boys and 6 girls were from the single-sex classes. Table 11 shows the distribution of participants among the three classes in terms of those on government support and reading level. The number of boys and girls in the coeducational class are shown separately. See Appendix C for a copy of the permission letter.

Table 11. Participants in the Study

Participants	Coeducation		Single Sex		Total
	Boys	Girls	Boys	Girls	
Number per Group	20	18	31	28	97
Government Support	17	9	11	7	44
Used one book per subject	20	13	15	10	58
Reading below grade four level.	17	10	13	6	46

The three teachers from the pilot study taught these classes. The male teacher taught the all-boys class and female teachers taught the other classes. A female teacher was asked to teach the co-educational class as this reflected the prevailing situation in Jamaica, in addition, previous researches have shown that boys in primary school respond more favourably to female teachers in a co-ed setting (Teape, 2015).

Layout of the Classrooms

The two classrooms at Bath Primary School were among three classrooms that occupied a rectangular shaped meeting hall. Two portable blackboards were used to separate one classroom from the other. Each classroom had four blackboards; two at the front and two at the back. The two blackboards approximately 2 meters wide and 1.25 metres in height, located at the front of the classroom, were used for class activities. The teacher's table was to the left, at the front of the room. Piles of students' notebooks were frequently seen on the teacher's table. In the all-boys' classroom, desk-chair combinations accommodating two students each, were placed along the walls to the right and the left of the room. These faced each across the room. Other similar seats were placed at the back of the room facing the blackboard. Single-seater desks and chairs facing the blackboard were in the middle of the classroom. The all-girls' classroom also had desk-chair combinations; some seating one student, others seating two students. The walls of the hall were made of solid building blocks along the lower level and decorative blocks at the upper level. These decorative blocks had large spaces which enhanced ventilation but proved to be a source of inconvenience whenever it rained. Charts and students work were displayed on the walls of the classrooms. See Appendix E and F for the layout of the classrooms.

The co-educational class at West Primary had only desk-chair combinations. The classroom had one white board approximately 4 meters wide and 1.25 metres in height, it was fixed to the front wall of the classroom. Two blackboards, at the back of the classroom were used to separate the grade four and the grade five classrooms. The blackboards at the back of the class were not used during the lessons. The teachers' table was to the left, at the front of the classroom and often had piles of students' notebooks submitted for marking. The walls were of construction similar to those at the other school and students work as well as charts displaying curriculum content, were similarly displayed. Please see Appendix D for layout of the classroom.

4.3.3 Implementation

The proposed study was conducted over a period of six months starting in October, 2014. During the first week, a pre-test was administered to obtain information on the academic status of students. The researcher administered the pre-test which lasted for 60 minutes. The researcher read each item aloud and gave students enough time to answer before moving on to the next item.

During the intervention, students received instruction in the open approach with open-ended problems for three days each week. See Table 12.

Table 12. Time Each Class had Open Approach Lessons

Time	Tuesday	Wednesday	Thursday	Friday
8:45 - 9:30	All -girls	Co-ed	All-boys	
9:30 -10:15	All-boys		All-girls	
10:15 - 10:30	BREAK			
10: 30 - 11:15		All-boy		
11:15 - 12:00	Co-ed	All-girl	Co-ed	
12:00 - 1:15	LUNCH			
3:30 - 5:00				Teachers' Meeting

Problems designed by the teachers during the training sessions, were presented during the lessons. These problems could be solved using a wide range of methods that required varying degrees of mathematical understanding. The goal of the instruction was for each student to improve in understanding by using his or her own problem solving strategies and discussing the various solutions presented with their peers. It was also intended for each child to become aware of his or her own way of thinking by reflecting on his or her own way of solving problems. Teachers adhered to the sequence in which topics should be taught as prescribed by the curriculum, but there was a change in timetable to facilitate observation of lessons.

Teachers met twice per month, on Fridays, to reflect on the lessons of the previous two weeks and to discuss lessons for the upcoming weeks. The three teachers followed the same lesson outline in order to ensure a similar classroom environment. Also, the lesson outline was similar to that of the Japanese lesson with introduction, development and reflection. The teachers also used *Hatsumon Kikan-shido, Neriage, Bansyo* and *Matome* throughout the lesson. Each lesson consisted of six sections as shown in Table 13. However, this format was not adhered to in all lessons as students' explorations occasionally lasted for more than the allotted time.

Table 13. Time Allotted for Different Sections of the Lesson

Part	Time	Activity
1	2 minutes	Problem introduced. Read aloud by students. Some students allowed to rephrase the problem statement
2	10 minutes	Students solve problem individually or in groups
3	15 minutes	Class discussion. Students explain their methods and solutions
4*	10 minutes	Extension to the problem (culminating activity)
5	3 minutes	Summary of lesson (by teacher)
6	5 minutes	Student's reflection

* Was not present in all lesson

At the end of each lesson, the researcher had a brief discussion with the teacher.

4.3.4 Instructional Tasks and Materials

The open-ended problems which were created by the teacher were subsequently vetted by the researcher. As prescribed by the conceptual framework, the task of the teacher was to establish an open environment by eliciting multiple solutions from students and to motivate them to participate in open discussion about the presented solutions. Student's activities included creating and sharing their own methods, giving reasons for an answer, working on extended tasks, finding underlying similarities or differences, making comparisons, rethinking and modifying their approach if necessary, and posing questions.

Students were given hands-on materials that they could use as aids to solve the problem or to demonstrate their thought process. In some lessons, items were placed on a desk in the classroom for students to use at their convenience. In other lessons relevant items were distributed to students, but this was done mainly during group work activities. Items and distribution patterns varied depending on the nature or content of the lesson.

4.3.5 Data Collection

Both qualitative and quantitative data were collected. Qualitative data were collected from observations notes and target lessons while quantitative data were collected from pre-test, post-test and target lessons.

4.3.5.1 Observation

Observation for gathering data was carried out in keeping with the aim of the research, the conceptual framework and the research questions. The study required data on the interactions between teacher and students, among students and between students and mathematics. The interactions between students and mathematics are usually subtle and require meticulous interpretations and analysis of oral statements, body language and gestures (Davis, 2006; Hoosain, 2001; Pirie & Schwarzenberger, 1988).

According to Pirie and Schwarzenberger (1988), classroom behaviour i.e. - behaviour of the teacher, behaviour of the student, and the interactions between teacher and student- can best be studied through naturalistic observation. Creswell (2009) supports this, noting that in observing, the researcher takes notes on the behaviours and activities of the participants at the research site. This researcher observed students in their natural classroom setting and took written notes in short-hand, unstructured form. Creswell (2009) described observation as an excellent instrument with which to gain a complete picture of any social phenomenon, such as the behaviour of learners in a classroom. Accordingly, the purpose of classroom observation in the current study was to determine what transpired in class during the open approach lessons. The primary focus of observation was on students' solution methods and processes and on interactions among students. Secondary focus was on interactions between students and the teacher. The researcher looked for patterns in behavior and processes as students were engaged in the solution of a problem, in small groups and during whole class discussions (Hoosain, 2001; Pirie & Schwarzenberger, 1988). Keen attention was given to student's oral and written expressions as well as their body language and the gestures (Davis, 2006; Hoosain, 2001; Pirie & Schwarzenberger, 1988). The researcher varied the focus of observation for each lesson. For example, for one lesson the focus was on students' interactions with each other while in another lesson, the focus was on students' body language. Students' level of participation was marked by counting the number of times they raised their hands or produced written or verbal comments. The short-hand-observation notes were rewritten giving more details at the end of each school day.

Target Lessons

For each month, a target lesson was identified. Here, the work done by all students was collected unlike for other lessons where only a sample of students' work was collected. The problems for these lessons were taken from the test instrument and given a contextualized name. For example, a problem about dividing whole numbers was renamed "Beach Trip" and was based on students and teachers travelling to the beach. Table 14 shows the problems given in the target lessons. Observations of these lessons focused on interactions among students and between individual student and mathematics. Observations focused on the type of strategies students used in solving these problems and on behaviours (gestures, body language, speech, interactions) that influenced the amount and type of solutions students created. Both quantitative and qualitative data were collected from these lessons.

Table 14. Items from Test Instrument Given during Target Lesson

Month	Item on Test	Example of Concepts Tested	Name Given in Class
1	Item 6	Counting, adding, subtracting	Eating Sweets
2	Item 9	Four operations, factors, multiples	At the Market
3	Item 7	Sequences, ordering, odd, even	Not on the Team
4	Item 10	Estimating, Decimals	At the Restaurant
5	Item 8	Dividing, rounding off, fraction	Beach Trip

4.3.5.2 The Test

The primary purpose of the test was to obtain quantitative evidence on changes in students' understanding during the intervention. The researcher developed the instrument as there was no prescribed test found in related literature that was suitable for measuring all concepts covered in this research. Students' solutions to each item on the test were matched to a predetermined rubric in order to assess their level of understanding of the concepts tested by the problem.

The test was used twice; once as a pre-test and again as a post-test. It was administered by the researcher on both occasions. Each open-ended item on the test served to measure students' understanding of its mathematical concept(s). A mathematics teacher from a school that did not participate in the study verified that the items were sufficiently complex for fourth-grade students, could be solved in multiple ways, and drew on realistic contexts.

Table 15 shows the concepts that each item on the test measured, the source of the item (Jamaican textbooks) and previous studies with similar open-ended items. See Appendix A for the test items.

Table 15. Textbook Problems Modified to Open-Ended Items

Item on Test	Main Concept Tested	Understanding Tested	Open or Closed	Source	Literature with similar item
1A	Addition	Procedural	Closed	Isaacs et al. (2003.p.18)	
1B	Subtraction	Procedural	Closed	Isaacs et al. (2003.p.21)	
1C	Multiplication	Procedural	Closed	Trotman & Severin, (2005. p.89)	
1D	Division	Procedural	Closed	Trotman & Severin, (2005.p.90)	
2A	Odd/Even	Procedural	Closed	Isaacs et al. (2003. p.6)	
2B	Odd/Even, less than and greater than	Connection (Math world)	Open	Isaacs et al. (2003.p.6)	Husain et al. (2012)
3A	Expansion	Procedural	Closed	Trotman & Severin, (2005.p.5)	
3B	Expanded form, place value, 4 basic operations.	Principle of Underlying procedure	Open	Trotman & Severin, (2005.p.5)	Husain et al. (2012)
4A	Place value, comparison	Procedural	Closed	Trotman & Severin, (2005.p.29)	
4B	Place value, comparison	Connection (Math world)	Open	Trotman & Severin, (2005.p.29)	
5A	Addition, fraction	Procedural	Closed	Trotman & Severin, (2005.p.19)	
5B	Addition, fraction, multiples, factors	Principle of Underlying procedure	Open	Trotman & Severin, (2005.p.20)	Lin et al. (2013)
6	Addition Equivalency, Subtraction,	Connections/ Underlying meaning	Open	Isaacs et al. (2003, p.81)	
7	Sequence	Knowing Why/ Connections	Open	James & Constantine (2005. p.140)	Klavir & Hershkovitz, (2008)
8	Dividing, estimation	Application (real world)	Open	Trotman & Severin (2005. P.97)	Cai, (1995)
9	Counting, four operations, estimation	Application (real world)	Open	James & Constantine, (2005. p.139)	
10	Four operations, estimating, decimal	Application/ Knowing Why/ Connection	Open	Trotman & Severin (2005. P. 56)	Kabiri & Smith, (2003)

Factor Analysis

A factor analysis was done using the 17 items on the test. All 17 items correlated at .30 or greater with at least one other item, suggesting reasonable factorability. The Kaiser-Meyer-Olkin measure of sampling adequacy was .82 which is above the recommended value of .60. The Bartlett's test of sphericity was significant at a value of 415.58 at $p < .05$. These indicators suggest that a factor analysis can be conducted with all 17 items and that the test is valid for measuring procedural and conceptual understanding.

The factor analysis shows two factors having an eigenvalue greater than 1.00. The first factor was associated with closed items (1, 2A, 3A, 4A, and 5A) and the second factor was associated with open-ended items (2B, 3B, 4B, 5, 6, 7, 8, 9 and 10). This shows that items were grouped according to their intended target, namely procedural understanding and conceptual understanding respectively. Both factors lie within the cumulative percentage of 65.5 percent which is greater than the 60 percent acceptable value. The scores of the open-ended problems were separated from the scores of the closed questions. A factor analysis of the open items was carried out. Factor analysis and scree plot of open items shows two factors having eigenvalues greater than 1.00. The two factor solutions at 5.20 and 1.12 which lie in the cumulative percentage of 70.22 percent as revealed in component matrix in Appendix G. Items, 2b, 3b, 5b, 6, 7, 8 and 10 showed high correlation in factor 1. Items 2b, 4b, and 9 showed correlation for measuring underlying principle of "knowing why a procedure works". The initial eigenvalues showed that the first factor explained 57.8% of the variance, and the second factor, 70.22% of the variance.

Normality Test

A normality test was done to ensure compliance with reliability and validity parameters. Razali and Wah (2011) state that interpretation and inferences are reliable and valid if the assumption of normality is not violated. A Shapiro-Wilk test ($p. > .05$) (Razali & Wah, 2011) and a visual inspection of the histograms, normal Q-Q plots and box plots showed that the data were approximately normally distributed for the co-ed and single-sex classes. For the co-ed class, skewness was .68 (standard error (S.E.) = .38) and kurtosis was -1.123 (S.E.=.75). For the all-boys class, skewness was -1.17(S.E. = .42) and kurtosis was .538 (S.E.= .82). For the all-girls class, skewness was .078 (S.E. =.44) and kurtosis was -.684 (S.E. = .86). This indicates that the data are a little skewed and kurtotic, but they do not differ significantly from normality. A Levene's test (Homogeneity of Variance) verified that there is equality of variance among students in the three classes (Gastwirth, Gel & Miao, 2009).

4.3.6 Data Analysis

Data analysis did not follow in a linear fashion nor occurs simultaneously with data collection. Intense data analysis took place after the data collection was completed (Merriam & Merriam, 1998).

4.3.6.1 Analysis of Quantitative Data

The test consisted of both closed and open-ended items. Solutions to closed items were assessed differently from solutions to open-ended items. Closed items were awarded a total of two marks; one for a correct answer and one for correct procedure. Open-ended items were scored by comparing students' solutions to a pre-prepared rubric for each item. The Statistical Package for the Social Sciences (SPSS) was used to assist in descriptive and inferential statistical analyses of the data. The one way analysis of variance (ANOVA) tests were used for comparing mean values of test items. Results from quantitative data were used to answer research questions one, two and three.

4.3.6.2 Analysis of Qualitative Data

Observation notes were written as jottings, phrases and short sentences while at the research site and later rewritten in detail using Microsoft Word. The researcher followed five phases of data analysis for observations, namely: (a) organizing the data; (b) immersion in the data; (c) coding the data; (d) generating themes and categories and, (e) offering interpretations (Bogan & Biklen, 1998, Marshall & Rossman, 2006; Creswell, 2009). Results from qualitative data were used to answer research questions four and five.

Organizing the Data

The initial intention was to choose the five target lessons for in-depth analysis because they had both qualitative and qualitative data for the three classes. To ensure that the target lessons were a valid representation of all lessons conducted during their respective month, the following process was undertaken. First, three electronic folders were created and given names as Single-boys class, Single-girls class and Co-ed class. The lessons pertaining to each class were inserted in the respective folder. All lessons were skimmed and placed in chronological order. For example; the topic of "Fraction" was the 19th topic in all three folders and was given the name "Fraction: all boys", "Fraction: all girls" and "Fraction: Co-ed" in their respective folders. Lessons that did not appear in all three folders were removed. There was a total of 28 matching lessons covering the first five months in each folder. (The sixth month was excluded because there was no lesson in the Number Strand that was common in all three classes.) The next step was to select the lesson that was most suitable for representing all the lessons within a given month. This was done by comparing the target lesson with the other lessons conducted within that month. It was decided that each target lesson could represent lessons for its respective month except for the lesson on "Eating at a Restaurant". This lesson was removed because a large portion of it was on decimals. A lesson on equivalent fractions was used as the representative lesson for that month instead. Observation notes were revisited and an addition lesson from each class was selected because it revealed characteristics or events unique to that particular class. For the all-boys class, the lesson requiring students to find the fraction of the surface of a regular football which was covered with pentagonal shapes, was chosen because it was one of the exemplary lessons in which boys developed their own strategies for solving problems. For the all-girls class, a lesson on creating problems from a given prompt was selected because they did much better than the boys on these problems. For the co-ed class, a lesson on generating equivalent fractions was chosen because it was one of the exemplary lessons during the final stage of the research. Six lessons per class were therefore selected for in-depth analysis making a total of 18 lessons.

Immersion in the Data

The researcher read the 18 lessons (six per class) selected for in-depth analysis three times in an effort to gain familiarity with content for comparison, thus facilitating the extraction of codes. That is, the three lessons for the first month were read consecutively, followed by lessons for second month to the fifth month. This process was repeated two times. During the second reading, the researcher made analytical notes by highlighting sentences and phrases that were common or deemed to be important. Copies of selected transcripts of some target lessons were also given to two researchers in mathematics education who also made analytical notes. Analytical notes that were common among the three researchers were chosen as interpretation of data. The third reading was done to confirm correctness of analytical notes and to convert notes to codes.

Coding the Data

The goal was to see if there were any differences or similarities in students' understanding when they were exposed to open approach method and if this difference was skewed towards a particular gender or class setting. In order to do this, analytical notes were converted to codes. If an analytical note occurred in three lessons or more it was written as a code. In addition, statements deemed to be significant in revealing students thought process and relevant situations in line with the research framework were coded. That is, for individual students, verbal and written expressions as well as body language that had impact on generating solutions or understanding were coded (Davis, 2006). Discourse among two or more students was coded (Pirie & Schwarzenberger, 1988). Sentences reflecting how students generated solutions when working in groups and on how group work affected individual student's understanding were coded. The aim was to see how these interactions helped students to understand their own thought processes and solutions.

Generating Themes and Categories

Similarities between codes were used for grouping them into themes. Each theme was given a description based on the similarity (Creswell, 2009). For example, posing questions or making statements of assent or dissent, were placed in a theme called "communication" because they contributed to class discussion. Themes were further grouped into two categories: "Open Environment" and "Students Responses", in accordance with the research framework. The category "Open Environment" described themes that reflected mainly what the teacher did to support students' understanding (Pirie & Schwarzenberger, 1988; Becker & Shimada, 1997). The "Students Response" category described students' behaviours that reflected understanding of a mathematical concept (Davis, 2006; Hoosain, 2001). A sample of coded observation notes is given below. This is followed by Table 16 showing the codes, themes and theme descriptions. The complete list of themes and categories is given in the following chapter.

		Excerpt from observation notes	**Code**
68	Dave:	I added 4, five times because there are five bananas for $4.00 each.	Strategy
69	Clive:	Why did you add and not multiply?	Asking questions
70	Dave:	because adding is easier.	Defending choices
75	Clive:	I do not agree, I think multiplying is easier.	Disagreeing
76	Clive:	I multiply 4 and 5 and I got $20.	Alternative solutions
83	Andy:	But the answers are the same, how comes?	Asking questions
86	Jerry:	Because multiply is adding in a short form.	Providing explanation
87	Jerry:	But what if we did it a different way? I grouped two bananas and two banana plus 1.	Alternative strategy
101	Andy:	Can we try it bigger numbers too?	Asking questions

Table 16. Examples of Codes and Themes

Code Examples	Code Description	Themes	Theme Description
I do not agree	Politely disagreeing and offering alternatives	Communication	Open discourse that promotes understanding, elicits multiple solutions, and accommodates various views.
Why did you add and not multiply?	Freedom to ask questions and make comments about solutions		
It (this) looks like …..	Describing and comparing experiences	Fluency in Solution	Producing multiple solutions
What if I …	Exploring, suggesting alternatives solutions		
Can we try bigger number?	Exploring alternatives solutions.	Flexibility in Solution	Using different mathematical concepts and ideas to solve the problem.

Some description of codes applied to more than one theme; as can be seen with "exploring alternatives" being reflected in both fluency and flexibility themes.

Offering Interpretations

Interpretation of data was carried out using the created themes under each category. Data were interpreted to show solutions to the research questions, to find significance and meaning in the open approach and to form a basis upon which conclusions could be drawn. Information was obtained from students' gestures, body language, oral and written communication (Davis, 2006; Hoosain, 2001; Williams, 2000) as well as class discussions and interactions between students (Hoosain, 2001; Pirie & Schwarzenberger, 1988). The solutions students produced to open-ended problems in each lesson were assessed in relation to fluency, flexibility elegance and originality.

Validating the Accuracy of the Findings

The researcher deliberately avoided controlling the research conditions and concentrated on recording the complexity of situational contexts and inter-relations as they occurred naturally (Marshall & Rossman, 2006). To ensure credibility in this study, the researcher used persistent observation for a period long enough to make the identification of salient issues possible (Mertens, 2005). Another method used to ensure trustworthiness of data was that the preliminary interpretations from observations were shared with teachers who had an opportunity to provide their input regarding interpretations. Additionally, two graduate students and a retired principal, gave their input regarding the interpretation of observation notes. Consolidated input from teachers and other researchers adds to the credibility of data analysis.

4.4 Summary

This chapter described the methods and processes used in this study. The mixed method study was conducted at the grade four level, in two elementary schools in Westmoreland Jamaica over a six-month period beginning in October 2014. The study looked at the impact on students' conceptual understanding as indicated by their solutions to open-ended mathematical problems. Comparison of students' responses was done based on gender as well as class setting. Participants were of similar cognitive levels, race, age and socio economic backgrounds. Both the pre-test and the post-test were administered to all students and all participated in the process of intervention. During intervention, the researcher observed lessons which were conducted for three days each week. Observations notes were then analysed and interpreted. Various computer software (Microsoft Word, Microsoft Excel and SPSS) were used to organise and categorize the data for analysis.

CHAPTER FIVE:

LESSONS ANALYSIS

The purpose of this chapter is to outline the process of analysing the observation notes. The summarised observation and analytical notes for a target lesson as conducted in the three classes are discussed. The chapter begins with a description of instruction, followed by the analysis process of the selected lesson; then culminates with the discussions on the general differences and similarities among the three classes.

5.1 Description of Instruction

The characterizations presented here, are not intended to describe one single lesson; instead, they depict instruction that typically occurred in lessons conducted in the different classes. The duration of a typical lesson was 45 minutes. If the lesson was the first session of the day, 5 to 10 minutes were used for roll call and collection of lunch money. To start the lesson, the teacher asked two or three questions to have students reflect on the concepts and content taught during the previous lesson. This was coded as linking content from one lesson with another. Showing the link between lesson content reinforces the concept that caused such link which leads to a deeper understanding of the concept. The topic of the lesson was introduced with a story, a picture prompt, or a demonstration by the teacher. This usually established a link between the topic and real-life situations. Connections such as these are deemed useful for generating interest in the lesson and for enhancing students' understanding of the related concept. For example, a lesson involving addition and subtraction was introduced with a picture prompt depicting a market scene. The teacher wrote the task on the board, asked the class to read it aloud twice, then asked two or three students to rephrase the task using their own words. The teacher at times asked a few students to suggest possible solutions before commencing individual work.

Teachers walked among the desks and observed students as they work. During this time, they used questions to help students to clarify their own understanding and reason for their chosen strategy. Teachers sought to identify strengths and weaknesses of each child so that appropriate support can be given to improve understanding. A student's reason for choosing a particular strategy and the teacher's knowledge of the difficulties that the student previously experienced, were used to guide the teacher in deciding the nature of the support to be offered to that student. Some students, though facing challenges, were allowed to work through these challenges on their own while others were given various degrees of support.

Two types of group activities were done. In some lessons, students were allowed to form collaborative four-member groups to solve the open-ended problem. The interactions of this type of group lasted between 20 and 25 minutes. In this group work, students had the option of first solving the problem individually then taking turns to explain their solution process to each other. They could alternatively decide to work together to create a method that was to be used by everyone in the group. The latter approach was frequently employed. The second format of group activity lasted for a shorter period and often involved students moving about the class. Occasionally, when a difficult question was posed during discussion, students were asked to seek the opinions of others before they decided on a final answer. During this time, students were allowed to move about the room and to talk with more than one person. This often resulted in students gathering in clusters to listen to and share their opinions with each other. Such interactions lasted for a maximum of two minutes with six or less students participating in each group. The short collaboration time allowed students to be exposed to various opinions and to compare these with their own points of view. It also allowed students to be more reflective and more analytical which enhanced their understanding of the concepts involved.

For class discussion, three or four students were asked to share their solutions with the class. These solutions are written on the board were organized in such a way that the similarities or differences among them could be readily identified. The presenter would first explain the method then answer any questions about the method, which were posed by either his/her peers or the teacher. As these explanations were given, selected students were asked to repeat or to rephrase or to comment on what the presenter said. In most cases, the class was asked if they agreed or disagreed with what was presented, and were asked to give supporting reasons. The teacher recorded salient points from the discussion on the board and highlighted their significance. For example; "A number times one, gives the number" or "Multiplying a number by 'y' gives the same result as adding the number 'y' number of times." These statements were sometimes used as axioms and repeated each time that they formed the basis of a solution. Generally, the teacher guided students into transitioning from the visual observation to the abstract thought.

Lessons ended with teacher recapping relevant information and procedures, giving additional information where necessary and asking questions to confirm students' understanding. Some students were asked to verbalize what they learnt from the lesson and these statements were also recorded on the board. Students then copied relevant statements from the board or made journal entries reflecting what they had learnt.

5.2 Background of the Selected Target Lesson.

In one of their bi-monthly meetings while discussing the teaching of fractions, the teachers decided to conduct pre-requisite lessons before teaching the desired topics on fractions in the grade four curriculum. Formative assessment revealed that most students in their classes lacked the necessary knowledge for adding fractions; therefore, three pre-requisite lessons were created to reinforce or ascertain previous knowledge. (It is often necessary for teachers to teach or re-teach certain content due to these aforementioned reasons.) The first lesson looked at defining and identifying fractions; the second focused on describing equivalent fractions and the third looked at ways of creating equivalent fractions. The second lesson on describing equivalent fractions was chosen as the lesson to represent other lessons for that month.

The topic of the sample lessons was *Equivalent Fractions*. The lesson in the all-boys class is presented first, followed by the all-girls class and the co-ed class. Observation notes and codes were summarise d to provide more concise information for the reader. An analysis of each lesson is provided at the end of each excerpt. In the excerpts; O.C. means observer's comment. "T" means teacher and "S" means students.

5.2.1 The All-boys' Class

	Introduction	**Codes**
008	The lesson began with recapping of the previous lesson.	Linking to previous lesson
009	T: Do you remember what we did in math class yesterday?	Eliciting responses
010	S: Yes sir.	
012	Students began saying what they remembered from the lesson.	Communication between T and S
013	Floyd: A fraction has two parts, a top number and bottom number.	Meaning of fraction
015	Kevin: The bottom number tells how many parts the whole has.	

020	T: What do we call the top number?	Using math terms
021	S: Numerator.	Using math terms
022	T: What do we call the bottom number?	Using math terms
023	S: Denominator	Using math terms
024	O.C. Students know these terms but still do not use them when required. Teachers seemed to realize this and was trying to get them to use these terms.	
025	T: Anything else?	Open question
026	James: We sang the fraction song	
027	The class went into a brief uproar.	Enjoyment
036	T: O.K. Let's sing it.	Class participation
037	Students and teacher sung a song about fraction.	

Introducing the Problem

043	T: Today we will continue looking at fractions.	
044	T: Remember when Jake was saying how ½ is a big fraction.	Connecting to previous lesson
045	S: yes sir.	
046	T: Well here is the question for today.	Introducing the problem
047	Jake and Paul are talking. Jake said ½ is bigger than 2/4 and Paul said 2/4 is bigger than ½. Who is correct? Explain.	Using students names, Motivating
048	Students read the problem aloud.	
054	T: Earl, what can you say about the problem?	Eliciting responses
055	Earl: Jake and Paul are saying different things.	Student's interpretation
056	Dale: They are saying one fraction is bigger than the other.	Student's interpretation
057	T: Who thinks ½ is bigger?	Stimulating responses
058	About six students raised their hand.	Think time
060	T: Who thinks 2/4 is bigger?	Giving opinions
061	About 9 students raised their hands	
064	T: ok. You must show me something to make me believe you are right. You can draw diagrams, use numbers or fold these square papers, but I want you to explore the problem and tell me what you find out.	Proof, flexibility in solution
065	At this point he gave each student a square sheet of paper.	

Solution Process

068	Student began working, the teacher walked between the desks to observe students' work.	Evaluating Students

093	James folded a square paper in four equal parts and another square paper in two.	Students' strategy-folding
094	He shaded two parts out of four and one part out of two respectively.	Freedom in strategy
096	He looked puzzled.	Confusion
097	He stared at the papers for a while, then called the teacher.	Asking for help.
098	James: They are the same size. (He said questioningly.)	Confusion
099	T: Why do you say that?	Open question, why…..
100	James: Because this part (pointing to ½) and this part (pointing to the 2/4) are same size?	Defending choices
101	He still had a puzzling look on his face, as if to say why are they the same	Rethinking
102	T: Are you sure?	Confirming idea
103	James: I think so (in an uncertain tone)	Uncertainty in understanding
105	T: Can you do it another way? Try to do it another way and see what happens.	Changing the approach
106	The teacher continued to move about the class.	Evaluating students
107	Students were using different method to try and solve the problem.	Multiple representations
108	O.C. I think they heard the suggestion the teacher gave to James as he had spoken loudly enough.	
110	He stopped to talk with Ted who was drawing in his book.	Strategy, using diagram
111	Ted: I folded, but I still do not know so I drew this.	Using multiple strategy to confirm idea
112	T: I see, what fraction is that?	Confirming ideas
113	Ted wrote 2/4 in his book and said 2/4	
114	T: Where is half, can you show me half?	Probing responses from students
115	Ted drew another square and divided it in two parts (see Figure 5).	Strategy

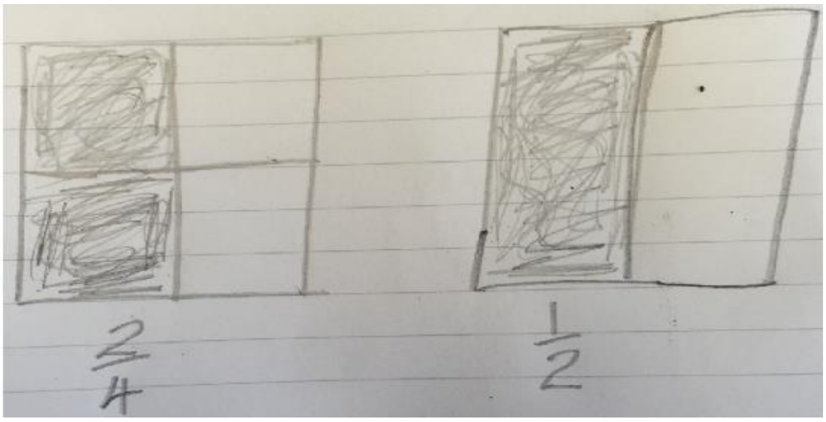


Figure 5. Ted's Diagram

116	Ted: This is a half.	
117	T: Which one do you think is bigger?	Comparing
118	Ted: 2/4	Justifying conclusion
119	T: Why?	Defending choices

120	Ted: Because this is 2 and the other is 1. Two is bigger.	Confirming understanding
121	T: Look back at your folded paper? Is it bigger there?	
122	Ted: No, and this is also two parts	Re-thinking idea
123	Ted: Hmmmm…	Re-Thinking idea
124	The teacher waited for a while…	Verifying idea
125	T: So, what can you say?	
126	Ted: I do not know	Being stuck
127	T: Think about it some more, try to imagine you are sharing a bun.	Using concrete objects
135	The teacher went over to Gary.	
136	Gary: I am looking for which is bigger.	Explaining process
137	T: And...	Encouraging explanation
138	Gary: I do not know.	Being stuck
139	Teacher: O.k. let's try this, what do you know so far?	Offering support in understanding
140	Gary: This is half because it's half of the paper.	
141	T: Right, very good, anything else?	
142	Gary: But if I write the fraction its two out of four	Reflecting on solution process
143	T: O.K. what can you say then about ½ and 2/4?	
145	T: So anything comes to mind. What have you decided?	Encouraging deeper exploration
146	Gary. Its looks like… (He paused then said) … I am not sure	Uncertainty
147	T: You are a bright boy, something will come to you. But you are on the right track	Motivating, encouraging
150	The teacher looked at Scott's work	Evaluating
151	Scott: I folded this way.	Different strategy
152	T: What can you say about it? What did you learn?	Probing for understanding
153	Scott: They are the same	
154	T: How do you know?	Probing for explanation
155	Scott put one paper over the other and try to match the shaded regions (see Figure 6).	Unique explanation
		Originality. Strategy

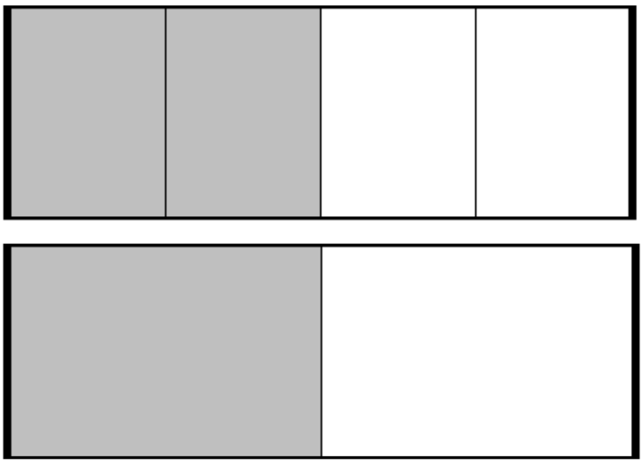


Figure 6. Scott's Diagram

156	T: So…	Clarifying
157	Scott: They match, so they are the same	Explanation
159	Gary: I see! Shouting out.	Satisfaction/Eureka
160	T: What have you found? Teacher moved to his desk	Probing for understanding
161	Gary: The same part is for ½ and 2/4	Explanation
162	Scott: Yes, they are equal. (Scott and others also went to his desk.)	
163	The teacher announced that students should write a mathematical statement or an alphanumeric statement about their answer.	Agreement
165	Teacher moved to Oral who was talking with Earl.	Guiding explanation. MW
166	Oral: Sir, he said they are the same but I do not think so	Pair discussion
171	T: Who can explain it to Oral?	Communication
172	Students from neighbouring desks were looking and listening to the conversation.	Eliciting response
173	Kevin: I did this but…	Uncertainty
174	He showed a square divided into four parts with two parts shaded.	Offering support
175	T: You said "but..." what do you want to say?	Strategy, solution
176	Kevin: Here is 2/4 but…	Explanation
177	T: But…?	Clarifying ideas
178	Kevin: Its ½ (see Figure 7).	Equivalent/ fraction

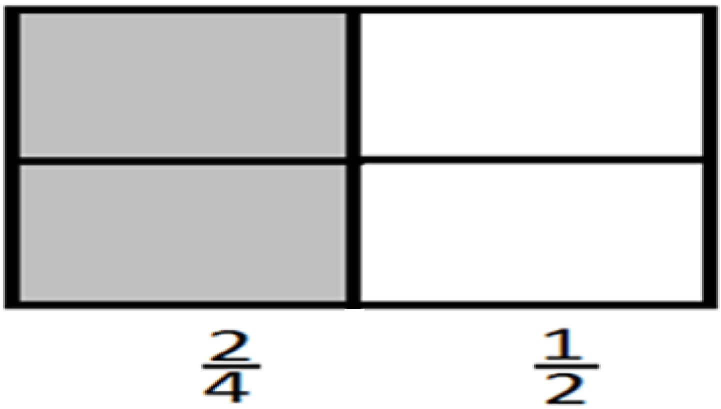
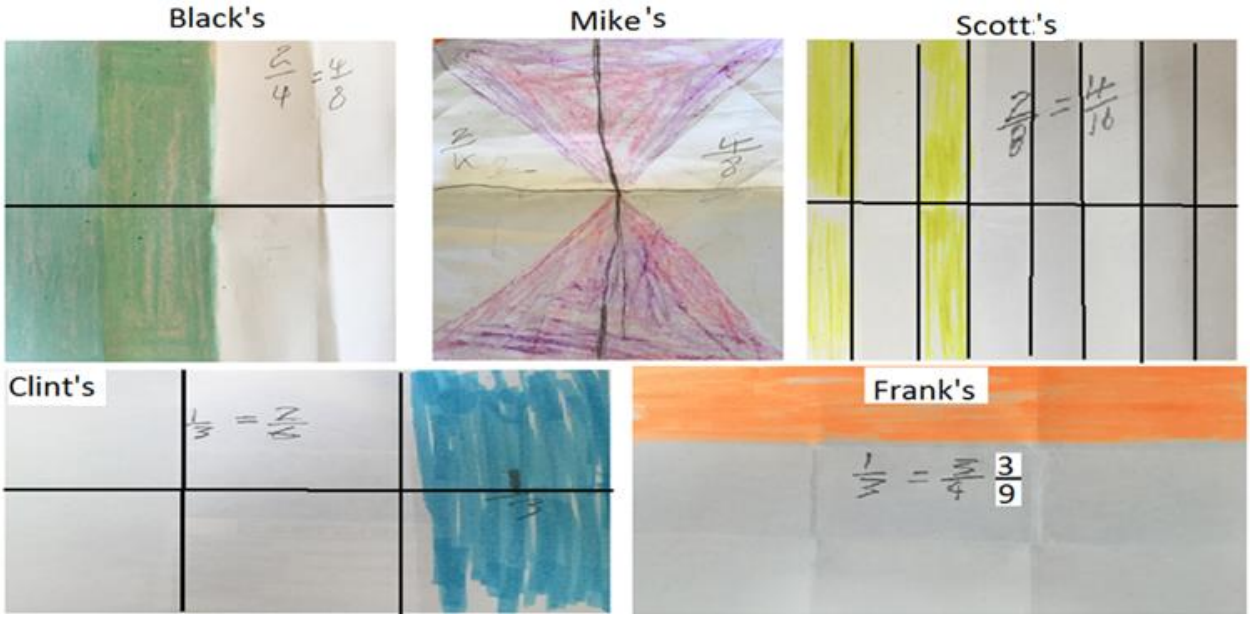


Figure 7. Kevin's Diagram

179	Earl: See, ½ is the same as 2/4	Eureka, understanding others strategy
180	O.C. The teacher did not correct this statement.	Lack of correction
181	Oral: I do not understand. How are they the same?	Asking questions
182	T: Ryan do you have something to add?	
183	Ryan covered the shaded part and asked (Speaking to Oral) what part is this?	Explanation, elegance,
184	Oral: Half	Explanation, elegance, Creative idea
185	Ryan removed his hand	Unique explanation
186	Ryan: Now count how many parts out of 4 is shaded.	
187	Oral: 1, 2	

188	Ryan covered the shaded area again and asked “What fraction is shaded?”	Understanding of fractions
189	Oral: I see! It’s the same part. ½ is same as 2/4!	Eureka moment
192	By this time, all students in the class heard that the fractions are the same size.	Satisfaction
193	The teacher told students who were finished to write a statement about what they discovered in their books.	Opportunity to explore. Strengthening connections.
194	(Students had ½ = 2/4, or ½ equal 2/4, ½ and 2/4 are the same).	
196	T: We use the word “equivalent” instead of equal. Why?	Discussion
197	O.C. Silence	
198	T: Hint, look at the numbers in the fractions.	Providing support
199	Scott: The numbers are different.	Eliciting response
200	T: Right. The numbers are different	
201	Mike: Oh… the size is the same.	
202	T: Yes, they have same size (he wrote the word “size” beside “same” on the board and underlined it.	Board writing
203	Students: Oh…	Eureka
204	T: Use these terms when you explain your work.	



*Folding areas are marked by inserted lines

Figure 8. Examples of Students’ Work

Analysis of the All-boy’s Class

The teacher began the lesson by recapping the previous day’s lesson. This helped students to see the link between the day’s lessons and the previous one. The teacher was able to do a brief analysis of what students had garnered from previous lessons. Three important points from students' responses were written on the board:

- A fraction is a part of an equally divided whole.
- The top number (of a fraction) is called the numerator.
- The bottom number (of a fraction) is called the denominator.

The song was a good way to ignite students' interest. It provided enjoyment for students. Also, students tend to remember information in songs more readily than they do speech (Matalon, 1997).

The teacher introduced the problem in a mathematical context. However, using the names of students in the class, helped the students to think of the problem in their own context and this encouraged discussion. The problem was open in that students could use different ways to show their solution. The teacher confirmed that students knew what was expected of them by asking Earl and others to interpret the problem. He further included class participation by asking the class to guess which fraction was bigger. This action provided hints and acted as a stimulus to help students solve the given problem.

Even though James had the correct answer, it was not accepted as a solution because he was unable to give an explanation for the answer. In an attempt to assist James to form a connection with the concept he was learning, the teacher directed him to think of a practical example. This is corroborated by the Hoosain's (2001) and the Davis' (2006) models. Here they list "Identifying Examples" as the first step in building one's understanding. In order to build James' understanding, the teacher helped him to identify common characteristics between the new and old examples. Using the common characteristics between two examples to create third example that reflects the same commonalities is one way of showing the understanding of a concept. Since James was uncertain of his answer, finding more examples would help to see the connections more clearly. Students were guided by words such as “so” and “why” to get them to think more about what they were doing and why they did it. These also caused students to think about the connections and to justify why their strategy works (Davis, 2006). An example of this is seen in the discussion between Ted and the teacher. The teacher asked Ted to justify his solution and explain why he thinks his answer is correct. Questions in which students were asked to justify their thoughts process were recorded more than 6 times in this lesson. These questions allowed students to think about the “why” of an action or response and encouraged them to offer a logical defence. It was common for teachers to use the questions to force students to defend their answers.

During individual work, the teacher supported each student by providing information when necessary. He waited on James to think about what to do next before offering a suggestion. The teacher waited while James formulated a mental connection between his disjointed knowledge to create a logical path to the solutions. In doing so he became aware of different connections with the concept. The more connections of a concept that a student can create, the more they demonstrate their understanding of the concept. Another noticeable “move” in this lesson was the shift in terminology. The teacher used the term “equal” at the beginning of the lesson and the term “equivalent” towards the end. This is an example of how the teacher helped students to use mathematical terms and to think more about the abstract world of mathematics. He allowed students to use the terms they were familiar with (equal) before introducing the new term (equivalent). Also, the definition of the concept did not come at the beginning of the lesson as in the expository approach. Here, teachers allowed the definition to evolve from students working with the concept. This reflects the sequential order of Davis' (2006) model for developing understanding. Moving from identifying examples (step 1 in level 1) to defining the concept (step 8 in level 2).

Level 1: Students understand a concept to the extent that they can make the following moves: 1 Give or identify examples of the concepts 2 Defend choices of examples of the concept 3 give or identify non examples of the concept 4 Defend choices of non-examples of the concept
Level 2: Characteristics of the concept 5 Identify things that are necessarily true about examples of the concept 6 determine properties sufficient to make something an example of the concept 7 Tell how one concept is like (or unlike) another concepts 8 Define the concept 9 Recognize the applicability of the concept in unfamiliar contexts

Figure 9. Adaptation of Davis (2006) Model of Conceptual Understanding

The researcher garnered additional information about the lesson by speaking with the teacher after the class. The teacher said he allowed the lesson to develop at a timely pace because the concept of equivalent fraction is important and understanding its foundation is key for successful operations with fractions. He did not attempt to help Ted nor Gary when they were stuck, because he knew they had perseverance and could figure out a way to overcome their struggle by themselves before giving up. Many scholars (Brooks & Brooks, 1999) encourage productive struggle in problem solving. A teacher will need to exercise patience to allow students to be confused and uncertain before discovering new perspective. This philosophy is supported in the theory of constructivism. Brooks and Brooks (1999) state that constructivism claims that real understanding, which must be complexly connected with prior learning, can only develop through the active discovering, explaining, and testing of relationships by students themselves. Here the teacher needs to be aware of both the student's ability and temperament. The duration of the productive struggle allowed by the teacher before offering assistance is also important. In this example, the teacher exercised patience and displayed a knowledge of the ability of each student. Interpretation of students' solutions in terms of fluency, flexibility and originality follows.

Fluency: Twenty-eight of the thirty-one students used the paper folding strategy (probably because paper was given to them to use). However, the students folded the papers differently. Twenty-two students folded the papers in plus sign - see Ryan's solutions. Six students use one direction folds to form rectangles- Scott's solution. Towards the end of the lesson however, other types of folding patterns were generated by students- see Figure 9. As a result of exposure to other ideas presented by their peers and participating in discussions, students were able to develop a greater understanding of the concept. As each member of the group developed greater understanding of the concepts, he/she was able to contribute more in-depth ideas, create more connections and produce more sophisticated ways of folding to show equivalency of fractions.

Flexibility: Three types of solution methods were revealed in the observation notes; using paper folding (28 students), drawing (25 students) and reasoning (9 students). In reasoning, students commented that 2/4 is bigger than 1/2 because the numbers are bigger. Folding the paper revealed something contrary to their original understanding, that is, the fractions were of same value. In doing self-reflection (Hoosain, 2001), students were faced with the dilemma of having to reject their first personal idea and to accept the results that the paper folding strategy revealed. Confronted by an opposing idea, and being unable or unwilling to reject their initial opinion, some students resorted to saying "I don't understand". From a conceptual point of view, it was difficult for students to grasp the idea that even though one fraction has larger numbers, both fractions represent the same value. These “I do not understand’ statements occurred frequently in lessons and can be used as indicators of level 1 understanding of not being able to defend one’s choice but unable or unwilling to accept the presented alternative (Davis, 2006). In teaching, a deliberate attempt should be made to assist the child in making that critical decision to give up his initial idea. With some individuals, this can prove to be a difficult task. Davis (2006) recommends providing additional examples to reinforce the idea. Hoosain (2001) suggests teaching for relational understanding. In the open approach both Davis’s and Hoosain’s suggestions can be applied simultaneously by referring to the solutions of other students in the class or by creating other solutions.

Originality: Solutions which evolved towards to end of the lesson became more sophisticated. Two solutions, Scott’s and Ryan’s, can be seen as unique or sophisticated ways of showing that the size of the fractions are the same. Scott placed one folded paper over the other to show that they had matching shaded regions. This kind of solution requires an understanding of the underlying structure of the concept. By covering one section of the paper, Ryan forced Oral to focus on the exposed section, then by removing his hand, a new perception of the whole paper was gained. This also shows an understanding of the underlying structure of the concept. Ryan was also able to elegantly explain to Oral so that he too could see the connections between ½ and 2/4. It should be noted that Ryan was considered to be a slow non-reader of the group, yet he understood the underlying meaning to the extent that he was able to produce such original explanation elegantly. This shows that the open approach is able to support students in understanding mathematical concepts regardless of their reading ability. This phenomenon was also discussed in Munroe (2016a) where students who were of low reading ability displayed greater understanding of mathematical concepts.

5.2.2 The All-girls' Class

	Introduction	**Code**
005	T: Yesterday we talked about sharing.	Recapping previous lesson
006	If I want to share this pencil for two students,	
007	Where should I cut it so that they both have the same size of pencil?	Daily life experience
008	Students: In the middle, halfway.	Daily life experience
011	What fraction of the pencil is this? (Pointing to the left half.)	
012	Students: A half.	
013	T: Very good. So yesterday we talked about fractions like these.	Linking to previous lesson
014	Who can tell me what they remember about fractions?	Linking to previous lesson
015	Some students raised their hands.	
016	The teacher called on some students whose hands were raised	
017	Rachel: A fraction is part of a whole	Eliciting responses
018	T: And….?	Partial understanding
019	Tamara: The whole has equal parts	Eliciting responses
020	Pam: a fraction is a part of a whole	Eliciting responses
021	T: Ok. Please remember that the whole is divided into…	Providing assistance
022	Students: Equal parts.	
025	The teacher called on other students, but they simply repeated the first two statements or said something similar.	

Introducing the Problem

028	T: Today we are going to look at some special fractions.	Introduction
029	The teacher wrote on the board, "Barbie is sharing chocolate with her friend. She ate ½ and gave her friend 2/4. Did anyone eat more, who eat more chocolate?"	Daily life experience
034	T: Do you understand the question?	Clarifying Question, Eliciting responses
036	Gabriel: Yes miss, we should say who ate more chocolate.	
037	Sheral: Barbie eat more chocolate because it's hers.	
042	T: Use the papers to help you solve the question. You can also draw in your book. (She said this while issuing two small square sheets of papers to students.)	Providing materials to elicit multiple responses, understanding in Math world
043	T: In how many parts should we fold the paper?	Providing support
044	Students: 2, 4.	

Solution Process

045	The teacher began to observe student's while they folded the pieces of paper.	Evaluating Students
046	Teacher spoke with Sara	Offer support
047	Sara: I think Barbie ate more because it's her chocolate.	Non math reason, experience
049	T: I see, can you show me? How did you fold your paper?	Clarifying idea
050	Sara folded one paper into two and the other into four parts (the Plus sign fold)	Strategy
051	T: O.k. and what can you say?	Clarifying idea
052	Sara: This is for Barbie (pointing to one side of the paper)	Defending choices
053	And this is for her friend (pointing to the other paper.	
054	T: How do you know that Barbie's is bigger?	
055	Sara: Because it's her chocolate.	
057	T: I want you to show me using the papers. Show that Barbie's part is bigger.	Guiding reasoning
058	Sara paused to think.	Guiding reasoning
059	T: Think about it, I will come back for an answer.	
060	The teacher talked with Rose. Rose and Britney were working together.	Pair work
061	Rose: Miss we know that ½ and 2/4 are equal (they had their notebook open to show the multiplication's table)	
062	T: I see, so what does that tell you about the chocolate Barbie and her friend ate?	Relating to real world
064	Britney: We learn it in grade three miss, 2/4 = ½ and 3/6 = 1/2.	Non math reason, experience
065	She said it scornful confidence.	Guided understanding
066	T: Yes, but why do we say 2/4 is equivalent to ½?	
067	Britney: Because they…	Pausing during explanation. Assessing one's own reasoning
068	T: what do you think you can do? Britney: We could use another method miss?	Alternative method
069	Most students folded two papers;	Fraction
070	One into two equal parts and the other into four equal parts.	Strategy
071	They shaded one side out of the 2 and 2 out of 4 parts respectively,	Strategy
072	But they did not recognize anything different or similar with their fractions.	
073	Some students had folded one strip of paper and coloured respective sides differently.	Strategy 2
074		Strategy 3
075	They too did not notice anything significant.	
076	Janet: Miss, the fractions are the same size (she said hesitantly)	Giving reasons
077	T: what makes you say that?	
078	Janet: They look the same, but	
079	T: But...?	Encouraging further explanation
080	Janet: I do not know miss, are they the same?	Uncertainty
081	T: You tell me,	Facilitating reason
082	Janet looked puzzled.	

083	Gayle raised her hand to indicate that she needed to speak with the teacher.	
084	Gayle: the fractions are the same miss (she said without conviction)	
085	T: Are you sure?	Uncertainty
086	Gayle: No miss,	
087	T: What can you do to be sure?	Being stuck
088	Gayle: I do not know.	
089	T: What do you see form what you have?	Facilitating understanding
090	Gayle: The fractions are different but the size is the same?	
091	T: What can you say about the size Barbie and her friend ate?	
092	Gayle: Barbie and her friend ate the same (size) chocolate?	
093	T: O.K. so what about the fractions then?	
094	Gayle: They are the same miss? (Still in a quizzical tone of voice)	
095	Some students were whispering to each other that the fractions are the same.	
097	T: Who thinks they have an answer. (Speaking to the class.)	
098	Mary went to the board and drew her diagram.	Alternative strategy
099	The teacher directed her regarding which side of the squares to shade.	Board writing
100	O.C. She is using the board more efficiently and thinking ahead of students.	
101	Mary: This is what Barbie ate (pointing to the half and this is what her friend ate (pointing to the quarters.)	Explanation
102	Mary's Diagram	Discussion

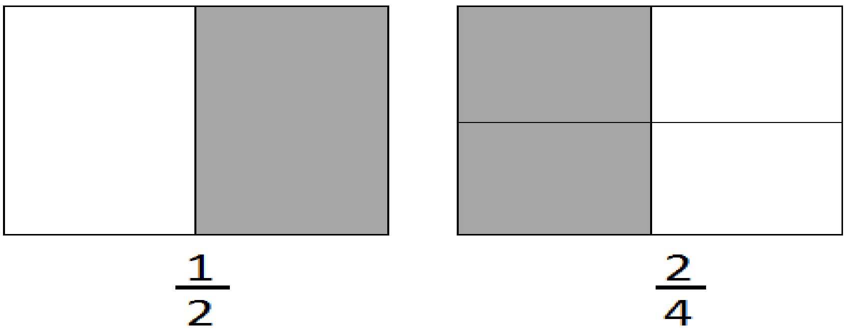


Figure 10. Mary's Diagram

103	Jane suggested that Mary use one paper (Diagram) While Mary was shading.	
104	Teacher: Jane what do you mean? Can you explain it on the board?	
105	Jane went and erase the diagram showing ½.	
106	Jane: Miss, if this is the chocolate,	Different strategy
107	Barbie ate this part (shaded part) and her friend ate that part (the unshaded part)	Persuasive argument
108	Students: oh, I see!	Eureka
110	Because it's one chocolate.	
111	Rachel: It's one chocolate so we should use one diagram.	Logical reasoning
112	T: What do you notice with the fractions now?	Persuasive argument

113	Pam: They look the same miss?	
114	Teacher writes on the board. “Same size”	Board writing
115	T: Can you say anything else about the fractions?	Open Question
116	Sheral: The two of them (fractions) are the same size.	
118	T: What can we say about the fractions ½ and 2/4?	Realization, eureka
119	Britney: They are equal.	
120	Teacher wrote on the board: two fractions are equivalent if they have the same size.	
121	Shelly: Yes, I remember! In the multiplication table 2/4 equals ½.	Connection
122	Some students looked at the multiplication table in their notebooks.	Connection
123	They showed each other how to locate equivalent fractions.	Connection
124	T: What other fractions that you think are equivalent to ½?	Eliciting responses
125	Students: Students were naming other fractions equivalent to ½.	
126	T: I would like you the fold these papers to show me two other fractions that are equivalent.	Extension to the problem
130	A major part of work on the board was erased to accommodate other presentations.	

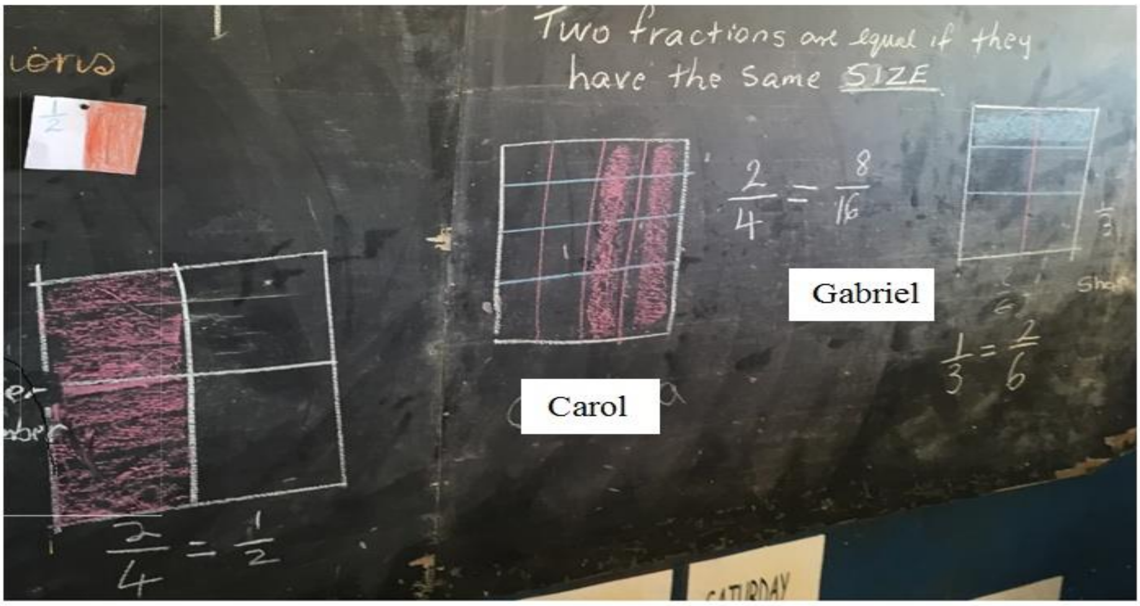


Figure 11. Blackboard Showing Two Students’ Work

Discussion on Presented Fractions.

131	The teacher moved about the class to observe students at work.	Evaluating Students
132	Most students folded the paper to show ½ and 4/8, or 2/4 = 8/16.	Multiple responses
134	Each member of the group had the same fraction.	Sharing
143	Teacher asked Gabriel to explain.	Evaluating Students
144	Gabriel: Miss, I will fold it in threes	Strategy
145	T: o.k. show me.	Encouraging further explanation
146	Gabriel folded her paper in three parts	Facilitating learning,
147	T: now what will you do?	Guiding question
148	Gabriel looked uncertain.	Facilitating learning

149	After about 40 seconds, the teacher suggested colouring the sections.	Facilitating understanding
150	Gabriel shaded 1/3 of the paper.	
151	Gabriel: Fold it again?	
152	T: yes	
153	Gabriel fold the paper in the same direction that she folded it before.	Solution strategy
154	T: can you fold it in another direction?	Offering Suggestions
155	Gabriel folded the paper again in another direction.	Facilitating understanding
156	T: what now?	
157	Gabriel said this is now 2. (Pointing to the shaded part)	
158	Teacher: yes, and….	
159	Gabriel: And this is 6. (She counted all the sections of the paper)	
160	Teacher: so what fraction do you get?	
161	Gabriel: 2/6 she wrote it on the paper. Is that right miss?	Uncertainty
162	T: do you think it is correct?	
163	Gabriel: yes miss, T: yes it is correct.	
164	Other students came to look at what Gabriel did.	
165	The teacher asked Gabriel and Carol to show their work on the board.	Motivating
172	Carol explains her diagram	
173	I first fold my paper in four like so...	Alternative solution
174	O.C. she spoke with more confidence than Gabriel	Confidence, mannerism
175	She showed a rectangle divided into four parts on the board	Fraction (1)
176	I shaded here and here which is two out of four.	Explanation
177	Then I fold it the other way like so….	
178	(again moving her hand over the board)	
179	And count, 1, 2, 3…	
180	This gives me 8/16. So 2/4 = 8/16.	
183	T: How do we know they are equal?	Clarifying idea, Eliciting responses
184	Students. They have the same size.	Responses
201	(Other students gave similar explanations)	
202	The teacher asked students to say what they learnt from the lesson and listed students responses on the board	

Analysis of the All-girls' Class

The teacher drew on student's experiences and everyday knowledge about a pencil to arouse their interest in the lesson. Some students carried only one pencil to school. If that pencil gets lost, they end up purchasing a new one or receiving a fraction of their friend's pencil. This occurs frequently, therefore students had sufficient experience with sharing a pencil and could easily relate to the scenario. However, using a picture of a pencil would have helped students to better visualize the situation and would have taken less time than did drawing a pencil on the board.

The teacher used the name of a popular doll (Barbie), to help students identify with the problem. She did not ask students to restate the problem, but offered a starting point by hinting at folding the paper into twos and fours. She may have thought that the problem was easy for students to understand and did not require verification. This may also explain the wording she used in the problem itself. The problem did not explicitly state comparing the two fractions (½ and 2/4). However, the sum of the two fractions would have been compared to one whole which in turn would achieve the desired outcome of showing that ½ and 2/4 are the same size or equivalent. This teacher had high expectations of her students, hence, she often overlooked minute details.

The problem was stated in a closed format and developed accordingly in that most students (more than 80%) folded the paper in the same pattern and had the same explanation. At the beginning of the solution process, even though students were told to attempt different methods to solve the problem, they used paper folding only and most students used the same pattern.

It was observed that students responded tentatively in most cases. They did not accept an answer as correct until it was confirmed by the teacher. It is better for students to have confidence in their solution rather than to rely on the teacher. This confidence comes when students truly understand the concept and how it is used. When this was pointed out to the teacher during the post-session discussion, she also expressed her concern and added that "her girls usually have great confidence in their work". However, she was comparing students working in the traditional approach with working in the open approach. In the traditional approach, students are given an example to follow. They displayed confidence because they knew they had followed the given example. This confidence is in their understanding of the "procedure" and not in their understanding of the "concept". In the open approach no example is given and students are required to use their own experience and knowledge of the concept to create a solution. Increasing students' understanding of mathematical concepts may also increase their confidence in doing math. However, having confidence does not necessarily mean that one has conceptual understanding as it is possible for an individual to confidently defend a position which is erroneous. For example, Sara was confident that Barbie ate the larger portion (line 047), but she was also wrong.

Understanding of procedure is different from understanding of a concept (Davis, 2006). Some students were aware that 2/4 is equivalent to ½, but had difficulty explaining why. Their only defence was that they learnt it in grade three. Encouraged by the teacher to use the folded papers and diagrams, students eventually could explain why the fractions are equivalent. This is a typical case highlighting the difference between procedural understanding and conceptual understanding.

The teacher initiated an extension of the problem. This was to provide students with the opportunity to further explore the concept with a wider range of numbers. The extended problem was given in a simple open-ended format. In this lesson, the extended problem was presented in the format of a culminating activity. Students were able to create equivalent fractions easily, but again, variations were few. Some students replicated the work of others that the teacher had sanctioned or produced work similar to what was discussed earlier.

Students in this class seemed to prefer being given direct instructions with step by step solution guidelines.

Carol, Gabby and Vicky were asked to show their work on the board and they became excited about this. They erased a section of the board so that they could draw big, bold diagrams. This is interpreted as a sign of confidence.

Fluency: Seven students folded one paper into four parts and five students folded one paper into two parts. The other 15 students folded both papers, one into two and the other into four equal parts. Student's reservations prevented them from presenting multiple solutions. Most students only used paper folding or made only one attempt at finding a solution. There are two interpretations for this; (1) they did not know what to do; (2) they knew what to do but were hesitant about doing it. The first interpretation means that students had no prior experiences with equivalent fractions or they had forgotten what they had previously learnt. The fact that some students in the class were able to identify equivalent fractions, suggests that the class had prior exposure to the content. In any case, the open-ended problem here presented, was a good way of developing the understanding of the concept. This conclusion could be drawn because students were able to produce multiple ways of showing equivalency towards the end of the lesson but were unable to do so at the beginning.

Secondly, students' hesitation may be interpreted as a lack of confidence or their inability to deviate from the traditional approach in which only one correct answer was required. Here they may have thought that one representation was enough and that no further exploration was required. However, this is unlikely since, up to this point, students had been using open approach for more than three months and would have been accustomed to presenting multiple solutions to a problem.

Flexibility: Three types of solution methods were revealed in the observation notes; using reasoning, paper folding and diagrams. Reasoning based on everyday experiences was used at the beginning of the lesson. All students eventually used the paper folding strategy probably because it was recommended by the teacher, because it was easier or because other were doing it. Some students used diagrams after others drew diagrams on the board.

Elegance: Carol's explanation reflects some level of elegance in explanation. Her step by step explanation was delivered in a timely manner. Her choice of words and gestures also help students to understanding her solution.

5.2.3 The Co-educational Class: Equivalent Fraction

	Introduction	**Code**
009	T: Yesterday we looked at…	
010	Students: Fractions	Link to previous lesson
011	T: And what did we say about fractions?	
012	T: What do you remember?	Eliciting response
017	Gabby: Fraction is a part of a whole.	Definition of a fraction
021	Samantha: The whole is in equal parts	Recalling information
026	T: Anything else?	Eliciting responses
027	Oliver: Bottom number, denominator	Recalling information about fractions
028	Jake: Top number	
033	Some students: Numerator	
034	T: Can you tell me when we use fractions?	Applying a concept
036	Trish: ½	
037	Teacher: Good Trish. That is an example of a fraction	Motivating
039	Tell me something we do where we can use fractions to explain it.	Rephrasing
040	Trish: Eating Chocolate	
041	T: Ok. Let's say use chocolate.	Using students' comments

Introducing the Problem

042	T: Today we will be looking at a special type of fraction.	Problem introduction
044	Trish is sharing a chocolate. She eat ½ and gave 2/4 to Kim. Who ate the bigger part, if any did?	Daily-life experience
048	T: Rosey, what should we do?	
049	Rosey: We should tell, who eat the bigger part, Trish or Kim.	Verifying
052	T: Who thinks Trish ate the bigger part?	Stimulating thinking
053	5 students raised their hands.	Stimulating thinking
054	T: Who thinks Kim ate the bigger part?	Stimulating thinking
055	8 students raised their hands.	Stimulating thinking
056	T: Sally, why do you think 2/4 is bigger?	Justifying
057	Sally: Because it has bigger numbers.	Giving reason
058	Many students agreed with her	Defending choices
059	T: Ok.	
060	Brady. 2 is bigger than 1 and 4 is bigger than 2 so 2/4 is bigger than ½.	Giving reasoning
061	T: do we agree	
	O.C. some students said yes.	
062	T: Troy, why do you think ½ is the bigger fraction?	Clarifying idea
063	Troy: Because ½ is always bigger.	Defending choices
064	Some students agreed with Troy.	
065	T: Explain. What do you mean?	
066	Troy: On the fraction chart, ½ is the bigger part, the others are smaller. (Show chart)	Proving
067	Gordon: But miss, the fraction on the chart is ¼ not 2/4, It has four 1/4's.	Counter argument
068	T: Hmm is see.	
069	Do you see that Troy? Come and show him Gordon.	Facilitating discussion
070	Gordon went to the chart and pointed to ¼	Proving
071	Gordon: this part is only one but here (he point to the 2/4 on the board)	Counter argument. Comparing
072	It's 2 out of 4.	Persuasive argument
073	This (2/4) is different.)	Persuasive argument
074	Troy: I still say ½ is bigger than 2/4.	Un-willing to change idea
075	T: I will not say anymore, you can solve the problem in groups or by yourself.	
076	Try to show your answer by doing it in two different ways.	Multiple representations
077	You can use these square papers to help you.	Manipulatives/concrete

Solution Process

079	The teacher observed students as they worked.	Evaluating students
080	Dave left his seat to work with Clive.	Pair work
083	They were folding the paper to show four parts.	Freedom in strategy
084	O.C. This is the same folding pattern seen in the other classes.	
085	It was expected.	
086	The teacher talked with Gail:	Individual evaluation
087	Gail: Miss I fold this one like this (in halves) and this one like this (in quarters.)	
088	T: And…	Encouraging further explanation
089	Gail paused and said.	
090	I do not know miss. I want to try drawing	I want to try
091	T: O.K.	Offering suggestion
092	The teacher went to Aman's desks	Individual Evaluation
093	T: What have you done so far?	
094	Aman drew two squares in his book.	Strategy
095	He had divided one into two and the other into four parts but the square he had divided into two was bigger than the other.	Strategy
096	Aman: I put it into two parts first, then four parts.	Strategy
097	Then I shade this part (two out of four)	Fraction
098	T: What did you find?	Strategy
099	Aman: ½ is bigger miss.	
100	T: I see, I think it is better if you draw this square as big as this one.	Offering suggestion
101	Aman: O.k. miss, I will try that.	Rethinking strategy
102	Tom and Luke were working together.	Pair work
103	Luke: They are the same miss,	
104	T: Why do you say that?	Asking clarifying Questions
105	Luke: This side is for Trish and this side is for Kim. They look the same (Luke was showing a diagram in his book).	Fraction (1) Providing reasons
106	Some students were finished and were discussing their answer.	
107	T: Those who has an answer,	Deepen thinking in math world

108	Write a mathematical sentence or a word sentence about what you found.	Deepen thinking in math world
109	After about two minutes the teacher asked some students to show their work on the board.	
110	Brad drew two squares, divided one in two and the other in four parts.	Strategy
111	Brad: This is Trish’s ½ and this is Kim’s 2/4.	Discussion
112	Kyle: He should have one diagram miss?	
113	T: We should wait until he is finished, well any way, what do you think Brad?	
114	Brad: No its two fractions	
115	Kyle: But it is one chocolate miss.	
116	Gabby: But we cannot show the two fractions on the same paper.	
117	Miss gives us two papers so we should use two.	
119	T: Now, now, you do not have to use the two papers at once.	
120	Did anyone use one paper?	
127	T: Michelle: we used one paper.	Alternative solution
128	First we folded it in four parts like this. (Using vertical and horizontal folding).	
129	Michelle: We tear half for Trish and half left for Kim.	Providing unique solution
131	The teacher look pleased.	
133	Kyle: I do not understand, where is 2/4?	Clarifying idea
134	Jerry: probably we could try drawing. He drew a square on the board, divided it into 4 parts.	I want to try/ Alternative
135	Jerry. The fraction for one part in the square is ¼	Students' Explanation
137	He wrote ¼ in each of the four parts in the square.	Fraction
138	Jerry: Here is 2/4.	
139	T: How did you get 2/4?	Checking students thinking
140	Jerry paused to think, and then counted two sections in the square.	
141	Troy: Miss could he add, ¼ and ¼.	Fraction. connecting,
142	This was another good way of seeing it.	
145	T: Very good Troy, that is what we are actually doing, she showed it on the board.	Motivating
160	Jerry: This is 2/4 but if you tear it you get ½ of the paper.	Exploring connection/ if statement

161	S: Oh, I see,	Eureka, realization
162	Kyle: tore his paper and said, Yes, it's true!	verifying, checking
163	Some students tore their paper and tried to match the shaded parts.	
164	Sally: They are the same size. (She said this surprisingly).	Realization
165	S: They are the same size,	
166	Jerry: Yes, 2/4 is ½	
167	The teacher repeated Jerry's statement and wrote it on the board.	
168	Rachel: 2/4 is supposed to be bigger, the numbers are bigger.	Asking why
169	T: Let's look at what the numbers mean.	
170	Let's look at what Luke did. (See Luke's diagram)	Offering support
174	The teacher went and wrote on the board ½ and drew a line linking it to ¼ plus ¼ equal 2/4.	Offering support
175	T: Do you see anything? Who can tell me what they notice?	Eliciting responses Making deduction
176	Abbie: The numbers are different but they are same…	Deduction
177	T: Same…?	Clarifying
178	Michelle: Same size.	
179	S: Oh… I see…	
180	Students repeated "the numbers are different but the sizes are the same."	
182	Oliver: Miss my diagram is different.	Multiple strategies/satisfaction
183	The teacher drew Oliver's diagram on the board while Oliver explained.	Each contribution valued
184	T: Very good. Do we understand Oliver's diagram?	Motivating, Confirming
185	S: Yes miss.	
186	T: What does 2/4 mean?	
187	S: 2 out of 4 equal parts	
188	T: And if I take off these two parts? What fraction of the chocolate would I eat?	Reinforcing ideas
189	She covered the shaded section on the diagram.	
190	S: ½	
195	T: These types of fractions are given a special name. Can anyone tell me?	Stimulating thinking
196	Abbie: Equal fractions.	

197	T: Yes, we call them Equivalent fractions. Then she wrote the term on the board.	
200	Bev: Miss, only 2/4 equal ½?	Freedom to ask questions
201	T: What do you think class? Are there other fractions equivalent to ½?	Passing on question to the class
202	S: Yes miss.	
203	T: Take some time to discuss it with others.	Brief discussion time
204	T: I would like you to find your own two equivalent fractions.	Extension
205	You can use these extra sheets of square papers if you like or you can draw them.	
212	Kasandra was asked to show her work on the board.	
213	Kasandra: I folded my paper in three parts.	Students' explanation,
214	Then I shade here. (Pointing to the first part.)	
215	T: What fraction is that shaded part. (Speaking to the class)	
216	S: 1/3 miss	
217	Kasandra: Then I fold it the other way and it looks like this.	
218	She drew a horizontal line in the middle of the diagram.	
219	Now here is 2/ 6.	
220	T: What fraction is this part now?	
221	S: 2/6	Fraction Concept (1)
222	T: What can we say about this?	Students' explanation
223	Kasandra wrote 1/3 = 2/6	
224	Michelle: Miss, the size did not change.	
225	T: Did you see that class, the size did not change.	Verifying understanding
226	If she divided it in nine parts, would the size change?	
227	S: No Miss?	
228	Other students stood and showed their paper or just called out the fractions they had. See Figure 12	

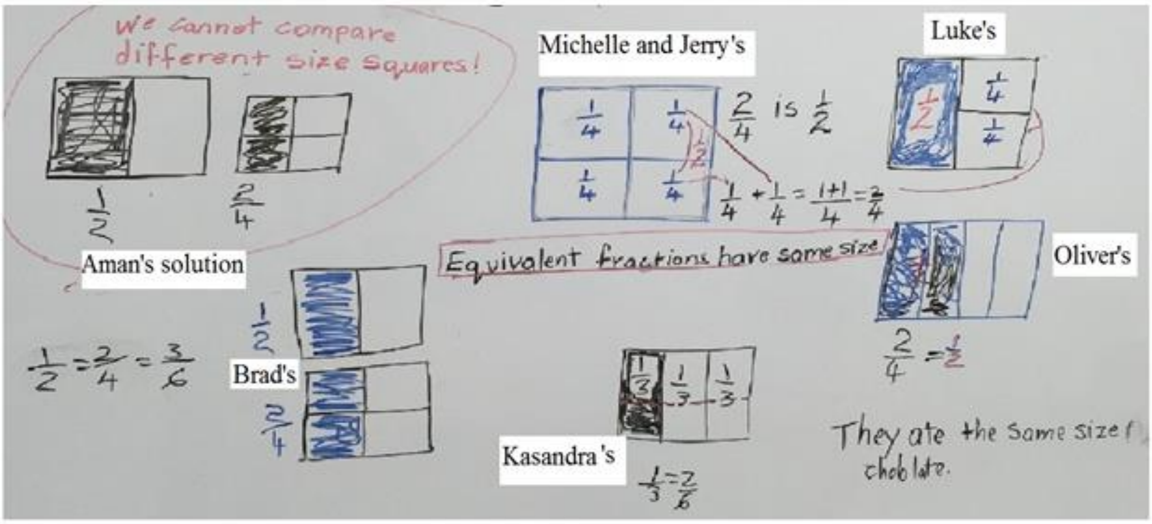


Figure 12. Diagram of the Board at the End of the Class

Reflection

234	T: What did you learn for today's lesson?	Eliciting responses
235	Gayle: Equivalent fractions have the same size.	Students reflection
237	Sally: Fractions have two parts	Students reflection
238	Kim: Equivalent fractions have same area.	Feeling of satisfaction
239	Dave: ½ is equivalent to 2/4	Feeling of satisfaction
240	Samantha: there are equivalent fractions to others.	Students reflection
241	T: How do we know that fractions are equivalent?	Guiding reflection
242	S: They have the same area.	Students reflection
243	T: O.k. show me for your homework.	
244	The teacher asked the class "How do you know when two fractions are equivalent?"	Giving assignment

Analysis of the Co-ed Class

The teacher began the lesson in the form of an interaction between teacher and student, where students completed the teacher's statement. This was a typical form of teaching seen in the three classes. Teachers ending their statements with "what" and "is" which are seen by students as prompts to give an answer or to complete the statement. The teacher asked students to give real examples of situations in which they needed to use fractions. This helped students to see the application of mathematics in their everyday lives (Hoosain, 2001; Munroe, 2015b).

The teacher "piggybacked" on Trish's suggestion about sharing a chocolate to make the transition into the introduction of the problem. She used the students' names in the problem - Trish and Kim. This helped students to identify with the problem. She allowed students to say what the question was asking, and encouraged them to think further by asking them to choose who ate the bigger piece.

The problem is open in that students could decide on their own method for arriving at a solution. Similar to the other classes, the teacher gave students paper but also suggested that they could use other methods to solve the problem.

Students were given the option of working individually or in groups and assistance was given as was required. Aman's diagram, unequal squares, showed a low understanding of comparison. Three other students made an error similar to Aman's - That of drawing diagrams of unequal sizes and attempting to compare them. The lesson was about comparing fractions but the students' method suggested a lack of understanding about making comparisons. The use of open-ended problems allows the teacher to assess students understanding of more than one concepts simultaneously. In this study the ability to recognize the relationship between concepts is an indication of the student's level of conceptual understanding. Teaching in the open approach allows students to see the relationships between the concept they are learning and other concepts in mathematics. This gives the teacher an opportunity to assess how well student understood the concept and its connection with other related concepts. In the lesson about equivalent fractions, students' lack of understanding of how to compare two quantities was revealed. The teacher interacted with each student individually and helped each to overcome the challenges faced. A class discussion of these students' misconceptions may have been helpful to other students, but this was not done. Nevertheless, the information gathered from these situations will be useful to the teacher when she plans lessons in the future.

Boys and girls participated equally in the discussions during this class and each child was given an equal opportunity to participate regardless of their location in the room. It must be noted here that in the traditional classroom, students closest to the teacher are usually the ones who participate in class discussions as they are usually the more advanced students. A layout of the class including seating arrangements is given in Appendix D and this may be used to verify the respective positions of students who contributed to the discourse. This type of discussion homogeneity was first observed in the co-ed class, then the all-boys class and finally, the all-girls' class. The homogeneity in class discussion highlights the type of classroom environment that is created with the open approach. Chevannes (2003) stated that, in the Jamaican classroom, it was not that boys were underperforming but that they were under -participating. Lack of participation resulted in lack of understanding and hence, poor performance. He blamed poor teaching methods for the boys' lack of participation and suggested a change to more student-oriented teaching. In this research, boys were participating to the point where they occasionally dominated class discussions. The resulting increase in understanding may be accredited to their participation in class discourse.

Ideas that emerged from class discussion were recorded on the board. The teacher systematically organised the board's content for visual effect. For example, she drew lines to connect solutions that were similar in content or method-see Figure 12.

As did occur in the all-boys' class, the teacher used a suggestion from a student to introduce the extension of the problem. Extensions to the problem were a common occurrence in all three classes and were done to provide students with the opportunity to solve a wide range of problems associated to the learnt concept (Davis, 2006; Hoosain, 2001).

The lesson ended with a homework assignment to help students think about the problem and their definition of "equivalent fractions". Some students completed the assignment during the session. Figure 13 below shows the solutions of two students, Michelle (a girl) and Jerry (a boy). Even though these two students were sitting beside each other and worked together on the paper folding activity, they had different diagrams to show their answer for the assignment. The collaborative work enhanced both students understanding of equivalent fraction but each student maintained his or her own unique way of thinking and solving the problem.

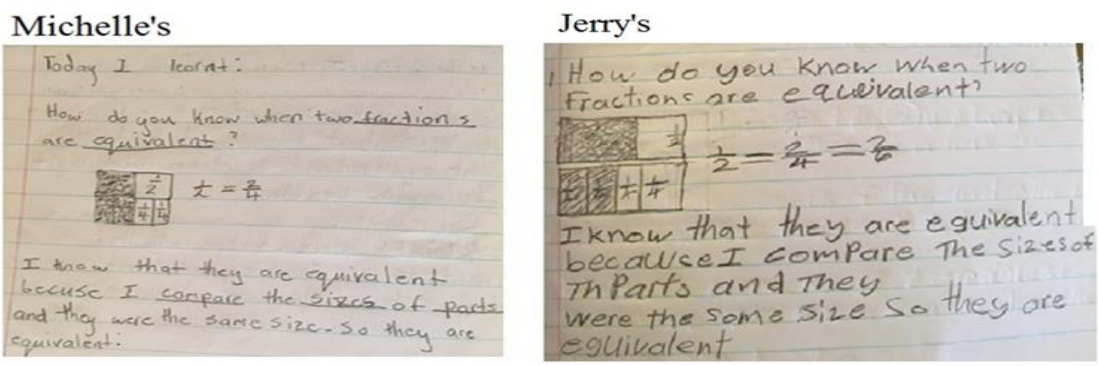


Figure 13. Two Students' Solutions to Assignment

Fluency and flexibility: Thirty-five students (17 boys and 18 girls) used paper folding, while thirty-one (18 boys and 13 girls) had correct diagrams. As Figure 13 reveals, students drew different types of diagrams to show that ½ and 2/4 were equivalent. The variety of solutions presented by students suggests that they were fairly comfortable working with equivalent fractions and representing them in different ways. Here the conclusion may be that students had an appreciable level of understanding of the concept. When compared with students in the all-girls class, students here seem to have had a greater level of understanding of equivalent fractions.

The students of this class used both paper folding and the drawing of diagrams to arrive at a solution. One method was used to confirm the other. More patterns in folding as well as in the diagrams drawn were revealed when students were asked to explore different fractions other than ½ and 2/4.

Originality: Michelle and Jerry gave the most original explanations when they tore the paper to show the similarity between ½ and 2/4. Tearing the paper in half and matching the torn pieces vividly show that the part shaded as "2/4" is the same size as the other. This uniqueness shows not only understanding of the concept but also creativity in thinking.

5.3 Differences among the Three Classes

A comparison of classes revealed that there were more meaningful class discussions in the all-boys and the co-ed classes than in the all-girls class. In the co-ed class, students listened to each other's comments and tried to contribute to the discussion. In the all-boys class, students helped each other and preferred to use manipulatives to assist each other in groups when solving problems. These students were creative in their solution methods as they drew diagrams and created songs about what they were learning. Students in the all-girls class preferred writing in their books and were cautious in their approach to solving open-ended questions. They liked using the board to explain their solutions, but unlike girls in the co-ed group, girls in the single-sex group preferred to simply copy a solution from a classmate rather than discussing how to arrive at a solution. Even when discussions were had, it was, in most cases, the more advanced students that arrived at a solution. This was subsequently copied by the others. This tendency to copy information was however more prevalent at the beginning of the period of intervention and was not as common among co-ed girls. At first, these students copied work without understanding the meaning behind the operation. This, however, changed during the research period as the teacher constantly insisted that students be able to explain their methods and their solutions.

Girls in the co-ed-class also liked working in groups. However, on many occasions students in the same group had different solutions, indicating some "independence" on the part of the students. Towards the end of the intervention, the girls in the single-sex class became more proficient in producing solutions and explaining the use of a concept in daily life. That is, girls in the single-sex class had a greater increase in fluency and elegance than did their counterparts. Co-ed girls on the other hand, showed greater increase in originality.

The boys in both groups were known to have difficulty with reading and writing; therefore, it was not surprising that they needed more time to complete writing activities. As a result, the teacher of the all-boys' class resorted to writing shorter sentences on the board.

Alternatively, in some lessons, students were allowed to stand and explain their work with no requirement for them to write on the board.

5.4 General Gender Comparison

In both class settings, girls were organised and formulated step by step plan to solve the problem; whereas, boys were more impetuous and often applied the trial and error strategy without thinking about efficiency or correctness. Students from both single-gender classes engaged in teasing, power struggle and group control. For, example, in the all-girls class, conflict sometimes occurred if two girls with dominating personalities were in the same group.

Appealing to the sex-role stereotypes which defines how male and female should behave, impacted on how the students participated in the lesson. The male teacher encouraged boys in the single-sex class by telling them they were competing against the girls in the single-sex group. This tactic was used to eliminate unwanted behaviours and to promote desired ones. If the teacher of the co-ed group began the lesson by saying, "I think this will be interesting for the boys", she would elicit more responses from the boys in the class. If the teacher said "I think this lesson will be more interesting for the girls", little change was seen among the girls and boys tended not to show much interest in the lesson. Another important observation was the use of the "I can't" statement. "I can't do it" was common among both sexes but for different reasons. Boys tended to say "I can't" when they were asked to write down what they did. Girls tended to say "I can't" when they were asked to do something or to show what they were thinking. Girls said they liked to copy work from the board and doing what teacher told them to do. On the other hand, most boys said they preferred to try for themselves as it was boring when teacher told them everything.

5.5 Similarities among the Three Classes

As stated before, the teachers' bimonthly meetings were conducted in an effort to ensure that the topics taught and the activities carried out were similar. They however, planned their lessons separately. In all three classes, the lessons began with recapping information on fractions. Students in the all-girls and the all-boys classes were asked to give examples of situations where knowledge of, or the use of fractions was necessary. Teachers supported students by providing opportunities for them to write, draw and talk about their solution process. This discussion was important and helped students to; (1) see other ways of solving the problem (2) compare their solution with those of others, (3) critique their solutions and those of their peers (Nohda, 1991). The other similarities observed among the classes will be discussed under their respective themes in Chapter 6 of this dissertation.

5.6 Generating Themes and Categories

The codes, similar to those of the right side of the observation notes in section 5.2 were compared and those that appeared in lessons from the three classes were selected. Codes were created to show what actions students took to indicate their understanding. For example, an indicator of fluency is to generate multiple examples and non-examples. Therefore, actions and behaviours which show that the child is creating or has created multiple solutions were coded. Codes were also created from actions and behaviours which indicated an attempt at producing multiple solutions and suggested an increase in understanding. Fifty-six codes were obtained. Codes were re-examined and selected using the criteria of *relevance* and *significance*. Codes that showed some relation to the conceptual framework were selected based on the criterion of *relevance* and codes that occurred in more than 60% of the observation notes were selected based on *significance*. This activity reduced the number of codes to 48. Fifteen codes referring to concepts were separated and the remaining 33 codes were grouped into themes. Themes that showed references to the intended environment were grouped together. Themes that reflected characteristics that were deemed relevant to the classroom environment that was to be created by the open approach, were grouped together. While teachers had been previously informed of these characteristics it was uncertain how they would be established and implemented in each classroom setting. Actions that highlighted any of these characteristics were coded as “open environment”. The remaining themes were categorized as “students’ responses” suggesting the possible impact this environment had on students’ understanding of mathematics. Table 17 below shows the codes, themes and categories obtained (the table covers 2 pages in landscape layout). A detailed description and discussion of each theme is given in chapter six.

Table 17. Codes, Themes and Categories

Code Examples (Descriptive)	Theme Description	Theme	Category Description	
I do not agree with you because my answer is different. (Agreeing and disagreeing. Offering alternative solutions.)	Open discourse that promotes understanding by offering multiple solutions and divergence of views.	Communication	This category suggest behaviour, reaction, activities and responses of students as they solve open-ended problems. Students communicate their strategies and ideas with each other. They change their ideas based on ideas from their classmates. Other times, they defended their ideas with logical arguments.	STUDENTS RESPONSES
It is better if you multiply. (Freedom to ask questions and make comments about mathematics)				
Communication among students and between students and teacher				
This can go with ….. (recognizing relationships)	Forming patterns and establishing relationships among ideas that produce different solutions. Identifying connections within a domain	Fluency		
"What if" sentences (exploration)				
I can also try… (generate examples)				
This can also be… (Apply a concept in multiple situations)	Showing the ability to view the problem from different angles in order to provide multiple types and forms of solutions and applications.	Flexibility		
If… then… statements (exploration base on intuition)				
Can we try…? Alternative solutions				
Identifying properties of a concept.	Showing the ability to think outside of the box, checking and justifying one's own hypothesis. Identifying connections across domains.	Originality		
This is true because… (Give plausible justification and explanations.)				
Recognizing and manipulate underlying structure (e.g. if 5 + 8 = 13, then 13 -5 = 8)				
Action used in calculation process (e.g. Using strokes)	Heuristics. Way of solving. Actions (mental, physical) performed during the solution process. Recognizing the value of different strategies.	Strategies		
Applying mathematics to real life				
"This does not look correct" (monitor process)				

Code Examples (Descriptive)	Theme Description	Theme	Category Description	Category
Teacher encourages the use of multiple representations.	Actions or behaviours that promote interest and participation in the lesson.	Stimulate discussion	These themes describe the type of environment that promotes the understanding of mathematical concepts. While the teacher may be the main initiator of many of these activities, students themselves could also initiate or create such opportunities for each other. Therefore the category is referred to as an environment rather than teacher's instruction.	OPEN ENVIRONMENT
All students are able to feel a sense of accomplishment from finding a correct solution.				
There is freedom of expressing oneself without the fear of being ridiculed.				
All contribution are accorded equal value.				
Teacher allows students to work at their own pace.	Assisted students to see Why in their chosen method. Allowing them to use their own chosen method and own way of thinking.	Support understanding		
Teacher asks students to justify their chosen solutions.				
Teacher assists students to clarify their own solution methods.				
Teacher supports both presenter and listener by repeating solutions for clarification and justification.				
Teacher uses and encourages the use of practical examples.	Giving and eliciting from students multiple examples for applying a concept in to everyday life. Showing the necessity of an idea through practical use.	Apply mathematical concept to real-life		
Teacher and students use manipulatives.				
Various applications of one concept are discussed.				
Students are free to explore different extensions to the problem.	Providing opportunity for students to explore wide range of ideas with numbers to identify connections among them. Emphasizing correct calculations.	Deepen mathematical understanding in the mathematics world		
Teacher asks students to define a concept.				
Teacher guides students to explore connections between two strategies or solutions.				
Discussions transition from concrete to abstract.				
Teacher assesses students individually.	Assess student progress continuously and encourage them to monitor and reflect on their own solution process. Ensuring calculations are correct.	Assess continuously		
Teacher assesses students collectively.				
Teacher allows students to give comments on the solution of their peers.				
Teacher reminds students to monitor their own solution process and progress.				

CHAPTER SIX:

RESULTS AND DISCUSSION

This chapter reports on the impact of the open approach teaching method on grade four students' understanding of mathematical concepts in the Number Strand. Ninety-seven (97) students from two schools in rural Jamaica took part in the study. Participants were given a pre-test, followed by lessons conducted with the open approach teaching method and a post-test. Data were collected by observations of lessons and the results from the tests. This chapter presents findings from quantitative and qualitative data.

6.1 Results from Quantitative Data.

It was important to test whether the groups were statistically similar before comparing them. The Leven's test was used to calculate homogeneity of variance between classes and between the genders. Homogeneity between the groups suggests that the students in the classes are statistically similar and can be compared (Gastwirth, Gel & Miao, 2009). The homogeneity of variance between the classes on the post-test was .347 (p = .708) and between the genders was at .329 (p =.073). This means that the variances are equal for the classes and for genders. The scores for boys and girls in the co-ed class where separated and a Levene's test of homogeneity for the four groups was done. For the post-test, the analysis yielded a calculated F-value of .126 which is less than the critical value of 3.01 and a P-value of .158 which is greater than level of .05 - see Table 18. The interpretation is that there is no significance in the variations between the groups; the groups are statistically similar and can be compared.

Table 18. Test of Homogeneity of Variance Between the Groups

		Sum of Squares	df	Mean Square	Levene's Statistic	F-value	P-value
Pre Test	Between Groups	101.43	3	33.81	0.29	0.81	0.84
	Within Groups	3861.12	93	41.52			
	Total	3962.56	96				
Post Test	Between Groups	25.76	3	8.59	1.78	0.13	0.16
	Within Groups	6351.79	93	68.30			
	Total	6377.55	96				

6.1.1 Research Question One.

What is the difference between the mean score of open-ended items on the pre-test and post-test for each group of students?

The ANOVA in SPSS was used to compare students' mean scores on the tests-see Table 19.

Table 19. Mean Scores of Open-ended Items for Students on the Pre-test and Post-test

Group	**N**	**Pre-Test Mean**	**SD**	**Post-test Mean**	**SD**	**Pre-Post Diff.**	**T-Stat.**	**P-Value**
Single Boys	31	17.32	4.64	21.01	6.26	3.69	8.82	0.00
Co-ed Boys*	20	17.42	3.59	22.95	4.65	5.53	9.83	0.00
Single Girls	28	18.61	5.09	20.14	7.07	1.53	2.91	0.00
Co-ed Girls *	18	17.67	4.12	20.67	8.53	3.00	2.22	0.02

* The table shows separate scores for boys and girls in the co-ed class.

For all groups, the absolute value of t-stat is higher than the critical value and p-value is less than alpha ($p < .05$) showing that there is a significant difference between the pre-test and post-test. Boys in the co-ed class had the largest increase on the post-test while girls in the single-sex class had the lowest increase. This result suggests that the open approach may be most favourable for boys in the co-ed class.

The standard deviation for boys is also lower than that of girls which is interpreted as most boys having scores that are close to the mean. Girls in the co-ed class had the largest standard deviation which is interpreted as a wider spread about the mean. Interpreting the standard deviation with respect to students' understanding would suggests that there is a different range of understanding among the boys from among the girls. There is a similar level of conceptual understanding among the boys, whereas some girls have high understanding while others have very low understanding of mathematical concepts.

6.1.2 Research Question Two.

What is the difference between genders or class settings in the mean score of closed items compared with open-ended items on the pre-test and post-test?

The ANOVA in SPSS was used to compare students' mean scores on the tests-Table 20.

Table 20. Mean Score of Closed and Open items of Students on Pre-Post-test

Group	**Pre- Test**			**Post-Test**		
	Closed Items	Open Items	Total	Closed Items	Open Items	Total
Single Boys	11.71	17.32	29.03	11.05	21.01	32.06
Co-ed Boys*	11.15	17.40	28.55	10.48	22.90	33.38
Single Girls	12.53	18.61	31.14	13.03	20.14	33.18
Co-ed Girls*	12.44	17.67	30.11	12.55	20.17	32.72

* The table shows separate scores for boys and girls in the co-ed class.

The results as obtained in SPSS show that girls (co-ed and single) had significantly higher scores on the closed items ($p < .05$) on both the pre-test and post-test. Boys, obtained significantly higher scores than girls on the open-ended items ($p < .05$). On the total score of the post test, co-ed boys and single-sex girls obtained higher scores than co-ed girls and single-sex boys, but this was not significant.

A comparison between scores obtained on closed items of the pre-test and post-test showed little or no improvement for all groups. Based on discussions in previous studies (e.g. Rittle-Johnson & Alibali, 1999), it was expected that students would show greater improvement in their procedural understanding. One factor that may have contributed to this result was the time allotted for answering closed items. Students were given one to two minutes to answer closed items but up to five minutes to answer open-ended items. A review of students' response sheet shows that some students, especially boys, omitted some closed items. Few of these students obtained high scores on the closed items they answers but this was negated when the mean score was calculated. Another reason may have been the scoring of the items. Students were given one mark for the correct answer and one mark for correct procedure. Students may have shown greater increase on their post test result with a wider scale rubric.

6.1.3 Research Question Three.

When comparing students' responses for each item on the post-test, is there a difference between the genders or class settings?

A Tukey ad hoc test in SPSS with multiple comparison was used for analysing students' performance on each test item. The performance of each group on each item was compared with the other three groups in terms of mean difference, standard error and significance. Table 21 shows items that have a significant value between two or more groups. The column showing significance is highlighted. Omitted items had no significance between groups.

Table 21. Multiple Comparisons between Genders on the Post-test

Tukey HSD							
Dependent Variable			Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Item 1C	Single Girls	Co-ed Boys	.750*	.163	.000	.321	1.178
	Co-ed Girls	Co-ed Boys	.500*	.181	.035	.024	.975
		Single Boys	.471*	.165	.027	.038	.903
Item 1D	Single Girls	Single Boys	.722*	.156	.000	.313	1.131
Item 3A	Single Girls	Co-ed Boys	.514*	.175	.022	.054	.973
Item 3B	Co-ed Boys	Single Girls	.707*	.237	.019	.086	1.328
Item 4A	Single Girls	Co-ed Boys	.150*	.056	.048	.001	.299
Item 4B	Single Boys	Co-ed girls	.646*	.222	.024	.063	1.230
	Co-ed Boys	Single Girls	.642*	.220	.022	.066	1.219
		Co-ed girls	.888*	.244	.003	.249	1.528
Item 5B	Single Boys	Single Girls	.619*	.229	.040	.020	1.219
	Co-ed Boys	Single Girls	.821*	.257	.010	.148	1.494
Item 7	Single Boys	Single Girls	.831*	.273	.016	.117	1.546
	Co-ed Boys	Single Girls	.978*	.306	.010	.176	1.780
Item 8	Co-ed Boys	Single Girls	1.228*	.297	.000	.451	2.005
		Co-ed girls	1.244*	.329	.002	.381	2.107
		Single boys	1.617*	.237	.000	.995	2.240
Item 9	Single Girls	Co-ed Boys	1.564*	.267	.000	.865	2.263
		Single boys	1.736*	.270	.000	1.029	2.444
	Co-ed Girls	Co-ed Boys	1.683*	.296	.000	.907	2.459

The Tukey's ad hoc test shows no significant difference between class settings, that is, between girls in the single-sex class and girls in co-ed class or between boys in single-sex class and boys in co-ed class.

With regards to gender; for closed items, there was significant difference in favour of girls in single-sex class when compared with boys in co-ed class on items 1C, 1D, 3A and 4A. Also, in favour of girls in co-ed class when compared with boys in co-ed class and boys in single-sex class on items 1C.

For open-ended items, the significant difference between the genders is broken down as follows:

- Boys in the single-sex class over girls in single-sex class on items 5B and 7
- Boys in co-ed class over girls in single-sex class on items 3B, 4B, 5B, 7 and 8.
- Girls in both class settings over boys in both class settings on item 9.

Boys had significantly higher scores on items that tested students understanding on *principles underlying procedure* (Items 3B and 5B) and on *knowing why* a calculation is important (item 7). The genders scored differently on items that required *real world applications*. This was due to the structure of the item and skill required for completing the item successfully. Boys had higher scores on items that required more spatial skills while girls scored higher on items that required more verbal skills. This result is similar to that of previous studies which argued that boys are be more spatial and girls are more verbal (Gurain 2006; McNeil, 2008).

In summary, it can be said that the quantitative findings show that generally, boys in the co-ed class benefitted the most from the intervention as their mean difference between pre-test and post-test was the greatest among the groups. Girls tended to do better than boys on closed items showing that they may have greater procedural understanding. This result reflects the current trend seen on tests in Jamaica. On the other hand, boys did better than girls on most open-ended items showing that they may have greater conceptual understanding. In terms of class setting, girls in the single-sex class did better than girls in the co-ed class on most items on the test, but these differences were not significant. Boys in the co-ed class also did better than boys in the single-sex class on most items on the test, but these differences were not significant.

6.2 Findings from Qualitative Data

Observation notes were coded based on similarities among the three classes. This gave 33 codes. These 33 codes were grouped into themes and the themes further grouped into categories. The two categories looked at students' understanding of mathematical concepts and on the classroom environment that enhances such understanding. Presented below is a summary of the results according to the themes identified.

6.2.1 Research Question Four.

What gender-specific differences and similarities reflecting conceptual understanding of mathematics are displayed among students as they respond to open-ended problems in the open approach?

Since the question is stated with regards to open-ended problems, the themes were given names reflecting the assessment rubric for open-ended items, i.e. fluency, flexibility, originality. The term "elegance" is included in the theme "Communication". The word "communication" was used because it involved not only student's explanations but their contribution to class discourse. Behaviours perceived to have an impact on students' conceptual understanding were placed under these themes for discussion. The themes, shown in Table 22, summarise the collective responses seen among students in the three classrooms.

Table 22. Themes from Observing Students Behaviour during the Solution Process

Themes	Theme Description	Code Examples
Communication	Open discourse that promotes understanding by eliciting multiple solutions and facilitating divergent views	I do not agree with you because my answer is different. (Agreeing and disagreeing. Offering alternatives solutions.)
		It is better if you multiply. (Freedom to ask questions and make comments about mathematics)
		Communication among students and between students and teacher
Fluency	Forming patterns and establishing relationships among ideas that produce different solutions. Identifying connections within a domain	This can go with ….. (recognizing relationships)
		"What if……?" sentences (exploration)
		"I can also try…" (generate examples)
Flexibility	Showing the ability to view the problem from different angles in order to provide multiple types and forms of solutions and applications.	"This can also be…" (Apply a concept in multiple situations)
		"If… then…" statements (exploration base on intuition)
		"Can we try…?" Alternative solutions
Originality	Showing the ability to think outside of the box, checking and justifying one's own hypothesis. Identifying connections across domains.	Identifying properties of a concept.
		"This is true because…" (Give plausible justification and explanations.)
		Recognizing and manipulating underlying structure.
Strategies	Heuristics. Way of solving. Actions (mental, physical) performed during the solution process. Recognizing the value of different strategies.	Action used in calculation process (e.g. Using strokes)
		Applying mathematics to real life
		"This does not look correct" (monitoring one's own solution process and answers)

6.2.1.1 Communication

Prior to the period of intervention, students had difficulty communicating with each other because some students lied about their solution. This they did deliberately to prevent their fellow classmates from finding the correct answer first. Though this was not common practice, it was enough to cause distrust in the single-sex classes. Additionally, most students did not actively listen to their classmates but simply held their hand up, waiting for a chance to speak. Others who were overly eager usually spoke out of turn. The teachers continuously emphasised the social norms established during the first week and this had the effect of gradually correcting the issues of lying. The teachers encouraged students to listen to each other by asking them to repeat, rephrase or state whether they agreed with solutions that their classmates presented. This helped students to listen keenly to their classmates' solutions. Towards the end of instruction of the study there was a parallel structure and smooth flow of the discourse as students took a somewhat equal roles in presenting and listening to each other in turns. An example of class discourse towards the end of the intervention is given in the excerpt below. The excerpt was taken from the co-ed class. In the previous lesson, students were asked to explore the concept of equivalent fractions. Using the strategy of folding strips of paper, students discovered that "equivalent fractions" may be represented by "equal areas". In this lesson, students explored ways to generate equivalent fractions. At the beginning of the lesson, students and teacher listed equivalent fractions for one - half (1/2) on the board: "$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$". Students were asked to explore patterns among these fractions. Further in the lesson, students were asked to see if their pattern could be generalised. Students were allowed to work individually, in pairs or groups and to use manipulatives such as fractions bars and counters as they saw fit. The excerpt begins at the point in the lesson where some students were asked to use the board to present their respective solutions. (O.C. means observer's comment.). Abbie was asked to present her solution first- see Figure 14.

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$$

Figure 14. Abbie's Solution

58 Abbie: I think if we add the fractions like this, we can get 2/4.
59 Patrick: Yes, yes, that's right, that is how I did it too. And 2/4 + 2/4 equal 4/8
60 (O.C. Patrick and other students showed excitement because they had the similar solutions to what Abbie did.)
64 Janet: No, the denominator should be 2. She (Abbie) should have 2/2.
65 Abbie: I know that, but I'm just saying, if we want equivalent fractions, we can add fractions like this.
66 Tom: But it's not correct. It should be 2/2.
T (Teacher): (speaking to Abbie) Did you share anything with someone today or this week?
72 Rachel: Miss I gave her piece of my Catch (chocolate bar).
T: Let's say that Rachel gave you half of the chocolate.
73 You ate half and Rachel ate half. Did you have any chocolate left?
74 Abbie: No miss, we ate all of it.
75 T: So if we add ½ and ½ we get…?
76 Some students: One whole.
77 Teacher drew a diagram on the board with explanation that ½ plus ½ gives 1.
78 She emphasized that the whole is divided into two parts not four parts.
81 Patrick: Oh, I see, so ½ plus ½ = 1.
83 Abbie: Yes miss, I see that, but…

Troy explains his method

$$\boxed{\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}}$$

Figure 15. Troy's Solution

122 Troy: I wrote the top numbers first, 1 2, 3
123 The class joined in with 4, 5, 6…

124 Troy: Then I wrote the bottom numbers, 2, 4,
125 The class joined in with 6, 8, 10, 12
126 T: What do you notice with the numerators?
127 S (Student): They are counting
128 T: Yes, counting by one. And what about the denominators?
129 S: They are even numbers.
130 T: Yes, so they are counting by….
131 S: by 2.
132 The teacher wrote this on the board (she rewrote Troy's method to the top right side on the board, Abbie's method was on the left side)
133 T: Do you notice anything else?
134 The class was silent.
135 T: What if I put 7 as my numerator? What would the denominator be?
136 Troy: 14 miss,
137 S: 14
138 T: What if I put 20 as my denominator, can you tell me what the numerator will be?
141 Some students began to write in their books, others tried to calculate mentally.
142 Rage: 10 miss.
143 T: How did you get 10? What exactly did you do?
144 Rage went to the board and extended Troy's solution to 10/20.
145 T: Very good. Anything else we could do? Did anyone do something else?
146 (Silence)
147 T: Look at 10 and 20. What can you say about them?
148 Troy: 2 × 10 is 20
149 T: Yes, anything else? Look at the other numbers.
150 Kyle: Miss the bottom number is 2 times the top number.
151 Teacher speaking to Kyle: say denominator and numerator.
152 Kyle: The denominator is two times the numerator.
T: Show us on the board.

$$\frac{1}{2} = \frac{2}{2x2} = \frac{3}{3x2} = \frac{4}{4x2} = \frac{5}{5x2}$$

Figure 16. Kyle's Solution

(Kyle explains his method)
156 Kyle: If the number at the top is 1, then the number at the bottom is 2 times 1. This one (pointing to 2/4). 4 is 2 times 2. For this one, (pointing to 3/6), 3 is at the numerator, so 2 times 3 is 6, and 6 is at the denominator.
158 *O.C.* The teacher had interjected to remind Kyle to use the terms numerator and denominator.
159 S: Oh! I see, yes.., I got it.
160 Bryan: And 8 is 2 x 4.
161 T: very good, Carol and Aman go and show the denominators for 7 and 8.

162 They used strokes to help them calculate 2 x 7 and 2 x 8 respectively (see Figure 27).
163 T: Very good. So if I have 40 for my numerator, what is the denominator?
164 Some students raised their hands almost immediately, however, the teacher waited for other students and called on Francis who raised his hand after about 20 seconds
165 Sally: 80 miss.
T: Very good Sally.

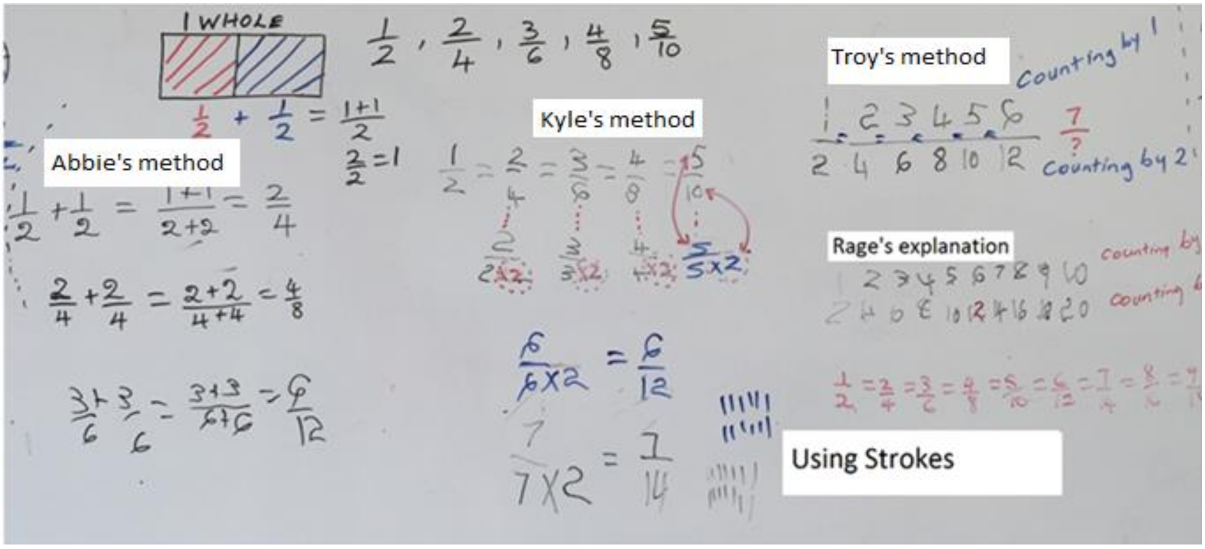


Figure 17. Whiteboard Showing Students' Solution to Finding Equivalent Fractions

Communication can help everyone in the classroom to understand a given concept or method. This is because it clarifies contrasting approaches which help students to clarify why some solutions are correct and others are not. Abbie presented a method that was common among most students in the class. This may have been the reason the teacher asked her to show the method on the board. Other students were able to identify the error in the calculation and were bold enough to point it out. At the beginning of the intervention, students most likely, would have allowed her to continue in the incorrect way of thinking. However, here towards the end of the intervention, students pointed out that the calculation was wrong. This shift shows caring for one's classmate and wanting them to succeed. It seems that before, students were competing to arrive at only one correct answer and had the mind-set that only one student could succeed in obtaining that answer. With the open approach, however, every student can create his method and gain the feeling of accomplishment. This caused students to change from competing against each other to working together. This shift in mind-set and the feeling of confidence gained from solving the problem correctly cause students to become more vocal and share their opinions in the class. The discussions in the classes became enriched when more students shared their opinions and solutions. Also, the teacher was better able to identify and correct errors and misconceptions while supporting correct calculations and mathematical reasoning.

Another point to note from the excerpt is the way in which the teacher dealt with the incorrect solution in the lesson. Highlighting the errors in the solution can help students to know why one solution is correct and the other incorrect. In these cases, teachers also refer to real life application to help students to see the logic or lack thereof behind their calculations. Here as well, the teacher called on Abbie's experience about sharing chocolate with a classmate, to help her understand why adding two halves gives one whole and not $\frac{2}{4}$. The teacher showed mathematically, the correct way of adding fractions and this revealed the error in Abbie's method. Conceptual understanding requires interpreting what the numbers mean which helps students to know why a calculation is necessary and what type of answer (a bigger number or a smaller number) should be obtained for a solution.

In general, a change in the type of discourse was observed among the three classes. At the beginning of the intervention students reported their solution in a rehearsed narrative, but towards the end of the intervention, the individual child had transitioned from merely reporting on an item to a more confident and persuasive defence of a choice or course of action. An example of this is seen in the discussion about Abbie's strategy. Abbie presented her solution and attempted to persuade the class by using the phrase I know we cannot add fractions this way but, "if we want equivalent fractions" we can use this method. She tried to persuade the class to look at getting equivalent fraction and not on the correctness of the calculations. The rest of the class was trying to persuade her that the correctness of the calculation is important, and since the calculations were wrong, then the method also was wrong. This type of discourse was not observed at the beginning of the intervention. The change in the type of discourse was due to a change in the social environment. Students felt more confident to share their opinions and solutions. Likewise, students felt more confident to ask questions about what they did not understand. This caused students to think about their reason and how their method works, giving a detailed description of their thought processes so that others could understand the method and would be able to apply such method. It must also be noted that students often rose in defence of their peers with whom they agreed or who had an approach or a solution similar to their own. These types of discourses are helpful, as they normally revealed the correctness or incorrectness of a given method or solution, causing the students to better understand the concept.

It was observed that the communication in the classrooms increased the understanding of students individually and as a group. Consider a student with no understanding or with partial understanding of a concept in a given problem. This student is still able to find one or more solutions to the open-ended problem (Nohda, 2000). Being able to solve the problem increases the child's confidence. The increase in self-confidence sparks curiosity to explore the problem further and this leads to more discoveries. The child then shares these discoveries with the group, discussing and comparing his/her solution with those of other students. Such discourse helps both the presenter and the audience to clarify their respective thought processes, explore other ways of solving a problem and enhances understanding.

(Cai, 1995; Hoosain, 2001). Active listening also improves understanding (Pehkonen & Ahtee, 2005).

Gender Comparison on Communication

Girls displayed higher communication skills than boys. Regarding written communication, girls' solutions tended to be more detailed and in some cases included context. This difference was attributed to students' writing skills, as girls were more adept at writing. This is similar to what was observed in the quantitative results where girls showed higher understanding on questions that required verbal skills. Also, girls tended to pay more attention to details in their writing. For example, the solutions of boys are often without units whereas those of girls had units. In terms of verbal communication, girls were also more eloquent than boys and had less struggle expressing their opinions and thoughts. Toward the beginning of instruction, girls dominated discourse in the co-ed class, but towards the end of instruction, boys participated in class discussion as much as girls did and in some instances they spoke more than the girls did. The increase in boy's participation was due to interest and growth in self-confidence.

Class Setting Comparison

There was not much difference among the students in the different classroom settings. It was expected that students in the single-sex classes would show more willingness to communicate their thoughts due to the absence of the opposite sex. Observation on the other hand, shows that towards the beginning of instruction, in both single-sex classes, it was only students who were considered to be “bright" who participated in the discussions. Over time, other students began to give reports on behalf of the group, asking and answering questions during class discussions. For boys in both class settings, the opportunity to communicate their thoughts orally was welcomed because many of them became frustrated when trying to write down their thought process. Knowing that some students were unable to read and write properly, the teachers often asked the entire class to read the problem aloud and purposely asked some non-readers to rephrase the problem in their own words. Because students were free to choose their own way of solving the problem, some boys resorted to using diagrams instead of sentences. This was interpreted to mean that boys were less willing or less able to write sentences. As boys having less interest in writing sentences. Girls in the single-sex class used words that gave clearer meaning than those in the co-ed class, but written statements were similar in length and detail; likewise, the logical flow of explanations was somewhat similar. This was interpreted as girls in the single-sex class having greater elegance than those in co-ed class, but this difference was not significant.

6.2.1.2 Fluency in Solution

There was an increase in the number of solutions produced towards the end of the intervention than at the beginning. This increase was observed in all three classes. Fluency reflected students' understanding of a concept in that, through listing many examples (Davis, 2006), students were better able to identify patterns or anomalies in their solutions. Discussing or exploring the reasons for these patterns or anomalies helps students to understand the concept better. For example, the teacher may ask students to list examples of fractions. Based on the premise that a fraction is a part of an equally divided whole or that a fraction has a numerator and a denominator the following list is given: $\frac{1}{3}, \frac{2}{5}, \frac{6}{17}, \frac{7}{3}, \frac{5}{9}$. (The list can be created by one student in his/her book or may be written on the board from many students' suggestions). On closer examination, one realizes that $\frac{7}{3}$ is different from the other fractions. Exploring why $\frac{7}{3}$ (and other fractions with larger numerators) are considered to be fractions even though they have bigger numerators would help students to have a wider understanding of fractions. In most classrooms, a pictorial representation such as that seen in Figure 18 is given. This is accompanied by an explanation about adding wholes divided into the same number of parts as the proper fraction together.

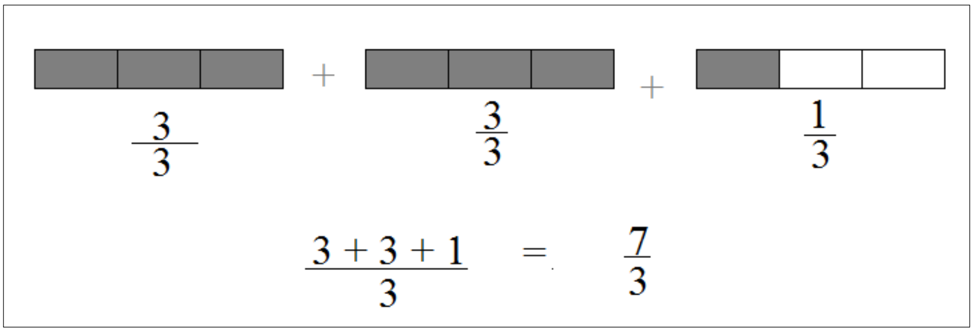


Figure 18. Explanation of an Improper Fraction

An explanation such as this allows students to realise that the definition of a "whole" has not changed and a fraction is still a number that lies between 0 and 1. Students are also able to make other discoveries such as:

(1) When the same number is used as the numerator and denominator, the fraction is equal to 1 whole.
(2) An improper fraction ($\frac{7}{3}$) can also be written as a mixed number ($2\frac{1}{3}$).

In case two, students are also able to see that the same fraction can be written in two ways which assists them in converting one to another.

Here the discussion flowed from listing fractions to explaining why a fraction can have a bigger numerator, to showing that fractions with bigger numerators can also be written as mixed numbers.

During the intervention, it was noticed that students' conceptual understanding increased when they searched for more solutions. Students who were able to produce many solutions used connection, exploration and reasoning during the solution process. The term "connection" was used to describe situations in which students were able to identify something in the problem that they were working out, which was similar to a previous problem or experience. This gave them an idea about how to solve the current problem, or it gave them a new way to look at the problem.

The analysis reveals that the phrases that were coded as reflecting fluency came after students had one or more solutions to the problems. That is, searching for new ways to generate solutions forced students to think of previous problems with similar solutions which revealed connecting links between concepts in the problem. This could be interpreted as using greater mental power from continuous reflection on the problem. Thinking about the problem situation for an extended period of time helps students to see more connections to the problem and this increases their understanding. This is one of the uniqueness of open-ended problems. Consider the problem "list eleven numbers that can be rounded off to 50". Eleven was purposely chosen to encourage students to include at least one decimal number in their solution. It would require most fourth graders in Jamaica to reflect on the problem for a while before realizing that decimal numbers could also be included in the answer. In solving this problem, Luke, an average student from the co-ed class listed the following numbers as his solution on the board:

First line: 51, 52, 53, 54
Second Line: 48, 49, 46, 45, 47
Third line: 50
Fourth line: 50.3 (with assistance from teacher)

In presenting he said "first I wrote this (first line) then, I remember this number (pointing to 48), and I guess that these could also be used. 50 is in the middle so I say 50 can also round off to 50." Hypothesising Luke's thought process from the patterns in his solutions one could deduce that at first he, realized that the numbers 51, 52, 53, and 54 were possible solutions because they were easier for him to recall. Reflecting on the problem helped him to connect an experience with 48. Thinking more about why 48 was possible allowed him to try 49 then 46 then 45 and 47. He did not say why these numbers were chosen. He was stuck again, and so he reflected again to see if he could find more connections or patterns. The counting pattern in the numbers helped him to deduce that 50 itself could also be rounded off to 50. Prompts from the teacher and possible class discussion helped him to find the 11th number of 50.3. Working on this problem, Luke gained deeper understanding of rounding numbers at each reflection stage. He learnt from other students' solutions during class discussion that 50.1 and 50.2 were also possible answers. The class discussion did not go beyond one decimal place.

After working with open-ended problems for two months, it was observed that when given a problem, the first strategy used by students was brainstorming. They tried to think of as many solutions as possible without thinking about how they were connected or about why they may be correct or incorrect. While brainstorming was a good technique to generate multiple solutions, discussion of these solutions was necessary to develop students understanding.

The rational for using logical thinking (reasoning) in the fluency category was that the in-depth analysis of some students' behaviour showed a link between understanding and logical thinking. On discovering something new or on grasping a concept, students used statements such as "Oh, I see", or "I get it now" or "So that's why…" These "*eureka"* moments were recorded as signs of enlightenment through logical reasoning or from exploring and were recorded as signs of growth in understanding. These phrases were recorded in the co-ed group more than the single-sex classes. Another aspect of reasoning observed was students' ability to rethink and modify their thought process to create more solutions. This example was seen in Luke's solution above. Writing the list of numbers in chronological order from 45 to 54, he realised that the number 50 was missing. Therefore, he reasoned that 50 could be a possible solution.

Gender and Class Comparison on Fluency.

Boys showed greater fluency than girls in most lessons. Students in the co-ed class had marginally higher fluency than those in single-sex group on more than 50% of the target lessons. However, it was more difficult to decide which gender showed greater fluency with understanding.

Students' solutions on the "Beach Trip" problem can be used as an example here. The problem asked students to calculate how many buses were needed to carry 110 people to the beach. Figure 19 shows examples of students' responses to this problem. Students from both genders had similar errors in their calculations and explanations.

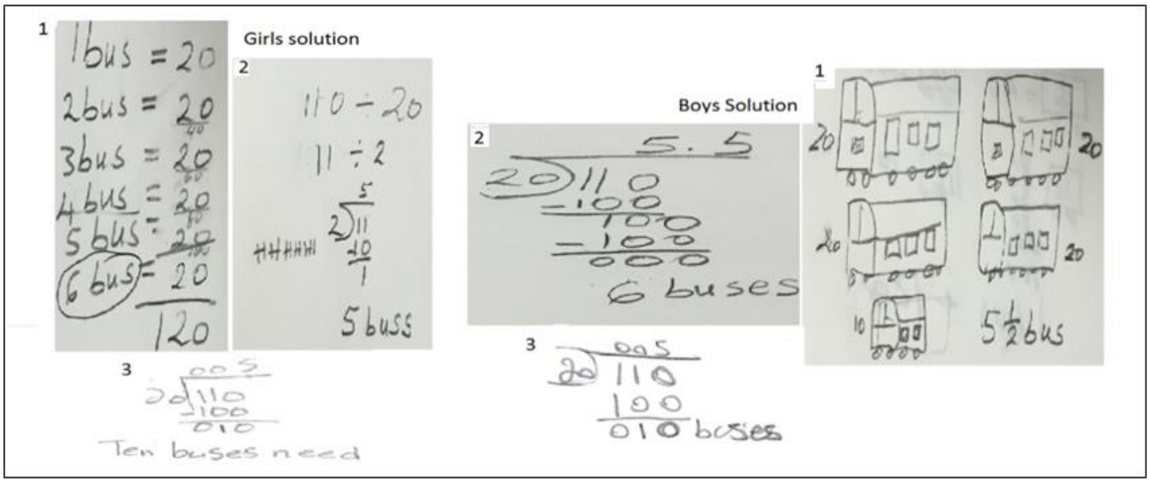


Figure 19. Students Solution to Road Trip Problem

The solutions marked "1" shows students' original approach to solving the problem. Avoiding the traditional way, these students created their own unique way of solving the problem. The solution marked “1” for both the girl and the boy show that students considered the capacity of one bus and gradually added buses until the quota was reached. They showed students ability to connect practical situation with abstract problems. Also, the students’ unique solutions shows that they (both the boy and the girl) had an understanding of the underlying meaning of division. The boy had four instead of five buses. When asked by the teacher why he drew four and a half buses, but had five as his answer, he responded that after drawing four buses he knew in his head that there should be one more so he did not need to draw it. He used the diagrams as a form of a scaffold to assist in counting. This aid was removed once he reached his goal. He did not need to continue with the diagrams as he was now able to proceed mentally with the computation. This too shows the students ability to conduct complex computations mentally. Another student showed understanding by linking the problem to real life situation by giving five buses and one car as his answer. He connected the solution to the practicality of everyday life and concluded that five buses and a car would be cheaper than six buses. Students who used the traditional method (solutions marked "2") were thinking in the mathematics world. Though some application of understanding was observed as some gave six buses as their answer, most showed no sign of connecting their solution to the real world as they did not see the impracticality of "½ a bus". This result was also observed in Cai (1995). She described her scale of measuring students understanding as "0 = no understanding, 1 = beginning understanding, 2 = some understanding, 3 = nearly complete understanding and 4 = complete and correct understanding" (p.2). However, she did not say where on the scale students with these errors would be placed. Students who gave solution "3" had procedural understanding but not conceptual understanding. This research would place these students at level 2 "Some understanding" since they had procedural understanding but had poor conceptual understanding of the division concept. This is an example of how the understanding of the concept is linked with the procedure used in solving the problem. Guiding students to apply mathematics in real-life situations helps to increase their understanding of mathematical concepts. Though this error was not seen among many students it was produced by students in each class setting and from both genders. The situation of common errors between genders and among the groups occurred frequently, making it difficult to say which gender or class setting had fluency with understanding.

6.2.1.3 Flexibility in Solution

Flexibility refers to the number of different concepts employed in the solution presented by a given student. In order to give a clearer view of this category, analysis of the following target lesson will be discussed. In this lesson, students were asked to solve the problem:

> *"Which number does not belong to the group and why? 2, 8, 9, 18. Give as many solutions as you can."*

Flexibility was calculated separately for each number. A student's solutions were grouped according to the concept being explained and one point given for each category (see Table 23). For example, one student selected the number 18 as not belonging to the group using the following three explanations;

1. It is the only number bigger than 10.
2. It is the only number that can be divided by 6 without leaving a remainder.
3. It is the only one with two place values.

The first and third responses were categorised as place value and were placed in one category while the second was characterized as "Division/Factors". This student had two categories and was given two marks. The reasons given for each number is shown in Table 23.

Table 23. Assessing Flexibility of Students' Solutions to an Open-Ended Problem

Which number does not belong 2, 8, 9, 18.And Why?					
For 2	**Concept**	Single-Boys	Co-ed Boys	Single-Girls	Co-ed Girls
		%	%	%	%
2 has only two factors	Factors	90	85	95	95
2 is prime	Prime numbers	0	60	0	0
No other number can divide into 2 (except 1)	Multiples	42	30	0	0
For 8					
8 is not in 2 x 9 =18	Multiplication	100	100	100	100
Only one divisible by 4	Division/Factor	68	35	35	39
For 9					
The only odd number/is not even	Odd and even	100	100	100	100
Factors are same (3 x 3). Square number	Square Numbers	42	20	62	43
The only one with odd number factors (3)	Factors	0	40	0	67
For 18					
Has two digits	Place Value	100	100	100	100
Divisible by 6	Division/Factor	0	60	0	0

Gender Comparison on Flexibility

Boys identified more concepts than girls in their respective classes, but co-ed girls had a similar level of concept identification with single-sex boys. Boys did better that girls both when the problem was given in class and when given on the test. The question was characterized as testing "connection knowledge" and "knowing why." It required students to use their knowledge on number properties to create a criteria that would eliminate one member from the set. Boys were able to do this more successfully than girls.

Class Comparison on Flexibility.

Responses of students in the co-ed class covered more concepts than those in the single-sex classes. This was the usual trend among the classes with flexibility theme. Even though boys in the single-sex class sometimes produced higher scores in fluency theme than the other classes, their scores were normally less in the flexibility theme. The reason for this may be due to students' thinking about the same concept for producing solutions. For example the three reasons given by one boy for excluding the number 9 were:

1. Nine is the only odd number.
2. All others are even except nine.
3. Nine is not divisible by 2.

All three reasons would fall in the same category thus attracting only one mark. When the codes for flexibility was applied to students' responses, there were more codes for boys in the co-ed class than for boys in the single sex class. Codes such as "what if…" or "I can try…" that reflect a shift in students reasoning were seen more among boys in the co-ed group. This shift in students thought process which leads them to create other solutions, is an indication that their understanding of the concept is being developed. This shift occurs among most students but more so among students with multi-directional reasoning than among those with uni-directional reasoning. The terms "Uni-directional reasoning" and "multi-directional reasoning" were coined after observing diagrams drawn by non-readers in the co-ed class over a period of time. At first, these non-readers were thought of as having the same level of understanding. However, analysis of students' responses reveals that some students' diagrams often have one object while other students' diagrams had multiple objects –see Figure 20. Students with uni-directional reasoning could concentrate on one object in their mind while those with multi-directional reasoning could think about multiple objects at a time.

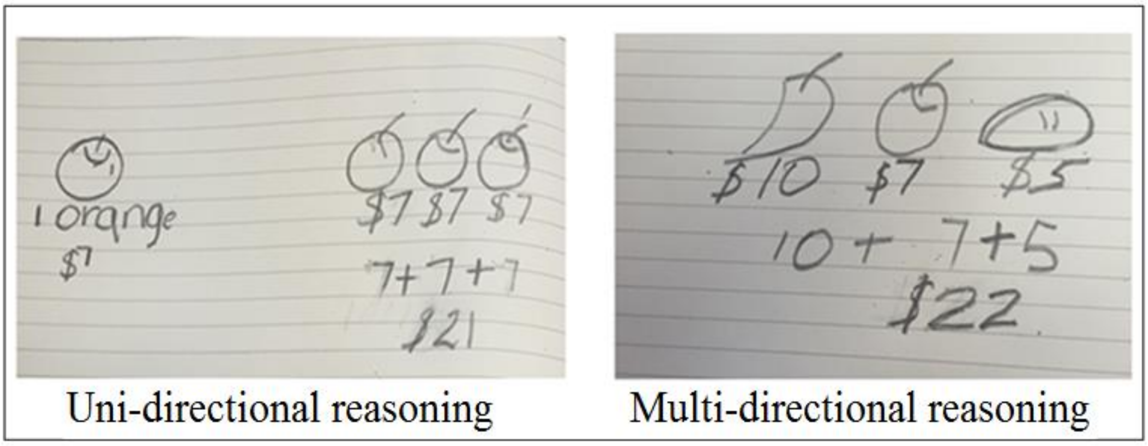


Figure 20. Students with Uni-Directional Reasoning and Multi-Directional Reasoning

This conclusion was confirmed in the all-boys class then in the all-girls class. A group in the all-boys class transferred the associative property learnt in adding whole numbers to adding decimals and in adding percentages. This led the researcher to conclude that multi-directional reasoning spans topics and may even go across strands. Students with uni-directional reasoning can understand a concept within the topic or domain in which it is learnt while students with multi-directional reasoning can transfer learning to other topics and possibly domains; thus, widening the scope of thinking. Students with multi-directional reasoning had a more elaborate mental structure of connections which made them more capable of responding to problems with a greater degree of flexibility. Observing these students in other lessons revealed that they produced solutions involving various context more often than those with uni-directional reasoning. Open-ended problems allow students to connect the same concept (for example the concept equivalence) across different topics and this will widen their understanding and increase their learning.

Students with multi-directional reasoning were able to create new strategies and to adopt strategies that were used by others. In doing this, they were able to "build" connections or relationships among solutions, as well as between a solution and a problem. This ability was seen more among boys than girls. For example, in Table 23 above, more boys created solutions with factors and multiples. Boys were able to "build" a connection using previously learnt knowledge which helped them to create factors of a number. More students showed evidence of multi-directional reasoning toward the end of the study indicating that this skill can be learnt through guidance and practice.

Comparing the flexibility of a class at the beginning of the instruction with that at the end of instruction showed improvement in all three classes. The all-girls class had the lowest degree of flexibility and this was attributed to their tendency to copy the exact work form each other.

6.2.1.4 Originality

This describes deep understanding used by students to synthesise ideas and to create new unique solutions. It was difficult to determine what caused students to produce original ideas and solutions. For situations that could be coded, it was observed that as students thought about solving a problem, they usually uttered statements such as "If this is so, then….", "If I change this …..to … then ...", and "We can also use … to…." These statements suggest that students re-organised their ideas based on their understanding and began exploring new connections. This understanding helps them to identify new relationships leading to new solutions being produced. There were many occasions, however, where these statements were used but the student did not produce an original idea. It was observed that students became more willing to express new unique ideas as the research progressed and that the more a student explored the problem, the more likely they were to produce a unique solution. This was accredited to increased confidence in solving open-ended problems and reduced fear of being ridiculed for offering a wrong answer. Becker and Shimada's (1997) explanation of originality as an "insightful" idea, would suggest that originality evolves from very clear understanding or deep understanding. It was observed that students whose solutions reflected originality were able to identify properties of the concept or could recognise and manipulate the underlying structure of the concept. It was expected that there would be some continuity among those students who produced original solutions; however, this was not the case. In fact, very few students produced more than five original ideas throughout the six-month research. Furthermore, original solutions were mainly ad hoc, one-off solutions. Original ideas were produced by students regardless of reading level or general academic level, It was as though each student operated at a certain level of understanding for most topics, but occasionally, there was a "mental leap" caused by deep understanding which allowed the student to produce a unique solution. This mental leap could not be considered as a guess as it goes beyond providing a one-word answer to explaining the underlying meaning of a concept. The reason for this "mental leap" is unknown, but it does show that students' level of understanding varied with the topic.

Gender Comparison on Originality

In general, boys were more inclined to create original ideas and solution than did the girls. This difference was accredited to their predisposition to taking risks and attempting to solve the problem from different angles. The more solutions one produced for a problem, the more likely it is for one or more of those solutions to be unique Girls were reserved and more tentative with the solution attempts, in spite of the “safety” in the environment created by the open approach.

Class Comparison on Originality

The overall difference among the classes with regard to originality, showed that boys in the co-ed class had more original ideas than those in the single-sex class, while girls in the single-sex class had more original ideas than those in the co-ed class.

6.2.1.5 Strategies

Three main strategies - finger counting, use of strokes and mental computations- were used for interim calculations among students in the three classes. Interim calculations are those done at a particular stage during the solution process. For example, in adding 125 and 28, an interim calculation would be; 5 plus 8. Students tended to call the larger number first regardless of the type of strategy they were using or the order in which the numbers appeared. Students who used finger counting would touch the chest, say “8” then continue counting 9, 10, 11…until they have counted five fingers. Students using strokes would flip to the last page in their writing book, write down eight strokes then five strokes and then count them all together. Others would do the addition mentally and write down 13 with no physical display of how the computation was done.

At first glance, it may seem like students choose strategies randomly; however, closer observation revealed that students purposely choose different strategies depending on the type of calculations they intend to perform. The finger counting strategy was only used when one of the addends was less than 10. Strokes were used for larger numbers up to about 30. Students wrote strokes in a pattern that allowed for easy calculation. In figure 18, students did not write 12 or 14 strokes in one line but wrote two sets of six and seven strokes respectively. Using strategies in this manner suggests forethought which can be interpreted as knowing why they are using such a strategy.

The use of interim strategies also revealed students understanding or misunderstanding of concepts. For example, the teacher of the co-ed class was able to identify students' misunderstanding of subtraction based on the way they used finger counting. That is, if faced with a situation such as 5 - 8, some students, mainly girls would say "8", then count backwards as 7, 6, 5, 4 and 3. These students followed a pattern they learnt in addition, where the larger number is spoken first. However, they were solving a different problem of 8 - 5. The teacher used this opportunity to help students to identify the difference between adding and subtraction with regards to commutative properties. Students realised that while the position of the numbers in an addition problem are interchangeable; the position of numbers in subtraction are not interchangeable. It was difficult to decipher students' thought process when they used mental computations. When asked how they got the answer, students were unable to explain and often made statements such as "I just know it" or "Teacher told me" or "I learned it before".

Strategies – A comparison Between the Genders

Observation notes showed that more boys than girls used finger counting and strokes in their calculations. More girls than boys used mental computation in their calculations. Girls were better at repeating the time table and statements made by the teacher. Boys, on the other hand, tended to rephrase teacher statements and sometimes left out crucial details. The researcher did not readily attribute these omissions as lack of understanding, as in many cases, even though boys left out important information in their explanation, they did show their understanding based on their calculation and how they presented their ideas. For example, students may say a fraction is a part of a whole, but if shown a diagram of an unequally divided whole they explain that it is not a fraction because the whole is not divided equally.

Class Comparison on Strategies

More boys in the single-sex class than those in co-ed class use strokes and finger counting, but this difference was marginal. Boys in the co-ed group used memory more than those in the single-sex class. Girls in the co-ed class used strokes and finger counting than those in single-sex classes. However, girls in the single-sex class used memory more than those in the co-ed class.

Main Strategies Students Used During the Solution Process.

An analysis was done on the strategies students used to solve the problems in the target lessons. The main strategies seen among students were: using a diagram, guess and check, retrieval/memory, using a benchmark and disproving, identifying patterns and using examples.

The main strategies seen among students can be placed into two groups: Those that reflect applications in the mathematics world and those concerning the real world. Strategies listed as a reflecting application in the mathematics world were traditional strategies such as “guess and check” and “benchmark”. Strategies that showed application to real-life were “using a diagram”, “retrieval from experience” or “creating a hypothetical situation”.

Boys used diagrams, guess and check, disproving and finding patterns more than girls did. There was no difference between the genders with using benchmark and using examples. Girls used more retrieval strategies than boys. Class comparison shows that girls in the co-ed class used more guess and check than those in the single-sex class. There was no difference between co-ed class and the all-boys class in using strategies. Students' use of strategies also provided information on their level of understanding. In most cases, students chose strategies that they understood. This action suggests that students are aware of the limits of the calculation ability. The open approach open students understanding of their own way of this and ability. This knowledge is helpful for when performing calculations especially on test. Based on the strategies students used, one may conclude that boys more than girls applied their thinking to real world context; however, based on class discussions there was no difference between the genders or in class setting regarding the application of the concept to real-world context.

6.2.2 Research Question Five.

How do teachers create a learning environment that supports students' understanding of mathematical concepts in the open approach?

The themes that were reflected in the learning environment are shown in Table 24. The themes were created from eighteen codes reflecting the type of psychosocial surroundings in which students interact with mathematics. They describe the actions that the teacher takes in an effort to support students as they solve open-ended problems. Further discussion on each theme follows. While a teacher may be the main initiator of the process, students themselves may also initiate actions that support and facilitate the learning process of their peers.

Table 24. Themes in the Open Approach Learning Environment

Theme	Theme Description	Code Examples (Descriptive)
Stimulate discussion	Actions or behaviours that promote interest and participation in the lesson.	Teacher encourages the use of multiple representations.
		All students are able to feel a sense of accomplishment from finding a correct solution.
		There is freedom of expressing oneself without the fear of being ridiculed.
		All contribution are accorded equal value.
Support understanding	Assists students to explain the "why" in their chosen method. Allowing them to use their chosen method.	Teacher allows students to work at their own pace.
		Teacher asks students to justify their chosen solutions.
		Teacher assists students to clarify their own solution methods.
		Teacher supports both presenter and listeners by repeating solutions for clarification and justification.
Apply mathematical concept to real-life situations	Giving and eliciting from students multiple examples for applying a concept in to everyday life. Showing the relevance of an idea through practical use.	Teacher uses and encourages the use of practical examples.
		Teacher and students use manipulatives.
		Various applications of one concept are discussed.
Deepen mathematical understanding in the mathematics world	Providing opportunity for students to explore wide range of ideas with numbers to identify connections among them. Emphasizing correct calculations.	Students are free to explore different extensions to the problem.
		Teacher asks students to define a concept.
		Teacher guides students to explore connections between two strategies or solutions.
		Discussions transition from concrete to abstract.
Assess continuously	Assess student progress continuously and encourage them to monitor and reflect on their own solution process ensuring calculations are correct.	Teacher assesses students individually.
		Teacher assesses students collectively.
		Teacher allows students to give comments on the solutions of their peers.
		Teacher reminds students to monitor their own solution process and progress

6.4.2.1. Stimulate Discussions

Teachers not only monitored students' level of engagement but also encouraged, facilitated and acknowledged their input into class discussions. Some strategies that were common among the classes included (1) calling on different students to share their solutions to a question, (2) asking a student to repeat explanations given by peers, and (3) asking one student to show a solution on the board then asking another to explain it. Teachers motivated students to participate in lessons by establishing a zero tolerance approach to mockery and sarcasm. This made it easier for students to express themselves freely without the fear of being ridiculed by their peers. The elimination of this deterrent, caused students to become more vocal in lessons and to contribute more to class discussion. In order to prevent one group of students dominating the discussions, teachers often gave the class time to think before answering a question. Also, in most situations, all answers were accepted, except for the ones that where a repetition of what was already written on the board.

Care was taken to correct mistakes and misunderstanding. The teacher did not immediately identify the right answer, but accepted and recorded on the board, all responses presented, then allowed students to express their views on each. . An example of this was seen in section 6.2.1 above with Abbie's method for generating equivalent fractions. In order to have a full understanding of why students make the mistakes they do, the teachers allowed students to perform the full calculations and explanation of their work. In this way, it was easier to differentiate between a simple mistake and a misconception. In the above case mentioned. Abbie was willing to make a "mistake" to accomplish her goal. She however, did not see her solution as a "mistake" because she was not asked to add fractions. In her view, her solution was another way of producing equivalent fractions which was the desired goal. Also, Abbie did not show misconception of fractions or adding fractions. The discussion of her "mistake" however, proved to be beneficial to the class. Other students may have used a similar method to Abbie's due to misconception of adding fractions. The teacher took the opportunity to reinforce the correct procedures for adding fractions.

It was also seen that the teacher questioned the students as a means to guiding them to figure out their mistakes. In cases like these, the students would realize a calculation error or a gap in their reasoning. In the case of a misconception however, the student would not be able to see his or her mistake and would repeat the mistake when given a similar problem. In the co-ed class for example, most girls had the misconception that in subtraction, the larger number should always be written first. Therefore regardless of the order of the numbers given they always subtracted the smaller from the larger. The discussion about the commutative property with regards to addition and subtraction concepts was useful in dispelling this misconception.

It was observed that the teachers were patient with students and encouraged them to be patient with their classmates. During class discussion, students shared their opinions, explained the mathematical premise for their solution, and asked questions in an effort to better understand the solution of their peers. Students were taught to use encouraging words when responding to questions from their peers. Therefore, words such as “stupid’ or its synonyms were taboo. Teachers elicited information from different students so that no individual or no group of students dominated the discussion.

No suggestion given by students was seen as insignificant and all responses presented were acknowledged and accepted. The teachers encouraged students to be creative and expressive about their ideas. When a student was presenting his/her solutions, teachers offered assistance, asking questions for clarification and providing words or phrases to help the child express himself/herself more eloquently. For example, the teacher may ask questions such as: “What are you doing?” – For clarification and “Why are you doing that?” – To establish relevance. Other questions asked during class were “How do you know that’s the answer?”… “Is there another way to solve this problem?”… And “Did you learn anything new?”

Multiple representations were used to initiate discussion and to help students to see both incorrect and correct ways of solving a problem. The teachers prompted students to talk by probing until students could reasonably explain their understanding. They ask students "Why?" until students adequately explained or supported their ideas. As teachers probed, students reflected on their methods and clarified their thoughts. Additionally, teachers encouraged students to respond, and to listen to each other. The teachers also listened to students’ discussions as it occurred.

6.2.2.2 Support Understanding

Teachers supported students’ understanding individually, in small groups, and as a whole class. Teachers moved about the class and offered support to students when necessary. Teachers offered individual support by reminding students of previous problem-solving situations. Additionally, teachers used words such as “so…”, “then…”, and “therefore...” to prompt students into giving more information or to think more deeply about what they were doing. Lampert (1990) also suggests such ways of assisting and encouraging students.

Teachers also offered support by asking questions such as; “Have you considered…?” or “How about…?” These questions prompt the child to think deeper about his solutions and seek to find more connections with the concept. An example of this was seen in the co-ed class discussed in Chapter 5. The teacher suggested that Ted drew diagrams of equal sizes for comparison. During class discussions, students were encouraged to think about strategies and concepts by repeating or restating explanations offered by their peers. Students' solutions were arranged on the board in such a pattern that comparisons could readily be made. That is, strategies or solutions that were similar were placed beside each other. This helped students to identify similarities in the concepts used. For example, students were able to see the connection between the Least Common Multiple (LCM) method and using equivalent fractions for adding fractions after they were written adjacent to each other on the board.

Teacher repeated exactly what the presenting students said in an effort to have them listen to their own line of thought. This helped students to think about their expressions and to restate if necessary. On some occasions the teacher repeated a student's presentation more slowly, giving both the presenter and class more time to think about and mentally process that input. A step-by-step clarification of students' description of a solution in this manner encouraged thoughtful reflection at a timely pace and allowed all students to focus on one stage of the solution process at a time. In the following excerpt, students were exploring ways to show the value of each digit in the number 2568. One student presented the following solution. The teacher repeated the student's explanation at a timely pace.

$$
\begin{array}{lr}
1000 + 1000 & = 2000 \\
100 + 100 + 100 + 100 + 100 + 100 & = \ \ 500 \\
10 \ + \ 10 + \ 10 + \ \ 10 \ + 10 + 10 & = \ \ \ 60 \\
1 + 1 \ + 1 + 1 + 1 + 1 + 1 + 1 & = \ + \ \ 8 \\
\hline
 & 2568
\end{array}
$$

Figure 21. Student's Solution to a Place Value Problem

114. Student(S): I went, "Two thousand plus five hundred plus six plus eight.
115. Teacher (T): O.k., so you did it like this: Two thousand, where did you get the 2?
116. S: From the thousand column
139. T: And this is 500, where did you get it from?
140. S: And 500 from here (pointing the 5 in the hundred column)
142. T: Did you see that class?
143. S: Yes miss
144. T: But is this five 100. Let's count them
145. Class(C): 1, 2, 3, 4, 5
147. T: And where did you get these 10s?
148. S: From the six here (pointing to the 6 in the number 2568)
149. T: Class, how many 10s does he have here?
150. C: 6
151. T: So that would make?
152. S: 60s.
155. T: Very good class. Do you understand it?
156. C: Yes miss.
157. T: So here he has? (Pointing to the digit 8)
158. C: Eight ones
159. T: And that should go where?
160. S: In the ones column.

This slow revision of the process provided clarification of the method and made it more comprehensible to the class. This provided support both to the presenter as well as to the other members of the class. The presenter was able to critically reflect on his thought process and the members of the class could assimilate the method in a timely manner. These actions assisted students in understanding why an action was taken and to understand its importance to the process. In the example above, students could easily see that there are two, 1000s in the number. That is 1000 + 1000 gives 2000. Teachers sometimes repeated students' explanation and stressed words or phrases that they wanted the class to focus on. These words were written on the board with different coloured chalk or they were underlined.

Teachers waited patiently for students to provide their explanation. Though this may seem tedious or unnecessary, it was a critical part of helping students to reflect and to organise their thoughts. Being patient with students as they go through this process is one characteristic of effective teaching.

Another key strategy used by teachers in supporting students' mathematical understanding was actively listening to them. This however, was not an easy task for teachers as they had to listen to and respond to the multiplicity of ideas and mathematical concepts that may be presented in the classroom. Teachers needed to make a conscious shift from telling to facilitating, and this skill improved as the research progressed. They moved from listening for what they thought students should say - selective listening, to listening with an open mind. Here, they tried not to make premature guesses or assumptions about what students were thinking but tried instead to understand the student's holistic thought process. They confirmed students' ideas by repeating or rephrasing presented statements. The teachers developed their skills to the point where they sought to understand what the student said and did and to uncover the essence and sources of their ideas. The ways in which a teacher might respond to students' mathematical activity is, of course, dependent on what it is that the teacher hears, sees and interprets in that activity in the first place. For example, in one of the lessons, boys were asked to determine what fraction of surface of a football was covered in pentagons. During the lesson the teacher asked one group what they were doing to find the answer. A student of the group replied "we will count the shapes." The teacher interpreted this as counting the pentagons. After some confusions and subsequent clarification, he found out that students meant counting the total number of both pentagons and hexagons. Since teachers were required to solve the problem prior to the class, they would have entered the class knowing many possible solutions. The challenge therefore was discarding their preconceived ideas and concentrating on understanding the student's ideas and their points of view. In the open approach lesson, it is the teacher who listens and seeks to understand students; not the other way around. The goal of teachers' listening was to find a way to develop students' thought process and understanding and to facilitate the sharing of such understanding with the class so that everyone can benefit from the experience.

Figure 22. Boys Counting the Number of Shapes on the Surface of a Football

Listening to the explanation students give reveals aspects of their understandings and dispositions towards mathematics in a way that written work alone does not allow. Students' explanations and discussions give important insights into their relationships with mathematics. These relationships include the students' way of thinking and understanding, and their beliefs about mathematics and mathematical concepts. Such knowledge is useful when planning lessons as well as when assessing individuals in the classroom.

Teaching in the open approach is more than simply giving students a problem and a predetermined method of solving such problem. Teaching with open-ended problems is an intricate process which requires careful planning and clear objectives. The teacher and more so the students can easily be confused when faced with multiple strategies or solutions to a problem. It is very important to have a clear knowledge of the focus of the lesson and clear ideas on how to move forward. The role of the teacher is to support students as they seek to organise a desired path in the web of presented ideas so that each student can gain something substantial from the process. This can be done by giving students sufficient time to interact with mathematics, to organise their thoughts and to verbalize their way of thinking. The teacher should listen keenly to what the student is saying, and help the child to correct misconceptions, eliminate assumptions and to make discoveries by providing real and abstract examples of connections between mathematical concepts.

6.2.2.3 Apply Mathematical Concept to Real Life Situations

Students' concept image is linked to their everyday experience with such concepts; this experience provides them with greater opportunities to form connections and this increases their understanding of the concept. In order to know the extent of students' understanding, it is necessary to observe how they apply what they learn to real-life situations. To help students form more connections with a concept and to see its application to daily life, teachers often introduced lessons by making reference to a practical, familiar situation in which the use of the concept is required. For example, a lesson on fraction may begin with a scenario about sharing a cake into equal parts. References to daily life application of a concept were also made at various points during the lesson. Teachers often used phrases such as "Have you ever……….. at home?" or "This is what it means when you …." Students are also asked to give practical examples to support their ideas. In doing so, the student connects mathematics to a daily life experience and this connection further enhances understanding of such concept. Being exposed to different ways of applying new knowledge helped students to think more widely about what they have learnt. The excerpt in section 6.2.1.1 shows how the teacher used a practical example of sharing sweets between two people to enhance students' understanding of "adding two halves". Later in the lesson, students talked about sharing a cake equally between two people and among four people. Applying mathematics to real-world context has been successfully argued to be an effective way for developing conceptual understanding (Hancock, 1995; Hoosain, 2001).

Hands-on materials were also used to show examples of a given idea or solutions path and to develop both procedural and conceptual understanding. For example, in helping a student to solve a question about the cost of five bananas, the teacher carried banana shaped objects to a student's desk and helped her as she tried to determine the cost of one banana, two bananas up to five bananas. Using hands-on material to stimulate understanding, especially in young children was a common feature for Piaget (1977) and his contemporaries. Likewise, it is common practice in today's classroom. However, finding the appropriate teaching materials was sometimes challenging. Teachers were often called upon to make their teaching aids or to use their personal funds to purchase materials for the class. During the study, teachers placed available manipulatives on a separate table in the classroom and students were encourage to use these materials on their own initiative, reflecting the intended openness of the learning environment.

Another strategy the teachers used was to highlight the mathematical concepts which evolve from the narratives of students' experiences. Helping students to connect an experience with the learning of a concept in this way aided retention and understanding (Hoosain, 2001).

The teachers’ aim was to show students why a calculation is necessary and how it is connected to everyday life situations. In doing so, they helped to increase students’ understanding of the concept. It was observed that deep conceptual understanding that builds on children’s everyday experiences and focuses on ideas that underlie computations, helped students to see the necessity of such computations.

6.2.2.4 Deepen Mathematical Understanding in the Mathematics World

Problems were phrased in such a way that every student could find at least one solution. Students however, were required to apply their own knowledge, experiences and preferences in order to gain more solutions. They were free to choose their own strategy or strategies and to explore the problem as deeply as their level of cognition would allow. With this, the more advanced students usually transitioned to the consideration or application of more concepts than their peers. During discussions, as different patterns emerged, the teacher would question students about the relationships among these concepts.

Teachers helped students to connect concepts in the mathematics world by encouraging them to compare the different presented solutions and make analytical decisions on best possible solutions. In this study, students used their unique strategies to solve problems. This does not mean that all strategies were equally good. The teachers examined all strategies and asked students to evaluate the different strategies for their advantages and disadvantages. The teachers ensured that incorrect strategies, reasoning and solutions were properly identified, and that the reason they were incorrect was stated clearly. In some lessons, correct and incorrect solutions were written on different sections of the board so that students could differentiate between them. Also, the teachers were strategic in the order with which they allowed students to discuss the different methods. For example, the teacher purposefully allowed Abbie in section 6.2.1.1 to show her solutions to the class for her fellow students to evaluate it for its merits and demerits. Students used their understanding to determine that the method was not mathematically sound in spite of the fact that it served Abbie's purpose. Teachers also indicated exactly where the error in students reasoning or solutions laid. Recognising the error and understanding its source, does aid in deeper understanding of the related principles.

Class discussions often progressed to higher levels of thought as students were directed to focus on emerging patterns or trends. Students were free to adopt the method presented by their classmates, but they should be able to explain it. Students were also allowed to build on each other's method during discussions. In the excerpt presented in section 6.2.1.1, Kyle's method evolved from Troy's method. In Kyle's method, students' attention was directed to the pattern of counting the numerator by one and the denominator by two, as they tried to obtain fractions equivalent to one-half (1/2). Troy's method introduced another dimension of comparing the numerator with the denominator and recognising the factor of two between them. While some students found it easy to grasp the idea of counting; Troy's method introduced a higher level of mathematical thought by comparing the two counting patterns and deducing the common trends between them. This form of interaction is made possible by allowing students to explore mathematics in their way and to share their responses. Sharing solutions in this manner builds students understanding of mathematical concepts (Choppin, 2007). Moving from lower to higher level of thought was also observed in the teachers' questioning technique. Questions toward the beginning of the lesson were specific and concrete, whereas, those toward the end of the lesson were general and abstract. This approach was used whether the teacher was dealing with an individual student, small groups or the class as a whole.

Another way teachers helped students to operate in the mathematics world, was to encourage them to use mathematical symbols and terms in their explanations. In the lesson from which the excerpt given above was taken, teachers constantly asked students to use the terms “numerator” and “denominator” instead of “top number” and “bottom number”.

Supporting students’ understanding in the mathematics world includes helping them to define the concept. The term “define” here does not necessarily mean using verbal statements to describe the concept, although that may also be necessary. Defining a concept could also mean showing rather than telling. An example of this is seen in figure 20 which relates to a lesson on place value. The teacher was assisting students to “see” the concept of place value by showing what exactly it means to say two thousands five hundred and sixty eight. Mathematics has many abstract concepts that may be difficult to “show”, however, separating the components and explaining how they are fitted together helps students to see the big picture of how the concept is connected to other concepts.

6.2.2.5 Assess Formatively

Teachers maintained a high level of expectation from all students in the class and provided feedback to students when necessary. The teachers focused more on how students grasped the functional use of concepts rather than the speed at which they obtained solutions. During desk work, the teacher questioned students to ensure clear and accurate understanding of the concept. During class discussion the teacher listened keenly to students’ explanations and asked clarifying questions to encourage both the speaker and the rest of the class to provide clearer explanations. This continuous assessment allowed them to correct misunderstandings quickly. The role of the teacher in assessment is twofold. First, the teacher checks the mathematical integrity of the students' solutions and ensures that each calculation and reason for a calculation is mathematically sound. The second role is to understand the students' solutions process from the student's point of view. In seeing the problem from students' point of view, the teacher can offer suggestions on ways in which the student's strategy could become more efficient.

Students were encouraged to assess the presentation of their peers by saying whether they agreed or disagreed with it. Along with this, students listened to each other's solutions and asked critical questions or made comments about the ideas being presented. The purpose was to encourage students to listen keenly and to try to understand their classmates' solutions process. This allowed students to gain a better understanding of the concept and to further modify their own if necessary. Students assessed their learning by making journal entries about what they learnt or what they did not fully understand. This too is of significance as self-evaluation facilitates mental growth and development (Nohda, 2000).

6.3 The Open Approach: Impact on Society

Learning in the open approach produced changes in students' behaviour that extended beyond the mathematics classroom. For example, observation of students from the co-ed class at play, showed boys and girls cooperating with each other and accommodating each other more as the research progressed. The practice of listening to and appreciating the opinions of others, is a desirable trait that is developed by the implementation of the open approach. This change in attitude towards each other, will, over time, have a positive impact on dispute resolution, gender relations and gender equity in the home, in the workplace, and in the wider society.

Additionally, the teacher of the co-ed class noted changes in students' behaviour in other subject areas. She said that in science lessons for example, students were now asking more "Why?" questions and they wanted to try to solve the problem using their methods before the teacher gave the solution. This was borne out in one session where students suggested that they tried to make their own pulleys for an assignment rather than having the teacher making one model for the class. This attitude of critically analysing and being confident enough to offer viable alternatives will serve students well as they progress to higher levels of learning and as they take their place as citizens of a dynamic society. Individuals, who passively accept all that is thrust at them by leaders or by the circumstances of life, will not have any meaningful impact on the growth and development of the society in which they live. Teachers of the other classes stated that students showed more interest in school and were more motivated to do their work. The major changes common among the three classes were:

Students were now listening to each other's opinions. More students were now willing to take responsibility for their learning. The latter was evidenced by the fact that students were more prepared for lessons. They were more equipped with the resources required for the lesson, they no longer gathered for long periods of time sharpening pencils at the dust bin nor did they need to be told to take out their books for school work. In addition, there were fewer incidences of undesirable behaviour in the classroom. These social behaviours suggest a positive change in students' attitude toward learning and towards their peers which results in them gaining more from the teaching – learning process. Taking responsibility for one's actions and for one's progress, when transferred to the wider society, is a valuable asset as students learn to become more independent learners, problem solvers and active participants in an evolving society.. According to Benjamin (2011), employers have long being asking for individuals who can work on their own initiative and who can solve work related problems as they arise. The open approach can be seen as an important step towards achieving this end.

Group work as practiced in the open approach, encourages altruism and discourages the negatives of competitiveness. Through cooperation, the group succeeds and this success is experienced by all. Each member of the group is made to feel valued and his contribution is seen as an important part of the process regardless of gender or stage of cognitive development. The recognition that the group succeeds or fails together, encourages each individual to give of his best for the benefit of the group.

If teachers continue implementing the open approach teaching method in schools, these skills and attitude learned in this open environment would serve to prepare students for the world of work. Girls will become more confident, independent thinkers and will display an increased ability to solve problems in their daily lives. The altruistic characteristics that boys develop during the executions of these lessons will serve as an aid for them to live in peace and harmony with their neighbours. This will also reduce the undesirable behaviours displayed by Jamaican young men.

6.4 Limitations of the Study

1. This study focused on only one rural area in Jamaica, so results may not be generalizable to other locations or may only apply to areas with conditions similar to where the research was conducted.

2. Interpretation of qualitative data can be subjective; however, this subjectivity was reduced by having different individuals providing independent interpretations of data. The researcher sought the interpretations of observation notes from six other individuals: two researchers, one former principal and the three teachers from the research sites.

3. Three teachers participated in the research. The differences in teaching ability may have affected the learning outcomes and the research results. An attempt was made to control this by having the three teachers participating in a four-day training sessions before the intervention so that they followed the same teaching pattern. Meeting twice per month helped teachers to synchronize the lesson activities. The teachers discussed different ways of introducing the problem, expected student responses and how to manage difficult situations (academic and behavioural) that may arise in each lesson. This provided a way for lessons delivery to be somewhat similar, as was observed.

6.5 Summary

Open-ended mathematics problems as used in this study can accommodate students at varying levels of understanding, helping each child to produce at least one solution. In most cases, one student could produce multiple solutions. Students worked at their own pace to solve and reflect on their solutions. Being able to solve the problem caused an increased confidence in students which in turn motivated them to search for more solutions. The students identified more connections with the concepts which each correct solution produced. Most students were able to produce multiple solutions to a problem. Students were allowed to present their solutions to the class and to see other solutions presented by their classmates. As students compared and discussed the different solutions, the connections between the solutions become clearer and students become familiar with the underlying concepts involved. Greater understanding was gained by connecting newly learnt concepts to everyday experiences and with previously learnt knowledge.

The support given to students as they solved open-ended problems also played an important role in developing students' understanding. In all the classes, learning was facilitated in an environment where the opinion of each person was valued and respected and where students had the freedom to explore and make mistakes without being ridiculed. Students were supported in clarifying and justifying their solution method, to explore connections and to reflect on their solutions process. This psychosocial environment gave students a sense of freedom to explore the problem. Because the environment played such a keen role in aiding students' development in conceptual understanding, the researcher would like to propose the themes in the ‘‘open environment category’’ as desired characteristics of the open approach classroom. However, further research may be needed to refine them so that they can be explained in greater detail and applied more successfully.

CHAPTER SEVEN:

CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

7.1 Overview of the Study

There is a paradigm shift from procedural knowledge to conceptual understanding in the teaching and learning of mathematics in Jamaica. A review of the literature on how conceptual understanding is being enhanced in other countries revealed four strategies commonly employed: RME, Hands-on, Problem Solving and Open Approach. Jamaica however, has its own unique cultural and social challenges which have impacted schooling at all levels. Among the issues that were of interest to this research was the unusual phenomenon of girls outperforming boys in mathematics at the primary level. This study suggests that the open approach with open-ended problems creates a more equitable classroom environment for both boys and girls and promotes conceptual understanding in students from both genders. Jamaica's view on the understanding of mathematical concepts involves connecting such concepts to other concepts in mathematics as well as in the context of daily life.

Implementing the open approach in Jamaica requires students to apply the concept in and out of the classroom. The open approach was modified to include application of mathematical concepts in students' daily lives. Here, students are encouraged to apply their knowledge and experiences inside and outside of the classroom when solving a problem. In the open approach teaching method, emphasis is placed on viewing or solving a given problem from different perspectives, as well as on the representation of mathematical content and principles in different ways. The practice of relating a given problem to everyday experiences assists students in discovering “new” connections and relationships between concepts as well as deepening their own understanding of the content. Additionally, when they are required to provide convincing arguments to support their theory or the strategies they use in solving a problem, their understanding of the related content is further enhanced. The teacher, in facilitating the learning process, guides students along the solution path that they, the students have chosen and encourages them to defend their theories using evidence and reasoning. On the other hand, students are required to listen to and consider the ideas of others while they are equally given an opportunity to offer counter proposals if they can present arguments to this effect. Some schools sought to implement these policies in their normal co-educational classes, but few decided to separate students into gender-based classes. This study incorporated these “ideals” by systematically applying the open approach teaching method in both single-sex and co-educational classrooms.

The overall purpose of this study was to apply the open-approach in Jamaican classrooms and to observe how the environment that it creates impacted students’ understanding of mathematical concepts. In this context, the issue of gender-specific responses was also examined.

Participants were organised into three classes, (co-ed, all-boys and all-girls respectively), and a pre-test administered. This was followed by six months of teaching intervention using the Open Approach teaching method and the study culminating with the administration of a post-test. Throughout the period of intervention, target lessons were conducted as a part of the formative process. The differences with these target lessons were that the problem was taken from the test and reconstructed to reflect an everyday situation with which the students were familiar, and work from all students were collected at the end of that lesson.

Observations were recorded; results were examined, and a comparison of the responses among the classes as well as between the genders was made from both quantitative and qualitative data.

7.2 Conclusions

This study yielded four key findings. First, the use of the open approach with open-ended problems impacted positively on students understanding of mathematical concepts regardless of gender or classroom setting. This was evidenced by the fact that all groups had an increase in performance on the post-test when compared to the pre-test, and all were able to produce more solution strategies at the end of the study than they did at the beginning. It was also evident that through the open approach, the teacher was able to develop students' understanding of mathematical concept regardless of their reading abilities and perceived competence in mathematics.

The second discovery was that boys had higher averages and displayed greater understanding of concepts than the girls did. Girls showed a greater tendency towards using traditional methods but showed little understanding of the method they used. Girls obtained higher scores than boys on closed items, but boys got higher scores than girls on open-ended items. Students' scores on closed items reflected the current trend in students' performance on national tests in Jamaica. These tests contain closed items in the form of multiple choice and "short answer" questions. From the solution strategies proposed and the arguments presented during class discourse in this study, it was concluded that boys were more adept at creating their own way of solving a given problem and were more predisposed to taking risks. This result is similar to that of previous studies such as Fennema and Tartre (1985) and Gallagher (1998) who suggested that females were more likely to solve problems with procedures learnt in the classroom and while boys tended to solve problems on their own. Kimball (1989) reported that females preferred to learn mathematics conversationally and collaboratively, this too was seen in this current study.

The third observation was that class setting did have an impact on the academic outcomes of lessons. Boys in the co-ed class displayed greater understanding and had more solution methods than boys in the single-sex class but for most test items there was no significance. The girls in the single-sex class showed greater understanding and had more solutions method than girls in co-ed class but this too was varied and had no significance on most items. A greater improvement on the post-test was expected based the positive change in students behaviour described in the observation. One factor that may have prevented students from performance on the test was the time duration for doing. Environment and personal factors on the day of the test may have also affected students' performance. Factors and Students' prior exposure to open-ended items in the pilot study may have contributed to marginal difference between pre and post test results. A larger difference may have been obtained if the pre-test was administered at the beginning of the pilot study rather than at the beginning of the main study. Regardless of whether or not a larger difference would have been obtained, this difference remains in favour of boys over girls generally, boys in the co-ed class over those in single-sex class and girls in the single sex class over girls in co-ed class. Currently, there is a move toward single-sex classes in many primary schools in Jamaica (Hibbert, 2015). The results of this study suggest that this may not be the best move for some schools in Jamaica. However, so far, schools that have reported an increase in boys' performance with single-sex classes are located in urban areas. This research was conducted with students in rural schools. Another difference between the studies is that this research focused on understanding of mathematics while the move toward single-sex classes targets procedural understanding in traditional classroom setting.

Finally, it was determined that the classroom environment created by the open approach had a positive impact on students' understanding. This environment is characterized by student-autonomy, stimulating discourse, a multiplicity of ideas, interconnectedness of concepts, thoughtful reflection and relevance to daily life. The themes in the environment category can be developed in a framework as the targeted characteristics of the open approach environment.

7.3 Implications

The reader should be careful when attempting to make generalisations about the implications and applications of these findings. It must be noted that each context is defined by its own idiosyncrasies and any attempt at replication or transference, should be mindful of this.

For the purpose of national development, consensus must be arrived at regarding the preferred outcomes of education. The system should be made to facilitate students evolving as "productive thinkers" rather than as mere "test takers" or "skilled performers". In order to achieve this end, lessons should be adapted to conform to the principles and constructs of understanding of mathematical concepts first.

The open approach teaching method can be used to help students understand mathematics. It was noticed that when given open-ended questions, students' first method of choice reflected a low level of understanding of the problem. The quest to find multiple solutions characterized by reflection and discussion, and which encourages diversity of thought, led students to a greater understanding of the problem and caused them to develop more sophisticated solutions. This demonstrates that if our students are given time and opportunity to explore a problem, then learning is enhanced.

This research supports the ideas put forward in the document*, The New National Mathematics Policy Guidelines*. In the current study, students approached the same problem from different perspectives and developed representation of mathematics in different ways. The teacher knew when to allow productive struggle and when to provide support to help the child along in the process. Being a guide to learning, the teacher allowed students the freedom to become flexible and resourceful problem solvers which resulted in students developing and understanding of mathematical concepts. Students were able to gain a greater understanding of mathematics by creating and discovering their own solutions. It is necessary for educators in Jamaica to consider the findings of this study and to make the necessary adjustments to existing philosophies and procedures in order to maximize the benefits derived by students from the system of education within which they are engaged. Classroom procedures should be elevated above the simple memorization and recall of facts. If students are to be prepared for the workforce of the future, they should be trained to become citizens who can reason and think critically when faced with non-routine problems. This is achieved by an understanding of the inter-connecting network of basic mathematical concepts.

7.4 Recommendations

The following are offered as recommendations to be implemented by teachers, principals, and other stakeholders, with a view to enhancing the process of education. In an age of advancing technology, mathematical competence has evolved from memorizations of facts and rote procedures to a display of more in-depth understanding. Mathematics classrooms should reflect the same.

The open approach reflects the true nature of the outside world. For example, asking students to calculate the cost of travelling from point A to point B, using a specific mode of transportation, does not reflect the true nature of daily life. There are feasible options, and in reality, one weighs all the option and compares these for cost, efficiency, comfort and utility. The options are varied; the answers are many; the procedure is “messy”, unpredictable, and sometimes time-consuming, but a higher level of understanding is involved. The open approach reflects the reality of everyday life. The ability of students to perform at higher level of thought is sometimes underestimated by teachers and parents alike, and this can be detrimental to their education. It is better for students to be provided with opportunities where they can extend their understanding and go beyond the norm, than for their chances of growth to be limited because of the teacher’s perception or misinterpretation of their abilities.

This research also showed that the role of the teacher should change. The teacher has to become more of a facilitator and guide rather than a dictator and dispenser of knowledge. The study provides information on ways the teacher can facilitate students’ mathematical understanding by the use of the open approach. This involves the creation of an empowering environment and the use of well-crafted problems which allow the students to utilise their knowledge base. If teachers solely use the traditional approach to teaching, then they do not allow students to construct meaningful understanding of mathematical concepts and students learning may be retarded.

A recommendation is that this approach to teaching be included in policy as well as incorporated into the curriculum during the formative years so that the skills involved can be developed and fostered over time.

All national tests should include more open-ended items. These items are better able to assess students' understanding. Rather than being required to circle a letter in response to a multiple choice item, students are to be given the opportunity to display their understanding of a concept using words and diagrams. In addition, having more open-ended items on national tests would make it necessary for teachers to include more open-ended problems in the classroom. Teachers indicated that they and their school are judged by the number of students who are able to "pass" national tests; therefore they teach based on the skills that they know the tests require; this is primarily the skill to recall facts If national test included more open-ended items, then teachers would include more open-ended items in their lessons.

Another recommendation based on the results of this study is that students be assisted in the development of their skills of articulation. While writing explanations about the steps taken in solving a problem may be appropriate for grades 5 and 6, students in the lower grades may find this more difficult. Some students, especially boys, in the study had difficulty expressing their understanding. Assisting them to verbalise and write down their thought processes will help them to develop a deeper understanding of mathematical content.

7.5 Recommendations for Teachers

7.5.1 Solve the problem first. The teacher should familiarise himself/herself with the problem and its solutions by solving it before giving it to the students. Being aware of the basic parameters of the problem enhances the ability of the teacher to help struggling students. The teacher should also create extensions to the problem and try to view the problem from different perspectives. A rubric of students' responses should be prepared in advance and this can be done simultaneously with solving the problem.

7.5.2 Know your students: Supporting students in the understanding of mathematics requires differentiating instructions on an individual level. It is important to get to know students as early and quickly as possible. The teacher should be aware of each child's learning style, temperament and mannerism in order to differentiate instruction appropriately and accurately.

7.5.3 Move from concrete to abstract. Introducing the problem in a real world context and moving from "the concrete" to "the abstract' proved to be an effective strategy**.** This is especially useful when teaching small children. Seeing the practical application of what they are doing makes mathematics real and relevant to them.

7.5.4 Encourage group work and discourse. Because open-ended problems allow multiple solutions, they offer an ideal context for students to learn from each other and discuss solutions.

7.5.5 Avoid giving answers and hints too early. "Productive Struggle" is a necessary ingredient for growth. By holding off on giving hints, the teacher is encouraging students to think for themselves, and this can promote understanding. Coaxing, and prodding is to be used as is necessary. Each student may need a different form of support. One may need confirmation that he is on the right track while another may need a hint about what to do next. Knowing when to give support to each student and the type of support needed may require new learning by the teacher but this is possible through effort and practice.

7.5.6 Be conscious of the type of questions you are asking. Using questions correctly was one of the important aspects of classroom discourse. It was highlighted that teachers used different types of questions depending on their purpose. It is recommended that teachers be conscious of this action and think about the purpose of asking the question before articulating it.

7.6 Recommendations for Future Research

1. The first suggestion for a future study is to replicate this study within an overcrowded classroom and/or a multi-grade classroom. It should be interesting the see if the required environment can be established in an overcrowded classroom and the type of discussion that would emerge from a multi-grade class.

2. A research comparing a treatment class with a control class on how the open-approach develop students understanding of mathematical concept.

3. It is also recommended that a research be conducted in Jamaica, examining teachers' attitudes toward teaching with the open approach.

REFERENCES

Alkin, M.C. (Ed.). (1992). Teaching effectiveness. *The Encyclopaedia of Educational Research (6th ed.)* (Vo1.4, pp.1373- 1388). New York: Macmillan.

Ball, D. L. (199 l). Teaching mathematics for understanding: What do teachers need to know

about subject matter? *In. M.M. Kennedy (Ed.), Teaching Academic Subjects to Diverse Learners (pp. 63-84).* New York: Teachers College Press.

Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for Research in Mathematics Education, 21*(1), 47-60.

Becker, J. P., & Shimada, S. (1997). *The open-ended approach: A new proposal for teaching mathematics*: Reston, Virginia. Mathematics National Council of Teachers of Mathematics, INC.

Becker, J. P., Sawada, T., & Shimizu, Y. (1999). Some findings of the US-Japan cross-cultural research on students' problem-solving behaviours. *International Comparisons in Mathematics Education*, 121-139.

Becker, J., P., Silver, A.,S., Kantowski,G.,M., Travers,J., K., &Wilson,W.,J., (1990). Some observations of mathematics teaching in Japanese elementary and junior high schools. *The Arithmetic Teacher,Vol.38, No.2*, 12-21.

Becker, J. R. (2003). Gender and mathematics: An issue for the twenty-first century. *Teaching Children Mathematics, 9(8),* 470-73.

Beller, M., & Gafni, N. (1994). International perspectives on the schooling and learning achievement of girls and boys as revealed in the 1991. *International Assessment of Educational Progress (Iaep).* National Institute for Testing and Evaluation.

Ben-Chaim, D., Lappen, G. and Houang, R. T. (1988). The effect of instruction on spatial visualization skills of middle school boys and girls. *American Educational Research Journal, 25(1)*, 51-71.

Ben-Shakhar, G., & Sinai, Y. (1991). Gender differences in multiple-choice tests: The role of differential guessing tendencies. *Journal of Educational Measurement, 28(1), 23-35.*

Benbow, C. P. and Stanley, J. C. (1980). Sex differences in mathematical ability: Facts or artefact? *Science, 210(12*), 1262-1264.

Benjamin, T. (2011, October 16). Teacher deficiencies multiply math misery. *Jamaica Gleaner*. Retrieved from http://jamaica-gleaner.com.

Benjamin, T. (2015, March 26). Math teacher problems hurting students. *Jamaica Gleaner.* Retrieved from http://jamaica-gleaner.com.

Berthold, K., & Renkl, A. (2009). Instructional aids to support a conceptual understanding of multiple representations. *Journal of Educational Psychology*, 101(1), 70-87.

Bessudnov, A., & Makarov, A. (2015). School context and gender differences in mathematical performance among school graduates in Russia. *International Studies in Sociology of Education*, *25*(1), 63-81.

Bishop, A. (1991). *Mathematical enculturation: A cultural perspective on mathematics education (Vol. 6*). Springer Science & Business Media.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*, 41-62.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*, 239-258.

Boaler, J. (2008). When politics took the place of inquiry: A response to the National Mathematics Advisory Panel's report of instructional practices. *Educational Researcher, 3*, 588-594.

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research for education: An Introduction to theory and methods* (3rd ed.).Boston.

Brooks, J. G., & Brooks, M. G. (1999). *In search of understanding: The case for constructivist classrooms.* ASCD.

Brown, J., & Chevannes, B. (1998). *"Why man stay so": An examination of gender socialization in the Caribbean*. University of the West Indies.

Buddo, C. (2013). Evaluating mathematics teaching through the lens of some grade 4 students. *International Journal of Education and Research. Vol. 1 No. 8.* ISSN: 2201-6333

Buddo, C. (2015). Teachers' reflections on their implementation of a revised primary mathematics curriculum. *International Journal of Education and Research. Vol. 3 No. 12.*

Burns, M. (2000). *About teaching mathematics: A K-8 resource*. White Plains, NY: Math Solutions Publications.

Byers, V., & Herscovics, N. (1977). Understanding school mathematics. *Mathematics Teaching*, *81*, 24-7.

Campbell, W. (2015, October 13). Arrest the flight of math teachers. *Jamaica Observer*. Retrieved from http://www.jamaicaobserver.com.

Cai, J. (1995). Exploring gender differences in solving open-ended mathematical problems. Paper presented at the *Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, pp.1-9.

Chan, C. M. E. (2007). Using open-ended mathematics problems: A classroom experience (Primary). *In C. Shegar & R. B. A. Rahim (Eds.), Redesigning pedagogy: Voices of Practitioners* (pp. 129-146).

Charles, R., & Lester, Jr., F. (1984). An evaluation of a process-oriented instructional program in mathematical problem solving in grades 5 and 7. *Journal for Research in Mathematics Education, 15*, 15-34.

Chevannes, B. (2003). Boys left out, gender achievements and prospects in education: the gap report, part one. Available online at: http://www.ungei.org/gap/interviewsChevannes.html

Choppin, J. (2007). *The interplay between discourse patterns and curricular resources: The impact of dialogic tendencies on the activation of resources in a mathematics reform curriculum.* Paper presented at the American Educational Research Association Annual Chicago, Illinois. Retrieved from http://www.rochester.edu/warner/papers/2007_aera_chopin_interplay.pdf

Clarke, C. (2005). Socialization and teacher expectations of Jamaican boys in schools: The need for a responsive teacher preparation program. *International Journal of Educational Policy, Research, and Practice: Conceptualizing Childhood Studies, 5(4),* 3-34.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist, 23*(2), 87-103.

Cockcroft, W. H. (1982). *Mathematics counts*. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools. London: HMSO.

Creswell, J. W. (2009). Editorial: Mapping the field of mixed methods research. *Journal of Mixed Methods Research*, *3*(2), 95-108.

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, *34*(4), 344-377.

Davis, E. J. (2006). A model for understanding in mathematics. *Mathematics Teaching in the Middle School, 12(4),* 190-197.

Davis, R. (2004).*Task Force on Educational Reform Report*. Jamaica Ministry of Education, Youth & Culture, Jamaica.

Devens-Seligman, C. (2007). *Mathematical problem solving: Its effect on achievement and attitudes of elementary school students* (Doctoral Dissertation, Claremont)

Dickinson, P., & Hough, S. (2012). Using realistic mathematics education in UK classrooms. *Centre for Mathematics Education, Manchester Metropolitan University, Manchester, UK*.

Else-Quest, N. M., Hyde, J. S., & Linn, M. C. (2010). Cross-national patterns of gender differences in mathematics: a meta-analysis. *Psychological Bulletin*, *136*(1), 103 - 127.

Evans, H. (1999). Gender differences in education in Jamaica. *Office of the UNESCO Representative in the Caribbean.*

Fennema, E., Carpenter, T., Jacobs, V., Franke, M., & Levi, L.W. (1998). Longitudinal study of gender differences in young children's mathematical thinking. *Educational Researcher, 27*(5), 6-11. Retrieved from http://www.jstor.org/stable/1176733

Fennema, E. & Tartre, L. A. (1985). The use of spatial visualization in mathematics by girls and boys. *Journal for Research in Mathematics Education*. 16(3), 184-206.

Foong, P.Y. (2002). Using short open-ended mathematics questions to promote thinking and understanding. *In A. Rogerson (Ed.), Proceedings of the International Conference: The Humanistic Renaissance in Mathematics Education* (pp.135-140).

Francis P. (2006, September 15). Poor CXC results - Jamaican students lag behind region in maths, English. *Jamaica Gleaner.* Retrieve from http://old.jamaicagleaner.com/gleaner/20060915/lead/lead1.html.

Gallagher, A. M. (1998). Gender and antecedents of performance in mathematics testing. *Teachers College Record, 100,* 297-314.

Gastwirth, J. L., Gel, Y. R., & Miao, W. (2009). The impact of Levene's test of equality of variances on statistical theory and practice. *Statistical Science, 24,* 343-360.

Goetz, A. (2005). Using *open*-ended problems for assessment. *Mathematics Teacher*, 9*9(1),* 12-17.

Gurian, M. (2006). *The wonder of boys.* New York, NY*:* Teacher.

Halpern, D. F. (2000). *Sex differences in cognitive abilities* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Hancock, C. L. (1995). Enhancing mathematics learning with open-ended questions. *The Mathematics Teacher, 88*(6), 496-499

Hashimoto, Y., & Becker, J. (1999). The open approach to teaching mathematics- creating a culture of mathematics in the classroom: Japan. In L. J. Sheffield (Ed.), *Developing Mathematically Promising Students* (pp. 101-120). Reston, VA: National Council of Teachers of Mathematics.

Haylock, D. (1 982). Understanding in mathematics: Making connections. *Mathematics Teaching, (98*)54- 56.

Hellekant, J. (I 994). Are multiple-choice tests unfair to girls? *System, 22*, 349-352.

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education*, 242-273.

Hibbert, K. (2015, February, 01) Allman town primary's success experiment. Jamaica observer. Retrieved from *http://www.jamaicaobserver.com/magazines/career/Allman-Town-Primary-s-success-experiment_18311257*

Hitz, H., & Scanlon, D. (2001). Effects of instructional methodologies on student achievement, attitude and retention. *Paper presented at the 28th Annual National Agricultural Education Research Conference.* Retrieved from http://aaae.okstate.edu/proceedings/2001/hitz.pdf

Hofman, R. H., Hofman, W. A., & Guldemond, H. (2001). Social context effects on pupils' perception of school. *Learning and Instruction, 11(3*), 171-194.

Hoosain, E. (2001). What does it mean to understand mathematics? *Humanistic Mathematics Network Journal, 25*, 20-22.

Hubbard, L., & Datnow, A. (2005). Do single-sex schools improve the education of low-income and minority students? An investigation of California's public single-gender academies. *Anthropology & Education Quarterly*, 115-131

Husain, H., Bais, B., Hussain, A., & Samad, S. A. (2012). How to construct open ended questions. *Procedia-Social and Behavioural Sciences*, *60*, 456-462.

Ikeda, T. (2010). Roots of the open-ended approach. *Special Issue (EARCOME 5) Mathematics Education Theories for Lesson Study: Problem Solving Approach and the Curriculum through Extension and Integration* (p.6). Bunshuodo

Insatusho.

Inprasitha, M. (2004). Teaching by Open-Approach Method in Japanese Mathematics Classroom. KKU Journal of Mathematics Education, pp.1-17 Vol. 1 No. 1.

Khon Kaen, Thailand. (in Thai) Inprasitha, M. (2006). Open-ended approach and teacher education *(Special Issue on The APEC-TSUKUBA International Conference" Innovative Teaching Mathematics through Lesson Study" January 15-20, 2006 Tokyo, JAPAN)*--(APEC Symposium: Innovative teaching mathematics through Lesson Study). 筑波数学教育研究, (25), 169-177.

Inprasitha, M. et.al. (2007). Preparing the context for using Japanese Teacher Professional Development "Lesson Study" in Thai. *Proceedings of JSN 1. Pp*. 152-163.

Inprasitha, M. (2011). One feature of adaptive lesson study in thailand: designing a Learning unit. *Journal of Science and Mathematics Education in Southeast Asia*, *34*(1), 47-66.

Isaacs, R.A., Johnson, S., Pinnock. M.L., (2003). *The new integrated approach mathematics workbook 4*. Mandeville, Jamaica. Mid-Island Educators.

Jackson, C. (2002). Can single-sex classes in co-educational schools enhance the learning experiences of girls and/or boys? An exploration of pupils' perceptions. *British Educational Research Journal*, *28*(1), 37-48. doi: 10.1080/01411920120109739.

James & Constantine, (2005). *Primary Mathematics for Jamaica Grade 3.* Ginn & Company, Oxford. Eniath's Printing Company. Trinidad and Tobago.

Jones, I., Inglis, M., Gilmore, C., & Hodgen, J. (2013). Measuring conceptual understanding: The case of fractions. *In A. M. Lindmeier & A. Heinze (Eds.),* *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 113–120). Kiel, Germany: IGPME

Kabiri, M. S., & Smith, N. L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics Teaching in the Middle School, 9(3*), 186-192.

Kieran, C. (1994). Doing and seeing things differently: A 25-year retrospective of mathematics education research on learning. *Journal for Research in Mathematics Education*, *25*(6), 583-607.

Kilpatrick, J. (2014). Competency frameworks in mathematics education. *In Encyclopaedia of Mathematics Education* (pp. 85-87). Springer Netherlands.

Kilpatrick, J., Swafford, J., & Findel , B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kimball, M. M. (1989). A new perspective on women's math achievement. *Psychological Bulletin, 107*(2), 198-214.

King, R. (1990). *A Study of the History of Schooling in Jamaica, 1866–1962*. *UWI,* Mona Planning and Estimates.

Klavir, R., & Hershkovitz, S. (2008). Teaching and evaluating open-ended problems. *International Journal for Mathematics Teaching and Learning*. Retrieved from http://www.cimt.plymouth.ac.uk/journal/default.htm

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. (2011). Indicators of creativity in mathematical problem posing: How indicative are they? *In M. Avotiņa, D. Bonka, H. Meissner, L. Ramāna, L. Sheffield & E. Velikova (Eds.), Proceedings of the 6th International Conference Creativity in Mathematics Education and the Education of Gifted Students* (pp. 120-125). Latvia: Latvia University

Korn, J. (2014). Teaching conceptual understanding of mathematics via a hands-on approach. *Senior Honors Theses*. http://digitalcommons.liberty.edu/honors/476

Kosyvas, G. (2016). Levels of arithmetic reasoning in solving an open-ended problem. *International Journal of Mathematical Education in Science and Technology*, *47*(3), 356-372.

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. Theory into practice, 41(4), 212-218.

Krutetskii, V. A. (1976). In Kilpatric J. & Wirszup I. *The psychology of math. Abilities in School children.* The University of Chicago Press.

Kwon, O. N., Park, J. H., & Park, J. S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review, 7* (1), 51-61.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction*, *3*(4), 305-342.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-63.

Laine, A., Knavery, L., Pehkonen, E., Ahtee, M., Heinilä, L., & Hannula, M. S. (2012). Third-graders' problem solving performance and teachers' actions. *In Proceedings of the ProMath meeting in Umeå (ed. T. Bergqvist)* (pp. 69-81).

Laine, A., Näveri, L., Ahtee, M., & Pehkonen, E. (2014). Development of Finnish elementary pupils' problem-solving skills in mathematics. *CEPS Journal, 4(3*), 111-129.

Leahey, E., & Guo, G. (2001). Gender differences in mathematical trajectories. *Social forces*, *80*(2), 713-732.

Lewis, C. (2002). *Lesson study: A handbook of teacher led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C. (2004). Lesson study. *In Easton, L. B. Powerful designs for professional learning.* Oxford, OH: National Staff Development Council.

Lewis, C. (2006). Lesson study in North America: Progress and challenges. Lesson study: *International perspective on policy and practice,* 7-36.

Lin, C. Y., Becker, J., Ko, Y. Y., & Byun, M. R. (2013). Enhancing pre-service teachers' fraction knowledge through open approach instruction. *The Journal of Mathematical Behaviour, 32(3),* 309-330.

Lubienski, S.T. (2000). A clash of social cultures? Students' expediencies in a discussion-intensive seventh-grade mathematics classroom. *The Elementary School Journal, 100(4),* 337-403.

Maccoby, E. E., & Jacklin, C. N. (1974). *The psychology of sex differences (Vol. 1).* Stanford University Press.

Mansor, R., Halim, L., & Osman, K. (2010). Teachers' knowledge that promote students' conceptual understanding. *Procedia-Social and Behavioural Sciences*, 9, 1835-1839.

Marshall, C., & Rossman, G. B. (2006). *Designing qualitative research.* Sage: California.

Matalon, B. A. (Ed.). (1994). *The psychology of learning: An introduction.* Teacher education development department. The University of the West Indies Press.

McFarland, M., Benson, A. M., & McFarland, B. (2011). Comparing achievement scores of students in gender specific classrooms with students in traditional classrooms. *International Journal of Psychology*, *8*, 99-114.

McNeil, M. (2008). Single-sex schooling gets new showcase: South Carolina's top education official sees a state-wide push for single-gender programs as a way to boost public school choice and scores. *Education Week, 27(36),* 20, 22.

Mertens, D.M. (2005). *Research methods in education and psychology: Integrating diversity with quantitative and qualitative approaches.* (2nd ed.) Thousand Oaks: Sage.

Merriam, S. B., & Merriam, S. B. (1998). Qualitative research and case study applications in education. San Francisco, CA: Jossey-Bass.

Meyer, D. 2010. Math class need a makeover. TED Conference. Retrieved from http://www.ted.com/talks/dan_meyer_math_curriculum_makeover#t-573955.

Miller, E. (1997). *Jamaican primary education: A review of policy-relevant studies*. Green Lizard Press.

Ministry of Education (1999). *Revised Primary Curriculum Guide Grades 1-3.* Retrieved from http://www.moe.gov.jm/sites/default/files/GuideGrade1-3.pdf.

Ministry of Education (1999). *Revised Primary Curriculum Guide Grade 4.* Retrieved from http://www.moe.gov.jm/sites/default/files/GuideGrade4.pdf.

Ministry of Education (2011). *National comprehensive numeracy programme*. Kingston.

Ministry of Education [MOE] (2013). National mathematics policy guidelines. Kingston.

Ministry of Education [MOE] (2014). Educational statistics, 2013-2014. Kingston.

Ministry of Education [MOE] (2015). Educational statistics, 2014-2015. Kingston.

Moreno, R., & Mayer, R. E. (1999). Gender differences in responding to open-ended problem-solving questions. *Learning and Individual Differences, 11(4)*, 355-364.

Moskal, B.M. (2000). Understanding student responses to open-ended tasks. *Mathematics Teaching in the Middle School, 5(8)*, pp.500-505.

Moyston, L. (2011, December 03). Destination education. *Jamaica Observer*. Retrieved from http://www.jamaicaobserver.com/columns/Destination-education_10235521

Moyston, L. (2014, May 20) Education: revolution and change. *Jamaica Observer*. Retrieved fromhttp://www.jamaicaobserver.com/columns/Education--revolution-and-change_16694842

Mulholland, J., Hansen, P., & Kaminski, E. (2004). Do single-gender classrooms in coeducational settings address boys' underachievement? *An Australian study. Educational Studies, 30(1*), 19-32.

Munroe, L. (2015). Observations of classroom practice…using the open approach to teach mathematics in a grade six class in Japan. *Paper presented at EARCOME 7,* Philippines.

Munroe, L. (2016a). Assessment of a problem posing task in a Jamaican grade four mathematics classroom. *Journal of Mathematics Education at Teachers College*, *Vol. 7 (1*) pp.51-58.

Munroe, L. (2016b). Impact of open approach on students’ understanding of mathematical concepts: A gender comparison. *Japan Academic Society of Mathematics Education, Vol. 22 (2)* pp.85-96.

Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, the Netherlands.

National Council of Teachers of Mathematics. [NCTM] (1980). *An agenda for action*: Recommendations for school mathematics of the 1980s. Reston, VA

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: A

National Research Council. Everybody counts: *A report to the nation on the future of mathematics education.* Washington, D.C. National Research Council, 1989.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89(6)*, 351-363.

Nohda, N. (1991). Paradigm of the 'open-approach' method in mathematics teaching: Focus on mathematical problem solving. *ZDM.* 32-37.

Nohda, N. (1993). How to link affective and cognitive aspects in mathematics class, *Proceedings of the Seventeenth International Conference for the Psychology of Mathematics Education, vol. I,* 8-10.

Nohda, N. (1995). Teaching and evaluating using “open-ended problem” in classroom. *International Reviews on Mathematical Education 27 (2)*, 57–61.

Nodha, N. (1999) Mathematical teaching by open-approach method in Japanese classroom activities. *Japan Society of Mathematical Education.* Pp.31-36. University of Tsukuba.

Nohda, N. (2000). Teaching by open-approach method in Japanese mathematics classroom. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education (vol. 1*, (pp. 39–53)). Japan: PME, Hiroshima University.

Nodha, N. (2001) Mathematical teaching and learning by open approach method: Focusing on quantitative and qualitative assessments. *Japan Society of Mathematical Education.* Pp.125-130. University of Tsukuba.

Nuttall, R. L., Casey, M. B., & Pezaris, E. (2005). Spatial ability as a mediator of gender differences on mathematics tests: A biological-environmental framework. Cambridge University Press.

Organization for economic co-operation and development (OECD). (2009). PISA, 2006 technical report.

Ontario Ministry of Education. (2011). *Capacity building series provoking student thinking/deepening conceptual understanding in the mathematics classroom*. Eight tips for asking effective questions.

O'Tuel, F., & Bullard, R. (1995). *Developing higher-order thinking in the content areas:K-12*. Victoria: Hawker Brownlow Education.

Pehkonen, E. (1997). Use of open-ended problems in mathematics classroom. *Research Report 176*. University of Helsinki, Dept. of Teacher Education, PO Box 38 (Ratakatu 6A), Helsinki 00014, Finland.

Pehkonen, E. (2014). Open problems as a means for promoting mathematical thinking and understanding. In András Ambrus & Éva Vásárhelyi (Eds.), Problem Solving in

Mathematics Education. *Paper presented at Proceedings of the 15th ProMath conference in Eger, Budapest* (pp.152–162).

Pehkonen, E., & Ahtee, M. (2005). The key element for good teaching: teachers' listening. *In Proceedings of the 4th Mediterranean conference on mathematics education* (pp. 641-650).

Perkins, D. (1998). What is understanding? *In M. S. Wiske (Ed.), Teaching for understanding: Linking research with practice* (pp. 39–57). San Francisco: Jossey-Bass.

Piaget, J. (1977). *The development of thought: equilibration of cognitive structures*. New

York: Viking Press.

Pirie, S. E. B., & Schwarzenberger, R. L. E. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in mathematics*, *19*(4), 459-470.

Polya, G. (2004). *How to Solve It.* Princeton, NJ: Princeton University Press. *(Original work published in 1945)*

Powell, a. B., Maher, c. A., & Alston, a. S. (2004). *Ideas, sense making, and the early development of reasoning in an informal mathematics setting1*. North American chapter of the international group for the psychology of mathematics education October 2004 Toronto, Ontario, Canada, 586.

Razali, N. M., & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-darling tests. *Journal of statistical modelling and analytics*, *2*(1), 21-33.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, 91(1), 175-189

Rittle-Johnson, B., Matthews, P., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence: A construct-modelling approach. *Journal of Educational Psychology*, 103, 85–104.

Sanchez W.B., 2013, Open-ended questions and the process standards. *Mathematics Teacher Vol.107, No.3* October 2013.

Santos-Trigo, M. (1996). An exploration of strategies used by students to solve problems with multiple ways of solution. *Journal of Mathematical Behaviour, 15*, 263-284.

Sax, L. (2005). *Why gender matters: What parents and teachers need to know about the emerging science of sex differences*. New York, NY: Doubleday.

Seng, S., & Chan, B. (2000). Spatial ability and mathematical performance: Gender differences in an elementary school. Eastlansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction).

Shimada, S. (ed.). (1977). *Open-ended approach in arithmetic and mathematics: A new plan for improvement of lessons*. Tokyo: Mizuumi Shobou. (In Japanese).

Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.

Silver, E. A. 1995. The nature and use of open problems in mathematics education: Mathematical and pedagogical perspectives. *ZMD Vol 2.* p. 67- 71

Silver, E. A. & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics, 12(3),* 129-135

Singapore Ministry of Education (2010). Parliament Replies. Retrieved from *http://www.moe.gov.sg/media/parliamentary-replies/2010/04/teach-less-learn-more.php*

Stewart, s. (2015) Schooling and coloniality: conditions underlying ‘extra lessons’ in Jamaica. *Postcolonial directions in education, 4(1),* 25-52.

Strong, S. G. (2009). *How do students experience open-ended math problems?* Dissertation.

Sullivan, A. (2009). Academic self-concept, gender and single-sex schooling. *British Educational Research Journal 35*:259-288.

Sullivan, M. (2009). *Connecting boys with books 2: Closing the reading gap* (Vol. 2). American Library Association.

Sullivan, P., Bourke, D., & Scott, A. (1997). Learning mathematics through exploration of open-ended tasks: Describing the activity of classroom participants. *In E. Pekhonnen (Ed.), Use of open-ended problems in mathematics classrooms* (pp. 88–106). Helsinki: University of Helsinki.

Sullivan, P., & Lilburn, P. (1997). *Open-ended maths activities: Using" good" questions to enhance learning*. Oxford University Press.

Sullivan, P., & Siemon, D. (2003). The potential of open-ended mathematics tasks for overcoming barriers to learning. In *Merga 26 (6-10 July)* (Vol. 2, pp. 813-816). Deakin University.

Skemp, R. (1976). Instrumental understanding and relational understanding. *Mathematics Teaching*, *77*, 20-26.

Skemp, R. R. (1987). *The psychology of learning mathematics*. Psychology Press.

Tartre, L. A., & Fennema, E. (1995). Mathematics achievement and gender: A longitudinal study of selected cognitive and affective variables [Grades 6–12]. *Educational studies in mathematics*, *28*(3), 199-217.

Teape, K. W. (2015). Male mathematics teachers and their influence on the expected performance of male students in mathematics in Jamaica. *The Bridge: Journal of Educational Research-Informed Practice. Volume 2*, Issue 2: Special Edition, ISSN 2056-6670

Trotman, S., & Severin, S. (2005). *Primary mathematics for Jamaica grade 4.* Ginn & Company, Oxford. Eniath's Printing Company. Trinidad and Tobago.

Tyson, E. (2012, March 4). Education transformation stillborn. *Jamaica Gleaner*. Retrieved from http://jamaica-gleaner.com/gleaner/20120304/cleisure/cleisure5.html

Ueda, A. (2011). Professional development of the teacher form point of terminologies which are used in lesson study in Japan. *Comparative International Studies of Teacher Professional Development and Best Teaching Practice in Professional Development System.* CNUE International Conference on Education. October 14 - 15, 2011 pp 235-245.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In Encyclopaedia of mathematics education (pp. 521-525). Springer Netherlands.

van Oers, B. (2002). Educational forms of initiation in mathematical culture. *In Learning Discourse (pp. 59-85).* Springer Netherlands.

Von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. *Constructivist views on the teaching and learning of mathematics* (Monograph #4). Reston, VA: NCTM.

Welsh, R. O. (2012). Overcoming smallness through education development: a comparative analysis of Jamaica and Singapore. Current Issues in Comparative Education; Vol. 15 Issue 1, p114-131.

Wester, A., & Henriksson, W. (2000). The interaction between item format and gender differences in mathematics performance based on TIMSS data. Studies in Educational Evaluation, 26(1), 79-90.

Williams, C. (2008). Realising rights through social guarantees: The case of Jamaica. Final Report submitted to the World Bank. Retrieved from http://siteresources.worldbank.org/EXTSOCIALDEV/Resources/317739-1168615404141/3328201-1192042053459/Jamaica.pdf

Williams, G. (2000). Collaborative problem solving and discovered complexity. In J. Bana & A.Chapman (Eds.), Mathematics education beyond 2000 (pp. 656-663). Perth, Australia: Mathematics Education Research Group of Australasia

Williams-Raynor, (2011). Many J'can schools not child-friendly. *Jamaica Observer*. Retrieved from http://www.jamaicaobserver.com/mobile/career/Many-J-can-schools-not-child-friendly_10201685

Wu, H. (1994). The Role of Open-ended problems in mathematics education. *Journal of Mathematical Behaviour, 13*, (pp.115-128).

Yackel, E., & Cobb, P. (1996). Socio-mathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education*, 458-477.

Zhu, Z. (2007). Gender differences in mathematical problem solving patterns: A review of literature. *International Education Journal, 8*(2), 187-203.

Zimmermann, B. 2010. “Open ended problem solving in mathematics instruction and some perspectives on research question” revisited – new bricks from the wall? In: *Problem Solving in Mathematics Education. Proceedings of the 11th ProMath conference in Buda-pest* (eds. A. Ambrus & E. Vasarhelyi), 143–157. Eötvös Lorand University.

Appendix A: The Test
Total 50 Marks

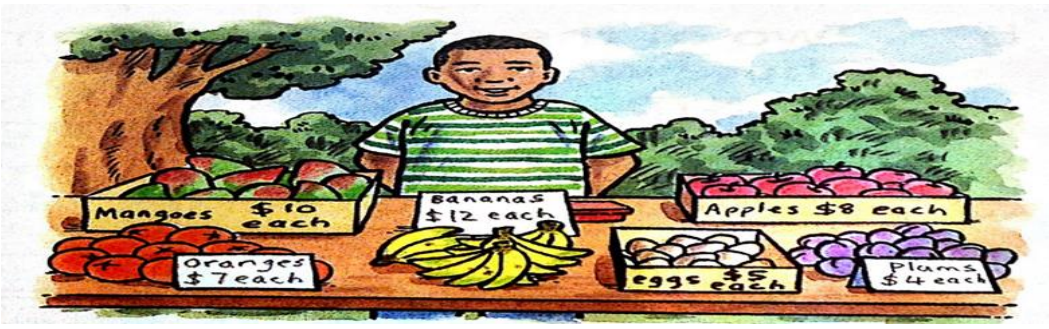
Name:_______________________________Age:_____Gender_________________School: __________________

1. Solve the following:
   a) 126 + 247 **(2Marks)** b) 582 – 367 **(2 Marks)** c) 13 x 5 **(2 Marks)** d) 246 ÷ 2 **(2 Marks)**

2a. Circle the odd numbers in the set 21, 22, 23, 24. **(2 Marks)**
2b. Write four even numbers in counting order that are less than 20. **(3 Marks)**
3. Expand 56? ____ **(2 Marks)**
3b. Tom expanded 327 as 3 x 100 + 2 x 1 + 7. Is he correct? Why/why not? **(3marks)**
4a. What is the place value of 3 in the number 238? ____________________ **(2 Marks)**
4b. Write three numbers with the digits 3, 5, 2 and 7 with 5 in the tens place. **(4 Marks)**
5a. Solve $\frac{1}{4} + \frac{2}{4}$ **(2 Marks)**
5b. Solve: How would you explain to add 1/3 and 1/4, you can use diagrams, objects or words. **(4 Marks)**
6. What two numbers you can put in the space so that the sentence is correct? __+ ___ = 25 **(4 Marks)**
7. Which number does not belong to the group 2, 8, 9 and 18? And why? Give as many solutions as necessary: **(4 Marks)**

8. Students and teachers at Bath Elementary School will go to the beach by bus. There is 110 students and teachers. Each bus holds 20 people. How many buses are needed? **(4 Marks)**

9. Write three math questions using the information in the picture below. Solve the questions you wrote. **(4 Marks)**



10. Kyle's mother gave him $700 for his lunch. For today's lunch he wants to order one box lunch, two snacks and a drink. Find three different meals that Kyle could choose. Show your calculations. **(4 Marks)**

Todays' Menu					
Box lunches		**Snacks**		**Drinks**	
Chicken	$200	Pudding	$150	Apple Juice	$130
Fish	$350	Cake	$170	Water	$70
Pork	$300	Sweets	$140	Soda	$90
Beef	$350	Ice Cream	$120	Orange Juice	$110

Appendix B: Observation Guide

Name of School *Pseudo name*____________ Time__________ Date_______________

Number of students________ # of boys________# of girls_________

Duration of lesson: _____________________**Topic/ Question:** __________________________

Focus of Observation: ____

Activities/Behaviours/expressions	Action –Reason- transition knowledge
Lesson introduced with real world connections or not.	
How did students relate to introduction?	
Concept(s) revealed in introduction	
How did the teacher stimulate students?	
How did students react?	
Significant points from introduction	
Body language/gestures during individual work.	
Teacher's action that motivated students during individual work?	
How many students showed signs of frustration?	
How students did overcame obstacles?	
What actions indicated thinking/understanding	
How does the teacher motivate slow learners?	
What actions of the teacher motivated struggling students?	
How did students benefitted from group work?	
What common strategies were seen and among which specific group of students	
What actions did students used when calculating	
What did students do to get more solutions?	
What tell that students know which concept they were working with?	
How was concept linked to experience?	
What Indicated that the students were gaining understanding?	

What actions suggest that students were reflecting		
Does the teacher use body language or facial expression to guide students?		
How did students benefitted from pair/group work?		
What were the common characteristics about students? who lead the discussions and gave explanations.		
What body language are students using to solve the problems		
How did students behave towards each other?		
Compare body language of a student who seeks help from others and those who work alone.		
Which students used the provided materials? Reasons.		
How did the teaching material help with the students understanding of the lesson?		
Students show interest by conducting exploration and investigation.		
Difference in strategies used by students		
How well does the teacher organise information on the board?		
Describe the discussion among students and between students and teacher.		
How many students could explain their solution?		
What words do students use in their explanation		
How many students commented on other's solution		
How many students change their way of doing the math.		
Describe the environment		
Fluency	Boys	Girls
Flexibility		
Originality		
Elegance		

Appendix C: Permission Letter

HIROSHIMA UNIVERSITY
Graduate School for International Development and Cooperation
1-5-1 Kagamiyama Higashi-Hiroshima, Hiroshima 739-8529 JAPAN
PHONE +81-82-424-6909 FAX +81-82-424-6904

Date

Dear Parents.

My name is Lloyd Munroe. I am a Jamaican studying at Hiroshima University in Japan. I am conducting a research study with primary school children. Name of principal has allowed me to contact you to request permission for your child to participate in the research study.

The research study is about helping children to understand mathematics better by allowing them to use different methods to solve math problems. Students can therefore choose the method they like and use it to solve the problem. The research will develop critical and creative thinking in students.

Your child will not miss any class time by participating in the research study. The results of the research will not be used for students' school grades. If a report of this study is published, or the results are presented at a professional conference, no one will know of yours child identity.

This research is important as the data collected may lead to increased understanding about how students learn mathematics. This will enable teachers to plan better lessons and teach mathematics more effectively. Therefore, some lessons will be video and audio recorded.

If you have any questions about the research, you may contact me at munroelloyd@yahoo.com

Please sign and return the attached permission slip if you are willing to have your child participate. Your support is greatly appreciated. Thank you.

Yours sincerely,

____

…………………………..…Please remove this section and return it to the school………………………………

I ____
Name of parent or guardian

Give permission for my child____
Name of child

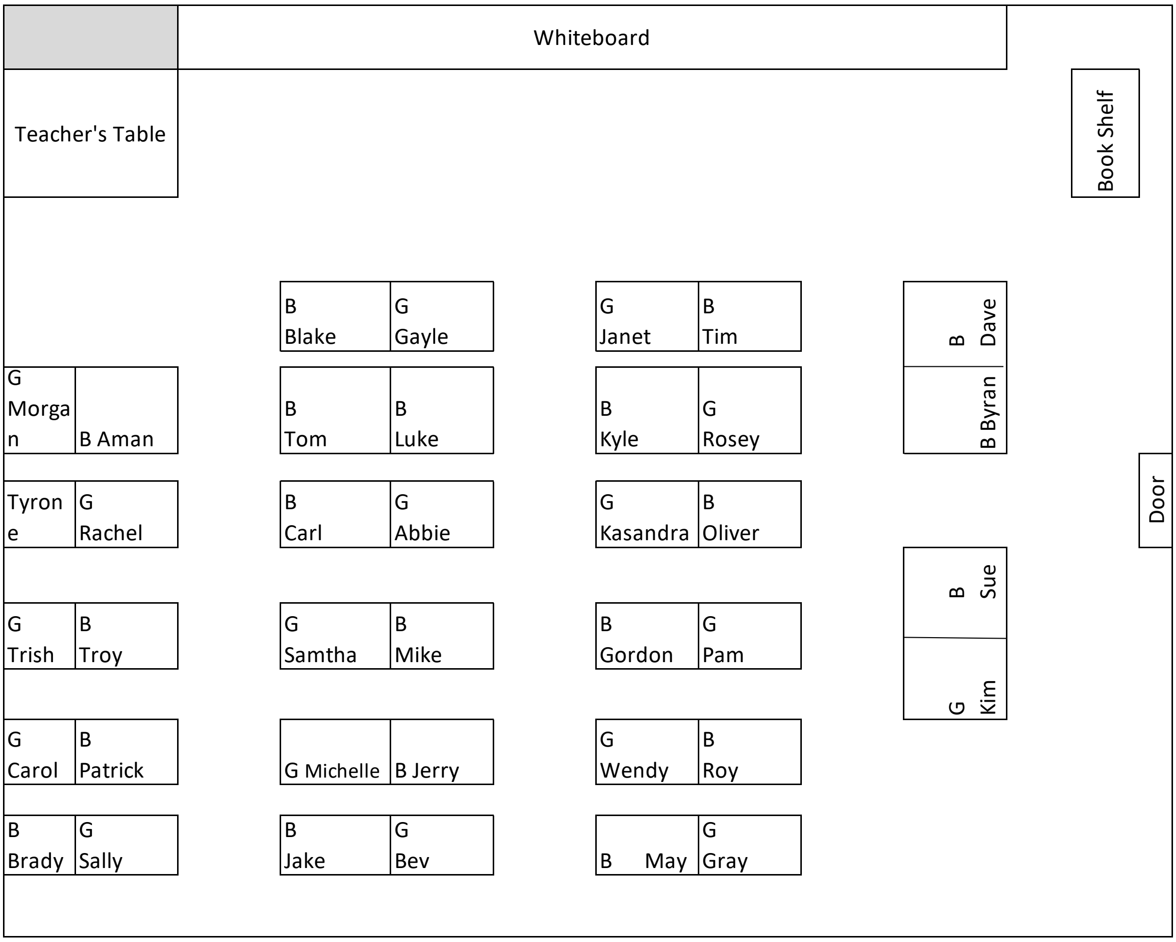
of____
Name of School

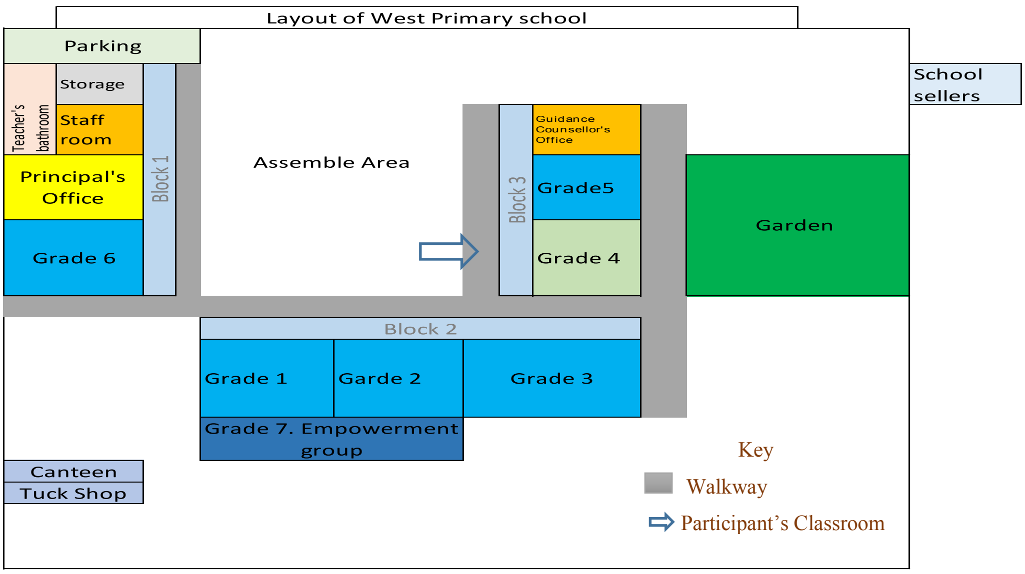
In grade __________________

To participate in the research study for the school year 2014 to 2015.

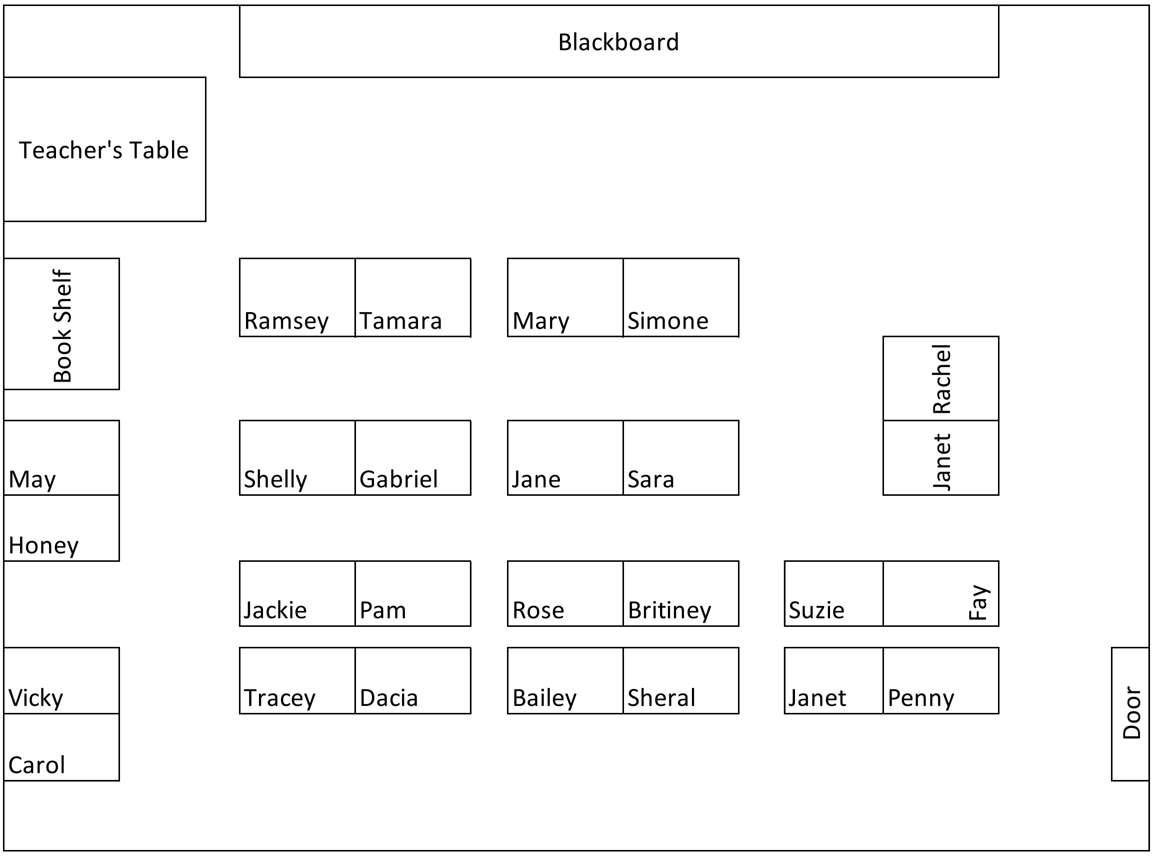
Signature of Parent or Guardian _______________________________ Date ___________________

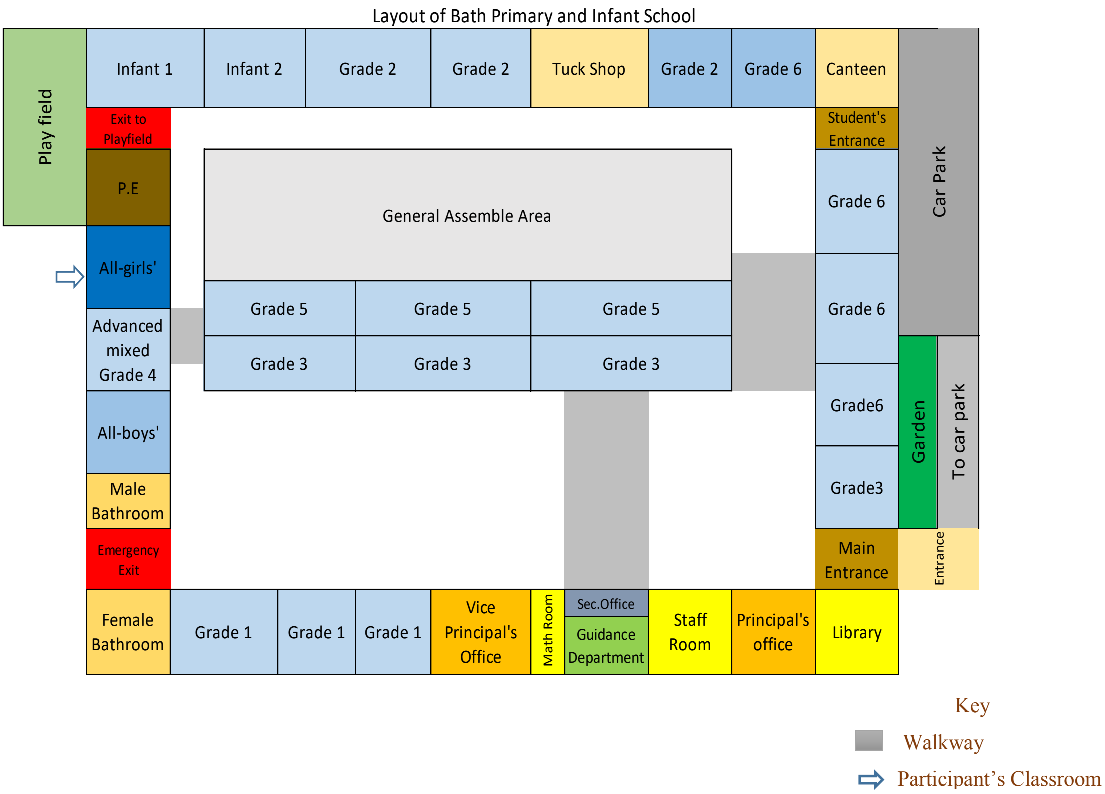
Appendix D: Layout of the Co-educational Class



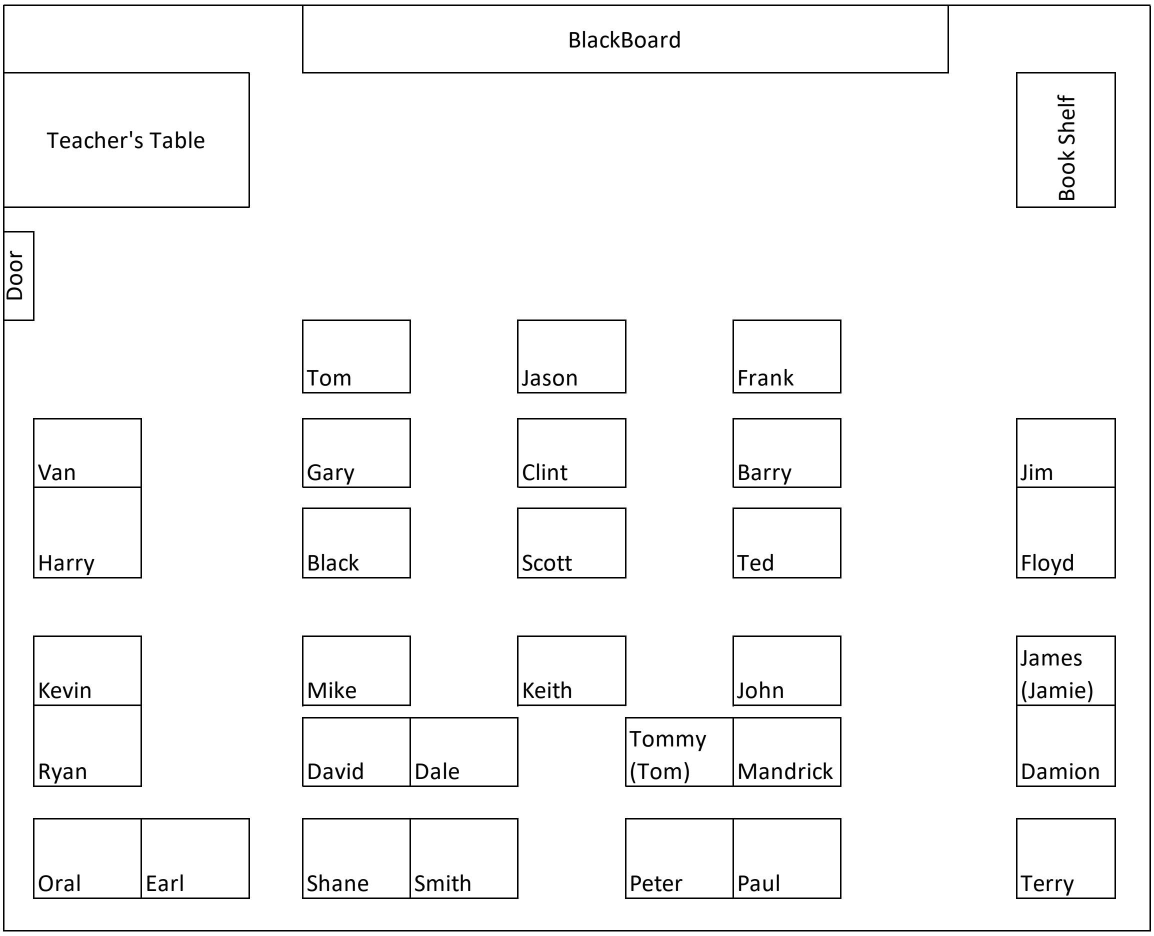


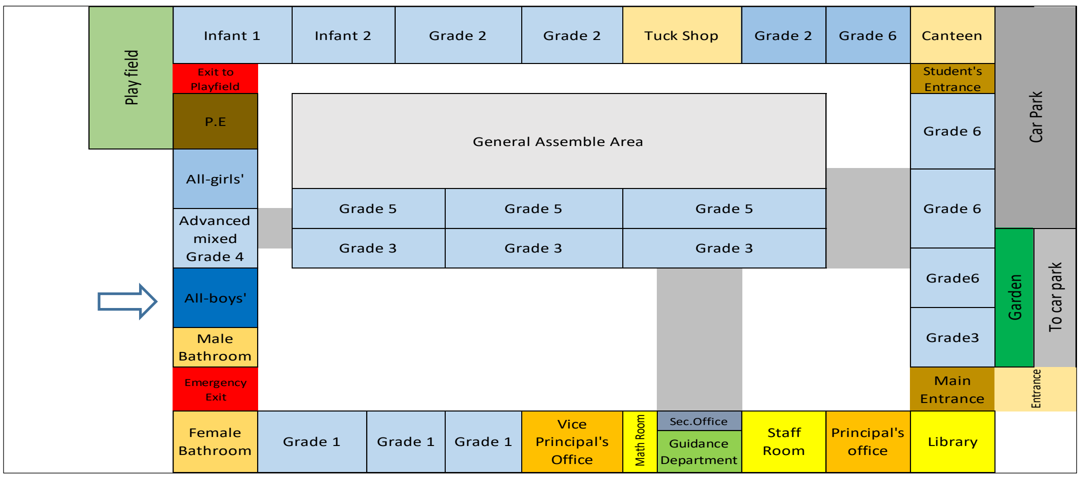
Appendix E: Layout of the All-girls' Class.





Appendix F: Layout of the All-boy's Class





Key

Walkway

Participant's Classroom

Appendix G: Factor Analysis

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	5.202	57.802	57.802	5.202	57.802	57.802
2	1.118	12.422	70.224	1.118	12.422	70.224
3	.762	8.470	78.695			
4	.585	6.497	85.192			
5	.425	4.725	89.917			
6	.295	3.273	93.190			
7	.242	2.690	95.880			
8	.193	2.143	98.023			
9	.178	1.977	100.000			

Extraction Method: Principal Component Analysis.

Correlation Matrix[a]

		Q2B	Q3B	Q4B	Q5B	Q6	Q7	Q8	Q9	Q10
Correlation	Q2B	1.000	.562	.497	.456	.703	.468	.605	.241	.483
	Q3B	.562	1.000	.666	.580	.584	.559	.562	.002	.551
	Q4B	.497	.666	1.000	.738	.586	.490	.563	-.077	.482
	Q5B	.456	.580	.738	1.000	.599	.580	.567	.016	.579
	Q6	.703	.584	.586	.599	1.000	.698	.785	.092	.613
	Q7	.468	.559	.490	.580	.698	1.000	.750	.020	.736
	Q8	.605	.562	.563	.567	.785	.750	1.000	.014	.697
	Q9	.241	.002	-.077	.016	.092	.020	.014	1.000	.081
	Q10	.483	.551	.482	.579	.613	.736	.697	.081	1.000
Sig. (1-tailed)	Q2B		.000	.000	.000	.000	.000	.000	.009	.000
	Q3B	.000		.000	.000	.000	.000	.000	.494	.000
	Q4B	.000	.000		.000	.000	.000	.000	.227	.000
	Q5B	.000	.000	.000		.000	.000	.000	.439	.000
	Q6	.000	.000	.000	.000		.000	.000	.186	.000
	Q7	.000	.000	.000	.000	.000		.000	.422	.000
	Q8	.000	.000	.000	.000	.000	.000		.446	.000
	Q9	.009	.494	.227	.439	.186	.422	.446		.215
	Q10	.000	.000	.000	.000	.000	.000	.000	.215	

a. Determinant = .003

Rotated Component Matrix[a]

	Component	
	1	2
Q2B	.785	-.027
Q3B	.794	-.022
Q4B	.238	.745
Q5B	.799	.150
Q6	.833	.215
Q7	.834	.109
Q8	.857	.067
Q9	-.080	.832
Q10	.802	.079

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 3 iterations.

Descriptive Statistics

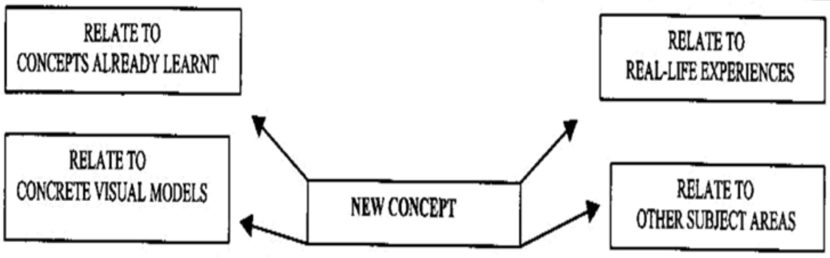
	Mean	Std. Deviation	Analysis N
Q2B	2.0206	1.33837	97
Q3B	1.7113	.85341	97
Q4B	2.0825	.81228	97
Q5B	2.3299	.90969	97
Q6	2.4845	1.11909	97
Q7	2.8247	1.13661	97
Q8	2.5155	1.18245	97
Q9	3.0412	1.22404	97
Q10	2.8247	1.02073	97

Appendix H: Grade Four Curriculum Term One: Numbers

TERMS USED IN THE CURRICULUM GUIDES

1.	THE ATTAINMENT TARGET	-	describes what pupils of different abilities and maturity levels should know and understand, and show by their behaviour what they value at the end of each level.
2.	THE OBJECTIVE	-	indicates in measurable terms, what pupils should be able to do, in relation to specific lessons or set of lessons. They are derived from the attainment targets, and reflect what is to be achieved during the particular level or stage.
3.	THE FOCUS QUESTION	-	serves to define the scope and sequence of the unit. It gives structure and focus to the unit by ensuring that the essential concepts within the topic are addressed.
4.	KEY VOCABULARY OR CONCEPTS	-	are those essential or pivotal terms introduced during the course of the unit. They will become, if they were not before, part of the pupils' active vocabulary.
5.	THE PROCEDURES/ACTIVITIES	-	present the actual experiences in which the pupils will engage in order to achieve the stated objectives.
6.	THE SKILLS	-	indicate what distinctly and specifically, the pupils will be able to do during the course of the unit. They indicate the dexterities or abilities the pupils are in the process of acquiring, and are expressed as verbs in the continuous tense.
7.	THE ASSESSMENT	-	is evidence of learning, that is, process development, conceptual insight, and knowledge. Assessment tasks result in a tangible product, an observed performance or a combination of both.
8.	THE EVALUATION	-	provides the criteria to guide the teacher in determining the level of performance by the pupils, that is, for assessing the products or performance presented.

- Numeracy IS a broad way of thinking that brings together pieces of relevant information from many places to shed light in problem situations.
- Numeracy IS an understanding of concepts within Mathematics related to one's real life experiences.
- Numeracy IS being able to communicate in a language of words and symbols concerning things you observe, investigate, conjecture and test.



Four relational contexts for aiding the understanding of new concepts

Primary level pupils must be enabled to think for themselves. This is where the teacher's understanding of his/her role is critical. When each new concept is introduced, to ensure that it is properly received, it must be related to the four different contexts as shown in the diagram. If it is not, there will be serious consequences for the understanding of the concept. Pupils will tend to isolate individual concepts and not transfer meaning easily from one area to another, within Mathematics, or across subject boundaries.

This means that:

1. Formal timetable divisions must become less rigid so that applications from Science, Language, Social Studies and the Fine Arts be incorporated into Mathematics teaching.
2. Materials must be available for teachers and pupils to use to demonstrate, discover and explore concepts and mathematical relationships.
3. The classroom must be a window into the world. The environment in the schoolyard, the local and wider community must be related to what happens in class. Mathematics is all around us. Problems which arise naturally from the environment instead of from the textbook can often provide a more stimulating focus for instruction.
4. Teachers must be acutely aware of the previous knowledge pupils have, so that they do not repeat where it is not required but they can make connections whenever possible. Faulty concepts can often be corrected naturally in this way without any formal remediation being necessary.

Teaching activities should be varied with opportunities for individual, pair, group and whole class work. Group activities offer a greater possibility of interaction, communication and informal evaluation by the teacher of how well concepts are being grasped. Well organised groups encourage talking, listening, tolerance, co-operation, and self-control skills which are critical to personal growth. Group success can be a valuable source of increased self-confidence for the individual who does not yet excel when working alone.

Any preconception on the part of teacher, parents or pupils that Mathematics is a subject only for the especially gifted must be fought at all costs. Such ideas may become self-fulfilling; the truth is that, in the twenty-first century those with little or no mathematical background will be increasingly marginalised.

New learning situations with active students, stimulating scenarios for problem-solving, opportunities for observation, discussion, analysis, summarizing, reasoning (in a relaxed setting without time constraints), testing conjectures, framing problems, and the exploration of pupils' own ideas are now seen as being more important than the traditional stress on 'the right answer'. At every possible opportunity, credit and praise need to be given for thinking and reasoning even when answers are incorrect. With the stress now being on the higher-level skills, the role of computation has changed. Laborious written calculations were once the norm both in the classroom and outside. No longer is this the case. The definition of computation must be broadened. Mental work, calculator use and deciding whether an exact figure is necessary or an estimated value will suffice, are central ideas.

Once pupils have shown themselves able to perform a certain sort of computation it is counterproductive to continue drilling them with ever more difficult numbers. Drill exercises can quickly lead to boredom. Teachers should be encouraged to stimulate pupils to observe patterns that enhance mental calculation as it is the quickest and often easiest way to calculate or at least to check calculations. In all of this, computation is a routine tool of Mathematics and the subject must be seen as something far greater, richer and nobler than mere calculations.

Caution: PLEASE do not get so involved in the Number strand, even in grade 1, to the exclusion of the rest of the curriculum, which undoubtedly includes the more lively areas of the subject. Move from strand to strand on a regular basis, or combine items from different strands into one lesson, series of lessons or unit of work.

MATHEMATICS **GRADE FOUR**

Unit Title: NUMBER **Term: ONE** **Unit: ONE** **Duration: FOUR WEEKS**

FOCUS QUESTIONS:

1. **How do I know the value of a number?**
2. **How do I apply fraction ideas to real life situations?**
3. **How can I estimate and verify my answers?**
4. **How can I apply fraction ideas to the solution of practical problems?**

ATTAINMENT TARGETS	OBJECTIVES	KEY VOCABULARY/ CONCEPTS
	At the end of this unit, pupils will:	
• Know and use the values of numerals and associate them with their names, numbers and ordinals	• identify the value of numbers to seven digits. • distinguish between value, place value and face value of a number. • investigate the base ten place value system when it is extended to show tenths and hundredths.	number value to seven digits decimal and common fractions number numeral digit value place value
• Demonstrate the understanding of fraction ideas	• apply equivalence to the addition and subtraction of fractions. • identify and use the various fractional numbers (whole number, proper, improper and mixed fractions). • name fractional numbers with denominator 10 or 100 in decimal form and vice versa. • recognize like fractions (fractions with equal denominators). • order fractions with different numerators and different denominators. • add or subtract fractional numbers with equal denominators without renaming where possible (e.g. 2 3/4 - 1 1/4; 3 1/5 + 3/5). • complete sequences of fractional numbers in decimal form counting by tenths or hundredths. • subtract a fractional number less than 1 or a mixed number from a whole number.	commutative associative difference ten/tenth hundred fraction fractional equivalent fraction numerator denominator half/halves, fourth, quarter, etc. whole number mixed number like fractions

ATTAINMENT TARGETS	OBJECTIVES	KEY VOCABULARY/ CONCEPTS
	• add or subtract 2-digit numbers mentally. • name whole numbers as fractions. • solve problems involving the addition or subtraction of like fractional numbers. • apply the four operations to problems involving decimal fractions (including money). • estimate and check answers to computations. use the properties of addition and subtraction (commutative, associative). • add or subtract fractional numbers with equal denominators when the sum is less than, equal to or greater than one.	order ordinal estimate guess approximate renaming base ten decimal sequence data information solution
• Explain the process of the basic operations, use estimation appropriately, and demonstrate proficiency with basic facts	• differentiate between the use of addition and multiplication, subtraction and division in solving problems. • identify 'the hidden question' in a 2-step problem. • write a 2-step problem from information given. • write mathematical sentences for a 2-step problem. • select data relevant to a problem when finding its solution.	

ACTIVITY PLAN

Focus Question 1. **How do I know the value of a number?**

Objectives: **Pupils will:**

- identify the value of numbers to seven digits.
- distinguish between value, place value and face value of a number.
- investigate the base ten place value system when it is extended to show tenths and hundredths.

PROCEDURES/ACTIVITIES	SKILLS	ASSESSMENT
Pupils will: 1. use manipulatives to demonstrate then read and write 5, 6, and 7-place numeral as in "Activity 4-1" in the "Primary Mathematics Teachers' Guide". Research and discuss instances in which large numbers play significant roles (e.g. figures in a budget, buying a car, census taking). For assessment use teacher-made rating scale.	• Depicting large numbers	• Research findings
2. investigate number patterns on 0-99 chart as seen in "Collections 3-6" pp. 57-66.	• Investigating patterns	
3. develop the place value concept using the idea of trading tokens of different values such as the "X-change activity" in "Activity Booklet 4-6" pp. 69-70. (Teacher will modify this activity to include tenths and hundredths).	• Naming digits	
4. explain, after investigation, how in the base ten place-value system, each succeeding digit is a tenth of the preceding digit.	• Distinguishing between values	
5. investigate the value, place value and face value of numbers (e.g. the "2" in 24 has value 20, occupies a place value 10 and has face value 2).		• Number value
6. be involved in a competition to add or subtract 2-digit numbers mentally. Explain their methods and mental processes.	• Recalling addition and subtraction facts	
Evaluation: Were pupils able to: • read/write large numbers correctly? • identify instances when large numbers are used? • distinguish between the value, place value and/or the face value of any given digit?	**Materials/Resources:** RU – "Collections 3-6" "Primary Mathematics Teachers' Guide" "Activity Booklet 4-6" Place value chart 0-99 chart coloured tokens	

ACTIVITY PLAN

Focus Question 2. **How do I apply fraction ideas to real life situations?**

Objectives: **Pupils will:**

- apply equivalence to the addition and subtraction of fractions.
- identify and use the various fractional numbers (whole number, proper, improper and mixed).
- name fractional numbers with denominator 10 or 100 in decimal form and vice versa.
- recognize like fractions (fractions with equal denominators).
- order fractions with different numerators and different denominators.
- add or subtract fractional numbers with equal denominators without renaming where possible (e.g. 2 3/4 - 1 1/4; 3 1/5 + 3/5).
- complete sequences of fractional numbers in decimal form counting by tenths or hundredths.
- subtract a fractional number less than 1 or a mixed number from a whole number.

PROCEDURES/ACTIVITIES	SKILLS	ASSESSMENT
Pupils will: 1. build fraction charts to develop the concept of fractions. A basic one is shown in "Activity Booklet 4-6" p 12. For assessment see 'rate series' in "Seeing Fractions".		• Rate (fraction) series
2. investigate relationships between fractions using Circular Fraction Kit - "Take it and Make It" p. 18.		
3. in groups, use manipulatives, record their results, explain their processes to bring out the need for a common denominator. Perform addition/subtraction on fractional numbers to include equivalent fractions, decimals, whole numbers and mixed numbers using real world situations.	• Explaining processes • Recording results	
4. play "Fill in the Blanks" game converting from simple fractions to simple fractions over ten or a hundred to decimal fractions and vice versa. Extend the activity to include mixed numbers.	• Recognizing like fractions • Investigating tenths and hundredths	• Recognition of equivalent fractions
Evaluation: Were pupils able to: • build a rate-series and explain the meaning of "a term" in the series ? • match equal representations of fractions ?	**Materials/Resources:** "Take It and Make It" RU – "Seeing Fractions" Crayons Split pins Cardboard	

ACTIVITY PLAN

Focus Question 2.

PROCEDURES/ACTIVITIES	SKILLS	ASSESSMENT
5. use paper folding to divide one fractional number by another as shown in "Activity Booklet 4-6" p 45.	• Folding paper accurately	• Problem solving using picture/diagram
6. use paper folding and shading to practise multiplying fractions as in "Activity Booklet 4-6" p 46.	• Computing fractions	
7. discuss real life situations in which rates are used (e.g. kilometres per hour; number of sweets for five dollars; $80 pay per hour) and express rates as fractions as shown in "Seeing Fractions" pp 43-49.		
8. work in groups with a set of fractions with different numerators/denominators to compare and order fractions giving their reasons and checking results with the use of calculators as in "Activity 4-5" in the "Primary Mathematics Teachers' Guide".	• Ordering fractions • Explaining processes • Recording results	
9. fill in missing values in sequences of various kinds (decimals, common fractions, ascending and descending).	• Computing fractions • Sequencing numbers	• Completed sequences
Evaluation: Were pupils able to: • represent information with a diagram/picture? • complete the number sequences accurately?	**Materials/Resources:** "Activity Booklet 4-6" RU – "Seeing Fractions" Worksheets "Primary Mathematics Teachers' Guide"	

MATHEMATICS **GRADE FOUR** **NUMBER** **TERM ONE** **UNIT ONE**

ACTIVITY PLAN

Focus Question 3. **How can I estimate and verify my answers?**

Objectives: **Pupils will:**

- add or subtract 2-digit numbers mentally.
- name whole numbers as fractions.
- solve problems involving the addition or subtraction of like fractional numbers.
- apply the four operations to problems involving decimal fractions (including money).
- estimate and check answers to computations.

PROCEDURES/ACTIVITIES	SKILLS	ASSESSMENT
Pupils will: 1. work in pairs/groups to write two-digit numbers then exchange and mentally add or subtract. Share results and methods used. e.g. 39 + 48 is approximately 40 + 50 (+1 and +2) = 90 (then take away 3) Result is 87	• Obeying rules • Co-operating with peers • Solving addition and subtraction problems • Estimating solutions	• Results of mental calculations
2. use race to a dollar/metre in "Get to the Point" pp 31-44 to consolidate the decimal fraction concept. (The teacher will modify the rules to include the four operations.) Discuss and justify which operation is best used at a particular time. Work in pairs/groups to check each other's responses to ensure correct calculations when the respective operations are used (underscoring the various ways in which a whole number may be written i.e. 3, 3.00, 3/1).	• Checking answers • Rounding off numbers • Renaming quantities	• Discussions
3. explain and solve problems involving like fractions such as those in "Caribbean Primary Mathematics" – Level 4 pp 96, 97 and 132.	• Justifying solutions	• Solutions to worksheet problems
Evaluation: Were pupils able to: • give correct answers (from mental work)? • explain their methods to others? • complete the worksheet successfully?	**Materials/Resources:** RU – "Get to the Point" "Caribbean Primary Mathematics" – Level 4 Photocopied game sheets Dice Cardboard	

ACTIVITY PLAN

Focus Question 4. **How can I apply fraction ideas to the solution of practical problems?**

Objectives: **Pupils will:**

- use the properties of addition and subtraction (commutative, associative).
- add or subtract fractional numbers with equal denominators when the sum is less than, equal to or greater than one.
- differentiate between the use of addition and multiplication, subtraction and division in solving problems.
- identify 'the hidden question' in a 2-step problem.
- write a 2-step problem from information given.
- write mathematical sentences for a 2-step problem.
- select data relevant to a problem when finding its solution.

PROCEDURES/ACTIVITIES	SKILLS	ASSESSMENT
Pupils will: 1. write individually/in pairs/in groups 2-step story problems from real life information. Exchange these problems and write mathematical statements for them.	• Composing 2-step problems	• Problems accurately written
2. develop from the 2-step problems "hidden questions".	• Determining hidden questions	• Hidden questions
3. write possible solutions to these questions and have other individuals/pairs/groups check to ensure their appropriateness (2-step, correct hidden question, correct solution). e.g. Carl had \$10 and paid \$1.74 for a pen and \$1.25 for a notebook. (2-step problem). Mathematical statement: \$10 - (\$1.74 + \$1.25) Possible questions: (a) What was his change? (b) How many more pens/books could he have bought? (c) How much more would he need to purchase 5 pens and 6 books?	• Composing mathematical statements	• Mathematical statements

Evaluation:	Materials/Resources:
Were pupils able to: • write a 2-step problem? • write a problem with a hidden question? • identify the steps and any hidden question for a problem and write a suitable mathematical statement?	Worksheets

Mathematics Attainment Targets and Objectives

GRADE 4

Review where necessary, the crucial objectives from grade 3.

NUMBER

Place value

- Distinguish between value, place value and face value of a number. (e.g. the '2' in 24 has value 20, occupies a place having place value 10 and has a face value 2)
- Read and write 5-, 6- and 7-place numerals
- Identify the value of numbers up to 7 digits

Addition and Subtraction ideas

- Use the properties of addition and subtraction (commutative and associative)
- Add or subtract 2-digit numbers mentally

Calculator

- Identify and use the keys on a pocket calculator
- Check answers using a calculator
- Investigate numbers and number patterns using a calculator

Multiplication and Division ideas

- Use the terms dividend, quotient, divisor, remainder in a division sentence correctly
- Discover, memorise and recall all multiplication and division facts up to at least 10 x 10 = 100
- Multiply a 4-digit number by a 1-digit number renaming in any of the 3 places
- Identify and correct wrong answers in problems involving multiplication and division
- Divide so that zero is the quotient
- Divide a 3- or 4-digit number so that zero is a digit in the tens or hundreds place or both in the quotient
- Test whether or not a number is divisible by 2, 3 or 4
- Mentally multiply a 2-digit number by a 1-digit number
- Multiply a number by a multiple of ten
- Multiply a 2- or 3-digit number by a 2-digit number (including money)
- Divide numbers of up to 5-digits by numbers of up to 2-digits
- Express the answers to a division problem with a remainder as a mixed number

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Problem solving

- Differentiate between the use of addition and multiplication, subtraction and division in problem situations
- Estimate and check answers to computations/problems
- Identify 'the hidden question' in a 2-step problem
- Write a 2-step problem from information given
- Write mathematical sentences for a 2-step problem
- Select data relevant to a problem when finding its solution

Fractions, including decimal form

- Apply equivalence to the addition and subtraction of fractions
- Name whole numbers as fractions
- Identify fractional numbers greate. than one
- Use the mixed form to write fractional numbers greater than one
- Recognize like fractions (fractions with equal denominators)
- Reinforce the ordering of fractions with different numerators and different denominators
- Add or subtract fractional numbers with equal denominators when the sum is less than, equal to or greater than one
- Add or subtract fractional numbers with equal denominators without renaming where possible (e.g. 2 ¾ - 1 ¼; 3 1/5 + 4 3/5)
- Solve problems involving the addition or subtraction of like fractional numbers
- Extend the base ten place value system to include tenths and hundredths
- Name fractional numbers with denominator 10 or 100 in decimal form and vice versa
- Complete sequences of fractional numbers in decimal form counting by tenths or hundredths
- Subtract a fractional number less than 1 or a mixed number from a whole number
- Apply the four operations to worded problems involving decimal fractions (including money)

MEASUREMENT

Time

- Read and write time using the hour, minute format e.g. 2:45 p.m.
- Know the relationships among units of time

Length

- Estimate, measure and record distances in metres and centimetres, in centimetres or to the nearest centimetre
- Solve problems using information on a road map
- Estimate and read distances recorded in kilometres on a road map
- Convert measurements from kilometres to metres and vice versa
- Write lengths (metres and centimetres, centimetres) in terms of a metre using decimal form

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Area
- Compare and contrast units of length and units of area
- Explain the difference between units of length and units of area
- Measure areas using unit squares

Mass
- Read a scale shown in a measurement situation using kilograms, kilograms and grams or grams
- Convert measurements from kilograms or kilograms and grams into grams and vice versa
- Know that 1 000 kg = 1 t
- Convert measurements from kilograms into tonnes and vice versa

Capacity
- Estimate and measure capacity or volume, using litres and/or millilitres
- Know that 1 000 m*l* = 1 *l*
- Convert measurements from litres or litre and millilitres into millilitres and vice versa

Temperature
- Estimate and measure temperature in degrees Celsius
- Tell the difference between two temperatures both above zero
- Tell the temperature which is a given number of degrees warmer or cooler than a given temperature

General
- Know the general meaning of the prefixes centi- and kilo-
- Identify the appropriate unit:- kilogram, gram, tonne, litre, millilitre, for a given measurement situation
- Know the relationship between the units having the prefixes centi- and kilo- and the main units gram, metre and litre

GEOMETRY
Angle ideas
- Identify and name rays and associate them with the formation of angles
- Associate the idea of a 'turn' with the formation of an angle
- Use capital/common letters to name angles/rays
- Recognize right angles when drawn or seen in the environment
- Identify angles less than, greater than and equal to a right angle
- Identify angles from different perspectives and orientations

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Polygons
- Differentiate between polygons and non-polygons
- Recognize and draw the following polygons:- triangle, square, rectangle and irregular quadrilaterals
- Identify rectangles from a set of quadrilaterals
- Draw polygons to a reasonable degree of accuracy where the length of sides is given
- Find the perimeter of polygons

Symmetry and Reflection
- Equate symmetry with reflection
- Identify the mirror line of a reflection
- Identify the mirror line as being a line of symmetry
- Know that any diameter of a circle is a line of symmetry
- Identify the possible lines of symmetry in common shapes and objects

Other ideas
- Identify, draw and/or describe parallel, perpendicular and intersecting line segments
- Identify congruent shapes and objects and say why they are congruent
- Make and explore combinations of geometric shapes

ALGEBRA
n–sentences
- Write n-sentences for problems
- Identify the correct operation (s) to be used in solving a problem
- Find replacements for variables that make number sentences true
- Demonstrate the principles of substitution in simple formulae
- Express simple sentences and word problems as algebraic expressions
- Solve word problems using algebraic expressions
- Write one or two-step problems based on information given in a story
- Then write the correct n-sentence and solve the problem

STATISTICS AND PROBABILITY
Graphs
- Present data using pictographs and bar graphs
- Convert a pictograph into a bar graph and vice versa
- Read and interpret bar, line, circle graphs and pictographs

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Mean average

- Find the mean of a set of data
- Find the total set, given the mean and the number of addends
- Solve problems based on the mean

Sample and Population ideas

- Explain the idea of 'a sample'
- Distinguish between a 'fair' (random) sample and a biased sample
- Use sampling techniques to collect information or conduct a survey
- Classify and sort collected data
- Explain the concept of 'population'
- Identify the population in any given problem situation
- Collect numeric data based on interviews and observation

Experimental probability

- List the possible expected values of an experiment
- Make predictions regarding the outcomes of experiments and record the results explaining any differences
- Distinguish between 'fair' and 'unfair' events