広島大学学位請求論文

Higgs sector of Dirac neutrino mass model of Davidson and Logan
（ダヴィドソンとローガンのディラックニュートリノ質量模型のヒッグスセクター）

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広島大学大学院理学研究科
物理科学専攻

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主論文

Higgs sector of Dirac neutrino mass model of Davidson and Logan

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Abstract

Xf tuvez Ejsbd ofvusjop n btt n pefmpg Ebwjetpo boe Mphbo0 Ui jt jt ui f n pefmui bu jouspevdft b ofx I jhht epvcmfu boe fyqmbjot ui f psjhjo pg tn bmmofvusjop n btt x jui pvu sfrvjsjoh ujoz Zvl bx b dpvqmjoht0 Jo ui jt qbqfs- xf tuvez ux p btqfdut pg ui f n pefm0 Pof jt bcpvu ui f rvbouvn dpssfdujpo up ui f wbdvvn fyqfdubujpo wbmvf pg I jhht flfmet0 Xf efsjwf ui f fybdu gpsn vmbf gps ui f rvbouvn dpssfdujpo up wbdvvn fyqfdubujpo wbmvft0 Xf dbmdvmbuf ui fn ovn fsjdbmmz0 Bopui fs jt bcpvu ui f qspevdujpo pg ui f ofx I jhht qbsujdmft0 Xf efsjwf ui f qbjs qspevdujpo dsptt tfdujpo- e^+ , $e^- \simeq \ddot{\nu}_e$, e^- , H^+ , $X)X$ A A, h^* boe dbmdvmbuf ju ovn fsjdbmmz0

Acknowledgment

Ji bwf cffo tvqqpsufe boe fodpvsbhfe cz n boz ufbdi fst- gsjfoet- boe n z gbn jmz gps x sjujoh ui jt ui ftjt0 Ejtdvttjpo x jui Nbtbojsj Pl bx b- Ubl vzb Npsp–vn j- Upn pi jsp Jobhbl j- I jspzvl j Ubl bub boe Lfo.jdi j Jti jl bx b x bt wfsz i fmqgvmgps n f up fyufoe ui f l opx mfehf boe tl jmmt pg n z tuvez0 J ui bol bmmpg ui f n fn cfst pg ui f psfujdbmqbsujdmf qi ztjdt hspvq jo I jspti jn b Vojwfstuz0

Contents

Chapter 1

Introduction

Ijhht jt ejtdpwfsfe bu MID boe bmmuif qbsujdmft pg TN bsf gpvoe0 Ipxfwfs- jo uif gsbnfxpsl pg uif TN- uif psjhjo pg nbttjwf ofvusjopt dbo opu cf fyqmbjofe0 Ofvusjop ptdjmmbujpo ejtdpwfsfe bu TVQFS LBNJPLBOEF jnqmjft uibu ofvusjopt ep ibwf opo.{fsp nbtt boe ofx qiztjdt pg cfzpoe TN xijdi qspwjeft uifjs tnbmm nbttft jt sfrvjsfe0 Uifsf bsf uxp uzqft pg ofvusjop0 Pof jt b ofvusjop pg uif Nbkpsbob uzqf boe bopuifs jt b ofvusjop pg uif Ejsbd uzqf0 Uif Nbkpsbob ofvusjop jt b ofvusjop uibu jt jut pxo bouj.ofvusjop boe uif Ejsbd ofvusjop jt b ofvusjop uibu jt opu jut pxo bouj.ofvusjop0 Uifsf bsf nboz ofx qiztjdt npefmt xjui Nbkpsbob ofvusjopt0 Ofvusjopmftt epvcmf cfub efdbz uibu xfsf bo fwjefodf pg Nbkpsbob ofvusjop- ipxfwfs- ibt opu cffo gpvoe zfu0 Uifsfgpsf ju jt ofdfttbsz up dpotjefs npefmt xjui Ejsbd ofvusjopt0 Uif npefm pg Ebwjetpo boe Mbohbo jt pof pg tvdi npefmt hjwjoh tnbmm nbtt up Ejsbd ofvusjopt]2a0

Uif jefb pg Sfg0]2- 3a jt uibu b ofx Ijhht tfdups jt joUspevdfe up fyqmbjo uif psjhjo pg tnbmm ofvusjop nbtt0 Uif ofx Ijhht gjfme ibt b ujoz wbdvvn fyqfdubujpo wbmvf)WFW* dpnqbsfe up uibu pg uif TN Ijhht0 Tjodf ofvusjopt dpvqmf up pomz uif ofx Ijhht- uif psjhjo pg ofvusjop nbtt jt jut ujoz wbdvvn fyqfdubujpo wbmvf0 Uif wjsuvf pg uif npefm jt uibu pof eptf opu offe up joUspevdf wfsz tnbmm Zvlbxb dpvqmjoh gps ofvusjop nbtt0 Jo uif TN xjui uif Ejsbd ofvusjop nbtt- pof nvtu uvof uif Zvlbxb dpvqmjoh tp uibu ju jt psefs pg 21^{-11} gps 2fW ofvusjop nbtt0 Jo dpousbtu up uif TN- uif Zvlbxb dpvqmjoh pg uif ofx npefm dbo cf bt mbshf bt 21^{-3} jg uif wbdvvn fyqfdubujpo wbmvf pg uif ofx Ijhht gjfme jt psefs pg 2lfW0

Jo uijt qbqfs- xf tuvez uxp btqfdut pg uif npefm0 Pof jt bcpvu uif rvbouvn dpssfdujpo up uif wbdvvn fyqfdubujpo wbmvf pg Ijhht gjfmet]4a0 Bopuifs jt bcpvu uif qspevdujpo pg uif ofx Ijhht qbsujdmft]5a0

Jo uif gjstu qmbdf- xf tuvez uif hmpcbm njojnvn pg uif usff mfwfm Ijhht qpufoujbm cz fyqmjdjumz tpmwjoh uif tubujpobsz dpoejujpot0 Xf dbsfgvmmz fybnjof dpoejujpot uibu uif mbshf wbdvvn fyqfdubujpo wbmvf pg b TN mjlf Ijhht boe uif tnbmm wbdvvn fyqfdubujpo wbmvf pg b ofx Ijhht gjfme dbo cf sfbmj{fe bt uif hmpcbm njojnvn pg uif Ijhht qpufoujbm0 Uifsf bsf nboz tuvejft pg uif usff mfwfm Ijhht qpufoujbm pg hfofsbm uxp Ijhht epvcmfu npefm]6a.]22a0 Ju ibt cffo tipxo uibu uif dibshf ofvusbm wbdvvn jt mpxfs uibo uif dibshf csfbljoh wbdvvn]6a0 Bmtp uif wbdvvn fofshz ejggfsfodf pg uxp ofvusbm njojnb xbt efsjwfe]8- 9a0 Xf nblf vtf pg uif sftvmut boe jefoujgz uif wbdvvn pg uif qsftfou npefm0

Uif dpotusbjout po uif qbsbnfufst pg uif npefm gps xijdi uif eftjsfe wbdvvn dbo cf sfbmj{fe- bsf efsjwfe boe uifz bsf sfxsjuufo jo ufsnt pg Ijhht nbttft boe b gfx dpvqmjoh dpotubout xijdi dbo opu cf ejsfdumz sfmbufe up uif Ijhht nbttft0 Uiftf dpotusbjout bsf gvmmz vtfe xifo xf tuvez uif sbejbujwf dpssfdujpot up uif wbdvvn fyqfdubujpo wbmvft ovnfsjdbmmz0

Cfzpoe uif usff mfwfm- xf tuvez uif sbejbujwf dpssfdujpo up uif Ijhht qpufoujbm boe uif wbdvvn fyqfdubujpo wbmvft pg I jhht0 Tjodf ui f ofvusjop n bttft bsf qspqpsujpobmup ui f wbdvvn fyqfdubujpo wbmvf pg pof pg I jhht- pof dbo bmtp dpn qvuf ui f sbejbujwf dpssfdujpot up ofvusjop n bttft0 Bt bmsfbez opufe jo Sfg0]2a ui f sbejbujwf dpssfdujpo up ui f tpgumz csfbl joh n btt qbsbn fufs jt mphbsjui n jdbmmz ejwfshfou boe ju jt sfopsn bmj{fe n vmujqmjdbujwfmz0 Xf efsjwf ui f gpsn vmbf gps ui f pof mppq dpssfdufe wbdvvn fyqfdubujpo wbmvft gps ux p I jhht epvcmfut cz tuvezjoh pof mppq dpssfdufe ffifdujwf qpufoujbm0 Ui f dpssfdujpot bsf fwbmvbufe ovn fsjdbmmz cz fyqmpsjoh ui f qbsbn fufs sfhjpot bmmpxfe gspn ui f hmpcbmn jojn vn dpoejujpo gps ui f wbdvvn0 Xf ti px i px ui f sbejbujwf dpssfdujpot di bohf efqfoejoh po ui f fyusb I jhht tqfdusvn0 Ui f sbejbujwf dpssfdujpot bsf bmtp fwbmvbufe gps ui f dbtf ui bu b sfmbujpo bn poh ui f dpvqmjoh dpotubout jt tbujtflfe0

Tfdpoemz xf tuvez %ui f ofx I jhht qbjs qspevdujpo%- xi jdi jt b qi fopn fob dmptfmz sfmbufe up ui f n fdi bojtn hfofsbujoh ui f tn bmmWFW0 Ui f ofx I jhht i bt b ofx V)2* di bshf boe ui f V)2* tzn n fusz hfofsbufe cz ui f di bshf jt fyqmjdjumz cspl fo0 Ui fsfgpsf- ui f tn bmmWFW pg ui f ofx I jhht csfbl t V)2* tzn n fusz0 Jo ui f tzn n fusjd mjn ju- ui f WFW wbojti ft0

Jo ui f n pefmmboz V)2* di bshf.wjpmbujoh qspdftt jt tvqqsfttfe cz ui f ujoz WFW0 Ui jt bmtp jn qmjft ui bu ui f qspcbcjmjuz bn qmjuvef jt tvqqsfttfe boe jt qspqpsujpobmup ofvusjop n btt0 Bo fybn qmf pg b tvqqsfttfe qspdftt jt b tjohmfu tfdpoe I jhht qspevdujpo xjui hbvhf cptpo gvtjpo0 Jo dpousbtu up ui f tjohmfu tfdpoe I jhht qspevdujpo- ui f qbjs qspevdujpo pg ui f tfdpoe I jhht jt b V)2* di bshf dpotfswjoh qspdftt0 Ui fsfgpsf- ju jt opu tvqqsfttfe0 Ui f qspdfttft jo ui jt dbufhpsz bsf $Z^*)\gamma^{**} \simeq H^+ ,\ H^-$- $W^+ ,\ W^- \simeq H^+ ,\ H^-$- boe $W^+ ,\ Z \simeq H^+ ,\ X)X$ A A, h*- xi fsf H^+- A- boe h efopuf ui f di bshfe I jhht- DQ.pee I jhht- boe DQ.fwfo I jhht jo ui f tfdpoe I jhht epvcmfu- sftqfdujwfmz0 Jo pvs xpsl - jo e^+e^- dpmmjtjpot- ui f qbjs qspevdujpo pg ui f di bshfe I jhht)H^+* boe ofvusbmI jhht)X* jo ui f tfdpoe I jhht epvcmfu jt tuvejfe0 Xf efsjwf ui f qbjs qspevdujpo dsptt tfdujpo- $e^+ ,\ e^- \simeq \ddot{\nu}_e ,\ e^- ,\ H^+ ,\ X)X$ A A, h*0 Jo ui f MI D tfuvq- ui f di bshfe I jhht qbjs qspevdujpo $p ,\ p \simeq Z^*)\gamma^{**} \simeq H^+ ,\ H^-$ jt tuvejfe jo Sfg0]3a0 Jo Sfg0]23a- wfdups cptpo gvtjpo joup ui f mjhi u DQ.fwfo I jhht qbjst jt tuvejfe bu ui f MI D0 Jo Sfg0]24a- ej.I jhht qspevdujpo jo wbsjpvt tdfobsjpt jt ejtdvttfe0 Jo Sfg0]25a- ui f tuboebse n pefmI jhht cptpo qbjs qspevdujpo jt tuvejfe0 Jo beejujpo- tff Sfg0]26a gps ui f sbujp pg ui f dsptt tfdujpo pg ui f tjohmfu I jhht cptpo boe ui f qbjs qspevdujpo dsptt tfdujpo jo ui f dpoufyu pg ui f tuboebse n pefm0

Xf ejtdvtt ui f tjhobuvsf pg ofx I jhht qbjs qspevdujpo xjui ui f ovn fsjdbmsftvmu0 Xf dpotjefs b qspdftt $e^+ ,\ e^- \simeq \ddot{\nu}_e ,\ e^- ,\ H^+ ,\ X \simeq \ddot{\nu}_e ,\ e^- ,\ l^+\nu_l ,\ \nu_k\ddot{\nu}_k$ boe dpn qbsf ju xjui $e^+ ,\ e^- \simeq \ddot{\nu}_e ,\ e^- ,\ W^+ ,\ Z \simeq \ddot{\nu}_e ,\ e^- ,\ l^+\nu_l ,\ \nu_k\ddot{\nu}_k$0

Ui f qbqfs jt pshboj{fe bt gpmmpx t0 Jo di bqufs 3- xf fyqmbjo ui f n pefmpg Ebwjetpo boe Mphbo0 Jo di bqufs 4- xf ejtdvtt rvbouvn dpssfdujpo up ujoz wbdvvn fyqfdubujpo wbmvf jo ui f n pefm0 Jo di bqufs 5- xf ejtdvtt di bshfe I jhht boe ofvusbmI jhht qbjs qspevdujpo pg xfbl hbvhf cptpot gvtjpo qspdftt jo e^+e^- dpmmjtjpo0 Jo di bqufs 6 jt efwpufe up dpodmvtjpot boe ejtdvttjpot0

Chapter 2

Dirac neutrino mass model of Davidson and Logan

2.1 The model

Jo ui f n pefmpg Ebwjetpo boe Mphbo]2a- jo beejujpo up ui f flfm dpoufou pg ui f TN- b ofx tdbmbs epvcmfu ff$_2$ xjui ui f tbn f hbvhf rvbouvn ovn cfst bt ui f TN I jhht epvcmfu ff$_1$ boe uisff hbvhf tjohmfu sjhi u. i boefe ofvusjop flfmet ν_{R_i} bsf jouspevdfe0 Ui f sjhi u.i boefe ofvusjopt gpsn Ejsbd qbsujdmft xjui ui f uisff mfgu.i boefe ofvusjopt pg ui f TN0B hmpcbmV)2* tzn n fusz jt jouspevdfe boe ui fo bmmTN flfmet bsf tjohmfut boe ui f ofx flfmet ff$_2$ boe ν_{R_i} dbssz di bshf , 20 N bkpsbob n btt ufsn t gps ui f ν_{R_i} bsf gpscjeefo cz ui f V)2* tzn n fusz0 Ui fo pomz ff$_2$ dpvqmft up sjhi u.i boefe ofvusjopt0 Ui f Zvl bxb Mbhsbohjbo xi jdi jt jowbsjbou voefs V)2* usbotgpsn bujpo cfdpn ft-

$$\{ \quad A \quad y^d_{ij}\bar{d}_{R_i}ff^\dagger_1 Q_{L_j} \quad y^u_{ij}\bar{u}_{R_i}ff^\dagger_1 Q_{L_j}$$
$$y^l_{ij}\bar{e}_{R_i}ff^\dagger_1 L_{L_j} \quad y^\nu_{ij}\bar{\nu}_{R_i}ff^\dagger_2 L_{L_j}\,, \quad h.c. \qquad)302*$$

Jg ui f V)2* tzn n fusz jt vocspl fo- ui f wfw pg ui f ofx tdbmbs ff$_2$ wbojti ft boe ui f ofvusjopt cfdpn f tusjdumz n bttmftt]27a0

Jo psefs up hfofsbuf tn bmmEjsbd ofvusjop n bttft xjui pvu ujoz Zvl bxb dpvqmjoht y^ν- ff$_2$ n vtu i bwf b tn bmmWFW0 Up pcubjo ui f tn bmmWFW- ui f hmpcbmV)2* tzn n fusz jt fyqmjdjumz cspl fo xjui b tpgumz V)2* csfbl joh ejn fotjpo.3 ufsn 0 Ui jt ufsn jo ui f I jhht qpufoujbmi bt ui f gpsn $m^2_{12}ff^\dagger_1 ff_2$0 Ui jt sftvmut jo b sfmbujpo Sfg0]28a bn poh WFWt pg ui f uxp I jhht epvcmfut-

$$v_2 \; A \; \frac{m^2_{12} v_1}{m^2_A}, \qquad)303*$$

xi fsf v_1 efopuft ui f WFW pg ff$_1$ boe m_A jt ui f n btt pg ui f ofvusbmqtfveptdbmbs I jhht0 Jo psefs up bdi jfwf $v_2 \gg$fW gps $m_A \gg$211HfW- m^2_{12} jt pg psefs)b gfx i voesfe l fW*^{2}0 Bo fyusfn fmz mjhi u tdbmbs jt opu qsftfou jo ui f n pefmcfdbvtf ui f dpfggjdjfou pg b ejn fotjpo.3 ufsn pg ui f gpsn ff$^\dagger_2$ff$_2$ jt mbshf boe qptjujwf0

2.2 Lagrangian

Jo ui jt tfdujpo- xf qsftfou ui f Mbhsbohjbo gps ui f n pefmjo ufsn t pg n btt fjhfotubuft gps ui f ux p I jhht epvcmfut0

$$\{ \text{ A } \{_Y \, , \ \{_H \, , \ \{_G, \qquad)304*$$

x i fsf $\{_Y$- $\{_H$ boe $\{_G$ dpssftqpoe up Zvl bx b qbsu- I jhht qpufoujbmqbsu boe Hbvhf.I jhht qbsu sftqfd. ujwfm0

$\{_G$: Gauge-Higgs part

Jo ui jt tvctfdujpo- xf qsftfou ui f Mbhsbohjbo gps ui f hbvhf.I jhht tfdups0

Ux p I jhht epvcmfut bsf qbsbn fufsj–fe bt-

$$\text{ff}_1 \text{ A} \Big) \begin{matrix} H^+ \text{tjo}\,\beta \\ \frac{v}{\sqrt{2}}\text{dpt}\,\beta \, , \ \frac{h\sin\gamma + H\cos\gamma - iA\sin\beta}{\sqrt{2}} \end{matrix} \Big\{ \qquad)305*$$

$$\text{ff}_2 \text{ A} \Big) \begin{matrix} H^+ \text{dpt}\,\beta \\ \frac{v}{\sqrt{2}}\text{tjo}\,\beta \, , \ \frac{h\cos\gamma - H\sin\gamma + iA\cos\beta}{\sqrt{2}} \end{matrix} \Big\{ , \qquad)306*$$

x i fsf γ jt b n jyjoh bohmf gps DQ.fwfo I jhht0 Pof dbo x sjuf ui f dpwbsjbou efsjwbujwf gps fmfdusp.x fbl hbvhf hspvq-

$$D_\mu \text{ A } \partial_\mu \, , \ i\frac{g}{\sqrt{3}} \Big) \begin{matrix} 1 & W_\mu^+ \\ W_\mu^- & 1 \end{matrix} \Big[\, , \ i\frac{g}{3\,\text{dpt}\,\theta_W} Z_\mu \Big) \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix} \Big[\, , \ ie)A_\mu \quad \text{ubo}\,\theta_W Z_\mu * \Big) \begin{matrix} 2 & 1 \\ 1 & 1 \end{matrix} \Big[\, . \qquad)307*$$

Ui fo $\sqrt{D_\mu \text{ff}_i}^{\,2}\sqrt{}$ jt-

$$\sqrt{D_\mu \text{ff}_1}^{\,2}\sqrt{} \text{ A } \text{tjo}^2\beta\,\partial^\mu H^- \partial_\mu H^+ \, , \ \frac{2}{3}\}\partial^\mu)h\,\text{tjo}\,\gamma\,, \ H\,\text{dpt}\,\gamma*\partial_\mu)h\,\text{tjo}\,\gamma\,, \ H\,\text{dpt}\,\gamma*, \ \partial^\mu A \partial_\mu A\,\text{tjo}^2\beta|$$

$$, \ \frac{v^2\text{dpt}^2\beta}{5}\Big) W^{-\mu}W_\mu^+ \, , \ \frac{2}{3\,\text{dpt}^2\theta_W} Z^\mu Z_\mu \Big[$$

$$, \ \frac{vg\,\text{dpt}\,\beta}{3}\Big\} g\Big) W^{-\mu}W_\mu^+ \, , \ \frac{2}{3\,\text{dpt}^2\theta_W} Z^\mu Z_\mu \Big[\ )h\,\text{tjo}\,\gamma\,, \ H\,\text{dpt}\,\gamma*$$

$$, \ \frac{\text{tjo}\,\beta}{\text{dpt}\,\theta_W} Z^\mu \partial_\mu A \quad e\,\text{tjo}\,\beta)A^\mu \quad \text{ubo}\,\theta_W Z^\mu*)W_\mu^- H^+ \, , \ W_\mu^+ H^- * \sqrt{}$$

$$, \ \Big]\frac{g^2}{5}\Big) W^{-\mu}W_\mu^+ \, , \ \frac{2}{3\,\text{dpt}^2\theta} Z^\mu Z_\mu \Big[\ h^2\,\text{tjo}^2\gamma\,, \ H^2\,\text{dpt}^2\gamma\,, \ A^2\,\text{tjo}^2\beta\,, \ 3hH\,\text{tjo}\,\gamma\,\text{dpt}\,\gamma[$$

$$, \ \frac{g\,\text{tjo}^2\beta}{3}\Big\} W^{+\mu})H^-\partial_\mu A \quad A\partial_\mu H^-*, \ W^{-\mu})H^+\partial_\mu A \quad A\partial_\mu H^+*\sqrt{}$$

$$\frac{g\,\text{tjo}\,\beta}{3\,\text{dpt}\,\theta_W} Z^\mu\}A\partial_\mu)h\,\text{tjo}\,\gamma\,, \ H\,\text{dpt}\,\gamma* \quad)h\,\text{tjo}\,\gamma\,, \ H\,\text{dpt}\,\gamma*\partial_\mu A\langle$$

$$, \ \text{tjo}^2\beta\Big\}\frac{g^2}{3} W^{-\mu}W_\mu^+ \, , \ \frac{g^2}{5\,\text{dpt}^2\theta_W} Z^\mu Z_\mu \, , \ \frac{ge}{\text{dpt}\,\theta_W} Z^\mu)A_\mu \quad \text{ubo}\,\theta_W Z_\mu*$$

$$, \ e^2)A^\mu \quad \text{ubo}\,\theta_W Z^\mu*)A_\mu \quad \text{ubo}\,\theta_W Z_\mu*\sqrt{} H^- H^+$$

$$\frac{eg\,\mathrm{tjo}\,\beta}{3})A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*)W^{-}_{\mu}H^{+}\,,\;\; W^{+}_{\mu}H^{-}*)h\,\mathrm{tjo}\,\gamma\,,\;\; H\,\mathrm{dpt}\,\gamma*\;\Big\{$$

$$,\; i\Big]\frac{vg\,\mathrm{dpt}\,\beta\,\mathrm{tjo}\,\beta}{3}\Big\}W^{-\mu}\partial_{\mu}H^{+} \quad W^{+\mu}\partial_{\mu}H^{-}|$$

$$,\; \frac{g\,\mathrm{tjo}\,\beta}{3}\Big\}W^{+\mu}\Big)H^{-}\partial_{\mu})h\,\mathrm{tjo}\,\gamma\,,\;\; H\,\mathrm{dpt}\,\gamma* \quad)h\,\mathrm{tjo}\,\gamma\,,\;\; H\,\mathrm{dpt}\,\gamma*\partial_{\mu}H^{-}\Big($$

$$,\; W^{-\mu}\Big))h\,\mathrm{tjo}\,\gamma\,,\;\; H\,\mathrm{dpt}\,\gamma*\partial_{\mu}H^{+} \quad H^{+}\partial_{\mu})h\,\mathrm{tjo}\,\gamma\,,\;\; H\,\mathrm{dpt}\,\gamma*\Big(\surd$$

$$,\; \frac{g\,\mathrm{tjo}^{2}\,\beta}{3\,\mathrm{dpt}\,\theta_W}Z^{\mu})H^{+}\partial_{\mu}H^{-} \quad H^{-}\partial_{\mu}H^{+}*$$

$$,\; e\,\mathrm{tjo}^{2}\,\beta)A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*\}\;H^{+}\partial_{\mu}H^{-} \quad H^{-}\partial_{\mu}H^{+} \quad \frac{g}{3})W^{-}_{\mu}H^{+} \quad W^{+}_{\mu}H^{-}*A\,\langle\;\Big\{, \qquad)308*$$

$$\underset{\surd}{D}_{\mu}\mathrm{ff}_{2}\,\underset{\surd}{^{2}} \quad \mathrm{A} \quad \mathrm{dpt}^{2}\,\beta\partial^{\mu}H^{-}\partial_{\mu}H^{+}\,,\;\; \frac{2}{3}\Big\}\partial^{\mu})h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\partial_{\mu})h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*,\;\; \partial^{\mu}A\partial_{\mu}A\,\mathrm{dpt}^{2}\,\beta|$$

$$,\; \frac{v^{2}\,\mathrm{tjo}^{2}\,\beta}{5}\Big)W^{-\mu}W^{+}_{\mu}\,,\;\; \frac{2}{3\,\mathrm{dpt}^{2}\,\theta_W}Z^{\mu}Z_{\mu}\Big[$$

$$,\; \frac{vg\,\mathrm{tjo}\,\beta}{3}\Big\}g\Big)W^{-\mu}W^{+}_{\mu}\,,\;\; \frac{2}{3\,\mathrm{dpt}^{2}\,\theta_W}Z^{\mu}Z_{\mu}\Big[\;)h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*$$

$$\frac{\mathrm{dpt}\,\beta}{\mathrm{dpt}\,\theta_W}Z^{\mu}\partial_{\mu}A\,,\;\; e\,\mathrm{dpt}\,\beta)A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*)W^{-}_{\mu}H^{+}\,,\;\; W^{+}_{\mu}H^{-}*\Big($$

$$,\; \Big]\frac{g^{2}}{5}\Big)W^{-\mu}W^{+}_{\mu}\,,\;\; \frac{2}{3\,\mathrm{dpt}^{2}\,\theta}Z^{\mu}Z_{\mu}\Big[\;\; h^{2}\,\mathrm{dpt}^{2}\,\gamma\,,\;\; H^{2}\,\mathrm{tjo}^{2}\,\gamma\,,\;\; A^{2}\,\mathrm{dpt}^{2}\,\beta \quad 3hH\,\mathrm{tjo}\,\gamma\,\mathrm{dpt}\,\gamma[$$

$$,\; \frac{g\,\mathrm{dpt}^{2}\,\beta}{3}\Big\}W^{+\mu})H^{-}\partial_{\mu}A \quad A\partial_{\mu}H^{-}*,\;\; W^{-\mu})H^{+}\partial_{\mu}A \quad A\partial_{\mu}H^{+}*\surd$$

$$,\; \frac{g\,\mathrm{dpt}\,\beta}{3\,\mathrm{dpt}\,\theta_W}Z^{\mu}\}A\partial_{\mu})h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma* \quad)h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\partial_{\mu}A\langle$$

$$,\; \mathrm{dpt}^{2}\,\beta\Big\}\frac{g^{2}}{3}W^{-\mu}W^{+}_{\mu}\,,\;\; \frac{g^{2}}{5\,\mathrm{dpt}^{2}\,\theta_W}Z^{\mu}Z_{\mu}\,,\;\; \frac{ge}{\mathrm{dpt}\,\theta_W}Z^{\mu})A_{\mu} \quad \mathrm{ubo}\,\theta_W Z_{\mu}*$$

$$,\; e^{2})A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*)A_{\mu} \quad \mathrm{ubo}\,\theta_W Z_{\mu}*\surd H^{-}H^{+}$$

$$,\; \frac{eg\,\mathrm{dpt}\,\beta}{3})A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*)W^{-}_{\mu}H^{+}\,,\;\; W^{+}_{\mu}H^{-}*)h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\;\Big\{$$

$$,\; i\Big]\frac{vg\,\mathrm{dpt}\,\beta\,\mathrm{tjo}\,\beta}{3}\Big\}W^{+\mu}\partial_{\mu}H^{-} \quad W^{-\mu}\partial_{\mu}H^{+}|$$

$$,\; \frac{g\,\mathrm{dpt}\,\beta}{3}\Big\}W^{+\mu}\Big))h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\partial_{\mu}H^{-} \quad H^{-}\partial_{\mu})h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\Big($$

$$,\; W^{-\mu}\Big)H^{+}\partial_{\mu})h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma* \quad)h\,\mathrm{dpt}\,\gamma \quad H\,\mathrm{tjo}\,\gamma*\partial_{\mu}H^{+}\Big(\surd$$

$$,\; \frac{g\,\mathrm{dpt}^{2}\,\beta}{3\,\mathrm{dpt}\,\theta_W}Z^{\mu})H^{+}\partial_{\mu}H^{-} \quad H^{-}\partial_{\mu}H^{+}*$$

$$,\; e\,\mathrm{dpt}^{2}\,\beta)A^{\mu} \quad \mathrm{ubo}\,\theta_W Z^{\mu}*\}\;H^{+}\partial_{\mu}H^{-} \quad H^{-}\partial_{\mu}H^{+} \quad \frac{g}{3})W^{-}_{\mu}H^{+} \quad W^{+}_{\mu}H^{-}*A\,\langle\;\Big\{. \qquad)309*$$

Vtjoh Fr0)308* boe Fr0)309*- pof dbo xsjuf uif Mbhsbohjbo pg I jhht tfdups bt-

$$
\begin{aligned}
\sqrt{D_\mu ff_1}^2_{\sqrt{}} , \sqrt{D_\mu ff_2}^2_{\sqrt{}} \quad A \quad & \frac{2}{3})\partial^\mu h \partial_\mu h \ , \ \partial^\mu H \partial_\mu H \ , \ \partial^\mu A \partial_\mu A* , \ \partial^\mu H^- \partial_\mu H^+ \\
, \quad & \frac{v^2}{5}\Big) W^{-\mu} W^+_\mu \ , \ \frac{2}{3\,\text{dpt}^2\,\theta_W} Z^\mu Z_\mu \Big[\\
, \quad & \frac{vg^2}{3}\Big) W^{-\mu} W^+_\mu \ , \ \frac{2}{3\,\text{dpt}^2\,\theta_W} Z^\mu Z_\mu \Big[\ \}h\,\text{tjo})\beta \ , \ \gamma* , \ H\,\text{dpt})\beta \ , \ \gamma*\langle \\
, \quad & \Big]\frac{g^2}{5}\Big) W^{-\mu} W^+_\mu \ , \ \frac{2}{3\,\text{dpt}^2\,\theta_W} Z^\mu Z_\mu \Big[\ )h^2 \ , \ H^2 \ , \ A^2* \\
, \quad & \frac{g}{3}\Big\} W^{+\mu})H^- \partial_\mu A \quad A\partial_\mu H^-* , \ W^{-\mu})H^+ \partial_\mu A \quad A \partial_\mu H^+ *\sqrt{} \\
, \quad & \frac{g}{3\,\text{dpt}\,\theta_W} Z^\mu \ \}A\partial_\mu h \quad h \partial_\mu A*\text{dpt})\beta \ , \ \gamma* \quad)A\partial_\mu H \quad H\partial_\mu A*\text{tjo})\beta \ , \ \gamma*\langle \\
, \quad & \Big\}\frac{g^2}{3} W^{-\mu} W^+_\mu \ , \ \frac{g^2\,\text{dpt}^2\,3\theta_W}{5\,\text{dpt}^2\,\theta_W} Z^\mu Z_\mu \ , \ g^2\,\text{ubo}\,\theta_W\,\text{dpt}\,3\theta_W A^\mu Z_\mu \ , \ e^2 A^\mu A_\mu \Big(\ H^- H^+ \\
, \quad & \frac{eg}{3})A^\mu \quad \text{ubo}\,\theta_W Z^\mu*)W^-_\mu H^+ \ , \ W^+_\mu H^- *\}h\,\text{dpt})\beta \ , \ \gamma* \quad H\,\text{tjo})\beta \ , \ \gamma*\langle \ \Big\{ \qquad)30 * \\
, \quad & i\Big]\frac{g}{3}\Big\}\Big)h\,\text{dpt})\beta \ , \ \gamma* \quad H\,\text{tjo})\beta \ , \ \gamma*\Big()W^{+\mu}\partial_\mu H^- \quad W^{-\mu}\partial_\mu H^+* \\
, \quad & \Big)\partial^\mu h\,\text{dpt})\beta \ , \ \gamma* \quad \partial^\mu H\,\text{tjo})\beta \ , \ \gamma*\Big()W^-_\mu H^+ \quad W^+_\mu H^- *\sqrt{} \qquad)3021* \\
, \quad & \Big)eA^\mu \ , \ \frac{g\,\text{dpt}\,3\theta_W}{3\,\text{dpt}\,\theta_W} Z^\mu \Big[\ )H^+ \partial_\mu H^- \quad H^- \partial_\mu H^+ * \\
& \frac{eg}{3})A^\mu \quad \text{ubo}\,\theta_W Z^\mu*)W^-_\mu H^+ \quad W^+_\mu H^- * A \ \Big\{ . \qquad)3022*
\end{aligned}
$$

Ui jt dpn qmfuft uif efsjwbujpo pg uif Mbhsbohjbo gps Hbvhf.I jhht tfdups0 Xf vtf uif Mbhsbohjbo boe efsjwf Gfzonbo svmft0

$\{_Y$: Yukawa part

Zvl bxb qbsu $\{_Y$ jt xsjuufo jo Fr0)302*0 Ui f ui sff ufsn t pg cfhjoojoht bsf ui f tbn f bt TN0 Xf fyqboe ui f gpvsui ufsn vtjoh Fr0)306*0

$$
\begin{aligned}
y^\nu_{ij}\ddot{\nu}_{R_i} ff^\dagger_2 L_{L_j} \ , \ h.c. \quad A \quad & \nu_i\Big)\frac{m_{\nu i}}{v}\Big(\ddot{\nu}_i \frac{\text{dpt}\,\gamma h \quad \text{tjo}\,\gamma H}{\text{tjo}\,\beta} \quad i\nu_i\Big)\frac{m_{\nu_i}}{v}\Big(\gamma_5 \ddot{\nu}_i\,\text{dpu}\beta A \\
, \quad & ^\dagger \bar{3}\,\text{dpu}\beta \ddot{l}_i V_{ij}\Big)\frac{m_{\nu_j}}{v}\Big(\nu_{R_j} H^+ \ , \ h.c., \qquad)3023*
\end{aligned}
$$

x i fsf m_ν efopuft ui f ofvusjop n bttft boe V efopuft ui f N bl j—Obl bhbx b—Tbl bub)N OT* n busjy-

$$
V \ A \Bigg) \begin{matrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} \quad c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} \quad s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} \quad c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} \quad s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{matrix} \ , \qquad)3024*
$$

x i fsf c_{ij} A dpt θ_{ij}- s_{ij} A tjo θ_{ij}0 θ_{ij} efopuf ui f n jyjoh bohmft0 δ jt ui f DQ wjpmbujpo qi btf0 Ui fz ubl f ui fjs wbmvft x jui jo ui f sbohf $]1, 3\pi a$

{ $_H$: **Higgs potential part**

I jhht qpufoujbm qbsu { $_H$ frvbmt up uif n jovt pg uif usff mfwfm qpufoujbm V_{tree}0

$$\{_H \text{ A } \; V_{tree} \text{ A } \Big) \int_{i=1,2} \Big) m_{ii}^2 ff_i^\dagger ff_i \; , \; \frac{\lambda_i}{3}) ff_i^\dagger ff_i *^2 \Big[\quad) m_{12}^2 ff_1^\dagger ff_2 \; , \; h.c.* , \; \lambda_3) ff_1^\dagger ff_1 *) ff_2^\dagger ff_2 * , \; \lambda_4 \; ff_1^\dagger ff_2 \;^2 \quad)3025*$$

I fsf xf qbsbn fufsj{f uif uxp I jhht TV)3* epvcmfut-

$$ff_1 \text{ A } \frac{2}{\sqrt{3}} \Big) \begin{matrix} \phi_1^1 \; , \; i\phi_1^2 \\ \phi_1^3 \; , \; i\phi_1^4 \end{matrix} \Big[\; , \quad ff_2 \text{ A } \frac{2}{\sqrt{3}} \Big) \begin{matrix} \phi_2^1 \; , \; i\phi_2^2 \\ \phi_2^3 \; , \; i\phi_2^4 \end{matrix} \Big[\; , \qquad)3026*$$

xi fsf ff$_1$ 't wbdvvn fyqfdubujpo wbmvf jt ofbsmz frvbm up uif fmfduspxfbl csfbljoh tdbmf boe uif tfdpoe I jhht ff$_2$ i bt b tn bmm wbdvvn fyqfdubujpo wbmvf xi jdi hjwft sjtf up ofvusjop n btt0 V)2*$'$ di bshf jt bttjhofe up uif tfdpoe I jhht0 Ui f V)2*$'$ hmpcbm tzn n fusz jt cspl fo tpgumz xjui uif ufsn $m_{12}^2$0 Jo ui jt qbqfs- xf jouspevdf uif gpmmpxjoh sfbm P)5* sfqsftfoubujpo gps fbdi epvcmfu- cfdbvtf ui jt qbsbn fusj{bujpo jt dpowfojfou xi fo dpn qvujoh uif pof mppq dpssfdufe ffifdujwf qpufoujbm0

$$\phi_1^\alpha \text{ A } \Big) \begin{matrix} \phi_1^1 \\ \phi_1^2 \\ \phi_1^3 \\ \phi_1^4 \end{matrix} \sum , \quad \phi_2^\alpha \text{ A } \Big) \begin{matrix} \phi_2^1 \\ \phi_2^2 \\ \phi_2^3 \\ \phi_2^4 \end{matrix} \sum , \quad \phi_1^\alpha \text{ A } \Big) \begin{matrix} \phi_1^2 \\ \phi_1^1 \\ \phi_1^4 \\ \phi_1^3 \end{matrix} \sum . \qquad)3027*$$

Vtjoh uif opubujpo bcpwf- uif usff mfwfm ffifdujwf qpufoujbm jouspevdfe jo Fr0)3025* dbo cf xsjuufo bt-

$$\begin{aligned} V_{tree} \quad \text{A} \quad & \frac{m_{11}^2}{3} \int_{a=1}^{4}) \phi_1^a *^2 \; , \; \frac{m_{22}^2}{3} \int_{a=1}^{4}) \phi_2^a *^2 \quad m_{12}^2 \int_{a=1}^{4} \phi_1^a \phi_2^a \\ & , \; \frac{\lambda_1}{9} \Big) \int_{a=1}^{4} {\phi_1^a}^2 \Big\{^2 \; , \; \frac{\lambda_2}{9} \Big) \int_{a=1}^{4} {\phi_2^a}^2 \Big\{^2 \; , \; \frac{\lambda_3}{5} \Big) \int_{a=1}^{4} {\phi_1^a}^2 \Big\{ \Big) \int_{a=1}^{4} {\phi_2^a}^2 \Big\{ \\ & , \; \frac{\lambda_4}{5} \Big) \Big) \int_{a=1}^{4} \phi_1^a \phi_2^a \Big\{^2 \; , \; \Big) \int_{a=1}^{4} \phi_1^a \phi_2^a \Big\{^2 \qquad)3028* \end{aligned}$$

xi fsf pof dbo di pptf m_{12}^2 sfbm boe qptjujwf0 X jui uif opubujpo pg Fr0)3027*- uif tpgumz cspl fo hmpcbm tzn n fusz V)2*$'$ dpssftqpoet up uif gpmmpxjoh usbotgpsn bujpo po ϕ_2^a -

$$\phi_2' \text{ A } O_{U(1)'} \phi_2 \text{ A } \Big) \begin{matrix} \text{dpt}\,\phi & \text{tjo}\,\phi & 1 & 1 \\ \text{tjo}\,\phi & \text{dpt}\,\phi & 1 & 1 \\ 1 & 1 & \text{dpt}\,\phi & \text{tjo}\,\phi \\ 1 & 1 & \text{tjo}\,\phi & \text{dpt}\,\phi \end{matrix} \sum \phi_2 . \qquad)3029*$$

ϕ_1 epft opu usbotgpsn voefs V)2*$'$ 0 Ui fsfgpsf V)2*$'$ jt cspl fo tpgumz xi fo m_{12}^2 epft opu wbojti 0

X jui pvu mptt pg hfofsbmjuz- pof dbo di pptf uif wbdvvn fyqfdubujpo wbmvft pg I jhht xjui uif gpsn hjwfo bt-

$$< \phi_1 > \text{A} \Big) \begin{matrix} 1 \\ 1 \\ v\,\text{dpt}\,\beta \\ 1 \end{matrix} \sum , \; < \phi_2 > \text{A} \Big) \begin{matrix} v\,\text{tjo}\,\beta\,\text{tjo}\,\alpha\,\text{dpt}\,\theta' \\ v\,\text{tjo}\,\beta\,\text{tjo}\,\alpha\,\text{tjo}\,\theta' \\ v\,\text{tjo}\,\beta\,\text{dpt}\,\alpha\,\text{dpt}\,\theta' \\ v\,\text{tjo}\,\beta\,\text{dpt}\,\alpha\,\text{tjo}\,\theta' \end{matrix} \sum , \qquad)302:*$$

xifsf uif sbohf gps θ' jt $]1, 3\pi$* boe uif sbohf gps β boe α jt $]1, \frac{\pi}{2}$a0 Xf dbmmuif gpvs psefs qbsbnfufst bt φ_I A)$v, \beta, \alpha, \theta'$*,)$I$ A $2, 3, 4, 5$*0 Xifo m_{12} wbojtift- cz ubljoh ϕ A θ' jo Fr0)3029*- pof dbo spubuf θ' bxbz jo Fr0)302:*0 Gps uif nptu hfofsbmdbtf- jo upubmuifsf bsf gpvs joefqfoefou psefs qbsbnfufst xifo V)2*$'$ tznnfusz jt cspl fo0

Chapter 3

Quantum correction to tiny vacuum expectation value in the model of Davidson and Logan

Jo ui jt di bqufs- xf tuvez ui f tubcjmjuz pg ui f wbdvvn bhbjotu ui f rvbouvn dpssfdujpot0 Ui jt i bt cffo tuvejfe fyufotjwfmz jo Sfg0]4a

3.1 Tree level potential

Jo ui jt tfdujpo- xf ejtdvtt ui f tubcjmjuz pg ui f wbdvvn pg ui f usff mfwfm qpufoujbm jo Fr 0)3025*0 Xf bmtp vtf ui f qbsbn fusj–bujpo jo ufsn t pg $)v, \beta, \alpha, \theta'*$ pg Fr 0)302: * gps WFW pg ui f ux p epvcmfu I jhhtft0

Ui f dpotusbjout po ui f rvbsujd dpvqmjoht gspn dpoejujpo ui bu ui f usff mfwfm qpufoujbm jt cpvoefe cfmpx- bsf efsjwfe jo Sfg0]2a-]6a-]29a0

$$\lambda_1 > 1, \ \lambda_2 > 1, \qquad)402*$$

$$\sqrt{\lambda_1 \lambda_2} \geq \lambda_3, \qquad)403*$$

$$\sqrt{\lambda_1 \lambda_2} \geq \lambda_3 \ , \ \ \lambda_4. \qquad)404*$$

Jo beejujpo up ui f dpoejujpot po ui f rvbsujd ufsn t- pof dbo dpotusbjo ui f qbsbn fufst jodmvejoh ui f rvbesbujd ufsn t tp ui bu ui f eftjsfe wbdvvn tbujtgjft ui f hmpcbm njojn vn dpoejujpot pg ui f qpufoujbm0 Bcpvu ui f hmpcbm njojn vn pg ui f usff qpufoujbm- ju xbt ti px o ui bu ui f ofvusbm dibshf ofvusbm wbdvvn jt mpx fs ui bo ui bu pg ui f di bshf csfbl joh wbdvvn]6a0 Xf ui fsfgpsf tfu α –fsp0 Xf bmtp sfrvjsf ui f wbdvvn fyqfdubujpo wbmvf pg ui f tfdpoe I jhht jt n vdi tn bmmfs ui bo ui bu pg ui f gjstu I jhht- x i jdi jn qmjft ui bu ubo β jt tn bmm0 Jo ufsn t pg ui f qbsbn fusj–bujpo jo Fr 0)302: * x jui α A 1- ui f qpufoujbm dbo cf x sjuufo bt-

$$V_{tree})v, \beta, \theta'* \text{ A } A)\beta*v^4 \ , \ \ B)\beta, \theta'*v^2, \qquad)405*$$

x i fsf-

$$A)\beta* \quad \text{A} \quad \frac{\lambda_1}{9} \, \text{dpt}^4 \beta \ , \ \ \frac{\lambda_2}{9} \, \text{tjo}^4 \beta \ , \ \ \Big) \frac{\lambda_3}{5} \ , \ \ \frac{\lambda_4}{5} \Big[\ \text{dpt}^2 \beta \, \text{tjo}^2 \beta,$$

$$B)\beta,\theta'* \quad A \quad \frac{m_{11}^2}{3}\,\mathrm{dpt}^2\beta\,, \;\; \frac{m_{22}^2}{3}\,\mathrm{tjo}^2\beta \quad m_{12}^2\,\mathrm{dpt}\,\theta'\,\mathrm{dpt}\,\beta\,\mathrm{tjo}\,\beta. \qquad)406*$$

Xf flstufloe ui f hmpcbmn jojn vn pg V_{tree}0 Ui f tubujpobsz dpoejujpot $\frac{\partial V_{tree}}{\partial\varphi_I}$ A $1)I$ A $2,3,5*$- bsf xsjuufo bt-

$$v)3Av^2\,,\;\; B* \text{ A } 1, \qquad)407*$$

$$3r_4 \text{ A } \mathrm{tjo}\,3\beta\frac{)2 \quad r_1r_2*\mathrm{dpt}\,3\beta\,, \;\; r_2 \quad r_1r_3}{r_2\,\mathrm{dpt}^2\,3\beta)r_3\,, \;\; 2*\mathrm{dpt}\,3\beta r_2}, \qquad)408*$$

$$m_{12}^2\,\mathrm{tjo}\,\theta'\,\mathrm{tjo}\,3\beta \text{ A } 1, \qquad)409*$$

xi fsf $r_i)i$ A $2 \gg 5*$ bsf efflofe bt-

$$r_1 \quad A \quad \frac{m_{11}^2 \quad m_{22}^2}{m_{11}^2\,, \;\; m_{22}^2},$$
$$r_2 \quad A \quad \frac{\lambda_1 \quad \lambda_2}{\lambda_1\,, \;\; \lambda_2 \quad 3\lambda_3 \quad 3\lambda_4},$$
$$r_3 \quad A \quad \frac{\lambda_1\,, \;\; \lambda_2\,, \;\; 3\lambda_3\,, \;\; 3\lambda_4}{\lambda_1\,, \;\; \lambda_2 \quad 3\lambda_3 \quad 3\lambda_4},$$
$$r_4 \quad A \quad \frac{m_{12}^2\,\mathrm{dpt}\,\theta'}{m_{11}^2\,, \;\; m_{22}^2}. \qquad)40\,*$$

Ui f tubujpobsz dpoejujpot Fr0)407* boe Fr0)408* dpssftqpoe up Fr0)E04* pg Sfg0]9a0 I fsf xf tpmwf ui fn fyqmjdjumz cz usfbujoh ui f tpgu csfbl joh ufsn m_{12} bt qfsuvscbujpo0 Ui f opo.–fsp tpmvujpo gps v^2 jo Fr0)407* jt xsjuufo bt-

$$v^2 \text{ A } \frac{B}{3A} \text{ A } \quad 5\frac{m_{11}^2\,, \;\; m_{22}^2}{\lambda_1\,, \;\; \lambda_2 \quad \lambda_{34}}\frac{2\,, \;\; r_1\,\mathrm{dpt}\,3\beta \quad 3r_4\,\mathrm{tjo}\,3\beta}{\mathrm{dpt}^2\,3\beta\,, \;\; r_3\,, \;\; 3r_2\,\mathrm{dpt}\,3\beta}, \qquad)4021*$$

xi fsf λ_{34} A $\lambda_3\,, \;\; \lambda_4$0 Tvctujuvujoh ju joup V_{tree}- pof pcubjot-

$$V_{tree} \sim V_{min.} \text{ A } \quad \frac{)m_{11}^2\,, \;\; m_{22}^2*^2}{3)\lambda_1\,, \;\; \lambda_2 \quad \lambda_{34}*}\frac{)2\,, \;\; r_1\,\mathrm{dpt}\,3\beta \quad 3r_4\,\mathrm{tjo}\,3\beta*^2}{\mathrm{dpt}^2\,3\beta\,, \;\; r_3\,, \;\; 3r_2\,\mathrm{dpt}\,3\beta}, \qquad)4022*$$

Gps opo.–fsp m_{12}^2 boe tjo 3β- ui f tpmvujpo pg Fr0)409* jt tjo θ' A 10 Pof tujmmoffet up floe β bn poh ui f tpmvujpot pg Fr0)408*- xi jdi rfibet up ui f n jojn vn pg $V_{min.}$0 Xf tpmwf Fr0)408* boe efufsn jof β cz usfbujoh r_4 $)m_{12}^2*$ bt b tn bmmfyqbotjpo qbsbn fufs0 Pof dbo fbtjmz floe ui f bqqspyjn buf tpmvujpot bt-

$$\left\{\begin{array}{ll})2*\mathrm{tjo}\,\beta \text{ A } \frac{\lambda_1 m_{12}^2}{|m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}|}, & \mathrm{dpt}\,\theta' \text{ A } \mathrm{tjho})m_{22}^2\lambda_1 \quad m_{11}^2\lambda_{34}*, \\ )3*\mathrm{dpt}\,\beta \text{ A } \frac{\lambda_2 m_{12}^2}{|m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}|}, & \mathrm{dpt}\,\theta' \text{ A } \mathrm{tjho})m_{11}^2\lambda_2 \quad m_{22}^2\lambda_{34}*, \\ )4*\mathrm{dpt}\,3\beta \text{ A } \frac{m_{11}^2(\lambda_{34}+\lambda_2) - m_{22}^2(\lambda_{34}+\lambda_1)}{m_{11}^2(-\lambda_{34}+\lambda_2) + m_{22}^2(-\lambda_{34}+\lambda_1)}\,, \;\; O)r_4*, & \end{array}\right. \qquad)4023*$$

Dpssftqpoejoh up fbdi tpmvujpo-)2*≫)4* pg Fr0)4023*- ui f wbdvvn fyqfdubujpo wbmvf v^2 boe ui f n jo. jn vn pg ui f qpufoujbmbsf pcubjofe0

$$)v^2, V_{tree}* \text{ A } \left\{\begin{array}{l})2*\Big) \frac{2m_{11}^2}{\lambda_1}\,, \;\; 3\lambda_1)m_{22}^2 \quad m_{11}^2*\Big)\frac{m_{12}^2}{m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}}\Big(^2, \quad \frac{m_{11}^4}{2\lambda_1}\,, \;\; \frac{m_{12}^4 m_{11}^2}{m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}}\Big[\,, \\ )3*\Big) \frac{2m_{22}^2}{\lambda_2}\,, \;\; 3\lambda_2)m_{11}^2 \quad m_{22}^2*\Big)\frac{m_{12}^2}{m_{11}^2\lambda_1 - m_{22}^2\lambda_{34}}\Big(^2, \quad \frac{m_{22}^4}{2\lambda_2}\,, \;\; \frac{m_{12}^4 m_{22}^2}{m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}}\Big[\,, \\ )4*\Big)3\frac{(\lambda_{34}-\lambda_2)m_{11}^2 + (\lambda 34 - \lambda_1)m_{22}^2}{\lambda_1\lambda_2 - \lambda_{34}^2}\,, \;\; O)r_4*, \quad \frac{\lambda_2 m_{11}^4 - 2m_{11}^2 m_{22}^2\lambda_{34} + \lambda_1 m_{22}^2}{2(\lambda_1\lambda_2 - \lambda_{34}^2)}\,, \;\; O)r_4*\Big(. \end{array}\right. \qquad)4024*$$

Uif mfbejoh ufsnt pg uif wbdvvn fyqfdubujpo wbmvft bhsff xjui uiptf pcubjofe jo Z_2 tznnfusjd npefm

)2*tjo β A $O)r_4$*	$\frac{m_{11}^4}{2\lambda_1}$	$\frac{m_{12}^4}{\lambda_3+\lambda_4-\frac{m_{22}^2}{m_{11}^2}\lambda_1}$
)3*dpt β A $O)r_4$*	$\frac{m_{22}^4}{2\lambda_2}$	$\frac{m_{12}^4}{\lambda_3+\lambda_4-\frac{m_{11}^2}{m_{22}^2}\lambda_2}$
)4*dpt 3β A O)2*	$\frac{\lambda_1 m_{11}^4-2m_{11}^2m_{22}^2(\lambda_3+\lambda_4)+\lambda_2 m_{22}^4}{2\{\lambda_1\lambda_2-(\lambda_3+\lambda_4)^2\}}$	

Ubcmf 402; Dmbttjgjdbujpo pg uif tpmvujpot xjui opo–fsp tjo β pg uif tubujpobsz dpoejujpot pgI jhht qpufoujbm0 Gps)4*- $O)r_4$* dpssfdujpo jt opu tipxo0

	dpt θ' A 1
)5*tjo β A 1	$\frac{m_{11}^4}{2\lambda_1}$
)6*dpt β A 1	$\frac{m_{22}^4}{2\lambda_2}$

Ubcmf 403; Dmbttjgjdbujpo pg uif tpmvujpot xjui tjo 3β A 10

]2: b0 Jg tjo 3β A 1- uifo r_4 nvtu cf wbojtijoh boe dpt θ' A 1 gspn Fr0)408* boe Fr0)409*0 Uif wbdvvn fofshjft pg uif opo.–fsp tjo 3β tpmvujpot bsf tipxo jo ubcmf 04020 Jo ubcmf 0402- uif wbdvvn fofshjft pg uif tpmvujpot xjui tjo 3β A 1 bsf tvnnbsj–fe0

Ofyu xf efsjwf uif dpotusbjout po uif qbsbnfufst tp uibu uif tpmvujpo dpssftqpoejoh up)2* jo ubcmf 0402 cfdpnft uif hmpcbm njojnvn pg uif qpufoujbm0 Tjodf uif puifs dbtft)3*.)6* ep opu ibwf eftjsfe qspqfsujft - xf sftusjdu uif qbsbnfufs tqbdf tp uibu uiftf tpmvujpot dbo opu cf b hmpcbm njojnvn0 Tjodf v nvtu ibwf mbshf qptjujwf wbdvvn fyqfdubujpo wbmvf- m_{11}^2 nvtu cf ofhbujwf0 Jo psefs uibu uif wbdvvn fofshz pg)2* jt mpxfs uibo uibu pg)5* -

$$m_{22}^2\lambda_1 \quad m_{11}^2\lambda_{34} > 1, \quad)\text{dpt}\,\theta' \text{ A } 2*. \qquad)4025*$$

Xifo Fr0)4025* jt tbujtgjfe boe uif tpmvujpo)2* epft fyjtu- pof dbo tipx uibu uif wbdvvn fofshz pg tpmvujpo)4* jt ijhifs uibo uibu pg)2*0 Gvsuifsnpsf xifo $m_{22}^2 > 1$- uif tpmvujpot dpssftqpoejoh up)3* boe)6* bsf opu sfbmj–fe0 Uifo pof dbo tubuf uif sfhjpo pg qbsbnfufs tqbdf xijdi jt dpotjtufou xjui uif dbtf uibu uif wbdvvn)2* cfdpnft hmpcbm njojnvn jt-

$$m_{11}^2 < 1, \quad m_{22}^2 > 1, \quad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2}\lambda_1. \qquad)4026*$$

Ofyu xf dpotjefs uif dbtf xjui ofhbujwf $m_{22}^2$0 Jo uijt dbtf xf jnqptf uif beejujpobm dpoejujpo tp uibu uif wbdvvn fofshjft dpssftqpoejoh up)3* boe)6* bsf ijhifs uibo uibu pg)2*0

$$\frac{m_{11}^4}{\lambda_1} > \frac{m_{22}^4}{\lambda_2}. \qquad)4027*$$

Uifo uif dpoejujpo gps)2* jt hmpcbm njojnvn jo uijt dbtf jt-

$$m_{11}^2 < 1, \quad m_{22}^2 < 1, \quad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2}\lambda_1, \quad \lambda_2\frac{m_{11}^2}{m_{22}^2} > \lambda_1\frac{m_{22}^2}{m_{11}^2}. \qquad)4028*$$

Jo uif gpmmpxjoh tfdujpot- xf fyqmpsf uif sfhjpot gps uif qbsbnfufst pcubjofe jo Fr0)4026*- Fr0)4028*- Fr0)403* boe Fr0)4041*0

3.2 One loop correction

Jo uijt tfdujpo- xf efsjwf uif fggfdujwf qpufoujbm xjui jo pof mppq bqqspyjnbujpo0 Uif tubujpobsz dpoejujpo xjui sftqfdu up uif psefs qbsbnfufst efufsnjof uif wbdvvn fyqfdubujpo wbmvf pg uif I jhht gjfme vq up pof mppq mfwfm0 Uifo pof dbo bshvf xifuifs uif usff mfwfm wbdvvn jt tubcmf bhbjotu rvbouvn dpssfdujpo0 Npsfpwfs xf dbo rvboujubujwfmz tuvez uif tj–f pg uif dpssfdujpot0

3.2.1 Effective potential in one loop and renormalization

Xf jouspevdf b sfbm tdbmbs gjfmet xjui fjhiu dpn qpofout bt- ϕ_i A $)\phi_1^1, \phi_1^2, \phi_1^3, \phi_1^4, \phi_1^5, \phi_1^6, \phi_1^7, \phi_1^{8*T},)i$ A $2 \gg$ 9*0 Xjui uif opubujpo bcpwf- uif pof mppq fggfdujwf bdujpo jt hjwfo bt-

$$
\begin{aligned}
{}^{1loop}_{eff} \quad & \text{A} \quad i\frac{2}{3}\,\text{mph efu}D^{-1})\phi*, \\
D^{-1} \quad & \text{A} \quad \Box\,,\ \ M_T^2, \qquad)4029*
\end{aligned}
$$

xifsf M_T^2 jt uif n btt trvbsfe n busjy pg uif I jhht qpufoujbm-

$$
\begin{aligned}
M_T^2 \quad \text{A} \quad & M^2)\phi*, \ \Bigg) \begin{matrix} m_{11}^2 * \ 2 & 1 \\ 1 & m_{22}^2 * \ 2 \end{matrix} \Bigg[\quad m_{12}^2\sigma_1, \\
M^2)\phi*_{ij} \quad \text{A} \quad & \frac{\partial^2 V_{tree}^{(4)}}{\partial\phi_i\partial\phi_j}, \qquad)402{:}*
\end{aligned}
$$

xifsf 2)1* efopuft 5 × 5 voju)–fsp* n busjy0 σ_1 jt efgjofe bt-

$$
\sigma_1 \ \text{A} \Bigg) \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix} \Bigg[. \qquad)4031*
$$

Jo Fr0)4031*- 2)1* bmtp efopuft b gpvs cz gpvs voju)–fsp* n busjy0 Jo n pejgjfe n jojn bm tvcusbdujpo tdifn f- uif gjojuf qbsu pg uif pof mppq fggfdujwf qpufoujbm cfdpn ft-

$$
\begin{aligned}
V_{1loop} \quad \text{A} \quad & \frac{\mu^{4-d}}{3} \Big[\ \frac{d^d k}{)3\pi^{*d} i}\,\text{Us mph})M_T^2 \quad k^2*, \ \ V_c, \\
\text{A} \quad & \frac{2}{75\pi^2}\text{Us} \Big\} M_T^4 \Big)\,\text{mph}\,\frac{M_T^2}{\mu^2} \quad \frac{4}{3}\Big[\Big(. \qquad)4032*
\end{aligned}
$$

V_c efopuft uif dpvoufs ufsn t boe uif efsjwbujpo pg V_c dbo cf gpvoe jo Bqqfoejy B0

3.2.2 One loop corrections to the vacuum expectation values

Jo uijt tvctfdujpo- xf dpn qvuf uif pof mppq dpssfdujpot up uif wbdvvn fyqfdubujpo wbmvft0 Vtjoh uif tzn n fusz pg uif n pefm jo hfofsbm pof dbo dippptf φ_I A $)v, \beta, \alpha, \theta'*$ bt uif wbdvvn fyqfdubujpo wbmvft pgI jhht qpufoujbm0 Ui fjs wbmvft bsf pcubjofe bt ui f tubujpobsz qpjout pg ui f pof mppq dpssfdufe ffifdujwf qpufoujbm V A V_{tree} , V_{1loop}-

$$\frac{\partial V}{\partial \varphi_I} \text{ A } 1. \qquad)4033*$$

Cz efopujoh ui f wbdvvn fyqfdubujpo wbmvft bt tvn pg ui f usff mfwfmpoft boe ui f pof mppq dpssfdujpot up ui fn =φ_I A $\varphi_I^{(0)}$, $\varphi_I^{(1)}$- xf efsjwf ui f pof mppq dpssfdufe qbsut0 Ui f efsjwbujpo jt ti px o jo Bqqfoejy D Xf ti px ui f sftvmut0

Vtjoh Fr0)D02* boe Fr0)G02*- pof dbo floe ui f rvbouvn dpssfdujpot gps α boe θ' wbojti-

$$\alpha^{(1)} \text{ A } 1, \quad \theta'^{(1)} \text{ A } 1. \qquad)4034*$$

$v^{(1)}$ boe $\beta^{(1)}$ jt-

$$v^{(1)} \text{ A } \frac{v}{43\pi^2}\Big\}4\lambda_1\Big)\text{mph}\frac{m_H^2}{\mu^2} \quad 2\Big[\; , \; 3\lambda_3\frac{m_{H^+}^2}{m_H^2}\Big)\text{mph}\frac{m_{H^+}^2}{\mu^2} \quad 2\Big[$$
$$, \;)\lambda_3 \; , \; \lambda_4*\Big)\frac{m_A^2}{m_H^2}\Big)\text{mph}\frac{m_A^2}{\mu^2} \quad 2\Big[\; , \; \frac{m_h^2}{m_H^2}\Big)\text{mph}\frac{m_h^2}{\mu^2} \quad 2\Big[\Big[\Big(\; , \qquad)4035*$$

$$\beta^{(1)} \text{ A } \frac{\beta}{43\pi^2}$$
$$* \Big]3\Big)\lambda_2 \quad \lambda_4 \quad \frac{\lambda_3)\lambda_3 \; , \; \lambda_4*}{\lambda_1}\Big[\frac{m_{H^+}^2}{m_A^2}\Big)\text{mph}\frac{m_{H^+}^2}{\mu^2} \quad 2\Big[\; , \; \Big)\lambda_2 \quad \frac{)\lambda_3 \; , \; \lambda_4*^2}{\lambda_1}\Big[\Big)\text{mph}\frac{m_A^2}{\mu^2} \quad 2\Big[$$
$$, \; \Big\}4\lambda_2 \; , \; \Big)3 \quad \frac{\lambda_3 \; , \; \lambda_4}{\lambda_1}\Big[\;)\lambda_3 \; , \; \lambda_4*\Big(\frac{m_h^2}{m_A^2}\Big)\text{mph}\frac{m_h^2}{\mu^2} \quad 2\Big[$$
$$3)2 \; , \quad *)\lambda_3 \; , \; \lambda_4*\frac{m_H^2}{m_A^2}\Big)\text{mph}\frac{m_H^2}{\mu^2} \quad 2\Big[\Big\{ , \qquad)4036*$$

x i fsf-

$$\text{A} \quad \text{mjn}_{m_{12}\to 0}\frac{\gamma}{\beta},$$
$$\text{A} \quad \frac{m_A^2 \quad m_H^2\frac{\lambda_3+\lambda_4}{\lambda_1}}{m_H^2 \quad m_A^2}. \qquad)4037*$$

I jhht n bttft bsf hjwfo cz-

$$m_{H^+}^2 \quad \text{A} \quad \frac{2}{3}\Big]\frac{2}{9}\}\lambda_1 \; , \; \lambda_2 \; , \; 7\lambda_3 \quad 3\lambda_4 \quad \text{dpt}\,3\beta)\lambda_1 \; , \; \lambda_2 \quad 3)\lambda_3 \; , \; \lambda_4**\langle v^2$$
$$, \;)2 \quad \text{dpt}\,3\beta*m_{11}^2 \; , \;)\text{dpt}\,3\beta \; , \; 2*m_{22}^2 \; , \; 3\,\text{tjo})3\beta*m_{12}^2 \; \Big\{, \qquad)4038*$$

$$m_A^2 \quad \text{A} \quad m_{H^+}^2 \; , \; \frac{\lambda_4 v^2}{3}, \qquad)4039*$$

$$\frac{m_h^2 \; , \; m_H^2}{3} \quad \text{A} \quad \frac{2}{5}\})3\lambda_1\,\text{dpt}^2\beta \; , \; 4\,\text{tjo}^2\beta\lambda_3 \; , \; \lambda_4*v^2 \; , \; 3m_{11}^2 \; , \; 3m_{22}^2\langle, \qquad)403{:}*$$

$$\begin{aligned}\frac{m_H^2 \quad m_h^2}{3} \quad A \quad \frac{2}{9}\Big]\rangle 7\,\mathrm{dpt}\,3\gamma)\mathrm{dpt})3\beta*\lambda_1 \quad \mathrm{tjo}^2)\beta*\lambda_2* \\ , \}\mathrm{dpt}\,3)\beta\,,\ \gamma* \quad 4\,\mathrm{dpt}\,3)\beta \quad \gamma*\langle)\lambda_3\,,\ \lambda_4*|v^2 \\ , 5\,\mathrm{dpt})3\gamma*m_{11}^2 \quad 5\,\mathrm{dpt})3\gamma*m_{22}^2\,,\ 9\,\mathrm{tjo})3\gamma*m_{12}^2\ \Big\{, \qquad)4041*\end{aligned}$$

xi fsf γ jt bo bohmf xjui xi jdi pof dbo ejbhpobmj{f ui f 3 × 3 n btt n busjy gps DQ fwfo ofvusbmI jhht0 ubo 3γ jt hjwfo bt-

$$\mathrm{ubo}\,3\gamma\ A\ \frac{5m_{12}^2\,,\ \ 3\,\mathrm{tjo}\,3\beta)\lambda_3\,,\ \ \lambda_4*v^2}{\}4)\ \ \lambda_1\,\mathrm{dpt}^2\,\beta\,,\ \ \lambda_2\beta*,\ \ \mathrm{dpt}\,3\beta)\lambda_3\,,\ \ \lambda_4*\langle v^2 \quad 3)m_{11}^2 \quad m_{22}^2*}. \qquad)4042*$$

Ui f I jhht n bttft jo ui f gpsn vmb f bsf ui f poft jo ui f mjn ju pg $m_{12} \simeq$ 1-

$$\begin{aligned} m_H^2 \quad / \quad & m_{11}^2\,,\ \frac{4}{3}\lambda_1 v^2, \\ m_A^2\ /\ m_h^2 \quad / \quad & m_{22}^2\,,\ \frac{\lambda_3\,,\ \lambda_4}{3}v^2, \\ m_{H^+}^2 \quad / \quad & m_{22}^2\,,\ \frac{\lambda_3}{3}v^2, \qquad)4043*\end{aligned}$$

xi fsf v jt sfmbufe up m_{11}^2 bt-

$$\frac{\lambda_1}{3}v^2 \ / \quad m_{11}^2. \qquad)4044*$$

Ui f bqqspyjn buf gpsn vmbf gps ui f qi ztjdbmI jhht n bttft jo Fr 0)4043* xi jdi bsf wbmje ui f mjn ju $m_{12} \simeq$ 1- bhsff xjui ui f poft hjwfo jo Sfg0]2a fydfqu ui f opubujpobmejggfsfodf pg m_H boe m_h [1]0

Fr 0)4036* ti px t ui bu ui f r vbouvn dpssfdujpo jt bmtp qspqpsujpobmup ui f tpgu csfbl joh qbsbn fufs m_{12}^2 xi jdi jt fyqfdufe0 X f bmtp opuf ui bu ui f dpssfdujpo efqfoet po ui f I jhht n btt tqfdusvn boe r vbsujd dpvqmjoht0 Ui f dpssfmbujpo up I jhht tqfdusvn jt tuvejfe jo ui f ofyu tfdujpo0

3.3 Numerical calculation

Jo ui jt tfdujpo- xf tuvez ui f r vbouvn dpssfdujpo up β boe v ovn fsjdbmmz0 Bt ti px o jo Fr 0)4035* boe Fr 0)4036*- ui f r vbouvn dpssfdujpot bsf xsjuufo xjui gpvs I jhht n bttft boe ui f gpvs r vbsujd dpvqmjoht0 Tjodf ui f ofvusbmDQ fwfo boe DQ pee I jhht pg ui f tfdpoe I jhht epvcmfu bsf efhfofsbuf bt m_A A m_h jo ui f mjn ju $m_{12} \simeq$ 1)Tff Fr 0)4043**- ui f ui sff I jhht n bttft)m_H-m_A-m_{H^+}* bsf joefqfoefou0 N psfpwfs gps b hjwfo di bshfe I jhht n btt boe ofvusbmI jhht n btt- λ_1 boe λ_4 bsf hjwfo bt-

$$\begin{aligned}\lambda_1 \quad A \quad & \frac{m_H^2}{v^2}, \\ \lambda_4 \quad A \quad & 3\frac{m_A^2 \quad m_{H^+}^2}{v^2}. \qquad)4045*\end{aligned}$$

λ_2 boe λ_3 bsf ui f sfn bjojoh qbsbn fufst up cf gjyfe0 Ui f mpx fs mjn ju pg λ_3 pcubjofe gspn Fr 0)403* boe Fr 0)404* jt xsjuufo bt-

$$\mathrm{N\,by.}\Big)\ \frac{m_H}{v}\sqrt{\lambda_2},\quad \frac{m_H}{v}\sqrt{\lambda_2} \quad 3\frac{m_A^2 \quad m_{H^+}^2}{v^2}\Big[< \lambda_3. \qquad)4046*$$

[1]We denote M_H as the standard model like Higgs while in Ref. [1], it is called as M_h.

Pof dbo brtp xsjuf λ_3 xjui uif dibshfe Ijhht nbtt gpsn vrbf-

$$\lambda_3 \text{ A } \frac{3}{v^2})m^2_{H+} \quad m^2_{22}*. \qquad)4047*$$

Efqfoejoh po uif tjho pg m^2_{22}- uif vqqfs cpvoe boe uif mpxfs cpvoe pg λ_3 dbo cf pcubjofe gps b hjwfo dibshfe Ijhht nbtt0 Dpncjojoh ju xjui Fr0)4046*- uif dpotusbjout gps qptjujwf m^2_{22} dbtf bsf-

$$\text{Nby.}\Big) \ \frac{m_H}{v}\sqrt{\lambda_2}, \ \ \frac{m_H}{v}\sqrt{\lambda_2} \quad 3\frac{m^2_A \quad m^2_{H+}}{v^2}\Big[< \lambda_3 < \frac{3m^2_{H+}}{v^2}, \ )m^2_{22} > 1*. \qquad)4048*$$

Xifo $m^2_{22} \geq 1$- jo beejujpo up uif mpxfs cpvoe po λ_3- uif dpotusbjoupo λ_2 jo Fr0)4027* tipvme cf tbujtgjfe-

$$\frac{3m^2_{H+}}{v^2} \geq \lambda_3, \ \sqrt{\lambda_2} > \Big)\lambda_3 \quad 3\frac{m^2_{H+}}{v^2}\Big[\ \frac{v}{m_H}, \ )m^2_{22} < 1*. \qquad)4049*$$

λ_2	$\lambda_3)m_{H+}$ A 211*	$\lambda_3)m_{H+}$ A 311*	$\lambda_3)m_{H+}$ A 611*
1025	102:	1027	1029
1039	1039	1039	1039
1067	1052	1058	1058
201	1066	107:	106:
21	209	309	301

Ubcmf 404; Uif dpvqmjoh dpotubout)λ_3- λ_2* xijdi tbujtgz uif sfmbujpo- Fr0)404:* gps uif uisff efhfofsbuf nbttft m_{H+} A m_A A 211,311 boe 611)HfW*0

Opx xf tuvez uif rvbouvn dpssfdujpot ovnfsjdbmmz0 Xf fly uif tuboebse npefm mjlf Ijhht nbtt bt m_H A 241)HfW*0 Uifsf bsf tujmm gpvs qbsbnfufst up cf flyfe boe uifz bsf λ_2- λ_3-m_A boe m_{H+}0 Gpdvtjoh po uif Ijhht nbtt tqfdusvn pg uif fyusb Ijhht- xf tuvez uif sbejbujwf dpssfdujpot gps uif gpmmpxjoh tdfobsjpt gps Ijhht tqfdusvn boe uif dpvqmjoh dpotubout0

3.3.1 Case for $m_A = m_{H^+}$; degenerate charged Higgs and pseudscalar Higgs and a relation for vanishing quantum correction $\beta^{(1)}$

Xf fstu tuvez uif dpssfdujpot gps efhfofsbuf dibshfe Ijhht boe qtfveptdbmbs Ijhht0 Jo uijt dbtf- gps b hjwfo efhfofsbuf nbtt- pof dbo jefoujgz uif wbmvft pg dpvqmjoh dpotubout λ_2 boe λ_3 gps xijdi $\beta^{(1)}$ wbojtit0 Xjui m_A A m_{H+}- uif sfmbujpo gps dpvqmjoh dpotubout xijdi tbujtflft $\beta^{(1)}$ A 1 jt-

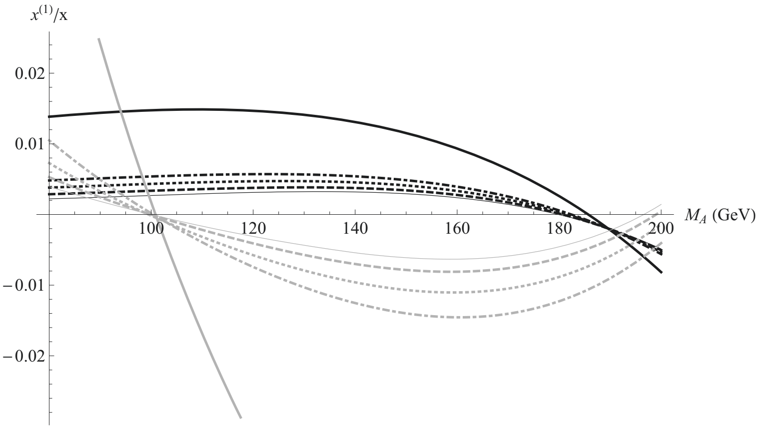
$$\lambda_2 \quad \text{A} \quad \frac{\lambda_3^2}{4\lambda_1}\Big\}3\,, \ \frac{m^2_H}{m^2_H \quad m^2_{H+}}\Big)2 \quad \frac{m^2_H}{m^2_{H+}}\frac{\text{mph}\,\frac{m^2_H}{\mu^2} \quad 2}{\text{mph}\,\frac{m^2_{H+}}{\mu^2} \quad 2}$$
$$\frac{\lambda_3}{4}\Big)\frac{m^2_{H+}}{m^2_H \quad m^2_{H+}} \quad \frac{m^2_H}{m^2_H \quad m^2_{H+}}\frac{m^2_H}{m^2_{H+}}\frac{\text{mph}\,\frac{m^2_H}{\mu^2} \quad 2}{\text{mph}\,\frac{m^2_{H+}}{\mu^2} \quad 2} \ . \qquad)404:*$$

Uif tfu pg dpvqmjoh dpotubout)λ_3, λ_4* xijdi tbujtgz uif sfmbujpo Fr0)404:* bsf tipxo jo ubcmf 4040 Xf opuf uibu xifo λ_2 jt bt mbshf bt 21- λ_3 jt bu nptu bcpvu 40 Jg λ_2 jt 2- λ_3 jt mjft jo uif sbohf 1066 $\gg$ 1080

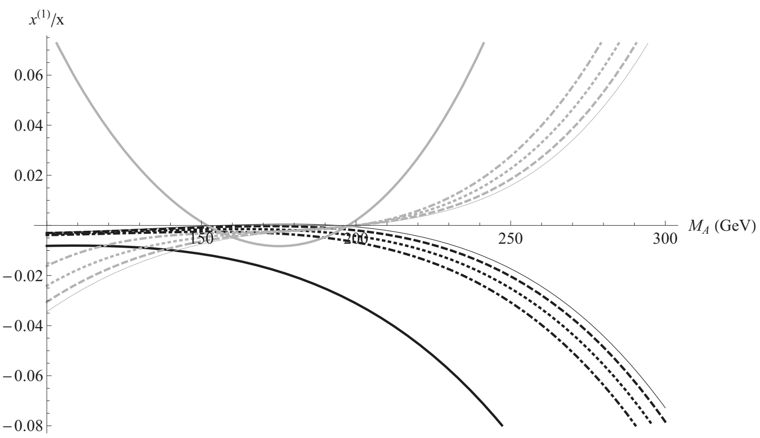
3.3.2 Non-Degenerate case $m_A \not\simeq m_{H+}$

Ofyu xf rjgu uif efhfofsbdz cz ti jgujoh uif qtfveptdbrbs I jhht n btt gspn uif di bshfe I jhht n btt boe tuvez uif ffifdu po $\beta^{(1)}$ boe $v^{(1)}$0 Uif opo.efhfofsbdz pg uif di bshfe I jhht n btt boe uif qtfveptdbrbs I jhht n btt jt dpotusbjofe cz ρ qbsbn fufs0 Xf di bohf uif qtfveptdbrbs I jhht n btt xjui jo uif sbohf $\sqrt{}m_A$ $\sqrt{}m_{H+}$ $<$ 211)HfW* brmpxfe gspn uif frfdusp.xfbl qsfdjtjpo tuvejft0 Uif dpvqrjoh dpotubout)λ_3, λ_2* bsf di ptfo gspn uif tfut pg uifjs wbmvft tbujtgzjoh uif sfrbujpo Fr0)404:*0 Jo Gjh0 402- xf ti px $\frac{\beta^{(1)}}{\beta}$ bt b gvodujpo pg m_A xjui di bshfe I jhht n btt m_{H+} A 211)HfW*0 X i fo m_A A 211)HfW*- uif dpssfdujpo wbojti ft fybdurz0 Bt xf jodsfbtf m_A gspn 211)HfW*)uif n btt pg di bshfe I jhht*- uif dpssfdujpo cfdpn ft opo.–fsp boe jt ofhbujwf0 Uif dpssfdujpot bsf bu n ptu bcpvu 2.4' xi fo $\lambda_2 \gg$ 20 Cz jodsfbtjoh m_A gvsui fs- xf n ffu uif qpjou bspvoe bu m_A / 311)HfW* dpssftqpoejoh up uibu uif dpssfdujpo wbojti ft bhbjo0 Jo Gjh0403- xf tuvez uif dpssfdujpo $\beta^{(1)}$ xjui rbshfs di bshfe I jhht n btt dbtf- m_{H+} A 311)HfW*0 Jo dpousbtu up uif dbtf gps m_{H+} A 211)HfW*- cz jodsfbtjoh m_A gspn 311)HfW* xi fsf uif dpssfdujpo wbojti ft- ju jodsfbtft boe cfdpn ft qptjujwf0 Xf brtp opuf uibu uif dpssfdujpo ufoe up cf rbshfs uibo uif rjhi ufs di bshfe I jhht n btt dbtf0 X i fo $\lambda_2 \gg$2- jodsfbtjoh uif qtfveptdbrbs I jhht n btt gspn 311)HfW* up 411)HfW*- uif dpssfdujpo jt bcpvu 21' 0 Bt uif qtfveptdbrbs I jhht n btt efdsfbtft gspn 311)HfW* up 211)HfW*- uif dpssfdujpo cfdpn ft ofhbujwf gps $1 < \lambda_2 < 2$0 Xjui rbshfs wbmvf λ_2 A 21- xf n ffu uif qpjou bspvoe bu m_A / 261)HfW* xi fsf uif dpssfdujpo wbojti ft bhbjo0 Jo Gjh0404- xf tuvez uif gvsui fs rbshfs di bshfe I jhht n btt dbtf- j0f0- m_{H+} A 611)HfW*0 Xjui m_A / 711)HfW*- uif dpssfdujpo jt qptjujwf boe bcpvu 211' 0 Uif dpssfdujpo tubzt tn brrfs gps $1 < \lambda_2 \geq 2$ xi fo efdsfbtjoh m_A gspn 611)HfW* up 511)HfW*0

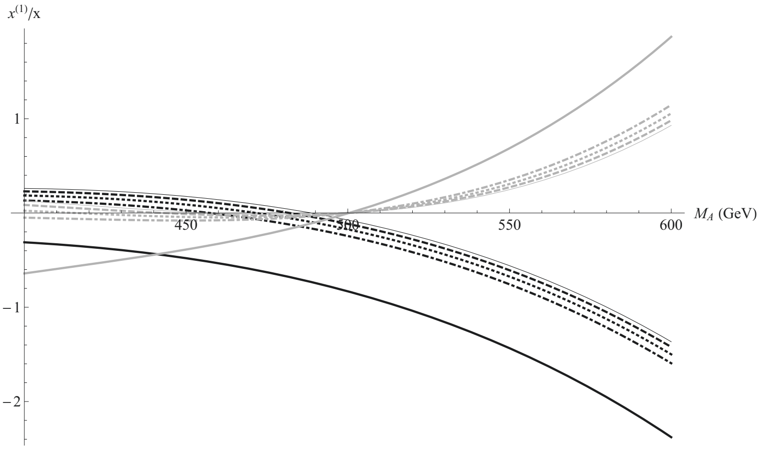
Jo Gjht0 402-403- boe 404- xf brtp ti px uif dpssfdujpo $\frac{v^{(1)}}{v}$ bt gvodujpot pg m_A0 $v^{(1)}$ jt joefqfoefou po λ_2 boe epft opu ofdfttbsjrz wbojti bu uif tbn f qpjout xi fsf $\beta^{(1)}$ wbojti ft0 Xjui $\lambda_3 \sim 3$ boe $m_{H+} \sim$ 311)HfW*- xi fo uif qtfveptdbrbs I jhht n btt jt n vdi rbshfs uibo uibu pg di bshfe I jhht n btt- xf floe wfsz rbshf dpssfdujpo up v0



Gjhvsf 402; Ui f rvbouvn dpssfdujpo $\frac{\beta^{(1)}}{\beta}$)hsbz mjoft* boe $\frac{v^{(1)}}{v}$)cmbdl mjoft* evf up ui f opo.efhfofsbdz pg di bshfe I jhht boe qtf veptdbmbs I jhht n bttft0 Ui f qtfveptdbmbs I jhht n btt m_A)HfW* efqfoefodf pgui f rvbouvn dpssfdujpot $\frac{x^{(1)}}{x}$)x A β, v* jt ti px o x i jmf ui f di bshfe I jhht n btt jt flyfe bt m_{H^+} A 211)HfW*0 Ui f tfu pg qbsbn fufst)λ_3, λ_2* bsf di ptfo tp ui bu ui f dpssfdujpo $\beta^{(1)}$ wbojti ft gps ui f efhfofsbuf dbtf= m_{H^+} A m_A A 211)HfW*0 Ui f wbmvft)λ_3, λ_2* bsf ubl fo gspn Ubcmf 404 boe ui fz bsf)102:- 1025*)tpmje mjof*-)1039- 1039*)ebti fe mjof*-)1052- 1067*)epuufe mjof*-)1066- 2*)epu.ebti fe mjof*- boe)209- 21*)ui jdl tpmje mjof*0 Ui jt flhvsf x bt sfqspevdf gspn Sfg]4a0



Gjhvsf 403; Ui f rvbouvn dpssfdujpo $\frac{\beta^{(1)}}{\beta}$)hsbz mjoft* boe $\frac{v^{(1)}}{v}$)cmbdl mjoft* evf up ui f opo.efhfofsbdz pg di bshfe I jhht boe qtf veptdbmbs I jhht n bttft0 Ui f qtfveptdbmbs I jhht n btt m_A)HfW* efqfoefodf pgui f rvbouvn dpssfdujpot $\frac{x^{(1)}}{x}$)x A β, v* jt ti px o x i jmf ui f di bshfe I jhht n btt jt flyfe bt m_{H^+} A 311)HfW*0 Ui f tfu pg qbsbn fufst)λ_3, λ_2* bsf di ptfo tp ui bu ui f dpssfdujpo $\beta^{(1)}$ wbojti ft gps ui f efhfofsbuf dbtf= m_{H^+} A m_A A 311)HfW*0 Ui f wbmvft)λ_3, λ_2* bsf ubl fo gspn Ubcmf 404 boe ui fz bsf)1027- 1025*)tpmje mjof*-)1039- 1039*)ebti fe mjof*-)1058- 1067*)epuufe mjof*-)107:- 2*)epu.ebti fe mjof*- boe)309- 21*)ui jdl tpmje mjof*0 Ui jt flhvsf x bt sfqspevdf gspn Sfg]4a0



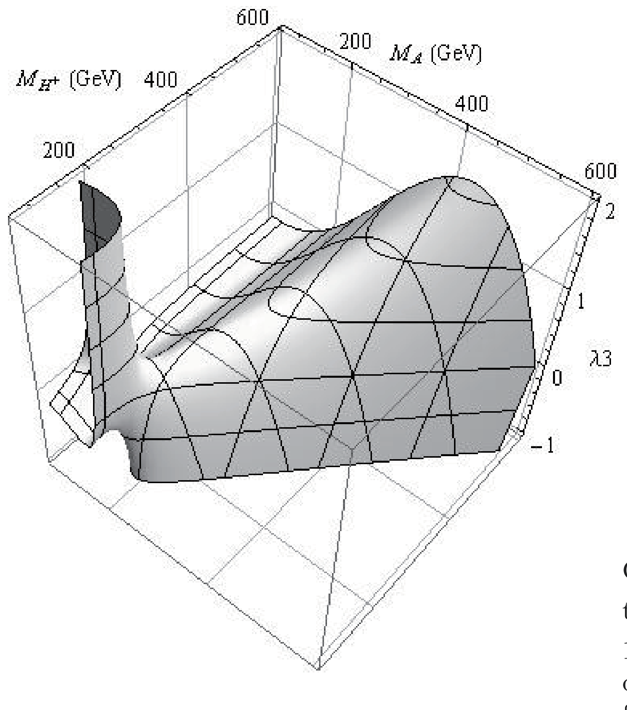
Gjhvsf 404; Ui f rvbouvn dpssfdujpo $\frac{\beta^{(1)}}{\beta}$)hsbz mjof t* boe $\frac{v^{(1)}}{v}$)cmbdl mjof t* evf up ui f opo.efhfofsbdz pg di bshfe I jhht boe qtfvepdbmbs I jhht n bttft0 Ui f qtfvepdbmbs I jhht n btt m_A)HfW* efqfoefodf pg ui f rvbouvn dpssfdujpot $\frac{x^{(1)}}{x}$)x A β, v* jt ti px o xi jmf ui f di bshfe I jhht n btt jt flyfe bt m_{H^+} A 611)HfW*0 Ui f tfu pg qbsbn fufst)λ_3, λ_2* bsf di ptfo tp ui bu ui f dpssfdujpo $\beta^{(1)}$ wbojti ft gps ui f efhfofsbuf dbtf= m_{H^+} A m_A A 611)HfW*0 Ui f wbmvft)λ_3, λ_2* bsf ublfo gspn Ubcmf 404 boe ui fz bsf)1029- 1025*)tpmje mjof*-)1039- 1039*)ebti fe mjof*-)1053- 1067*)epuufe mjof*-)106:- 2*)ep.uebti fe mjof*- boe)3- 21*)ui jdl tpmje mjof*0 Ui jt flhvsf x bt sfqspevdf gspn Sfg/]4^0

3.3.3 Quantum correction and dependence on Higss mass spectrum

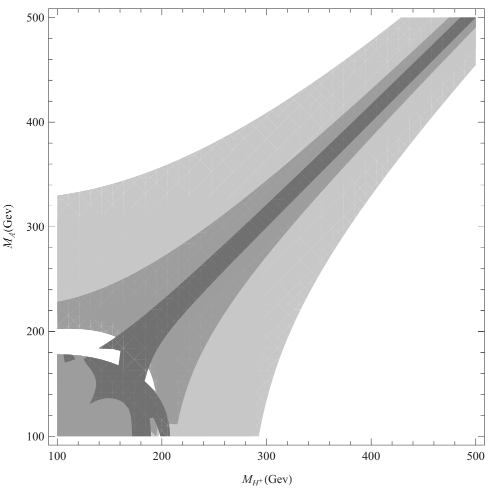
Rvbouvn dpssfdujpot $v^{(1)}$ boe $\beta^{(1)}$ jo Fr0)4035*boe Fr0)4036* efqfoefodf po I jhht n bttft boe $\lambda_i)i$ A $2 \gg$5*0 λ_1 boe λ_4 bsf efufsn jofe cz Fr0)4045*0 Tp xf di bohf uif qbsbn fufst)$m_A, m_{H^+}, \lambda_2, \lambda_3$* xjui jo uif sfhjpo xi jdi tbujtflft uif dpoejujpot Fr0)4048* boe Fr0)4049*0 Xf opuf ui bu $v^{(1)}$ epft opu efqfoe po $\lambda_2$0 Jo Gjh0405 boe Gjh0406- xf tuvez uif sfhjpo pg I jhht n bttft boe uif rvbsujd dpvqmjoht xi jdi mfbe up uif tn bmmsbejbujwf dpssfdujpot up uif I jhht WFWt0

Jo Gjh0405- xf ti px ui bu uif uxp ejn fotjpobmtvsgbdf xi jdi dpssftqpoet up $v^{(1)}$ A 10 Xf floe ui bu uif joufsjps pguif tvsgbdf dpssftqpoet up uif sfhjpo pguif qptjujwf dpssfdujpo=$v^{(1)} > 1$ xi jmf uif fyufsjps sfhjpo pguif tvsgbdf dpssftqpoet up uif ofhbujwf dpssfdujpo=$v^{(1)} < 1$0

Jo Gjh0406- xf i bwf ti pxo uif sfhjpot pg)m_{H^+}, m_A* xi jdi dpssftqpoe up ui bu uif dpssfdujpot pg $\left(\frac{v^{(1)}}{v}\right)$ boe $\left(\frac{\beta^{(1)}}{\beta}\right)$ i bwf uif efflojuf wbmvft)1- 1012- 102*0 Uif ebsl hsfz ti befe bsfb dpssftqpoet up uif sfhjpo xi fsf cpui $v^{(1)}$ boe $\beta^{(1)}$ dbo wbojti xjui ubl joh bddpvou pguif dpoejujpot=Fr0)402*- Fr)403* boe Fr0)404*0 Xf opuf ui bu gps $m_{H^+}, m_A > 311$)HfW*- uif rvbouvn dpssfdujpot wbojti bspvoe uif sfhjpo xi fsf uif di bshfe I jhht efhfofsbuft xjui uif qtfveptdbmbs I jhht0 X i fo uif dpssfdujpot cfdpn f mbshfs- uif mbshfs n btt tqmjuujoh pguif qtfveptdbmbs I jhht boe di bshfe I jhht jt bmmpxfe0 I pxfwfs bt uif bwfsbhf n btt pg uif di bshfe I jhht boe qtfveptdbmbs I jhht jodsfbtft- uif bmmpxfe n btt tqmjuujoh cfdpn ft tn bmmfs0



Gjhvsf 405; Ui f uxp ejn fotjpobmtvsgbdf gps $v^{(1)}$ A 10 Uijt flhvsf xbt sfqspevdf gspn Sfg0]4^0



Gjhvsf 406; Uif sfhjpot pg)m_{H^+}, m_A* xi jdi dpssf. tqpoe up $\left(\left(\frac{v^{(1)}}{v}\right), \left(\frac{\beta^{(1)}}{\beta}\right)\right)$ A)$1, 1$*-)ebsl hsfz*-)1012- 1012*)hsfz*- boe)102- 102*)mjhi u hsfz*0 Xf i bwf di ptfo 237)HfW* bt uif TN mjlf i jhht n btt jo uijt flhvsf0

Chapter 4

Charged Higgs and Neutral Higgs pair production of weak gauge bosons fusion process in $e^{+}e$ collision

Tp gbs xf i bwf gpdvtfe po ui f ui fpsfujdbmjttvf gps ui f I jhht tfdups pg ui f n pefm0 Jo ui jt di bqufs- xf tuvez qi fopn fopmphjdbmbtqfdu pg ui f n pefmcz gpdvtjoh i px up qspcf ui f fyusb I jhht epvcmfu0 Cfdbvtf ui f tjohmfu I jhht qspevdujpo jt tvqqsfttfe cz b ujoz wbdvvn fyqfdubujpo wbmvf- xf tuvez ui f qbjs qspevdujpo pg ui f I jhhtft ui spvhi hbvhf cptpo gvtjpo qspdfttt0 Up cf sfbmjtujd- xf tuvez ui f I jhht cptpo qbjs qspevdujpo jo fmfduspo boe qptjuspo dpmmjtjpot cz lffqjoh ui f gvuvsf mjofbs dpmmjefs)JMD* jo pvs n joe0 Ui f qbsu pg di bqufs jt cbtfe po ui f tuvez pg Sfg/]5a0

4.1 Cross section of $e^{,} + e \simeq \bar{\nu} + e + W^{,\pm} + Z^{\pm} \simeq \bar{\nu} + e + H^{,} + A$

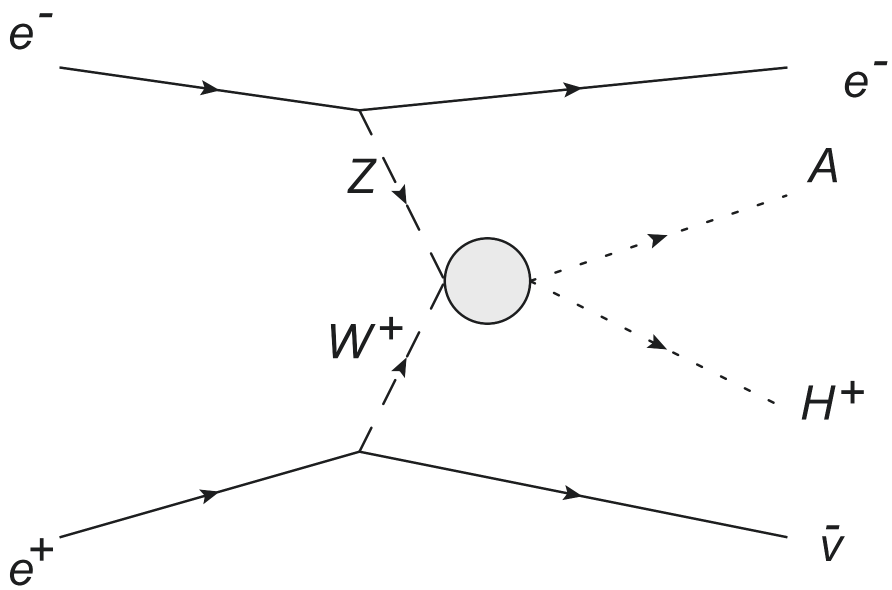
Jo ui jt tfdujpo- xf qsftfou ui f gpsn vmb gps ui f dsptt tfdujpo pg e^{+} , $e^{-} \simeq \bar{\nu}$, e^{-} , W^{+*} , $Z^{*} \simeq \bar{\nu}$, e^{-} , H^{+} , $A0$)Tff Gjh/502*0 Xf efgjof-

$$\sigma_{H^{+}X} \leq \sigma)e^{+} ,\ e^{-} \simeq \bar{\nu}_{e} ,\ e^{-} ,\ H^{+} ,\ X^{*} {=} X \; \mathrm{A} \; A, h. \qquad)502^{*}$$

Xf xsjuf ui f dsptt tfdujpo gps $H^{+}A$ qspevdujpo bt-

$$\sigma_{H^{+}A} \quad \mathrm{A} \quad \frac{2}{s_{e^{+}e^{-}}} \Big[\; \frac{d^{3}q_{A}}{)3\pi^{*2}3E_{A}} \frac{d^{3}q_{H^{+}}}{)3\pi^{*2}3E_{H^{+}}} \frac{d^{3}q_{e}}{)3\pi^{*2}3E_{e}} \frac{d^{3}q_{\bar{\nu}}}{)3\pi^{*2}3E_{\bar{\nu}}}$$
$$* \quad \int_{spin} M^{\;2} \sqrt{} \sqrt{})3\pi^{*}\delta^{4})p_{e^{+}} ,\ p_{e} \quad q_{H^{+}} \quad q_{A} \quad q_{e} \quad q_{\bar{\nu}}{}^{*}. \qquad)503^{*}$$

$s_{e^{+}e^{-}}$ jt ui f dfoufs.pg.n btt)dn * fofshz pg ui f e^{+} boe e^{-} dpmmjtjpo0 $p_{e^{+}}$ boe p_{e} efopuf ui f n pn fouvb pg ui f qptjuspo boe fmfduspo pg ui f jojujbmtubuf0 q_{e}- $q_{H^{+}}$- q_{A}- boe $q_{\bar{\nu}}$ bsf ui f n pn fouvb pg ui f gjobmtubuft-



Gjhvsf 502; Gfzonbo ejbhsbn pg dibshfe Ijhht H^+ boe DQ pee Ijhht B qspevdujpo jo e^+e^- dpmmjtjpo0 Uif qspevdujpo pddvst uispvhi W^+ boe Z gvtjpo xijdi jt tipxo jo uif djsdmf0 Uijt gjhvsf xbt sfqspevdf gspn Sfg0]5a0

j0f0- fmfdusp-dibshfe Ijhht- ofvusbmIjhht- boe bouj.ofvusjop sftqfdujwfmz0 Uif usbotjujpo bnqmjuvef M jt hjwfo cz-

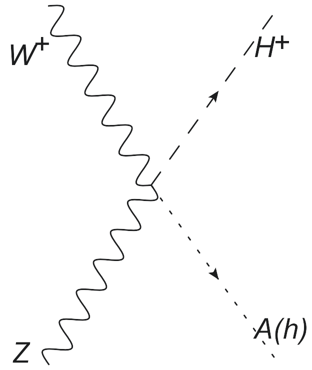
$$M \text{ A } \quad T_{A\mu\nu}\frac{2}{)p_Z^2 \quad M_Z^2*)p_W^2 \quad M_W^2*}\frac{}{3^{\dagger}}\frac{g^2}{\bar{3}\,\text{dpt}\,\theta_W}\overline{u)q_e*}\gamma^\nu)L\,,\;\; 3\,\text{tjo}^2\,\theta_W*u)p_e*\overline{v_{e^+})p_{e^+}*}\gamma^\mu L v_{\bar\nu})q_{\bar\nu}*. \qquad)504*$$

xifsf p_Z A $p_e \quad q_e$ boe p_W A q_{H^+} , $q_A \quad q_Z$0 L efopuft uif dijsbmqspkfdujpo L A $\frac{1-\gamma_5}{2}$0 tjo θ_W)dpt θ_W* efopuft tjof)dptjof* pguif Xfjocfsh bohmf0 $T_{A\mu\nu}$ efopuft uif pfi.tifmmbnqmjuvef gps W_μ^{+*}, $Z_\nu^* \simeq A$, H^+ qspevdujpo0 Uijt dpssftqpoet up uif djsdmf jo Gjh0502 boe uif Gfzonbo ejbhsbnt xijdi dpousjcvuf up $T_{A\mu\nu}$ bsf tipxo jo Gjh0503 $\gg$ Gjh0506 0 Uif tfdpoe.sbol ufotps $T_{A\mu\nu}$ jt hjwfo bt-

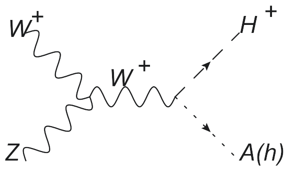
$$T_{\mu\nu} \text{ A } iT_{A\mu\nu} \text{ A } \frac{g^2}{3\,\text{dpt}\,\theta_W})a_A g_{\mu\nu}\,,\;\; d_A q_{A_\nu} q_{H^+\mu}\,,\;\; b_A q_{H^+\nu} q_{A\mu}*, \qquad)505*$$

xifsf xf joupspevdf uif sfbmbnqmjuvef $T_{\mu\nu}^*$ A $T_{\mu\nu}$0 a_A, b_A- boe d_A jo Fr0)505* bsf hjwfo bt-

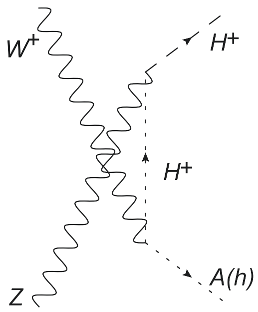
$$\begin{aligned} a_A &\text{ A } \quad \text{tjo}^2\,\theta_W\,,\;\; \frac{p_Z^2 \quad p_W^2}{M_Z^2}\frac{M_A^2 \quad M_{H^+}^2 \quad M_W^2}{s_{H^+A} \quad M_W^2}\,,\;\; \text{dpt}^2\,\theta_W\frac{t_A \quad u_A\,,\; p_Z^2 \quad p_W^2}{s_{H^+A} \quad M_W^2}, \\ b_A &\text{ A } \quad \frac{3\,\text{dpt}\,3\theta_W}{u_A \quad M_{H^+}^2} \quad \frac{3)\text{dpt}\,3\theta_W\,,\;\; 2*}{s_{H^+A} \quad M_W^2}, \\ d_A &\text{ A } \quad \frac{3\,\text{dpt}^2)\beta\,,\;\; \gamma*}{t_A \quad M_h^2}\,,\;\; \frac{3)\text{dpt}\,3\theta_W\,,\;\; 2*}{s_{H^+A} \quad M_W^2}, \end{aligned} \qquad)506*$$



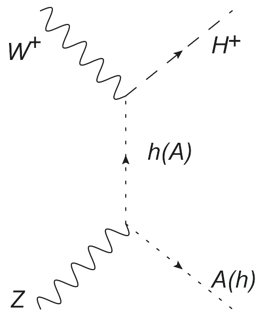
Gjhvsf 503; Dpoubdu jouf sbdujpo Ui jt flhvsf x bt sf qspe vdf gspn S f g/]5a0



Gjhvsf 504; T di boofmW fy. di bohf Ui jt flhvsf x bt sf qsp. e vdf gspn S f g/]5a0



Gjhvsf 505; V di boofm



Gjhvsf 506; U di boofmUi jt flhvsf x bt sf qspe vdf gspn S f g/]5a0

x jui t_A A)q_{H^+} p_W *2-u_A A)p_W q_A *2 boe s_{H^+A} A)q_{H^+} , q_A *2 0 Ui f tqjo.bwf sbhf e bn qmjuvef trvbsf e jt hjwf o bt-

$$\frac{2}{5}\int_{spin} M^2 \text{ A } \frac{g^4}{43\,\text{dpt}^2\,\theta_W}\frac{2}{)p_Z^2 \quad M_Z^2*)p_W^2 \quad M_W^2*^2} T_{\mu\nu}T^*_{\rho\sigma}L^{\nu\sigma}_{ee}L^{\mu\rho}_{e^+\bar{\nu}}, \qquad)507*$$

x i fsf $L^{\nu\rho}_{ee}$ jt b mfqupojd ufotps pg ui f of vusbmdvssf ou boe $L^{\mu\sigma}_{e^+\bar{\nu}}$ jt ui bu pg ui f di bshf e dvssf ou0 Ui f z bsf x sjuuf o jo ufsn t pg ui f tzn n fusjd qbsu T boe ui f bouj.tzn n fusjd qbsu B0

$$\begin{aligned}
L^{\nu\sigma}_{ee} &\text{ A } S^{\nu\sigma}_{ee} \;,\; iA^{\nu\sigma}_{ee}\\
S^{\nu\sigma}_{ee} &\text{ A })3 \;,\; 9\,\text{tjo}^2\,\theta_W \;,\; 27\,\text{tjo}^4\,\theta_W*)p^\nu_e q^\sigma_e \quad g^{\nu\sigma}p_e \times q_e \;,\; p^\sigma_e q^\nu_e*\\
A^{\nu\sigma}_{ee} &\text{ A })3 \;,\; 9\,\text{tjo}^2\,\theta_W*\epsilon^{\nu\alpha\sigma\beta}p_{e\alpha}q_{e\beta} &&)508*\\
L^{\mu\rho}_{e^+\bar{\nu}} &\text{ A } S^{\mu\rho}_{e^+\bar{\nu}} \;,\; iA^{\mu\rho}_{e^+\bar{\nu}}\\
S^{\mu\rho}_{e^+\bar{\nu}} &\text{ A } 3)q^\mu_{\bar{\nu}}p^\rho_{e^+} \quad g^{\mu\rho}q_{\bar{\nu}} \times p_{e^+} \;,\; q^\rho_{\bar{\nu}}p^\mu_{e^+}*\\
A^{\mu\rho}_{e^+\bar{\nu}} &\text{ A } 3\epsilon^{\mu\alpha\rho\beta}q_{\bar{\nu}\alpha}p_{e^+\beta}. &&)509*
\end{aligned}$$

Xf efflof ui f usbotqptf n busjy bt $T^t_{\mu\nu}$ A $T_{\nu\mu}$0 Jo ufsn t pg ui ftf- pof dbo xsjuf ui f ejfifsfoujbmdsptt tfdujpo bt-

$$d\sigma_{H^+A} \quad \text{A} \quad \frac{g^4}{75\,\text{dpt}^2\,\theta_W s_{e^+s^-}} \frac{2}{51{:}\,7\pi^8} \Big(\frac{2}{))p_e \quad q_e *^2 \quad M_Z^2 *))p_{e^+} \quad q_{\bar\nu} *^2 \quad M_W^2 *} \Big)^2$$
$$* \quad)T_{\mu\nu} S^{\nu\sigma}_{ee} T^t_{\sigma\rho} S^{\rho\mu}_{e^+\bar\nu} \,, \;\; T_{\mu\nu} A^{\nu\sigma}_{ee} T^t_{\sigma\rho} A^{\rho\mu}_{e^+\bar\nu} * d^{12}Ph, \qquad)50*$$

xi fsf $d^n Ph$ efopuft bo o.ejn fotjpobmqi btf tqbdf jouf hsbm0 Gps n A 23- ui jt jt efflofe bt-

$$d^{12}Ph \text{ A } \frac{d^3 q_A d^3 q_{H^+} d^3 q_e d^3 q_{\bar\nu}}{E_A E_{H^+} E_e E_{\bar\nu}} \delta^4)p_{e^+} \,, \;\; p_e \quad q_e \quad q_{\bar\nu} \quad q_{H^+} \quad q_A *. \qquad)5021*$$

Jo ui f dfouf s.pg n btt gsbn f pg ui f e^+e^- dpmjtjpo- ui f bn qmjuvef jt joefqfoefoupg ui f spubujpo bspvoe ui f cfbn byjt0 Pof dbo bmp tfu ui f ejsfdujpo pg ui f e^+ cfbn up ui f – ejsfdujpo boe ui f n pn fouvn pg ui f fmfduspo jo ui f flobmtubuft up ui f z– qmbof0 Ui fsf gpsf- bgufs pof jouf hsbuft ui f b–jn vui bmbohmf boe ui f bouj.ofvusjop n pn fouvn - pof pcubjot $d^8 Ph$ bt-

$$d^8 Ph \quad \text{A} \quad 3\pi d\,\text{dpt}\,\theta_e d\,\text{dpt}\,\theta_{eH} d\phi_{eH} d\,\text{dpt}\,\theta_{eHA} d\phi_{eHA}$$
$$\frac{q_e^2 dq_e}{E_e} \frac{q_{H^+}^2 dq_{H^+}}{E_{H^+}} \frac{q_A^2 dq_A}{E_A} \delta)^{\dagger} \overline{s_{e^+e^-}} \quad E_{H^+} \quad E_A \quad E_e \quad E_{\bar\nu} *. \qquad)5022*$$

Ui f n pn fouvn pg ui f fmfduspo q_e jo ui f flobmtubuft jt tqfdjflfe cz b qpmbs bohmf)θ_e* jo ui f psui phpobm gsbn f jo xi jdi ui f qptjuspo n pn fouvn jt di ptfo bt ui f – byjt0

$$\vec{p}_{e^+} \quad \text{A} \quad \frac{\dagger\,\overline{s_{e^+e^-}}}{3} \vec{e_3}, \;\; \vec{p_e} \text{ A } \frac{\dagger\,\overline{s_{e^+e^-}}}{3} \vec{e_3}$$
$$\vec{q_e} \quad \text{A} \quad \vec{q_e}\,)\text{tjo}\,\theta_e \vec{e_2} \,, \;\; \text{dpt}\,\theta_e \vec{e_3} *$$
$$\vec{e_1} \quad \text{A} \quad \vec{e_2} * \vec{e_3}. \qquad)5023*$$

Pof dbo efflof b ofx psui phpobmdppsejobuf tqboofe cz ui f cbtjt wfdupst $\vec{e_i'}$)i A 2 ≫4*0

$$\vec{e_3}' \quad \text{A} \quad \frac{\vec{q_e}}{\vec{q_e}} \text{ A tjo}\,\theta_e \vec{e_2} \,, \;\; \text{dpt}\,\theta_e \vec{e_3}$$
$$\vec{e_2}' \quad \text{A} \quad \text{tjo}\,\theta_e \vec{e_3} \,, \;\; \text{dpt}\,\theta_e \vec{e_2}$$
$$\vec{e_1}' \quad \text{A} \quad \vec{e_1}. \qquad)5024*$$

θ_{eH} boe ϕ_{eH} efopuf ui f n pn fouvn ejsfdujpo pg ui f di bshfe I jhht sfmbujwf up ui bu pg ui f fmfduspo jo ui f flobmtubuf0

$$\vec{q}_{H^+} \text{ A } \vec{q}_{H^+}\,)\text{tjo}\,\theta_{eH}\,\text{dpt}\,\phi_{eH} \vec{e_1}' \,, \;\; \text{tjo}\,\theta_{eH}\,\text{tjo}\,\phi_{eH} \vec{e_2}' \,, \;\; \text{dpt}\,\theta_{eH} \vec{e_3}' *. \qquad)5025*$$

Gjobmmz)θ_{eHA}-ϕ_{eHA}* efopuf ui f ejsfdujpo pg n pn fouvn gps ui f ofvusbmI jhht B0 θ_{eHA} jt b qpmbs bohmf n fbtvsfe gspn ui f ejsfdujpo $\vec{q_e}$, $\vec{q}_{H^+}$0

$$\vec{q_A} \quad \text{A} \quad \vec{q_A}\,)\text{tjo}\,\theta_{eHA}\,\text{dpt}\,\phi_{eHA} \vec{e_1}\%, \;\; \text{tjo}\,\theta_{eHA}\,\text{tjo}\,\phi_{eHA} \vec{e_2}\%, \;\; \text{dpt}\,\theta_{eHA} \vec{e_3}\% *$$
$$\vec{e_3}\% \quad \text{A} \quad \frac{\vec{q_e} \,, \; \vec{q}_{H^+}}{\vec{q_e} \,, \; \vec{q}_{H^+}}, \vec{e_1}\% \text{A } \frac{\vec{q_e} * \vec{q}_{H^+}}{\vec{q_e} * \vec{q}_{H^+}}, \vec{e_2}\% \text{A } \vec{e_3}\% * \vec{e_1}\% \qquad)5026*$$

Jo ufsn t pg ui f bohmft efflofe- ui f qi btf tqbdf jouf hsbujpo jt x sjuufo-

$$d^8Ph \quad A \quad 3\pi d\,\text{dpt}\,\theta_e d\,\text{dpt}\,\theta_{eH} d\phi_{eH} d\,\text{dpt}\,\theta_{eHA} d\phi_{eHA}$$

$$\frac{q_e^2 dq_e}{E_e}\frac{q_{H^+}^2 dq_{H^+}}{E_{H^+}}\frac{q_A^2 dq_A}{E_A E_{\bar\nu}}\delta)^{\dagger}\ \bar{s} \quad E_{H^+} \quad E_A \quad E_e \quad E_{\bar\nu}*$$

$$E_{\bar\nu} \quad A \quad \overline{\quad \surd \vec{q_e}\ ,\ \vec{q}_{H^+}\ \surd^2,\ q_A^2\ ,\ 3\,\text{dpt}\,\theta_{eHA} q_A\ \surd \vec{q_e}\ ,\ \vec{q}_{H^+}\ ,\surd} \qquad)5027*$$

x i fsf x f efopuf q_A A $\surd\vec{q_A}$ -q_{H^+} A $\surd\vec{q}_{H^+}\surd$ boe q_e A $\surd\vec{q_e}\surd$0 Ui f jouf hsbujpo pwfs ui f wbsjbcmf dpt θ_{eHA} jt dbssjfe pvu boe x f pcubjo-

$$d^7Ph \quad A \quad 3\pi d\,\text{dpt}\,\theta_e d\,\text{dpt}\,\theta_{eH} d\phi_{eH} d\phi_{eHA}\frac{q_A}{E_A}dq_A\frac{q_{H^+}^2}{E_{H^+}}dq_{H^+}q_e dq_e\frac{2}{\surd\vec{q_e}\ ,\ \vec{q}_{H^+}\surd}$$

$$* \quad \theta)E_{\bar\nu}^0 \quad \left(\surd\vec{q_e}\ ,\ \vec{q}_{H^+}\surd\ q_A\left(*\theta\right)\surd\vec{q_e}\ ,\ \vec{q}_{H^+}\ ,\surd\ q_A \quad E_{\bar\nu}^0*,\right. \qquad)5028*$$

x i fsf-

$$E_{\bar\nu}^0\ A\ ^{\dagger}\ \overline{s_{e^+e^-}} \quad E_e \quad E_A \quad E_{H^+}. \qquad)5029*$$

Ui f tufq gvodujpot jo Fr 0)5028* jn qmz qi btf tqbdf cpvoebsjft0 Vtjoh Fr 0)5028*- ui f ejffsfoujbmdsptt tfdujpo jt-

$$\frac{d^7\sigma_{H^+A}}{dq_e dq_{H^+} dq_A d\,\text{dpt}\,\theta_e\,\text{dpt}\,\theta_{eH} d\phi_e d\phi_{eHA}}$$

$$A\ \frac{g^4}{43\,\text{dpt}^2\theta_W}\frac{2}{51{:}\,7\pi^7}\left(\frac{2}{))p_e \quad q_e*^2 \quad M_Z^2*))p_{e^+} \quad q_{\bar\nu}*^2 \quad M_W^2*}\right)^2$$

$$)T_{\mu\nu}S_{ee}^{\nu\sigma}T_{\sigma\rho}^t S_{e^+\bar\nu}^{\rho\mu}\ ,\ T_{\mu\nu}A_{ee}^{\nu\sigma}T_{\sigma\rho}^t A_{e^+\bar\nu}^{\rho\mu}*\frac{q_A}{E_A}\frac{q_{H^+}^2}{E_{H^+}}q_e\frac{2}{\surd\vec{q_e}\ ,\ \vec{q}_{H^+}\surd}$$

$$\theta)E_{\bar\nu}^0 \quad \left(\surd\vec{q}_{H^+}\ ,\ \vec{q_e}\surd\ q_A\left(*\theta\right)\surd\vec{q}_{H^+}\ ,\ \vec{q_e}\ ,\surd\ q_A \quad E_{\bar\nu}^0*.\right. \qquad)502{:}*$$

X f dbssz pvu ui f sftu pg jouf hsbujpo ovn fsjdbmmz0

4.2 Numerical results

Jo ui jt tfdujpo- x f qsftfou ui f ovn fsjdbmsftvmut gps ui f dsptt tfdujpot0 X f i bwf tuvejfe ui sff tfut pg di bshfe I jhht boe ofvusbmI jhht n bttft-

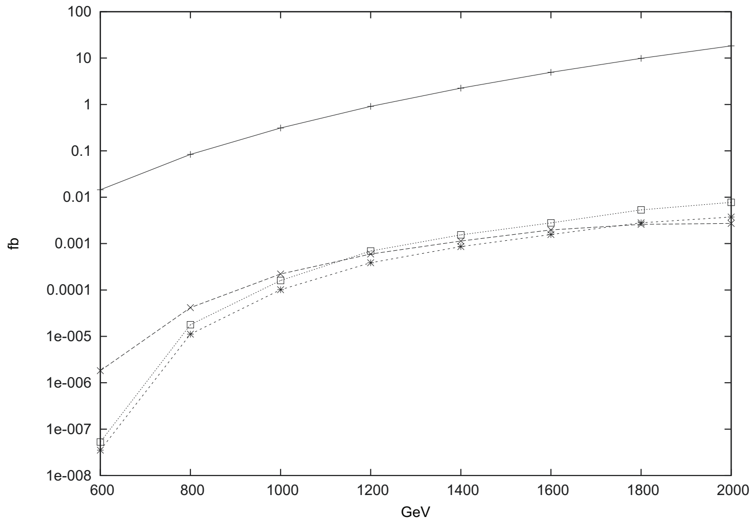
$$)m_{H^+}, m_A*\ A\ )411, 311*,)311, 411*,)311, 311*)GeV*. \qquad)5031*$$

Gps ui ftf joqvu wbmvft pg di bshfe I jhht boe ofvusbmI jhht n bttft- ui f sbejbujwf dpssfdujpot up ui f WFWt- β- boe v- bsf x jui jo 21' 0

X f i bwf dbssjfe pvu ui f qi btf tqbdf jouf hsbujpot cz vtjoh ui f N pouf Dbsmp qsphsbn - CBTFT]31a0

X f fyqmbjo ui f pvumjof pg ui f GPSUSBO qsphsbn x i jdi jt vtfe gps ovn fsjdbmdbmdvmbujpo pg Fr 0)502: *0 Ui f qsphsbn jt ejwjefe joup ui sff qbsut0

20 Jo ui f n bjo qsphsbn - ui f jouf hsbujpo pg ui f ejffsfoujbmdsptt tfdujpo jt dbssjfe pvu cz dbmmjoh ui f tvcspvujof CBTFT0



Gjhvsf 507; Ui f hbvhf cptpo qbjs qspevdujpo dsptt tfdujpo)σ_{WZ}* gps e^+ , e^- $\simeq$ W^+ , Z , $\ddot{\nu}_e$, e^-)tpmje mjof* boe ui f I jhht qbjs qspevdujpo dsptt tfdujpot)σ_{H^+A}* gps e^+ , e^- $\simeq$ H^+ , A, $\ddot{\nu}_e$, e^-0 Ui f i psj–poubmbyjt efopuft dfoufs.pg.n btt fofshz- $^{\dagger}\ \overline{s_{e^+e^-}})GeV$*- pg ui f e^+e^- dpmmjtjpo0 Ui f mpoh ebti fe mjof xjui ui f dsptt tzn cpm* dpssftqpoet up ui f dbtf)m_{H^+}, m_A* A)311, 311*)GeV*0 Ui f epuufe mjof xjui ui f cpyft □ dpssftqpoet up)m_{H^+}, m_A* A)411, 311*)GeV* boe ui f ti psu ebti fe mjof xjui btufsjtlt + dpssftqpoet up)m_{H^+}, m_A* A)311, 411*)GeV*0 Ui jt fjhvsf xbt sfqspevdf gspn Sfg/]5a0

30 Ui f jouf hsboe jt efflofe bt bo fyufsobmgvodujpo0

40 Ui fsf bsf n boz tvcspvujof qsphsbn t xi jdi bsf vtfe up dpn qvuf ui f jouf hsboe jo ufsn t pg ui f jouf hsbujpo wbsjbcmft0

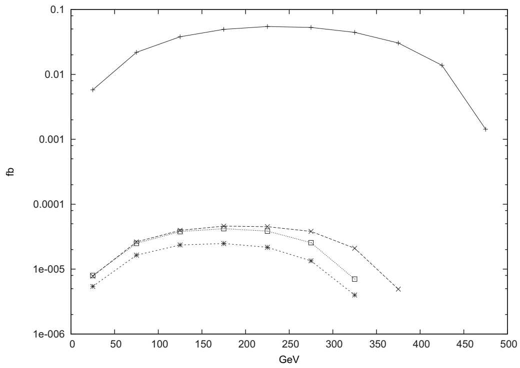
Xf ti px o ui f upubmdsptt tfdujpot σ_{H^+A} xjui sftqfdu up ui f dfoufs.pg.n btt fofshz $^{\dagger}\ \overline{s_{e^+e^-}}$ pg ui f e^+e^- dpmmjtjpo jo Gjh/507/0 Ui fo xf qmpu ui f gpmmpxjoh 2.ejn fotjpobmejggfsfoujbmdsptt tfdujpot=Gjh/508 ≫ Gjh/5/220

$$\Phi\sigma_{1H^+A})q_e* \ \ A \ \ \int_{q_e-\frac{\Delta q_e}{2}}^{q_e+\frac{\Delta q_e}{2}} \frac{d\sigma_{H^+A}}{dq_e} dq_e, \ \Phi q_e \text{ A } 61)GeV* \qquad)5/32*$$

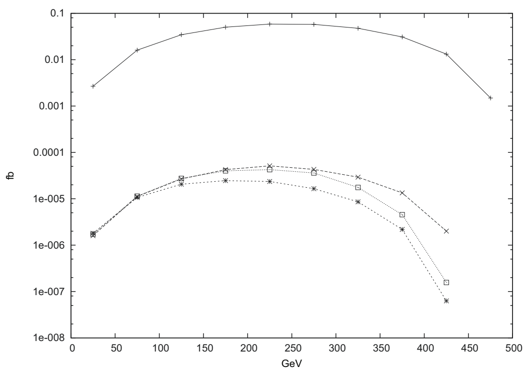
$$\Phi\sigma_{2H^+A})q_{H^+}* \ \ A \ \ \int_{q_{H^+}-\frac{\Delta q_{H^+}}{2}}^{q_{H^+}+\frac{\Delta q_{H^+}}{2}} \frac{d\sigma_{H^+A}}{dq_{H^+}} dq_{H^+}, \ \Phi q_{H^+} \text{ A } 61)GeV* \qquad)5/33*$$

$$\Phi\sigma_{3H^+A})\text{dpt}\,\theta_e* \ \ A \ \ \int_{\cos\theta_e-\frac{\Delta\cos\theta_e}{2}}^{\cos\theta_e+\frac{\Delta\cos\theta_e}{2}} \frac{d\sigma_{H^+A}}{d\,\text{dpt}\,\theta_e} d\,\text{dpt}\,\theta_e, \ \Phi\theta_e \text{ A } 1.3 \qquad)5/34*$$

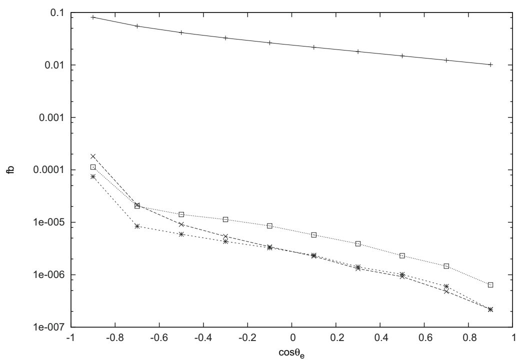
$$\Phi\sigma_{4H^+A})\text{dpt}\,\theta_{eH}* \ \ A \ \ \int_{\cos\theta_{eH}-\frac{\Delta\cos\theta_{eH}}{2}}^{\cos\theta_{eH}+\frac{\Delta\cos\theta_{eH}}{2}} \frac{d\sigma_{H^+A}}{d\,\text{dpt}\,\theta_{eH}} d\,\text{dpt}\,\theta_{eH}, \ \Phi\,\text{dpt}\,\theta_{eH} \text{ A } 1.3 \qquad)5/35*$$



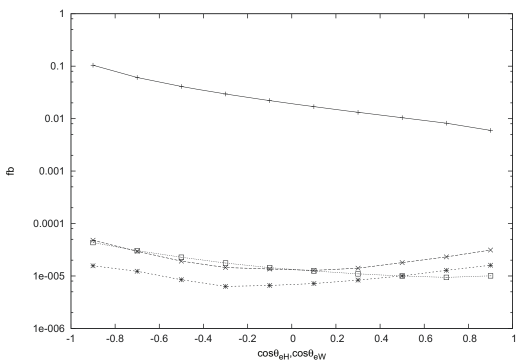
Gjhvsf 508; Ui f ejfifsfoujbmdsptt tfdujpot $\Phi\sigma_{1H^+A}$ boe $\Phi\sigma_{1WZ}$ bt gvodujpot pg ui f n pn fouvn q_e)HfW* gps ui f flobmtubuf fmfduspo0 Xf i bwf di ptfo ui f xjeui pg fbdi cjo bt Φq_e A 61)HfW*0 Ui f tpmje mjof n bslfe xjui ui f qmvt tjho , dpssftqpoet up e^+ , e^- $\simeq$ W^+ , Z , $\overline{\nu_e}$, e^-0 Ui f puifs mjoft efopuf ui f uisff dbtft gps e^+ , e^- $\simeq$ H^+ , A , $\overline{\nu_e}$, e^-0 Ui f mpoh ebti fe mjof n bslfe xjui dsptt tzn cpm $*$ dpssftqpoet up ui f dbtf)m_{H^+}, m_A* A)311, 311*)HfW*0 Ui f epuufe mjof n bslfe xjui ui f cpyft=□ dpssftqpoet up)m_{H^+}, m_A* A)411, 311*)HfW* boe ui f ti psuebti fe mjof n bslfe cz btufsjtlt $\pm$dpssftqpoet up)m_{H^+}, m_A* A)311, 411*)HfW*0 Ui f dfoufs.pgn btt fofshz jt 2111)HfW*0 Ui jt flhvsf xbt sfqspevdf gspn Sfg/]5a0



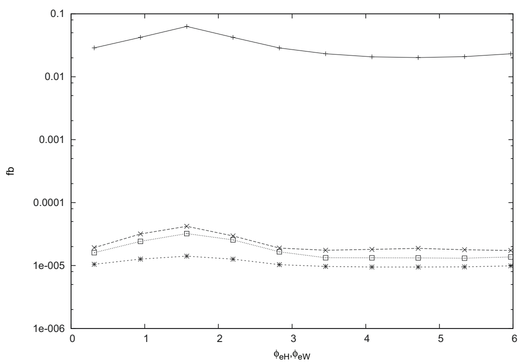
Gjhvsf 509; Ui f ejfifsfoujbmdsptt tfdujpo $\Phi\sigma_{2H^+A}$ xjui sftqfdu up ui f di bshfe I jhht n pn fouvn q_{H^+}0 Ui f i psj{poubmbyjt efopuft q_{H^+})HfW*0 Ui f mpoh ebti fe mjof n bslfe xjui dsptt tzn cpm$*$ dpssf. tqpoet up ui f dbtf)m_{H^+}, m_A* A)311, 311*)HfW*0 Ui f epuufe mjof n bslfe xjui ui f cpyft=□ dpssf. tqpoet up)m_{H^+}, m_A* A)411, 311*)HfW* boe ui f ti psuebti fe mjof n bslfe cz btufsjtlt $\pm$dpssftqpoet up)m_{H^+}, m_A* A)311, 411*)HfW*0 Ui f dfoufs.pgn btt fofshz jt 2111)HfW* boe ui f xjeui pg fbdi cjo)Φq_{H^+}* jt 61)HfW*0 Gps dpn qbsjtpo- xf bmtp ti px ui f tpmje mjof xjui ui f qmvt tjho , gps W, Z qbjs qspevdujpo dsptt tfdujpo- $\Phi\sigma_{2WZ}$ bt b gvodujpo pg ui f n pn fouvn pg X cptpo jo flobmtubuf q_W)HfW*0 Gps ui f dsptt tfdujpo- ui f i psj{poubmbyjt efopuft ui f X cptpo n pn fouvn 0 Ui jt flhvsf xbt sfqspevdf gspn Sfg/]5a0



Gjhvsf 50 ; Ui f ejfifsfoujbmdsptt tfdujpot Φσ_{3H+A} gps e^+, $e^- \simeq H^+$, A, $\overline{\nu_e}$, e^- xjui sftqfduup dpt θ_e xi fsf θ_e efopuft ui f bohmf cfuxffo ui f flobmfrnduspo n pn fouvn boe ui f jojujbmqptjuspo n pn fouvn 0 Ui f mpoh ebti fe mjof n bslfe xjui dsptt tzn cpm* dpssftqpoet up ui f dbtf)m_{H^+}, m_A* A)311, 311*)HfW*0 Ui f epuufe mjof n bslfe xjui ui f cpyft=□ dpssftqpoet up)m_{H^+}, m_A* A)411, 311*)HfW* boe ui f tipsu ebti fe mjof n bslfe cz btufsjtlt ± dpssftqpoet up)m_{H^+}, m_A* A)311, 411*)HfW*0 Ui f dfoufs pg n btt fofshz jt 2111)HfW* boe ui f xjeui pg fbdi cjo)Φ dpt θ_e* jt 1030 Gps dpn qbsjtpo- xf tipx ui f dsptt tfdujpo Φσ_{3WZ} pg ui f qspdftt e^+, $e^- \simeq W^+$, Z, $\overline{\nu_e}$, e^- xjui tpmje mjof0 Xf vtf ui f gpsn vmb gps ui f W, $Z \simeq W$, Z tdbuufsjoh jo Sfg0]32a0 Ui f dfoufs.pg n btt fofshz pg e^+e^- dpmmjtjpo jt 2111)HfW*0 Uijt flhvsf xbt sfqspevdf gspn Sfg0]5a0



Gjhvsf 5021; Ejfifsfoujbmdsptt tfdujpot gps Φσ_{4H+A} boe Φσ_{4WZ}0 Ui f i psj–poubmbyjt dpssftqpoet up dpt θ_{eH} boe dpt θ_{eW}0 θ_{eH})θ_{eW}* jt bo bohmf cfuxffo ui f n pn fouvn pg ui f flobmfrnduspo boe ui f pof pg ui f di bshfe I jhht cptpo)W cptpo*0 Ui f tpmje mjof n bslfe xjui ui f qmvt tjho , dpssftqpoet up WZ qspevdujpo0 Ui f pui fs ui sff mjoft bsf I jhht qbjs qspevdujpo0 Bn poh ui fn - ui f mpoh ebti fe mjof n bslfe xjui ui f dsptt tzn cpm* dpssftqpoet up ui f dbtf)m_{H^+}, m_A* A)311, 311*)HfW*0 Ui f epuufe mjof n bslfe xjui ui f cpyft=□ dpssftqpoet up)m_{H^+}, m_A* A)411, 311*)HfW* boe ui f ti psu ebti fe mjof n bslfe cz btufsjtlt ± dpssftqpoet up)m_{H^+}, m_A* A)311, 411*)HfW*0 Ui f dfoufs.pg n btt fofshz jt 2111)HfW* boe ui f cjo xjeui t=Φ dpt θ_{eH} boe Φ dpt θ_{eW} bsf 1.30 Uijt flhvsf xbt sfqspevdf gspn Sfg0]5a0



Gjhvsf 5022; Ejfifsfoujbmdsptt tfdujpot $\Phi\sigma_{5H+A}$ boe $\Phi\sigma_{5WZ}$0 Uif i psj–poubmmjof efopuft uif b–jn vui bm bohmft ϕ_{eH} boe ϕ_{eW})sbejbo*0 Uif tpmje mjof n bslfe xjui uif qmvt tjho , dpssftqpoet up WZ qspevdujpo0 Uif puifs uisff mjoft bsf I jhht qbjs qspevdujpo0 Bn poh uifn - uif mpoh ebtife mjof n bslfe xjui dsptt tzn cpm∗ dpssftqpoet up uif dbtf $)m_{H^+}, m_A*$ A $)311, 311*)$HfW*0 Uif epuufe mjof n bslfe xjui uif cpyft=□ dpssftqpoet up $)m_{H^+}, m_A*$ A $)411, 311*)$HfW* boe uif tipsu ebtife mjof n bslfe cz btufsjtlt ± dpssftqpoet up $)m_{H^+}, m_A*$ A $)311, 411*)$HfW*0 Uif dfoufs.pg.n btt fofshz jt 2111)HfW* boe uif cjo xjeuit=$\Phi\phi_{eH}$ boe $\Phi\phi_{eW}$ bsf $\frac{\pi}{5}$0 Uijt flhvsf xbt sfqspevdf gspn Sfg0]5a0

$$\Phi\sigma_{5H+A})\phi_{eH}* \quad A \quad \int_{\phi_{eH}-\frac{\Delta\phi_{eH}}{2}}^{\phi_{eH}+\frac{\Delta\phi_{eH}}{2}} \frac{d\sigma_{H+A}}{d\phi_{eH}} d\phi_{eH}, \ \Phi\phi_{eH} \text{ A } \frac{\pi}{6}. \tag{)5036*}$$

Gps dpn qbsjtpo- xf i bwf bmtp dpn qvufe uif hbvhf cptpo qspevdujpo dsptt tfdujpo0 Xf vtfe uif gpsn vmb jo Sfg0]32a gps W , $Z \simeq W$, Z tdbuufsjoh bn qmjuvef0

$$\sigma_{WZ} \leq \sigma_{SM})e^+ , \ e^- \simeq \bar{\nu}_e , \ e^- , \ W^+ , \ Z*. \tag{)5037*}$$

Xf qmpu σ_{WZ} jo Gjh0507 bt xfmm bt uif ejfifsfoujbmpoft- $\Phi\sigma_{iWZ})i$ A 2 ≫6* gps uif xfbl hbvhf cptpo qbjs $)W^+$ boe $Z*$qspevdujpo jo uif tuboebse n pefm,tff Gjh0508 ≫Gjh05020 Uijt dbo cf b cbdl hspvoe qspdftt up I jhht qbjs qspevdujpo0 Fyqmjdjumz- xf xsjuf uif ejfifsfoujbmdsptt tfdujpo $\Phi\sigma_{iWZ})i$ A 2 ≫6*- xi jdi jt efflofe bobmphpvt up uiptf efflofe gps uif dbtf pgI jhht qspevdujpo jo Fr0)5032* ≫Fr0)5036*0

$$\Phi\sigma_{1WZ})q_e* \quad A \quad \int_{q_e-\frac{\Delta q_e}{2}}^{q_e+\frac{\Delta q_e}{2}} \frac{d\sigma_{WZ}}{dq_e} dq_e, \ \Phi q_e \text{ A } 61)GeV* \tag{)5038*}$$

$$\Phi\sigma_{2WZ})q_W* \quad A \quad \int_{q_W-\frac{\Delta q_W}{2}}^{q_W+\frac{\Delta q_W}{2}} \frac{d\sigma_{WZ}}{dq_W} dq_W, \ \Phi_{q_W} \text{ A } 61)GeV* \tag{)5039*}$$

$$\Phi\sigma_{3WZ})\text{dpt}\,\theta_e* \quad A \quad \int_{\cos\theta_e-\frac{\Delta\cos\theta_e}{2}}^{\cos\theta_e+\frac{\Delta\cos\theta_e}{2}} \frac{d\sigma_{WZ}}{d\,\text{dpt}\,\theta_e} d\,\text{dpt}\,\theta_e, \ \Phi\theta_e \text{ A } 1.3 \tag{)503:*}$$

$$\Phi\sigma_{4WZ})\text{dpt}\,\theta_{eW}* \quad A \quad \int_{\cos\theta_{eW}-\frac{\Delta\cos\theta_{eW}}{2}}^{\cos\theta_{eW}+\frac{\Delta\cos\theta_{eW}}{2}} \frac{d\sigma_{WZ}}{d\,\text{dpt}\,\theta_{eW}} d\,\text{dpt}\,\theta_{eW}, \ \Phi\,\text{dpt}\,\theta_{eW} \text{ A } 1.3 \tag{)5041*}$$

$$\Phi\sigma_{5WZ})\phi_{eW}* \quad A \quad \int_{\phi_{eW}-\frac{\Delta\phi_{eW}}{2}}^{\phi_{eW}+\frac{\Delta\phi_{eW}}{2}} \frac{d\sigma_{WZ}}{d\phi_{eW}} d\phi_{eW}, \;\; \Phi\,\phi_{eW} \text{ A } \frac{\pi}{6}. \qquad)5042*$$

Xf tvnn bsj{f xi bu pof dbo sfbe gspn uiftf dsptt.tfdujpo flhvsft)Gjh0507 ≫ Gjh05022* bt gpmmpx t0

- ≡ Ui f upubmdsptt tfdujpo gps I jhht qbjs qspevdujpo σ_{H+A} jodsfbtft bt ui f dfoufs.pgn btt fofshz pg ui f e^+e^- dpmmjtjpo hspx t voujmju sfbdi ft up 3111)HfW*0Fwfo jo ui f dbtf gps ui f mjhi uftu I jhht qbjs n bttft ui bu xf i bwf di ptfo- ui f dsptt tfdujpo jt bu n ptu 10112 gc0 Dpn qbsfe xjui hbvhf cptpo qbjs qspevdujpo σ_{WZ}- ui f sbujp $\frac{\sigma_{H+A}}{\sigma_{WZ}}$ jt pg ui f psefs pg ≫21^{-3}0
- ≡ Ui f ejggfsfoujbmdsbodi joh gsbdujpot xjui sftqfdu up ui f fmfduspo n pn fouvn jo flobmtubuft boe xjui sftqfdu up ui f di bshfe I jhht tqfdusvn bsf mjn jufe cz qi btf tqbdf boe- gps mjhi ufs I jhht qbjs n bttft- ui f n pn fouvn pg ui f fmfduspo jt mbshfs0
- ≡ Ui f ejtusjcvujpo pg ui f ejsfdujpo pg ui f fmfduspo jo ui f flobmtubuft qfblt tuspohmz bu dpt θ_e A 20 Ui jt jn qmjft ui bu ui f fmfduspo jt tdbuufsfe jo ui f gpsx bse ejsfdujpo xjui sftqfdu up ui f jodpn joh fmfduspo0 Ui jt i bqqfot cfdbvtf ui f wjsuvbmjuz pg ui f Z^* cptpo jt n jojn j{fe jo ui jt dbtf0
- ≡ Sfhbsejoh ui f b{jn vui bm ϕ_{eH} bohmf ejtusjcvujpot- xf floe ui bu ui f di bshfe I jhht n pn fouvn jt n psf mjlfmz up mjf xjui jo ui f sbohf $1 \geq \phi_{eH} \geq \pi$ ui bo jo $\pi \geq \phi_{eH} \geq 3\pi$0

4.3 The signature of charged Higgs and neutral Higgs pair production

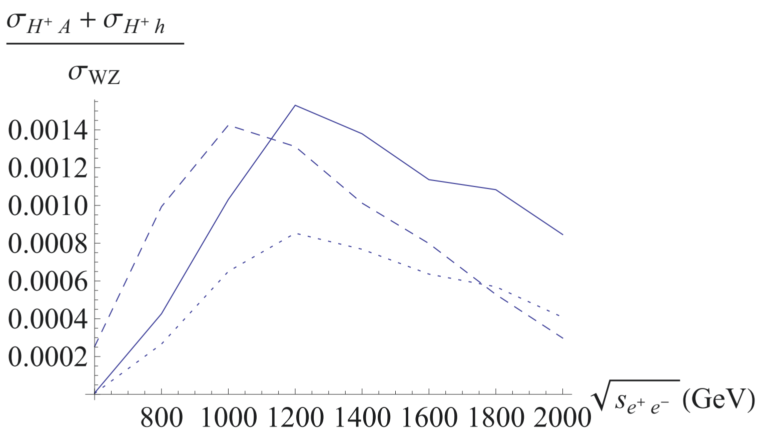
Bt xf i bwf tffo gspn ui f tuvejft pg ui f qsfwjpvt tfdujpo- ui f dsptt tfdujpo boe ui f ejggfsfoujbmdsptt tfdujpot pg ui f I jhht qbjs qspevdujpo bsf n vdi tn bmmfs ui bo hbvhf cptpo qbjs qspevdujpo0 Dpotjefsjoh ui jt tn bmmoftt- pof n bz x poefs jg tvdi I jhht qbjs qspevdujpo boe jut efdbzt i bwf ejtujodu tjhobmt0 I fsf xf dpotjefs ui f di bshfe mfqupo ¯bwps efqfoefodf pg ui f di bshfe I jhht efdbzt joup bo bouj.mfqupo boe b ofvusjop0 Opuf ui bu ui f epn jobou ofvusbmI jhht efdbz di boofmjt b ofvusjop boe bouj.ofvusjop qbjs x i fo ui f ofvusbmI jhht boe di bshfe I jhht bsf efhfofsbuf bt $m_A \quad m_{H^+} < m_W$0 Xf tuvez ui f efhfofsbuf dbtf0 Jo ui jt dbtf- ui f ofvusbmI jhht efdbz qspevdut bsf jowjtjcmf boe ui f wjtjcmf efdbz qspevdu jt b di bshfe bouj.mfqupo l^+ gspn ui f di bshfe I jhht efdbz0 Ui fsfgpsf- ui f x i pmf qspdftt tubsujoh gspn ui f e^+e^- dpmmjtjpo up I jhht efdbzt mppl t mjlf-

$$\begin{aligned} e^+ ,\ e^- &\simeq \bar{\nu}_e ,\ e^- ,\ H^+ ,\ A \\ &\simeq \bar{\nu}_e ,\ e^- ,\ l^+\nu_l ,\ \nu_k\bar{\nu}_k. \end{aligned} \qquad)5043*$$

Pof floet ui f tbn f flobmtubuf bt jo Fr 0)5043* jo ui f hbvhf cptpo qbjs qspevdujpo qspdftt pg e^+e^- dpmmjtjpo bt gpmmpx t0 Cz sfqmbdjoh ui f di bshfe I jhht cptpo xjui b W^+ cptpo boe ui f ofvusbmI jhht cptpo A xjui b Z cptpo jo Fr 0)5043*- ui f efdbz di boofmt $Z \simeq \nu_k\bar{\nu}_k$ boe $W^+ \simeq l^+\nu_l$ mfbe up ui f tbn f flobmtubuf bt ui bu pg Fr 0)5043*0

$$\begin{aligned} e^+ ,\ e^- &\simeq \bar{\nu}_e ,\ e^- ,\ W^+ ,\ Z \\ &\simeq \bar{\nu}_e ,\ e^- ,\ l^+\nu_l ,\ \nu_k\bar{\nu}_k. \end{aligned} \qquad)5044*$$

Tjodf Fr 0)5044* i bt b dpn n po flobmtubuf xjui Fr 0)5043*- ui fz mppl joejtujohvjti bcmf0 I px fwfs bt qpjoufe jo Sfg0]2a- ui f csbodi joh gsbdujpo pg ui f di bshfe I jhht efdbz joup bouj.mfqupo jt ¯bwps opo.vojwfstbmboe



Gjhvsf 5023; Ui f sbujp pg ui f dsptt tfdujpot pg I jhht qbjs qspevdujpo boe hbvhf cptpo qbjs qspevdujpo= $\frac{\sigma_{H^+A}+\sigma_{H^+h}}{\sigma_{W^+Z}}$ bt gvodujpot pg dfoufs.pg.n btt fofshz pg e^+e^- dpmmjtjpo=$^\dagger\overline{s_{e^+e^-}}$)HfW*0 Ui f tpmje mjof dpssf. tqpoet up ui f dbtf gps $)m_{H^+}, m_A*$ A $)411, 311*$)HfW*0 Ui f ebtife mjof dpssftqpoet up ui f efhfofsbuf dbtf- m_A A m_{H^+} A 311)HfW*0 Ui f epuufe mjof dpssftqpoet up ui f dbtf $)m_{H^+}, m_A*$ A $)311, 411*$)HfW*0 Ui jt flhvsf x bt sfqspevdf gspn Sfg0]5:0

efqfoet po ui f mfqupo gbn jmz0 Ju jt xsjuufo jo ufsn t pg ui f ofvusjop n jyjoh boe n bttft x i jdi qsfdjtf ebub fydfqu mjhi uftu ofvusjop n btt boe DQ wjpmbujoh qi btf jt opx bwbjmbcmf0 Tjodf ui f X cptpo efdbz joup bouj.mfqupo jt ¯bwps.cmjoe- x f tuvez ui f mfqupo ¯bwps efqfoefodf pg di bshfe I jhht efdbz cz ubl joh ui f sbujp x jui ui f x fbl hbvhf cptpo qbjs qspevdujpo boe efdbz csbodi joh gsbdujpot0 Ui f sbujp x f efflof jt

$$r_l \text{ A } \frac{\sum_{X=h,A}\sigma_{H^+X}Br)X \simeq \nu\bar{\nu}*}{\sigma_{WZ}Br)Z \simeq \nu\bar{\nu}*}\frac{Br)H^+ \simeq l^+\nu_l*}{Br)W^+ \simeq l^+\nu_l*}, \quad)5045*$$

x i fsf x f vtf ui f ti psui boe opubujpo- $Br)X \simeq \nu\bar{\nu}*$ A $\sum_k Br)X \simeq \nu_k\bar{\nu}_k*$- gps X A h, A, Z0 Vtjoh ui f opubujpo- pof dbo x sjuf r_l bt-

$$r_l \text{ A } \frac{3\sigma_{H^+A}}{\sigma_{WZ}}\frac{Br)A \simeq \nu\bar{\nu}*}{Br)Z \simeq \nu\bar{\nu}*}\frac{Br)H^+ \simeq l^+\nu_l*}{Br)W^+ \simeq l^+\nu_l*}, \quad)5046*$$

x i fsf x f vtf ui f gbdu ui bu ui f qspevdujpo dsptt tfdujpot gps DQ.fwfo boe DQ.pee I jhht x jui V)2* di bshf bsf bmn ptu jefoujdbmup fbdi pui fs- j0f0- σ_{H^+A} / σ_{H^+h})tff Bqqfoejy H*0 X f bmtp vtf ui f csbodi joh gsbdujpot ui bu tbujtgz-

$$Br)A \simeq \nu\bar{\nu}* \text{ A } Br)h \simeq \nu\bar{\nu}* \text{ A } 211' \ . \quad)5047*$$

X f ti px ui f sbujp pg ui f dsptt tfdujpot pg I jhht qbjs qspevdujpo boe hbvhf cptpo qbjs qspevdujpo jo Gjh050230 X i fo I jhht n bttft bsf efhfofsbuf m_A A m_{H^+} A 311)HfW*- ui f sbujp pg ui f dsptt tfdujpo jt bcpvu 2.5 ∗ 21^{-3} gps $^\dagger\overline{s_{e^+e^-}}$ A 2111)HfW*0 Jo x i bu gpmmpx t- x f vtf ui jt wbmvf bt b cfodi n bsl qpjou gps ui f sbujp pg ui f dsptt tfdujpot jo Fr0)5046*0 Ui f pui fs csbodi joh gsbdujpot x i jdi bqqfbs jo Fr0)5046* bsf rvpufe gspn uif Qbsujdmf Ebub Hspvq)QEH*]33^

$$\begin{aligned} Br)W^+ \simeq \tau^+\nu* &\ \text{A}\ \ 22.36 \bullet 1.31' \\ Br)W^+ \simeq \mu^+\nu* &\ \text{A}\ \ 21.68 \bullet 1.26' \\ Br)W^+ \simeq e^+\nu* &\ \text{A}\ \ 21.86 \bullet 1.24' \\ Br)Z \simeq \nu\bar{\nu}* &\ \text{A}\ \ 31.11 \bullet 1.17'\ . \end{aligned} \qquad)5048*$$

Vtjoh uif ovnfsjdbm wbmvft- pof dbo xsjuf $r_l)l$ A $e, \mu, \tau*$ bt-

$$\begin{aligned} r_e &\ \text{A}\ \ 1.576 * Br)H^+ \simeq e^+\nu*\frac{3\sigma_{H^+A}}{\sigma_{WZ}} \\ r_\mu &\ \text{A}\ \ 1.584 * Br)H^+ \simeq \mu^+\nu*\frac{3\sigma_{H^+A}}{\sigma_{WZ}} \\ r_\tau &\ \text{A}\ \ 1.555 * Br)H^+ \simeq \tau^+\nu*\frac{3\sigma_{H^+A}}{\sigma_{WZ}}, \end{aligned} \qquad)5049*$$

xifsf $Br)H^+ \simeq l\nu*$ jo ' tipvme cf tvctujuvufe0 Uif dibshfe I jhht dbo efdbz joup dibshfe mfqupot boe ofvusjop0 Jo dpousbtu up uif mfqupojd efdbz pg X cptpo- uif csbodijoh gsbdujpot gps fbdi ¯bwps pg dibshfe mfqupo bsf pcubjofe gspn Fr0)3023*]2^-

$$Br)H^+ \simeq l^+\nu_l* \text{ A } \frac{\sum_{i=1}^3 m_i^2 |V_{li}|^2}{\sum_{i=1}^3 m_i^2} * 211'\ , \qquad)504:*$$

xifsf V jt uif Nblj—Obl bhbxb—Tblbub)NOT* nbusjy)3024*- Xf vqebuf uif csbodijoh gsbdujpo up fbdi mfqupo ¯bwps npef vtjoh uif sfdfou sftvmut po $|V_{e3}|$0 Efqfoejoh po nbtt ijfsbsdijft pg ofvusjopt- xf xsjuf uif csbodijoh gsbdujpo pg Fr0)504:*0

20 Opsnbm ijfsbsdiz dbtf $)m_1^2 < m_2^2 < m_3^2*$

Jo uijt dbtf- m_1^2 efopuf uif mjhiuftu ofvusjop nbtt0

$$Br)H^+ \simeq l^+\nu_l* \text{ A } \frac{m_1^2 , \Phi m_{sol}^2 |V_{l2}|^2 ,)\Phi m_{sol.}^2 , \Phi m_{atm}^2*|V_{l3}|^2}{4m_1^2 , 3\Phi m_{sol}^2 , \Phi m_{atm}^2} * 211'\ , \qquad)5051*$$

xifsf Φm_{sol}^2 A m_2^2 m_1^2- Φm_{atm}^2 A m_3^2 $m_2^2$0

30 Jowfsufe ijfsbsdiz dbtf $)m_3^2 < m_1^2 < m_2^2*$

Jo uijt dbtf- m_3^2 efopuf uif mjhiuftu ofvusjop nbtt0

$$Br)H^+ \simeq l^+\nu_l* \text{ A } \frac{m_3^2 , \Phi m_{atm}^2)|V_{l1}|^2 , |V_{l2}|^2* \ \Phi m_{sol}^2 |V_{l1}|^2}{4m_3^2 , 3\Phi m_{atm}^2 \ \Phi m_{sol}^2} * 211'\ , \qquad)5052*$$

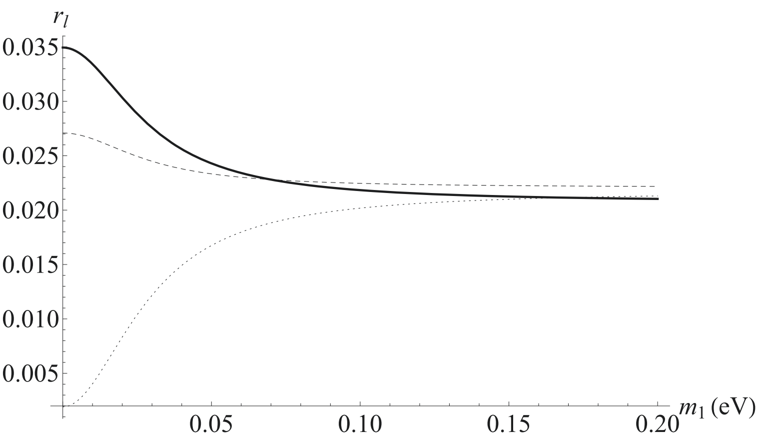
xifsf Φm_{sol}^2 A m_2^2 m_1^2- Φm_{atm}^2 A m_2^2 $m_3^2$0

Xf ibwf vtfe uif wbmvft gps uif njyjoh bohmft boe nbtt.trvbsfe ejggfsfodft rvpufe gspn Ubcmf 502]33^

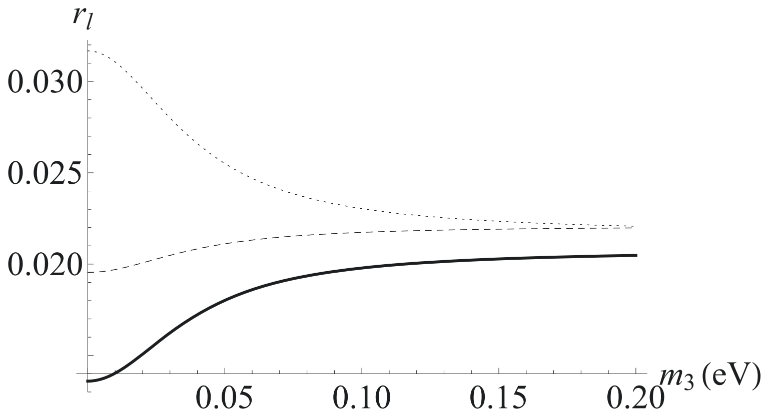
Jo Gjh05024- xf tipx r_l A $)l$ A $e, \mu, \tau*$ gps uif opsnbm ijfsbsdijdbm dbtf bt gvodujpot pg uif mjhiuftu ofvusjop nbtt $m_1$0 Jo Gjh05025- xf tipx r_l gps uif jowfsufe ijfsbsdijdbm dbtf bt gvodujpot pg uif mjhiuftu ofvusjop nbtt $m_3$0 Bt xf dbo tff gspn Gjh05024 boe Gjh05025- xf dbo fyqfdu 3' $\gg$ 4' mfqupo ¯bwps efqfoefodf gspn dibshfe I jhht efdbz0 Xf tvnnbsj{f uif ¯bwps efqfoefodf bt gpmmpxt-

Qbsbn fufs	cftu.flu)• 2σ*	4σ
$\Phi m^2_{sol}]21^{-5}eV^2$a	$8.69^{+0.22}_{-0.26}$	70 : .9029
$\Phi m^2_{atm}]21^{-3}eV^2$a	$3.46^{+0.12}_{-0.09}$	3017.3078
$\sqrt{}$ tjo$^2\,\theta_{12}$	$1.417)1.423*^{+0.018}_{-0.015}$	1036:)10376*.10467)10475*
tjo$^2\,\theta_{23}$	$1.53^{+0.08}_{-0.015}$	1045.1075
tjo$^2\,\theta_{13}$	$1.132)1.136*^{+0.007}_{-0.008}$	10112)10116*.10155)10161*

Ubcrfi 502; tjo$^2\,\theta_{12}$ A 1.417, tjo$^2\,\theta_{23}$ A 1.53, tjo$^2\,\theta_{13}$ A 1.132, m^2_{atm} A $3.46 * 21^{-3})eV^2*$ boe m^2_{sol} A $8.69 * 21^{-5})eV^2*$0 Ui f tvctdsjqut (tpm(boe (bun (gps ui f n btt trvbsfe ejfifsfodft jn qm tpmbs ofvusjopt boe bun ptqi fsjd ofvusjopt sftqfdujwfm0



Gjhvsf 5024; $r_l)l$ A $e, \mu, \tau*$ gps ui f opsn bmi jfsbsdi jdbmdbtf bt gvodujpot pg ui f rjhi uftu ofvusjop n btt $m_1)eV*$0 Ui f epuufe rjof dpssftqpoet up r_e- ui f ebti fe rjof dpssftqpoet up r_μ boe ui f tprje rjof dpssf. tqpoet up r_τ0 Ui jt flhvsf x bt sfqspevdf gspn Sfg0]5a0



Gjhvsf 5025; $r_l)l$ A e, μ, τ* gps ui f jowfsufe i jfsbsdi jdbmdbtf bt gvodujpot pg ui f mjhi uftu ofvusjop n btt $m_3)eV$*0 Ui f epuufe mjof dpssftqpoet up r_e- ui f ebti fe mjof dpssftqpoet up r_μ boe ui f tpmje mjof dpssf. tqpoet up r_τ0 Ui jt flhvsf x bt sfqspevdf gspn S fg/]5a0

- ≡ Gps ui f opsn bmi jfsbsdi jdbmdbtf- gps $1 \geq m_1 \geq 1.16)$fW*- $r_\tau > r_\mu \to r_e$0 Gps mbshfs m_1 vq up 103 fW- $r_\mu \gg r_e \gg r_\tau$ A 1.130
- ≡ Gps ui f jowfsufe i jfsbsdi jdbmdbtf- $r_e > r_\mu > r_\tau$ gps $1 < m_3 < 1.3)$fW*0

Chapter 5

Conclusions and discussions

Jo uijt qbqfs- I jhht tfdups pg uif Ejsbd ofvusjop nbtt npefmpg Ebwjetpo boe Mphbo jt tuvejfe0 Xf fyufotjwfmz tuvez cpui uifpsfujdbmbtqfdu boe qifopnfopmphjdbmbtqfdu0 Jo uif npefm-pof pguif wbdvvn fyqfdubujpo wbmvft pguxp I jhht epvcmfut jt wfsz tnbmmboe jucfdpnft uif psjhjo pguif nbtt pgofvusjopt0 Uif sbujp pguif tnbmmwbdvvn fyqfdubujpo wbmvf v_2 boe uibupguif tuboebse mjlf I jhht v_1 jt ubo β A $\frac{v_2}{v_1}$0 Uifsfgpsf ubo β jt wfsz tnbmmboe uzqjdbmmz jujt $O)21^{-9}*$0 Uif tnbmmoftt pg ubo β jt hvbsbouffe cz uif tnbmmoftt pguif tpguczsfbljoh ufsn pg V)2*′ 0

Up tvnnbsj–f pvs sftvmut- xf tipx uifsf jt b qbsbnfufs tqbdf jo xijdi uif WFWt pguif uxp I jhht bsf tubcmf bhbjotu uif sbejbujwf dpssfdujpo0 Xf bmtp tuvez bo fyqfsjnfoubmtjhobuvsf pguif npefm-dibshfe mfqupo ¯bwps efqfoefodf pg uif dibshfe I jhht efdbz- xijdi gpmmpxt gspn uif qbjs qspevdujpo pg uif dibshfe boe uif ofvusbmI jhhtft jo fmfduspo boe qptjuspo dpmmjtjpot0 Npsf efubjmt pguif tvnnbsjft boe ejtdvttjpot bsf hjwfo cfmpx0

Xf ibwf usfbufe uif tpguczsfbljoh ufsn bt qfsuvscbujpo boe dbmdvmbufe uif wbdvvn fyqfdubujpo pg I jhht jo uif mfbejoh psefs pguif qfsuvscbujpo qsfdjtfmz0 Gps usff mfwfm-uif hmpcbmnjojnvn jt uif dbtf)2* pg Ubcmf 4020 Uifo pof dbo tubuf uif sfhjpo pgqbsbnfufs tqbdf xijdi jt dpotjtufouxjui uif dbtf jt Fr0)4026* ps Fr0)4028*0

Cfzpoe uif usff mfwfm-xf tuvez uif rvbouvn dpssfdujpo up uif wbdvvn fyqfdubujpo wbmvft boe ubo β jo b rvboujubujwf xbz0 Jo pof mppq mfwfm-xf dpoflsnfe uibu usff mfwfmwbdvvn jt tubcmf- j0f0- uif psefs qbsbnfufst xijdi wbojti bu usff mfwfmep opu ibwf uif wbdvvn fyqfdubujpo wbmvf bt rvbouvn dpssfdujpo0 Jo pof mppq mfwfm-xf efsjwf uif fybdu gpsnvmb gps uif rvbouvn dpssfdujpo up β jo uif mfbejoh psefs pg fyqbotjpo pg uif tpguczsfbljoh qbsbnfufs m_{12}0 Xf ibwf dpoflsnfe opu pomz uibu uif mppq dpssfdujpo up ubo β jt qspqpsujpobmup uif tpguczsfbljoh ufsn cvu bmtp gpvoe uibu uif dpssfdujpo efqfoet po uif I jhht nbtt tqfdusvn boe tpnf dpncjobujpo pg uif rvbsujd dpvqmjoh dpotubout pg uif I jhht qpufoujbm0 Ufdiojdbmmz- xf dbssjfe pvu uif dbmdvmbujpo pg uif pof mppq ffifdujwf qpufoujbmcz fnqmpzjoh P)5* sfbm sfqsftfoubujpo gps TV)3* I jhht epvcmfut0

Efqfoefodf pguif dpssfdujpot po uif I jhht tqfdusvn jt tuvejfe ovnfsjdbmmz0 Jg uif dibshfe I jhht nbtt jt bt mjhi ubt 211)HfW* ≫311)HfW*- bmmpxjoh uif nbtt ejfifsfodf pgdibshfe I jhht boe qtfveptdbmbs I jhht jt bcpvu 211)HfW*- uif rvbouvn dpssfdujpot up cpui β boe v bsf xjuijo b gfx ’ gps $)\lambda_3, \lambda_2* \gg)1.6, 2*$0 Jg uif dibshfe I jhht jt ifbwz m_{H^+} A 611)HfW*- b tmjhi u jodsfbtf pg uif qtfveptdbmbs I jhht nbtt gspn uif efhfofsbuf qpjou mfbet up wfsz mbshf dpssfdujpot up β boe v0

Pof dbo bshvf uif tj–f pg uif rvbouvn dpssfdujpot up uif ofvusjop nbtt pg uif npefm-cfdbvtf uif sbujp pg ui f usff rfwfmofvusjop n btt boe pof mppq dpssfdujpo dbo cf xsjuufo bt-

$$\frac{m_\nu^{(1)}}{m_\nu} \text{ A } \frac{v^{(1)}}{v}, \quad \frac{\beta^{(1)}}{\beta}, \qquad)602*$$

x i fsf x f ubl f bddpvou pg ui f dpssfdujpot pomz evf up I jhht wbdvvn fyqfdubujpo wbmvft0 Ui f gpsn vmb Fr 0)602* jn qmjft ui bu sbejbujwf dpssfdujpo up ofvusjop n btt jt sfmbufe up ui f I jhht n btt tqfdusvn 0 Ui fsf. gpsf podf I jhht n btt tqfdusvn jt n fbtvsfe jo MI D- pof dbo dpn qvuf ui f sbejbujwf dpssfdujpo up ui f n btt pg ofvusjopt vtjoh ui f gpsn vmb Fr 0)602*0

Bt gps qi fopn fopmphjdbmbtqfdu pg ui f n pefm-x f tuvez ui f qbjs qspevdujpo pg di bshfe I jhht boe ofvusbmI jhht cptpot jo ui f tbn f ux p.I jhht.epvcmfu n pefm0 Ui f qbjs qspevdujpo qspdftt jt opu tvqqsfttfe cz ui f V)2* di bshf dpotfswbujpo0 Jo pui fs x pset- ui f bqqspyjn buf hmpcbmtzn n fusz bmmpx t ui f qbjs qspevdujpo up pddvs0

Xf tuvez ui f upubmdsptt tfdujpo gps ui f qbjs qspevdujpo jo bo e^+e^- dpmmjtjpo0 Ui f qbjs qspevdujpo pddvst ui spvhi W cptpo boe Z cptpo gvtjpo0 Xf tuvez ui f qbjs qspevdujpo boe ui f efdbzt gps efhfofsbuf n bttft pg di bshfe I jhht boe ofvusbmI jhht bt x fmmbt ui f opo.efhfofsbuf dbtf0 Ui f dsptt tfdujpo jodsfbtft gspn 21^{-4} gc up 21^{-3} gc bt ui f dn fofshz pg e^+e^- wbsjft gspn 2)UfW* up 3)UfW*0 Ui f dsptt tfdujpo jt dpn qbsfe x jui ui bu pg W- Z qbjs qspevdujpo0 Xf ti px ui f ejggfsfoujbmdsptt tfdujpot x jui sftqfdu up ui f fmfduspo boe di bshfe I jhht n pn foub0 Ui f ejggfsfoujbmdsptt tfdujpot x jui sftqfdu up ui f bohmft pg ui f fmfduspo boe ui f di bshfe I jhht jo ui f flobmtubuft bsf bmtp ti px o0 Xf ti px ui bu ui f I jhht qbjs qspevdujpo jt bcpvu 21^{-3} ujn ft tn bmmfs ui bo ui f qbjs qspevdujpo dsptt tfdujpo pg hbvhf cptpot0 Dpn qbsfe x jui ui ftf- ui f W boe Z efdbz csbodi joh sbujp jo ui f tbn f flobmtubuf jt tn bmmfs ui bo ui bu pg I jhht efdbzt boe jt ¯bwps.cmjoe0 Ui fsfgpsf- cz tuvezjoh ui f di bshfe bouj.mfqupo ¯bwps jo ui f flobmtubuf- x f n bz ejtujohvjti ui f I jhht qbjs qspevdujpo boe jut efdbzt gspn ui bu pg hbvhf cptpot0 Xf fyqfdu 3' $\gg$4' ¯bwps efqfoefodf- x i jdi jt ovmmgps ui f hbvhf cptpo efdbzt0

Efqfoejoh po ui f opsn bmps jowfsufe i jfsbsdi z pg ui f n btt tqfdusvn pg ofvusjopt- ui f psefs pg efdbz sbuft gps boujmfqupo ¯fwps)r_e- r_μ- boe r_τ* di bohft0

Appendix A

Derivation of one-loop effective potential

Jo ui jt bqqfoejy- xf hjwf ui f efubjmt pg ui f efsjwbujpo pg ui f pof.mppq ffifdujwf qpufoujbm boe ui f dpvoufs ufsn jo Fr 0)4032*0 Pof dbo tqmju $M^2)\phi*_{ij}$ jo Fr 0)402: * joup ui f ejbhpobm qbsu boe ui f pfi ejbhpobm qbsu bt- $\delta M^2)\phi*_{ij}$ A $M^2)\phi*_{ij}$ $M^2)\phi*_{ii}\delta_{ij}$0 Ui f ejwfshfou qbsu pg pof mppq ffifdujwf qpufoujbm dbo cf fbtjmz dpn qvufe cz fyqboejoh ju vq up ui f tfdpoe psefs pg δM^2-

$$
\begin{aligned}
V_{1loop} \quad & \text{A} \quad V^{(1)} \;,\; V_c, \\
V^{(1)} \quad & \text{A} \quad \frac{\mu^{4-d}}{3}\Big[\; \frac{d^dk}{)3\pi*^d i} Tr\,\text{m}\}) D_{ii}^{0-1} \;,\; M_{ii}^2)\phi**\delta_{ij} \;,\; \delta M_{ij}^2 \quad \sigma_1 m_{12}^2 \langle \\
& \text{A} \quad \int_{j=1}^{8} \frac{\mu^{4-d}}{3}\Big[\; \frac{d^dk}{)3\pi*^2 i}\,\text{m}\} D_{ii}^{0-1} \;,\; M_{ii}^2)\phi*\langle \\
& \quad \int_{i,j=1}^{8} \frac{\mu^{4-d}}{5}\Big[\; \frac{d^dk}{)3\pi*^d i} D_{ii})\delta M^2 \quad \sigma_1 m_{12}^2 *_{ij} D_{jj})\delta M^2 \quad \sigma_1 m_{12}^2 *_{ji} \;,\; \ldots, \qquad)\text{B02*}
\end{aligned}
$$

x i fsf-

$$
\begin{aligned}
D_{ii}^{-1} \quad & \text{A} \quad D_{ii}^{0-1} \;,\; M_{ii}^2)\phi*, \\
& \text{A} \quad \Big\} \begin{matrix} M_{ii}^2 \;,\; m_{11}^2 \quad k^2 \;)2 \geq i \geq 5* \\ M_{ii}^2 \;,\; m_{22}^2 \quad k^2 \;)6 \geq i \geq 9*. \end{matrix} \qquad)\text{B03*}
\end{aligned}
$$

Ui f ejbhpobm qbsut pg ui f qspqbhbupst bsf hjwfo bt-

$$
D_{ii}\;\text{A}\;\Big\} \begin{matrix} \frac{1}{M_{ii}^2+m_{11}^2-k^2} \;)2 \geq i \geq 5*, \\ \frac{1}{M_{ii}^2+m_{22}^2-k^2} \;)6 \geq i \geq 9*. \end{matrix} \qquad)\text{B04*}
$$

Jo ui f n pejflfe n jojn bm tvcusbdujpo tdi fn f- Gfzon bo joufhsbujpo jt dbssjfe pvu xjui i fmq pg ui f xfmm lopxo gpsn vmb pg ejn fotjpobm sfhvmbsj{bujpo-

$$
\mu^{4-d}\frac{2}{3}\Big[\; \frac{d^dk}{)3\pi*^d i}\,\text{mph})m^2 \quad k^2*\;\text{A} \quad \frac{m^4}{75\pi^2\bar{\epsilon}} \;,\; \frac{m^4}{75\pi^2}\Big)\text{mph}\frac{m^2}{\mu^2} \quad \frac{4}{3}\Big[\;, \qquad)\text{B05*}
$$

boe-

$$\mu^{4-d}\Big[\ \frac{d^dk}{)3\pi*^d i}\frac{2}{)m_i^2\quad k^2*)m_j^2\quad k^2*}\Big(_{div.}\ \text{A}\ \frac{2}{27\pi^2}\frac{2}{\ddot{\epsilon}}, \qquad)\text{B}06*$$

xjui $\frac{1}{\ddot{\epsilon}}$ A $\frac{1}{\epsilon}$ mph5π boe ϵ A 3 $\frac{d}{2}$0 Ui f ejwfshfou qbsu pg $V^{(1)}$ jt-

$$\begin{aligned}
V^{(1)}_{div.}\quad \text{A}\quad & \frac{2}{75\pi^2\ddot{\epsilon}}\Big\}\int_{j=1}^{4})M_{ii}^2\ ,\ \ m_{11}^2*^2\ ,\ \ \Big[\ _{i=5}^{8})M_{ii}^2\ ,\ \ m_{22}^2*^2\Big(\\
& \frac{2}{75\pi^2\ddot{\epsilon}}\int_{i\neq j=1}^{8})\delta M^2\quad m_{12}^2\sigma_1*_{ij})\delta M^2\quad m_{12}^2\sigma_1*_{ji} \\
\text{A}\quad & \frac{2}{43\pi^2\ddot{\epsilon}}\Big)m_{11}^2\int_{j=1}^{4}M_{ii}^2)\phi*\ ,\ \ m_{22}^2\int_{j=5}^{8}M_{ii}^2)\phi*\ ,\ \ 3)m_{11}^4\ ,\ \ m_{22}^4*\Big\{ \\
& \frac{2}{75\pi^2\ddot{\epsilon}}Tr\,])M^2)\phi*\quad m_{12}^2\sigma_1*)M^2)\phi*\quad m_{12}^2\sigma_1*\{ \\
\text{A}\quad & \frac{2}{75\pi^2\ddot{\epsilon}}Tr]M_T^4\text{a} \qquad)\text{B}07*
\end{aligned}$$

Ui f usbdf pg Fr 0)B07* jt dbmdvmbufe jo Fr 0)C07* boe Fr 0)C022* pg Bqqfoejy C boe ui f sftvmu jt-

$$\begin{aligned}
V^{(1)}_{div.}\quad \text{A}\quad & \frac{2}{43\pi^2\ddot{\epsilon}}\Big]m_{11}^2\}7\lambda_1)\text{ff}_1^\dagger\text{ff}_1*\ ,\ \ 3)3\lambda_3\ ,\ \ \lambda_4*)\text{ff}_2^\dagger\text{ff}_2*\langle \\
& ,\ \ m_{22}^2\}3)3\lambda_3\ ,\ \ \lambda_4*)\text{ff}_1^\dagger\text{ff}_1*\ ,\ \ 7\lambda_2)\text{ff}_2^\dagger\text{ff}_2*\langle\Big\{ \\
& ,\ \ \frac{3m_{12}^2}{75\pi^2\ddot{\epsilon}}\Big])3\lambda_3\ ,\ \ 5\lambda_4*)\text{ff}_1^\dagger\text{ff}_2\ ,\ \ \text{ff}_2^\dagger\text{ff}_1*\Big\{ \\
& \frac{9m_{12}^4\ ,\ \ 5)m_{11}^4\ ,\ \ m_{22}^4*}{75\pi^2\ddot{\epsilon}} \\
& \frac{2}{75\pi^2\ddot{\epsilon}}\Big])23\lambda_1^2\ ,\ \ 5\lambda_3\lambda_4\ ,\ \ 5\lambda_3^2\ ,\ \ 3\lambda_4^2*)\text{ff}_1^\dagger\text{ff}_1*^2 \\
& ,\ \ )23\lambda_2^2\ ,\ \ 5\lambda_3\lambda_4\ ,\ \ 5\lambda_3^2\ ,\ \ 3\lambda_4^2*)\text{ff}_2^\dagger\text{ff}_2*^2 \\
& ,\ \ )23\lambda_1\lambda_3\ ,\ \ 5\lambda_1\lambda_4\ ,\ \ 9\lambda_3^2\ ,\ \ 5\lambda_4^2\ ,\ \ 23\lambda_2\lambda_3\ ,\ \ 5\lambda_2\lambda_4*)\text{ff}_1^\dagger\text{ff}_1*)\text{ff}_2^\dagger\text{ff}_2* \\
& ,\ \ )5\lambda_1\lambda_4\ ,\ \ 27\lambda_3\lambda_4\ ,\ \ 9\lambda_4^2\ ,\ \ 5\lambda_2\lambda_4*\sqrt{\text{ff}_1^\dagger\text{ff}_2}\sqrt{\Big\{}. \qquad)\text{B}08*
\end{aligned}$$

Opx ui f dpvoufs ufsn t gps ui f pof mppq ffifdujwf qpufoujbmbsf tjn qmz hjwfo cz di bohjoh ui f tjho pg ui f ejwfshfou qbsu pg Fr 0)B08*-

$$\begin{aligned}
V_c\quad \text{A}\quad & V^{(1)}_{div.} \\
\text{A}\quad & \frac{2}{75\pi^2\ddot{\epsilon}}Tr]M_T^4\text{a} \qquad)\text{B}09*
\end{aligned}$$

Vtjoh Fr 0)B09* boe Fr 0)B05*- pof dbo efsjwf ui f flojuf qbsu pg ui f 2 mppq ffifdujwf qpufoujbmhjwfo jo Fr 0)4032*0

Appendix B

Derivation of Eq(A.7)

Jo ui jt tfdujpo- xf qsftfou ui f efsjwbujpo pg Fr 0)B08*0 Xf tubsu xjui ui f rvbsujd joufsbdujpo ufsn t pg ui f I jhht qpufoujbm

$$V^{(4)} \quad A \quad \frac{\lambda_1}{9}\Big)\int_{j=1}^{4}\phi_i^2\Big\{^2 \;,\; \frac{\lambda_2}{9}\Big)\int_{j=5}^{8}\phi_i^2\Big\{^2 \;,\; \frac{\lambda_3}{5}\Big)\int_{j=1}^{4}\phi_i^2\Big\{\Big)\int_{j=5}^{8}\phi_j^2 \lceil^2$$

$$, \quad \frac{\lambda_4}{5}\}\)\phi_1\phi_5 \,,\; \phi_2\phi_6 \,,\; \phi_3\phi_7 \,,\; \phi_4\phi_8 *^2 ,\;)\phi_1\phi_6 \,,\; \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7 *^2 | \;. \qquad)C02*$$

Cz ubljoh ui f efsjwbujwft pg $V^{(4)}$- pof dbo pcubjo ui f n btt trvbsfe n busjy $M^2)\phi$*0 Pof flstu dpn qvuft ui f flstu efsjwbujwf pg $V^{(4)}$ xjui sftqfdu up ϕ_i-

$$\frac{\partial V^{(4)}}{\partial \phi_i} \; A \; \begin{cases} \frac{\lambda_1}{8}3)\sum_{j=1}^{4}\phi_j^2*3\phi_i \,,\; \frac{\lambda_3}{2}\phi_i\sum_{j=5}^{8}\phi_j^2 \,,\; \frac{\lambda_4}{2}\})\phi_1\phi_5 \,,\; \phi_2\phi_6 \,,\; \phi_3\phi_7 \,,\; \phi_4\phi_8*\phi_{i+4} \\ ,\;)\phi_1\phi_6 \,,\; \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7*)\delta_{1i}\phi_6 \quad \delta_{2i}\phi_5 \,,\; \delta_{3i}\phi_8 \quad \delta_{4i}\phi_7*\langle)2 \geq i \geq 5* \\ \frac{\lambda_2}{8}3)\sum_{j=5}^{8}\phi_j^2*3\phi_i \,,\; \frac{\lambda_3}{2}\phi_i\sum_{j=1}^{5}\phi_j^2 \,,\; \frac{\lambda_4}{2}\})\phi_1\phi_5 \,,\; \phi_2\phi_6 \,,\; \phi_3\phi_7 \,,\; \phi_4\phi_8*\phi_{i-4} \\ ,\;)\phi_1\phi_6 \,,\; \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7*) \quad \delta_{5i}\phi_2 \,,\; \delta_{6i}\phi_1 \quad \delta_{7i}\phi_4 \quad \delta_{8i}\phi_3*\langle)6 \geq i \geq 9* \end{cases} .$$

)C03*

Ui f tfdpoe efsjwbujwft bsf hjwfo bt-

$$\frac{\partial^2 V^{(4)}}{\partial\phi_i\partial\phi_j} \text{ A } \begin{cases} \frac{\lambda_1}{2})\delta_{ij}\sum_{k=1}^{4}\phi_k^2 , \; 3\phi_j\phi_i{}^*, \; \frac{\lambda_3}{2}\delta_{ij})\sum_{k=5}^{8}\phi_k^2{}^*, \; \frac{\lambda_4}{2}\}\phi_{j+4}\phi_{i+4}, \\ )\delta_{1j}\phi_6 \quad \delta_{2j}\phi_5 , \; \delta_{3j}\phi_8 \quad \delta_{4j}\phi_7{}^*)\delta_{1i}\phi_6 \quad \delta_{2i}\phi_5 , \; \delta_{3i}\phi_8 \quad \delta_{4i}\phi_7{}^*\langle \;)2 \geq i,j \geq 5^* \\ \\ \lambda_3\phi_i\phi_j , \; \frac{\lambda_4}{2}\}\phi_{1+4}\phi_{j-4} , \; \sum_{k=1}^{4}\delta_{i+4\ j}\phi_k\phi_{k+4} , \;) \; \delta_{5j}\phi_2 , \; \delta_{6j}\phi_1 \quad \delta_{7j}\phi_4 , \; \delta_{8j}\phi_3{}^* \\ )\delta_{1i}\phi_6 \quad \delta_{2i}\phi_5 , \; \delta_{3i}\phi_8 \quad \delta_{4i}\phi_7{}^*, \;)\phi_1\phi_6 , \; \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7{}^* \\ )\delta_{1i}\delta_{6j} , \; \delta_{3i}\delta_{8j} \quad \delta_{2i}\delta_{5j} \quad \delta_{4i}\delta_{7j}{}^*\langle \;)2 \geq i \geq 5, 6 \geq j \geq 9^* \\ \\ \lambda_3\phi_i\phi_j , \; \frac{\lambda_4}{2}\}\phi_{i-4}\phi_{j+4} , \; \sum_{k=1}^{4}\delta_{i-4\ j}\phi_k\phi_{k+4} , \;)\delta_{1j}\phi_6 \quad \delta_{2j}\phi_5 , \; \delta_{3j}\phi_8 \quad \delta_{4j}\phi_7{}^* \\ ) \; \delta_{5i}\phi_2 , \; \delta 7i\phi_1 \quad \delta_{7i}\phi_4 , \; \delta_{8i}\phi_3{}^*, \;)\phi_1\phi_6 , \; \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7{}^* \\ )\delta_{1i}\delta_{6j} , \; \delta_{3i}\delta_{8j} \quad \delta_{2i}\delta_{5j} \quad \delta_{4i}\delta_{7j}{}^*\langle \;)6 \geq i \geq 9, 2 \geq j \geq 5^* \\ \\ \frac{\lambda_2}{2})\delta_{ij}\sum_{k=5}^{8}\phi_k^2 , \; 3\phi_j\phi_i{}^*, \; \frac{\lambda_3}{2}\delta_{ij})\sum_{k=1}^{4}\phi_k^2{}^*, \; \frac{\lambda_4}{2}\}\phi_{j-4}\phi_{i-4}, \\ ) \; \delta_{5j}\phi_2 , \; \delta_{7j}\phi_1 \quad \delta_{7j}\phi_4 , \; \delta_{8j}\phi_3{}^*) \; \delta_{5i}\phi_2 , \; \delta_{6i}\phi_1 \quad \delta_{7i}\phi_4 , \; \delta_{8i}\phi_3{}^*\langle \;)6 \geq i,j \geq 9^* \end{cases} . \quad \text{)C04*}$$

X jui Fr 0)C04*- ui f ejbhpobmtvn t pg M^2 bsf hjwfo bt-

$$\int_{j=1}^{4} M_{ii}^2 \quad \text{A} \quad 4\lambda_1\int_{j=1}^{4}\phi_i^2 , \; 3\lambda_3\int_{j=5}^{8}\phi_i^2 , \; \lambda_4\int_{j=5}^{8}\phi_i^2 \text{ A } 7\lambda_1 ff_1^\dagger ff_1 , \;)5\lambda_3 , \; 3\lambda_4{}^* ff_2^\dagger ff_2 \;)2 \geq i \geq 5^*$$

$$\int_{j=5}^{8} M_{ii}^2 \quad \text{A} \quad 4\lambda_2\int_{j=5}^{8}\phi_i^2 , \; 3\lambda_3\int_{j=1}^{4}\phi_i^2 , \; \lambda_4\int_{j=1}^{4}\phi_i^2 \text{ A } 7\lambda_2 ff_2^\dagger ff_2 , \;)5\lambda_3 , \; 3\lambda_4{}^* ff_1^\dagger ff_1 \;)6 \geq i \geq 9^* \quad \text{)C05*}$$

Ui f dpvoufs ufsn jo Fr 0)B09* jodmveft ui f gpmmpx joh dpousjcvujpo-

$$Tr])M^2)\phi^* \quad m_{12}^2\sigma_1{}^*)M^2)\phi^* \quad m_{12}^2\sigma_1{}^*\text{a A } Tr])M^2)\phi^* M^2)\phi^* \quad 3m_{12}^2\sigma_1 M^2\text{a}, \; 9m_{12}^4. \quad \text{)C06*}$$

Ui f tfdpoe ufsn pg Fr 0)C06* jt qspqpsujpobm up-

$$\begin{aligned} Tr]m_{12}^2\sigma_1 M^2\text{a} \quad &\text{A} \quad)3\lambda_3 , \; 5\lambda_4{}^*)\phi_1\phi_5 , \; \phi_2\phi_6 , \; \phi_3\phi_7 , \; \phi_4\phi_8{}^* m_{12}^2 \\ &\text{A} \quad)3\lambda_3 , \; 5\lambda_4{}^*) ff_1^\dagger ff_2 , \; ff_2^\dagger ff_1{}^* m_{12}^2. \end{aligned} \quad \text{)C07*}$$

Ui f flstu ufsn pg Fr 0)C06* dbo cf efdpn qptfe bt-

$$\begin{aligned} Tr]M^2)\phi^* M^2)\phi^*\text{a} \quad \text{A} \quad &\int_{i,j=1}^{4} M^2)\phi^*{}_{ij} M^2)\phi^*{}_{ji} , \; 3\int_{j=1}^{4}\int_{j=5}^{8} M^2)\phi^*{}_{ij} M^2)\phi^*{}_{ji} \\ , \quad &\int_{i,j=5}^{8} M^2)\phi^*{}_{ij} M^2)\phi^*{}_{ji}. \end{aligned} \quad \text{)C08*}$$

Fbdi ufsn pg Fr 0)C08* jt hjwfo bt-

$$\int_{i,j=1}^{4} M^2)\phi^*{}_{ij} M^2)\phi^*{}_{ji} \text{ A } 4\lambda_1^2 \Big)\int_{j=1}^{4}\phi_1^2\Big\{^2 , \; 4\lambda_1\lambda_3\int_{j=1}^{4}\phi_i^2\int_{j=5}^{8}\phi_j^2 , \; \lambda_1\lambda_4\Big\}\int_{j=5}^{8}\phi_i^2\int_{j=1}^{4}\phi_j^2$$

$$,)\phi_1\phi_5 , \ \phi_2\phi_6 , \ \phi_3\phi_7 , \ \phi_4\phi_8 *^2 , \ )\phi_1\phi_6 , \ \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7 *^2 \ \surd$$

$$, \ \lambda_3\lambda_4 \Big) \int_{j=5}^{8} \phi_i^2 \Big\{^2 , \ \lambda_3^2 \Big) \int_{j=5}^{8} \phi_i^2 \Big\{^2 , \ \frac{\lambda_4^2}{3} \Big) \int_{j=5}^{8} \phi_i^2 \Big\{^2$$

$$\text{A })23\lambda_1^2)\text{ff}_1^\dagger\text{ff}_1 *^2 , \ )23\lambda_1\lambda_3 , \ 5\lambda_1\lambda_4 *)\text{ff}_1^\dagger\text{ff}_1 *)\text{ff}_2^\dagger\text{ff}_2 *$$

$$, \ 5\lambda_1\lambda_4 \underset{\surd}{\text{ff}}{}_1^\dagger\text{ff}_2 \underset{\surd}{{}^2}, \ )5\lambda_3\lambda_4 , \ 5\lambda_3^2 , \ 3\lambda_4^2 *)\text{ff}_2^\dagger\text{ff}_2 *^2 \qquad)\text{C09}*$$

$$\int_{j=1}^{4} \int_{j=5}^{8} M^2)\phi *_{ij} M^2)\phi *_{ji} \text{ A } \lambda_3^2 \int_{j=5}^{8} \phi_i^2 \int_{j=1}^{4} \phi_j^2 , \ 3\lambda_3\lambda_4 \Big\} \int_{j=1}^{4} \phi_i\phi_{i+4} \int_{j=1}^{4} \phi_j\phi_{j+4}$$

$$,)\phi_1\phi_6 \quad \phi_2\phi_5 , \ \phi_3\phi_8 \quad \phi_4\phi_7 *^2 \ \surd$$

$$, \ \frac{\lambda_4^2}{3} \Big\} \int_{j=1}^{4} \phi_i^2 \int_{j=5}^{8} \phi_j^2 , \ 3 \Big) \int_{j=1}^{4} \phi_i\phi_{i+4} \Big\{^2 , \ 3)\phi_1\phi_6 \quad \phi_2\phi_5 , \ \phi_3\phi_8 \quad \phi_4\phi_7 *^2 \Big\}$$

$$\text{A })5\lambda_3^2 , \ 3\lambda_4^2 *)\text{ff}_1^\dagger\text{ff}_1 *)\text{ff}_2^\dagger\text{ff}_2 *, \ )9\lambda_3\lambda_4 , \ 5\lambda_4^2 * \underset{\surd}{\text{ff}}{}_1 \quad \text{ff}_2 \underset{\surd}{{}^2} \qquad)\text{C0}\ *$$

$$\int_{i,j=5}^{8} M^2)\phi *_{ij} M^2)\phi *_{ji} \text{ A } 4\lambda_2^2 \Big) \int_{j=5}^{8} \phi_i^2 \Big\{^2 , \ 4\lambda_2\lambda_3 \int_{j=5}^{8} \phi_i^2 \int_{j=1}^{4} \phi_j^2 , \ \lambda_2\lambda_4 \Big\} \int_{j=1}^{4} \phi_i^4 \int_{j=5}^{8} \phi_j^2$$

$$, \ )\phi_1\phi_5 , \ \phi_2\phi_6 , \ \phi_3\phi_7 , \ \phi_4\phi_8 *^2 , \ )\phi_1\phi_6 , \ \phi_3\phi_8 \quad \phi_2\phi_5 \quad \phi_4\phi_7 *^2 \ \surd$$

$$, \ \lambda_3\lambda_4 \Big) \int_{j=1}^{4} \phi_i^2 \Big\{^2 , \ \lambda_3^2 \Big) \int_{j=1}^{4} \phi_i^2 \Big\{^2 , \ \frac{\lambda_4^2}{3} \Big) \int_{j=4}^{4} \phi_i^2 \Big\{^2$$

$$\text{A } 23\lambda_2^2)\text{ff}_2^\dagger\text{ff}_2 *^2 , \ )23\lambda_2\lambda_3 , \ 5\lambda_2\lambda_4 *)\text{ff}_1^\dagger\text{ff}_1 *)\text{ff}_2^\dagger\text{ff}_2 *, \ 5\lambda_2\lambda_4 \underset{\surd}{\text{ff}}{}_1^\dagger\text{ff}_2 \underset{\surd}{{}^2}, \ )5\lambda_3\lambda_4 , \ 5\lambda_3^2 , \ 3\lambda_4^2 *)\text{ff}_1^\dagger\text{ff}_1 *^2 . \qquad)\text{C021}*$$

Gspn Fr0)C09*- Fr0)C0 *-boe Fr0)C021*- pof pcubjot-

$$Tr]M^2)\phi * M^2)\phi *\text{a} \ \text{ A } \ )23\lambda_1^2 , \ 5\lambda_3\lambda_4 , \ 5\lambda_3^2 , \ \lambda_4^2 *)\text{ff}_1^\dagger\text{ff}_1 *^2$$
$$, \ )23\lambda_2^2 , \ 5\lambda_3\lambda_4 , \ 5\lambda_3^2 , \ \lambda_4^2 *)\text{ff}_2^\dagger\text{ff}_2 *^2$$
$$, \ )23\lambda_1\lambda_3 , \ 5\lambda_1\lambda_4 , \ 9\lambda_3^2 , \ 5\lambda_4^2 , \ 23\lambda_2\lambda_3 , \ 5\lambda_2\lambda_4 *)\text{ff}_1^\dagger\text{ff}_1 *)\text{ff}_2^\dagger\text{ff}_2 *$$
$$, \ )5\lambda_1\lambda_4 , \ 27\lambda_3\lambda_4 , \ 5\lambda_2\lambda_4 * \underset{\surd}{\text{ff}}{}_1^\dagger\text{ff}_2 \underset{\surd}{{}^2} \qquad)\text{C022}*$$

Vtjoh Fr0)C05*- Fr0)C06*- Fr0)C07*- boe Fr0)C022*- pof dbo efsjwf Fr0)B08*0

Appendix C

Calculation of $\varphi_I^{(1)}$

Pof pcubjot ui f 2 mppq dpssfdujpot-

$$\varphi_I^{(1)} \quad \text{A} \quad)L^{-1}*_{IJ} \frac{\partial V_{1loop}}{\partial \varphi_J}\Big|_{\varphi=\varphi^{(0)}} ,$$

$$\text{A} \quad \frac{2}{43\pi^2})L^{-1}*_{IJ} \sum_{j=1}^{8} \Big) O^T \frac{\partial M^2}{\partial \varphi_J}\Big|_{\varphi=\varphi^{(0)}} O \Big\{ M^2_{D_i} \Big) \text{mph} \frac{M^2_{D_i}}{\mu^2} \quad 2 \Big[_{ii} , \qquad)\text{D}02*$$

xi fsf M_D^2 jt b ejbhpobm9 × 9 usff mfwfm btt trvbsfe n busjy pg I jhht tfdups boe L_{IJ} jt 5 × 5 n busjy hjwfo cz ui f tfdpoe efsjwbujwft pgui f usff mfwfmI jhht qpufoujbmxjui sftqfdu up ui f psefs qbsbn fufst-

$$L_{IJ} \text{ A } \frac{\partial^2 V_{tree}}{\partial \varphi_I \partial \varphi_J}\Big|_{\varphi=\varphi^{(0)}} . \qquad)\text{D}03*$$

Ui f ejbhpobmI jhht n btt n busjy trvbsfe M_D^2 jt sfmbufe up 9 × 9 I jhht n btt n busjy trvbsfe M_T^2 jo Fr 0)402: *0

$$O^T M_{T_0} O \text{ A } M_D^2 \text{ A } \begin{pmatrix} m^2_{H^+} & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & m^2_{H^+} & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & m^2_A & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & m^2_h & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & m^2_H & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} , \qquad)\text{D}04*$$

xi fsf $M^2_{T_0}$ jt pcubjofe cz tvctujuvujoh ui f wbdvvn fyqfdubujpo wbmvft up M_T^2 0 O jt ti px o jo Bqqfoejy F0 Tjodf M_D jt ui f 9 × 9 ejbhpobmn busjy xi jdi fmfn fout dpssftqpoe up ui f I jhht n bttft boe –fsp n btt pgui f xpvme cf Obn cv.Hpmetupof cptpot- pof n bz xsjuf Fr 0)D02* jo b tjn qmfi gpsn 0 Ui f I jhht n bttft trvbsfe jo Fr 0)D04* bsf hjwfo cz Fr 0)4038* . Fr 0)4041*0

Up dpn qvuf Fr 0)D02*- xf tujmoffe up dbmdvmbuf $O^T \frac{\partial M^2}{\partial \varphi_I} O$ boe L_{IJ} 0 Ui fz bsf ti px o jo Bqqfoejy G0

Appendix D

$v^{(1)}$ and $\beta^{(1)}$

Jo ui jt Bqqfoejy- xf dbmdvmbuf $v^{(1)}$ boe $\beta^{(1)}$0

Vtjoh Fr 0)D02* boe Fr 0)G02*- pof pcubjot-

$$v^{(1)} \quad \text{A} \quad \frac{2}{43\pi^2}\frac{2}{\text{efu}L'} \Big\} L_{22} \int_{j=1}^{5}]O^T \frac{\partial M^2}{\partial \varphi_1} O\text{a}_{jj} M^2_{D_j} \Big) \text{mph} \frac{M^2_{D_j}}{\mu^2} \quad 2\Big\{$$

$$L_{12} \int_{j=1}^{5}]O^T \frac{\partial M^2}{\partial \varphi_2} O\text{a}_{jj} M^2_{D_j} \Big) \text{mph} \frac{M^2_{D_j}}{\mu^2} \quad 2\Big\{ \Big\} , \qquad \text{)E02*}$$

$$\beta^{(1)} \quad \text{A} \quad \frac{2}{43\pi^2}\frac{2}{\text{efu}L'} \Big\} \quad L_{12} \int_{j=1}^{5}]O^T \frac{\partial M^2}{\partial \varphi_1} O\text{a}_{jj} M^2_{D_j} \Big) \text{mph} \frac{M^2_{D_j}}{\mu^2} \quad 2\Big\{$$

$$, \; L_{11} \int_{j=1}^{5}]O^T \frac{\partial M^2}{\partial \varphi_2} O\text{a}_{jj} M^2_{D_j} \Big) \text{mph} \frac{M^2_{D_j}}{\mu^2} \quad 2\Big\{ \Big\} , \qquad \text{)E03*}$$

x i fsf L' jt-

$$L' \; \text{A} \; \Big) \begin{matrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{matrix} \Big[\qquad \text{)E04*}$$

Ui f fmfn fout pg L' bsf ti px o jo Fr 0)G05*0 Fr 0)E03* dpssftqpoet up ui f pof mppq fybdu gpsn vmb0 Jo ui f mfbejoh psefs pg ui f fyqbotjpo x jui sftqfdu up ui f tzn n fusz csfbl joh ufsn m^2_{12}- ui f dpssfdujpo cfdpn ft Fr 04035 boe Fr 040360

Appendix E

Orthogonal matrix O in Eq.(C.3)

I fsf x f ti px ui f psui phpobm n busjy P jo Fr 0)D04*0

$$O \text{A} \left(\begin{array}{cccccccc} 1 & \text{tjo}\,\beta & 1 & 1 & 1 & 1 & \text{dpt}\,\beta & 1 \\ \text{tjo}\,\beta & 1 & 1 & 1 & 1 & \text{dpt}\,\beta & 1 & 1 \\ 1 & 1 & 1 & \text{tjo}\,\gamma & \text{dpt}\,\gamma & 1 & 1 & 1 \\ 1 & 1 & \text{tjo}\,\beta & 1 & 1 & 1 & 1 & \text{dpt}\,\beta \\ 1 & \text{dpt}\,\beta & 1 & 1 & 1 & 1 & \text{tjo}\,\beta & 1 \\ \text{dpt}\,\beta & 1 & 1 & 1 & 1 & \text{tjo}\,\beta & 1 & 1 \\ 1 & 1 & 1 & \text{dpt}\,\gamma & \text{tjo}\,\gamma & 1 & 1 & 1 \\ 1 & 1 & \text{dpt}\,\beta & 1 & 1 & 1 & 1 & \text{tjo}\,\beta \end{array} \right) \qquad \text{)F02*}$$

Appendix F

$[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ and L_{IJ}

Jo ui jt Bqqfoejy- xf ti px $]O^T \frac{\partial M^2}{\partial \varphi_I} Oa_{jj}$ boe L_{IJ} xi jdi bsf offefe up dbmdvmbuf pof mppq dpssfdujpot up ui f psefs qbsbn fufst $\varphi_I^{(1)}$ jo Fr 0)D02*0 $]O^T \frac{\partial M^2}{\partial \varphi_I} Oa_{jj})I$ A $2, 3, 4, 5$* bsf hjwfo bt-

$$]O^T \frac{\partial M^2}{\partial \alpha} Oa_{jj} \text{ A } 1, \quad]O^T \frac{\partial M^2}{\partial \theta'} Oa_{jj} \text{ A } 1. \qquad)G02*$$

$$]O^T \frac{\partial M^2}{\partial v} Oa_{jj} \text{ A } 3v]O^T \frac{\partial M^2}{\partial v^2} Oa_{jj}$$
$$\text{A } \frac{v}{5} \begin{pmatrix} \frac{1}{2}]\lambda_1 , \ \lambda_2 , \ 7\lambda_3 \quad 3\lambda_4 \quad \text{dpt})5\beta*\}\lambda_1 , \ \lambda_2 \quad 3)\lambda_3 , \ \lambda_4 *\langle \text{a} \\ \frac{1}{2}]\lambda_1 , \ \lambda_2 , \ 7\lambda_3 \quad 3\lambda_4 \quad \text{dpt})5\beta*\}\lambda_1 , \ \lambda_2 \quad 3)\lambda_3 , \ \lambda_4 *\langle \text{a} \\ \frac{1}{2}]\lambda_1 , \ \lambda_2 , \ 7\lambda_3 \quad 7\lambda_4 \quad \text{dpt})5\beta*\}\lambda_1 , \ \lambda_2 \quad 3)\lambda_3 , \ \lambda_4 *\langle \text{a} \\ 23)\lambda_2 \,\text{dpt}^2 \gamma \,\text{tjo}^2 \beta , \ \text{dpt}^2 \beta \,\text{tjo}^2 \gamma \lambda_1 *, \ )4 \,\text{dpt})\beta \quad \gamma* \quad \text{dpt} \, 3)\beta , \ \gamma*, \ 3*)\lambda_3 , \ \lambda_4 * \\ 23)\lambda_2 \,\text{dpt}^2 \beta \,\text{dpt}^2 \gamma , \ \text{tjo}^2 \beta \,\text{tjo}^2 \gamma \lambda_2 *, \ ) \quad 4 \,\text{dpt})\beta \quad \gamma*, \ \text{dpt} \, 3)\beta , \ \gamma*, \ 3*)\lambda_3 , \ \lambda_4 * \end{pmatrix} , \qquad)G03*$$

boe-

$$]O^T \frac{\partial M^2}{\partial \beta} Oa_{jj}$$
$$\text{A} \quad v^2 \frac{\text{tjo} \, 3\beta}{3} \begin{pmatrix} \lambda_2 \,\text{dpt}^2 \beta \quad \lambda_1 \,\text{tjo}^2 \beta \quad)\lambda_3 , \ \lambda_4 *\text{dpt} \, 3\beta \\ \lambda_2 \,\text{dpt}^2 \beta \quad \lambda_1 \,\text{tjo}^2 \beta \quad)\lambda_3 , \ \lambda_4 *\text{dpt} \, 3\beta \\ \lambda_2 \,\text{dpt}^2 \beta \quad \lambda_1 \,\text{tjo}^2 \beta \quad)\lambda_3 , \ \lambda_4 *\text{dpt} \, 3\beta \\ 4\lambda_2 \,\text{dpt}^2 \gamma \quad 3\lambda_1 \,\text{tjo}^2 \gamma , \ \frac{1}{2 \sin 2\beta} \text{tjo} \, 3)\beta , \ \gamma* \quad 4 \,\text{tjo} \, 3)\beta \quad \gamma*)\lambda_3 , \ \lambda_4 * \\ 4\lambda_1 \,\text{dpt}^2 \gamma \quad 3\lambda_2 \,\text{tjo}^2 \gamma , \ \frac{1}{2 \sin 2\beta} \text{tjo} \, 3)\beta , \ \gamma* \quad 4 \,\text{tjo} \, 3)\beta \quad \gamma*)\lambda_3 , \ \lambda_4 * \end{pmatrix} . \qquad)G04*$$

Ofyu xf ti px L_{IJ} jo Fr 0)D03*0 Opuf ui bu L_{IJ} jt tzn n fusjd L_{IJ} A L_{JI} boe jut opo.{fsp fmfn fout bsf-

$$L_{11} \quad \text{A} \quad \text{dpt}^2 \beta m_{11}^2 , \ \text{tjo}^2 \beta m_{22}^2 \quad 3 \,\text{dpt} \, \beta \,\text{tjo} \, \beta m_{12}$$

$$, \quad \frac{2}{3}]4v^2\}\lambda_1 \,\mathrm{dpt}^4 \beta \,,\ \ \mathrm{tjo}^2 \beta)3)\lambda_3 \,,\ \ \lambda_4 * \mathrm{dpt}^2 \beta \,,\ \ \mathrm{tjo}^2 \beta \lambda_2 * \langle \{ $$

$$ L_{22} \quad \mathrm{A} \quad v^2\} \quad \frac{\mathrm{dpt}\,5\beta}{5})\lambda_1 \,,\ \ \lambda_2 \quad 3)\lambda_3 \,,\ \ \lambda_4 ** v^2 \,,\ \ \frac{\mathrm{dpt}\,3\beta}{5})\lambda_2 \quad \lambda_1 * v^2 $$

$$, \quad 3m_{12}^2 \,\mathrm{tjo}\,3\beta \quad \mathrm{dpt}\,3\beta)m_{11} \quad m_{22}*\langle $$

$$ L_{12} \quad \mathrm{A} \quad L_{21} \ \mathrm{A}\ v\} \quad \frac{\mathrm{tjo}\,5\beta}{5})\lambda_1 \,,\ \ \lambda_2 \quad 3)\lambda_3 \,,\ \ \lambda_4 ** v^2 \,,\ \ \frac{2}{3}\,\mathrm{tjo}\,3\beta)\lambda_2 \quad \lambda_1 * v^2 $$

$$ 3m_{12}^2 \,\mathrm{dpt}\,3\beta \quad \mathrm{tjo}\,3\beta)m_{11}^2 \quad m_{22}^2*\langle $$

$$ L_{33} \quad \mathrm{A} \quad \frac{2}{9}v^2 \,\mathrm{tjo}\,3\beta)v^2 \,\mathrm{tjo}\,3\beta\lambda_4 \quad 5m_{12}^2* $$

$$ L_{44} \quad \mathrm{A} \quad v^2 \,\mathrm{dpt}\,\beta \,\mathrm{tjo}\,\beta m_{12}^2. \qquad)\mathrm{G}5* $$

Appendix G

Amplitude of $W^{+\pm} + Z^{\pm} \simeq\ H^{+} + h$

Jo ui jt bqqfoejy- xf ti px ui f pfi.ti fmmdi bshfe I jhht boe DQ.fwfo ofvusbmI jhht)h* cptpo qspevdujpo bn qmjuvef gps hbvhf cptpo gvtjpo W^{+*} , $Z^{*} \simeq H^{+}$, h0

$$T_{h\mu\nu}\ \mathrm{A}\ \frac{g^2\,\mathrm{dpt})\beta\ ,\ \gamma^*}{3\,\mathrm{dpt}\,\theta_W})a_h g_{\mu\nu}\ ,\ d_h q_{h\nu} q_{H^+\mu}\ ,\ b_h q_{H^+\nu} q_{h\mu}^*, \qquad)H02^*$$

x i fsf x f dpn qvuf ui f gpvs Gfzon bo ejbhsbn t dpssftqpoejoh up- ui f dpoubdu joufsbdujpo)Gjh0503*- ui f T di boofmW^{+} fydi bohf)Gjh0504* ui f V di boofmdi bshfe I jhht fydi bohf)Gjh0 505*- boe ui f U di boofm DQ.pee I jhht)A* fydi bohf)Gjh0506*0 a_h- b_h- boe d_h jo Fr 0)H02* bsf hjwfo bt-

$$\begin{aligned}
a_h\ &\mathrm{A}\quad \mathrm{tjo}^2\,\theta_W\ \frac{p_Z^2\quad p_W^2}{M_Z^2}\frac{M_h^2\quad M_{H^+}^2\quad M_W^2}{s_{H^+h}\quad M_W^2}\quad \mathrm{dpt}^2\,\theta_W \frac{t_h\quad u_h\ ,\ p_Z^2\quad p_W^2}{s_{H^+h}\quad M_W^2}\\
b_h\ &\mathrm{A}\quad \frac{3\,\mathrm{dpt}\,3\theta_W}{u_h\quad M_{H^+}^2}\ ,\ \frac{3)\mathrm{dpt}\,3\theta_W\ ,\ 2^*}{s_{H^+h}\quad M_W^2}\\
d_h\ &\mathrm{A}\quad \frac{3}{t_h\quad M_A^2}\quad \frac{3)\mathrm{dpt}\,3\theta_W\ ,\ 2^*}{s_{H^+h}\quad M_W^2}, \qquad)H03^*
\end{aligned}$$

x jui t_h A)q_{H^+} p_W*^{2}- u_h A)p_W q_h*^{2} boe s_{H^+h} A)q_{H^+} , q_h*^{2}0 Cz ubl joh ui f wbojti joh mjn ju pg ui f V)2* csfbl joh ufsn - jof0- $m_{12} \simeq$ 1- β boe γ wbojti 0 Opuf brtp ui bu- jo ui jt mjn ju- pof dbo ti px m_h A m_A boe $iT_{A\mu\nu}$ A $T_{h\mu\nu}$ x jui ui f bqqspqsjbuf sfqmbdfn fou $q_A \simeq q_h$)tff Fr 0)505**0 Ui fsfgpsf jo ui jt mjn ju ui f qspevdujpo bn qmjuvef t gps H^+A boe H^+h bsf jefoujdbmup fbdi pui fs- σ_{H^+A} A σ_{H^+h}0

References

]2a Ti bjofo N0Ebwjetpo boe I fbui fs F0Mphbo0 *Phys. Rev.*- E91;1: 6119- 311: 0

]3a Ti bjofo N0Ebwjetpo boe I fbui fs F0Mphbo0 *Phys. Rev.*- E93;226142- 31210

]4a U0Npsp–vn j- I 0Ubl bub boe L0Ubn bj- Qi zt0Sfw E 96 166113)3123*0Dpqzsjhi u)3123* cz ui f Bn fsjdbo Qi ztjdbmTpdjfuz-Qi ztjdbmSfwjfx E9: -18: : 12)F*)3125*0Dpqzsjhi u)3125* cz ui f Bn fsjdbo Qi ztjdbmTpdjfuz0

]5a U0Npsp–vn j boe L0Ubn bj- QUFQ **2013**)3124* : - 1: 14C130Dpqzsjhi u)3124* cz Pygpse Vojwfstuz Qsftt- Fssbuvn ; Qsph0Ui fps0Fyq0Qi zt03125- 15: 3120Dpqzsjhi u)3125* cz Pygpse Vojwfstuz Qsftt0

]6a Q0N0Gfssfjsb- S0Tbouept- boe B0Cbssptp0 *Phys. Lett.*- C714;32: ˜33: - 31150

]7a N0Nbojbujt- B0wpo Nbouf vfifm-P0Obdi un boo- boe G0Obhfm0 *Eur. Phys. J.*- D59;916˜934- 31170

]8a B0Cbssptp- Q0N0Gfssfjsb- boe S0Tbouept0 *Phys. Lett.*- C763;292˜2: 4- 31180

]9a J0G0Hjo–cvsh boe L0B0Lbojti fw0 *Phys. Rev.*- E87;1: 6124- 31180

]: a J0Q0Jwbopw0 *Acta Phys. Polon.*- C51;389: ˜3918- 311: 0

]21a Tbrbi Obtsj boe Ti fsjg Npvttb0 *Mod. Phys. Lett.*- B28;882˜889- 31130

]22a H0D0Csbodp fu bm0 Ui fpsz boe qi fopn fopmphz pg uxp.I jhht.epvcmfu n pefmt0 31220

]23a U0Gjhz- Npe0Qi zt0Mfuu0B **23**- 2: 72)3119*0

]24a N0K0Epmbo- D0Fohfmsu boe N0Tqboopxtl z- Qi zt0Sfw0E **87**- 166113)3124*0

]25a B0Qbqbfgtubui jpv- M0M0Zboh boe K0[vsjub- Qi zt0Sfw0E **87**- 122412)3124*0

]26a G0Hpfsu–- B0Qbqbfgtubui jpv- M0M0Zboh boe K0[vsjub- KI FQ **1306**- 127)3124*0

]27a Q0Gbzfu0 *Nucl. Phys.*- C89;25- 2: 85=B0Cbssptp boe K0Q0Tjmwb- *Phys. Rev.* E 61;5692- 2: : 50

]28a F0Nb0 *Phys. Rev. Lett.*- 97;3613- 31120

]29a Di sjtupqi fs U0I jmm-Di voh Ohpd Mfvoh- boe Tvn bui j Sbp0 *Nucl. Phys.*- C373;628- 2: 960

]2: a Nbsjb Lsbxd–zl boe Epspub Tpl pmpx tl b0 Ui f Jofsu Epvcmfu N pefmboe fwpmvujpo pg ui f Vojwfstf0 31220

]31a T0L bx bcbub- *Comput. Phys. Commun.* 99;41: - 2: : 60

]32a U0Cbi ojl - bsYjw;: 821376]i fq.qi a0

]33a K0Cfsjohfs fu0bm0 *Phys. Rev.* E97;121112- 31230

公表論文

(1) Quantum correction to tiny vacuum expectation value in two Higgs doublet model for Dirac neutrino mass.
Kotaro Tamai,Takuya Morozumi,Hiroyuki Takata
Physical Review D,85, 055002 (2012)1-13.

(2) Charged Higgs and neutral Higgs pair production of the weak gauge boson fusion process in electron-positron collisions.
Kotaro Tamai,Takuya Morozumi
Progress of Theoretical and Experimental Physics,093B02(2013)1-16.

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Quantum correction to the tiny vacuum expectation value in the two-Higgs-doublet-model for the Dirac neutrino mass

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We study a Dirac neutrino mass model of Davidson and Logan. In the model, the smallness of the neutrino mass is originated from the small vacuum expectation value of the second Higgs of two Higgs doublets. We study the one-loop effective potential of the Higgs sector and examine how the small vacuum expectation is stable under the radiative correction. By deriving formulas of the radiative correction, we numerically study how large the one-loop correction is and show how it depends on the quadratic mass terms and quartic couplings of the Higgs potential. The correction changes depending on the various scenarios for extra Higgs mass spectrum.

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I. INTRODUCTION

The smallness of the neutrino mass compared with the other quarks and leptons is one of the mysteries of nature. Recently, a new mechanism generating small Dirac mass terms for neutrino has been proposed [1–3]. The similar mechanism generating the small neutrino Dirac mass term for the TeV seesaw mechanism is also proposed in [4] and phenomenology is studied in [5,6]. There are also models with radiatively generated Dirac mass term in [7,8]. The interesting feature of the model proposed in [1,2] is the tiny vacuum expectation value for an extra Higgs SU(2) doublet [9]. The small neutrino mass is realized without introducing tiny Yukawa coupling for neutrinos. A softly broken global U(1) symmetry guarantees the tiny vacuum expectation value for the extra doublet. In addition to the small softly breaking mass parameter, the mass squared parameter for the extra Higgs is chosen to be positive so that the light pseudo Nambu-Goldstone bosons due to the softly broken global symmetry do not appear. This is a contrast to the mass squared parameter for the standard model like Higgs boson.

In the present paper, we study the global minimum of the tree level Higgs potential by explicitly solving the stationary conditions. There are many studies of the tree level Higgs potential of general two Higgs doublet model [10–15]. (See also [16] for recent review of two Higgs doublet model). It has been shown that the charge neutral vacuum is lower than the charge breaking vacuum [10]. Also, the vacuum energy difference of two neutral minima was derived [12,14]. We make use of the results and identify the vacuum of the present model. When the U(1) symmetry breaking term is turned off, the tree level Higgs potential and the phase structure of the present model is rather similar to the model with Z_2 discrete symmetry [17,18]. In contrast to Z_2 symmetric case, it is essential to keep the soft breaking term when finding the true vacuum. If we set the symmetry-breaking term at zero, then the order parameter corresponding to the softly broken U(1) symmetry becomes redundant parameter and can not be determined. We treat the soft breaking term as small expansion parameter and obtain the vacuum expectation values and the vacuum energies in terms of the parameters of the Higgs potential.

The constraints on the parameters of the model for which the desired vacuum can be realized are derived and they are rewritten in terms of Higgs masses and a few coupling constants, which can not be directly related to the Higgs masses. These constraints are fully used when we study the radiative corrections to the vacuum expectation values numerically.

Beyond the tree level, we study the radiative correction to the Higgs potential and the vacuum expectation values of Higgs. Since the neutrino masses are proportional to the vacuum expectation value of one of Higgs, one can also compute the radiative corrections to neutrino masses. As already noted in [1], the radiative correction to the softly breaking mass parameter is logarithmically divergent and it is renormalized multiplicatively. We derive the formulas for the one-loop corrected vacuum expectation values for two Higgs doublets by studying one-loop corrected effective potential. The corrections are evaluated numerically by exploring the parameter regions allowed from the global minimum condition for the vacuum. We show how the radiative corrections change depending on the extra Higgs spectrum. The radiative corrections are also evaluated for the case that a relation among the coupling constants is satisfied.

The paper is organized as follows. In Sec. II, we derive the condition for the desired vacuum being global minimum. In Sec. III, one-loop effective potential is derived, and one-loop corrections to the vacuum expectation values are obtained in Sec. IV. In Sec. V, the corrections are evaluated numerically for various choices of parameters of the Higgs potential. Section VI is devoted to summary and discussion.

II. MODEL FOR DIRAC NEUTRINO WITH A TINY VACUUM EXPECTATION VALUE

The model of the Dirac neutrino is proposed in [1]. In [1], two Higgs SU(2) doublets are introduced,

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1^1 + i\phi_1^2 \\ \phi_1^3 + i\phi_1^4 \end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2^1 + i\phi_2^2 \\ \phi_2^3 + i\phi_2^4 \end{pmatrix}, \quad (1)$$

where Φ_1's vacuum expectation value is nearly equal to the electroweak breaking scale and the second Higgs Φ_2 has a small vacuum expectation value, which gives rise to neutrino mass. The Higgs potential in [1] is:

$$V_{\rm tree} = \sum_{i=1,2}\left(m_{ii}^2\Phi_i^\dagger\Phi_i + \frac{\lambda_i}{2}(\Phi_i^\dagger\Phi_i)^2\right) - (m_{12}^2\Phi_1^\dagger\Phi_2 + {\rm H.c.}) + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4|\Phi_1^\dagger\Phi_2|^2. \quad (2)$$

$U(1)'$ charge is assigned to the second Higgs. The $U(1)'$ global symmetry is broken softly with the term m_{12}^2. In this paper, we introduce the following real O(4) representation for each doublet, because this parametrization is convenient when computing the one-loop corrected effective potential.

$$\phi_1^a = \begin{pmatrix} \phi_1^1 \\ \phi_1^2 \\ \phi_1^3 \\ \phi_1^4 \end{pmatrix}, \qquad \phi_2^a = \begin{pmatrix} \phi_2^1 \\ \phi_2^2 \\ \phi_2^3 \\ \phi_2^4 \end{pmatrix}, \qquad \tilde{\phi}_1^a = \begin{pmatrix} -\phi_1^2 \\ \phi_1^1 \\ -\phi_1^4 \\ \phi_1^3 \end{pmatrix}. \quad (3)$$

Using the notation above, the tree level effective potential introduced in Eq. (2) can be written as:

$$\begin{aligned} V_{\rm tree} &= m_{11}^2\frac{1}{2}\sum_{a=1}^4(\phi_1^a)^2 + m_{22}^2\frac{1}{2}\sum_{a=1}^4(\phi_2^a)^2 \\ &\quad - m_{12}^2\sum_{a=1}^4\phi_1^a\phi_2^a + \frac{\lambda_1}{8}\left(\sum_{a=1}^4\phi_1^{a2}\right)^2 + \frac{\lambda_2}{8}\left(\sum_{a=1}^4\phi_2^{a2}\right)^2 \\ &\quad + \frac{\lambda_3}{4}\left(\sum_{a=1}^4\phi_1^{a2}\right)\left(\sum_{a=1}^4\phi_2^{a2}\right) + \frac{\lambda_4}{4}\left(\left(\sum_{a=1}^4\phi_1^a\phi_2^a\right)^2 + \left(\sum_{a=1}^4\tilde{\phi}_1^a\phi_2^a\right)^2\right), \end{aligned} \quad (4)$$

where one can choose m_{12}^2 real and positive. With the notation of Eq. (3), the softly broken global symmetry $U(1)'$ corresponds to the following transformation on ϕ_2^a:

$$\phi_2' = O_{U(1)'}\phi_2 = \begin{pmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}\phi_2. \quad (5)$$

ϕ_1 does not transform under $U(1)'$. Therefore, $U(1)'$ is broken softly when m_{12}^2 does not vanish. Without loss of generality, one can choose the vacuum expectation values of Higgs with the form given as

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ 0 \\ v\cos\beta \\ 0 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} v\sin\beta\sin\alpha\cos\theta' \\ -v\sin\beta\sin\alpha\sin\theta' \\ v\sin\beta\cos\alpha\cos\theta' \\ -v\sin\beta\cos\alpha\sin\theta' \end{pmatrix}, \quad (6)$$

where the range for θ' is $[0, 2\pi)$ and the range for β and α is $[0, \frac{\pi}{2}]$. We call the four order parameters as $\varphi_I = (v, \beta, \alpha, \theta')$, $(I = 1, 2, 3, 4)$. When m_{12} vanishes, by taking $\phi = \theta'$ in Eq. (5), one can rotate θ' away in Eq. (6). For the most general case, in total, there are four independent order parameters when $U(1)'$ symmetry is broken.

For completeness of our discussion, we give the constraints on the quartic couplings from condition that the tree level potential is the bounded below[1,10,19]:

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \quad (7)$$

$$-\sqrt{\lambda_1\lambda_2} \le \lambda_3, \quad (8)$$

$$-\sqrt{\lambda_1\lambda_2} \le \lambda_3 + \lambda_4. \quad (9)$$

In addition to the conditions on the quartic terms, one can constrain the parameters, including the quadratic terms so that the desired vacuum satisfies the global minimum conditions of the potential. About the global minimum of the tree potential, it was shown that the energy of charge neutral vacuum is lower than that of the charge-breaking vacuum [10]. We therefore set α zero. We also require the vacuum expectation value of the second Higgs is much smaller than that of the first Higgs, which implies that $\tan\beta$ is small. In terms of the parametrization in Eq. (6) with $\alpha = 0$, the potential can be written as

$$V_{\rm tree}(v, \beta, \theta') = A(\beta)v^4 + B(\beta, \theta')v^2, \quad (10)$$

where

$$\begin{aligned} A(\beta) &= \frac{\lambda_1}{8}\cos^4\beta + \frac{\lambda_2}{8}\sin^4\beta + \left(\frac{\lambda_3}{4} + \frac{\lambda_4}{4}\right)\cos^2\beta\sin^2\beta, \\ B(\beta, \theta') &= \frac{m_{11}^2}{2}\cos^2\beta + \frac{m_{22}^2}{2}\sin^2\beta - m_{12}^2\cos\theta'\cos\beta\sin\beta. \end{aligned} \quad (11)$$

We first find the global minimum of $V_{\rm tree}$. The stationary conditions $\frac{\partial V_{\rm tree}}{\partial\varphi_I} = 0$ $(I = 1, 2, 4)$, are written as

$$v(2Av^2 + B) = 0, \quad (12)$$

$$2r_4 = \sin 2\beta\frac{(1 - r_1 r_2)\cos 2\beta + r_2 - r_1 r_3}{r_2\cos^2 2\beta + (r_3 + 1)\cos 2\beta + r_2}, \quad (13)$$

$$m_{12}^2\sin\theta'\sin 2\beta = 0, \quad (14)$$

where $r_i (i = 1 \sim 4)$ are defined as,

$$r_1 = \frac{m_{11}^2 - m_{22}^2}{m_{11}^2 + m_{22}^2}, \qquad r_2 = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4}, \qquad r_3 = \frac{\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4}{\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4}, \qquad r_4 = \frac{m_{12}^2 \cos\theta'}{m_{11}^2 + m_{22}^2}. \tag{15}$$

The stationary conditions in Eq. (12) and (13) correspond to Eq. (36) of [14]. Here we solve them explicitly by treating the soft breaking term m_{12} as perturbation. The nonzero solution for v^2 in Eq. (12) is written as

$$v^2 = -\frac{B}{2A} = -4\frac{m_{11}^2 + m_{22}^2}{\lambda_1 + \lambda_2 - 2\lambda_{34}} \frac{1 + r_1\cos2\beta - 2r_4\sin2\beta}{\cos^2 2\beta + r_3 + 2r_2\cos2\beta}, \tag{16}$$

where $\lambda_{34} = \lambda_3 + \lambda_4$. Substituting it into $V_{\rm tree}$, one obtains,

$$V_{\rm tree} \geq V_{\rm min} = -\frac{(m_{11}^2 + m_{22}^2)^2}{2(\lambda_1 + \lambda_2 - 2\lambda_{34})} \times \frac{(1 + r_1\cos2\beta - 2r_4\sin2\beta)^2}{\cos^2 2\beta + 2r_2\cos2\beta + r_3}. \tag{17}$$

For nonzero m_{12}^2 and $\sin2\beta$, the solution of Eq. (14) is $\sin\theta' = 0$. One still needs to find β among the solutions of Eq. (13), which leads to the minimum of $V_{\rm min}$. We solve Eq. (13) and determine β by treating $r_4(m_{12}^2)$ as a small expansion parameter. One can easily find the approximate solutions as:

$$\begin{cases} (1)\,\sin\beta = \frac{\lambda_1 m_{12}^2}{|m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}|}, & \cos\theta' = \mathrm{sign}(m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}), \\ (2)\,\cos\beta = \frac{\lambda_2 m_{12}^2}{|m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}|}, & \cos\theta' = \mathrm{sign}(m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}), \\ (3)\,\cos2\beta = \frac{m_{11}^2(\lambda_{34}+\lambda_2) - m_{22}^2(\lambda_{34}+\lambda_1)}{m_{11}^2(-\lambda_{34}+\lambda_2) + m_{22}^2(-\lambda_{34}+\lambda_1)} + O(r_4). & \end{cases} \tag{18}$$

Corresponding to each solution, (1) $\sim$ (3) of Eq. (18), the vacuum expectation value v^2 and the minimum of the potential are obtained.

$$(v^2, V_{\rm min}) = \begin{cases} (1)\left(-\frac{2m_{11}^2}{\lambda_1} + 2\lambda_1(m_{22}^2 - m_{11}^2)\left(\frac{m_{12}^2}{m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}}\right)^2, -\frac{m_{11}^4}{2\lambda_1} + \frac{m_{12}^4 m_{11}^2}{m_{22}^2\lambda_1 - m_{11}^2\lambda_{34}}\right), \\ (2)\left(-\frac{2m_{22}^2}{\lambda_2} + 2\lambda_2(m_{11}^2 - m_{22}^2)\left(\frac{m_{12}^2}{m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}}\right)^2, -\frac{m_{22}^4}{2\lambda_2} + \frac{m_{12}^4 m_{22}^2}{m_{11}^2\lambda_2 - m_{22}^2\lambda_{34}}\right), \\ (3)\left(2\frac{(\lambda_{34}-\lambda_2)m_{11}^2 + (\lambda_{34}-\lambda_1)m_{22}^2}{\lambda_1\lambda_2 - \lambda_{34}^2} + O(r_4), -\frac{\lambda_2 m_{11}^4 - 2m_{11}^2 m_{22}^2\lambda_{34} + \lambda_1 m_{22}^4}{2(\lambda_1\lambda_2 - \lambda_{34}^2)} + O(r_4)\right). \end{cases} \tag{19}$$

The leading terms of the vacuum expectation values agree with those obtained in Z_2 symmetric model [18]. If $\sin2\beta = 0$, then r_4 must be vanishing and $\cos\theta' = 0$ from Eq. (13) and (14). The vacuum energies of the nonzero $\sin2\beta$ solutions are shown in Tables I. In Table II, the vacuum energies of the solutions with $\sin2\beta = 0$ are summarized.

Next, we derive the constraints on the parameters so that the solution corresponding to (1) in Table I becomes the global minimum of the potential. Since the other cases (2)–(5) do not have desired properties, we restrict the parameter space so that these solutions can not be a global minimum. Since v must have large positive vacuum expectation value, m_{11}^2 must be negative. In order that the vacuum energy of (1) is lower than that of (4),

$$m_{22}^2\lambda_1 - m_{11}^2\lambda_{34} > 0, \qquad (\cos\theta' = 1). \tag{20}$$

When Eq. (20) is satisfied and the solution (1) does exist, one can show that the vacuum energy of solution (3) is higher than that of (1). Furthermore, when $m_{22}^2 > 0$, the solutions corresponding to (2) and (5) are not realized. Then one can state the region of parameter space, which

TABLE I. Classification of the solutions with nonzero $\sin2\beta$ of the stationary conditions of Higgs potential. For (3), $O(r_4)$ correction is not shown.

(1) $\sin\beta = O(r_4)$	$-\frac{m_{11}^4}{2\lambda_1} - \frac{m_{12}^4}{\lambda_3 + \lambda_4 - \frac{m_{22}^2}{m_{11}^2}\lambda_1}$
(2) $\cos\beta = O(r_4)$	$-\frac{m_{22}^4}{2\lambda_2} - \frac{m_{12}^4}{\lambda_3 + \lambda_4 - \frac{m_{11}^2}{m_{22}^2}\lambda_2}$
(3) $\cos2\beta = O(1)$	$-\frac{\lambda_1 m_{11}^4 - 2m_{11}^2 m_{22}^2(\lambda_3+\lambda_4) + \lambda_2 m_{22}^4}{2(\lambda_1\lambda_2 - (\lambda_3+\lambda_4)^2)}$

TABLE II. Classification of the solutions with $\sin2\beta = 0$.

	$\cos\theta' = 0$
(4) $\sin\beta = 0$	$-\frac{m_{11}^4}{2\lambda_1}$
(5) $\cos\beta = 0$	$-\frac{m_{22}^4}{2\lambda_2}$

is consistent with the case that the vacuum (1) becomes global minimum is

$$m_{11}^2 < 0, \qquad m_{22}^2 > 0, \qquad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2}\lambda_1. \tag{21}$$

Next, we consider the case with negative m_{22}^2. In this case, we impose the additional condition so that the vacuum energies corresponding to (2) and (5) are higher than that of (1):

$$\frac{m_{11}^4}{\lambda_1} > \frac{m_{22}^4}{\lambda_2}. \tag{22}$$

Then, the condition for (1) is global minimum in this case is

$$m_{11}^2 < 0, \qquad m_{22}^2 < 0, \qquad \lambda_{34} > \frac{m_{22}^2}{m_{11}^2}\lambda_1, \\ \lambda_2 \frac{m_{11}^2}{m_{22}^2} > \lambda_1 \frac{m_{22}^2}{m_{11}^2}. \tag{23}$$

In the following sections, we explore the regions for the parameters obtained in Eq. (21), (23), (8), and (9).

III. EFFECTIVE POTENTIAL IN ONE-LOOP AND RENORMALIZATION

In this section, we derive the effective potential within one-loop approximation. We introduce a real scalar fields with eight components as $\phi^i = (\phi_1^1, \phi_1^2, \phi_1^3, \phi_1^4, \phi_2^1, \phi_2^2, \phi_2^3, \phi_2^4)^T$, $(i = 1 \sim 8)$. With the notation above, the one-loop effective action is given as

$$\Gamma_{\rm eff}^{\rm 1loop} = i\frac{1}{2}\,{\rm lndet}D^{-1}(\phi), \qquad D^{-1} = \Box + M_T^2, \tag{24}$$

where M_T^2 is the mass squared matrix of the Higgs potential,

$$M_T^2 = M^2(\phi) + \begin{pmatrix} m_{11}^2 \times 1 & 0 \\ 0 & m_{22}^2 \times 1 \end{pmatrix} - m_{12}^2\sigma_1, \\ M^2(\phi)_{ij} = \frac{\partial^2 V_{\rm tree}^{(4)}}{\partial\phi_i \partial\phi_j}, \tag{25}$$

and where 1(0) denotes 4×4 unit (zero) matrix. σ_1 is defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{26}$$

In Eq. (26), 1(0) also denotes a four by four unit (zero) matrix. In modified minimal subtraction scheme, the finite part of the one-loop effective potential becomes

$$V_{\rm 1loop} = \frac{\mu^{4-d}}{2}\int \frac{d^d k}{(2\pi)^d i}\,{\rm TrLn}(M_T^2 - k^2) + V_c, \\ = \frac{1}{64\pi^2}\,{\rm Tr}\Big(M_T^4\Big({\rm Ln}\frac{M_T^2}{\mu^2} - \frac{3}{2}\Big)\Big). \tag{27}$$

V_c denotes the counterterms and the derivation of V_c can be found in Appendix A.

IV. ONE-LOOP CORRECTIONS TO THE VACUUM EXPECTATION VALUES

In this section, we compute the one-loop corrections to the vacuum expectation values. Using the symmetry of the model, in general, one can choose $\varphi_I = (v, \beta, \alpha, \theta')$ as the vacuum expectation values of Higgs potential. Their values are obtained as the stationary points of the one-loop corrected effective potential $V = V_{\rm tree} + V_{\rm 1loop}$,

$$\frac{\partial V}{\partial \varphi_I} = 0. \tag{28}$$

By denoting the vacuum expectation values as sum of the tree level ones and the one-loop corrections to them, $\varphi_I = \varphi_I^{(0)} + \varphi_I^{(1)}$, one obtains the one-loop corrections,

$$\varphi_I^{(1)} = -(L^{-1})_{IJ}\frac{\partial V_{\rm 1loop}}{\partial \varphi_J}\Big|_{\varphi=\varphi^{(0)}}, \\ = -\frac{1}{32\pi^2}(L^{-1})_{IJ}\sum_{i=1}^{8}\Big(O^T\frac{\partial M^2}{\partial\varphi_J}\Big|_{\varphi=\varphi^{(0)}}O\Big)_{ii} \\ \times M_{Di}^2\Big({\rm ln}\frac{M_{Di}^2}{\mu^2} - 1\Big), \tag{29}$$

where M_D^2 is a diagonal 8×8 tree level mass squared matrix of Higgs sector and L_{IJ} is 4×4 matrix given by the second derivatives of the tree level Higgs potential with respect to the order parameters,

$$L_{IJ} = \frac{\partial^2 V_{\rm tree}}{\partial\varphi_I \partial\varphi_J}\Big|_{\varphi=\varphi^{(0)}}. \tag{30}$$

The diagonal Higgs mass matrix squared M_D^2 is related to 8×8 Higgs mass matrix squared M_T^2 in Eq. (25).

$$O^T M_{T0}^2 O = M_D^2 \\ = \begin{pmatrix} M_{H^+}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{H^+}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_h^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_H^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{31}$$

where M_{T0}^2 is obtained by substituting the vacuum expectation values to M_T^2. O is shown in Appendix D. Since M_D is the 8×8 diagonal matrix which elements correspond to the Higgs masses and zero mass of the would be Nambu-Goldstone bosons, one may write Eq. (29) in a simple form. The Higgs masses squared in Eq. (31) are given by

$$M_{H^+}^2 = \frac{1}{2}\Big[\frac{1}{8}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4)))v^2 + (1 - \cos(2\beta))m_{11}^2 + (\cos(2\beta) + 1)m_{22}^2 + 2\sin(2\beta)m_{12}^2\Big],$$

$$M_A^2 = M_{H^+}^2 + \frac{\lambda_4 v^2}{2}, \qquad \frac{M_h^2 + M_H^2}{2} = \frac{1}{4}((3\lambda_1\cos^2(\beta) + 3\sin^2(\beta)\lambda_2 + \lambda_3 + \lambda_4)v^2 + 2m_{11}^2 + 2m_{22}^2),$$

$$\frac{M_H^2 - M_h^2}{2} = \frac{1}{8}\Big[\{6\cos(2\gamma)(\cos^2(\beta)\lambda_1 - \sin^2(\beta)\lambda_2) + (\cos(2(\beta+\gamma)) - 3\cos(2(\beta-\gamma)))(\lambda_3 + \lambda_4)\}v^2 + 4\cos(2\gamma)m_{11}^2 - 4\cos(2\gamma)m_{22}^2 + 8\sin(2\gamma)m_{12}^2\Big], \tag{32}$$

where γ is an angle with which one can diagonalize the 2×2 mass matrix for *CP*-even neutral Higgs. tan2γ is given as

$$\tan 2\gamma = \frac{-4m_{12}^2 + 2\sin 2\beta(\lambda_3 + \lambda_4)v^2}{(3(-\lambda_1\cos^2\beta + \lambda_2\sin^2\beta) + \cos 2\beta(\lambda_3 + \lambda_4))v^2 - 2(m_{11}^2 - m_{22}^2)}. \tag{33}$$

To compute Eq. (29), we still need to calculate $O^T \frac{\partial M^2}{\partial \varphi_I} O$ and L_{IJ}. They are shown in Appendix C. Using Eqs. (29) and (C1), one can find the quantum corrections for α and θ' vanish:

$$\alpha^{(1)} = 0, \qquad \theta'^{(1)} = 0. \tag{34}$$

For $v^{(1)}$ and $\beta^{(1)}$, one obtains,

$$\begin{aligned} v^{(1)} &= -\frac{1}{32\pi^2}\frac{1}{\det L'}\Big(L_{22}\sum_{j=1}^{5}\Big[O^T\frac{\partial M^2}{\partial\varphi_1}O\Big]_{jj}M_{Dj}^2\Big(\ln\frac{M_{Dj}^2}{\mu^2} - 1\Big) - L_{12}\sum_{j=1}^{5}\Big[O^T\frac{\partial M^2}{\partial\varphi_2}O\Big]_{jj}M_{Dj}^2\Big(\ln\frac{M_{Dj}^2}{\mu^2} - 1\Big)\Big),\\ \beta^{(1)} &= -\frac{1}{32\pi^2}\frac{1}{\det L'}\Big(-L_{12}\sum_{j=1}^{5}\Big[O^T\frac{\partial M^2}{\partial\varphi_1}O\Big]_{jj}M_{Dj}^2\Big(\ln\frac{M_{Dj}^2}{\mu^2} - 1\Big) + L_{11}\sum_{j=1}^{5}\Big[O^T\frac{\partial M^2}{\partial\varphi_2}O\Big]_{jj}M_{Dj}^2\Big(\ln\frac{M_{Dj}^2}{\mu^2} - 1\Big)\Big), \end{aligned} \tag{35}$$

where L' is

$$L' = \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix}. \tag{36}$$

The elements of L' are shown in Eq. (C4). Equation (35) corresponds to the one-loop exact formulas and is a main result of the present paper. In the leading order of the expansion with respect to the symmetry breaking term m_{12}^2, the correction to v becomes

$$v^{(1)} = -\frac{v}{32\pi^2}\Big\{3\lambda_1\Big(\ln\frac{M_H^2}{\mu^2} - 1\Big) + 2\lambda_3\frac{M_{H^+}^2}{M_H^2}\Big(\ln\frac{M_{H^+}^2}{\mu^2} - 1\Big) + (\lambda_3 + \lambda_4)\Big(\frac{M_A^2}{M_H^2}\Big(\ln\frac{M_A^2}{\mu^2} - 1\Big) + \frac{M_h^2}{M_H^2}\Big(\ln\frac{M_h^2}{\mu^2} - 1\Big)\Big)\Big\}. \tag{37}$$

The Higgs masses in the formulas are the ones in the limit of $m_{12} \to 0$,

$$M_H^2 \simeq m_{11}^2 + \frac{3}{2}\lambda_1 v^2, \qquad M_A^2 \simeq M_h^2 \simeq m_{22}^2 + \frac{\lambda_3 + \lambda_4}{2}v^2, \qquad M_{H^+}^2 \simeq m_{22}^2 + \frac{\lambda_3}{2}v^2, \tag{38}$$

where v is related to m_{11}^2 as,

$$\frac{\lambda_1}{2}v^2 \simeq -m_{11}^2. \tag{39}$$

The approximate formulas for the physical Higgs masses in Eq. (38), which are valid to the limit $m_{12} \to 0$, agree with the ones given in [1] except the notational difference of M_H and M_h. The one-loop correction to β in the leading order expansion of m_{12}^2 is given as

$$\beta^{(1)} = -\frac{\beta}{32\pi^2}\Big\{2\Big(\lambda_2 - \lambda_4 - \frac{\lambda_3(\lambda_3+\lambda_4)}{\lambda_1}\Big)\frac{M_{H^+}^2}{M_A^2}\Big(\ln\frac{M_{H^+}^2}{\mu^2} - 1\Big) + \Big(\lambda_2 - \frac{(\lambda_3+\lambda_4)^2}{\lambda_1}\Big)\Big(\ln\frac{M_A^2}{\mu^2} - 1\Big)$$
$$+ \Big(3\lambda_2 + \Big(2\Gamma - \frac{\lambda_3+\lambda_4}{\lambda_1}\Big)(\lambda_3+\lambda_4)\Big)\frac{M_h^2}{M_A^2}\Big(\ln\frac{M_h^2}{\mu^2} - 1\Big) - 2(1+\Gamma)(\lambda_3+\lambda_4)\frac{M_H^2}{M_A^2}\Big(\ln\frac{M_H^2}{\mu^2} - 1\Big)\Big\}, \quad (40)$$

where

$$\Gamma = \lim_{m_{12}\to 0}\frac{\gamma}{\beta} = \frac{M_A^2 - M_H^2\frac{\lambda_3+\lambda_4}{\lambda_1}}{M_H^2 - M_A^2}. \quad (41)$$

Equation (40) shows that the quantum correction is also proportional to the soft-breaking parameter m_{12}^2, which is expected. We also note that the correction depends on the Higgs mass spectrum and quartic couplings. The correlation to Higgs spectrum is studied in the next section.

V. NUMERICAL CALCULATION

In this section, we study the quantum correction to β and v numerically. As shown in Eq. (37) and (40), the quantum corrections are written with four Higgs masses and the four quartic couplings. Since the neutral CP even and CP -odd Higgs of the second Higgs doublet are degenerate as $M_A = M_h$ in the limit $m_{12} \to 0$ (See Eq. (38)), the three Higgs masses (M_H, M_A, M_{H^+}) are independent. Moreover, for a given charged Higgs mass and neutral Higgs mass, λ_1 and λ_4 are given as

$$\lambda_1 = \frac{M_H^2}{v^2}, \qquad \lambda_4 = 2\frac{M_A^2 - M_{H^+}^2}{v^2}. \quad (42)$$

λ_2 and λ_3 are the remaining parameters to be fixed. The lower limit of λ_3 obtained from Eq. (8) and (9) is written as

$$\mathrm{Max}\Big(-\frac{M_H}{v}\sqrt{\lambda_2}, -\frac{M_H}{v}\sqrt{\lambda_2} - 2\frac{M_A^2 - M_{H^+}^2}{v^2}\Big) < \lambda_3. \quad (43)$$

One can also write λ_3 with the charged Higgs mass formulas,

$$\lambda_3 = \frac{2}{v^2}(M_{H^+}^2 - m_{22}^2). \quad (44)$$

Depending on the sign of m_{22}^2, the upper bound and the lower bound of λ_3 can be obtained for a given charged Higgs mass. Combining it with Eq. (43), the constraints for positive m_{22}^2 case are,

$$\mathrm{Max}\Big(-\frac{M_H}{v}\sqrt{\lambda_2}, -\frac{M_H}{v}\sqrt{\lambda_2} - 2\frac{M_A^2 - M_{H^+}^2}{v^2}\Big) < \lambda_3 < \frac{2M_{H^+}^2}{v^2}, \qquad (m_{22}^2 > 0). \quad (45)$$

When $m_{22}^2 \le 0$, in addition to the lower bound on λ_3, the constraint on λ_2 in Eq. (22) should be satisfied:

$$\frac{2M_{H^+}^2}{v^2} \le \lambda_3, \qquad \sqrt{\lambda_2} > \Big(\lambda_3 - 2\frac{M_{H^+}^2}{v^2}\Big)\frac{v}{M_H}, \qquad (m_{22}^2 < 0). \quad (46)$$

Now we study the quantum corrections numerically. We fix the standard model like Higgs mass as $M_H = 130$ (GeV). There are still four parameters to be fixed and they are λ_2, λ_3, M_A, and M_{H^+}. Focusing on the Higgs mass spectrum of the extra Higgs, we study the radiative corrections for the following scenarios for Higgs spectrum and the coupling constants.

A. Case for $M_A = M_{H^+}$; degenerate charged Higgs and pseudoscalar Higgs and a relation for vanishing quantum correction $\beta^{(1)}$

We first study the corrections for degenerate charged Higgs and pseudoscalar Higgs. In this case, for a given degenerate mass, one can identify the values of coupling constants λ_2 and λ_3, for which $\beta^{(1)}$ vanishes. With $M_A = M_{H^+}$, the relation for coupling constants which satisfies $\beta^{(1)} = 0$ is

$$\lambda_2 = \frac{\lambda_3^2}{3\lambda_1}\left\{2 + \frac{M_H^2}{M_H^2 - M_{H^+}^2}\left(1 - \frac{M_H^2}{M_{H^+}^2}\frac{\log\frac{M_H^2}{\mu^2} - 1}{\log\frac{M_{H^+}^2}{\mu^2} - 1}\right)\right\} - \frac{\lambda_3}{3}\left(\frac{M_{H^+}^2}{M_H^2 - M_{H^+}^2} - \frac{M_H^2}{M_H^2 - M_{H^+}^2}\frac{M_H^2}{M_{H^+}^2}\frac{\log\frac{M_H^2}{\mu^2} - 1}{\log\frac{M_{H^+}^2}{\mu^2} - 1}\right). \quad (47)$$

The set of coupling constants (λ_3, λ_2), which satisfy the relation Eq. (47), are shown in Table III. We note that when λ_2 is as large as 10, λ_3 is at most about 3. If λ_2 is 1, λ_3 is lies in the range $0.55 \sim 0.7$.

TABLE III. The coupling constants (λ_3, λ_2) which satisfy the relation, Eq. (47) for the three degenerate masses $M_{H^+} = M_A = 100$, 200 and 500 (GeV).

λ_2	λ_3 ($M_{H^+} = 100$)	λ_3 ($M_{H^+} = 200$)	λ_3 ($M_{H^+} = 500$)
0.14	0.19	0.16	0.18
0.28	0.28	0.28	0.28
0.56	0.41	0.47	0.42
1.0	0.55	0.69	0.59
10	1.8	2.8	2.0

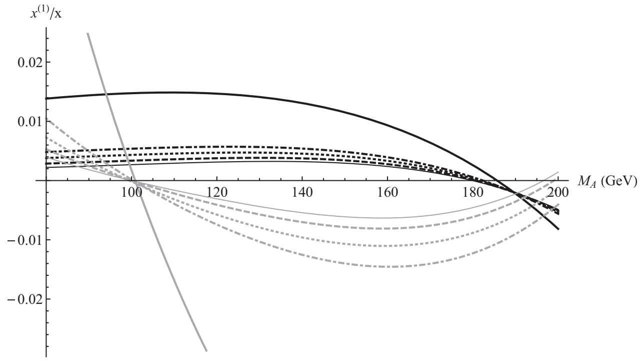

FIG. 1. The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudoscalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ $(x = \beta, v)$ is shown, while the charged Higgs mass is fixed as $M_{H^+} = 100$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 100$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are (0.19, 0.14) (solid line), (0.28, 0.28) (dashed line), (0.41, 0.56) (dotted line), (0.55, 1) (dotdashed line), and (1.8, 10) (thick solid line).

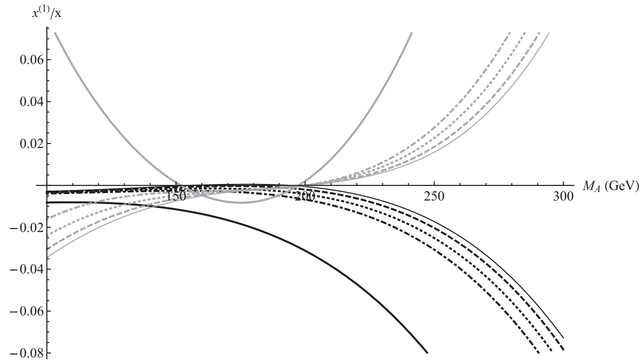

FIG. 2. The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudoscalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ $(x = \beta, v)$ is shown while the charged Higgs mass is fixed as $M_{H^+} = 200$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 200$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are (0.16, 0.14) (solid line), (0.28, 0.28) (dashed line), (0.47, 0.56) (dotted line), (0.69, 1) (dotdashed line), and (2.8, 10) (thick solid line).

B. Non-Degenerate case $M_A \neq M_{H^+}$ with the coupling constants satisfying Eq. (47)

Next we lift the degeneracy by shifting the pseudoscalar Higgs mass from the charged Higgs mass and study the effect on $\beta^{(1)}$ and $v^{(1)}$. The nondegeneracy of the charged Higgs mass and the pseudoscalar Higgs mass is constrained by ρ parameter. We change the pseudoscalar Higgs mass within the range $|M_A - M_{H^+}| < 100$ (GeV) allowed from the electro-weak precision studies. The coupling constants (λ_3, λ_2) are chosen from the sets of their values satisfying the relation Eq. (47). In Fig. 1, we show $\frac{\beta^{(1)}}{\beta}$ as a function of M_A with charged Higgs mass $M_{H^+} = 100$ (GeV). When $M_A = 100$ (GeV), the correction vanishes exactly. As we increase M_A from 100 (GeV) (the mass of charged Higgs), the correction becomes nonzero and is negative. The corrections are at most about 1.3% when $\lambda_2 \sim 1$. By increasing M_A further, we meet the point around at $M_A \simeq 200$ (GeV) corresponding to that the correction vanishes again. In Fig. 2, we study the correction $\beta^{(1)}$ with larger charged Higgs mass case, $M_{H^+} = 200$ (GeV). In contrast to the case for $M_{H^+} = 100$ (GeV), by increasing M_A from 200 (GeV) where the correction vanishes, it increases and becomes positive. We also note that the correction tends to be larger than the lighter charged Higgs mass case. When $\lambda_2 \sim 1$, increasing the pseudoscalar Higgs mass from 200 (GeV) to 300 (GeV), the correction is about 10%. As the pseudoscalar Higgs mass decreases from 200 (GeV) to 100 (GeV), the correction becomes negative for $0 < \lambda_2 \leq 1$. With the larger value $\lambda_2 = 10$, we meet the point around at $M_A \simeq 150$ (GeV) where the correction vanishes again. In Fig. 3, we study the further larger charged Higgs mass case, i.e., $M_{H^+} = 500$ (GeV). With $M_A \simeq 600$ (GeV), the correction is positive and about 100%. The correction stays small for $0 < \lambda_2 \leq 1$ when decreasing M_A from 500 (GeV) to 400 (GeV).

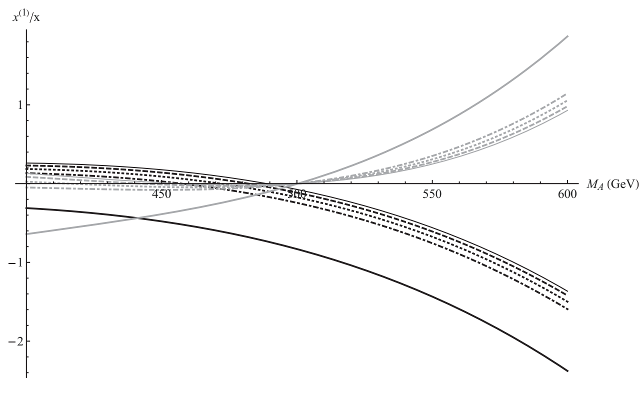

FIG. 3. The quantum correction $\frac{\beta^{(1)}}{\beta}$ (gray lines) and $\frac{v^{(1)}}{v}$ (black lines) due to the nondegeneracy of charged Higgs and pseudoscalar Higgs masses. The pseudoscalar Higgs mass M_A (GeV) dependence of the quantum corrections $\frac{x^{(1)}}{x}$ $(x = \beta, v)$ is shown while charged Higgs mass is fixed as $M_{H^+} = 500$ (GeV). The set of parameters (λ_3, λ_2) are chosen so that the correction $\beta^{(1)}$ vanishes for the degenerate case; $M_{H^+} = M_A = 500$ (GeV). The values (λ_3, λ_2) are taken from Table III and they are (0.18, 0.14) (solid line), (0.28, 0.28) (dashed line), (0.42, 0.56) (dotted line), (0.59, 1) (dotdashed line), and (2, 10) (thick solid line).

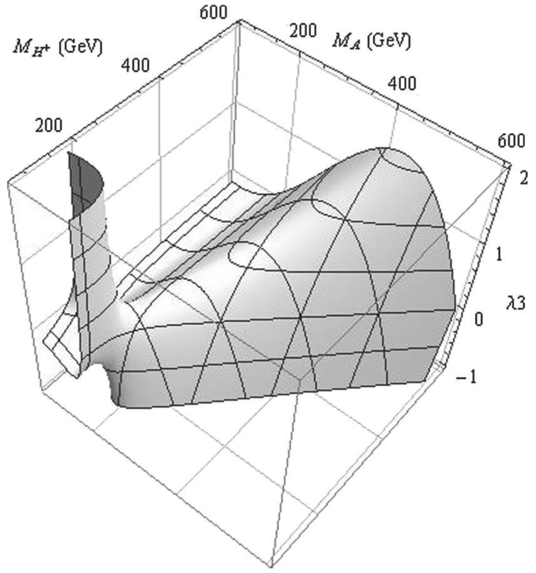

FIG. 4. The two dimensional surface for $v^{(1)} = 0$.

C. The correction $\frac{v^{(1)}}{v}$

In Figs. 1–3, we also show the correction $\frac{v^{(1)}}{v}$ as functions of M_A. $v^{(1)}$ is independent of λ_2 and does not necessarily vanish at the same points where $\beta^{(1)}$ vanishes. With $\lambda_3 \geq 2$ and $M_{H^+} \geq 200$ (GeV), when the pseudoscalar Higgs mass is much larger than that of charged Higgs mass; we find a very large correction to v. In Fig. 4, we show that the two dimensional surface, which corresponds to $v^{(1)} = 0$. We find that the interior of the surface corresponds to the region of the positive correction $v^{(1)} > 0$, while the exterior region of the surface corresponds to the negative correction $v^{(1)} < 0$.

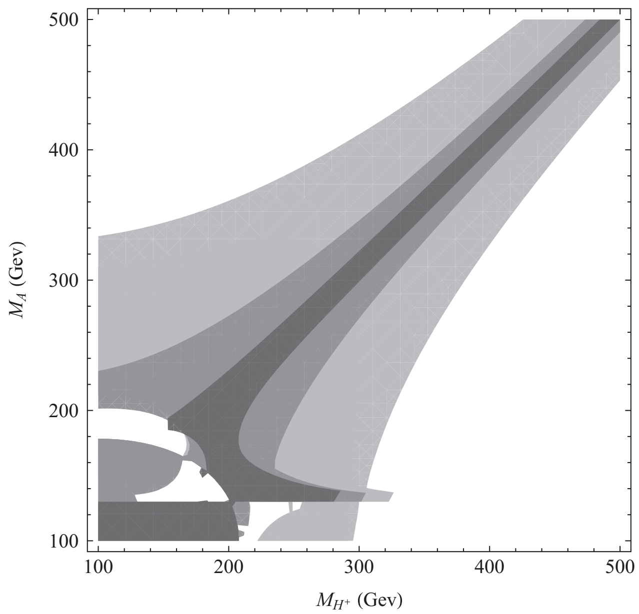

FIG. 5. The regions of (M_{H^+}, M_A), which correspond to $(|\frac{v^{(1)}}{v}|, |\frac{\beta^{(1)}}{\beta}|) = (0, 0)$ (dark gray), (0.01, 0.01) (gray), and (0.1, 0.1) (light gray).

In Fig. 5, we have shown the regions of (M_{H^+}, M_A) which correspond to that the corrections of $|v^{(1)}|$ and $|\beta^{(1)}|$ have the definite values (0, 0.01, 0.1). The dark gray shaded area corresponds to the region where both $v^{(1)}$ and $\beta^{(1)}$ can vanish with taking account of the conditions in Eqs. (7)–(9). We note that for $M_{H^+}, M_A > 200$ (GeV), the quantum corrections vanish around the region where the charged Higgs degenerates with the pseudoscalar Higgs. When the corrections become larger, the larger mass splitting of the pseudoscalar Higgs and charged Higgs is allowed. However, as the average mass of the charged Higgs and pseudoscalar Higgs increases, the allowed mass splitting becomes smaller.

VI. DISCUSSION AND CONCLUSION

In this paper, the Dirac neutrino mass model of Davidson and Logan is studied. In the model, one of the vacuum expectation values of two Higgs doublets is very small and it becomes the origin of the mass of neutrinos. The ratio of the small vacuum expectation value v_2 and that of the standard-like Higgs v_1 is $\tan\beta = \frac{v_2}{v_1}$. Therefore, $\tan\beta$ is very small and typically it is $O(10^{-9})$. The smallness of $\tan\beta$ is guaranteed by the smallness of the soft breaking term of $U(1)'$.

We have treated the soft-breaking term as perturbation and calculated, in particular, the vacuum expectation of Higgs in the leading order of the perturbation precisely. As summarized in Table I, only by including the soft breaking terms, one can argue which of the local minima minimizes the potential and becomes the global minimum. We have studied the global minimum of the tree-level Higgs potential, including the effect of the soft breaking term as perturbation.

Beyond the tree level, we study the quantum correction to the vacuum expectation values and $\tan\beta$ in a quantitative way. In one-loop level, we confirmed that tree-level vacuum is stable, i.e., the order parameters which vanish at tree level do not have the vacuum expectation value as quantum correction. In one-loop level, we derived the exact formulas for the quantum correction to β in the leading order of expansion of the soft breaking parameter m_{12}^2. We have confirmed not only that the loop correction to $\tan\beta$ is proportional to the soft breaking term, but also found that the correction depends on the Higgs mass spectrum and some combination of the quartic coupling constants of the Higgs potential. Technically, we carried out the calculation of the one-loop effective potential by employing O(4) real representation for SU(2) Higgs doublets.

Dependence of the corrections on the Higgs spectrum is studied numerically. We first derive a relation of the coupling constants, which corresponds to the condition that the correction to β vanishes for degenerate extra Higgs masses. Next, we study the effect of nondegeneracy of the charged Higgs and pseudoscalar Higgs on the correction. If the charged Higgs mass is as light as 100 (GeV) ~ 200 (GeV), allowing the mass difference of charged Higgs and pseudoscalar Higgs is about 100 (GeV), the quantum corrections to both β and v are within a few % for $(\lambda_3, \lambda_2) \sim (0.5, 1)$. If the charged Higgs is heavy $M_{H^+} = 500$ (GeV), a slight increase of the pseudoscalar Higgs mass from the degenerate point leads to very large corrections to β and v.

One can argue the size of the quantum corrections to the neutrino mass of the model, because the ratio of the tree level neutrino mass and one-loop correction can be written as

$$\frac{m_\nu^{(1)}}{m_\nu} = \frac{v^{(1)}}{v} + \frac{\beta^{(1)}}{\beta}, \tag{48}$$

where we take account of the corrections only due to Higgs vacuum expectation values. The formulas in Eq. (48) imply that radiative correction to neutrino mass is related to the Higgs mass spectrum. Therefore, once Higgs mass spectrum is measured in LHC, one can compute the radiative correction to the mass of neutrinos using the formulas of Eq. (48).

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Note added.—After submitting the paper, we became aware that the stability of the model studied in this paper was also discussed in [20]. Compared to their analysis, we derived the one-loop effective potential taking into account all the interactions of Higgs sector while they consider a part of the interactions and study the stability in a qualitative way. Using the effective potential, we carried out the quantitative analysis of the quantum corrections.

APPENDIX A: DERIVATION OF ONE-LOOP EFFECTIVE POTENTIAL

In this appendix, we give the details of the derivation of the one-loop effective potential and the counterterm in Eq. (27). One can split $M^2(\phi)_{ij}$ in Eq. (25) into the diagonal part and the off-diagonal part as $\delta M^2(\phi)_{ij} = M^2(\phi)_{ij} - M^2(\phi)_{ii}\delta_{ij}$. The divergent part of $V_{1\mathrm{loop}}$ can be easily computed by expanding it up to the second order of δM^2,

$$\begin{aligned} V_{1\mathrm{loop}} &= V^{(1)} + V_c, \\ V^{(1)} &= \frac{\mu^{4-d}}{2}\int \frac{d^dk}{(2\pi)^d i}\,\mathrm{TrLn}\{(D_{ii}^{0-1} + M_{ii}^2(\phi))\delta_{ij} + \delta M_{ij}^2 - \sigma_1 m_{12}^2\} \\ &= \sum_{i=1}^{8}\frac{\mu^{4-d}}{2}\int \frac{d^dk}{(2\pi)^d i}\ln\{D_{ii}^{0-1} + M_{ii}^2(\phi)\} - \sum_{i,j=1}^{8}\frac{\mu^{4-d}}{4}\int \frac{d^dk}{(2\pi)^d i} D_{ii}(\delta M^2 - \sigma_1 m_{12}^2)_{ij} D_{jj}(\delta M^2 - \sigma_1 m_{12}^2)_{ji} + \ldots, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} D_{ii}^{-1} &= D_{ii}^{0-1} + M_{ii}^2(\phi), \\ &= \begin{cases} M_{ii}^2 + m_{11}^2 - k^2 & (1 \le i \le 4), \\ M_{ii}^2 + m_{22}^2 - k^2 & (5 \le i \le 8). \end{cases} \end{aligned} \tag{A2}$$

The diagonal parts of the propagators are given as,

$$D_{ii} = \begin{cases} \frac{1}{M_{ii}^2 + m_{11}^2 - k^2} & (1 \le i \le 4), \\ \frac{1}{M_{ii}^2 + m_{22}^2 - k^2} & (5 \le i \le 8). \end{cases} \tag{A3}$$

In the modified minimal subtraction scheme, Feynman integration is carried out with help of the well known formulas of dimensional regularization

$$\begin{aligned} &\mu^{4-d}\frac{1}{2}\int \frac{d^dk}{(2\pi)^d i}\log(m^2 - k^2) \\ &= -\frac{1}{64\pi^2\bar{\epsilon}}m^4 + \frac{m^4}{64\pi^2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right), \end{aligned} \tag{A4}$$

and

$$\mu^{4-d}\int \frac{d^dk}{(2\pi)^d i}\frac{1}{(m_i^2 - k^2)(m_j^2 - k^2)}\bigg|_{\mathrm{div}} = \frac{1}{16\pi^2}\frac{1}{\bar{\epsilon}}, \tag{A5}$$

with $\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} - \log 4\pi$ and $\epsilon = 2 - \frac{d}{2}$. The divergent part of $V^{(1)}$ is

$$\begin{aligned}V^{(1)}_{\rm div} &= -\frac{1}{64\pi^2\bar{\epsilon}}\left\{\sum_{i=1}^{4}(M^2_{ii}+m^2_{11})^2+\sum_{i=5}^{8}(M^2_{ii}+m^2_{22})^2\right\}-\frac{1}{64\pi^2\bar{\epsilon}}\sum_{i\neq j=1}^{8}(\delta M^2-m^2_{12}\sigma_1)_{ij}(\delta M^2-m^2_{12}\sigma_1)_{ji},\\ &= -\frac{1}{32\pi^2\bar{\epsilon}}\left(m^2_{11}\sum_{i=1}^{4}M^2_{ii}(\phi)+m^2_{22}\sum_{i=5}^{8}M^2_{ii}(\phi)+2(m^4_{11}+m^4_{22})\right)-\frac{1}{64\pi^2\bar{\epsilon}}{\rm Tr}[(M^2(\phi)-m^2_{12}\sigma_1)(M^2(\phi)-m^2_{12}\sigma_1)],\\ &= -\frac{1}{64\pi^2\bar{\epsilon}}{\rm Tr}[M^4_T]. \end{aligned}\tag{A6}$$

The trace of Eq. (A6) is calculated in Eq. (B6) and (B11) of Appendix B, and the result is,

$$\begin{aligned}V^{(1)}_{\rm div} &= -\frac{1}{32\pi^2\bar{\epsilon}}[m^2_{11}\{6\lambda_1(\Phi_1^\dagger\Phi_1)+2(2\lambda_3+\lambda_4)(\Phi_2^\dagger\Phi_2)\}+m^2_{22}\{2(2\lambda_3+\lambda_4)(\Phi_1^\dagger\Phi_1)+6\lambda_2(\Phi_2^\dagger\Phi_2)\}]\\ &\quad+\frac{2m^2_{12}}{64\pi^2\bar{\epsilon}}[(2\lambda_3+4\lambda_4)(\Phi_1^\dagger\Phi_2+\Phi_2^\dagger\Phi_1)]-\frac{8m^4_{12}+4(m^4_{11}+m^4_{22})}{64\pi^2\bar{\epsilon}}\\ &\quad-\frac{1}{64\pi^2\bar{\epsilon}}\Big[(12\lambda_1^2+4\lambda_3\lambda_4+4\lambda_3^2+2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2+(12\lambda_2^2+4\lambda_3\lambda_4+4\lambda_3^2+2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2\\ &\quad+(12\lambda_1\lambda_3+4\lambda_1\lambda_4+8\lambda_3^2+4\lambda_4^2+12\lambda_2\lambda_3+4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\\ &\quad+(4\lambda_1\lambda_4+16\lambda_3\lambda_4+8\lambda_4^2+4\lambda_2\lambda_4)|\Phi_1^\dagger\Phi_2|^2\Big]. \end{aligned}\tag{A7}$$

Now the counterterms for the one-loop effective potential are simply given by changing the sign of the divergent part of Eq. (A7),

$$V_c=-V^{(1)}_{\rm div}=\frac{1}{64\pi^2\bar{\epsilon}}{\rm Tr}[M^4_T].\tag{A8}$$

Using Eq. (A8) and (A4), one can derive the finite part of the one-loop effective potential given in Eq. (27).

APPENDIX B: DERIVATION OF EQ. (A7)

In this section, we present the derivation of Eq. (A7). We start with the quartic interaction terms of the Higgs potential,

$$\begin{aligned}V^{(4)} &= \frac{\lambda_1}{8}\left(\sum_{i=1}^{4}\phi_i^2\right)^2+\frac{\lambda_2}{8}\left(\sum_{i=5}^{8}\phi_i^2\right)^2+\frac{\lambda_3}{4}\left(\sum_{i=1}^{4}\phi_i^2\right)\left(\sum_{j=5}^{8}\phi_j^2\right)\\ &\quad+\frac{\lambda_4}{4}((\phi_1\phi_5+\phi_2\phi_6+\phi_3\phi_7+\phi_4\phi_8)^2+(\phi_1\phi_6+\phi_3\phi_8-\phi_2\phi_5-\phi_4\phi_7)^2). \end{aligned}\tag{B1}$$

By taking the derivatives of $V^{(4)}$, one can obtain the mass squared matrix $M^2(\phi)$. One first computes the first derivative of $V^{(4)}$ with respect to ϕ_i,

$$\frac{\partial V^{(4)}}{\partial\phi_i}=\begin{cases}\frac{\lambda_1}{8}2\left(\sum_{j=1}^{4}\phi_j^2\right)2\phi_i+\frac{\lambda_3}{2}\phi_i\sum_{j=5}^{8}\phi_j^2+\frac{\lambda_4}{2}\{(\phi_1\phi_5+\phi_2\phi_6+\phi_3\phi_7+\phi_4\phi_8)\phi_{i+4}\\ \quad+(\phi_1\phi_6+\phi_3\phi_8-\phi_2\phi_5-\phi_4\phi_7)(\delta_{1i}\phi_6-\delta_{2i}\phi_5+\delta_{3i}\phi_8-\delta_{4i}\phi_7)\}, & (1\le i\le 4)\\ \frac{\lambda_2}{8}2\left(\sum_{j=5}^{8}\phi_j^2\right)2\phi_i+\frac{\lambda_3}{2}\phi_i\sum_{j=1}^{4}\phi_j^2+\frac{\lambda_4}{2}\{(\phi_1\phi_5+\phi_2\phi_6+\phi_3\phi_7+\phi_4\phi_8)\phi_{i-4}\\ \quad+(\phi_1\phi_6+\phi_3\phi_8-\phi_2\phi_5-\phi_4\phi_7)(-\delta_{5i}\phi_2+\delta_{6i}\phi_1-\delta_{7i}\phi_4+\delta_{8i}\phi_3)\}. & (5\le i\le 8).\end{cases}\tag{B2}$$

The second derivatives are given as

$$\frac{\partial^2 V^{(4)}}{\partial\phi_i\partial\phi_j} = \begin{cases} \frac{\lambda_1}{2}\left(\delta_{ij}\sum_{k=1}^4\phi_k^2 + 2\phi_j\phi_i\right) + \frac{\lambda_3}{2}\delta_{ij}\left(\sum_{k=5}^8\phi_k^2\right) + \frac{\lambda_4}{2}\{\phi_{j+4}\phi_{i+4} \\ \quad + (\delta_{1j}\phi_6 - \delta_{2j}\phi_5 + \delta_{3j}\phi_8 - \delta_{4j}\phi_7)(\delta_{1i}\phi_6 - \delta_{2i}\phi_5 + \delta_{3i}\phi_8 - \delta_{4i}\phi_7)\}, & (1\le i,j\le 4), \\ \lambda_3\phi_i\phi_j + \frac{\lambda_4}{2}\Big\{\phi_{i+4}\phi_{j-4} + \sum_{k=1}^4\delta_{i+4,j}\phi_k\phi_{k+4} + (-\delta_{5j}\phi_2 + \delta_{6j}\phi_1 - \delta_{7j}\phi_4 + \delta_{8j}\phi_3) \\ \quad \times(\delta_{1i}\phi_6 - \delta_{2i}\phi_5 + \delta_{3i}\phi_8 - \delta_{4i}\phi_7) + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7) \\ \quad \times(\delta_{1i}\delta_{6j} + \delta_{3i}\delta_{8j} - \delta_{2i}\delta_{5j} - \delta_{4i}\delta_{7j})\Big\}, & (1\le i\le 4, 5\le j\le 8), \\ \lambda_3\phi_i\phi_j + \frac{\lambda_4}{2}\Big\{\phi_{i-4}\phi_{j+4} + \sum_{k=1}^4\delta_{i-4,j}\phi_k\phi_{k+4} + (\delta_{1j}\phi_6 - \delta_{2j}\phi_5 + \delta_{3j}\phi_8 - \delta_{4j}\phi_7) \\ \quad \times(-\delta_{5i}\phi_2 + \delta_{6i}\phi_1 - \delta_{7i}\phi_4 + \delta_{8i}\phi_3) + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7) \\ \quad \times(\delta_{1i}\delta_{6j} + \delta_{3i}\delta_{8j} - \delta_{2i}\delta_{5j} - \delta_{4i}\delta_{7j})\Big\}, & (5\le i\le 8, 1\le j\le 4), \\ \frac{\lambda_2}{2}\left(\delta_{ij}\sum_{k=5}^8\phi_k^2 + 2\phi_j\phi_i\right) + \frac{\lambda_3}{2}\delta_{ij}\left(\sum_{k=1}^4\phi_k^2\right) \\ \quad + \frac{\lambda_4}{2}\{\phi_{j-4}\phi_{i-4} + (-\delta_{5j}\phi_2 + \delta_{6j}\phi_1 - \delta_{7j}\phi_4 + \delta_{8j}\phi_3) \\ \quad \times(-\delta_{5i}\phi_2 + \delta_{6i}\phi_1 - \delta_{7i}\phi_4 + \delta_{8i}\phi_3)\}, & (5\le i,j\le 8). \end{cases} \tag{B3}$$

With Eq. (B3), the diagonal sums of M^2 are given as

$$\sum_{i=1}^4 M_{ii}^2 = 3\lambda_1\sum_{i=1}^4\phi_i^2 + 2\lambda_3\sum_{i=5}^8\phi_i^2 + \lambda_4\sum_{i=5}^8\phi_i^2 = 6\lambda_1\Phi_1^\dagger\Phi_1 + (4\lambda_3 + 2\lambda_4)\Phi_2^\dagger\Phi_2, \qquad (1\le i\le 4),$$
$$\sum_{i=5}^8 M_{ii}^2 = 3\lambda_2\sum_{i=5}^8\phi_i^2 + 2\lambda_3\sum_{i=1}^4\phi_i^2 + \lambda_4\sum_{i=1}^4\phi_i^2 = 6\lambda_2\Phi_2^\dagger\Phi_2 + (4\lambda_3 + 2\lambda_4)\Phi_1^\dagger\Phi_1, \qquad (5\le i\le 8). \tag{B4}$$

The counterterm in Eq. (A8) includes the following contribution:

$$\mathrm{Tr}[(M^2(\phi) - m_{12}^2\sigma_1)(M^2(\phi) - m_{12}^2\sigma_1)] = \mathrm{Tr}[M^2(\phi)M^2(\phi) - 2m_{12}^2\sigma_1 M^2] + 8m_{12}^4. \tag{B5}$$

The second term of Eq. (B5) is proportional to

$$\mathrm{Tr}[m_{12}^2\sigma_1 M^2] = (2\lambda_3 + 4\lambda_4)(\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)m_{12}^2 = (2\lambda_3 + 4\lambda_4)(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)m_{12}^2. \tag{B6}$$

The first term of Eq. (B5) can be decomposed as

$$\mathrm{Tr}[M^2(\phi)M^2(\phi)] = \sum_{i,j=1}^4 M^2(\phi)_{ij}M^2(\phi)_{ji} + 2\sum_{i=1}^4\sum_{j=5}^8 M^2(\phi)_{ij}M^2(\phi)_{ji} + \sum_{i,j=5}^8 M^2(\phi)_{ij}M^2(\phi)_{ji}. \tag{B7}$$

Each term of Eq. (B7) is given as

$$\begin{aligned}\sum_{i,j=1}^4 M^2(\phi)_{ij}M^2(\phi)_{ji} &= 3\lambda_1^2\left(\sum_{i=1}^4\phi_i^2\right)^2 + 3\lambda_1\lambda_3\sum_{i=1}^4\phi_i^2\sum_{j=5}^8\phi_j^2 + \lambda_1\lambda_4\Big\{\sum_{i=5}^8\phi_i^2\sum_{j=1}^4\phi_j^2 + (\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)^2 \\ &\quad + (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)^2\Big\} + \lambda_3\lambda_4\left(\sum_{i=5}^8\phi_i^2\right)^2 + \lambda_3^2\left(\sum_{i=5}^8\phi_i^2\right)^2 + \frac{\lambda_4^2}{2}\left(\sum_{i=5}^8\phi_i^2\right)^2 \\ &= 12\lambda_1^2(\Phi_1^\dagger\Phi_1)^2 + (12\lambda_1\lambda_3 + 4\lambda_1\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 4\lambda_1\lambda_4|\Phi_1^\dagger\Phi_2|^2 \\ &\quad + (4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2,\end{aligned} \tag{B8}$$

$$\sum_{i=1}^{4}\sum_{j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji} = \lambda_3^2\sum_{i=5}^{8}\phi_i^2\sum_{j=1}^{4}\phi_j^2 + 2\lambda_3\lambda_4\Big\{\sum_{i=1}^{4}\phi_i\phi_{i+4}\sum_{j=1}^{4}\phi_j\phi_{j+4} + (\phi_1\phi_6 - \phi_2\phi_5 + \phi_3\phi_8 - \phi_4\phi_7)^2\Big\}$$
$$+ \frac{\lambda_4^2}{2}\Big\{\sum_{i=1}^{4}\phi_i^2\sum_{j=5}^{8}\phi_j^2 + 2\Big(\sum_{i=1}^{4}\phi_i\phi_{i+4}\Big)^2 + 2(\phi_1\phi_6 - \phi_2\phi_5 + \phi_3\phi_8 - \phi_4\phi_7)^2\Big\}$$
$$= (4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + (8\lambda_3\lambda_4 + 4\lambda_4^2)|\Phi_1^\dagger\Phi_2|^2, \tag{B9}$$

$$\sum_{i,j=5}^{8} M^2(\phi)_{ij}M^2(\phi)_{ji} = 3\lambda_2^2\Big(\sum_{i=5}^{8}\phi_i^2\Big)^2 + 3\lambda_2\lambda_3\sum_{i=5}^{8}\phi_i^2\sum_{j=1}^{4}\phi_j^2 + \lambda_2\lambda_4\Big\{\sum_{i=1}^{4}\phi_i^2\sum_{j=5}^{8}\phi_j^2 + (\phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8)^2$$
$$+ (\phi_1\phi_6 + \phi_3\phi_8 - \phi_2\phi_5 - \phi_4\phi_7)^2\Big\} + \lambda_3\lambda_4\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2 + \lambda_3^2\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2 + \frac{\lambda_4^2}{2}\Big(\sum_{i=1}^{4}\phi_i^2\Big)^2$$
$$= 12\lambda_2^2(\Phi_2^\dagger\Phi_2)^2 + (12\lambda_2\lambda_3 + 4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 4\lambda_2\lambda_4|\Phi_1^\dagger\Phi_2|^2$$
$$+ (4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2. \tag{B10}$$

From Eqs. (B8)–(B10), one obtains,

$$\mathrm{Tr}[M^2(\phi)M^2(\phi)] = (12\lambda_1^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_1^\dagger\Phi_1)^2 + (12\lambda_2^2 + 4\lambda_3\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2)(\Phi_2^\dagger\Phi_2)^2$$
$$+ (12\lambda_1\lambda_3 + 4\lambda_1\lambda_4 + 8\lambda_3^2 + 4\lambda_4^2 + 12\lambda_2\lambda_3 + 4\lambda_2\lambda_4)(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ (4\lambda_1\lambda_4 + 16\lambda_3\lambda_4 + 8\lambda_4^2 + 4\lambda_2\lambda_4)|\Phi_1^\dagger\Phi_2|^2. \tag{B11}$$

Using Eqs. (B4)–(B6) and (B11), one can derive Eq. (A7).

APPENDIX C: $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ AND L_{IJ}

In this appendix, we show $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ and L_{IJ}, which are needed to calculate one-loop corrections to the order parameters $\varphi_I^{(1)}$ in Eq. (29). $[O^T \frac{\partial M^2}{\partial \varphi_I} O]_{jj}$ $(I = 1, 2, 3, 4)$ are given as

$$\Big[O^T\frac{\partial M^2}{\partial\alpha}O\Big]_{jj} = 0, \qquad \Big[O^T\frac{\partial M^2}{\partial\theta'}O\Big]_{jj} = 0. \tag{C1}$$

$$\Big[O^T\frac{\partial M^2}{\partial v}O\Big]_{jj} = 2v\Big[O^T\frac{\partial M^2}{\partial v^2}O\Big]_{jj}$$
$$= \frac{v}{4}\begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 - 2\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ \frac{1}{2}(\lambda_1 + \lambda_2 + 6\lambda_3 + 6\lambda_4 - \cos(4\beta)(\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4))) \\ 12\{\lambda_2\cos^2\gamma\sin^2\beta + \cos^2\beta\sin^2\gamma\lambda_1\} + (3\cos2(\beta-\gamma) - \cos2(\beta+\gamma) + 2)(\lambda_3 + \lambda_4) \\ 12\{\lambda_1\cos^2\beta\cos^2\gamma + \sin^2\beta\sin^2\gamma\lambda_2\} + (-3\cos2(\beta-\gamma) + \cos2(\beta+\gamma) + 2)(\lambda_3 + \lambda_4) \end{pmatrix}, \tag{C2}$$

and

$$\Big[O^T\frac{\partial M^2}{\partial\beta}O\Big]_{jj} = v^2\frac{\sin2\beta}{2}\begin{pmatrix} \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ \lambda_2\cos^2(\beta) - \sin^2(\beta)\lambda_1 - \cos(2\beta)(\lambda_3 + \lambda_4) \\ 3\lambda_2\cos^2(\gamma) - 3\sin^2\gamma\lambda_1 + \frac{1}{2\sin2\beta}(\sin(2(\beta+\gamma)) - 3\sin(2(\beta-\gamma)))(\lambda_3 + \lambda_4) \\ -3\lambda_1\cos^2(\gamma) + 3\sin^2(\gamma)\lambda_2 - \frac{1}{2\sin2\beta}(\sin(2(\beta+\gamma)) - 3\sin(2(\beta-\gamma)))(\lambda_3 + \lambda_4) \end{pmatrix}. \tag{C3}$$

Next, we show L_{IJ} in Eq. (30). Note that L_{IJ} is symmetric $L_{IJ} = L_{JI}$ and its nonzero elements are:

$$\begin{aligned}
L_{11} &= \cos^2\beta m_{11}^2 + \sin^2\beta m_{22}^2 - 2\cos(\beta)\sin(\beta)m_{12}^2 + \frac{1}{2}[3v^2\{\lambda_1\cos^4(\beta) + \sin^2(\beta)(2(\lambda_3+\lambda_4)\cos^2(\beta) + \sin^2(\beta)\lambda_2)\}],\\
L_{22} &= v^2\left\{-\frac{\cos4\beta}{4}(\lambda_1+\lambda_2-2(\lambda_3+\lambda_4))v^2 + \frac{\cos2\beta}{4}(\lambda_2-\lambda_1)v^2 + 2m_{12}^2\sin2\beta - \cos2\beta(m_{11}^2 - m_{22}^2)\right\},\\
L_{12} &= L_{21} = v\left\{-\frac{\sin4\beta}{4}(\lambda_1+\lambda_2-2(\lambda_3+\lambda_4))v^2 + \frac{1}{2}\sin2\beta(\lambda_2-\lambda_1)v^2 - 2m_{12}^2\cos2\beta - \sin2\beta(m_{11}^2 - m_{22}^2)\right\}\\
L_{33} &= -\frac{1}{8}v^2\sin(2\beta)(v^2\sin(2\beta)\lambda_4 - 4m_{12}^2),\\
L_{44} &= v^2\cos(\beta)\sin(\beta)m_{12}^2.
\end{aligned} \tag{C4}$$

APPENDIX D: ORTHOGONAL MATRIX O IN EQ. (31)

Here we show the orthogonal matrix O in Eq. (31).

$$O = \begin{pmatrix}
0 & -\sin\beta & 0 & 0 & 0 & 0 & \cos\beta & 0\\
-\sin\beta & 0 & 0 & 0 & 0 & \cos\beta & 0 & 0\\
0 & 0 & 0 & \sin\gamma & \cos\gamma & 0 & 0 & 0\\
0 & 0 & -\sin\beta & 0 & 0 & 0 & 0 & \cos\beta\\
0 & \cos\beta & 0 & 0 & 0 & 0 & \sin\beta & 0\\
\cos\beta & 0 & 0 & 0 & 0 & \sin\beta & 0 & 0\\
0 & 0 & 0 & \cos\gamma & -\sin\gamma & 0 & 0 & 0\\
0 & 0 & \cos\beta & 0 & 0 & 0 & 0 & \sin\beta
\end{pmatrix}. \tag{D1}$$

[1] Shainen M. Davidson and Heather E. Logan, Phys. Rev. D **80**, 095008 (2009).
[2] Shainen M. Davidson and Heather E. Logan, Phys. Rev. D **82**, 115031 (2010).
[3] S. Gabriel and S. Nandi, Phys. Lett. B **655**, 141 (2007).
[4] Ernest Ma, Phys. Rev. Lett., **86**, 2502 (2001).
[5] Naoyuki Haba and Masaki Hirotsu, Eur. Phys. J. C **69**, 481 (2010).
[6] Naoyuki Haba and Koji Tsumura, J. High Energy Phys. 06 (2011) 068.
[7] Shinya Kanemura, Takehiro Nabeshima, and Hiroaki Sugiyama, Phys. Lett. B **703**, 66 (2011).
[8] Salah Nasri and Sherif Moussa, Mod. Phys. Lett. A **17**, 771 (2002).
[9] Michio Hashimoto and Shinya Kanemura, Phys. Rev. D **70**, 055006 (2004).
[10] P. M. Ferreira, R. Santos, and A. Barroso, Phys. Lett. B **603**, 219 (2004).
[11] M. Maniatis, A. von Manteuffel, O. Nachtmann, and F. Nagel, Eur. Phys. J. C **48**, 805 (2006).
[12] A. Barroso, P. M. Ferreira, and R. Santos, Phys. Lett. B **652**, 181 (2007).
[13] Igor P. Ivanov, Phys. Rev. D **77**, 015017 (2008).
[14] I. F. Ginzburg and K. A. Kanishev, Phys. Rev. D **76**, 095013 (2007).
[15] I. P. Ivanov, Acta Phys. Pol. B **40**, 2789 (2009).
[16] G. C. Branco *et al.*, arXiv:1106.0034.
[17] Michael Gustafsson, Proc. Sci., CHARGED2010 (2010) 030.
[18] Maria Krawczyk and Dorota Sokolowska, Fortschr. Phys. **59**, 1098 (2011).
[19] Christopher T. Hill, Chung Ngoc Leung, and Sumathi Rao, Nucl. Phys. **B262**, 517 (1985).
[20] Naoyuki Haba and Tomohiro Horita, Phys. Lett. B **705**, 98 (2011).

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Charged Higgs and neutral Higgs pair production of the weak gauge boson fusion process in electron-positron collisions

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Pair production of the neutral and charged Higgs bosons is a unique process that is a signature of the two-Higgs-doublet model. In this paper, we study the pair production and decays of the Higgses in the neutrinophilic two-Higgs-doublet model. The pair production occurs through the W and Z gauge boson fusion process. In the neutrinophilic model, the vacuum expectation value (VEV) of the second Higgs doublet is small and is proportional to the neutrino mass. The smallness of VEV is associated with the approximate global U(1) symmetry, which is slightly broken. Therefore, there is a suppression factor for the U(1) charge breaking process. The second Higgs doublet has U(1) charge; its single production from gauge boson fusion violates the U(1) charge conservation and is strongly suppressed. In contrast to the single production, the pair production of the Higgses conserves U(1) charge and the approximate symmetry does not forbid it. To search for the pair productions in a collider experiment, we study the production cross section of a pair of charged Higgs and neutral Higgs bosons in e^+e^- collisions with a center of energy from 600 GeV to 2000 GeV. The total cross section varies from 10^{-4} fb to 10^{-3} fb for the degenerate (200 GeV) charged and neutral Higgs mass case. The background process to the signal is the gauge boson pair $W^+ + Z$ production and their decays. We show that the signal over background ratio is about 2–3% by combining the cross section ratio with ratios of branching fractions.

Subject Index B40, B53

1. Introduction

While LHC have already started constraining many new physics models, there are a few aspects in the beyond-standard models into which future e^+e^- colliders [1,2] could make unique searches because of their clean environments. In this paper, we study the signature of the neutrinophilic two-Higgs-doublet model [3] in e^+e^- collisions by focusing on the pair production and decays of the charged Higgs and neutral Higgs bosons.

In the neutrinophilic model, a second Higgs doublet is introduced and the neutrino masses are generated from the tiny VEV (vacuum expectation value) of the second Higgs doublet. The new U(1) global symmetry is introduced. The second Higgs doublet and right-handed neutrinos have the U(1) charge $+1$ and the other fields do not have that charge. The U(1) global symmetry is approximate and is broken explicitly by the soft breaking bilinear term with respect to the second Higgs doublet and to

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the standard-model-like Higgs doublet. The tiny VEV of the second Higgs generated is proportional to the coefficient of the mass dimension two in the bilinear term.

In the model, any U(1) charge-violating process is suppressed by the tiny VEV. This also implies that the probability amplitude is suppressed and is proportional to neutrino mass. An example of a suppressed process is a single second Higgs production with gauge boson fusion. In contrast to the single second Higgs production, the pair production of the second Higgs is a U(1) charge conserving process. Therefore, it is not suppressed. The processes in this category are $Z^*(\gamma^*) \to H^+H^-$, $W^+ + W^- \to H^+ + H^-$, and $W^+ + Z \to H^+ + X$ $(X = A, h)$, where H^+, A, and h denote the charged Higgs, CP-odd Higgs, and CP-even Higgs in the second Higgs doublet, respectively.

In the LHC set-up, the charged Higgs pair production $p + p \to Z^*(\gamma^*) \to H^+ + H^-$ is studied in Ref. [4]. In Ref. [5], vector boson fusion into the light CP-even Higgs pairs is studied at the LHC. In Ref. [6], di-Higgs production in various scenarios is discussed. In Ref. [7], the standard model Higgs boson pair production is studied. In addition, see Ref. [8] for the ratio of the cross section of the single Higgs boson and the pair production cross section in the context of the standard model.

In our work, in e^+e^- collisions, the pair production of the charged Higgs (H^+) and neutral Higgs (X) in the second Higgs doublet is studied. We derive the pair production cross section, $e^+ + e^- \to \overline{\nu_e} + e^- + H^+ + X$ $(X = A, h)$.

The paper is organized as follows. In Sect. 2, we set up the Lagrangian that is used in the calculation of charged Higgs and neutral Higgs production. In Sect. 3, we derive the expression of the cross sections for pair production from $e^+ + e^-$ collisions. In Sect. 4, the cross sections, including the various differential cross sections, are numerically computed and compared to the standard model background cross section. In Sect. 5, the decays of the charged Higgs and neutral Higgs are discussed and the dependence on the charged lepton flavor in the final state is studied. Section 6 is devoted to the summary.

2. Two-Higgs-doublet model with softly broken global symmetry

In this section, we present the Lagrangian to set up the notation and also to display the interaction terms that are relevant to the calculation in later sections. The Higgs potential is given by [3]:

$$V_{\text{tree}} = \sum_{i=1,2} \left(m_{ii}^2 \Phi_i^\dagger \Phi_i + \frac{\lambda_i}{2} (\Phi_i^\dagger \Phi_i)^2 \right) - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.) + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2. \tag{1}$$

The two Higgs doublets in the unitary gauge are parameterized as [9]:

$$\Phi_1 = \begin{pmatrix} 0 \\ \dfrac{v \cos\beta}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} -\sin\beta H^+ \\ \dfrac{\sin\gamma h + \cos\gamma H - i \sin\beta A}{\sqrt{2}} \end{pmatrix},$$
$$\Phi_2 = \begin{pmatrix} 0 \\ \dfrac{v \sin\beta}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \cos\beta H^+ \\ \dfrac{\cos\gamma h - \sin\gamma H + i \cos\beta A}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

The new U(1) charge for Φ_1 (Φ_2) is $0(+1)$. The term proportional to m_{12} is the U(1) breaking term. H and h denote CP-even Higgses, A denotes a CP-odd Higgs. In our notation, H is close to the standard-model-like Higgs, a different notation from Ref. [3]. In most of the present paper, we follow the notation of Ref. [9]. $\tan\beta$ is the ratio of two VEVs and is given approximately as [3]:

$$\tan\beta = \frac{m_{12}^2}{m_A^2}. \tag{3}$$

v^2 is the squared sum of two VEVs. γ is the mixing angle of CP-even Higgses given by [9]:

$$\tan 2\gamma = -\frac{-4m_{12}^2 + 2\sin 2\beta(\lambda_3+\lambda_4)v^2}{(3(-\lambda_1\cos\beta^2 + \lambda_2\sin^2\beta) + \cos 2\beta(\lambda_3+\lambda_4))v^2 - 2(m_{11}^2 - m_{22}^2)}. \tag{4}$$

Then one can write the covariant derivative terms for the two doublets, including the electroweak interactions of the Higgses with gauge bosons:

$$\begin{aligned}\sum_{i=1,2} D_\mu\Phi_i^\dagger D^\mu\Phi_i \ni\; & gM_W\left(W_\mu^+W^{\mu-} + \frac{1}{2c_W^2}Z^\mu Z_\mu\right)(\sin(\beta+\gamma)h + \cos(\beta+\gamma)H)\\ & + \frac{g^2}{2}s_W(A_\mu - t_W Z_\mu)[(H^+W^{\mu-} + H^-W^{\mu+})(h\cos(\beta+\gamma) - H\sin(\beta+\gamma))\\ & - i(H^+W^{\mu-} - H^-W^{\mu+})A] + i\frac{g\cos 2\theta_W}{2\cos\theta_W}Z_\mu(\partial^\mu H^- H^+ - \partial^\mu H^+ H^-)\\ & + \frac{g\cos(\beta+\gamma)}{2\cos\theta_W}(\partial_\mu h A - \partial_\mu A h)Z^\mu + \left\{i\frac{g}{2}\cos(\beta+\gamma)W^{+\mu}(h\partial_\mu H^- - \partial_\mu h H^-)\right.\\ & \left. + \frac{g}{2}W^{+\mu}(H^-\partial_\mu A - A\partial_\mu H^-) + h.c.\right\}.\end{aligned} \tag{5}$$

One notes that a single CP-even Higgs boson (h or H) could be produced by the gauge boson fusion process $W^+ + W^-(Z+Z) \to h$ or H. There is no single CP-odd Higgs A production from gauge boson fusion. The absence of terms like $AW_\mu^+W^{-\mu}$ is due to CP symmetry. We also note that the CP-even Higgs h is mostly the real part of the down component of the second Higgs Φ_2. Its coupling to the gauge boson pair operators $W^{+\mu}W_\mu^-$ and $Z^\mu Z_\mu$ is suppressed as $\sin(\beta+\gamma)$. Since $\sin\beta$ and $\sin\gamma$ are suppressed to zero in the vanishing limit of the U(1) breaking term m_{12}, the gauge boson fusion to h is forbidden in the limit. As for the decays of charged Higgs and neutral Higgs, the Yukawa coupling to the right-handed neutrino is important. Assigning the U(1) charge +1 to the right-handed neutrino [3], it is written in terms of mass eigenstates as:

$$\begin{aligned}\mathcal{L}_Y &= -y_{\nu ij}\overline{\psi_i}\tilde{\Phi}_2\nu_{Rj}^0\\ &\ni -\overline{\nu_i}\left(\frac{m_{\nu i}}{v}\right)\nu_i\frac{\cos\gamma h - \sin\gamma H}{\sin\beta} + i\overline{\nu_i}\left(\frac{m_{\nu i}}{v}\right)\gamma_5\nu_i\cot\beta A\\ &\quad + \sqrt{2}\cot\beta\overline{l_i}V_{ij}\left(\frac{m_{\nu j}}{v}\right)\nu_{Rj}H^- + h.c.,\end{aligned} \tag{6}$$

where m_ν denotes the neutrino masses and V denotes the Maki–Nakagawa–Sakata (MNS) matrix.

3. Cross section of $e^+ + e^- \to \bar{\nu} + e^- + W^{+*} + Z^* \to \bar{\nu} + e^- + H^+ + A$

In this section, we present the formulae for the cross section of $e^+ + e^- \to \bar{\nu} + e^- + W^{+*} + Z^* \to \bar{\nu} + e^- + H^+ + A$ (see Fig. 1).

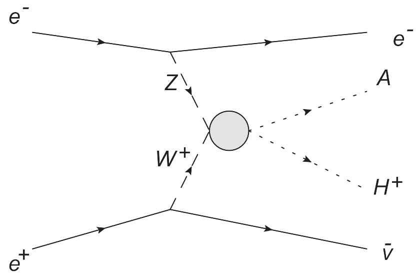

Fig. 1. Feynman diagram of charged Higgs H^+ and CP-odd Higgs A production in e^+e^- collisions. The production occurs through W^+ and Z fusion, which is shown by the circle.

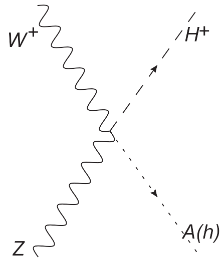

Fig. 2. Contact interaction.

We define

$$\sigma_{H^+X} \equiv \sigma(e^+ + e^- \to \overline{\nu_e} + e^- + H^+ + X);\, X = A, h. \tag{7}$$

We write the cross section for H^+A production as:

$$\sigma_{H^+A} = \frac{1}{2s_{e^+e^-}} \int \frac{d^3q_A}{(2\pi)^3 2E_A} \frac{d^3q_{H^+}}{(2\pi)^3(2E_{H^+})} \frac{d^3q_e}{(2\pi)^3 2E_e(q_e)} \frac{d^3q_\nu}{2E_{\bar{\nu}}} \times \frac{1}{4}\sum_{\text{spin}} |M|^2 (2\pi)\delta^4(p_{e^+} + p_e - q_{H^+} - q_A - q_e - q_{\bar{\nu}}). \tag{8}$$

$s_{e^+e^-}$ is the center-of-mass (cm) energy of the e^+ and e^- collision. p_{e^+} and p_e denote the momenta of the positron and electron of the initial state. q_e, q_{H^+}, q_A, and $q_{\bar{\nu}}$ are the momenta of the final states, i.e., electron, charged Higgs, neutral Higgs, and anti-neutrino respectively. The transition amplitude M is given by

$$M = -T_{A\mu\nu} \frac{1}{(p_Z^2 - M_Z^2)(p_W^2 - M_W^2)} \frac{g^2}{2\sqrt{2}c_W} \overline{u(q_e)}\gamma^\nu(L + 2s_W^2)u(p_e)\overline{v_{e^+}(p_{e^+})}\gamma^\mu L v_{\bar{\nu}}(q_{\bar{\nu}}), \tag{9}$$

where $p_Z = p_e - q_e$ and $p_W = q_{H^+} + q_A - p_Z$. L denotes the chiral projection $L = \frac{1-\gamma_5}{2}$. $s_W(c_W)$ denotes sine (cosine) of the Weinberg angle. $T_{A\mu\nu}$ denotes the off-shell amplitude for $W_\mu^{+*} + Z_\nu^* \to A + H^+$ production. This corresponds to the circle in Fig. 1, and the Feynman diagrams that contribute to $T^A_{\mu\nu}$ are shown in Figs. 2–5.

The second-rank tensor $T_{A\mu\nu}$ is given as:

$$T_{\mu\nu} = iT_{A\mu\nu} = \frac{g^2}{2\cos\theta_W}\left(a_A g_{\mu\nu} + d_A q_{A\nu} q_{H^+\mu} + b_A q_{H^+\nu} q_{A\mu}\right), \tag{10}$$

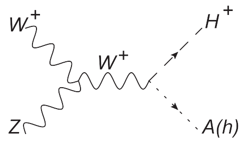

Fig. 3. S channel W exchange.

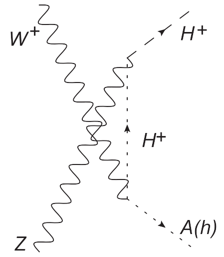

Fig. 4. U channel.

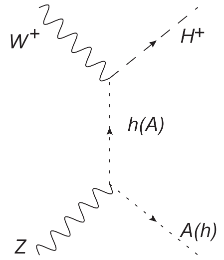

Fig. 5. T channel.

where we introduce the real amplitude $T^*_{\mu\nu} = T_{\mu\nu}$ (the on-shell case is shown in Ref. [10]). a_A, b_A, and d_A in Eq. (10) are given as:

$$a_A = s_W^2 + \frac{p_Z^2 - p_W^2}{M_Z^2}\frac{M_A^2 - M_{H^+}^2 - M_W^2}{s_{H^+A} - M_W^2} + c_W^2\frac{t_A - u_A + p_Z^2 - p_W^2}{s_{H^+A} - M_W^2},$$

$$b_A = -\frac{2\cos 2\theta_W}{u_A - M_{H^+}^2} - \frac{2(\cos 2\theta_W + 1)}{s_{H^+A} - M_W^2},$$

$$d_A = \frac{2\cos^2(\beta+\gamma)}{t_A - M_h^2} + \frac{2(\cos 2\theta_W + 1)}{s_{H^+A} - M_W^2}, \tag{11}$$

with $t_A = (q_{H^+} - p_W)^2$, $u_A = (p_W - q_A)^2$, and $s_{H^+A} = (q_{H^+} + q_A)^2$. The spin-averaged amplitude squared is given as:

$$\frac{1}{4}\sum_{\text{spin}} |M|^2 = \frac{g^4}{32c_W^2}\frac{1}{|(p_Z^2 - M_Z^2)(p_W^2 - M_W^2)|^2} T_{\mu\nu} T^*_{\rho\sigma} L^{\nu\sigma}_{ee} L^{\mu\rho}_{e^+\bar{\nu}}, \tag{12}$$

where $L^{\nu\rho}_{ee}$ is a leptonic tensor of the neutral current and $L^{\mu\sigma}_{e^+\bar{\nu}}$ is that of the charged current. They are written in terms of the symmetric part S and the anti-symmetric part A:

$$L^{\nu\sigma}_{ee} = S^{\nu\sigma}_{ee} + iA^{\nu\sigma}_{ee},$$

$$S^{\nu\sigma}_{ee} = (2 + 8s_W^2 + 16s_W^4)(p_e^\nu q_e^\sigma - g^{\nu\sigma} p_e \cdot q_e + p_e^\sigma q_e^\nu),$$

$$A^{\nu\sigma}_{ee} = (2 + 8s_W^2)\epsilon^{\nu\alpha\sigma\beta} p_{e\alpha} q_{e\beta}, \tag{13}$$

$$L^{\mu\rho}_{e^+\bar{\nu}} = S^{\mu\rho}_{e^+\bar{\nu}} + iA^{\mu\rho}_{e^+\bar{\nu}},$$

$$S^{\mu\rho}_{e^+\bar{\nu}} = 2(q_{\bar{\nu}}^\mu p_{e^+}^\rho - g^{\mu\rho} q_{\bar{\nu}} \cdot p_{e^+} + q_{\bar{\nu}}^\rho p_{e^+}^\mu),$$

$$A^{\mu\rho}_{e^+\bar{\nu}} = 2\epsilon^{\mu\alpha\rho\beta} q_{\bar{\nu}\alpha} p_{e^+\beta}. \tag{14}$$

We define the transpose matrix as $T^t_{\mu\nu} = T_{\nu\mu}$. In terms of these, one can write the differential cross section as:

$$d\sigma_{H^+A} = \frac{g^4}{64c_W^2 s_{e^+e^-}}\frac{1}{4096\pi^8}\left|\frac{1}{((p_e - q_e)^2 - M_Z^2)((p_e^+ - q_{\bar{\nu}})^2 - M_W^2)}\right|^2$$

$$\times (T_{\mu\nu} S^{\nu\sigma}_{ee} T^t_{\sigma\rho} S^{\rho\mu}_{e^+\bar{\nu}} + T_{\mu\nu} A^{\nu\sigma}_{ee} T^t_{\sigma\rho} A^{\rho\mu}_{e^+\bar{\nu}}) d^{12}Ph, \tag{15}$$

where $d^n Ph$ denotes an n-dimensional phase space integral. For $n = 12$, this is defined as:

$$d^{12}Ph = \frac{d^3q_A d^3q_{H^+} d^3q_e d^3q_{\bar{\nu}}}{E_A E_{H^+} E_e E_{\bar{\nu}}} \delta^4(p_{e^+} + p_e - q_e - q_{\bar{\nu}} - q_{H^+} - q_A). \tag{16}$$

In the center-of-mass frame of the e^+e^- collision, the amplitude is independent of the rotation around the beam axis. One can also set the direction of the e^+ beam to the z direction and the momentum of the electron in the final states to the yz plane. Therefore, after one integrates the azimuthal angle and the anti-neutrino momentum, one obtains d^8Ph as:

$$d^8Ph = 2\pi\, d\cos\theta_e\, d\cos\theta_{eH}\, d\phi_{eH}\, d\cos\theta_{eHA}\, d\phi_{eHA}$$

$$\times \frac{q_e^2 dq_e}{E_e}\frac{q_{H^+}^2 dq_{H^+}}{E_{H^+}}\frac{q_A^2 dq_A}{E_A}\delta(\sqrt{s} - E_{H^+} - E_A - E_e - E_{\bar{\nu}}). \tag{17}$$

The momentum of the electron q_e in the final states is specified by a polar angle (θ_e) in the orthogonal frame in which the positron momentum is chosen as the z axis:

$$\mathbf{p_{e^+}} = \frac{\sqrt{s_{e^+e^-}}}{2}\mathbf{e_3}, \quad \mathbf{p_e} = -\frac{\sqrt{s_{e^+e^-}}}{2}\mathbf{e_3},$$

$$\mathbf{q_e} = |\mathbf{q_e}|(\sin\theta_e \mathbf{e_2} + \cos\theta_e \mathbf{e_3}),$$

$$\mathbf{e_1} = \mathbf{e_2} \times \mathbf{e_3}. \tag{18}$$

One can define a new orthogonal coordinate spanned by the basis vectors $\mathbf{e}'_i$ ($i = 1$–3):

$$\mathbf{e}'_3 = \frac{\mathbf{q_e}}{|\mathbf{q_e}|} = \sin\theta_e \mathbf{e_2} + \cos\theta_e \mathbf{e_3},$$
$$\mathbf{e}'_2 = -\sin\theta_e \mathbf{e_3} + \cos\theta_e \mathbf{e_2},$$
$$\mathbf{e}'_1 = \mathbf{e_1}. \tag{19}$$

θ_{eH} and ϕ_{eH} denote the momentum direction of the charged Higgs relative to that of the electron in the final state:

$$\mathbf{q_{H^+}} = |\mathbf{q_{H^+}}|(\sin\theta_{eH}\cos\phi_{eH}\mathbf{e}'_1 + \sin\theta_{eH}\sin\phi_{eH}\mathbf{e}'_2 + \cos\theta_{eH}\mathbf{e}'_3). \tag{20}$$

Finally, $(\theta_{eHA}, \phi_{eHA})$ denote the direction of momentum for the neutral Higgs A. θ_{eHA} is a polar angle measured from the direction $\mathbf{q_e} + \mathbf{q_{H^+}}$:

$$\mathbf{q_A} = |q_A|(\sin\theta_{eHA}\cos\phi_{eHA}\mathbf{e}''_1 + \sin\theta_{eHA}\sin\phi_{eHA}\mathbf{e}''_2 + \cos\theta_{eHA}\mathbf{e}''_3), \tag{21}$$

$$\mathbf{e}''_3 = \frac{\mathbf{q_e} + \mathbf{q_{H^+}}}{|\mathbf{q_e} + \mathbf{q_{H^+}}|}, \mathbf{e}''_1 = \frac{\mathbf{q_e} \times \mathbf{q_{H^+}}}{|\mathbf{q_e} \times \mathbf{q_{H^+}}|}, \mathbf{e}''_2 = \mathbf{e}''_3 \times \mathbf{e}''_1. \tag{22}$$

In terms of the angles defined, the phase space integration is written:

$$d^8 Ph = 2\pi d\cos\theta_e d\cos\theta_{eH} d\phi_{eH} d\cos\theta_{eHA} d\phi_{eHA}$$
$$\times \frac{q_e^2 dq_e}{E_e}\frac{q_{H^+}^2 dq_{H^+}}{E_{H^+}}\frac{q_A^2 dq_A}{E_A E_{\bar\nu}}\delta(\sqrt{s} - E_{H^+} - E_A - E_q - E_{\bar\nu})$$
$$E_{\bar\nu} = \sqrt{|\mathbf{q_e} + \mathbf{q_{H^+}}|^2 + q_A^2 + 2\cos\theta_{eHA} q_A|\mathbf{q_e} + \mathbf{q_{H^+}}|}, \tag{23}$$

where we denote $q_A = |\mathbf{q_A}|$, $q_{H^+} = |\mathbf{q_{H^+}}|$, and $q_e = |\mathbf{q_e}|$. The integration over the variable $\cos\theta_{eHA}$ is carried out and we obtain:

$$d^7 Ph = 2\pi d\cos\theta_e d\cos\theta_{eH} d\phi_{eH} d\phi_{eHA}\frac{q_A}{E_A}dq_A\frac{q_{H^+}^2}{E_{H^+}}dq_{H^+}q_e dq_e\frac{1}{|\mathbf{q_e} + \mathbf{q_{H^+}}|}$$
$$\times\theta(E_{\bar\nu}^0 - ||\mathbf{q_{H^+}} + \mathbf{q_e}| - q_A|)\theta(|\mathbf{q_e} + \mathbf{q_{H^+}}| + q_A - E_{\bar\nu}^0), \tag{24}$$

where

$$E_{\bar\nu}^0 = \sqrt{s_{e^+e^-}} - E_e - E_A - E_{H^+}. \tag{25}$$

The step functions in Eq. (24) imply phase space boundaries. Using Eq. (24), the differential cross section is:

$$\frac{d^7\sigma_{H^+A}}{dq_e dq_{H^+} dq_A d\cos\theta_e d\cos\theta_{eH} d\phi_e d\phi_{eHA}} \tag{26}$$

$$= \frac{g^4}{32c_W^2 s}\frac{1}{4096\pi^7}\left|\frac{1}{((p_e - q_e)^2 - M_Z^2)((p_{e^+} - q_{\bar\nu})^2 - M_W^2)}\right|^2$$
$$\times (T_{\mu\nu}S_{ee}^{\nu\sigma}T_{\sigma\rho}^t S_{e^+\bar\nu}^{\rho\mu} + T_{\mu\nu}A_{ee}^{\nu\sigma}T_{\sigma\rho}^t A_{e^+\bar\nu}^{\rho\mu})\frac{q_A}{E_A}\frac{q_{H^+}^2}{E_{H^+}}q_e\frac{1}{|\mathbf{q_e} + \mathbf{q_{H^+}}|}$$
$$\times\theta(E_{\bar\nu}^0 - ||\mathbf{q_{H^+}} + \mathbf{q_e}| - q_A|)\theta(|\mathbf{q_e} + \mathbf{q_{H^+}}| + q_A - E_{\bar\nu}^0). \tag{27}$$

We carry out the rest of the integration numerically.

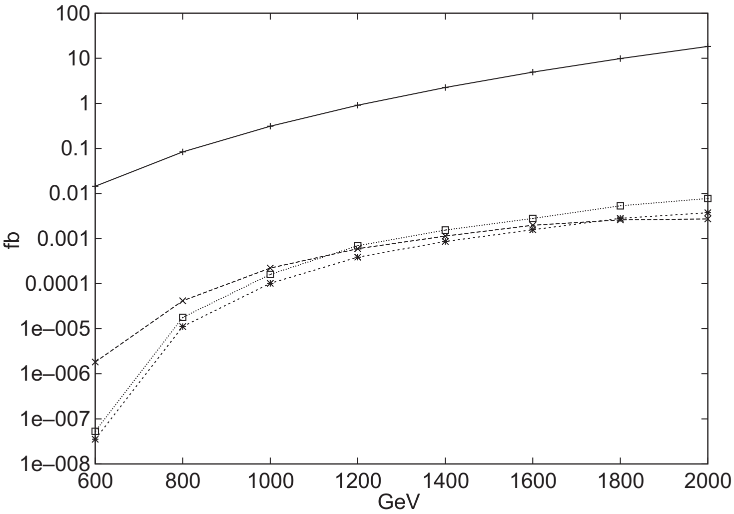

Fig. 6. The gauge boson pair production cross section (σ_{WZ}) for $e^+ + e^- \to W^+ + Z + \overline{\nu_e} + e^-$ (solid line) and the Higgs pair production cross sections (σ_{H^+A}) for $e^+ + e^- \to H^+ + A + \overline{\nu_e} + e^-$. The horizontal axis denotes center-of-mass energy, $\sqrt{s_{e^+e^-}}$ (GeV), of the e^+e^- collision. The long dashed line with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line with the boxes $\square$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line with asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV).

4. Numerical results

In this section, we present the numerical results for the cross sections. We have carried out the phase space integrations by using the Monte Carlo program, bases [11]. We have studied three sets of charged Higgs and neutral Higgs masses:

$$(m_{H^+}, m_A) = (300, 200), (200, 300), (200, 200) \text{ (GeV)}. \tag{28}$$

As shown in Ref. [9], for these input values of charged Higgs and neutral Higgs masses, the radiative corrections to the VEVs, β and v, are within 10%.

We show the total cross sections σ_{H^+A} with respect to the center-of-mass energy ($\sqrt{s_{e^+e^-}}$) of the e^+e^- collision in Fig. 6. Then we plot the following 1D differential cross sections in Figs. 7–11:

$$\Delta\sigma_{1H^+A}(q_e) = \int_{q_e-\frac{\Delta q_e}{2}}^{q_e+\frac{\Delta q_e}{2}} \frac{d\sigma_{H^+A}}{dq_e} dq_e, \quad \Delta q_e = 50 \text{ (GeV)}, \tag{29}$$

$$\Delta\sigma_{2H^+A}(q_{H^+}) = \int_{q_{H^+}-\frac{\Delta q_{H^+}}{2}}^{q_{H^+}+\frac{\Delta q_{H^+}}{2}} \frac{d\sigma_{H^+A}}{dq_{H^+}} dq_{H^+}, \quad \Delta q_{H^+} = 50 \text{ (GeV)}, \tag{30}$$

$$\Delta\sigma_{3H^+A}(\cos\theta_e) = \int_{\cos\theta_e-\frac{\Delta\cos\theta_e}{2}}^{\cos\theta_e+\frac{\Delta\cos\theta_e}{2}} \frac{d\sigma_{H^+A}}{d\cos\theta_e} d\cos\theta_e, \quad \Delta\cos\theta_e = 0.2, \tag{31}$$

$$\Delta\sigma_{4H^+A}(\cos\theta_{eH}) = \int_{\cos\theta_{eH}-\frac{\Delta\cos\theta_{eH}}{2}}^{\cos\theta_{eH}+\frac{\Delta\cos\theta_{eH}}{2}} \frac{d\sigma_{H^+A}}{d\cos\theta_{eH}} d\cos\theta_{eH}, \quad \Delta\cos\theta_{eH} = 0.2, \tag{32}$$

$$\Delta\sigma_{5H^+A}(\phi_{eH}) = \int_{\phi_{eH}-\frac{\Delta\phi_{eH}}{2}}^{\phi_{eH}+\frac{\Delta\phi_{eH}}{2}} \frac{d\sigma_{H^+A}}{d\phi_{eH}} d\phi_{eH}. \quad \Delta\phi_{eH} = \frac{\pi}{5}. \tag{33}$$

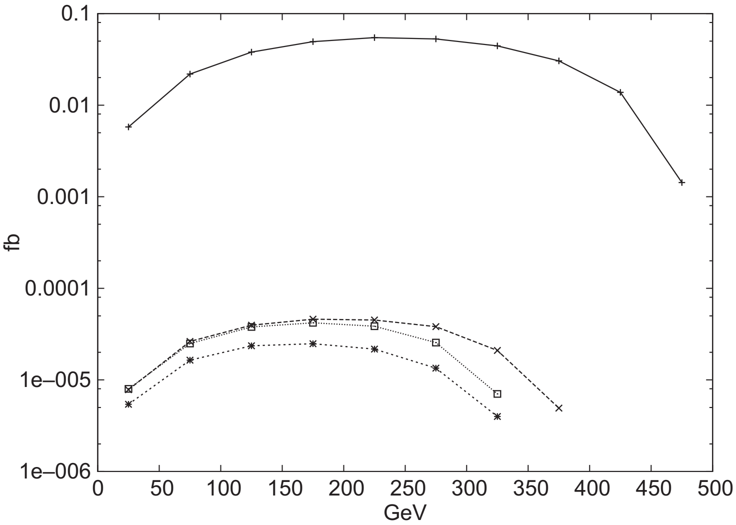

Fig. 7. The differential cross sections $\Delta\sigma_{1H^+A}$ and $\Delta\sigma_{1WZ}$ as functions of the momentum q_e (GeV) for the final state electron. We have chosen the width of each bin as $\Delta q_e = 50$ (GeV). The solid line marked with the plus sign $+$ corresponds to $e^+ + e^- \to W^+ + Z + \overline{\nu_e} + e^-$. The other lines denote the three cases for $e^+ + e^- \to H^+ + A + \overline{\nu_e} + e^-$. The long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center-of-mass energy is 1000 (GeV).

For comparison, we have also computed the gauge boson production cross section. We used the formulae in Ref. [12] for the $W + Z \to W + Z$ scattering amplitude:

$$\sigma_{WZ} \equiv \sigma_{SM}(e^+ + e^- \to \overline{\nu_e} + e^- + W^+ + Z). \tag{34}$$

We plot σ_{WZ} in Fig. 6 as well as the differential ones, $\Delta\sigma_{iWZ}$ ($i = 1$–5) for the weak gauge boson pair (W^+ and Z) production in the standard model; see Figs. 7–11. This can be a background process to Higgs pair production. Explicitly, we write the differential cross section $\Delta\sigma_{iWZ}$ ($i = 1$–5), which is defined analogous to those defined for the case of Higgs production in Eqs. (29)–(33):

$$\Delta\sigma_{1WZ}(q_e) = \int_{q_e-\frac{\Delta q_e}{2}}^{q_e+\frac{\Delta q_e}{2}} \frac{d\sigma_{WZ}}{dq_e} dq_e, \quad \Delta q_e = 50\,(\text{GeV}), \tag{35}$$

$$\Delta\sigma_{2WZ}(q_W) = \int_{q_W-\frac{\Delta q_W}{2}}^{q_W+\frac{\Delta q_W}{2}} \frac{d\sigma_{WZ}}{dq_W} dq_W, \quad \Delta q_W = 50\,(\text{GeV}), \tag{36}$$

$$\Delta\sigma_{3WZ}(\cos\theta_e) = \int_{\cos\theta_e-\frac{\Delta\cos\theta_e}{2}}^{\cos\theta_e+\frac{\Delta\cos\theta_e}{2}} \frac{d\sigma_{WZ}}{d\cos\theta_e} d\cos\theta_e, \quad \Delta\cos\theta_e = 0.2, \tag{37}$$

$$\Delta\sigma_{4WZ}(\cos\theta_{eW}) = \int_{\cos\theta_{eW}-\frac{\Delta\cos\theta_{eW}}{2}}^{\cos\theta_{eW}+\frac{\Delta\cos\theta_{eW}}{2}} \frac{d\sigma_{WZ}}{d\cos\theta_{eW}} d\cos\theta_{eW}, \quad \Delta\cos\theta_{eW} = 0.2, \tag{38}$$

$$\Delta\sigma_{5WZ}(\phi_{eW}) = \int_{\phi_{eW}-\frac{\Delta\phi_{eW}}{2}}^{\phi_{eW}+\frac{\Delta\phi_{eW}}{2}} \frac{d\sigma_{WZ}}{d\phi_{eW}} d\phi_{eW}, \quad \Delta\phi_{eW} = \frac{\pi}{5}. \tag{39}$$

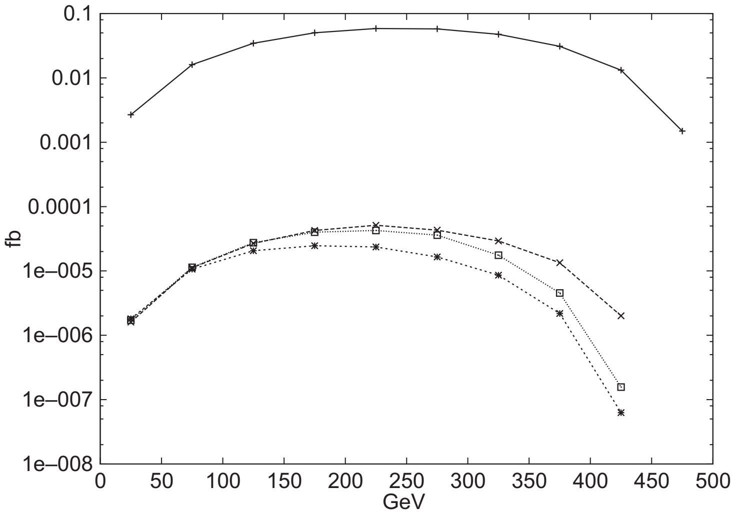

Fig. 8. The differential cross section $\Delta\sigma_{2H^+A}$ with respect to the charged Higgs momentum q_{H^+}. The horizontal axis denotes q_{H^+} (GeV). The long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center-of-mass energy is 1000 (GeV) and the width of each bin (Δq_{H^+}) is 50 (GeV). For comparison, we also show the solid line with the plus sign $+$ for the W, Z pair production cross section, $\Delta\sigma_{2WZ}$ as a function of the momentum of the W boson in the final state q_W (GeV). For the cross section, the horizontal axis denotes the W boson momentum.

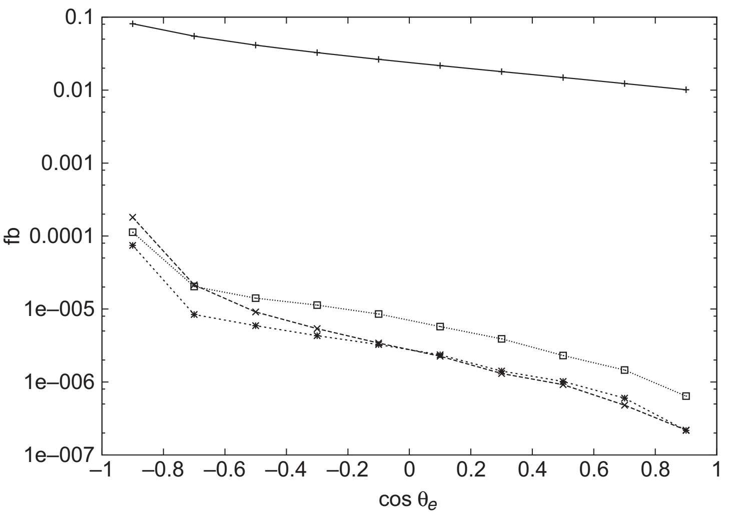

Fig. 9. The differential cross sections $\Delta\sigma_{3H^+A}$ for $e^+ + e^- \to H^+ + A + \overline{\nu_e} + e^-$ with respect to $\cos\theta_e$, where θ_e denotes the angle between the final electron momentum and the initial positron momentum. The long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center-of-mass energy is 1000 (GeV) and the width of each bin $(\Delta\cos\theta_e)$ is 0.2. For comparison, we show the cross section $\Delta\sigma_{3WZ}$ of the process $e^+ + e^- \to W^+ + Z + \overline{\nu_e} + e^-$ with a solid line. We use the formulae for the $W + Z \to W + Z$ scattering in Ref. [12]. The center-of-mass energy of the e^+e^- collision is 1000 (GeV).

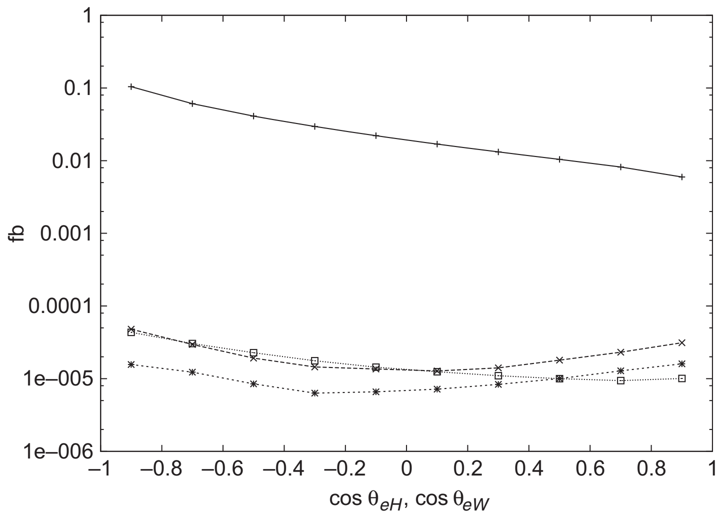

Fig. 10. Differential cross sections for $\Delta\sigma_{4H^+A}$ and $\Delta\sigma_{4WZ}$. The horizontal axis corresponds to $\cos\theta_{eH}$ and $\cos\theta_{eW}$. $\theta_{eH}(\theta_{eW})$ is the angle between the momentum of the final electron and that of the charged Higgs boson (W boson). The solid line marked with the plus sign $+$ corresponds to WZ production. The other three lines are Higgs pair production. Among them, the long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes $\square$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center-of-mass energy is 1000 (GeV) and the bin widths $\Delta\cos\theta_{eH}$ and $\Delta\cos\theta_{eW}$ are 0.2.

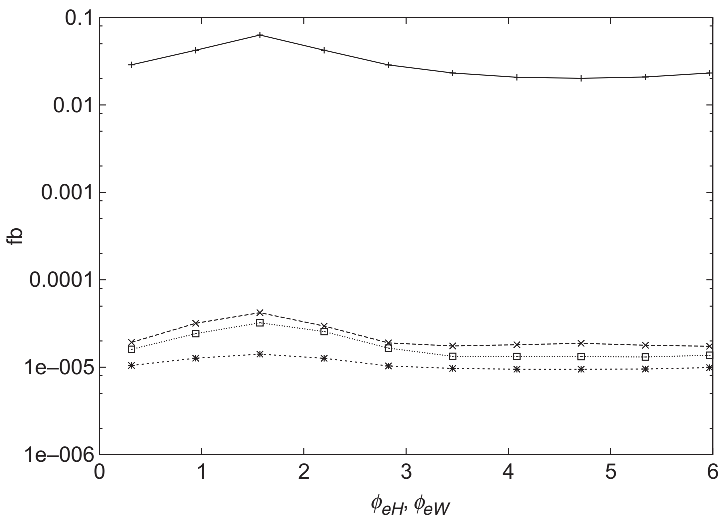

Fig. 11. Differential cross sections $\Delta\sigma_{5H^+A}$ and $\Delta\sigma_{5WZ}$. The horizontal line denotes the azimuthal angles ϕ_{eH} and ϕ_{eW} (radian). The solid line marked with the plus sign $+$ corresponds to WZ production. The other three lines are Higgs pair production. Among them, the long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes $\square$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center-of-mass energy is 1000 (GeV) and the bin widths $\Delta\phi_{eH}$ and $\Delta\phi_{eW}$ are $\frac{\pi}{5}$.

We summarize what one can read from these cross-section figures (Figs. 6–11) as follows:

- The total cross section for Higgs pair production σ_{H^+A} increases as the center-of-mass energy of the e^+e^- collision grows until it reaches to 2000 (GeV). Even in the case for the lightest Higgs pair masses that we have chosen, the cross section is at most 0.001 fb. Compared with gauge boson pair production σ_{WZ}, the ratio $\frac{\sigma_{H^+A}}{\sigma_{WZ}}$ is of the order of $\sim 10^{-3}$.
- The differential branching fractions with respect to the electron momentum in final states and with respect to the charged Higgs spectrum are limited by phase space and, for lighter Higgs pair masses, the momentum of the electron is larger.
- The distribution of the direction of the electron in the final states peaks strongly at $\cos\theta_e = -1$. This implies that the electron is scattered in the forward direction with respect to the incoming electron. This happens because the virtuality of the Z^* boson is minimized in this case.
- Regarding the azimuthal ϕ_{eH} angle distributions, we find that the charged Higgs momentum is more likely to lie within the range $0 \leq \phi_{eH} \leq \pi$ than in $\pi \leq \phi_{eH} < 2\pi$.

5. The signature of charged Higgs and neutral Higgs pair production

As we have seen from the studies of the previous section, the cross section and the differential cross sections of the Higgs pair production are much smaller than gauge boson pair production. Considering this smallness, one may wonder if such Higgs pair production and its decays have distinct signals. Here we consider the charged lepton flavor dependence of the charged Higgs decays into an anti-lepton and a neutrino. Note that the dominant neutral Higgs decay channel is a neutrino and anti-neutrino pair when the neutral Higgs and charged Higgs are degenerate as $|m_A - m_{H^+}| < m_W$. We study the degenerate case. In this case, the neutral Higgs decay products are invisible and the visible decay product is a charged anti-lepton l^+ from the charged Higgs decay. Therefore, the whole process starting from the e^+e^- collision to Higgs decays looks like:

$$\begin{aligned} e^+ + e^- &\to \overline{\nu_e} + e^- + H^+ + A \\ &\to \overline{\nu_e} + e^- + l^+\nu_l + \nu_k\overline{\nu}_k. \end{aligned} \tag{40}$$

One finds the same final state as in Eq. (40) in the gauge boson pair production process of the e^+e^- collision as follows. By replacing the charged Higgs boson with a W^+ boson and the neutral Higgs boson A with a Z boson in Eq. (40), the decay channels $Z \to \nu_k\overline{\nu_k}$ and $W^+ \to l^+\nu_l$ lead to the same final state as that of Eq. (40):

$$\begin{aligned} e^+ + e^- &\to \overline{\nu_e} + e^- + W^+ + Z \\ &\to \overline{\nu_e} + e^- + l^+\nu_l + \nu_k\overline{\nu_k}. \end{aligned} \tag{41}$$

Since Eq. (41) has a common final state with Eq. (40), they look indistinguishable. However, as pointed out in Ref. [3], the branching fraction of the charged Higgs decay into an anti-lepton is flavor non-universal and depends on the lepton family. It is written in terms of the neutrino mixings and masses, for which precise data, excluding the lightest neutrino mass and CP-violating phase, are now available. Since the W boson decay into an anti-lepton is flavor-blind, we study the lepton flavor dependence of charged Higgs decay by taking the ratio with the weak gauge boson pair production and decay branching fractions. The ratio we define is

$$r_l = \frac{\sum_{X=h,A}\sigma_{H^+X}\mathrm{Br}(X \to \nu\bar{\nu})}{\sigma_{WZ}\mathrm{Br}(Z \to \nu\bar{\nu})}\frac{\mathrm{Br}(H^+ \to l^+\nu_l)}{\mathrm{Br}(W^+ \to l^+\nu_l)}, \tag{42}$$

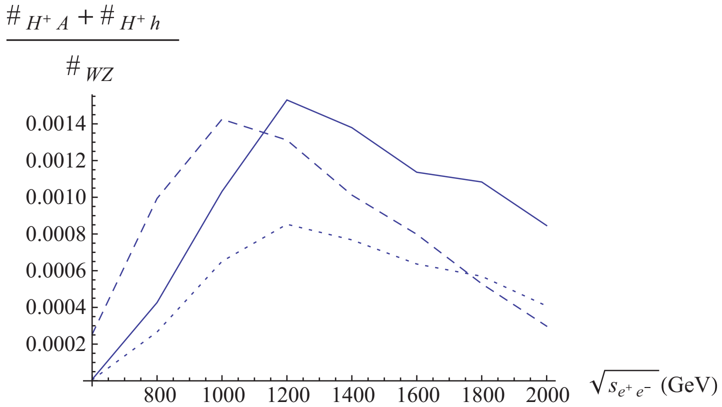

Fig. 12. The ratio of the cross sections of Higgs pair production and gauge boson pair production $\frac{\sigma_{H^+A}+\sigma_{H^+h}}{\sigma_{W^+Z}}$ as a function of the center-of-mass energy of the e^+e^- collision $\sqrt{s_{e^+e^-}}$ (GeV). The solid line corresponds to the case for $(m_{H^+}, m_A) = (300, 200)$ (GeV). The dashed line corresponds to the degenerate case, $m_A = m_{H^+} = 200$ (GeV). The dotted line corresponds to the case $(m_{H^+}, m_A) = (200, 300)$ (GeV).

where we use the shorthand notation $\mathrm{Br}(X \to \nu\bar{\nu}) = \sum_k \mathrm{Br}(X \to \nu_k\bar{\nu}_k)$ for $X = h, A, Z$. Using the notation, one can write r_l as

$$r_l = \frac{2\sigma_{H^+A}}{\sigma_{WZ}} \frac{\mathrm{Br}(A \to \nu\bar{\nu})}{\mathrm{Br}(Z \to \nu\bar{\nu})} \frac{\mathrm{Br}(H^+ \to l^+\nu_l)}{\mathrm{Br}(W^+ \to l^+\nu_l)}, \tag{43}$$

where we use the fact that the production cross sections for CP-even and CP-odd Higgs with $U(1)$ charge are almost identical to each other, i.e., $\sigma_{H^+A} \simeq \sigma_{H^+h}$ (see Appendix A). We also use the branching fractions that satisfy

$$\mathrm{Br}(A \to \nu\bar{\nu}) = \mathrm{Br}(h \to \nu\bar{\nu}) = 100\%. \tag{44}$$

We show the ratio of the cross sections in Fig. 12. When Higgs masses are degenerate, $m_A = m_{H^+} = 200$ (GeV), the ratio of the cross section is about 1.4×10^{-3} for $\sqrt{s_{e^+e^-}} = 1000$ (GeV). In what follows, we use this value as a benchmark point for the ratio of the cross sections in Eq. (43). The other branching fractions that appear in Eq. (43) are quoted from the Particle Data Group (PDG) [13]:

$$\begin{aligned} \mathrm{Br}(W^+ \to \tau^+\nu) &= 11.25 \pm 0.20\%, \\ \mathrm{Br}(W^+ \to \mu^+\nu) &= 10.57 \pm 0.15\%, \\ \mathrm{Br}(W^+ \to e^+\nu) &= 10.75 \pm 0.13\%, \\ \mathrm{Br}(Z \to \nu\bar{\nu}) &= 20.00 \pm 0.06\%. \end{aligned} \tag{45}$$

Using the numerical values, one can write $r_l (l = e, \mu, \tau)$ as:

$$\begin{aligned} r_e &= 0.465 \times \mathrm{Br}(H^+ \to e^+\nu)\frac{2\sigma_{H^+A}}{\sigma_{WZ}}, \\ r_\mu &= 0.473 \times \mathrm{Br}(H^+ \to \mu^+\nu)\frac{2\sigma_{H^+A}}{\sigma_{WZ}}, \\ r_\tau &= 0.444 \times \mathrm{Br}(H^+ \to \tau^+\nu)\frac{2\sigma_{H^+A}}{\sigma_{WZ}}, \end{aligned} \tag{46}$$

where $\mathrm{Br}(H^+ \to l\nu)$ in % should be substituted. The charged Higgs can decay into charged leptons and a neutrino. In contrast to the leptonic decay of the W boson, the branching fractions for each

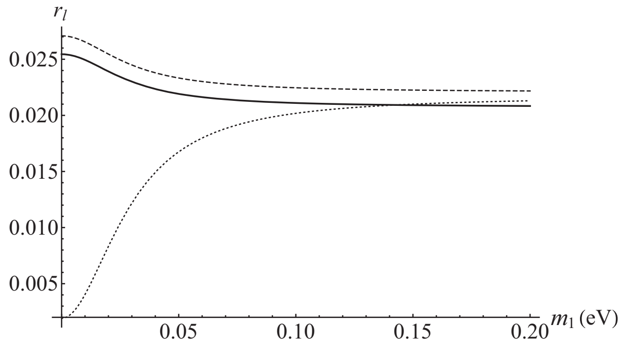

Fig. 13. r_l ($l = e, \mu, \tau$) for the normal hierarchical case as functions of the lightest neutrino mass m_1 (eV). The dotted line corresponds to r_e, the dashed line corresponds to r_μ, and the solid line corresponds to r_τ.

flavor of charged lepton are obtained from Eq. (6) [3]:

$$\mathrm{Br}(H^+ \to l^+ \nu_l) = \frac{\sum_{i=1}^{3} m_i^2 |V_{li}|^2}{\sum_{i=1}^{3} m_i^2} \times 100\%. \tag{47}$$

We update the branching fraction to each lepton flavor mode using the recent results on $|V_{e3}|$. For the normal hierarchy case, the branching fractions are written as:

$$\mathrm{Br}(H^+ \to l^+ \nu_l) = \frac{m_1^2 + \Delta m_{sol}^2 |V_{l2}|^2 + (\Delta m_{sol}^2 + \Delta m_{atm}^2)|V_{l3}|^2}{3m_1^2 + 2\Delta m_{sol}^2 + \Delta m_{atm}^2} \times 100\%. \tag{48}$$

In the formulae of Eq. (48), m_1 denotes the lightest neutrino mass. For the inverted hierarchical case, they are written as:

$$\mathrm{Br}(H^+ \to l^+ \nu_l) = \frac{m_3^2 + \Delta m_{atm}^2 (|V_{l1}|^2 + |V_{l2}|^2) - \Delta m_{sol}^2 |V_{l1}|^2}{3m_3^2 + 2\Delta m_{atm}^2 - \Delta m_{sol}^2} \times 100\%, \tag{49}$$

where m_3 denotes the lightest neutrino mass. We have used the following values for the mixing angles and mass-squared differences quoted from Table 13.7 in Sect. 13 of Neutrino Mass, Mixing, and Oscillation of Ref. [13]: $\sin^2\theta_{12} = 0.306$, $\sin^2\theta_{23} = 0.42$, $\sin^2\theta_{13} = 0.021$, $m_{atm}^2 = 2.35 \times 10^{-3}$ (eV^2), and $m_{sol}^2 = 7.58 \times 10^{-5}$ (eV^2). The subscripts 'sol' and 'atm' for the mass squared differences imply solar neutrinos and atmospheric neutrinos respectively. In Fig. 13, we show r_l ($l = e, \mu, \tau$) for the normal hierarchical case as functions of the lightest neutrino mass m_1. In Fig. 14, we show r_l for the inverted hierarchical case as functions of the lightest neutrino mass m_3. As we can see from Figs. 13 and 14, we can expect 2–3% lepton flavor dependence from charged Higgs decay. We summarize the flavor dependence as follows:

- For the normal hierarchical case, for $0 \le m_1 < 0.05$ (eV), $r_\mu > r_\tau \gg r_e$. For larger m_1 up to 0.2 eV, $r_\mu \sim r_e \sim r_\tau = 0.02$.
- For the inverted hierarchical case, $r_e > r_\mu > r_\tau$ for $0 < m_3 < 0.2$ eV.

6. Conclusions and discussions

In this paper, we study the pair production of charged Higgs and neutral Higgs bosons in the neutrinophilic two-Higgs-doublet model. The pair production process is not suppressed by the U(1) charge conservation. In other words, the approximate global symmetry allows the pair production to occur.

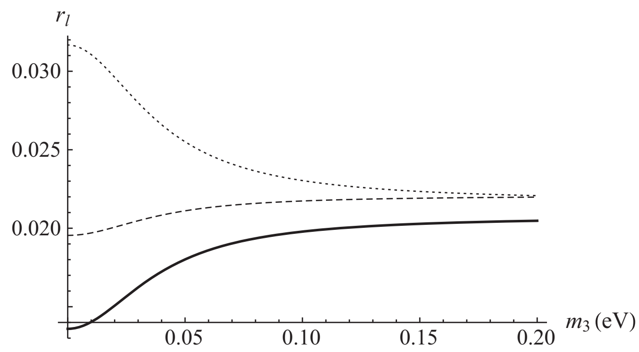

Fig. 14. r_l ($l = e, \mu, \tau$) for the inverted hierarchical case as functions of the lightest neutrino mass m_3 (eV). The dotted line corresponds to r_e, the dashed line corresponds to r_μ, and the solid line corresponds to r_τ.

We study the total cross section for the pair production in an e^+e^- collision. The pair production occurs through W boson and Z boson fusion. We study the pair production and the decays for degenerate masses of charged Higgs and neutral Higgs as well as the non-degenerate case. The cross section increases from 10^{-4} fb to 10^{-3} fb as the cm energy of e^+e^- varies from 1 (TeV) to 2 (TeV). The cross section is compared with that of W, Z pair production. We show that the Higgs pair production is about 10^{-3} times smaller than the pair production cross section of gauge bosons. Therefore, if Z decays invisibly into neutrino pairs and the W boson decays into an anti-lepton and a neutrino, the gauge boson pair production and its decays become a background to the signal. When the charged Higgs (H^+) and neutral Higgs ($X = A, h$) are degenerate as $|m_{H^+} - m_X| < M_W$, which is favored from the electroweak precision data, the charged Higgs dominantly decays into an anti-lepton and a neutrino and the neutral Higgs dominantly decays into a neutrino and anti-neutrino pair. Compared with these, the W and Z decay branching ratio in the same final state is smaller than that of Higgs decays and is flavor-blind. Therefore, by studying the charged anti-lepton flavor in the final state, we may distinguish the Higgs pair production and its decays from that of gauge bosons. We expect 2–3% flavor dependence, which is null for the gauge boson decays. Depending on the normal or inverted hierarchy of the mass spectrum of neutrinos, the order of r_e, r_μ, and r_τ changes. We show the differential cross sections with respect to the electron and charged Higgs momenta. The differential cross sections with respect to the angles of the electron and the charged Higgs in the final states are also shown. These are also important in identifying the signals.

Appendix. Amplitude of $W^{+*} + Z^* \to H^+ + h$

In this appendix, we show the off-shell charged Higgs and CP-even neutral Higgs (h) boson production amplitude for gauge boson fusion $W^{+*} + Z^* \to H^+ + h$:

$$T_{h\mu\nu} = \frac{g^2 \cos(\beta + \gamma)}{2 \cos \theta_W} \left(a_h g_{\mu\nu} + d_h q_{h\nu} q_{H^+\mu} + b_h q_{H^+\nu} q_{h\mu}\right), \tag{A1}$$

where we compute the four Feynman diagrams corresponding to the contact interaction (Fig. 2), the S channel W^+ exchange (Fig. 3), the U channel charged Higgs exchange (Fig. 4), and the T channel

CP-odd Higgs (A) exchange (Fig. 5). a_h, b_h, and d_h in Eq. (A1) are given as:

$$\begin{aligned} a_h &= -s_W^2 - \frac{p_Z^2 - p_W^2}{M_z^2}\frac{M_h^2 - M_{H^+}^2 - M_W^2}{s_{H^+h} - M_W^2} - c_W^2 \frac{t_h - u_h + p_Z^2 - p_W^2}{s_{H^+h} - M_W^2}, \\ b_h &= \frac{2\cos 2\theta_W}{u_h - M_{H^+}^2} + \frac{2(\cos 2\theta_W + 1)}{s_{H^+h} - M_W^2}, \\ d_h &= -\frac{2}{t_h - M_A^2} - \frac{2(\cos 2\theta_W + 1)}{s_{H^+h} - M_W^2}, \end{aligned} \tag{A2}$$

with $t_h = (q_{H^+} - p_W)^2$, $u_h = (p_W - q_h)^2$, and $s_{H^+h} = (q_{H^+} + q_h)^2$. By taking the vanishing limit of the U(1) breaking term, i.e., $m_{12} \to 0$, β and γ vanish. Note also that, in this limit, one can show $m_h = m_A$ and $-iT_{A\mu\nu} = T_{h\mu\nu}$ with the appropriate replacement $q_A \to q_h$ (see Eq. (10)). Therefore, in this limit, the production amplitudes for H^+A and H^+h are identical to each other, $\sigma_{H^+A} = \sigma_{H^+h}$.

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References

[1] T. Behnke et al., arXiv:1306.6327 [physics.acc-ph].
[2] H. Baer et al., arXiv:1306.6352 [hep-ph].
[3] S. M. Davidson and H. E. Logan, Phys. Rev. D **80**, 095008 (2009).
[4] S. M. Davidson and H. E. Logan, Phys. Rev. D **82**, 115031 (2010).
[5] T. Figy, Mod. Phys. Lett. A **23**, 1961 (2008).
[6] M. J. Dolan, C. Englert, and M. Spannowsky, Phys. Rev. D **87**, 055002 (2013).
[7] A. Papaefstathiou, L. L. Yang, and J. Zurita, Phys. Rev. D **87**, 011301 (2013).
[8] F. Goertz, A. Papaefstathiou, L. L. Yang, and J. Zurita, J. High Energy Phys. **1306**, 016 (2013).
[9] T. Morozumi, H. Takata, and K. Tamai, Phys. Rev. D **85**, 055002 (2012).
[10] T. Morozumi and K. Tamai, arXiv:1212.2138 [hep-ph].
[11] S. Kawabata, Comput. Phys. Commun. **88**, 309 (1995).
[12] T. Bahnik, arXiv:9710265 [hep-ph].
[13] J. Beringer et al., Phys. Rev. D **86**, 010001 (2012).