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Determinations of Rate Parameters in Drying-rate Equations of Water-absorbing Sponges

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(Figs. 1 ~ 10, Tables 1 ~ 2)

INTRODUCTION

In a previous paper[1], we studied the drying-rate equations based on the drying-shell models and the calculation methods of the rate parameters in the rate equations. In present paper, we took up the study of the drying of the water-absorbing sponges which are imagined fiver-foods. The imagined fiver-foods are simple samples which have no shrinkage of surface. The drying experiments of the water-absorbing sponges were performed for the sphere, the long-cylinder and the infinite-slab. The rate parameters in the drying-rate equations were determined for the drying of the water-absorbing sponges using a non-linear least square method.

EXPERIMENTAL METHODS AND RESULTS

1. Samples

As the samples of the imagined fiber-foods, the water absorbing sponges were used, those are assumed to the simple models which have no shrinkage of surface. The sponges are used for breaking out the cushions of chairs, and constitute polyurethane which has continuous pores. The diameters of the pores are 0.3 ~ 0.8 mm, and the thickness of the boundary tissues is about 0.1 mm.

The samples of the sponges were used as the shapes of the sphere, the long-cylinder and the infinite-slab. The water was absorbed in the sponges at a reduced pressure, and the falling water on the surface of the sponges was wiped with a filter paper. The samples are shown in Table 1.

Table 1. Experimental conditions

Run	Shapes	Diameter and Length (cm)	Dry- and Wet-bulb temperatures t_d(°C)	t_w(°C)	Air velocity u(cm/min) and Direction
1	sphere	1.58 X 1.58 X 1.58	40	29	3480 horiz. to axis
2	long-cylinder	0.64 X 0.64 X 3.51	〃	31	〃 〃
3	infinite-slab	0.42 X 1.93 X 3.89	〃	30	〃 〃
4	〃	〃	〃	30.5	3480 vert. to axis

2. Experimental apparatus and methods

Fig. 1 illustrates the flow sheet of the experimental apparatus used in the drying experiments. The cylindrical dryer is made from a steel tube of 10.5 cm I.D. and 40 cm in length. The upper part of the dryer is uncovered, and set up the balance. The lower part is attached to the dispersion plate which can disappeared the air flow distribution.

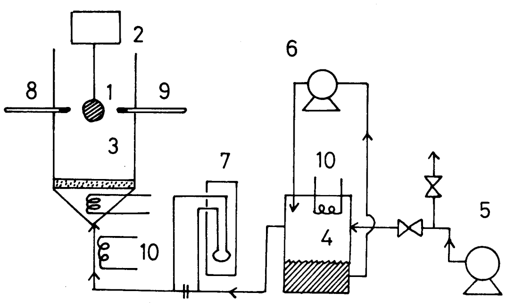

Fig.1. Flow sheet of the experimental apparatus.

1 sample
2 balance
3 cylindrical dryer
4 humidity controler
5 blower
6 gear pump
7 orifice flow meter
8 dry-bulb thermometer
9 wet-bulb 〃
10 electric heater

The humidity of the flowing out air from the blower is controled by the humidity controler which has the electric heater. At the upper part of the controler the heated water is sprayd. The water is circulated by the gear pump.

The humidity of the air is controled by the electric heater in the controler, and the temperature of the humidity controled air is controled by the electric heater in front of the dryer. The temperature and humidity of the air are controled automatically by two transistor relays based on the dry- and wet-bulb temperatures in the dryer. The velocity of the air is measured using the orifice flow meter.

3. Experimental results

Table 1 illustrates the experimental conditions. Air velocity is 3480 cm/min and the dry- and wet-bulb temperatures are 40 and about 30°C, respectively. The relations of weight of drying materials w(g) vs. drying time θ(min) were measured. The weights of the equilibrium states w_e(g) were considered as being the values that kept the balance unchanged for more than 60 min. The weights of the completely drying states w_d(g) were decided as being the values that dryed the equilibrium materials for 2 hrs at 105°C in another dryer.

The relations of w(g) vs. θ(min) for each sample are shown in Figs. 4 and 5, respectively. The values are shown with the 5 points averages of the original experimental values.

DETERMINATIONS OF RATE PARAMETERS AND DISCUSSIONS

1. Methods of determinations of rate parameters

The calculation methods of the rate parameters in the drying-rate equations were described in detail for the sphere, the long-cylinder and the infinite-slab in the former paper[1]. Then we described briefly the calculation methods of the rate parameters h_m(cm³-void/cm²·min) and k_m(cm³-void/cm·min) for the spherical materials at constant surface.

$$dr_c/d\theta = -\mathcal{R}/(4\pi r_c^2 \rho_h) \tag{1}$$

$$\mathcal{R} = 4\pi R_o^2 (c_s - c_g)/((1/h_m) + ((R_o - r_c)/((r_c/R_o)k_m))) \tag{2}$$

$$\text{where,} \quad \rho_h = (w_o - w_e)/V_o \tag{3}$$

The experimental data are given as the relations of the weight of materials w(g) vs. drying time θ(min). The radius of undrying-core r_c(cm) is obtained from w using the following equations.

$$X_w = (w_o - w)/(w_o - w_e) \tag{4}$$

$$r_c = (1 - X_w)^{1/3} R_o \tag{5}$$

To the standard deviations σ(cm) for r_c is minimized, we substituted Eq. (2) into Eq. (1), and integrated numerically $\theta = 0 \rightarrow \theta$; $r_c = R_o \rightarrow r_c$ using a Runge-Kutta-Gill method.

The equations of a non-linear least square method are shown as the following simultaneous equations.

$$(\mathbf{A} + \lambda \mathbf{D})\delta = \nu \mathbf{g} \tag{6}$$

The notations in Eq. (6) are the same as in the former paper[1]. The weighting factor λ is given 1.0 as the initial value, and is reduced to half for every iteration. The step size ν is given 1.0 as the initial value for every iteration, and when the value of the standard deviation σ decreases not less than the former iteration results, it is reduced by half until it decreases.

The initial values of the rate parameters h_m and k_m were given the values obtained from the integrated equations which were illustrated in the former paper[1] assuming the gas-film or shell diffusion controlling.

$$h_m = \rho_h(R_o^3 - r_c^3)/(3R_o^2(c_s - c_g)\theta)$$
$$= \rho_h R_o X_w/(3(c_s - c_g)\theta) \tag{7}$$
$$k_m = \rho_h(R_o^3 + 2r_c^3 - 3R_o r_c^2)/(6R_o(c_s - c_g)\theta)$$
$$= \rho_h R_o^2(1 - 3(1 - X_w)^{2/3} + 2(1 - X_w))/(6(c_s - c_g)\theta) \tag{8}$$

The initial values of h_m used the values at the smallest θ, and those of k_m used the values at the largest θ. The values of c_g and c_s (g-H_2O/cm^3-void) were obtained from following equations.

$$c_g = H/((2.24 \times 10^{-2})(H/18 + 1/29)) \tag{9}$$
$$c_s = H_s/((2.24 \times 10^{-2})(H_s/18 + 1/29)) \tag{10}$$

The values of X_w, w, W and $dW/d\theta$ compared to the observed values respectively, were calculated from r_c and $dr_c/d\theta$ using the following equations.

$$X_w = 1 - (r_c/R_o)^3 \tag{11}$$
$$w = w_o(1 - X_w) + w_e X_w \tag{12}$$
$$W = (w - w_d)/w_d \tag{13}$$
$$dW/d\theta = (3(1 - X_w)^{2/3}(w_o - w_e)/(R_o w_d))dr_c/d\theta \tag{14}$$

2. Calculated results and discussions

Table 2 illustrates the calculated results of the rate parameters h_m and k_m of the water-absorbing sponges for the sphere, the long-cylinder and the infinite-slab. The FACOM 230-60 digital computer in the Computation Center of Nagoya University was used for the calculations. The relations of the rate parameters h_m and k_m obtained from the integrated equations vs. the drying time θ are shown in Fig. 2. The rate parameters h_m and k_m at the larger and smaller θ respectively, are not desirable as shown in Fig. 2. In Table 2, the initial values of the rate parameters h_m and k_m are those calculated results at the smallest and largest θ respectively.

Table 2. Rate parameters h_m (cm^3-void/cm^2·min) and k_m (cm^3-void/cm·min) and standard deviations σ (–)

Run	Initial values			Number of	Calculated values		
	h_m	k_m	σ	interation	h_m	k_m	σ
1	39.7	12.8	0.1472	11	52.6	45.6	0.00614
2	42.3	4.17	0.0585	13	57.0	9.32	0.00352
3	43.5	2.79	0.0494	12	78.3	4.98	0.00707
4	48.9	4.41	0.0419	15	68.2	12.8	0.00142

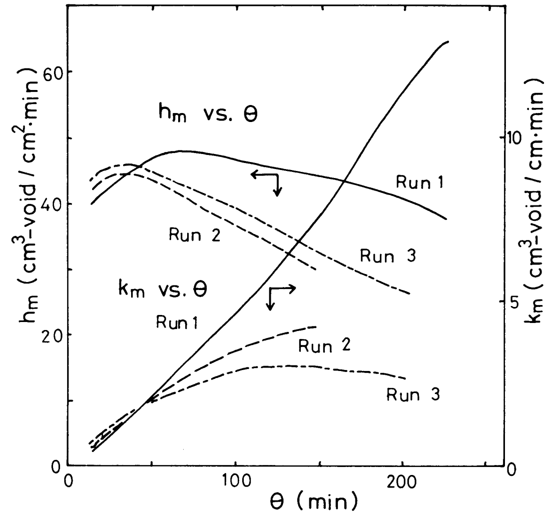

Fig. 2. Relations of rate parameters h_m and k_m vs. drying time θ.

In Table 2, the calculated values of the rate parameters h_m and k_m are the values obtained from a non-linear least square method assuming the gas-film and shell diffusion controllings. The relations of the rate parameters h_m and k_m and the standard deviation σ vs. the number of iteration k for the sphere are shown as an example in Fig. 3. The calculated results to compared with the observed values are illustrated by the solid lines in Figs. 4 and 5. The broken lines illustrate the results for the initial values. From Figs. 3 ~ 5, the best values of the rate parameters h_m and k_m obtained from the integrated equations do not satisfy the whole relation of w vs. θ. The insufficiency is due to the interrelation of the rate parameters h_m and k_m. The simulate calculation of a non-linear least square method using a electronic computer is necessary.

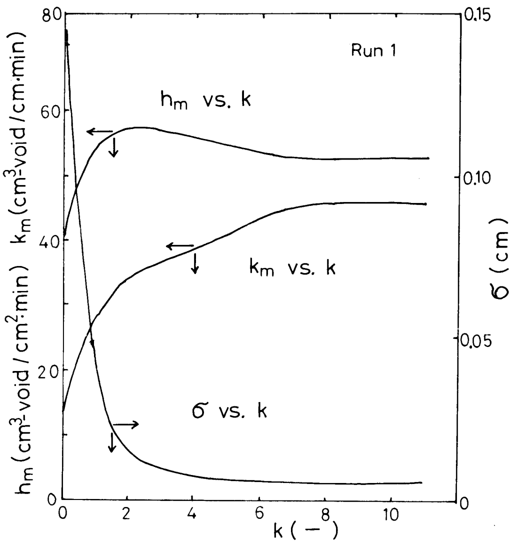

Fig. 3. Relations of rate parameters h_m, k_m and standard deviation σ vs. number of iteration k.

Fig. 6 shows the relations of the moisture content W(g-H_2O/g-D.M.), the radius of undrying-core r_c (cm), the drying-ratio X_w (−) and the drying-rate $dW/d\theta$ (g-H_2O/min·g-D.M.) vs. the drying time θ. In the relations of w, W, r_c and X_w except $dW/d\theta$, vs. θ, the calculated results agree with the experimental results. The reason of insufficiency for

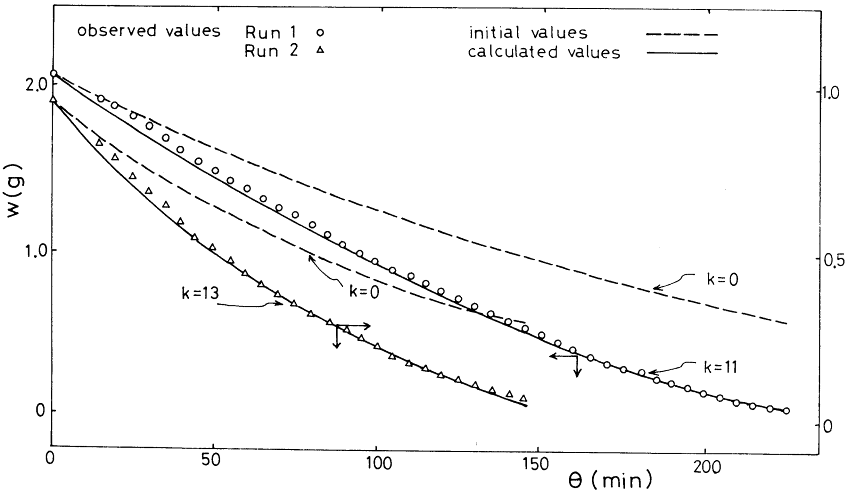

Fig. 4. Relations of weight of drying materials w vs. drying time θ.

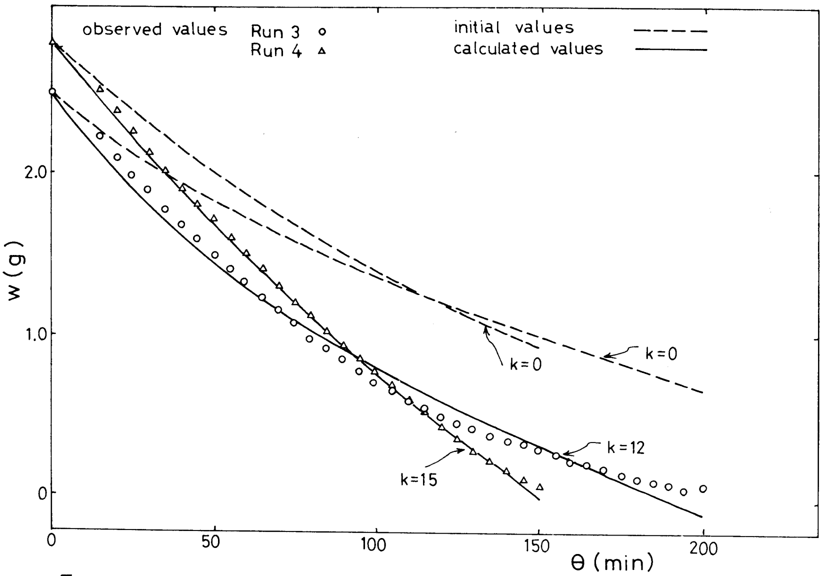

Fig. 5. Relations of weight of drying materials w vs. drying time θ.

the relation of $dW/d\theta$ vs. θ is due to the inaccuracy of the differentiation of the experimental data. If the standard deviations σ for $dr_c/d\theta$ or $dW/d\theta$ are minimized, the relation of $dW/d\theta$ vs. θ agrees well, but not to the original relation of w vs. θ.

The relations of $dW/d\theta$ vs. W are shown in Figs. 7 ~ 10. The results for the calculated and initial values are illustrated by the heavy solid and broken lines. The thin solid lines show the results for some fixed values of the rate parameters. In Figs. 7 ~ 10,

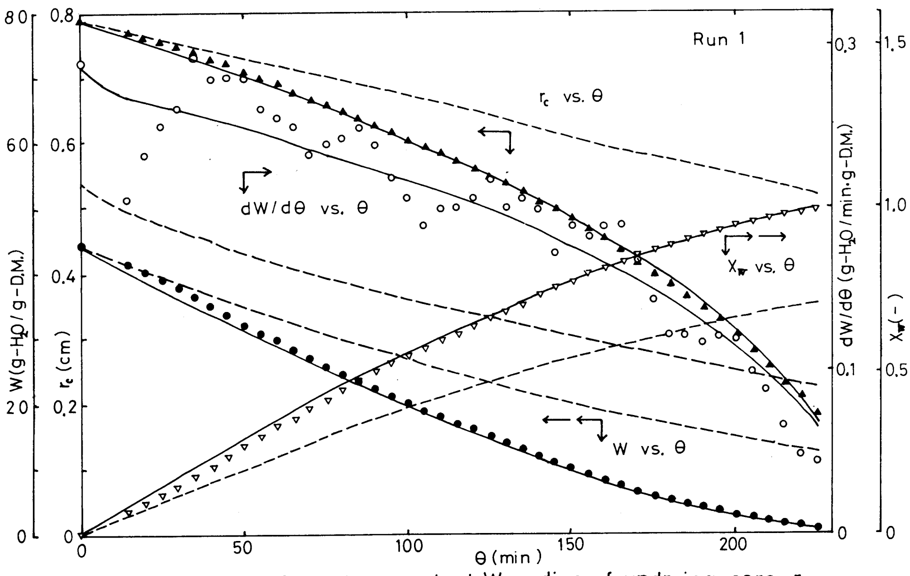

Fig. 6. Relations of moisture content W, radius of undrying-core r_c, drying-rate dW/dθ and drying-ratio X_w vs. drying time θ.

the results for the sphere are satisfactory, but the results for the long-cylinder and the infinite-slab are less satisfactory. The heavy chain lines in Figs. 8 ~ 10 illustrate the results of the rate parameters obtained from the sphere shown in Fig. 7.

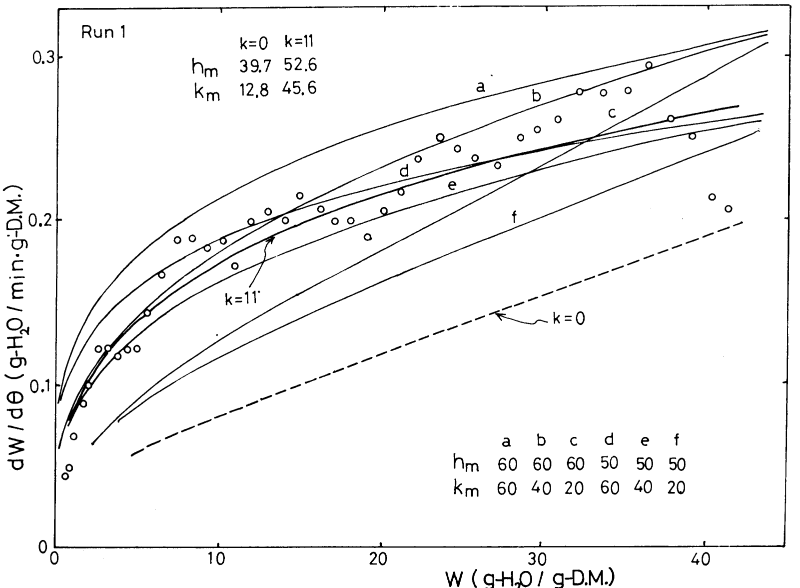

Fig. 7. Relations of drying-rate dW/dθ vs. moisture content W.

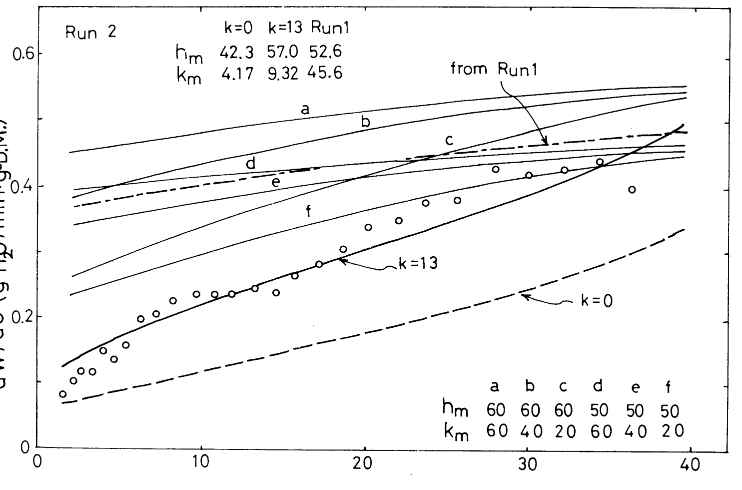

Fig. 8. Relations of drying-rate $dW/d\theta$ vs. moisture content W.

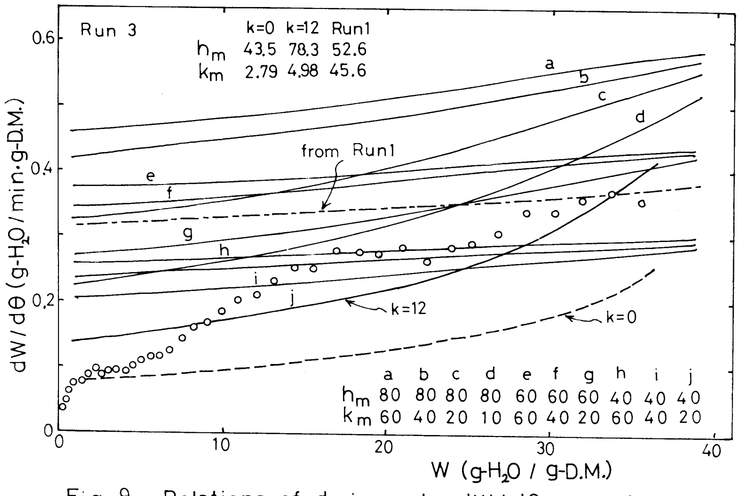

Fig. 9. Relations of drying-rate $dW/d\theta$ vs. moisture content W.

The observed results for the long-cylinder and the infinite-slab are smaller than the calculated results. The reason is that the pore diameters of sponges used in this study were relatively large, and the water cores of the long-cylinder and the infinite-slab were transformed into the cores of the ellipsoids and the disks. The observed surfaces of the long-cylinder and the infinite-slab are smaller than the theoretical surfaces of the shell-models, and decrease with the increase of θ. When the pore diameters of samples are not as small as in this paper, the results show that the sphere must be used.

The molecular diffusion coefficient of water vapor in air can be obtained from the

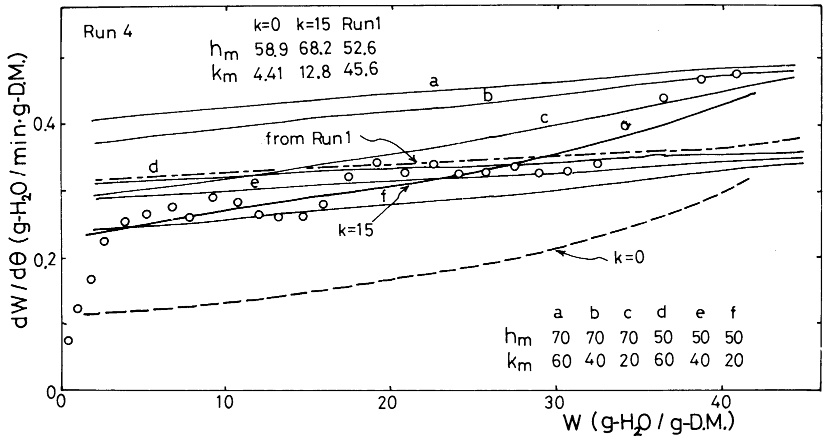

Fig. 10. Relations of drying-rate $dW/d\theta$ vs. moisture content W.

following equation.[2)]

$$k_m = 0.042T^{1.833}(1/M_1 + 1/M_2)^{1/2}/((T_C/P_C)_1{}^{1/3} + (T_C/P_C)_2{}^{1/3})^3 \quad (15)$$

where, T: absolute temperature (°K), T_C: critical temperature (°K),

P_C: critical pressure (atm), M: molecular weight (–)

The value of k_m obtained from Eq. (15) is 17.7 cm²/min at 35°C. The value of k_m in this study for the sphere in Table 2 is larger than the values obtained from Eq. (15). If the rate parameter k_m in this study isn't a merely experimental parameter but a diffusion coefficient, then a small quantity of water leaks out by capillary action.

If heat-transfer controlling steps are assumed, the rate parameters of the gas-film and shell heat-transfers h_h (cal/cm²·min·°C) and k_h (cal/cm·min·°C) are calculated by the following equations[1)] using the values of h_m and k_m.

$$h_h = h_m(\Delta H)(c_s - c_g)/(t_g - t_s) \quad (16)$$

$$k_h = k_m(\Delta H)(c_s - c_g)/(t_g - t_s) \quad (17)$$

If the values of h_m and k_m for the sphere in Table 2 are substituted into Eqs. (16) and (17), we obtain $h_h = 8.72 \times 10^{-2}$ cal/cm²·min·°C and $k_h = 7.56 \times 10^{-2}$ cal/cm·min·°C.

The rate parameter of h_h can be obtained from the following equation[3)] and so on.

$$Nu = 2 + 0.65Re^{1/2}Pr^{1/3} \quad (18)$$

where, Nu: Nusselt number (–), Re: Reynolds number (–),

Pr: Prandtl number (–)

The value of h_h obtained from Eq. (18) is 4.14×10^{-2} cal/cm²·min·°C for the sphere in Table 2 at 35°C. The value of h_h in this study is larger than the value obtained from Eq. (18). The reason is due to the diminution of the gas-film of sponges as compared with other smooth materials and so on.

The rate parameters of k_h for the water vapor and water can be obtained from the following equations[4),5)], respectively.

For water vapor;

$$k_h = \mu(c_p + 2.58/M) \tag{19}$$

where, μ : viscosity (g/cm·min), c_p : specific heat (cal/g·°C), M : molecular weight (–)

For water;

$$k_h = 0.0842(1 + 2.81 \times 10^{-3}(t - 20)) \tag{20}$$

where, t : temperature (°C)

The values of k_h for water vapor and water obtained from Eqs. (19) and (20) are 3.05×10^{-3} and 8.77×10^{-2} cal/cm·min·°C at 35°C, respectively. The value of h_h in this study is close to the latter value, and larger than the former value. These results show that most shell heat-transfer occurs at the solid and moving water by capillary action.

In above discussions, the driving force in the drying-rate equations assumed (c_s–c_g) and (t_g–t_s), but can assume another driving force too. The temperature transition of the undrying-core surface t_s(°C) was not considered in this paper. Then, the rate parameters h_m, k_m, h_h and k_h in this paper are merely experimental parameters, but can be adopted satisfactorily for the design of various drying apparatuses.

RESULTS

The rate parameters in the drying-rate equations of the water-absorbing sponges which are considered as imagined fiber-foods, were determined for the sphere, the long-cylinder and the infinite-slab using a non-linear least square method.

The relations of the weight of material w(g) vs. the drying time θ(min) which are calculated from the rate parameters h_m and k_m obtained independently from the integrated equations did not agree with the observed values. The reason is the inter-relation of the rate parameters. Simulate calculations of the rate parameters using a non-linear least square method were necessary.

The pore diameters of the sponges used in this study were not small and the water cores of the long-cylinder and the infinite-slab were transformed into ellipsoids and the disks. The calculated results for the long-cylinder and the infinite-slab did not agree with the observed values, but in the case of the sphere the results agreed. The rate parameters in this paper are merely experimental ones, but can be adopted satisfactorily for the design of various drying apparatuses.

SUMMARY

In a previous paper, we studied the drying-rate equations based on the drying-shell models and the calculation methods of the rate parameters in the rate equations. In present paper, the rate parameters of the water-absorbing sponges which are considered as imagined fiber-foods, were determined for the sphere, the long-cylinder and the infinite-slab using a non-linear least square method. The imagined fiver-foods are simple samples which have no shrinkage of surface.

The calculated values using the rate parameters obtained independently from the integrated equations did not agree with the observed values. Simulate calculations of the rate parameters using a non-linear least square method were necessary.

The pore diameters of the sponges used in this study were not small and the water cores of the long-cylinder and the infinite-slab were transformed into ellipsoids and the disks. The calculated results for the long-cylinder and the infinite-slab did not agree with the observed values, but in the case of the sphere the results agreed. The rate parameters in this paper are merely experimental ones, but can be adopted satisfactorily for the design of various drying apparatuses.

NOTATIONS

c_g and c_s : moisture concentrations of gas-film and undrying-core surfaces (g-H_2O/cm^3-void)

H and H_s : humidity and saturated humidity (g-H_2O/g-D.A.)

ΔH : latent heat of vaporization (cal/g-H_2O)

h_m and h_h : rate parameters of gas-film diffusion (cm^3-void/cm$^2\cdot$min) and heat transfer (cal/cm$^2\cdot$min$\cdot$°C)

k_m and k_h : rate parameters of shell diffusion (cm^3-void/cm$\cdot$min) and heat transfer (cal/cm$\cdot$min$\cdot$°C)

R : radius of sphere (cm), $\mathcal{R}$: drying-rate (g-H_2O/min)

r_c and x_c : radius and half-thickness of undrying-core of sphere, long-cylinder or infinite-slab (cm)

t_g and t_s : dry-bulb and undrying-core surface temperature (°C)

V, W, w and X_w : volume, moisture content, weight and drying-ratio of drying materials (cm^3), (g-H_2O/g-D.M.), (g) and (−)

$dW/d\theta$: drying-rate (g-H_2O/min$\cdot$g-D.M.), θ : drying time (min)

ρ_h : water concentration of vaporization (g-H_2O/cm^3)

Subscripts;

o, e and d : initial, equilibrium and completely drying states

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含水スポンジの乾燥速度式における速度パラメータの算出

久保田清・広田令子・鈴木寛一・保坂秀明

既報[1]において，殻状乾燥モデルに基づく乾燥速度式の設定および速度パラメータの算出方法について報告した。本報では，含水スポンジを仮想食品として考えて，乾燥実験を行ない，速度パラメータの算出をした。含水スポンジは，表面収縮をしなく殻状乾燥をするので，既報の各種の乾燥速度式を検討するのに都合のよい簡単なモデル体である。球，長い円柱および無限平板状の試料について検討した結果，つぎに示すような結論が得られた。

(1) ガス境膜および乾燥殻状部拡散律速をそれぞれ仮定して誘導された積分形[1]から求められた速度パラメータ h_m (cm³-void/cm²・min) および k_m (cm³-void/cm・min) を用いた計算結果は，試料の重さ w (g) 対 乾燥時間 θ (min) の関係で得られた実験データを満足しなかった。この原因は，速度パラメータの相関性によるものであり，非線形最小二乗法を用いた同時計算が必要であることが分った。

(2) 本研究で使用した含水スポンジは，細孔径が大きく，長い円柱および無限平板状の試料においては，未乾燥核が水の凝集力によりそれぞれ楕円体および円盤体となり，殻状乾燥モデルを満足する実験データが得られなかった。このような試料の場合に対しては，球状についての実験が必要であることが分った。

(3) 球状の試料については，実験データをよく満足する速度パラメータ h_m および k_m が得られた。従来の関係式から求められる各種の推算値と比較検討を行なった。理論的により意味のある速度パラメータを得て従来の値と比較検討をしていくためには，未乾燥核の温度変化を測定する実験などを行なっていくことが必要である。

本研究で示した方法で得られる乾燥速度式は，速度パラメータの意味が究明されていかない限り半理論式的なものであるが，当面する各種の乾燥装置の設計などに対しては簡単な取り扱いをしており有用なものである。