低エネルギー領域における陽子・陽子散乱の位相差分析とπ^0pp結合定数の精密決定

リムカイサン　ウィロート

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Phase-Shift Analysis of *pp* Scattering in Low-Energy Region and Precise Determination of $\pi^0 pp$ Coupling Constant

Viroj LIMKAISANG*)

Environmental and Material Science, Graduate School of Biosphere Sciences
Hiroshima University, Higashi-Hiroshima, 739-8521, Japan

Introduction

In the usual phase-shift analysis of nucleon-nucleon (*N-N*) scattering, which is called the modified phase-shift analysis (PSA), the peripheral part of the scattering amplitude in the outer region of the nuclear distance $r \geq 2.5$ fm is provied by the one-pion-exchange (OPE) contribution.[1] Even now, the method of evaluation of $g^2/4\pi$ differs slightly between groups.[2)-4)] The exact determination of $g^2/4\pi$ is impotant not only in nuclear physics but also in hadron physics.

In 1998, the *pp* analyzing power and spin correlation data between 200 and 450 MaV were measured by the IUCF group.[5] At $T_L = 25.68$ MeV, we have the extremely precise data of the spin-correlation obtained by the PSI group in 1994. The accumulation of *pp* scattering data at $T_L = 25\text{-}500$ MeV is very excellent.

Here we carry out the energy-independent PSA of *pp* scattering in the region $T_L = 25\text{-}500$ MeV using our proposed χ^2-mapping method in order to find both best fit solutions of the phase shifts and the peripheral amplitude with unfixed $g^2_{\pi^0 pp}/4\pi$ value, simultaneously.

The experimental data used in the analyses

The experimental data for *pp* scattering below the laboratory energy $T_L = 510$ MeV were collected from papers published between 1950 and 1998. This database consists of 1,477 $d\sigma/d\Omega$ data points, 1,326 $P(\theta)$ data points and 2,463 spin-correlation data points, which is a total of 5,266 data points.

In order to perform the single-energy PSA, we need a database that is close to "complete" at each energy. From the distribution maps, we find sufficient data at $T_L = 25, 50, 140, 210, 310, 400, 445$ and 500 MeV, which are selected as the energy points for our PSA. The observables and the correspondinng number

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*) Permanent address : Ratjamangala Institute of Technology, Nakornratchasima, Thailand.

of experimental data used in the present analysis at each energy are listed in Table I. The following criteria were adopted for the selection of the database at each energy. We take the energy bins $\Delta T_L = \pm 10$ MeV for the $d\sigma/d\Omega$ data selection and $\Delta T_L = \pm 5$ MeV for other spin-correlation data selections. The number of data points used in a single energy PSA at each energy is given in Table I.

Table I. The observables and the corresponding number of experimental data points used in the single-energy phase-shift analysis at each energy.

T_L (Mev)	25	50	140	210	310	400	445	500
Forward Obs.	4	4		2	2	4	3	3
$d\sigma/d\Omega$	23	72	47	7	14	39	43	97
P	24	39	105	58	82	118	103	189
R	8	5	14	15	27	29	22	43
A	5	10	12	14	24	29	19	48
R'			11	11				12
A'				5			2	16
D_{NN}	8	1	19	20	19	13	17	43
A_{NN}	1	3	2		57	58	59	34
A_{LL}					10	4	17	40
A_{SL}		1			40	40	54	23
A_{SS}	3	2			40	40	63	22
K_{NN}					8	16	28	24
M_{WSN}					8	16	24	24
M_{WKN}					8	16	21	21
D_{WS}							22	24
D_{WK}							24	24
Total	76	137	210	132	339	422	521	687

forward observables : σ_t, σ_Y, $\Delta\sigma_T$, $\Delta\sigma_L$.

Solution of phase shifts and $g^2_{\pi^0 pp}/4\pi$ coupling constant

The modified phase-shift analysis of NN scattering, in which the peripheral part of the amplitudes is evaluated using the one-pion exchange contribution, was proposed by Moravscik[6] and generalized to the inelastic region by Hoshizaki.[7] The scattering amplitude is represented by the partial wave amplitudes as follows:

$$M = \sum_{\ell(\ell \le \ell_0)} f_\ell(\delta_\ell, \eta_\ell) + \sum_{\ell_0 < \ell < \ell_1} f_\ell(\delta_\ell^{OPE}, \eta_\ell) + M^{OPE}(\ell \ge \ell_1). \tag{1}$$

Here, ℓ is an orbital angular momentum, δ_ℓ a phase shift, and η_ℓ a reflection parameter. The boundary angular momentum ℓ_0 corresponds to the impact parameter equal to 2.5 fm, which is the effective range of the one-pion-exchange potential. ℓ_1 is appropriately determined in the process of performing the PSA. The partial wave amplitudes with $\ell > \ell_0$ in Eq. (1) were calculated using the OPE amplitude.

We carried out the PSA by using the representation of the S matrices proposed by Matsuda and Watari[8] as follows. In the case $\ell = J$,

$$S_J = \eta_{\ell, J} \exp(2i\delta_{\ell, J}), \tag{2}$$

where $\delta_{\ell,J}$ is the nuclear-bar phase shift and $\eta_{\ell,J}$ the reflection parameter. In the case $\ell = J\pm1$,

$$S_J = \begin{bmatrix} \sqrt{1-|\rho_J|^2}\eta_-\exp(2i\delta_-) & i\rho_J\sqrt{\eta_-\eta_+}\exp[i(\delta_- + \delta_+)] \\ i\rho_J\sqrt{\eta_-\eta_+}\exp[i(\delta_- + \delta_+)] & \sqrt{1-|\rho_J|^2}\eta_+\exp(2i\delta_+) \end{bmatrix}, \quad (3)$$

where $\delta\pm \equiv \delta_{J\pm1,J}$ and $\eta\pm = \eta_{J\pm1,J}$ are the phase shifts and reflection parameters, and ρ_J is the parameter of the wave mixing. $\eta_{\ell,J} = 1.0$ in the energy region, where no inelastic channels are opened.

A best fit solution of partial wave amplitudes is obtained by varying the free parameters so as to minimize the χ^2 value:

$$\chi^2 = \sum_{i,j}\left[\frac{\theta^{th}_{i,j} - n_j\theta^{ex}_{i,j}}{n_j\Delta\theta^{ex}_{i,j}}\right]^2 + \sum_j\left[\frac{1-n_j}{\Delta n_j}\right]^2, \quad (4)$$

where $\theta^{ex}_{i,j}$ is the experimental datum for observable i from the jth experiment, with experimental error $\Delta\,\theta^{ex}_{i,j}$, and $\theta^{th}_{i,j}$ is its theoretical value. Here n_J and Δn_J are the experimental renormalization parameter and the statistical error assigned to the experimental data of the jth group. n_J is used as a free parameter only for some data regarding the differential cross-section, for which extreme differences among the data sets exist. In the χ^2-minimizing search, we determined reasonable values of the boundary angular momentum ℓ_0 and the conpling constant $g^2_{\pi^0pp}4/\pi$ (abbreviated as g^2) by our proposed χ^2-mapping method.

The value determined for $g^2_{\pi^0pp}/4\pi$ at each energy is given in Table II. The obtained solutions for phase shifts at T_L = 25, 50, 140, 210, 310, 400, 445 and 500 MeV are given in Table III.

We obtained almost unique solutions for the phase shifts at each energy below 310 MeV. Above 400 Me V, the reflection parameters $\eta_{\ell,J}$ for low partial waves must be searched with the phase shifts, owing to opening of inelastic channels. We need more data on many kinds of spin-correlation observables in this energy region in order to remove the ambiguities found in the solutions for $\delta(^3H_4)$ and ρ_4. The average value of $g^2_{\pi^0pp}/4\pi$ obtained in the present analysis is 13.52±0.23. The value obtained by the Nijmegen group[2)] is most consistent with our value. In the future, we will carry out PSA for *pp* scattering by using the χ^2 mapping method in the energy region T_L = 500-1000 MeV.

Table II. The $g^2_{\pi^0pp}/4\pi$ values determined by the χ^2 mapping method at each energy point.

T_L(Mev)	No. of δ,η	No. of data	χ^2	$g^2_{\pi^0pp}/4\pi$
25	4	76	89	13.47±0.21
50	9	137	150	13.53±0.07
140	11	210	229	13.54±0.22
210	12	132	173	13.56±0.36
310	11	339	434	13.58±0.35
400	14	422	543	13.49±0.14
445	15	521	910	13.45±0.29
500	17	687	1476	13.51±0.22
			(average)	13.52±0.23

Table III. The phase shifts obtained by the present single-energy PSA at T_L=25, 50, 140, 210,310, 400, 445 and 500 Me V. The phase shift values in the parentheses are those calculated by the one-pion exchange amplitude.

Waves	T_L=25 (MeV) χ^2/N_t=89/76	T_L=50 (MeV) χ^2/N_t=150/137	T_L=140 (MeV) χ^2/N_t=229/210
	δ	δ	δ
$^1S_0$	48.52±0.26	39.22±0.13	17.31±0.18
$^3P_0$	9.45±0.20	12.58±0.13	7.22±0.08
$^3P_1$	-4.70±0.18	-7.16±0.12	-16.84±0.20
$^3P_2$	2.36±0.12	5.66±0.08	13.61±0.17
ρ_2	(-0.014)	-0.058±0.10	-0.010±0.13
$^1D_2$	(0.56)	1.58±0.03	5.12±0.06
$^3F_2$	(0.11)	0.12±0.07	1.21±0.09
$^3F_3$	(-0.24)	-0.92±0.06	-1.95±0.08
$^3F_4$	(0.02)	0.04±0.04	0.97±0.06
ρ_4	(-0.001)	(-0.003)	-0.021±0.10
$^1G_4$	(0.04)	(0.15)	0.49±0.05
$^3H_4$	(0.01)	(0.03)	(0.19)

Waves	T_L=210 (MeV) χ^2/N_t=173/132	T_L=310 (MeV) χ^2/N_t=434/339
	δ	δ
$^1S_0$	4.68±0.52	-7.16±0.46
$^3P_0$	-2.85±0.47	-10.38±0.46
$^3P_1$	-22.73±0.31	-28.93±0.29
$^3P_2$	16.50±0.25	17.11±0.29
ρ_2	-0.088±0.28	-0.079±0.31
$^1D_2$	7.73±0.24	9.68±0.23
$^3F_2$	1.27±0.26	0.84±0.23
$^3F_3$	-2.60±0.29	-2.99±0.29
$^3F_4$	1.58±0.20	2.67±0.20
ρ_4	-0.048±0.25	-0.046±0.28
$^1G_4$	0.89±0.13	1.55±0.21
$^3H_4$	0.26±0.12	(0.55)

Waves	T_L=400 (MeV) χ^2/N_t=543/422		T_L=445 (MeV) χ^2/N_t=910/521	
	δ	η	δ	η
$^1S_0$	-13.34±0.41	1.0	-20.23±0.38	1.0
$^3P_0$	-18.05±0.29	1.0	-22.11±0.32	0.949±0.007
$^3P_1$	-33.89±0.32	1.0	-36.69±0.33	0.958±0.005
$^3P_2$	17.40±0.23	1.0	18.04±0.27	1.0
ρ_2	-0.033±0.31	0.0	-0.023±0.31	0.0
$^1D_2$	11.34±0.18	0.953±0.003	12.15±0.26	0.935±0.003
$^3F_2$	-0.22±0.23	1.0	-0.33±0.28	1.0
$^3F_3$	-2.59±0.23	0.995±0.001	-2.84±0.25	0.974±0.002
$^3F_4$	3.81±0.17	1.0	4.11±0.20	1.0
ρ_4	-0.057±0.023	0.0	-0.053±0.23	0.0
$^1G_4$	2.13±0.12	1.0	2.89±0.13	1.0
$^3H_4$	0.70±0.19	1.0	(0.79)	1.0

Waves	T_L=500 (MeV) χ^2/N_t=1476/687	
	δ	η
$^1S_0$	-24.34±0.38	1.0
$^3P_0$	-25.26±0.34	0.972±0.007
$^3P_1$	-39.48±0.30	0.961±0.003
$^3P_2$	18.79±0.22	1.0
ρ_2	-0.036±0.32	0.0
$^1D_2$	13.24±0.21	0.853±0.003
$^3F_2$	-0.63±0.21	1.0
$^3F_3$	-1.69±0.23	0.971±0.002
$^3F_4$	4.38±0.16	1.0
ρ_4	-0.053±0.26	0.0
$^1G_4$	2.78±0.15	0.999±0.001
$^3H_4$	0.52±0.18	1.0

N_t : Total number of the data.

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