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Studies of Cooking-rate Equations based on Water-soaking-shell Models

Kiyoshi KUBOTA, Kanichi SUZUKI, Hideaki HOSAKA,
Kazunori HIRONAKA and Masatoshi AKI

Department of Food Chemistry and Technology, Faculty of Fisheries and Animal Husbandry, Hiroshima University, Fukuyama

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(Figs. 1~5; Tables 1~4; Appendix)

INTRODUCTION

In previous papers[1),2)], we have studied the drying-rate equations based on the drying-shell models, and we determined the rate parameters in the rate equations of water-absorbing sponges. In this paper, we took up the water-soaking of foods by cooking, and we studied the cooking-rate equations.

In order to design and to control automatically various cooking apparatuses, it is necessary to determine the cooking-rate equations as approximating equations based on simple cooking models, and to simulate the rate parameters in the rate equations from the experimental data. In this paper, we postulated the cooking-rate equations based on the water-soaking-shell models with consideration of the expanding surface for a sphere, for a long-cylinder and for a infinite-slab, then we induced the integrated equations of the cooking-rate equations. Concequently the rate parameters in the cooking-rate equations were determined for the cooking of rice and so on, using a non-linear least square method[3),4)].

COOKING-RATE EQUATIONS

Cooking mechanisms of foods are generally complicated, the mechanisms of some water-soaking-foods such as rice and so on however are simple. We can consider simple shell models which have two simple idealized zones, the soaking-shell and the unsoaking-core. The shell models for catalytic reactions and so on have been reported[5)~9)] for the sphere, but for the cooking of foods which must be cosidered the expanding surface for the long-cylinder and infinite-slab too, they have not yet been reported on. We postulate the cooking-rate equations on the basis of the water-soaking-shell models, and we induce the integrated equations.

1. Cooking-rate equations for sphere

Fig. 1 illustrates the case of a spherical cooking material. The radius, the volume and the weight of the spherical material in the intermediate state, are symbolized by R(cm), V(cm^3) and w(g), and the radius of the unsoaking-core is represented by r_c(cm).

The subscripts o, e and d in these symbols signify the initial, equilibrium and completely drying states, respectively.

Thus, we may define the cooking-ratio $X_w(-)$ of material by the following equation.

$$X_w = (w - w_o)/(w_e - w_o) \quad (1)$$

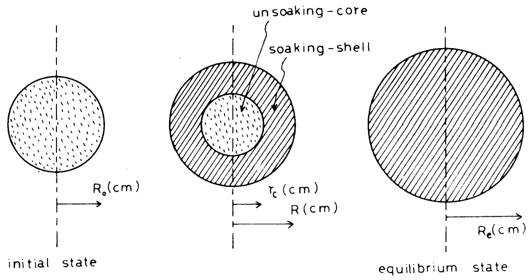

Fig. 1. Water-soaking-shell model for sphere.

In the intermediate state, if the water-soaking-shell and unsoaking-core states are the same the equilibrium and initial states, respectively, the relations of the radius of the unsoaking-core r_c, the cooking-ratio X_w and the radius of spherical material R become as follows:

$$r_c = (1 - X_w)^{1/3} R_o \quad (2)$$

$$R = (R_e^3 - ((R_e/R_o)^3 - 1) r_c^3)^{1/3} \quad (3)$$

The material balance equation of cooking rate is obtained, for the decreasing of a very small radius of unsoaking-core dr_c(cm), in a very short time internal $d\theta$(min), where $\mathcal{R}$(g-H_2O/min) is the cooking rate of the material.

$$dr_c/d\theta = -\mathcal{R}/(\pi r_c^2 \rho_h) \quad (4)$$

$$\text{where, } \rho_h = (w_e - w_o)/V_o \ (\text{g-}H_2O/\text{cm}^3) \quad (5)$$

For the diffusion controllings, when the water concentrations of the unsoaking-core and outer and inner parts of soaking-shell are c_o, c_e and c_i (g-H_2O/cm^3), respectively, the cooking-rate $\mathcal{R}$ can be expressed as follows:

For shell diffusion controlling:

$$\mathcal{R} = (4\pi R r_c/(R - r_c)) k_m (c_e - c_i) \quad (6)$$

where, k_m: rate parameter of shell diffusion (cm^2/min)

For reaction-rate controlling:

$$\mathcal{R} = 4\pi r_c^2 k_r (c_i - c_o) \quad (7)$$

where, k_r: rate parameter of reaction-rate (cm/min)

The reaction-rate of cooking in Eq. (7) is the rate of chemical and physical changes of material such as a α-conversion of starch and so on.

The cooking-rate equation for the diffusion controlling is obtained by combining Eqs. (6) and (7), and eliminating c_i.

$$\mathcal{R} = 4\pi R^2 (c_e - c_o)/(((R - r_c)/((r_c/R) k_m)) + (1/((r_c/R)^2 k_r))) \quad (8)$$

If the material is enclosed with water only, we do not need to consider a liquid-film diffusion, and then we can use Eq. (8). However, if the material is not enclosed with pure liquid, the liquid-film diffusion must be considered such as the gas-film diffusion in a drying operation[1),2)]. For extraction operations and so on, we must consider the liquid-film diffusion.

The cooking-rate equation for heat-transfer controlling is obtained in the same way as in the previous papers[1),2)]. The rates of the heat-transfers in the liquid-film and water-soaking-shell for a cooking operation are much larger than the rates in the gas-film and drying-shell for a drying operation, then we do not usually need to consider the heat-transfers. However, when the size of the material is much larger, the heat of reaction is much larger, and the temperature of the water is much higher than the temperature of the material, so we must consider the heat-transfer too.

2. Cooking-rate equations for long-cylinder

For a long-cylinderical material, the length, the radius and the radius of the unsoaking-core are given as L(cm), R(cm) and r_c(cm), respectively, hence the length is very much longer than the radius.

Thus, analogically to the sphere, the relations of the radius of the unsoaking-core r_c, the cooking-ratio X_w and the radius of the material R become as follows:

$$r_c = (1 - X_w)^{1/2} R_o \tag{9}$$

$$R = (R_e{}^2 - ((R_e/R_o)^2 - 1) r_c{}^2)^{1/2} \tag{10}$$

The material balance equation and the cooking-rate equations can be expressed as follows:

$$dr_c/d\theta = -\mathcal{R}/(2\pi r_c L \rho_h) \tag{11}$$

For shell diffusion controlling:

$$\mathcal{R} = (2\pi L/\ln(R/r_c)) k_m (c_e - c_i) \tag{12}$$

For reaction-rate controlling:

$$\mathcal{R} = 2\pi r_c L k_r (c_i - c_o) \tag{13}$$

The cooking-rate equation for diffusion controlling is obtained by combining Eqs. (12) and (13).

$$\mathcal{R} = 2\pi R L (c_e - c_o)/((1/((r_c/R)k_r)) + (R\ln(R/r_c)/k_m)) \tag{14}$$

3. Cooking-rate equations for infinite-slab

For an infinite-slab's material, the one side area, the half-thickness and the half-thickness of the unsoaking-core are given as A(cm^2), L(cm) and x_c (cm), respectively, hence the width and the longitudinal length are very much longer than the half-thickness.

Thus, analogically to the sphere, the relations of the half-thickness of the unsoaking-core x_c, the cooking-ratio X_w and the half-thickness of the material L become as follows:

$$x_c = (1 - X_w) L_o \tag{15}$$

$$L = L_e - ((L_e/L_o) - 1) x_c \tag{16}$$

The material balance equation and the cooking-rate equations can be expressed as follows:

$$dx_c/d\theta = -\mathcal{R}/(2A\rho_h) \tag{17}$$

For shell diffusion controlling:

$$\mathcal{R} = (2A/(L - x_c))k_m(c_e - c_i) \tag{18}$$

For reaction-rate controlling:

$$\mathcal{R} = 2Ak_r(c_i - c_o) \tag{19}$$

The cooking-rate equation for diffusion controlling is obtained by combining Eqs. (18) and (19).

$$\mathcal{R} = 2A(c_e - c_o)/((1/k_r) + ((L - x_c)/k_m)) \tag{20}$$

CALCULATION METHODS OF RATE PARAMETERS

1. Integrations of cooking-rate equations

For the cases of only a partly controlling, we could integrate the cooking-rate equations, and the rate parameters can be calculated easily in following explicit functions.

For the rate parameter k_m of spherical material, the integrated equation can be obtained by substituting Eqs. (3) and (6) into Eq. (4), and integrating $\theta = 0 \to \theta$, $r_c = R_o \to r_c$ hence $c_i = c_o$. For the rate parameter k_r, the equation can be obtained by substituting Eqs. (3) and (7) into Eq. (4), and integrating, hence $c_i = c_e$.

(1) Spherical material

For expanding surface:

$$k_m = \rho_h((R_o{}^2R_e{}^3 - r_c{}^2(R_e{}^3 - R_o{}^3)) - R_o(R_o{}^3R_e{}^3 - r_c{}^3(R_e{}^3 - R_o{}^3))^{2/3})$$
$$/(2(R_e{}^3 - R_o{}^3)(c_e - c_o)\theta) \tag{21}$$

$$k_r = \rho_h R_o(1 - (r_c/R_o))/((c_e - c_o)\theta)$$
$$= \rho_h R_o(1 - (1 - X_w)^{1/3})/((c_e - c_o)\theta) \tag{22}$$

For constant surface:

$$k_m = \rho_h(R_o{}^3 + 2r_c{}^3 - 3R_o r_c{}^2)/(6R_o(c_e - c_o)\theta)$$
$$= \rho_h R_o^2(1 - 3(1 - X_w)^{2/3} + 2(1 - X_w))/(6(c_e - c_o)\theta) \tag{23}$$

k_r = same to Eq.(22)

(2) Long-cylindrical material

For expanding surface:

$$k_m = \rho_h((R_o{}^2R_e{}^2 - r_c{}^2(R_e^2 - R_o{}^2))\ln(R_e^2(R_o{}^2 - r_c{}^2) + R_o{}^2r_c{}^2)$$
$$+2r_c{}^2(R_e{}^2 - R_o{}^2)\ln r_c - (4R_o{}^2R_e{}^2 - 2r_c{}^2(R_e^2 - R_o{}^2))\ln R_o)/(4(R_e^2 - R_o^2)$$
$$(c_e - c_o)\theta) \tag{24}$$

$$k_r = \rho_h R_o(1 - (r_c/R_o))/((c_e - c_o)\theta)$$

$$= \rho_h R_o(1 - (1 - X_w)^{1/2})/((c_e - c_o)\theta) \tag{25}$$

For constant surface:

$$k_m = \rho_h(R_o^2 - r_c^2 + 2r_c^2 \ln r_c - 2r_c^2 \ln R_o)/(4(c_e - c_o)\theta)$$
$$= \rho_h R_o^2 (X_w + (1 - X_w)\ln(1 - X_w))/(4(c_e - c_o)\theta) \quad (26)$$

k_e = same to Eq.(25)

(3) Infinite-slab's material

For expanding surface:

$$k_m = \rho_h L_e (L_o - x_c)^2/(2L_o(c_e - c_o)\theta)$$
$$= \rho_h L_o L_e X_w^2/(2(c_e - c_o)\theta) \quad (27)$$

$$k_r = \rho_h L_o (1 - (x_c/L_o))/((c_e - c_o)\theta)$$
$$= \rho_h L_o X_w/((c_e - c_o)\theta) \quad (28)$$

For constant surface:

$$k_m = \rho_h (L_o - x_c)^2/(2(c_e - c_o)\theta)$$
$$= \rho_h L_o^2 X_w^2/(2(c_e - c_o)\theta) \quad (29)$$

k_r = same to Eq. (28)

The values of c_o and c_e (g-H_2O/cm^3) are obtained from the following equations.

$$c_o = (w_o - w_d)/V_o \qquad c_e = (w_e - w_d)/V_e \quad (30)$$

2. Non-linear least square method

For the calculation of the rate parameters in the drying-rate equations[2], the calculated values using the rate parameters obtained from the integrated equations did not agree with the observed values. Then, the simulate calculations of the rate parameters using a non-linear least square method were necessary. In this paper, we calculated the rate parameters in the cooking-rate equations using a non-linear least square method[3],[4].

The subroutine programs for the calculation of the non-linear rate parameters are the same as in the previous paper[1]. The practical program for the spherical material at expanding surface and the calculated results for the cooking of rice are shown in Appendix (used of the Computation Center of Hiroshima University: TOSBAC 3400-14).

EXPERIMENTAL METHODS AND RESULTS

1. Samples and experimental methods

As the samples, we used a rice ("NAKATE-SHINSENBON" in 1974), *udon* "MARU UDON" by Sanuki Co., Ltd.) and *kishimen* ("NO.1. KISHIMEN" by Nisshin Co., Ltd.). The *udon* and *kishimen* are the round and flat noodles of wheat flour.

In the cooking of these samples, we observed the soaking-shell and unsoaking-core. In the cooking of rice, this phenomenon has been reported by a photographic method using coloring matters, and it has been known that the phenomenon is mainly caused by the water diffusion and not by the heat transfer [10].

Studies of the soaking and cooking of rice and so on have been followed by a total volume measuring method[11], a weighting method[12),13], a chemical method[14] and so on. In this paper, we used the weighing method because it is the most simple of all.

The samples were cooked with water in a beaker at a fixed temperature. The temperature of the water was controlled by an electric heater. The samples were poured quickly into the beaker, and cooked at a constant temperature for fixed periods.

The cooked samples were poured quickly into a cooled water containing pieces of ice. The surfaces of the cooled samples were wiped quickly by a filter paper, and then were transfused into a weighing tube. After this, the weights of the samples were weighed with a chemical balance. The weights of the completely drying state of the cooked samples were estimated as being the values of 12 hrs drying at 105 °C in a dryer.

The diameter, the length and the volume of samples were measured by a travelling microscope and a specific gravity bottle. By this method the errors of the observed values are large. Therefore, we repeated the experiment three times for each sample, and in each experiment we used 50 samples for rice and 10 samples for *udon* and *kishimen* respectively. The observed values used in this paper are the average values.

2. Experimental results

Table 1 illustrates the samples and experimental conditions. The initial weights, sizes and volumes of the samples for fixed periods were not constant, then we corrected the experimental data using the average values.

$$w' = w''(w_o/w_o'') \qquad w_d' = w_d(w_o/w_o'') \tag{31}$$

where, w_o'', w'' and w_d'' : original experimental values, w' and w_d': corrected values by $w_o'' \rightarrow w_o$, w_o: average value of w_o''

Because the samples dissolved in the cooking water, we corrected the experimental data using the weights of the completely drying states of the uncooked samples and cooked samples for fixed periods.

$$w = w' + (w_d - w_d')(\rho_e' / \rho_d') \tag{32}$$

where, w: corrected value by $w_d' \rightarrow w_d$, w_d: w_d' for uncooked samples, $\rho_e' = w_e'/V_e'$, $\rho_d' = w_d'/V_d'$

Table 1. Samples and experimental conditions

Run	Samples	Initial values					Cooking temp-erature(°C)	Completely drying values	
		Diameter and length (cm)			w_o (g)	v_o (cm^3)		w_d(g)	v_d(cm^3)
1	rice	0.203*,	0.277*,	0.506*	0.02135	0.01484	92.5	0.01813	0.01262
2	*udon*	0.192*,	0.192*,	2.48**	0.09751	0.07167	〃	0.08311	0.06402
3	*kishimen*	0.133**,	0.486**,	0.992**	0.08352	0.06418	〃	0.07441	0.06334

* : diameter. **: length.

The relations of weight of cooking materials w(g) vs. cooking time θ(min) for each sample are shown in Figs. 2 ~ 4.

DETERMINATION OF RATE PARAMETERS AND DISCUSSIONS

The weights of the equilibrium states w_e (g) were assumed as the unchanged values in the cooking periods of 100 ~ 120, 150 ~ 200 and 150 ~ 200 min for rice, *udon* and *kishimen* respectively. However, the unsoaking-core disappeard perfectly in the first period of 35, 40 and 40 min for rice, *udon* and *kishimen* respectively. Table 2 illustrates the experimental results assuming these equilibrium states.

Table 2. Experimental results

Run	Samples	Assuming equilibrium time θ_e (min)	Equilibrium values				
			Diameter and length (cm)			w_e (g)	v_e (cm^3)
1-A	rice	100 ~ 120	0.380,	0.519,	0.949	0.1030	0.09810
-B	〃	35	0.325,	0.444,	0.811	0.06659	0.06119
2-A	*udon*	150 ~ 200	0.458,	0.458,	3.13	0.5561	0.5163
-B	〃	40	0.374,	0.374,	3.19	0.3524	0.3497
3-A	*kishimen*	150 ~ 200	0.252,	0.980,	1.62	0.4598	0.4573
-B	〃	40	0.221,	0.820,	1.45	0.2728	0.2626

For the calculations of the rate parameters, rice, *udon* and *kishimen* were assumed as the shapes of the sphere, long-cylinder and infinite-slab, respectively. However, the lengths of *udon* and *kishimen* increased with the cooking, and these samples were not of the exact theoretical models of the long-cylinder and the infinite-slab. For *kishimen* the expanding-ratio of three sizes are nearly the same, then we assumed to sphere too. The diameter for the sphere was used sphere-volume-equivalent diameter.

Table 3 illustrates the calculated results of the rate parameters k_m (cm^2/min) and

Table 3. Rate parameters k_m (cm^2/min) and k_r (cm/min)

Run	Assuming shapes	Initial values			Number of iteration	Calculated values		
		k_m	k_r	σ		k_m	k_r	σ
1-A	sphere	5.09×10^{-4}	0.0168	0.0120	23	0.00692	0.00977	0.00241
-B	〃	7.85 〃	0.0194	0.0209	24	0.0289	0.0143	0.00805
2-A	cylinder	2.65 〃	0.0186	0.0112	12	0.000261	0.431	0.00250
-B	〃	6.57 〃	0.0245	0.0182	24	0.000749	0.146	0.00192
3-A	slab	1.94 〃	0.0149	0.0100	12	0.000223	0.299	0.00339
-B	〃	4.89 〃	0.0164	0.0146	13	0.000674	0.0424	0.00274
3-A	sphere	6.58 〃	0.0194	0.0248	11	0.000643	0.680	0.00807
-B	〃	18.8 〃	0.0224	0.0532	25	0.0169	0.0267	0.00851

$\sigma = (\sum_{i=1}^{n} (r_{c,obs} - r_{c,cal})_i^2 W_i / n)^{1/2}$: standard deviation.

W_i : weighting coefficient for $(r_c)_i$. : n number of experimental points.

k_r (cm/min). In Table 3. the initial values of k_m and k_r are the values calculated results from the integrated equations at the largest and smallest θ respectively. The calculated results compared to the observed values are illustrated by the solid lines in Figs. 2 ~ 5. The broken lines illustrate the results for the initial values. In Table 3, some of the calculated values of k_m and k_r differ too greatly from the initial values. This reason is due to the interrelation of k_m and k_r.

For the cooking of rice, the cooking rate was limited by the reaction-rate of the rice components with water at temperatures of 110 °C and below[9]. Then, we assumed too

Table 4. Rate parameter k_r (cm/min)

Run	Assuming shapes	Initial values k_r	Initial values σ	Number of iteration	Calculated values k_r	Calculated values σ
1-A	sphere	0.0168	0.0360	10	0.00913	0.00243
-B	〃	0.0194	0.0193	11	0.0148	0.00746
2-A	cylinder	0.0186	0.0985	12	0.00551	0.00846
-B	〃	0.0245	0.0349	13	0.0148	0.00477
3-A	slab	0.0149	0.0823	8	0.00472	0.0124
-B	〃	0.0164	0.0292	11	0.0105	0.00877
3-A	sphere	0.0194	0.0867	9	0.00872	0.0155
-B	〃	0.0224	0.0110	7	0.0239	0.00855

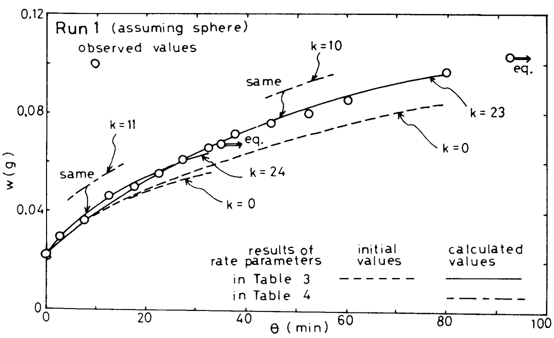

Fig. 2. Relations of weight of cooking rice w vs. cooking time θ.

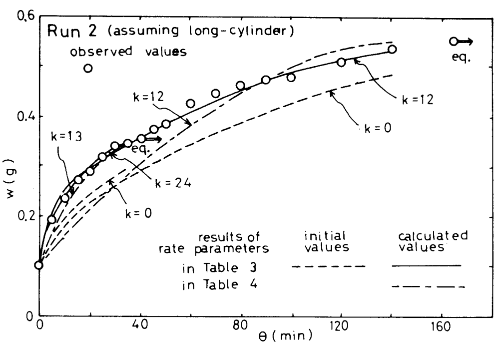

Fig. 3. Relations of weight of cooking *udon* w vs. cooking time θ

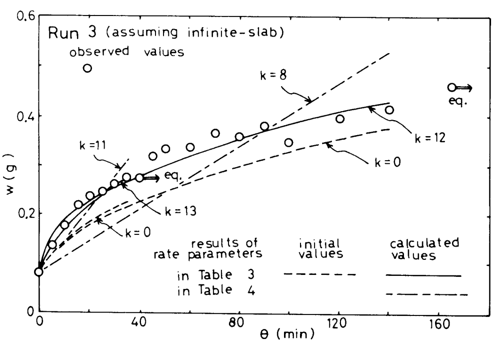

Fig. 4. Relations of weight of cooking *kishimen* w vs. cooking time θ.

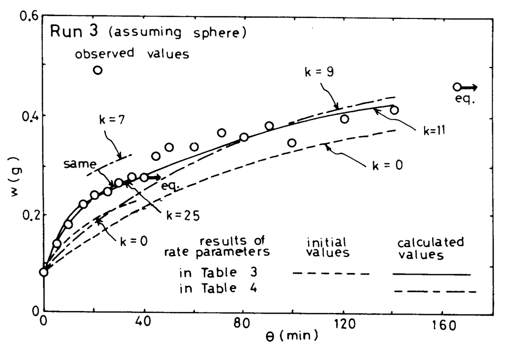

Fig. 5. Relations of weight of cooking *kishimen* w vs. cooking time θ.

that the cooking rate was limited by the reaction-rate only. Table 4 illustrates the calculated results of the rate parameter k_r. The calculated results compared to the observed values are illustrated by the chain lines in Figs. 2 ~ 5. In Figs. 2 ~ 5, the chain line for rice is satisfactory, but the results for *udon* and *kishimen* are not. The reason is that the shapes of *udon* and *kishimen* were not the exact theoretical models.

RESULTS

We postulated the cooking-rate equations based on the water-soaking-shell models, and then the calculation methods of the rate parameters were reported. The rate parameters in the cooking-rate equations of rice, *udon* and *kishimen* were determined assuming the sphere, the long-cylinder and the infinite-slab shapes.

The calculated results of rice using both the rate parameter k_r only and the rate parameters k_m and k_r were satisfactory with the observed values at 92.5 °C. However, the calculated results of *udon* and *kishimen* using the rate parameter k_r only, were not quite satisfactory. The reason was that the shapes of *udon* and *kishimen* were not the exact theoretical dimensions of the long-cylinder and the infinite-slab respectively.

SUMMARY

In order to design and to control automatically various cooking apparatuses, it is necessary to determine the cooking-rate equations and to obtain the rate parameters for the equations. In this study, we postulated the cooking-rate equations based on the water-soaking-shell models, and reported on the calculation methods of the rate parameters using a non-linear least square method.

The rate parameters in the cooking-rate equations of rice, *udon* and *kishimen* (round and flat noodles of wheat flour, respectively) were determined at 92.5 °C, assuming the sphere, the long-cylinder and the infinite-slab. The lengths of *udon* and *kishimen* increased by the cooking, then the calculated values of these rate parameters did not give the exact theoretical values.

The rate parameters in this paper are semi-theoretical or merely experimental ones, but may be adopted with satisfaction for the design of various cooking apparatuses.

NOTATIONS

A : one side area of infinite-slab (cm^2)

c_o, c_e and c_i : water concentrations of unsoaking-core and outer and inner parts of soaking-shell (g-H_2O/cm^3)

k_m and k_r : rate parameters of shell diffusion (cm^2/min) and of reaction-rate (cm/min)

L : length of long-cylinder of half-thickness of infinite-slab(cm)

R : radius of sphere or long-cylinder (cm)

r_c : radius of unsoaking-core of sphere or long-cylinder (cm)

$\mathcal{R}$: cooking-rate (g-H_2O/min)

V : volume of cooking material (cm^3)
w : weight of cooking material (g)
X_w : cooking-ratio (–)
x_c : half-thickness of unsoaking-core of infinite-slab (cm)
θ : cooking time (min)
ρ_h : increased water concentration by cooking (g-H_2O/cm^3)

Subscripts;

o, e and d : initial, equilibrium and completely drying states

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APPENDIX

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C       MAIN PROGRAM
C       CALCULATION OF RATE PARAMETERS IN COOKING-RATE EQUATION OF
C       SOAKING-SHELL MODEL (SPHERE, EXPANDING SURFACE)
C       NON-LINEAR LEAST SQUARE METHOD (HISENS),
C       FROM K.KUBOTA AND N.MORITA, COMPUT. CENTER NEWS OF NAGOYA UNIV.,
C       4(4), PP.318(1973)
C       FROM H.HOSAKA,K.KUBOTA AND K.SUZUKI, SHOKUHIN KOGAKU, PP.170,
C       KYORITSU SHUPPAN(1975)
C       N= TOTAL NUMBER OF EXPERIMENTAL POINT
C       T(N)= TIME, W(N)= WEIGHT, X(N)= COOKING-RATIO, R(N)= RADIUS
C       RC(N)= RADIUS OF UNSOAKING-CORE
C       AH(N),AK(N)= RATE PARAMETERS OF REACTION-RATE AND SHELL DIFFUSION,
C       FROM INTEGRATED EQUATIONS
C       K1,K= NUMBER OF INITIAL SETTING AND ITERATION ENDING POINTS
C       A(1,K),A(2,K)= RATE PARAMETERS OF REACTION-RATE AND SHELL
C       DIFFUSION, FROM NON-LINEAR LEAST SQUARE METHOD
C       SD(K)= STANDARD DEVIATION
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```
C      YC(1,N,K)= CALCULATION VALUES OF RC(N)
C      WC(N,K)= CALCULATION VALUES OF W(N)
C      XC(N,K)= CALCULATION VALUES OF X(N)
       DIMENSION T(70),W(70),X(70),RC(70),R(70),
      1AH(70),AK(70),YY(1,70),XX(1,70),A(2,26),HM(26),HS(26),WW(1,70),
      2YC(1,70,26),YCS(1,70,3),DD(2,3),SD(26),XC(70,26),WC(70,26)
       COMMON RO,RE,CO,CE,RH
       DATA HAA/0.0005/,HM(1),HS(1)/2*1.0/,HMM,HSS/2*0.5/,HST/0.005/,
      1EPS/1.0E-40/,MY,MX/2*1/,L/2/,NT/70/,KT/25/
       EXTERNAL SUB
   777 READ(5,10) N
       IF(N) 999,999,888
   888 READ(5,20) RO,RE,WO,WE,WD,CO,CE,RH
       READ(5,30) (T(I),I=1,N)
       READ(5,30) (W(I),I=1,N)
       DO 200 I=1,N
   200 X(I)=(W(I)-WO)/(WE-WO)
       WRITE(6,50) N,RO,RE,WO,WE,WD,CO,CE,RH
       WRITE(6,60) (T(I),I=1,N)
       WRITE(6,70) (W(I),I=1,N)
       WRITE(6,75) (X(I),I=1,N)
C      RATE PARAMETERS AH(N),AK(N)= FROM INTEGRATED EQUATIONS
       RE0=RE/RO
       RE1=1.0-RE0**3
       RE2=RO**3-RE**3
       DO 300 I=1,N
       AX=1.0-X(I)
       RC(I)=AX**(1.0/3.0)*RO
       R(I)=(RE**3+RE1*RC(I)**3)**(1.0/3.0)
       AH(I)=RH*RO*(1.0-RC(I)/RO)/((CE-CO)*T(I))
       AK(I)=RO*((RO*RE)**3+RE2*RC(I)**3)**(2.0/3.0)-RO**2*RE**3
      1-RE2*RC(I)**2
   300 AK(I)=RH*AK(I)/(2.0*(CE-CO)*RE2*T(I))
       WRITE(6,80) (AH(I),I=1,N)
       WRITE(6,85) (AK(I),I=1,N)
C      RATE PARAMETERS A(1,K),A(2,K)= FROM NON-LINEAR LEAST SQUARE METHOD
       L1=L+1
       K1=KT+1
       A(1,1)=AH(1)
       A(2,1)=AK(N)
       DO 400 I=1,N
       YY(1,I)=RC(I)
       XX(1,I)=T(I)
   400 WW(1,I)=1.0
       CALL HISENS(YY,XX,A,HAA,HM,HMM,HS,HSS,HST,WW,YC,YCS,DD,SD,
      1MY,MX,N,NT,L,L1,K,KT,K1,SUB,EPS,ILL)
       IF(ILL) 410,420,410
   410 WRITE(6,90) ILL
       GO TO 999
   420 WRITE(6,95)
       WRITE(6,96) (A(IA,K1),IA=1,L),SD(K1)
C      CALCULATED VALUES FROM OBTAINED RATE PARAMETERS
       DO 500 I=1,N
       XC(I,K1)=1.0-(YC(1,I,K1)/RO)**3
       WC(I,K1)=WO*(1.0-XC(I,K1))+WE*XC(I,K1)
       DO 500 IK=1,K
       XC(I,IK)=1.0-(YC(1,I,IK)/RO)**3
   500 WC(I,IK)=WO*(1.0-XC(I,IK))+WE*XC(I,IK)
       WRITE(6,97) (WC(I,K1),I=1,N)
       WRITE(6,98) (XC(I,K1),I=1,N)
       WRITE(6,99) K
       WRITE(6,96) (A(IA,K),IA=1,L),SD(K)
       WRITE(6,97) (WC(I,K),I=1,N)
       WRITE(6,98) (XC(I,K),I=1,N)
       GO TO 777
   999 STOP
    10 FORMAT(I4)
    20 FORMAT(5F8.0,3F12.0)
    30 FORMAT(10F8.0)
    50 FORMAT(1H1,26HN,RO,RE,WO,WE,WD,CO,CE,RH=/1H ,I4,9X,2E13.5/
      1(1H ,3E13.5))
    60 FORMAT(1H0,5HT(N)=/(1H ,5E13.5))
    70 FORMAT(1H0,5HW(N)=/(1H ,5E13.5))
    75 FORMAT(1H0,5HX(N)=/(1H ,5E13.5))
    80 FORMAT(1H0,6HAH(N)=/(1H ,5E13.5))
    85 FORMAT(1H0,6HAK(N)=/(1H ,5E13.5))
    90 FORMAT(1H0,4HILL=,I8)
    95 FORMAT(1H0,5HK=  0)
    96 FORMAT(1H0,13HA(L,K),SD(K)=/(1H ,5E13.5))
```

```
   97 FORMAT(1H0,8HWC(N,K)=/(1H ,5E13.5))
   98 FORMAT(1H0,8HXC(N,K)=/(1H ,5E13.5))
   99 FORMAT(1H0,2HK=,I3)
      END


      SUBROUTINE HISENS(Y,X,A,HAA,HM,HMM,HS,HSS,HST,W
     1,YC,YCS,DD,SD,MY,MX,N,NT,L,L1,K,KT,K1,SUB,EPS,ILL)
C     NON-LINEAR LEAST SQUARE METHOD
C     H.HOSAKA,K.KUBOTA AND K.SUZUKI, SHOKUHIN KOGAKU, PP.170,
C     KYORITSU SHUPPAN(1975)
      DIMENSION Y(MY,NT),X(MX,NT),A(L,K1),HM(K1),HS(K1),W(MY,NT)
     1,YC(MY,NT,K1),YCS(MY,NT,L1),DD(L,L1),SD(K1),HA(20)
     2,AS(20),DDS(20)
      IF(KT.GE.1.OR.MY.GE.1.OR.MX.GE.1.OR.N.GE.L.OR.L.GE.1.OR
     1.L.LE.20.OR.L.LT.L1.OR.KT.LT.K1.OR.EPS.GE.0.0) GO TO 200
      ILL=30000
      GO TO 999
  200 K=1
      HM(K1)=HM(1)
      HS(K1)=HS(1)
      SDS=0.0
      DO 10 IA=1,L
   10 A(IA,K1)=A(IA,1)
  888 CALL SUB(YC,X,A,MY,MX,N,NT,L,K,K1,ILL)
      IF(ILL) 300,400,300
  300 IF(SDS) 999,999,700
  400 D=0.0
      DO 20 I=1,N
      DO 20 J=1,MY
   20 D=D+((Y(J,I)-YC(J,I,K))*W(J,I))**2
      SD(K)=SQRT(D/(N*MY))
      IF(SDS) 999,500,600
  500 SDS=SD(K)
      SD(K1)=SD(K)
      DO 30 I=1,N
      DO 30 J=1,MY
      YCS(J,I,L1)=YC(J,I,K)
   30 YC(J,I,K1)=YC(J,I,K)
      DO 21 IA=1,L
   21 A(IA,K1)=A(IA,1)
      GO TO 777
  600 IF(SDS-SD(K)) 700,800,800
  700 HS(K)=HS(K)*HSS
      IF(HST-HS(K)) 666,999,999
  800 IF(KT-K) 999,999,900
  900 K=K+1
      HM(K)=HM(K-1)*HMM
      HS(K)=HS(1)
      SDS=SD(K-1)
      DO 40 IA=1,L
   40 A(IA,K)=A(IA,K-1)
      DO 50 I=1,N
      DO 50 J=1,MY
   50 YCS(J,I,L1)=YC(J,I,K-1)
  777 DO 60 IA=1,L
      AST=A(IA,K)
      HA(IA)=A(IA,K)*HAA
      A(IA,K)=A(IA,K)+HA(IA)
      CALL SUB(YC,X,A,MY,MX,N,NT,L,K,K1,ILL)
      IF(ILL) 999,1000,999
 1000 DO 70 I=1,N
      DO 70 J=1,MY
      YCS(J,I,IA)=YC(J,I,K)
   70 YCS(J,I,IA)=(YCS(J,I,IA)-YCS(J,I,L1))/HA(IA)
   60 A(IA,K)=AST
      DO 80 IA1=1,L
      DO 80 IA2=1,L
      DD(IA1,IA2)=0.0
      DO 80 I=1,N
      DO 80 J=1,MY
   80 DD(IA1,IA2)=DD(IA1,IA2)+YCS(J,I,IA1)*YCS(J,I,IA2)
     1*W(J,I)**2
      DO 90 IA=1,L
      DD(IA,IA)=DD(IA,IA)*(1.0+HM(K))
      DD(IA,L1)=0.0
      DO 90 I=1,N
      DO 90 J=1,MY
   90 DD(IA,L1)=DD(IA,L1)+YCS(J,I,IA)*(Y(J,I)-YCS(J,I,L1))
     1*W(J,I)**2
```

```
      IF(L-1) 999,333,222
  222 CALL GAUYOS(DD,L,L1,EPS,ILL)
      IF(ILL) 999,1100,999
  333 DD(1,2)=DD(1,2)/DD(1,1)
      ILL=0
 1100 DO 110 IA=1,L
      AS(IA)=A(IA,K)
  110 DDS(IA)=DD(IA,L1)
  666 DO 120 IA=1,L
      DD(IA,L1)=DDS(IA)*HS(K)
  120 A(IA,K)=AS(IA)+DD(IA,L1)
      GO TO 888
  999 RETURN
      END

      SUBROUTINE GAUYOS(A,N,N1,EPS,ILL)
C     GAUSS-JORDAN METHOD
C     CALCULATION OF SIMULTANEOUS LINEAR EQUATIONS
      DIMENSION A(N,N1)
      DO 10 K=1,N
      BIG=ABS(A(K,K))
      IP=K
      K1=K+1
      IF(K1.GT.N) GO TO 14
      DO 11 I=K1,N
      IF(ABS(A(I,K)).LE.BIG) GO TO 11
      BIG=ABS(A(I,K))
      IP=I
   11 CONTINUE
   14 IF(BIG.GE.EPS) GO TO 12
      ILL=1000
      GO TO 999
   12 IF(IP.EQ.K) GO TO 15
      DO 13 J=1,N1
      TEMP=A(K,J)
      A(K,J)=A(IP,J)
   13 A(IP,J)=TEMP
   15 W=A(K,K)
      DO 20 J=K1,N1
   20 A(K,J)=A(K,J)/W
      DO 30 I=1,N
      IF(I.EQ.K) GO TO 30
      W=A(I,K)
      DO 40 J=K1,N1
   40 A(I,J)=A(I,J)-W*A(K,J)
   30 CONTINUE
   10 CONTINUE
      ILL=0
  999 RETURN
      END

      SUBROUTINE SUB(YC,X,A,MY,MX,N,NT,L,K,K1,ILL)
C     CALCULATION OF EQUATIONS FOR SIMULATION
      DIMENSION YC(MY,NT,K1),X(MX,NT),A(L,K1),YY(1),YS(1),DY(1),AA(2)
     1,Q(1)
      COMMON RO,RE,CO,CE,RH
      EXTERNAL DIFEQ
      XX=0.0
      HX=X(1,N)/100.0
      DO 10 I=1,L
   10 AA(I)=A(I,K)
      DO 20 I=1,MY
      YY(I)=RO
   20 Q(I)=0.0
      DO 30 I=1,N
      IF(XX-X(1,I)) 40,50,50
   40 DO 60 J=1,MY
   60 YS(J)=YY(J)
      CALL URKGS(YY,DY,XX,HX,MY,AA,L,DIFEQ,Q)
      IF(XX-X(1,I)) 40,50,50
   50 DO 30 J=1,MY
   30 YC(J,I,K)=YY(J)-(YY(J)-YS(J))*(XX-X(1,I))/HX
      ILL=0
      RETURN
      END

      SUBROUTINE URKGS(Y,DY,X,HX,MY,A,L,DIFEQ,Q)
C     RUNGE-KUTTA-GILL METHOD
C     CALCULATION OF ORDINARY DIFFERENTIAL EQUATIONS
      DIMENSION Y(MY),DY(MY),A(L),Q(MY),T(20),R(20),P(6),B(4),C(4)
```

```
      DATA P(1),C(1),C(4)/3*0.0/,B(1)/1.0/
      DATA B(2)/0.2928932/,B(3)/1.707107/
      PX=X
      P(2)=0.5*HX
      P(3)=0.5*HX
      P(6)=0.5*HX
      P(4)=HX
      P(5)=HX
      B(4)=1.0/3.0
      C(2)=0.7071068*HX
      C(3)=-C(2)
      DO 100 J=1,4
      X=PX+P(J)
      CALL DIFEQ(Y,DY,X,MY,A,L)
      DO 100 I=1,MY
      T(I)=P(J+2)*DY(I)-Q(I)
      R(I)=B(J)*T(I)
      Y(I)=Y(I)+R(I)
  100 Q(I)=3.0*R(I)-T(I)+C(I)*DY(I)
      RETURN
      END

      SUBROUTINE DIFEQ(Y,DY,X,MY,A,L)
C     COOKING-RATE EQUATION (SPHERE, EXPANDING SURFACE)
C     Y(1)= RADIUS OF UNSOAKING-CORE
C     X= TIME
C     DY(1)= DIFFERENTIAL VALUE OF Y(1) RESPECT TO X
C     A(1),A(2)= RATE PARAMETERS OF REACTION-RATE AND SHELL DIFFUSION
      DIMENSION Y(MY),DY(MY),A(L)
      COMMON RO,RE,CO,CE,RH
      R=(RE**3+(1.0-(RE/RO)**3)*Y(1)**3)**(1.0/3.0)
      R1=1.0/((Y(1)/R)**2*A(1))
      R2=(R-Y(1))/((Y(1)/R)*A(2))
      R3=R**2*(CO-CE)/(R1+R2)
      DY(1)=R3/(RH*Y(1)**2)
      RETURN
      END
```

```
 N,RO,RE,WO,WE,WD,CO,CE,RH=
   12            0.15250E 00  0.28620E 00
   0.21350E-01   0.10298E 00  0.18130E-01
   0.21700E 00   0.86490E 00  0.55007E 01

 T(N)=
   0.25000E 01   0.75000E 01  0.12500E 02  0.17500E 02  0.22500E 02
   0.27500E 02   0.32500E 02  0.37500E 02  0.45000E 02  0.52500E 02
   0.60000E 02   0.80000E 02

 W(N)=
   0.29030E-01   0.36000E-01  0.43040E-01  0.49180E-01  0.54500E-01
   0.60120E-01   0.65220E-01  0.71140E-01  0.75620E-01  0.79810E-01
   0.85510E-01   0.97700E-01

 X(N)=
   0.94083E-01   0.17947E 00  0.26571E 00  0.34093E 00  0.40610E 00
   0.47495E 00   0.53742E 00  0.60995E 00  0.66483E 00  0.71616E 00
   0.78599E 00   0.93532E 00

 AH(N)=
   0.16779E-01   0.11015E-01  0.10133E-01  0.95995E-02  0.91745E-02
   0.90989E-02   0.90280E-02  0.92997E-02  0.87860E-02  0.84542E-02
   0.86713E-02   0.96876E-02

 AK(N)=
   0.21904E-03   0.23950E-03  0.29076E-03  0.32365E-03  0.34373E-03
   0.37241E-03   0.39442E-03  0.43225E-03  0.42447E-03  0.42105E-03
   0.44618E-03   0.50902E-03

 K=  0

 A(L,K),SD(K)=
   0.16779E-01   0.50902E-03  0.12040E-01

 WC(N,K)=
   0.27029E-01   0.34652E-01  0.40601E-01  0.45708E-01  0.50259E-01
   0.54396E-01   0.58202E-01  0.61731E-01  0.66585E-01  0.70987E-01
   0.74995E-01   0.84050E-01

 XC(N,K)=
   0.69572E-01   0.16296E 00  0.23584E 00  0.29840E 00  0.35415E 00
```

```
  0.40482E 00   0.45145E 00   0.49469E 00   0.55415E 00   0.60807E 00
  0.65717E 00   0.76810E 00

K= 23

A(L,K),SD(K)=
  0.97682E-02   0.69224E-02   0.24096E-02

WC(N,K)=
  0.25826E-01   0.34095E-01   0.41614E-01   0.48491E-01   0.54789E-01
  0.60555E-01   0.65827E-01   0.70634E-01   0.77032E-01   0.82528E-01
  0.87196E-01   0.96106E-01

XC(N,K)=
  0.54833E-01   0.15613E 00   0.24825E 00   0.33249E 00   0.40964E 00
  0.48028E 00   0.54486E 00   0.60375E 00   0.68213E 00   0.74946E 00
  0.80663E 00   0.91579E 00
```

殻状吸水モデルに基づくクッキング速度式に関する研究

久保田清・鈴木寛一・保坂秀明・弘中和憲・阿紀雅敏

大量連続の炊飯装置など各種食品のクッキング装置を設計し，制御化などを行なっていくためには，クッキング速度式を設定し，それに含まれている速度パラメータを求めていくことが必要である。

一般に食品のクッキング現象は複雑で，厳密な意味でのクッキング速度式を得ることは困難である。しかし，例えば炊飯などのように，クッキング現象を殻状吸水現象などによって近似的に置きかえができるような場合には，概括的にはなるがクッキング速度式を設定することが可能である。

本研究は，当面する炊飯装置設計などに対応するために，殻状吸水モデルに基づいたクッキング速度式を設定する研究を行ない，米飯などの実験データを例として，シミュレーションを行なった結果を示した。

(1) クッキング速度が，水の吸水殻状部拡散律速と水と固体食品との反応律速とによって支配されるものと考えて，クッキング速度式を球，長い円柱および薄い平板形について設定した。

(2) クッキング速度式に含まれる速度パラメータを求めるための非線形最小二乗法を使用した電子計算機プログラムを作成した。また，この計算のパラメータ初期値を求めるために，何れか一つの律速を仮定して，速度パラメータを陽関数的に求める積分式を誘導した。

(3) 米，うどんおよびきしめんを，近似的に殻状吸水モデルが適用できる球，長い円柱および薄い平板形の食品例として考えて，速度パラメータの算出例を示した。理論的により意味のある速度パラメータを得て検討していくためには，未吸水核の消失現象を仔細に研究する実験などを行なっていくことが必要である。

本研究で示したクッキング速度式は，吸水殻と未吸水核が現われる食品に適用できる。速度パラメータの意味が究明されない限り半理論式的なものであるが，当面する各種のクッキング装置の設計などに対しては簡単な取り扱いをしており有用なものである。