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The Equations regarding Temperature and Concentration of the Density and Viscosity of Sugar, Salt and Skim Milk Solutions

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(Figs. 1–10, Tables 1–3)

INTRODUCTION

The values of the density and the viscosity of liquid foods are important for designing various apparatuses. These values vary with the temperature and the concentration of solid components and so on. If the equations of these values regarding the temperature and the concentration can be obtained, the equations will be useful for various calculation of designing and so on, using a electronic computer. In this paper, we studied the equations in function of the temperature and the concentration on the density and viscosity of sugar, salt and these mixed skim milk solutions.

THE EQUATIONS FOR DENSITY

1. Data of density

In the previous paper[1,2], we have studied the flow behavior of sugar, salt and skim milk solutions. In these experiments, the density of the fluids has been measured at various temperatures and concentrations by using a pycnometer. We use these values in this study.

The samples for these studies were prepared commercially in Japan. The sugar, salt and skim milk are used by Mitsui Seito Co., Naikai Engyo Co. and Yukijirushi Nyugyo Co., respectively. The main components of the skim milk are; protein: 35.0 wt%, lipid: 1.0 wt%, non-fibrous carbohydrate: 52.0 wt%, ash: 8.0 wt% and water: 4.0 wt%.

The sample solutions used in this study were prepared in the desired concentrations from the solid matters, under vigorous agitation by stirring for 30 minutes at 30°C. The solution obtained was degassed with a laboratory vacuum drier for 20~30 minutes at 110 mmHg and 30°C. The values of the concentration of the prepared samples were determined by measuring the weight of the degassed solutions. The density was measured in a constant temperature water bath at a set temperature.

The values of the density of water at the temperatures of 0~100°C can be obtained from the tables in handbook[3]. However, the values of the density of sugar and salt solutions can be obtained only at 22°C and at 20, 25°C respectively in handbooks[3),4)].

The data of the density ρ (g/cm^3) vs. the temperature t(°C) of the water are plotted in Fig. 1. The relationships between the density ρ and the concentration S(wt%) of the

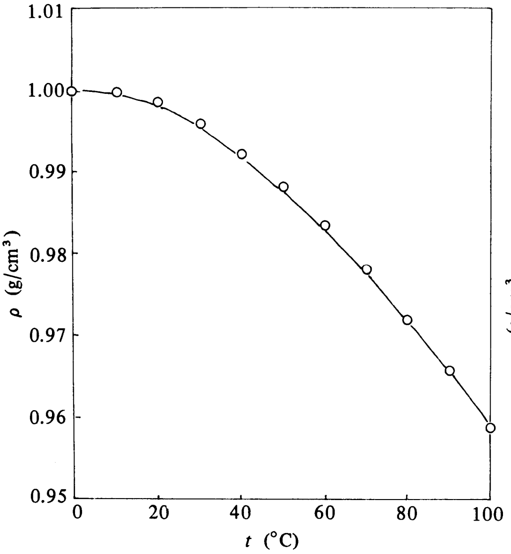

Fig. 1. Relations between the density ρ and the temperature t of water.

literature value[3] : ○

calculated value for Eq. (1) : ——

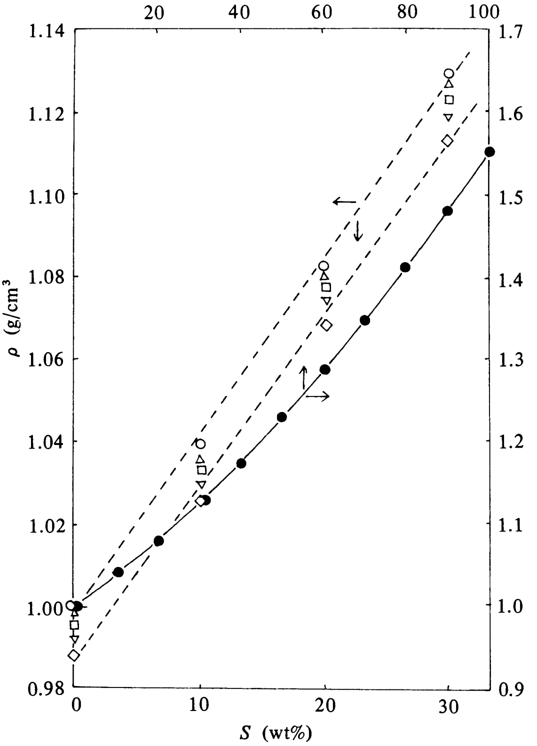

Fig. 2. Relations between the density ρ and the concentration S of sugar solutions.

observed value[2] : t(°C) 10 20 30 40 50

○ △ □ ▽ ◇

literature value[3] : t=22°C ●

calculated value :

for Eq. (1) : ——

for Eq. (10) at t=10,50 °C : -----

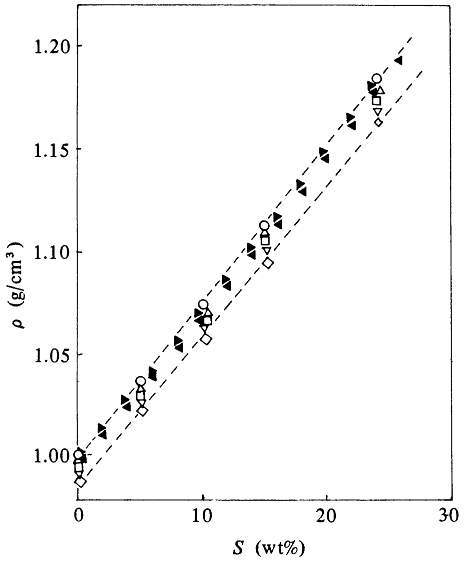

Fig. 3. Relations between the density ρ and the concentration S of salt solutions.

observed value[2] : t(°C) 10 20 30 40 50

○ △ □ ▽ ◇

literature value[3,4] : t=20,25°C

calculated value :

for Eq. (11) at t=10,50°C : ----

sugar, salt and skim milk solutions are plotted in Figs. 2~4, respectively. The relationships between the density ρ and the weight fraction x (−) of the sugar-skim milk solutions and the ones of the salt-skim milk and the salt-sugar solutions are plotted in Figs. 5~6, respectively.

From Figs. 1~4, it is seen that the values of the density do not vary remarkably with the temperature, but vary remarkably with the concentration. From Figs. 5~6, it is seen too that the values of the density of the mixed solutions can be obtained apploximately by using a linear interpolation method. Literature reference[4] shows that the linear interpolation me-

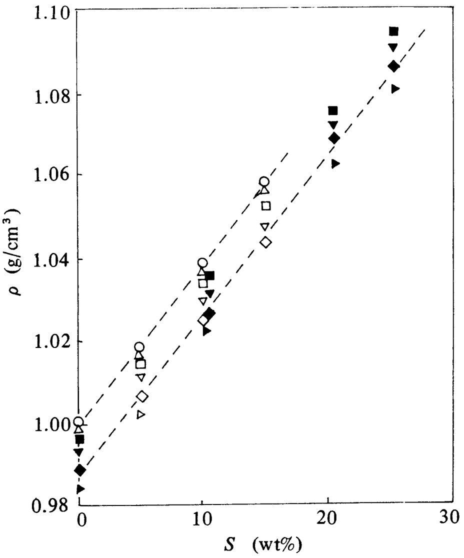

Fig. 4. Relations between the density ρ and the concentration S of skim milk solutions.

observed value$^{2)}$: t(°C) 10 20 30 40 50
○ △ □ ▽ ◇

observed value$^{1)}$: t(°C) 30 40 50 60
■ ▼ ◆ ▶

calculated value :
for Eq. (12) at t=10,50°C : ----

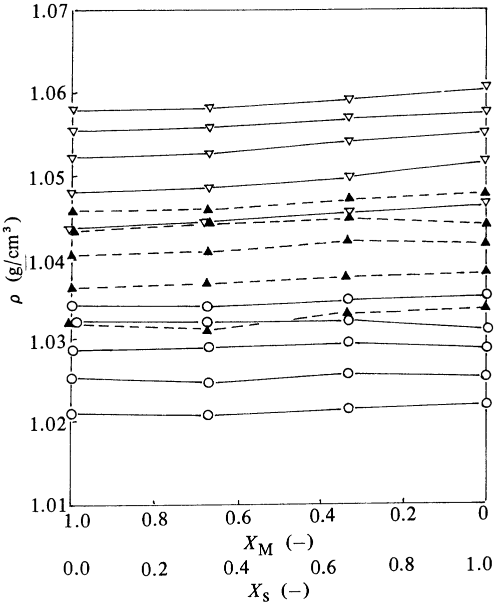

Fig. 5. Relations between the density ρ and the weight fraction X of sugar-skim milk solutions.

observed value (t=10~50°C)
S_t (wt%) : 9 12 15
–○– ---▲--- –▽–

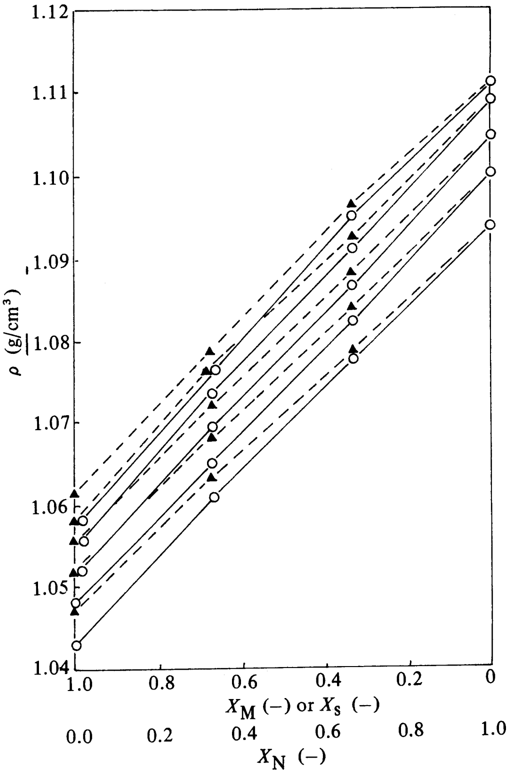

Fig. 6. Relations between the density ρ and the weight fraction X of salt-skim milk and salt-sugar solutions.

observed value (S_t=15wt%) :
salt-skim milk : –○–
salt-sugar : ---▲---

thod can be used apploximately for a concentration of up to 25 wt%, as our results show in Figs. 5~6.

2. Equations of density

The values of density of gases can be obtained usually from the equation of an ideal gas low. However, the values of liquids can not be usually obtained using simply the theoretical equation, and must be obtained from the tables or the figures. In this sence, we may make the simple equation as follows:

$$y = a + bx + cx^2 + dx^3 \tag{1}$$

$$y = a + bx + cx^2 \tag{2}$$

$$y = a + bx \tag{3}$$

$$y = ax^b + c \tag{4}$$

$$y = a(x + c)^b \tag{5}$$

$$y = 10^{ax} + c \tag{6}$$

Table 1. Calculated parameters and standard deviations of Eq. (1) for the data of density.

$\rho = a + bT + cT^2 + dT^3$ (g/cm³)

Samples (wt%)	T (°K)	a	b	c	d	$\sigma \times 10^4$
Water 3)	273.2 ~ 373.2	0.853	9.69×10^{-4}	-8.06×10^{-7}	-2.78×10^{-9}	4.00
Sugar (10)	283.2 ~ 323.2	0.804	2.80×10^{-3}	-9.94×10^{-6}	1.05×10^{-8}	3.52
Sugar (20)	283.2 ~ 323.2	0.914	5.67×10^{-4}	2.80×10^{-6}	-9.50×10^{-9}	3.66
Sugar (30)	283.2 ~ 323.2	0.995	9.82×10^{-4}	-9.60×10^{-7}	-2.94×10^{-9}	1.98
Salt (5)	283.2 ~ 323.2	0.921	1.38×10^{-3}	-4.32×10^{-6}	3.17×10^{-9}	1.74
Salt (10)	283.2 ~ 323.2	0.857	1.81×10^{-3}	-3.82×10^{-6}	3.43×10^{-9}	2.95
Salt (15)	283.2 ~ 323.2	1.25	-1.81×10^{-3}	8.75×10^{-6}	-1.43×10^{-8}	1.73
Salt (24)	283.2 ~ 323.2	1.14	7.97×10^{-4}	-2.30×10^{-6}	2.46×10^{-10}	1.77
Skim milk (5)	283.2 ~ 323.2	0.875	8.09×10^{-4}	1.16×10^{-7}	-4.20×10^{-9}	0.901
Skim milk (10)	283.2 ~ 323.2	0.819	1.84×10^{-3}	-4.01×10^{-6}	9.40×10^{-10}	1.04
Skim milk (15)	283.2 ~ 323.2	1.03	1.97×10^{-4}	5.91×10^{-7}	-3.34×10^{-9}	1.96
Skim milk (20.5)	303.2 ~ 333.2	0.635	3.32×10^{-3}	-6.67×10^{-6}	1.62×10^{-9}	0.581
Skim milk (25.3)	303.2 ~ 333.2	0.914	1.01×10^{-3}	2.48×10^{-7}	-5.38×10^{-9}	1.29

$\rho = a + bS + cS^2 + dS^3$ (g/cm³)

Samples (°C)	S (wt%)	a	b	c	d	$\sigma \times 10^4$
Sugar (22)3)	0 ~ 100	0.999	3.79×10^{-3}	1.60×10^{-5}	1.47×10^{-8}	1.78
Salt (20)3)	0 ~ 24	0.998	2.09×10^{-3}	1.16×10^{-5}	3.79×10^{-7}	0.277
Salt (25)4)	0 ~ 26	0.997	7.02×10^{-3}	1.25×10^{-5}	3.73×10^{-7}	0.261
Sugar (10)	0 ~ 30	1.00	2.67×10^{-3}	1.15×10^{-4}	-2.00×10^{-6}	0.0197
Sugar (20)	0 ~ 30	0.998	3.15×10^{-3}	7.03×10^{-5}	-1.08×10^{-6}	0.0348
Sugar (30)	0 ~ 30	0.996	3.21×10^{-3}	6.64×10^{-5}	-1.09×10^{-6}	0.139
Sugar (40)	0 ~ 30	0.992	3.15×10^{-3}	7.47×10^{-5}	-1.33×10^{-6}	0.0367
Sugar (50)	0 ~ 30	0.988	3.53×10^{-3}	2.96×10^{-5}	-2.79×10^{-7}	0.0888
Salt (10)	0 ~ 24	1.00	7.41×10^{-3}	-8.89×10^{-6}	8.34×10^{-7}	3.89
Salt (20)	0 ~ 24	0.998	6.69×10^{-3}	5.89×10^{-5}	-9.69×10^{-7}	4.52
Salt (30)	0 ~ 24	0.996	6.75×10^{-3}	4.39×10^{-5}	6.51×10^{-7}	2.38
Salt (40)	0 ~ 24	0.992	6.50×10^{-3}	6.12×10^{-5}	-1.09×10^{-6}	2.59
Salt (50)	0 ~ 24	0.988	6.76×10^{-3}	1.01×10^{-5}	4.87×10^{-7}	3.11
Skim milk (10)	0 ~ 15	1.00	3.28×10^{-3}	8.81×10^{-5}	-3.21×10^{-6}	0.0143
Skim milk (20)	0 ~ 15	0.998	3.30×10^{-3}	7.26×10^{-5}	-2.52×10^{-6}	0.0983
Skim milk (30)	0 ~ 25.3	0.996	3.56×10^{-3}	1.66×10^{-5}	-1.23×10^{-7}	2.39
Skim milk (40)	0 ~ 25.3	0.992	3.43×10^{-3}	2.73×10^{-5}	-3.58×10^{-7}	5.30
Skim milk (50)	0 ~ 25.3	0.988	3.45×10^{-3}	2.36×10^{-5}	-2.85×10^{-7}	6.53

Table 2. Standard deviations of Eqs. (2)~(8) for the data of density.

The values of $\sigma \times 10^4$ for the equations ρ vs. T

Samples (wt%)	Eq.(2)	Eq.(3)	Eq.(4)	Eq.(6)	Eq.(7)	Eq.(8)
Water[3)]	3.57	29.2	29.0	29.7	29.9	40.7
Sugar (10)	7.34	3.55	3.55	3.57	3.58	4.51
Sugar (20)	7.01	10.1	10.0	10.1	10.2	12.0
sugar (30)	2.71	6.16	6.06	6.23	6.29	8.50
Salt (5)	2.50	2.95	2.93	3.03	3.04	4.82
Salt (10)	2.92	6.11	6.06	6.20	6.22	8.24
Salt (15)	7.03	8.58	8.50	8.66	8.74	11.2
Salt (24)	1.66	2.28	2.22	1.44	2.46	5.28
Skim milk (5)	2.05	6.74	6.74	6.81	6.80	8.33
Skim milk (10)	1.50	6.13	6.12	6.23	1.22	8.07
Skim milk (15)	1.04	5.82	5.80	5.90	5.93	7.94
Skim milk (20.5)	1.36	5.52	5.48	5.62	5.68	6.94
Skim milk (25.3)	4.18	3.79	3.74	3.86	3.89	5.34

The values of $\sigma \times 10^4$ for the equations ρ vs. S.

Samples (°C)	Eq.(2)	Eq.(3)	Eq.(4)	Eq.(5)	Eq.(6)	Eq.(7)
Sugar (22)[3)]	3.64	149	32.0	145	7.99	48.4
Salt (20)[3)]	1.25	12.6	4.57	13.0	0.905	1.20
Salt (25)[4)]	1.48	15.7	5.51	15.6	1.30	1.31
Sugar (10	13.4	28.4	8.98	28.3	11.0	21.2
Sugar (20)	7.27	22.9	3.38	22.9	5.20	14.9
Sugar (30)	7.15	18.9	3.92	18.9	5.68	11.5
Sugar (40)	8.94	17.5	6.09	17.5	7.82	11.1
Sugar (50)	1.79	17.1	1.29	17.3	4.14	8.89
Salt (10)	4.71	13.9	7.17	13.8	4.13	7.80
Salt (20)	5.46	16.1	5.35	16.0	5.65	5.71
Salt (30)	3.26	13.3	3.16	13.2	37.5	4.54
Salt (40)	4.39	14.5	3.35	14.4	4.77	4.91
Salt (50)	3.51	17.5	6.69	17.5	3.64	3.86
Skim milk (10)	2.68	4.81	1.88	4.81	2.27	3.45
Skim milk (20)	2.01	4.48	1.23	4.48	1.59	3.00
Skim milk (30)	2.44	7.98	3.07	7.98	2.58	3.84
Skim milk (40)	5.51	10.4	5.85	10.4	5.58	6.90
Skim milk (50)	6.63	10.5	7.08	10.5	6.76	7.53

$$y = 10^{ax+b} \tag{7}$$

$$y = a \exp(b/x) \tag{8}$$

where, y is the density $\rho(g/cm^3)$, x is the temperature T(°K) or the concentration of solid matters S(wt%), and a, b, c and d are the parameters which can be obtained from the data. However, Eq. (8) of the Arrhenius type equation is not efficient for the relationships of ρ vs. S, because the denominator x has a value of zero. Eq.(5) can not be used for the relationships of ρ vs. T, because this equation can not express the increasing of the positive value of y with the increasing of x. The parameters a, b, c and d can be determined from the experimental results of ρ vs. T or of ρ vs. S. Eqs. (1)~(3) have been used for some solutions[5)], but not for sugar, salt and skim milk solutions. The parameters in Eqs. (1)~(3) and (8) are able to be solved by a linear least square method from the data, but the parameters in Eqs. (4)~(7) are not able to be solved.

Therefore, we calculated the parameters by a non-linear least square method using a digital electronic computer (The Computation Center of Hiroshima Univ., HITAC 8700–OS7). The subroutine program of a non-linear least square method employed herein has been descrived in detail elswhere[6)].

The following standard deviation σ was minimized.

$$\sigma = \left[\sum_{i=1}^{N} (y_{obs} - y_{cal})_i^2 W_i / N\right]^{1/2} \tag{9}$$

where, y_{obs} and y_{cal} are the observed and the calculated values of y, N is the number of experimental points and W_i is the weighing coefficient.

The values of the parameters in Eqs.(1)~(8) can be obtained from the data in Figs. 1~4. The values of the standard deviation for Eq. (1) were the smallest in all the results for Eq. (1)~(8). The values of the parameters in Eq.(1) and the standard deviation are listed in Table 1. Excellent equations could not be obtained for the relationships between the parameters and the concentrations or the temperatures in Table 1. Excellent relationships were obtained for the two parameters equations of Eqs. (3), (7) and (8). The values of standard deviation for Eqs. (2)~(8) are listed in Table 2.

In Table 2, the values of the standard deviation for the two parameters equations are not much larger than the values of the other equations. For our narrow experimental region, the simplest equation of Eq. (3) is most usefull. The results show as follows:

For sugar solution (T=283.2~323.2°K, S=0~30 wt%):

$$\rho = -3.49 \times 10^{-4} T + 4.26 \times 10^{-3} S + 1.098 \tag{10}$$

$$\sigma = 2.11 \times 10^{-3}$$

For salt solution (T=283.2~323.2°K, S=0~24 wt%):

$$\rho = -4.11 \times 10^{-4} T + 7.46 \times 10^{-3} S + 1.118 \tag{11}$$

$$\sigma = 1.99 \times 10^{-3}$$

For skim milk solution (T=283.2~323.2°K, S=0~25.3 wt%):

$$\rho = -3.39 \times 10^{-4} T + 3.88 \times 10^{-3} S + 1.096 \tag{12}$$

$$\sigma = 1.39 \times 10^{-3}$$

The following equation can be tried out, by using the non-linear least square method.

$$\rho = aT^n + bS^m + c \tag{13}$$

where, n and m are the added parameters. The results used Eq.(13) are shown as follow:

For sugar solution:

$$\rho = -3.48 \times 10^{-4} T^{1.00} + 3.11 \times 10^{-3} S^{1.09} + 1.100 \tag{14}$$

$\sigma = 9.72 \times 10^{-4}$

For salt solution:

$$\rho = -4.21 \times 10^{-4} T^{0.997} + 6.36 \times 10^{-3} S^{1.05} + 1.120 \tag{15}$$

$\sigma = 1.37 \times 10^{-3}$

For skim milk solution:

$$\rho = -3.27 \times 10^{-4} T^{1.01} + 3.40 \times 10^{-3} S^{1.04} + 1.099 \tag{16}$$

$\sigma = 1.04 \times 10^{-3}$

where, the conditions are similar to those of Eqs. (10)~(13). The values of the standard deviations of these equations are not much smaller than the values of the former equations of Eqs. (10)~(12), these are same to the type of Eq. (13) setted n=1.0 and m= 1.0. Therefore, the former equations of Eqs. (10)~(12) are approximately useful. The broken lines in Figs. 2~4 are respectively the calculated results of t=10 and 50°C for Eqs. (10)~(12). The solid corves in Figs. 1 and 2 are the calculated results for Eq. (1).

If the experimental regions of temperature and concentration are expanded, we can not use anymore Eqs. (10)~(12). We must use more complicated equations such as Eq. (1) show, with the solid curves in Figs. 1 and 2.

The relationships between the density and the concentration, temperature and weight fraction of the mixed solutions as shown in Figs. 5 and 6 are very complicated. Therefore, we considered that the density of the mixed solutions should be obtained from the following equation.

$$y = k_A X_A + k_B X_B + \delta_{AB} X_A X_B \tag{17}$$

where, y is the density ρ (g/cm^3) and X_A and X_B are the weight fractions of the solid matters A and B in the solutions, respectively. k_A and k_B (g/cm^3) are the values of the oensity of the solutions of solid matters A and B for the concentration of $S_t = S_A + S_B$ (wt%), respectively. The values of parameter δ_{AB} can be obtained from the relationships of ρ vs. X_A and X_B. The relationships of Figs. 5 and 6 make the linear approximation, then, we can give $\delta_{AB}=0$ in our experimental region.

THE EQUATIONS FOR VISCOSITY

1. Data of viscosity

In the previous papers[1,2], we have studied the flow behavior of sugar, salt and skim milk solutions. In these experiments, the viscosity of the fluids has been measured at various temperatures and concentrations by using a capillary tube viscometer. We use these values in this study.

The values of the viscosity of water at the temperatures of 0~100°C can be obtained from the tables in handbook and so on[3,7]. The values of the viscosity of sugar solution can be obtained at 20, 40 and 60 wt% in handbook and so on[3,7]. but the values of under 20 wt% can not be obtained. In the previous paper[2], we measured the viscosity of 10, 20 and 30 wt% sugar solutions at 10~50°C. The values of the salt solution can be obtained from 0 to 80°C for concentrations under 20 wt% in handbook[4)], but the values of upper 20 wt% can not be obtained at the temperature region. In the previous paper[2)], we measured the viscosity of 5, 10, 15 and 24 wt% salt solutions at 10~50°C, and in the previous papers[1,2)], we measured the viscosity of 5.0~25.3 wt% skim milk solutions at 10 or 39~50 or 60°C.

The data of the viscosity μ (g/cm·sec) vs. the temperature t(°C) of the water are plotted in Fig.7. The relationships between the viscosity μ and the concentration S (wt%) of sugar, salt and skim milk solutions are plotted in Figs. 8~10, respectively. In Figs. 8~10, our observed values are shown with the literature values.

The relationships between the viscosity μ and the weight fraction $X(-)$ of sugar-skim milk, salt-skim milk and salt-sugar solutions have been plotted in the previous paper[2)]. As these results are not used in this study, we omit their figures.

2. Equations of viscosity

The relationships between the viscosity and the temperature have been expressed using the Andrade or Arrhenius type equation as used in the previous papers[1,2)]. This equation is the same as Eq. (8). In Eq. (8), y is the viscosity μ (g/cm·sec) and x is the temperature T(°K). However, Eq. (8) of the Andrade type equation has not a high accuracy on the relationships of μ vs. T in water. Literature reference[8)] shows that the following

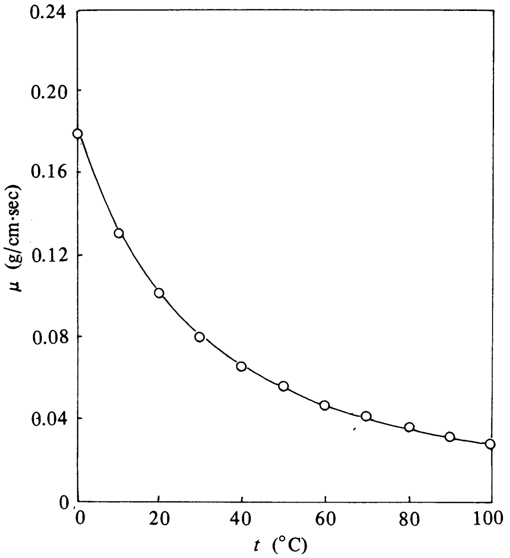

Fig. 7. Relations between the viscosity μ and the temperature t of water.

literature value[3,7)] : ○

calculated value for Eq. (18) : ——

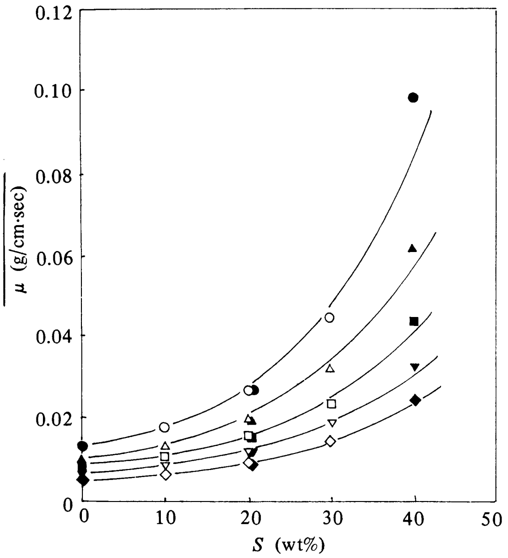

Fig. 8. Relations between the viscosity μ and the concentration S of sugar solutions.

observed value[2)] : t(°C) 10 20 30 40 50
○ △ □ ▽ ◇

literature value[3,7)] : t(°C) 10 20 30 40 50
● ▲ ■ ▼ ◆

calculated value for Eq. (18) : ——

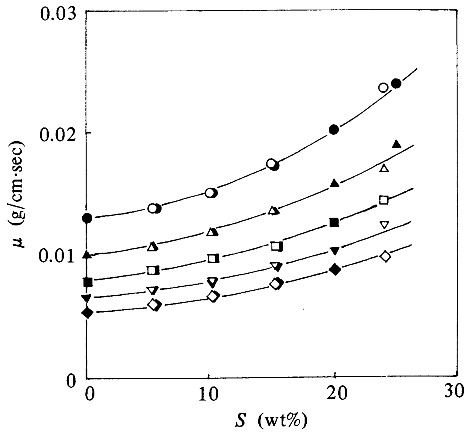

Fig. 9. Relations between the viscosity μ and the concentration S of salt solutions.

observed value2) : t(°C) 10 20 30 40 50
○ △ □ ▽ ◇
literature value4) : t(°C) 10 20 30 40 50
● ▲ ■ ▼ ◆
calculated value for Eq. (18) : ——

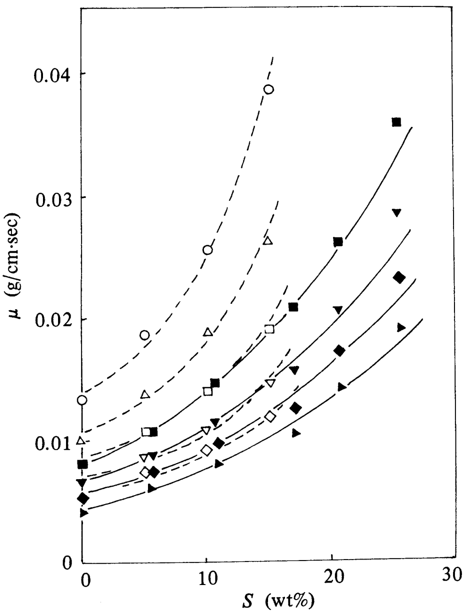

Fig. 10. Relations between the viscosity μ and the concentration S of skim milk solutions.

observed value2) : t(°C) 10 20 30 40 50
○ △ □ ▽ ◇
observed value1) : t(°C) 30 40 50 60
■ ▼ ◆ ▶
calculated value for Eq. (18) : —— - - -

equations must be used in water for the wide region of temperature.

$$\mu = a \exp(b/T^n) \qquad (18)$$

where, a, b and n are the parameters.

The values of the parameters in Eq. (18) can be obtained from the literature data and from our data in Figs. 7~10. The values of the parameters which fixed n=3 and the standard deviations which fixed n=3, 2 and 1 in Eq. (18) are listed in Table 3.

The parameters which fixed n=1, 2 and 3 in Eq. (18) are able to be obtained by a linear least square method. As the program used non-linear least square method have been made, then, we used this program.

For the wide region data, the values of the standard deviation which fixed n=3 are the smallest. However, for the narrow experimental region, these results can not be obtained cleary. The Andrade equation which fixed n=1 in Eq. (18) is apploximately useful. The equations regarding the temperature and the concentration of the viscosity which fixed n=1 in Eq. (18) were shown in the previous paper2).

The results which fixed n=3 in Eq. (18) by used the data in Figs. 8~10 are shown as follow:

Table 3. Calculated parameters which fixed n=3.0 and standard deviations which fixed n=3.0. 2.0, and 1.0 in Eq. (18).

$\mu = a \exp(b/T^n)$

Samples (wt%)	t (°C)	n=3.0		$\sigma \times 10^2$		
		a	b	n=3.0	n=2.0	n=1.0
Watar3,7)	0 ~100	8.95×10^{-4}	6.10×10^{7}	0.378	1.56	3.08
Sugar3,7) (20)	0 ~85	1.20×10^{-3}	7.04×10^{7}	0.528	3.11	6.01
Sugar3,7) (40)	0 ~100	1.65×10^{-3}	9.17×10^{7}	10.7	15.5	25.1
Sugar3,7) (60)	0 ~100	4.68×10^{-4}	1.50×10^{8}	168	268	2310
Salt4) (5)	0 ~80	1.06×10^{-3}	5.84×10^{7}	0.376	1.48	2.93
Salt4) (10)	0 ~80	1.24×10^{-3}	5.68×10^{7}	0.489	1.61	3.17
Salt4) (15)	0 ~80	1.38×10^{-3}	5.71×10^{7}	0.690	1.69	3.42
Salt4) (20)	0 ~80	1.54×10^{-3}	5.82×10^{7}	0.662	1.81	3.88
Sugar2) (10)	10 ~50	1.03×10^{-3}	6.47×10^{7}	0.364	0.838	1.50
Sugar2) (20)	10 ~50	1.32×10^{-4}	6.81×10^{7}	2.87	1.87	0.993
Sugar2) (30)	10 ~50	1.54×10^{-4}	7.64×10^{7}	2.29	0.602	1.54
Salt2) (5)	10 ~50	1.13×10^{-3}	5.66×10^{7}	1.44	1.69	2.02
Salt2) (10)	10 ~50	1.30×10^{-3}	5.57×10^{7}	0.688	0.239	0.484
Salt2) (15)	10 ~50	1.35×10^{-3}	5.80×10^{7}	0.462	0.686	1.24
Salt2) (24)	10 ~50	1.82×10^{-3}	5.78×10^{7}	2.05	2.27	2.74
Skim milk2) (5)	10 ~50	1.04×10^{-3}	6.50×10^{7}	1.49	2.13	2.82
Skim milk2) (10)	10 ~50	1.01×10^{-3}	7.33×10^{7}	1.09	1.77	2.70
Skim milk2) (15)	10 ~50	9.41×10^{-4}	8.40×10^{7}	2.63	4.22	17.5
Skim milk1) (5)	30 ~60	1.50×10^{-3}	5.41×10^{7}	1.42	4.05	1.82
Skim milk1) (10.4)	30 ~60	1.39×10^{-3}	6.55×10^{7}	2.35	2.57	2.82
Skim milk1) (17.0)	30 ~60	1.24×10^{-3}	7.81×10^{7}	1.36	1.74	2.18
Skim milk1) (20.5)	30 ~60	2.32×10^{-3}	6.71×10^{7}	2.31	2.25	2.34
Skim milk1) (25.3)	30 ~60	2.76×10^{-3}	7.14×10^{7}	0.838	0.0780	0.849

For sugar solution 2,3,7) (T=283.2~323.2°K, S=0~40 wt%):

$$a = 10^{4.58\times10^{-3}s^{1.15}-3.05}$$

$$b = 9.90 \times 10^{4} S^{1.51} + 6.10 \times 10^{7}$$

For salt solution2,4) (T=283.2~323.2°K, S=0~24 wt%):

$$a = 10^{3.59\times10^{-3}s^{1.33}-2.00}$$

$$b = 5.79 \times 10^{7}$$

For skim milk solution2) (T=283.2~323.2°K, S=0~15 wt%):

$$a = 9.77 \times 10^{-4}$$

$$b = 3.09 \times 10^{5} S^{1.59} + 6.10 \times 10^{7}$$

For skim milk solution1) (T=303.2~333.2°K, S=0~25.3 wt%):

$$a = 10^{2.83\times10^{-3}s^{0.94}-3.12}$$

$$b = 6.62 \times 10^{7}$$

The curves in Figs. 8~10 are the calculated results used above parameters, respectively. The curves in Figs. 7 are the calculated results which fixed n=3 in Eq. (18).

In Fig. 8, the calculated results for the higher concentrations do not agree well with the observed values. The values of the viscosity on the higher concentrations of sugar solutions are very complicated as shown on the higher standard deviations in Table 3. For the sugar solutions, the calculated results can be obtained well only under 30 wt%.

In Fig. 10, the solid curves do not agree well with the broken curves. The values of the viscosity on the higher concentrations of skim milk solutions are very complicated too. The calculated results do not agree with the observed values for the lower concentrations too. For skim milk solutions, the previous equations[1,2)] which fixed n=1 in Eq. (18) are more useful.

The relationships between the viscosity and the weight fraction of the mixed solutions can be expressed using the following equation which we obtained in the previous paper[2)].

For sugar, salt and skim milk mixed solutions (T=283.2~323.2°K, $S_t=S_S+S_N+S_M$ = 0~15 wt%) :

$$\mu=(k_S X_S+k_N X_N+k_M X_M)(1-3.4\times10^{-3}X_S X_M-8.4\times10^{-3}X_N X_M -4.2\times10^{-3}X_S X_N+1.67\times10^{-2}X_S X_N X_M)+2.7\times10^{-3}X_S X_M+7.1 \times10^{-3}X_N X_M+3.8\times10^{-3}X_S X_N-1.26\times10^{-2}X_S X_N X_M$$

where, k_S, k_N and k_M are the values of the viscosity of the solutions of solid matters S, N and M (sugar, salt and skim milk) for the concentration of $S_t=S_S+S_N+S_M$ (wt%), respectively.

SUMMARY

The values of the density and the viscosity are important for designing and controlling various apparatuses. In this paper, we studied the equations regarding the temperature and the concentration of the density and the viscosity of sugar, salt and skim milk solutions.

For the density, the following equations were approximately obtained.

$$\rho=aT+bS+c \qquad (g/cm^3)$$

For sugar solution (T=283.2~323.2°K, S=0~30 wt%):

$$a=-3.49\times10^{-4},\quad b=4.26\times10^{-3},\quad c=1.098$$

For salt solution (T=283.2~323.2°K, S=0~24 wt%):

$$a=-4.11\times10^{-4},\quad b=7.46\times10^{-3},\quad c=1.118$$

For skim milk solution (T=283.2~323.2°K, S=0~25.3wt%):

$$a=-3.39\times10^{-4},\quad b=3.88\times10^{-3},\quad c=1.096$$

For the mixed solutions:

$$\rho = k_S X_S + k_N X_N + k_M X_M$$

where, $X(-)$ is weight fractions of the solid matters in the solutions and k_S, k_N and k_M (g/cm³) are the values of the density of the solutions of solid matters S, N and M (sugar, salt and skim milk) for the concentration of $S_t = S_S + S_N + S_M$ (wt%), respectively.

For the viscosity, the following equations were approximately obtained at T=283.2~323.2 °K.

$$\mu = a \exp(b/T^n) \qquad (\mathrm{g/cm \cdot sec})$$

For sugar solution (S=0~30 wt%):

$$a = 10^{4.58\times10^{-3} S^{1.15} - 3.05}$$

$$b = 9.90 \times 10^4 S^{1.51} + 6.10 \times 10^7$$

$$n = 3$$

For salt solution (S=0~24 wt%):

$$a = 10^{3.59\times10^{-3} S^{1.33} - 2.00}$$

$$b = 5.79 \times 10^7$$

$$n = 3$$

For skim milk solution (S=0~15 wt%):

The previous results1,2) used Andrade equation are more useful.

For the mixed solutions (S_t=0~15 wt%):

The previous results2) can be used.

NOTATIONS

a, b, c, n and m : constants in equations
N : number of experimental points (−)
S : concentration of fluids (wt%)
T and t : temperature of fluids (°K) and (°C)
X : weight fraction of solid matters in fluids (−)
x : independent valiable of T or S
y : dependent valiable of ρ or μ
μ : viscosity of fluids (g/cm·sec)
ρ : density of fluids (g/cm³)
σ : standard deviation (g/cm³) or (g/cm·sec)

Subscripts;

t : total value
M, N, and S : skim milk, salt and sugar

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砂糖，食塩および脱脂粉乳水溶液の密度および粘度の温度，濃度関係式

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液状食品に関する各種装置の設計ならびに操作に関する研究を行なっていくためには，液状食品の密度，粘度などの物性値を温度，濃度などの条件関係式として表わしておくことが必要である。本研究では，砂糖，食塩，脱脂粉乳およびこれらの混合水溶液についての密度，粘度の測定値を例として，関係式の設定について検討した。

密度関係式は，条係範囲が広い場合には，温度，濃度についてそれぞれ2～3次多項式として表わす必要があるが，条件範囲が狭い場合には，次式を近似式として利用できる結果が得られた。

$$\rho = aT + bS + c$$

ここで，ρ（g／cm³）:密度，T（°K）:温度，S（wt％）：濃度である。混合水溶液に対しては，重量混合率を用いた加成性が得られる結果となった。

粘度関係式は，次式を近似式として利用できる結果が得られた。

$$\mu = a \exp(b/T^n)$$

ここで，μ（g／cm³・sec）：粘度である。水に近いもので，条件範囲が広いものでは，$n=3$として表わす必要があるが，条件範囲が広くない場合には，$n=1$としたAndradeの式が利用できる結果が得られた。aとbは，濃度範囲が広くない範囲で，次式が適用できる結果になった。

$$a = 10^{\alpha S^n + \beta}$$

$$b = \alpha' S^m + \beta'$$