An Explanation of Asymmetric Effects of Inflation Targeting*

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Abstract

Matsukawa, Okamura and Taki (2012) show that in a neoclassical Phillips curve model, two types of discretionary equilibria can be characterized as a deflationary equilibrium and a high-inflation equilibrium and the absolute value of feedback coefficient on the lagged output gap is larger for the former. This paper points out that inflation targeting can be supported by activist monetary policy in bringing inflation down, if the economy is in a high-inflation equilibrium, but not in getting out of the deflationary trap if the economy is in a deflationary equilibrium.

Ⅰ. Introduction

The inflation-targeting countries achieved success in lowering inflation from high levels, but the Bank of Japan had not adopted inflation targeting to escape from deflation before 2013. The purpose of this paper is to show that inflation targeting has asymmetric effects, depending on whether it is adopted to lower the inflation rate or to escape from a deflationary trap. In the latter case, it is not effective in bringing inflation up because active monetary policy cannot support it.

Inflation targeting is thought of as rule rather than discretion because it is a kind of commitment strategy which influences inflation expectations. A central bank under inflation-targeting regime, however, normally adjusts monetary policy more actively than before its adoption to enhance the outcome. This implies that inflation targeting is in fact accompanied by more active monetary policy than that adopted in the 1970s. Advocates of policy rules tend to overlook the policy measures accompanying inflation targeting, i.e., the comprehensive activist policy package. As a result, even though activist policies may have a greater effect on the outcome, it is interpreted as the result of inflation targeting, treated as a rule.

The failure of activist monetary policies in the late 1970s encouraged advocates of policy rules in the view that activist policies can lead the economy away from the optimal path. The model used in the pioneering work of Barro and Gordon (1983) (hereafter BG) explicitly demonstrated the superiority of the "rule" over "discretion," in terms of the social loss function. But, there is a contradiction between the prediction of BG model and the success of inflation targeting because more active policies accompanying inflation targeting resulted in lower rather than higher inflation rates.

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Lower inflation rates also arise in the kind of flexible approach to policymaking that has been characteristic of the Greenspan era (1987-2006), which has resulted in decreased price inflation and the volatility of output and employment.[1] How can economists specify the difference between the discretion in the Greenspan era and activist monetary policy before 1979 (at which Fed instituted a radical shift in policy)? Clarida et al. (2000) and Taylor (1999) claimed that the interest rate rule adopted in the Greenspan era can be characterized by an inflation coefficient greater than unity; see also Woodford (2003, Chapter 1). The responsiveness of the nominal interest rate to the inflation rate as well as the inclusion of the output gap in the Taylor rule can be viewed as a reflection of discretion and the difference in inflation coefficients implies that monetary policy in the Greenspan era is more active than activist monetary policy before 1979. In both inflation targeting and monetary policy in the Greenspan era (Taylor rule), more active policies, which we called G-active monetary policies in Matsukawa, Okamura and Taki (2012), resulted in better economic outcomes than the active policies before 1979, which we called conventionally active, or C-active monetary policies. One of the main purposes of Matsukawa, Okamura and Taki (2012) is to demonstrate the possibility of the prevalence of zero-inflation or deflation when the policy maker is G-active. This critical aspect contradicts the results commonly accepted in the context of the "rule versus discretion" literature; that is, activist policies result in high inflation, and therefore, a G-active policy maker would produce a hyperinflationary economy. To analyze the macroeconomic consequences of monetary policy regimes, this paper employs a simple model, in which monetary policy takes the form of inflation-rate feedback rules, instead of interest-rate feedback rules.[2] This simple model can capture the above-mentioned aspect. The differences between the expectations formation process and the timing of moves assumed in this model and those of the previous literature are essential for this purpose.

The rest of the paper is organized as follows. Section 2 describes the basic model and explains the expectations formation process and the relative timing of moves assumed in this paper. Section 3 shows that there are two types of discretionary equilibria－a deflationary and a high-inflationary equilibrium. It also analyzes the social optimum under commitment and rational expectations equilibrium. Section 4 shows asymmetric effects of inflation targeting. If inflation targeting is adopted in a deflationary equilibrium, it cannot be supported by activist monetary policy and cannot succeed in escaping from deflation. In contrast, if it is adopted in a high-inflationary equilibrium, activist monetary policy supports it. Finally, Section 5 concludes the paper.

Ⅱ. The Model

Since the model in this paper is the same as that presented in Matsukawa, Okamura and Taki (2012),

[1] It is still an unresolved issue to what extent the nonmechanical monetary policy of the Greenspan era was responsible for the favorable economic outcomes that ensued (Friedman (2006)). Blanchard and Simon (2001) pointed out that the secular decrease in output volatility, rather than monetary policy during the Greenspan era, accounts for the two long expansions of the US economy since the early 1980s. Orphanides (2004) shows that the responsiveness of the nominal interest rate to the inflation rate was also greater than unity before 1979.

[2] We have abstracted from a feedback from endogenous target variables to the short-term nominal interest rate that would occur if the government actually implemented the policy scheme.

this section briefly describes its framework and compares it to the model of Dittmar, Gavin, and Kydland (1999) (hereafter DGK). The model's five key elements are: a loss function that the central bank should use to evaluate alternative paths for the economy, an aggregate supply relation, expectations formation, a feedback rule for the inflation rate, and an assumption on the timing of moves.

In contrast to the DGK model, the public's expectations formation is based on the perceived decision rule of the central bank, as is the case in a linear-quadratic dynamic game. The timing of moves is also altered relative to the DGK model so that the central bank considers the public's expectations formation (not the expected inflation rate) when choosing a feedback rule. In other respects, the model is similar to the models used in a number of analyses of inflation targeting, including Svensson (1997, 1999), DGK, and Dittmar and Gavin (2000). The following table reports the full list of variables included for each of the equations.

Table

$0<\beta<1$	*discount rate*
y_τ	*output deviation from its natural rate (in logarithm) in period* τ
π_τ	*the rate of inflation in period* τ
$\bar{y}$	*the optimal level of the output gap*
π^*	*the optimal inflation rate*
$0\le\lambda\le1$	*the weight on output gap*
η_τ	*an iid technology shock*
σ_τ^2	*the variance in* τ
ρ	*the degree of output persistence*
α	*the slope of the short-run Phillips curve*
π_τ^e	*the public's expectation of* π *in period* τ *conditional on the information available in period* $\tau-1$
(A_1^e, A_2^e)	*the feedback coefficients anticipated by the public*
(A_1, A_2)	*the feedback coefficients chosen by the central bank*

The Central Bank's Intertemporal Loss Function

(1) $L_t=\sum_{\tau=t}^{\infty}\beta^{\tau-t}\ E_t\{\lambda(y_\tau-\bar{y})^2+(1-\lambda)(\pi_\tau-\pi^*)^2\}$,

The Short-Run Phillips Curve

(2) $y_\tau=\rho y_{\tau-1}+\alpha(\pi_\tau-\pi_\tau^e)+\eta_\tau$.

Neither the central bank nor the public can observe current realizations of η_τ.[3]

Expectations Formation of the Public

(3) $\pi_\tau=A_1^e+A_2^e y_{\tau-1}$,

[3] DGK specifies the information set of the central bank, which differs from that of the private sector. They assume that current realizations of supply shocks are in the information set of the central bank at the time it makes policy decisions, but not in that of the private sector (see Okamura, Matsukawa and Taki (2006)). In the present model, however, the central bank's information set does not include current realizations of supply shocks.

The Public's Expectation of π_τ in Period τ

(4) $\pi_\tau^e = A_1^e + A_2^e y_{\tau-1}$,

for every realized value of $y_{\tau-1}$.

The Central Bank's Reaction Function

Substituting (3) into (2) gives

(5) $y_\tau = -\alpha A_1^e + (\rho - \alpha A_2^e) y_{\tau-1} + \alpha \pi_\tau + \eta_\tau$.

The central bank's reaction function (her strategy) is assumed to take the form,

(6) $\pi_\tau = A_1 + A_2 y_{\tau-1}$.

The Timing of Moves

The last element of the model is the timing of moves. Here, we take advantage of game theory structure to discuss the interaction between the public's expectations formation and the central bank's choice of a feedback rule; the central bank chooses its strategy (feedback rule) *after* the public expects the inflation rate rule. In other words, the central bank considers the public's expectations formation when choosing a feedback rule, and in this sense, the central bank will always have discretion in adjusting its instrument.

Comparison to the Rational Expectations Model

Before turning to a discretionary regime, it is useful to consider a rational expectations equilibrium and the social optimum under commitment. Rational expectations hypothesis implies that the process of expectations formation can be described using the same feedback rule as that chosen by the central bank. Therefore,

(7) $\pi_\tau^e = A_1 + A_2 y_{\tau-1}$.

Rational expectations equilibrium is then defined simply by satisfaction of conditions (1), (2), and (6), along with $\pi_\tau^e = E_\tau \pi_\tau$. The calculations in DGK can be used to obtain the feedback coefficients chosen by the central bank in the rational expectations equilibrium, $(A_1^\#, A_2^\#)$:

(8) $A_1^\# = \dfrac{\alpha\lambda\bar{y}}{(1-\lambda)(1-\rho\beta)} + \pi^*$, $A_2^\# = \dfrac{-\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}$.

Note that in our model, (7) also holds in discretion equilibria, but $A_1 = A_1^e$ and $A_2 = A_2^e$ are equilibrium conditions and not assumptions.

The Social Optimum under Commitment

Next, consider the optimal reaction function under commitment. Suppose that the central bank is committed *ex ante* to a particular feedback rule. In other words, commitment to particular values of feedback coefficients (A_1, A_2) is assumed to be possible. As the central bank is able to affect private agents' expectations of its feedback coefficients under commitment, it follows that

$$A_1^e = A_1, \quad A_2^e = A_2.$$

Thus, we can write DGK's Lucas supply function as

(9) $y_\tau = \rho y_{\tau-1} + \eta_\tau$.

Substituting $y_\tau = \rho y_{\tau-1} + \eta_\tau$ and $\pi_\tau = A_1 + A_2 y_{\tau-1}$ for all $\tau \geq t$ into the intertemporal loss function (1), we obtain

$$(10)\quad L = \lambda[\frac{\beta\rho^2 y_{t-1}^2}{1-\beta\rho^2} - \frac{2\rho\bar{y}y_{t-1}}{1-\beta\rho} + \frac{\bar{y}^2}{1-\beta} + \frac{\sigma_\eta^2}{(1-\beta)(1-\beta\rho^2)}]$$

$$+(1-\lambda)[\frac{(A_1-\pi^*)^2}{1-\beta} + \frac{2(A_1-\pi^*)A_2 y_{t-1}}{1-\beta\rho} + \frac{A_2^2 y_{t-1}^2}{1-\beta\rho^2} + \frac{\beta A_2^2}{(1-\beta)(1-\beta\rho^2)}\sigma_\eta^2],$$

where σ_η^2 represents the variance of η_τ. Section A of Matsukawa, Okamura and Taki (2007)[4], presents details of the solution procedure.

The central bank's problem is to minimize (10) with respect to A_1 and A_2,where the minimization is subject to a given initial condition for y_{t-1}. Considering the results of BG, it is a reasonable conjecture that the feedback coefficients associated with the minimum of the central bank's loss function, denoted by (A_1^R, A_2^R), are:

$$(11)\quad A_1^R = \pi^*, \qquad A_2^R = 0,$$

which we call strict inflation targeting. Section B of Matsukawa Okamura and Taki (2007) present the proof of this result.

Since monetary policy cannot affect long-run average output in a world consistent with the natural-rate hypothesis, it is reasonable that the optimum rule involves setting (11).

Ⅳ. Discretionary Equilibria under G-Active and C-Active Monetary Policies

Since details of the solution procedure are presented in Appendix A, this section briefly summarizes the methodology to construct discretionary equilibria.

The Central Bank's Best Response

In a discretionary regime, the central bank chooses the optimum feedback rule, given the public's expectations of feedback coefficients. Using the first-order conditions, and eliminating the Lagrangian multiplier attached to constraint (5), we obtain an Euler equation of the form:

$$(12)\quad \lambda(y_\tau - \bar{y}) + \frac{(1-\lambda)(\pi_\tau - \pi^*)}{\alpha} - \frac{\beta(1-\lambda)(\rho - \alpha A_2^e)}{\alpha} E_\tau(\pi_{\tau+1} - \pi^*) = 0.$$

Substituting (5) and (6) for time $\tau + 1$ into (12), and equating coefficients, we obtain two constraints on the central bank's feedback coefficients A_1 and A_2:

$$(13)\quad A_2^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho - \alpha A_2^e)^2}{\alpha\beta(1-\lambda)(\rho - \alpha A_2^e)} A_2 - \frac{\lambda}{\beta(1-\lambda)} = 0,$$

[4] Matsukawa, Okamura and Taki (2007) is available on request.

(14) $A_1 = [\lambda\alpha^2 + (1-\lambda)\{1-\beta(\rho-\alpha A_2^e)(1+\alpha A_2)\}]^{-1}$

$$[\lambda\alpha(\alpha A_1^e + \bar{y}) + (1-\lambda)\pi^* - \beta(1-\lambda)(\rho-\alpha A_2^e)(\alpha A_1^e A_2 + \pi^*)],$$

which can be used in deriving the central bank's best-response function $(A_1, A_2) = B(A_1^e, A_2^e)$.

<u>Constructing Discretionary Equilibria</u>

Discretionary equilibria of the model, denoted by (A_1^D, A_2^D), are fixed points of the central bank's best-response function, i.e., $(A_1^D, A_2^D) = B(A_1^D, A_2^D)$, or equivalently,

(15) $(1-\lambda)(A_2^D)^2 + \dfrac{(1-\lambda)(1-\beta\rho^2)}{\alpha\beta\rho} A_2^D + \dfrac{\lambda}{\beta} = 0$,

(16) $A_1^D = \dfrac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}} + \pi^*$.

Matsukawa, Okamura and Taki (2012) pointed out that given the values for ρ, α, and β, the discriminant of (15), denoted by $\phi(\lambda, \rho, \alpha, \beta)$, changes sign from negative to positive as λ falls below some critical value ($\dfrac{\partial\phi}{\partial\lambda} < 0$, for $0 < \lambda < 1$). Furthermore, with a positive determinant ($\phi(\lambda, \rho, \alpha, \beta) \geq 0$), the quadratic equation (15) has two distinct negative roots, by Viète's formulas. That is, as long as the central bank does not place too much weight on output stability, there exist two discretionary equilibria.

<u>Two Types of Discretionary Equilibria</u>

Let A_2^{**} be the one with greater absolute value and A_2^* be the one with smaller absolute value. Denote the intercept associated with A_2^{**} and A_2^* as A_1^{**} and A_1^*. As in Matsukawa, Okamura and Taki (2012), we call (A_1^{**}, A_2^{**}) Greenspan-type active or G-active monetary policies and the resulting equilibrium is called the G-active equilibrium. Meanwhile, the discretionary monetary policy represented by (A_1^*, A_2^*) and the resulting equilibrium are called conventionally active or C-active monetary policy, and the C-active equilibrium.

In the case that $|A_2^{**}|$ is large enough to make the denominator of (16) negative, G-active monetary policy can bring about deflation, rather than high inflation in a discretionary equilibrium. In contrast, the resulting inflation rate in the C-active equilibrium is higher than that in the social optimum. Furthermore, as Matsukawa, Okamura and Taki (2012) pointed out, it can be shown that $0 < A_2^{\#} < A_2^*$ (see Appendix B).

<u>The Stability of Equilibria</u>

Differentiating (13) and (14) with respect to A_2^e and A_1^e, and evaluating them in the discretionary equlibria, we have: $\left.\dfrac{dA_2}{dA_2^e}\right|_{A_2^e = A_2^D} = -\dfrac{(1+\beta\rho^2)\alpha A_2^D}{(1-\beta^2\rho)(\rho-\alpha A_2^D)}$ and $\dfrac{\partial A_1}{\partial A_1^e} = \dfrac{\lambda\alpha^2 - \beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D}{\lambda\alpha^2 - \beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D + (1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}}$.

Since $A_2^{**} < -\dfrac{1-\beta\rho^2}{2\alpha\beta\rho}$ for a G-active equilibrium, we have: $0 < (1-\beta^2\rho)(\rho-\alpha A_2^{**}) < -(1+\beta\rho^2)\alpha A_2^{**}$, and $\left.\dfrac{dA_2}{dA_2^e}\right|_{A_2 = A_2^{**}} > 1$. For a C-active equilibrium, it holds that $A_2^* > -\dfrac{1-\beta\rho^2}{2\alpha\beta\rho}$. Then, we obtain: $(1-\beta^2\rho)(\rho-\alpha A_2^*) > -(1+\beta\rho^2)\alpha A_2^*$, and $0 < \left.\dfrac{dA_2}{dA_2^e}\right|_{A_2 = A_2^*} < 1$. Matsukawa, Okamura and Taki (2012) explore the implications of these results and illustrated a graphic example of discretionary equilibria with different stability properties. It is also straightforward to show: $\left.\dfrac{\partial A_1}{\partial A_1^e}\right|_{A_1 = A_1^{**},\, A_2 = A_2^{**}} > 1$, in a G-active equilibrium, and $0 < \left.\dfrac{\partial A_1}{\partial A_1^e}\right|_{A_1 = A_1^{**},\, A_2 = A_2^{**}} < 1$, in a C-active equilibrium[5]. For details, see Appendix C or Section F of Matsukawa, Okamura and Taki (2007).

V. Inflation Targeting and Activist Monetary Policy

Consider the shift of the central bank's loss function caused by changes in π^* or $\bar{y}$. An example is an increase in π^* that allows more room for the central bank to lower the real interest rate in an economy with deflationary bias. Another example is a decrease in $\bar{y}$ that reflects the recognition of long-run increases in natural unemployment rate.

Returning to the central bank's best-response function $(A_1, A_2) = B(A_1^e, A_2^e)$, the feedback coefficients, A_1 and A_2 are obtained from the recursive equations (13) and (14). These equations are recursive in the sense that A_2 affects A_1 via (14), but A_2 is determined exclusively by (13), in which A_1 is not included. Furthermore, the feedback coefficient A_1 given by (14) shifts with changes in π^* and $\bar{y}$. In contrast, the feedback coefficient, A_2 implicitly given by (13) remains unchanged because it depends neither on these variables nor on A_1.

The central bank tries to move from the initial discretionary equilibrium to the new equilibrium as soon as possible, but the public does not believe in the policy change at first. In this situation, an inflation targeting regime can be adopted to lead to a new discretionary equilibrium more quickly. In other words, the central bank attains the new equilibrium more quickly by shifting the public's expectations to the new equilibrium rate of inflation given by (16). Since the central bank's feedback coefficient chosen in the discretionary equilibrium A_2^D does not depend on π^*, $\bar{y}$, and A_1^D, even after adopting inflation targeting, the value of A_2^D remains the same as far as the central bank faces the same Phillips curve. Notice that unlike the case of the optimum with commitment (strict inflation targeting), the target inflation rate in the present context is not always equal to the optimum rate of inflation, π^* [6].

Now compare the two cases. In the first case, suppose that the economy is in a G-active equilibrium with deflationary bias, denoted by two asterisks (**). The central bank adopts inflation targeting to increase inflation expectations, and chooses a higher value of A_1. As stated above, the feedback coefficient A_2^{**} remains the same as far as the public continues to anticipate A_2^{**}. However, in this situation, A_1^e does not always catch up with the increase in A_1 immediately, because the public tends to expect a lower rate of inflation than the target level announced by the central bank: $A_1^e < A_1 < 0$. Since, $\frac{\partial A_1}{\partial A_1^e} > 1$ in a G-active equilibrium (see the previous section), the central bank's best response is to choose a lower rate of inflation than that expected by the public; thus, the inflation-targeting regime will break down, as is illustrated in Figure I where π^* is raised from 0.01 to 0.03 but the new G-active equilibrium, $(A_1^{**}, A_2^{**}) = (.0048, -2.6608)$ is unstable and unattainable.

In the second case, suppose that the economy is in a C-active equilibrium, denoted by an asterisk (*). Faced with high inflation, the central bank adopts inflation targeting to decrease inflation expectations and chooses a lower value of A_1. In this case, the public tends to expect a higher inflation rate than that

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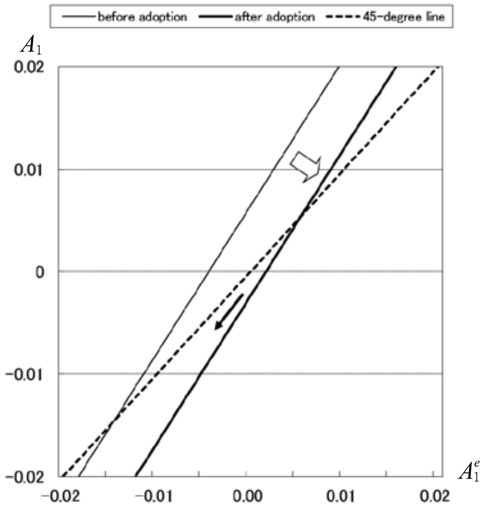
5 Our concept of stability differs from that of E-stability proposed by Evans and Honkapohja (2001). Rational expectations equilibrium is E-stable if the REE is locally asymptotically stable under the mapping from the perceived law of motion to the actual law of motion.

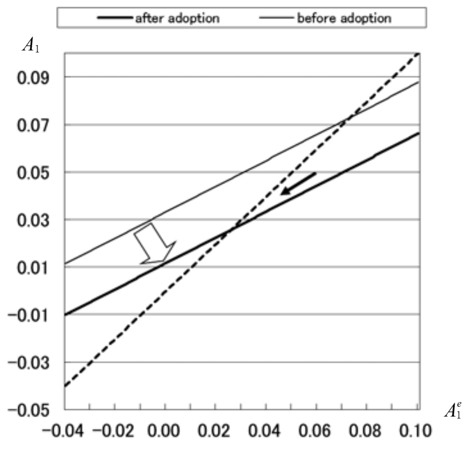
6 This is true as long as $\bar{y}$ differs from zero.

Figure Ⅰ The Central Bank's Best Response Before and After Adoption of Inflation Targeting
The Economy in a T-active Equilibrium
π^* is changed from 0.01 to 0.03
α =0.5, β = .99, ρ =0.5, λ =0.5



announced by the central bank. Since $0 < \frac{\partial A_1}{\partial A_1^e} < 1$ in a C-active equilibrium, the central bank's best response is to choose a lower inflation rate than that anticipated by the public. The economy approaches a new C-active equilibrium and activist monetary policy can support an inflation-targeting regime. Therefore, it succeeds in bringing inflation down, as is illustrated in Figure Ⅱ where the decrease in $\bar{y}$ from 0.04 to 0.01 lowers the equilibrium rate of inflation from .07308 to .01577. In this case, it is concluded that inflation targeting is effective in lowering inflation.

Figure Ⅱ The Central Bank's Best Response Before and After Adoption of Inflation Targeting
The Economy in a C-active Equilibrium
$\bar{y}$ is changed from 0.04 to 0.01
α =0.5, β = .99, ρ =0.5, λ =0.5



In summary, we have:

Theorem: If inflation targeting is adopted to lower the rate of inflation, it can be supported by activist monetary policy. In contrast, if it is adopted to escape from a deflationary trap, activist monetary policy cannot support it.

Ⅵ. Conclusion and Discussion

In this paper, we make use of game theory constructs to discuss the interaction between the public's expectations formation and the central bank's choice of a feedback rule. The contrast between the expectations formation process and the relative timing of moves assumed in this model and those assumed in the previous literature is essential for our result. Our main conclusions are as follows:

1. When the public anticipates more (less) active policy than that chosen in a G-active equilibrium, the best response for the central bank is to choose an even more (even less) active policy than that anticipated by the public. In contrast, when the public anticipates more (less) active policy than that chosen in a C-active equilibrium, the best response for the central bank is to choose a less (more) active policy than that anticipated by the public. In other words, G-active equilibrium is unstable and C-active equilibrium is stable.
2. Inflation targeting has asymmetric effects, depending on whether it is adopted to lower the inflation rate or to escape from a deflationary trap. In the former, it is effective in bringing inflation down because it can be supported by active monetary policy. In the latter case, however, active monetary policy cannot support it, and are not effective. Inflation targeting in this context differs from strict inflation targeting (the optimum rule under commitment).
3. Under commitment, the optimum rule is unique, and reduces to a strict inflation-targeting rule, whereby the inflation rate is kept at its optimum level at all times. Since monetary policy cannot affect the long-run average output in a world consistent with the natural-rate hypothesis, it is reasonable that the optimum rule involves a zero coefficient on the lagged output gap. The optimum rule under commitment is, however, time inconsistent.

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Technical Appendix

This technical appendix presents the proof of the propositions in the text as well as details of the derivations for some of the expressions that were asserted in Matsukawa, Okamura and Taki (2012). The equations appearing below without labels A, B, C... are referred to those in the text.

A. The Euler Equation and Discretionary Equilibria

Since the optimized rule sets $\pi_{\tau+1}$ as in (6), substitution of (5) into (6) leads to:

(A1) $\pi_{\tau+1} = A_1 + A_2 y_\tau$

$$= A_1 + A_2 \{-\alpha A_1^e + (\rho - \alpha A_2^e) y_{\tau-1} + \alpha \pi_\tau + \eta_\tau\}$$

$$= A_1 - \alpha A_1^e A_2 + A_2(\rho - \alpha A_2^e) y_{\tau-1} + \alpha A_2 \pi_\tau + A_2 \eta_\tau .$$

Then, substituting (5), (6) and (A1) into the Euler equation (12) yields:

$$\lambda\,[\rho y_{\tau-1} + \alpha\{\pi_\tau - (A_1^e + A_2^e y_{\tau-1})\} + \eta_\tau - \bar{y}] + \frac{1-\lambda}{\alpha}(A_1 + A_2 y_{\tau-1} - \pi^*)$$

$$-\frac{\beta(1-\lambda)(\rho-\alpha A_2^e)}{\alpha}E_\tau[\,A_1 - \alpha A_1^e A_2 + A_2(\rho - \alpha A_2^e)y_{\tau-1} + \alpha A_2\pi_\tau + A_2\eta_\tau - \pi^*] = 0,$$

$$\lambda\,[\rho y_{\tau-1} + \alpha\{A_1 + A_2 y_{\tau-1} - (A_1^e + A_2^e y_{\tau-1})\} + \eta_\tau - \bar{y}] +$$

$$\frac{1-\lambda}{\alpha}(A_1 + A_2 y_{\tau-1} - \pi^*) - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)}{\alpha}[A_1 - \alpha A_1^e A_2$$

$$+ A_2(\rho-\alpha A_2^e)y_{\tau-1} + \alpha A_2\{A_1 + A_2 y_{\tau-1}\} + A_2\eta_\tau - \pi^*] = 0,$$

(A2) $$[\lambda\{\alpha(A_1 - A_1^e) - \bar{y}\} + \frac{1-\lambda}{\alpha}(A_1 - \pi^*) - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)}{\alpha}(A_1 - \alpha A_1^e A_2 + \alpha A_2 A_1 - \pi^*)]$$

$$+[\lambda(\rho + \alpha A_2 - \alpha A_2^e) + \frac{1-\lambda}{\alpha}A_2 - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)}{\alpha}\{A_2(\rho-\alpha A_2^e) + \alpha A_2^2\}]\,y_{\tau-1} = 0.$$

By equating coefficients, we obtain values for A_1 and A_2 in terms of A_1^e, A_2^e and parameters of the model.

First, setting to zero the coefficient on $y_{\tau-1}$, we have:

$$-\beta(1-\lambda)(\rho-\alpha A_2^e)A_2^2 + \{\lambda\alpha + \frac{1-\lambda}{\alpha} - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)^2}{\alpha}\}A_2 + \lambda(\rho-\alpha A_2^e) = 0,$$

or equivalently,

(A3) $$A_2^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho-\alpha A_2^e)^2}{\alpha\beta(1-\lambda)(\rho-\alpha A_2^e)}A_2 - \frac{\lambda}{\beta(1-\lambda)} = 0.$$

If there exists an optimal value for A_2, it must satisfy the quadratic equation (A3). Since the discriminant of the quadratic equation (A3),

$$\{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho-\alpha A_2^e)^2\}^2 + 4\lambda\alpha^2\beta(1-\lambda)(\rho-\alpha A_2^e)^2$$

is positive, there are necessarily two distinct real roots. And since the constant term of the equation (A3), $-\frac{\lambda}{\beta(1-\lambda)}$ is negative, only one of them is negative and associated with the minimum expected discounted losses.

Second, equating the constant term of the equation (A2) to zero leads to

(A4) $$\lambda\{\alpha(A_1 - A_1^e) - \bar{y}\} + \frac{1-\lambda}{\alpha}(A_1 - \pi^*) - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)}{\alpha}(A_1 - \alpha A_1^e A_2 + \alpha A_2 A_1 - \pi^*) = 0,$$

or, equivalently,

(A5) $$[\lambda\alpha + \frac{1-\lambda}{\alpha} - \frac{\beta(1-\lambda)(\rho-\alpha A_2^e)(1+\alpha A_2)}{\alpha}]A_1 - \lambda(\alpha A_1^e + \bar{y}) - \frac{1-\lambda}{\alpha}\pi^*$$

$$+\beta(1-\lambda)(\rho-\alpha A_2^e)(A_1^e A_2 + \frac{\pi^*}{\alpha}) = 0.$$

Solving for A_1 gives:

(A6) $$A_1 = [\lambda\alpha^2 + (1-\lambda)\{1-\beta(\rho-\alpha A_2^e)(1+\alpha A_2)\}]^{-1}$$

$$[\lambda\alpha(\alpha A_1^e + \bar{y}) + (1-\lambda)\pi^* - \beta(1-\lambda)(\rho-\alpha A_2^e)(\alpha A_1^e A_2 + \pi^*)].$$

Note that the negative root of the quadratic equation (A3) can also be derived by solving the discounted algebraic matrix Riccati equation for which analytical solutions can be obtained (see Okamura, Matsukawa and Taki (2006), Appendix 2).

Setting $A_2^e = A_2 = A_2^D$ in (13) or (A3), it follows:

$$(A_2^D)^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho-\alpha A_2^D)^2}{\alpha\beta(1-\lambda)(\rho-\alpha A_2^D)}A_2^D - \frac{\lambda}{\beta(1-\lambda)} = 0.$$

Multiply this equation by $\alpha\beta(1-\lambda)(\rho-\alpha A_2^D)$ and obtain:

$$\alpha\beta(1-\lambda)(\rho-\alpha A_2^D)(A_2^D)^2 - \{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho-\alpha A_2^D)^2\}A_2^D - \alpha\lambda(\rho-\alpha A_2^D) = 0.$$

Rearranging terms yields:

$$\beta(1-\lambda)(\rho-\alpha A_2^D)[\alpha(A_2^D)^2 + (\rho-\alpha A_2^D)A_2^D] - (\alpha^2\lambda + 1 - \lambda)A_2^D - \alpha\lambda(\rho-\alpha A_2^D) = 0.$$

So we have:

$$-\alpha\beta\rho(1-\lambda)(A_2^D)^2 + (1-\lambda)(\beta\rho^2 - 1)A_2^D - \alpha\rho\lambda = 0,$$

(A7) $$(1-\lambda)(A_2^D)^2 + \frac{(1-\lambda)(1-\beta\rho^2)}{\alpha\beta\rho}A_2^D + \frac{\lambda}{\beta} = 0,$$

which is equivalent to (15).

Setting $A_2^e = A_2 = A_2^D$ and $A_1^e = A_1 = A_1^D$ in (A6), and after rearranging, we have:

$$-\lambda\bar{y} + \frac{1-\lambda}{\alpha}(A_1^D - \pi^*) - \frac{\beta(1-\lambda)(\rho-\alpha A_2^D)}{\alpha}(A_1^D - \pi^*) = 0.$$

Therefore

(A8) $$A_1^D = \frac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}} + \pi^*,$$

which is equivalent to (16).

B. Comparison of Rational Expectations Equilibrium and C-active Equilibrium

Rewrite the quadratic equation (15) as

(B1) $f(x) = x^2 - \frac{1-\beta\rho^2}{\alpha\beta\rho}x + \frac{\lambda}{\beta(1-\lambda)} = 0$.

Differentiate $f(x)$ with respect to x and obtain:

(B2) $f'(x) = 2x - \frac{1-\beta\rho^2}{\alpha\beta\rho}$.

Substituting $x = -\frac{\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}$ into (B1) and (B2), we have (see (8)):

(B3) $f(-\frac{\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}) = \frac{\alpha^2\lambda^2\rho^2}{(1-\lambda)(1-\rho^2\beta)} \geq 0$.

and

(B4) $f'(-\frac{\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}) = \frac{-2\alpha\rho\lambda}{(1-\lambda)(1-\rho^2\beta)} + \frac{1-\beta\rho^2}{\alpha\beta\rho} = \frac{-2\alpha^2\beta\rho^2\lambda + (1-\lambda)(1-\rho^2\beta)^2}{(1-\lambda)(1-\rho^2\beta)\alpha\beta\rho}$.

Since the discriminant of Eq. (18) is positive: $(\frac{1-\beta\rho^2}{\alpha\beta\rho})^2 - \frac{4\lambda}{\beta(1-\lambda)} > 0$, it follows: $(1-\lambda)(1-\rho^2\beta)^2 > 4\alpha^2\beta\rho^2$. Thus, the numerator of the right hand side of (B4) is positive, implying that $f'(-\frac{\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}) > 0$.

Denote C-active equilibrium as $x^* = \frac{-\frac{1-\beta\rho^2}{\alpha\beta\rho} + \sqrt{(\frac{1-\rho^2\beta}{\alpha\beta\rho})^2 - \frac{4\lambda}{\beta(1-\lambda)}}}{2}$, we have: $f(x^*) = 0$ and $f'(x^*) > 0$. Combined with the results obtained above, we conclude: $x^* < x^\# < 0$, or $A_2^* < A_2^\# < 0$ as asserted in the text and Matsukawa Okamura and Taki (2012).

C. The Stability of Equilibria

Rewrite the quadratic equation (15) as

(C1) $y^2 - b(x)y - c = 0$,

where $y = A_2$, $x = A_2^e$, $c = \frac{\lambda}{\beta(1-\lambda)}$ and $b(x) = \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho-\alpha x)^2}{\alpha\beta(1-\lambda)(\rho-\alpha x)}$. Since the negative root of (C1) is $z(x) = \frac{b(x) - \{b^2(x)+4c\}^{\frac{1}{2}}}{2}$, we have:

(C2) $\frac{dz}{dx} = \frac{b'(x)}{2}[1 - b(x)\{b^2(x)+4c\}^{-\frac{1}{2}}]$.

Given a fixed point (discretionary equilibrium) $\tilde{x}$ that satisfies: $z(\tilde{x}) = \tilde{x}$, it follows that: $\{b^2(\tilde{x}) + 4c\}^{\frac{1}{2}} = b(\tilde{x}) - 2\tilde{x}$. Substitution into (C2) gives:

(C3) $$\left.\frac{dz}{dx}\right|_{x=\tilde{x}}=\frac{\tilde{x}b'(\tilde{x})}{2\tilde{x}-b(\tilde{x})}$$

$$=\frac{\tilde{x}b'(\tilde{x})}{\tilde{x}+\frac{c}{\tilde{x}}}$$

$$=\frac{\frac{\beta(1-\lambda)(\rho-\alpha\tilde{x})^2+(\alpha^2\lambda+1-\lambda)}{\beta(1-\lambda)(\rho-\alpha\tilde{x})^2}\tilde{x}}{\frac{\beta(1-\lambda)\tilde{x}^2+\lambda}{\beta(1-\lambda)\tilde{x}}}$$

$$=\frac{\{\beta(1-\lambda)(\rho-\alpha\tilde{x})^2+(\alpha^2\lambda+1-\lambda)\}\tilde{x}^2}{\{\beta(1-\lambda)\tilde{x}^2+\lambda\}(\rho-\alpha\tilde{x})^2}$$

$$=\frac{\frac{(\rho-\alpha\tilde{x})\{-\alpha\beta(1-\lambda)\tilde{x}^2+\beta(1-\lambda)\rho\tilde{x}+\alpha\beta(1-\lambda)\tilde{x}^2+\beta(1-\lambda)(\rho-\alpha\lambda)\tilde{x}-\lambda\alpha\}}{\tilde{x}}}{-\frac{(1-\lambda)(1-\beta\rho^2)\tilde{x}}{\alpha\rho}(\rho-\alpha\tilde{x})^2}$$

$$=\frac{\frac{(\rho-\alpha\tilde{x})\{-\alpha\beta(1-\lambda)\tilde{x}^2+2\beta(1-\lambda)\rho\tilde{x}-\alpha\lambda\}}{\tilde{x}}}{-\frac{(1-\lambda)(1-\beta\rho^2)\tilde{x}}{\alpha\rho}(\rho-\alpha\tilde{x})^2}$$

$$=-\frac{(1+\beta\rho^2)\alpha\tilde{x}}{(1-\beta\rho^2)(\rho-\alpha\tilde{x})},$$

where (A3) and (A7) are used repeatedly.

(C3) implies that: $\left.\frac{dA_2}{dA_2^e}\right|_{A_2^e=A_2^D}=-\frac{(1+\beta\rho^2)\alpha A_2^D}{(1-\beta^2\rho)(\rho-\alpha A_2^D)}$. Since $A_2^{**}<-\frac{1-\beta\rho^2}{2\alpha\beta\rho}$ for the G-active equilibrium, we have: $0<(1-\beta^2\rho)(\rho-\alpha A_2^{**})<-(1+\beta\rho^2)\alpha A_2^{**}$, and $\left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^{**}}>1$. For the C-active equilibrium, it holds that: $A_2^*>-\frac{1-\beta\rho^2}{2\alpha\beta\rho}$. Then it follows that:

$(1-\beta^2\rho)(\rho-\alpha A_2^*)>-(1+\beta\rho^2)\alpha A_2^*$, and $0<\left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^*}<1$.

Next, differentiating (14) with respect to A_1^e, and evaluating the result in discretionary equlibria, we have: $\frac{\partial A_1}{\partial A_1^e}=\frac{\lambda\alpha^2-\beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D}{\lambda\alpha^2-\beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D+(1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}}$. Then it is straightforward to show: $\left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{**},A_2=A_2^{**}}>1$, in the G-active equilibrium, and $0<\left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{**},A_2=A_2^{**}}$, in the C-active equilibrium. The sequential nature of the solution allows us to conclude that the G-active (C-active) equilibrium is unstable (stable).