Doctoral Dissertation

EXAMINING VENEZUELAN SECONDARY SCHOOL MATHEMATICS TEACHERS' STATISTICAL KNOWLEDGE FOR TEACHING: FOCUSING ON THE INSTRUCTION OF VARIABILITY-RELATED CONCEPTS

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We hereby recommend that the dissertation by Mr. ORLANDO RAFAEL GONZÁLEZ GONZÁLEZ entitled "EXAMINING VENEZUELAN SECONDARY SCHOOL MATHEMATICS TEACHERS' STATISTICAL KNOWLEDGE FOR TEACHING: FOCUSING ON THE INSTRUCTION OF VARIABILITY-RELATED CONCEPTS" be accepted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN EDUCATION.

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Dedication

To my parents, Yolanda and Eliseo, in deep and sincere appreciation for their inexhaustible love and support, for having inculcated in me their belief in education as a way of ensuring success in life, and for having invigorated my pursuit of it.

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LIST OF ABBREVIATIONS

MKT. Mathematical Knowledge for Teaching

SKT. Statistical Knowledge for Teaching

SMK. Subject Matter Knowledge

PCK. Pedagogical Content Knowledge

CCK. Common Content Knowledge

SCK. Specialized Content Knowledge

HCK. Horizon Content Knowledge

KCS. Knowledge of Content and Students

KCT. Knowledge of Content and Teaching

KCC. Knowledge of Content and Curriculum

L.H. School. Lower High School

U.H. School. Upper High School

ABSTRACT

The fundamental role that statistics has in today's knowledge-based society is unquestionable. Given this, it is by no means surprising the incorporation of several statistical contents into the school mathematics curriculum at all levels—particularly at secondary school—by the recent curricular reforms that have been carried out in many countries. In the case of those statistical contents studied in secondary school mathematics—all of them related to the idea of variability—, they are likely to be the last exposure that many students will have to statistics, and then it is expected that a big proportion of those students will develop their statistical literacy skills, knowledge base, attitudes and beliefs about statistics from these courses. Thus, given the general agreement among specialists in the field of statistics education that acknowledgement, understanding, explanation, and quantification of the variability in data is fundamental to statistical literacy, and due to critical role played by secondary mathematics teachers in the promotion of statistical literacy among their students, it is natural the importance to conduct research on secondary mathematics teachers' professional knowledge to teach variability-related concepts, as an important first step in making any future improvement in the teaching and learning of statistics at any school level.

Despite all the aforementioned facts, there is a scarcity of studies focused on the professional knowledge entailed by teaching variability-related contents at secondary

school. This is particularly true in the case of Venezuela. In order to deal with this gap in the literature, a research having the following main objectives was performed:

- To propose a conceptual framework for *statistical knowledge for teaching*—henceforth SKT, the professional knowledge needed to carry out effectively the work of teaching statistics—, aiming to examine the professional competencies—i.e., professional knowledge and affective-motivational characteristics—held by secondary mathematics teachers to teach variability-related contents.

- To examine the knowledge base of SKT held particularly by Venezuelan in-service secondary school mathematics teachers, using a survey designed on the basis of the framework proposed in this research.

In order to develop a conceptual framework for SKT, an extensive literature review was conducted at first, aimed to identify several components that are agreed to be potential indicators of teachers' professional competencies for teaching statistics from the viewpoint of variability. From this review, eight traits were identified; two in the affective domain—*conceptions of variability* and *statistics-related beliefs*—, and six in the cognitive domain—*statistical literacy*, *specialized content knowledge*, *horizon content knowledge*, *knowledge of content and students*, *knowledge of content and teaching*, and *knowledge of content and curriculum*. Moreover, twelve indicators associated to the latter six traits—two per trait—were identified and listed, in order to provide a comprehensive assessment framework for teachers' professional knowledge to teach variability-related ideas at secondary level. The consideration of not only six cognitive traits simultaneously, but also of teachers' conceptions and personal beliefs on statistics teaching and learning—which have been highlighted to have an inextricable relation with teachers' knowledge and curriculum implementation—, is an original feature of this framework, which cannot be found within any of the few frameworks of SKT proposed to date.

Based on the conceptual framework previously outlined, a pen-and-paper instrument was developed. Such instrument, designed to be completed in one hour, was comprised of a task addressing—by comparing the histograms of two distributions—many variability-related ideas present in the secondary school mathematics curriculum. This task was accompanied by seven SKT-related questions, aimed to elicit and gather information about each one of the eight traits indicated previously.

The next phase of the research was carrying out the survey on a purposive sample of 53 Venezuelan in-service secondary school mathematics teachers working in the metropolitan area of Caracas, who were asked to anonymously and voluntary fill in the designed questionnaire between July and September 2012. After carrying out the survey, a qualitative analysis of the collected answers was made, using as assessment framework the aforementioned twelve indicators related to SKT. This analysis provided a comprehensive picture of the current state of Venezuelan secondary school mathematics teachers' knowledge base on SKT, conceptions of variability, and beliefs about statistics teaching and learning. Moreover, the analysis of the gathered data revealed a number of strengths and weaknesses of the surveyed teachers to effectively teach variability-related concepts in the area of descriptive statistics, as well as interesting relations among and within the cognitive and affective-motivational characteristics considered in this study. In addition, specific issues regarding the secondary school mathematics curriculum—at both intended and implemented levels—were identified; theoretical, methodological, and practical implications of the study findings were discussed; and recommendations for the systematic improvement of the statistical knowledge base for teacher education and the professional development of mathematics teacher educators regarding statistics in Venezuela were offered, based on all the information that emerged out of the aforementioned data analysis.

In its current form, the framework for SKT proposed here can provide a fair qualitative characterization of in-service secondary school mathematics teachers' SKT related to the instruction of statistics, in particular, variability-related contents. However, this study has the following limitations: (1) the developed survey instrument only deals with variability-related contents in the area of descriptive statistics, leaving aside probability and sampling, the other important statistical areas considered in secondary school mathematics; (2) prospective teachers were not included in the present study.

CHAPTER 1:

Introduction

1.1 Rationale and research motivation

In recent years, and aiming towards achieving statistical literacy (Gal, 2002), curricular reforms in many countries have included and emphasized the teaching of topics related to statistics at all school levels, particularly in the secondary school mathematics curriculum (e.g., Ministerio de Educación [ME], 1987, 1997; Centro Nacional para el Mejoramiento de la Enseñanza de la Ciencia y la Matemática [CENAMEC], 1991; National Council of Teachers of Mathematics [NCTM], 2000; American Statistical Association [ASA], 1991; American Association for the Advancement of Science [AAAS], 1993; Australian Education Council, 1994; School Curriculum and Assessment Authority & Curriculum and Assessment Authority for Wales, 1996; United Kingdom Department for Education and Employment, 1999; Ministry of Education of New Zealand, 2007; Ministry of Education, Culture, Sports, Science and Technology of Japan [MEXT], 2008a, 2008b, 2009). This responds to one of the main changes that has altered the modern world: the transformation of industrial economies and societies into knowledge- and information-based ones (Sagheb-Tehrani, 2006). Therefore, as we move towards an increasingly knowledge-based society (Gal, 2004; Willke, 2007), it is more important than ever for all citizens to be able to provide good evidence-based arguments, to critically evaluate data-based claims and arguments coming from different sources of information, to make optimal decisions on the basis of statistical and probabilistic information about policies and practices in the socio-political, industry, medicine, workplace, and consumer arenas (Wild & Pfannkuch, 1999; Franklin & Garfield, 2006; Kader & Perry, 2006), and to explain, judge, and analyze information (Rumsey, 2002), in order to intelligently participate in many fields of today's society (e.g., for businesspeople, physicians, politicians, lawyers) after the end of school, and ensure an increased influence on it (Watson & Callingham, 2003; Budgett & Pfankuch, 2007). All these skills—which are related to being statistically literate—are expected of citizens in information-laden societies, and are regarded in many countries as an expected outcome of compulsory schooling and as a necessary component of adults' numeracy and literacy (Gal, 2002, 2004). For that reason, promotion of statistical literacy has been recently sought by a number of school mathematics curricula worldwide, aiming at preparing students to encounter the needs of society when they complete their compulsory education (Watson & Callingham, 2003; Willke, 2007).

In the case of Venezuela, topics on statistics and probability were introduced into elementary (i.e., in Grades 1-6) and lower secondary education (i.e., in Grades 7-9) in 1985, while their introduction into upper secondary education (i.e., in Grades 10-11) occurred in 1972, year until which the study of such topics was left exclusively to university students (ME, 1972, 1987, 1997; CENAMEC, 1991). It is noticeable that variability—a property of a statistical object which accounts for its propensity to vary or change—arise naturally in many different ways in all the statistical topics included in the Venezuelan secondary school mathematics curriculum, as it happens in the case of other countries' curricula. This is not a surprise at all, since several researchers have regarded variability not only as a fundamental concept in statistics (e.g., Shaughnessy, 2007; Pfannkuch & Ben-Zvi, 2011; Gattuso & Ottaviani, 2011), but also as its raison d'être (e.g., Moore, 1990; Moore & Cobb, 1997; Shaughnessy & Ciancetta, 2001; Shaughnessy, 2007), with the acknowledgment and understanding of variability being fundamental for statistical literacy (Shaughnessy, 2008), particularly as the words "variable", "variation" or "vary" are a part of everyday language (Watson, 2006), as it is the ability to think statistically about varying outcomes and phenomena to make decisions not only at the individual level, but also for society in general. Therefore, nowadays Venezuelan secondary mathematics teachers—as their counterparts in other countries—must instruct several variability-related ideas—such as the ones of graphical representations of data, measures of variation, distribution and sampling—, and such work demands from them specific professional knowledge, skills and habits of mind, without which the aims of the mathematics curriculum regarding statistics education cannot be achieved.

1.2 Problem statement

An important implication can be drawn from the above rationale: since mathematics teachers' professional competence translates into gains in student achievement (Ball & Cohen, 1999; Supovitz, 2001; Borko, 2004); secondary school mathematics teachers, not only in Venezuela but in all countries, need to be prepared to teach and facilitate discussions with students about the statistical contents included in the mathematics curriculum. That is, teachers must, among other things, be able to appropriately answer the statistical problems that they assign to their students; understand the importance of students spending time formulating questions and collecting data; and move beyond mere numerical calculation of statistical measures or construction of tables and graphs, in order to implement different instructionally viable models for teaching statistical ideas related to variability (Newton, Dietiker & Horvath, 2011). Hence, the professional competence and prowess held by secondary school teachers to efficiently teach statistics and foster statistical literacy represent an immediate concern in the teaching of statistics at school level, which is natural, since these teacher must be, at least, statistically literate in order to develop students' statistical literacy, and professionally competent enough to provide them with knowledge, skills and habits of mind to understand, explain and use varying data and statistical information (Franklin et al., 2005).

The scarce literature on Venezuelan statistics education clearly points out that students finish compulsory schooling without learning even the basic notions of statistics included in the mathematics curriculum, and hence without acquiring the basic statistical literacy skills expected by official documents. Salcedo (2008) analyzed the results in the area of statistics obtained by students in Grades 3, 6 and 9 in 1998 at the only attempt of the National System of Measurement and Evaluation of Learning (known by its Spanish acronym as SINEA), as well as by students at Grade 11 in the 2006 Scholastic Aptitude Test, which was, until 2007, a national entrance examination to Venezuelan public universities. The results of SINEA indicate that less than 30% of the students at Grades 3, 6 and 9 answered correctly the questions in the section “Data Organization and Representation”, which was comprised of items related to statistics and probability. Similarly, the results of the 2006 Scholastic Aptitude Test show that the percentage of correct answers in the questions related to statistics and probability ranged between 7.5% and 36.6%. Moreover, in both examinations, the results in the items related to statistics and probability were the lowest ones, when compared with the results in other mathematical strands such as “Numbers”, “Operations”, “Geometry” and “Measurements”. Salcedo (2008) then concluded that mathematics teachers, in case they are working with topics in the strand “Statistics and Probability”, could be dealing with them with didactical and mathematical mistakes and shortcomings, which interfere with students’ understanding. The findings reported by Salcedo (2008) are in agreement with those of Tapia (2011), who found in his research with Venezuelan university students that those new admitted in the different engineering undergraduate programs at Venezuelan universities—particularly at Universidad Nacional Experimental de los Llanos Ezequiel Zamora (UNELLEZ)—demonstrate a disturbing lack of basic knowledge on statistics, especially those related to descriptive statistics, which were supposed to be studied by them at elementary and secondary school.

In order to improve classroom instruction and student achievement, a key factor to consider are teachers' professional competencies—i.e., professional knowledge and affective-motivational characteristics—, whose promotion is the central goal of mathematics teacher education and professional development (Ball & Cohen, 1999; Cohen & Hill, 2000; Corcoran, Shields, & Zucker, 1998; Darling-Hammond & McLaughlin, 1995; Elmore, 1997; Little, 1993; National Commission on Teaching and America's Future, 1996; Döhrmann, Kaiser & Blömeke, 2012). According to a number of researchers (e.g., Döhrmann et al., 2012; Tatto et al., 2012), "successful teaching depends on professional knowledge and teacher beliefs" (cf. Döhrmann et al., 2012, p. 327). This "professional knowledge"—which will be understood in the present study as *subject matter knowledge* (SMK) and *pedagogical content knowledge* (PCK) (cf. Ball, Thames & Phelps, 2008; Döhrmann et al., 2012)—represents an important part of the competence required of mathematics teachers for accomplishing an effective teaching and learning, as well as a fundamental criteria for effective teacher education.

Nevertheless, in the case of statistics education, scarce studies can be found in the literature focused on both the professional knowledge entailed by teaching variability-related contents to help students achieve the aims of statistics education (cf. Shaughnessy, 2007), the beliefs held by in-service teachers on statistics teaching and learning of such contents (cf. Pierce & Chick, 2011), and the conceptions of variability that teachers hold (cf. Batanero, Garfield, Ottaviani & Truran, 2001; Canada, 2006a, 2006b; Peters, 2009; Gonzalez, 2011; Isoda & Gonzalez, 2012), which are closely related to the previous two traits. This paucity of research in the aforementioned issues is particularly true in the case of Venezuela, country in which the few reported researches on statistics education to date have been centered on the statistical contents in the school curriculum (e.g., Salcedo, 2006) or on students' knowledge about statistics and probability (e.g., León, 2011), with no studies reported, to the knowledge of the present author, on teachers' professional knowledge and beliefs related to the teaching of statistics at any school level.

Thus, the problem this dissertation examines is the clarification of the current state of the professional knowledge base on statistics, conceptions of variability, and beliefs about teaching and learning of statistics, held by Venezuelan secondary school mathematics teachers. By knowledge base, I mean a "codified or codifiable aggregation" of knowledge, understanding, skills and disposition that teachers use to carry out their classroom responsibilities (Shulman, 1987, p. 4; Valli & Tom, 1988). This includes things we imagine "in the brain", but also includes skill (the ability to enact knowledge) and disposition (a propensity to act or not act on what one knows). A knowledge base for teaching categorizes knowledge and provides "a means of representing and communicating it" (Shulman, 1987, p. 4).

Thus, in order to provide a truly comprehensive picture of the current state of the aforementioned traits in the case of Venezuelan secondary school mathematics teachers, it will be necessary to identify relevant dimensions of professional competencies for teaching variability-related contents at secondary school level, and then to examine a sample of Venezuelan mathematics teachers looking for evidence of such dimensions.

1.3 Purpose of the research

Three key themes underpin the present research. The first is the increasing status of statistics in the secondary school mathematics curricula not only in Venezuela, but worldwide, which is clear from the recent inclusion of several statistical topics into them. This fact demands from mathematics teachers professional competencies that are necessary to carry out their complex and demanding role in achieving the goals of mathematics curriculum and promoting both statistical literacy and understanding of statistics in students. Second, the critical role that exposure to secondary school statistics plays in developing students' knowledge base, attitudes and beliefs about statistics. For many students, the exposure to statistics at secondary school mathematics might be the last and only statistics formal program taken by future users of statistics. Moreover, for those students moving on to tertiary education, prior exposure to statistics in secondary school would prepare them for statistics in college and university, since most of the topics discussed in statistics courses at tertiary level are advanced versions of the contents of probability and statistics studied in secondary school. Third, a lack of research efforts and a shared understanding about the professional knowledge needed to teach statistics at secondary school level, not only in Venezuela, but worldwide, due to the little research that has been done to address this topic, which results in a blurry picture about the current preparedness of our mathematics teachers to teach statistics at secondary school. In summary, the purpose of this research can be regarded as two-fold, with a theoretical as well as a theoretical-practical component, which are described as follows:

(1) To propose a conceptual framework for *statistical knowledge for teaching*—henceforth SKT, the professional knowledge needed to carry out effectively the work of teaching statistics—, aiming to examine the professional knowledge, conceptions of variability, and statistics-related beliefs held by secondary school mathematics teachers to teach variability-related contents.

(2) By using the framework for SKT proposed here, as well as a survey designed based on it, to examine qualitatively the knowledge base of SKT, conceptions of variability, and statistics-related beliefs held by Venezuelan in-service secondary school mathematics teachers, in order to clarify the current state of such traits in this group.

1.4 Research questions

In order to fulfill the purpose of this research, the following research questions were posed:

1. What is statistical knowledge for teaching, and what are the indicators that could serve to evaluate it in the case of teaching variability-related concepts?

2. On the basis of the conceptualization of SKT adopted by this research, what is the knowledge base of SKT that Venezuelan secondary school mathematics teachers have to teach variability-related concepts?

3. How do Venezuelan secondary school mathematics teachers conceptualize variability, and what beliefs about statistics, its teaching and learning, do they have?

1.5 Significance of the present study

Teachers must be conscious about the fact that secondary school statistics is especially important because it is expected that a big proportion of their students will form their knowledge base, attitudes and beliefs about statistics from secondary school mathematics courses. Thus, such courses serve a critical function, and obviously mathematics teachers have the potential to play a critical role in statistics education and the promotion of statistical literacy among secondary school students, which is fundamental for them to fully participate in todays' knowledge-based society. Then, since the extent to which mathematics teachers are adequately prepared to teach statistics at secondary school level is one of the major determinants to achieve the aims of the mathematics curriculum regarding statistics education, carrying out the present research is important, because the findings of this research will provide a valuable insight into the knowledge base of statistical knowledge for teaching of Venezuelan middle and high school teachers. Moreover, this research will provide valuable information about both how teachers conceptualize variability and think statistically when dealing with situations where variation and variability arise—such as interpretation and understanding of histograms, data manipulation, and interpretation of data distribution features, among others—, and how they believe that statistics can and should be learnt and taught, traits that are fundamental for the successful implementation of recent curricular reforms, and the teaching of statistics in ways that serve the present and future needs of students and the whole society in general. Then, this study helps to remedy a problem such as the evident absence of studies focusing on professional knowledge, conceptions of variability, and statistics-related beliefs of secondary school mathematics teachers that currently exists in the literature, which is particularly remarkable in the case of Venezuela. Moreover, by addressing the aforementioned research questions, the present study aims to unite research on teacher knowledge, research on teachers' beliefs, research on secondary school teachers, and research on statistics education in the domain of variability-related contents, areas in which a noticeable lack of study have been acknowledged in the literature by several concerned authors and statistics educators.

In this regard, currently there is a very limited literature on the professional knowledge needed to teach statistics at school level, in particular at secondary one, not only at Venezuela but worldwide. This is shown by an analysis of research literature, for example papers published in the *Journal of Mathematics Teacher Education*, as well as in survey papers and handbooks concerning mathematics education, which pay little attention to the teaching of statistics (Batanero, Burrill & Reading, 2011). Hence, it is by no means surprising the urgent call for increasing research on these areas made by a number of concerned researchers, particularly for studies on teachers' professional knowledge and practices while teaching variability (e.g., Sánchez, da Silva & Coutinho, 2011, p.219), as well as for teachers' beliefs on statistics itself and on what aspects of statistics should be taught in schools and how (e.g., Pierce & Chick, 2011, p.160). These numerous calls for research that have been recently made, as well as the paucity of research in the aforementioned issues acknowledged by several researchers and statistics educators, provide clear evidence of the need and importance of conducting research on the aforementioned issues, which would, like the current one, provide insights into the professional knowledge base that our secondary school teachers have to teach several statistical contents in which variability can be appreciated.

In addition, within the handful of models attempting to describe what content knowledge and pedagogical content knowledge is considered adequate and appropriate to teach statistics, it is evident how important—as well as complex—components that might influence or conform mathematics teachers' professional knowledge to teach statistics—such as teachers' conceptions of variability, statistics-related beliefs and knowledge of content and curriculum, among others—have been left out of consideration. Therefore, one of the main contributions of this research is the proposition of a novel conceptual framework of SKT that takes into consideration many components that have been left out by previous models, as well as survey instrument designed from both an extensive literature review and the proposed eightfold framework for SKT, in which are investigated SKT—which is delved into 6 categories—, conceptions of variability, and beliefs about teaching and learning of statistics. Then, this research contributes to fill a significant gap in the statistics education literature, and provides a valuable tool to examine and qualitatively assess mathematics teachers' competency and preparedness to teach statistical contents, particularly those related to variation and variability, a theme in which, to my knowledge, no research has been done in the case of Venezuela at any level of the educational system. This contribution is important as teachers' competency to teach statistics would be reflected on areas such as their teacher content understanding, beliefs, and teacher practice.

Professional knowledge in the preparation of teachers has been identified as a fundamental component of teacher education programs (Ball & McDiarmid, 1990). Therefore, even though this research focuses on in-service teachers, it may provide insight and have implications for policymakers, for those interested in improving teacher education, and for those who make decisions about the professional development of

Venezuelan teachers. Hence, the present study might be helpful to identify what topics in statistics and which instructional approaches need to be attended or included in pre-service and in-service teacher education programs, which would contribute to develop effective ways to train current and future teachers of statistics. All this, at the end, will result in an improvement of the professional development of pre-service and in-service Venezuelan teachers' SKT, and hence in the quality of statistical teaching in Venezuela.

Also, this research provides solid ground for researchers wishing to continue to detail the characteristics of how Venezuelan in-service middle and high school teachers give response to tasks where variation and variability in data plays a leading role.

In summary, while this study does claim neither to unveil completely how Venezuelan secondary school mathematics teachers acknowledge variability in particular settings, nor what kind of training they need in order to build and improve their current knowledge about the surveyed contents, it does give insights about their statistical literacy, reasoning and thinking that can help advise research, teaching, and curriculum development in the areas of secondary school statistics, as well as in pre-service and in-service teacher education programs.

CHAPTER 2:

Statistics Education in Venezuela

2.1 Contextual Background of the Education System in Venezuela

According to the Chapter III of the new Organic Law of Education—adopted in August 2009—, Venezuela's traditional or mainstream education system is organized by the following levels and modalities (D'Amico & González, 2006; D'Amico, Loreto & Mendoza, 2011):

- Initial education, serving children between birth and 5 years of age. The kindergarten stage, a three-year institution that begins at the age of three and continues until age five, is compulsory.

- Elementary school, a six-year institution for children between 6 and 11 years of age. It is divided into First stage (Grades 1, 2 and 3), and Second stage (Grade 4, 5 and 6).

- Secondary school, a five or six-year institution for youngsters between 12 and 17 years old. It is divided into two stages: the Basic (years 1, 2 and 3) and the Diversified (years 4–5 or 6) cycles. At the latter one, the student has the option to choose between one of the following three streams: Basic Sciences, Humanities (each of them lasting 2 years), or Professional (the only one lasting three years). At the professional level, the student can get a technical qualification in one of the following six specialties: Industrial, Commerce and Administrative Services, Farming and Livestock, Social work, Arts, Security and Defense (D'Amico & González, 2006, pp.64-65). Right after graduation at the Professional stream, students usually have the functional and technical knowledge and skills to work in the chosen area. Students graduating from the Basic Sciences or Humanities streams will be able to pursue higher education. The secondary education diploma is awarded after finishing this level.

- Higher education, which comprises university and technical college education. This stage serves youngsters from 18 years of age. Undergraduate studies at the government universities and technical colleges are free.

The modalities refer to other areas of education, such as special education, education for the arts, vocational education, military education, adult education, and off-campus education.

The main responsibility for this educational system rests in the hands of the Ministry of the People's Power for Education. Venezuela's education system follows a national curriculum for all subjects and school levels, and the Ministry of the People's Power for Education is responsible, among other functions, for designing and changing areas or subject matter in the curricula for each education level (Navarro & de la Cruz, 1998, pp.133-134).

This structure of the Venezuelan educational system has a philosophical underpinning in the Constitution of the Bolivarian Republic of Venezuela, aimed to promote endogenous and sustainable development, as well as to encourage Latin American and Caribbean integration, as it is stated in its Article 153. Moreover, in Venezuela, education is constitutionally understood to be a 'public service', which gives it a character of special interest to every citizen. In fact, according to the Articles 3, 102 and 103 of the Constitution of the Bolivarian Republic of Venezuela, education is a right ensured by the State, compulsory from initial education to the diversified secondary level, as well as free of charge in all government and local body educational institutions up to the undergraduate level (Constitución de la República Bolivariana de Venezuela [CRBV], 1999, pp.7, 38; D'Amico, Loreto & Mendoza, 2011, pp.92-93).

The "shift system" is a representation of one of the nuances in the Venezuelan educational system. This practice, although incongruent with regular Western systems of education, is seen also in other countries such as Singapore, Puerto Rico, Brazil, Mexico, India, Jamaica, Turkey and Senegal (Bray, 2000), mainly to combat large class sizes. Venezuelan schools can operate in three different ways: morning shift, afternoon shift, or all-day shift. At the primary level, the vast majority of students are taught in all-day shift schools. Morning or afternoon shifts are circumscribed to secondary schools. In Venezuela, a secondary school day is organized on a shift basis; that is, in separate morning and afternoon shifts. Therefore, it accommodates two different sets of students in one day. The first set attends school from early morning to midday—i.e., from 7 a.m. to 12:30 p.m—; while the other set attends from midday to late afternoon—i.e., from 12:30 p.m. to 6:00 p.m—, normally with several breaks between lessons. Both sets of pupils use the same school facilities and amenities. However, they are often taught by different teachers. Spanish is the only language of main instruction in schools allowed by the Venezuelan Ministry of Education.

In Venezuela, teacher salaries are low compared to salaries in other fields. A teacher fresh out of the university will perceive about US$470 working full-time for a public school. Furthermore, teachers do not get any salary or non-salary incentive by working in rural or remote areas. Because of these, and due to the "shift system" that characterizes Venezuelan educational system, many teachers work in more than one school, looking for augmenting their salary by attending one group in the morning and another in the afternoon. This means that novice teachers and teachers with few hours in one school are required to seek employment in multiple ones, managing their hours into morning and afternoon shifts at more than one location. Therefore, it is not rare to see a secondary school teacher working not only in more than one school, but also for different school systems—i.e., in the public state, municipal, and private school systems—, as well as in more than one school level—i.e., in lower and upper secondary school level.

In Venezuela, at elementary school, teachers teach most or all of the subjects, whereas at secondary school teachers are specialized in particular ones. Mathematics is a compulsory subject for Venezuelan students from elementary school through to secondary school. Mathematics teachers are trained to teach school mathematics in pre-service teacher programmes at universities and teacher colleges.

2.2 Statistics Curriculum in Venezuela

In Venezuela, statistics is taught as part of the mathematics curriculum at all levels, from elementary through Grade 11. Statistical contents are acknowledged in national curriculum documents as a single unit, for which—depending on the school level—conceptual, procedural and attitudinal contents, competencies and assessment indicators, and suggested assessment and methodological strategies, are outlined. A detailed description of how statistical contents are developed over elementary and secondary education in Venezuela follows.

2.2.1 Statistics in the Venezuelan elementary school mathematics curriculum

Topics on statistics and probability were introduced into the elementary school system in Venezuela for the first time in 1985, year before which the study of statistical topics was exclusive to students in the Diversified cycle of secondary school and in higher education. In the First stage of elementary school, statistical contents are studied as part of the mathematical strand “Statistics and Probability”, one of the five strands comprising the mathematics curriculum at that level (the others are “Meeting the Numbers”, “Starting to Make Calculations”, “Geometrical Bodies and Shapes” and “How Do We Measure?”). In the Second stage of elementary school, statistical contents are again studied as part of the mathematical strand “Statistics and Probability”, one of the five strands comprising the mathematics curriculum at that level (the others are “Numbers”, “Operations”, “Geometry” and “Measurements”).

The last reform to the elementary school mathematics curriculum, which was implemented in 1996, introduced some modifications pertinent to the study of statistics, particularly regarding to the arrangement of the contents. A comparison between the statistical contents present in the 1985 and 1996 curriculum designs is shown in Table 1.

As it can be appreciated in Table 1, the elementary school curriculum is spiral, so statistical contents are revisited several times over the course of schooling at increasingly deep and complex levels of understanding and reasoning, in order to consolidate students’ knowledge, conceptual understanding and skills.

2.2.1.1 Objectives and contents in the Venezuelan elementary school mathematics curriculum

Venezuelan mathematics curriculum for elementary school outlines ten general

Table 1: Statistical contents as in the last two reforms to the Venezuelan elementary school mathematics curriculum

Grade	1985 Reform	1996 Reform
1	Design and application of simple surveys, elaboration and interpretation of tables, bar graphs and pictograms.	No changes from the 1985 Reform.
2	Design and application of simple surveys, elaboration and interpretation of tables, bar graphs and pictograms.	Added to the contents in 1985 Reform: Notion of chance; impossible, sure, and probable events.
3	Design and application of surveys, elaboration and interpretation of tables, bar graphs and pictograms.	Added to the contents in 1985 Reform: Notion of chance, random experimentation, elaboration and interpretation of tables with data from a random experimentation.
4	Design and application of surveys, elaboration and interpretation of tables, bar graphs and pictograms.	Added to the contents in 1985 Reform: Finding of the mode in non-grouped data.
5	Arithmetic mean, mode, median. Notion of chance; impossible, sure, and probable events. Computation of probabilities based on classic definition.	Removed from the contents in 1985 Reform: Mode (moved to Grade 4) and median (moved to Grade 6).
6	Frequency distributions, elaboration and interpretation of tables, bar and circular graphs. Computation of probabilities based on the classic definition. Tree diagrams.	Added to the contents in 1985 Reform: Elaboration and interpretation of histograms, computation of the arithmetic mean and the median; basic ideas about counting.

objectives, within which one is explicitly related to the strand "Statistics and Probability": "Based on the study of notions of statistics and probability, the student is able to interpret situations from the daily life" (ME, 1987, 1997). Moreover, for each grade, conceptual, procedural and attitudinal contents, as well as competencies and assessment indicators, are outlined. Since the focus of this research is the secondary education, these objectives are not included here for discussion.

2.2.1.2 Methodological guidelines provided by the Venezuelan elementary school mathematics curriculum

For the First stage of elementary school, the current Venezuelan mathematics curriculum for elementary school does not provide any explicit methodological guidelines on how to teach the contents included in the strand “Statistics and Probability”. Then, since only a general objective; conceptual, procedural and attitudinal contents; and competencies and assessment indicators related to this strand are outlined, teachers have wide latitude for interpreting, instructionally deploying, and assessing such contents. In the case of the Second stage of elementary school, the current Venezuelan mathematics curriculum for elementary school provides some general orientations for teachers to implement it.

Such orientations, while promoting a student-centred lecture environment, also asks teachers to foster in students not only arithmetic and spatial abilities, but also the capacity to look for, verify and order information; the ability to find a particular solution from different solving methods; embed the teaching of mathematics into situations with cultural and social interest to the students, as well as into the daily life of the students; and the incorporation of new technologies (i.e., calculators and computers) as tools for simplifying calculations, looking for patterns, and carrying out mathematical experiments.

2.2.2 Statistics in the Venezuelan secondary school mathematics curriculum

The reform to the secondary school mathematics curriculum that took place at Venezuela in 1972 represents a milestone in the education system of the country: for first time, the study of topics on statistics and probability was introduced in the compulsory education. In that reform, which was influenced by the "New Math" movement, statistical topics and basic notions of probability were included in the Diversified cycle of secondary school. The next—and last—reform to the secondary school mathematics curriculum took place in 1990. Such reform make substantial changes to the arrangement and number of statistical topics studied in the Diversified cycle of secondary school. Regarding the Basic cycle of secondary school, it was considered as the Third stage of Elementary Education until 2009, when the new Organic Law of Education was enacted (D'Amico, Loreto & Mendoza, 2011). For that reason, the statistical contents studied at the first three years of secondary education in Venezuela are still those prescribed by the curricular reforms to the elementary school mathematics curriculum of 1985 and 1996. Table 2 shows the statistical contents after the last two curricular reforms to the Basic and Diversified cycles of secondary school.

The mathematics curriculum for the Basic cycle of secondary education does not explicitly specify any particular mathematical strand, like it is done at elementary school level. Nevertheless, five content domains can be appreciated: "Numerical sets and Algebraic Expressions", "Functions", "Geometry", "Statistics and Probability" and

Table 2: Statistical contents as in the last two reforms to the secondary school mathematics curriculum in Venezuela

Grade	1985 Reform	1996 Reform
7	Grouping data in intervals. Frequency distribution, absolute and relative frequency. Elaboration and interpretation of absolute frequency histograms. To solve problems applying basic ideas of chance and probability. Tree diagrams.	No changes from the 1985 Reform.
8	Computation of the median and the mode of frequency distributions of grouped and non-grouped data. To solve problems using the median and the mode of a frequency distribution of grouped and non-grouped data. To identify independent events. Computation of the compound probability of independent events.	No changes from the 1985 Reform.
9	To solve problems applying basic notions of statistics (arithmetic mean and mode of grouped and non-grouped data). To solve problems applying basic notions of probability (chance, classic probability, tree diagrams).	No changes from the 1985 Reform.
Grade	**1972 Reform**	**1990 Reform**
10	Random experiments. Statistical variable. Measurement scales. Sampling and population. Absolute and relative frequency. Frequency distribution of grouped data. Ogives and histograms. Measures of central tendency. Measures of variation.	All topics on statistics from the 1972 Reform were suppressed.
11	Probability, counting methods, combinatory methods, probability of co-occurrence of two independent events, conditional probability, application of Bayes' rule.	Added to the contents in 1985 Reform: Applications of Newton's binomial. Computation of the arithmetic mean, mode, median, range, variance, standard deviation, quartiles, deciles, percentiles. Normal curve.

"Introduction to Informatics". At the Diversified cycle of secondary school, the mathematics curriculum divides into five units of instruction the topics to be studied at each grade. Then, mathematics contents in Grade 10 are organized into "Real Functions", "Trigonometry", "Vectors in the Plane", "Set of Complex Numbers $\mathbb{C}$" and "Progressions (Number Sequences)"; while in Grade 11 mathematics contents are organized into "The $\mathbb{R}^3$ Vector Space", "Polynomials", "Inequalities", "Geometry" and "Probability, Statistics and Combinatorial Theory".

As the mathematics curriculum for elementary school, this one is also spiral in nature, for which the statistical ideas are present in the secondary school mathematics curriculum are revisited from grade to grade in successively sophisticated ways rather than repetitively (Lappan et al., 1996), the emphasis being on developing and refining application and problem solving skills.

2.2.2.1 Objectives and contents in the Venezuelan secondary school mathematics curriculum

Venezuelan mathematics curriculum for the Basic cycle of secondary school outlines thirty-eight overall objectives, within which five are explicitly related to statistics and probability (ME, 1987, 1997). Moreover, in the current mathematics curriculum for the Basic cycle of secondary school, for each grade, each instructional content is related to an overall objective, followed by specific objectives, which can be appreciated in Table 3.

Table 3: Overall and specific objectives related to the statistical contents included in the current mathematics curriculum for the Basic cycle of secondary school in Venezuela

Grade	Overall Objective	Specific Objectives
7	XI. To apply the concept of probability when posing and solving problems XII. To study basic notions of descriptive statistics.	28. To solve problems in which basic notions of probability could be used. 29. To represent events from a random event through tree diagrams. 30.1. To group statistical data in class intervals. 30.2. To determine the absolute and absolute cumulative frequency in a collection of grouped data. 31. To make absolute frequency histograms.
8	XIII. To study basic notions of compound probability. XIV. To study basic notions of descriptive statistics.	26.1. To identify when two events are independent. 26.2. To compute the compound probability of independent events. 27.1. To compute the arithmetic mean and the mode of a distribution of grouped data. 27.2. To solve problems in which the arithmetic mean and the mode of a distribution of grouped data could be used.
9	IX. To solve problems in which basic notions of statistics and probability could be used.	24.1. To solve problems in which basic notions of statistics could be used. 24.2. To solve problems in which basic notions of probability could be used.

In the case of the Diversified cycle of secondary school, the current Venezuelan mathematics curriculum specifies at its beginning the overall objectives for each grade, with the specific objectives stated at the beginning of each unit. Table 4 shows the overall and specific objectives related to the teaching of statistics present in the current secondary school mathematics curriculum in Venezuela.

Since objectives state the intended learning outcome to be demonstrated by the student after appropriate opportunities to learn (Krathwohl, 2002), it would be interesting

Table 4: Overall and specific objectives related to the statistical contents included in the current mathematics curriculum for the Diversified cycle of secondary school in Venezuela

Grade	Overall Objective	Specific Objectives
10	No topics on statistics and probability are included.	
11	To learn fundamental notions of probability theory to allow students to model situations of uncertainty.	5.1. Probability • The student will understand fundamental notions of probability theory. • The student will learn the concept of chance and will use it to model situations in which uncertainty or lack of knowledge are present. • The student will solve problems embedded in simple sample spaces, by using counting techniques and applying knowledge of combinatorial theory. 5.2. Newton's Binomial Theorem. • The student will learn the general formula of Newton's Binomial Theorem and will apply it, as well as knowledge of combinatorial theory, to expand powers of a binomial. 5.3. Numerical Methods. • The student will be able to describe, by the means of characteristic values, distributions of probability associated to experimental data.

to check the verbs used in the entire Venezuelan mathematics curriculum for secondary school. Through careful consideration of action verbs, I believe it is possible to accurately interpret the instructional action intended by the word that was chosen precisely for the expectation. Moreover, verbs are used in many mathematics programs to inform teachers about all instructional and assessment decisions (van de Walle, 2004, p.13); that is, verbs tell the teacher whether the teaching requires direct instruction, student experimentation and discovery, or a demonstration of problem-solving strategies. Therefore, since the verbs in the curriculum standards indicate the level of critical thinking expected of students, an analysis of verbs used in the Venezuelan secondary school mathematics curriculum will indicate what kinds of cognitive demands are stressed by them. In order to perform such analysis, the six hierarchical thinking levels in Benjamin Bloom's cumulative hierarchical theoretically-based taxonomy framework in the cognitive domain (under the headings of: knowledge, comprehension, application, analysis, synthesis, and evaluation) will be used (Athanassiou, McNett & Harvey, 2003; Anderson, 2005; Heward, 2006). The verbs used to describe the cognitive demands in the Venezuelan secondary school mathematics curriculum, as well as the analysis of such verbs using Bloom's taxonomy as assessment blueprint, are shown in Table 5.

Table 5: Analysis of the verbs used in the Venezuelan secondary school mathematics curriculum, using revised Bloom's taxonomy framework

Level	Verb	Frequency	Thinking level
Basic Cycle	Apply	1	Application
	Pose	1	Synthesis
	Solve	5	Application
	Study	3	Knowledge
	Use	5	Application
	Represent	1	Application
	Group	1	Analysis
	Determine	1	Evaluation
	Make (a graph)	1	Application
	Identify	1	Knowledge
	Compute	2	Application
Diversified Cycle	Learn	3	Knowledge
	Model	2	Synthesis
	Understand	1	Comprehension
	Use	2	Application
	Solve	1	Application
	Apply	2	Application
	Expand	1	Application
	Describe	1	Comprehension

From the 35 verbs used in the overall and specific objectives present in the Venezuelan secondary school mathematics curriculum, 30 of them (85.7%) are action verbs from the lower thinking levels (knowledge, 20.0%; comprehension, 5.7%, and application, 60.0%). This means that 85.7% of the cognitive demand that this standards place upon students are related to lower order thinking skills, which require from students to apply the knowledge ordinarily, which is not always associated with having high abilities. For those verbs with a cognitive demand related to higher order thinking skills, those related to "Synthesis" are the ones with higher frequency (analysis, 2.6%; synthesis, 5.7%; and evaluation, 2.6%). These skills, sometimes known as critical thinking skills, allow students to think convergently and divergently to investigate challenges and problems as well as to think in complex and creative ways (O'Tuel & Bullard, 1995). Then, since higher order learning is non-algorithmic, so there is no one pattern or procedure that must be followed (Resnick, 1987), and is characterized by open mindedness, the ability to explore alternative solutions is essential to thinking effectively (Tishman, Jay & Perkins, 1993), it is fair to say that, in regards to the learning of statistics, Venezuelan secondary school mathematics curriculum does not promote the critical thinking (stressed in the definition of statistical literacy), but focus on knowledge, comprehension and/or application instead.

2.2.2.2 Methodological guidelines provided by the Venezuelan secondary school mathematics curriculum

In the case of the Basic cycle of secondary school, the last two reforms to the current Venezuelan mathematics curriculum—in 1985 and 1996—mentioned quite detailed methodological suggestions for teachers in a topic-by-topic basis. Such suggestions promote a student-centred lecture environment, as well as emphasize the significance of trying to improve students' comprehension of statistics and probability contents by relating them to students' own environmental and social context (Salcedo, 2006). The use of concrete materials and drawing upon students' experiences as a starting point for teaching is suggested in both curriculums in order to promote more active participation in classroom activities. A summary of the suggested methodological strategies appearing in the current mathematics curriculum for the Basic cycle of secondary education, in relation to the objectives related to descriptive statistics, is shown in Table 6:

Table 6: Summary of the statistics-related methodological strategies provided by the Venezuelan mathematics curriculum for the Basic cycle of secondary education

Grade	Suggested Methodological Strategies
7	a) Motivate the process of data organization. b) Propose data gathering to students. c) Organize data. d) Obtain rules for a better data organization. e) Organize data in class intervals. f) Determine the absolute cumulative frequency. g) Inform students that organizing data in class intervals with their respective frequencies is called "frequency distribution". h) Guide the interpretation of data.
8	a) Review the concepts to be used (measures of central tendency). b) Obtain the midpoints of a frequency distribution. c) Determine the arithmetic mean for a grouped data set. d) Determine the mode of a distribution. e) Solve exercises and problems.
9	a) Solve problems regarding the environmental and social dimensions, in which measures of central tendency could be used. b) Solve problems regarding the environmental and social dimensions, in which notions of probability could be used. c) Solve problems using the reviewed contents in a) and b), in which the student justifies the used procedure and operations.

In the reform to the mathematics curriculum of 1985, the Ministry of Education also developed a teaching guide for the grades that are now known as Basic cycle of the secondary school. In such document, suggestions for introducing concepts and develop the expected skills in students are provided, which supplied strategies and guided practice for the teaching of a topic that was not taught before at that level in Venezuela. This document explicitly mentions, among others, some of the aspects that are sought to be reached through the study of mathematics at lower secondary level, such as (1) to guarantee students' acquisition of knowledge, capabilities and skills needed to their incorporation to the working life, and (2) to give a relevant place to problem solving, particularly tackling problems with statements that arouse interest or curiosity in the students. Furthermore, this document also mentions that the teacher is expected to stimulate and strengthen children's mathematics learning through practical experiences linked to knowledge about daily life, and in particular, environmental situations interesting to the students, through which they could appreciate and value nature and natural resources. Since this reform gave heavy emphasis on problem solving in the curriculum, it is natural that its teaching guide defined what a good problem is. According to this document, a good problem is one that has a real-life content, which formulation should start from the following criteria:

- Pupils' interests and activities.
- Life style of the community, region and country.
- Games and folklore.
- Goals of the lower secondary school curriculum.

- Current national and international events.

Then, here is more evident the student-centered nature of the Venezuelan mathematics curriculum for Basic cycle of secondary school.

The 1990 reform to the Venezuelan mathematics curriculum for Diversified cycle of secondary school included a special section devoted to methodological suggestions for teachers. Despite being general suggestions, this represented a considerable improvement in the curriculum, in comparison to the previous reform undertaken in 1972, in which only a handful of activities were recommended to teachers for their use in the classroom. A summary of the suggested methodological strategies appearing in the current mathematics curriculum for the Diversified cycle of secondary education, in relation to the objectives related to descriptive statistics, can be appreciated in Table 7.

Table 7: Summary of the statistics-related methodological strategies provided by the Venezuelan mathematics curriculum for the Diversified cycle of secondary education

Grade	Suggested Methodological Strategies
10	No topics on statistics and probability are included.
11	– We suggest to the teacher to solve problems with statements that would be of particular appeal to students. – Finally, we recommend to present student with innovative problems related to the daily life, which motivate them to analyze situations in which statistics could provide new parameters for the interpretation of results. These problems will be particularly useful to the student if they are solved using school-related data, economical phenomena, population analysis, averages, production data, and so on.

2.3 Research on statistics education at secondary school level in Venezuela

In the case of Venezuela, there is a paucity of research efforts investigating mathematics teachers' professional competence—i.e., professional knowledge and affective-motivational characteristics—to teach statistics at any school level, despite the increasing international need for understanding on this issue. In fact, the few research attempts on this area in Venezuela have focused on investigating aspects of teachers' content knowledge and teaching practice. In fact, among those few research efforts, there are some of them drawing conclusions about the current state of teaching and learning of statistics at school level in Venezuela by extrapolation from research on either individual perceptions of teacher preparation to teach statistics or students' performance and achievement in statistics at tertiary education. For example, León (2005), in her theoretical essay about statistical literacy and development of statistical thinking at school level, states that in Venezuela, in practice, the statistical contents included in the mathematics curricula are mostly dealt with in a mechanistic way, through the application of mathematical formulas, based on her experience as a teacher and teacher educator, with no data is given to support such a claim. According to her, Venezuelan mathematics teachers tend to conceive statistical literacy in a rather simplistic way, as the mere action of teaching repetitive calculations by well-defined procedures, instead of as a multifaceted set of skills that will prepare students to participate, understand and transform society; to be critical consumers of contextualized information; to be able to take a politically and socially compromised position in issues expressed in statistical terms; and to be able to understand clearly the real world, the social order and the human behavior, in which variability and uncertainty are omnipresent components. This view of teaching statistics is what Garfield (1995) calls a "cookbook" fashion; that is, teaching statistics focusing on mechanics instead of the logic of applications.

In a similar way to León (2005), Tapia (2011) reflects on the teaching of statistics at school level in Venezuela in his didactical proposal for a new way of teaching statistics at university level for students in the major of computer engineering. He highlights the fact that new students admitted in the different engineering undergraduate programs at Venezuelan universities—particularly at UNELLEZ—demonstrate a disturbing lack of basic knowledge on statistics, especially those related to descriptive statistics, which were supposed to be studied at elementary and secondary school. Based on this, Tapia affirms that in Venezuela, the teaching of statistics at elementary and secondary schools is limited to its mathematical aspects and carried out by professionals with a deterministic training, being this the reason why the random phenomena studied at school lack of importance and are covered only superficially or discarded at the end of the school year, resulting in students entering universities with a total lack of basic knowledge on statistics.

The research efforts made in examining aspects of Venezuelan mathematics teachers' professional competence to teach statistics at school level have been focused on the elementary school. In that regard, the studies carried out by Parra (1997), Sanoja (2012), and Sanoja and Ortíz (2013) were found.

Parra (1997), based upon her diagnosis of the situation of the teaching of statistics and probability at Second stage of elementary school (Grade 4, 5 and 6) in public and private institutions at the municipality of Maturín, Monagas State, concluded that teachers required to develop methodological strategies to achieve the goals of teaching statistics an probability at school, in particular to develop student's thought processes such as observation, comprehension, analysis, classification and synthesis, among others, as well as a participative, reflexive and critical dynamics in the classroom.

Sanoja (2012) carried out a study aimed to examine the process of teaching statistics in elementary school at Venezuela, focusing on aspects such as content knowledge, teaching planning and practice, and statistical thinking—in particular, on the different ways of thinking about data and its context in order to explore, properly analyze and understand them. To that end, lesson observation, interviews and surveys were carried out on all the teachers working at a government elementary school in Maracay, Aragua State. Sanoja concludes that teachers employ a traditional didactic approach, heavily influenced by the use of textbooks, which were found to show a trend of traditional didactics and even conceptual errors in some cases. In addition to this, teachers were found to lack of sound training in statistics and its teaching, which results in teachers teaching statistical contents as they see and understand them. This represents a problem, since there was evidence of misconceptions and difficulties regarding data visualization (basic concepts and data organization) and measures of central tendency (mean, median and mode). The surveyed teachers also expressed their interest in improving their teaching practice in statistics instruction.

Sanoja and Ortíz (2013) carried out a study to determine what statistical content knowledge was held by forty-eight teachers working at two government elementary schools in Maracay, Aragua State. Findings show evidence of misconceptions and difficulties in areas such as data visualization (basic concepts and data organization), measures of central tendency (mean, median and mode) and probability. In particular, the following shortcomings were observed: misconceptions regarding the concepts of variable and data; errors in associating particular kind of graphs with variables; difficulty reading and understanding line graphs and histograms; lack of understanding of the meaning of arithmetic mean, even though there was evidence that teachers knew how to compute it; lack of knowledge on how to calculate the median; mixing up the terms of mean and median; and lack of understanding of the meaning of possible event. Nevertheless, it should be emphasized that just as teachers showed some weaknesses in the use of some statistical concepts, they also showed some strengths, such as procedural mastery in constructing frequency distribution tables and double-entry frequency tables; mastery in constructing, reading and interpreting bar graphs; and mastery in the construction of pictograms.

A pair of studies on examining some aspects of Venezuelan mathematics teachers' professional competence to teach statistics at secondary school level was found. These were carried out by Salcedo (2008) and Santamaría and Sanoja (2013).

Salcedo (2008) surveyed forty-eight teachers working at the Basic and the Diversified cycles of secondary school at different institutions in the Venezuelan Capital District, in order to study some indicators of the current situation of teaching and learning of statistics at secondary school level in Venezuela. Results indicate that most of the surveyed teachers are not teaching the contents related to statistics and probability, and the few who do, ask students for a written report about definitions of statistical concepts. This observed practice of asking students to make written reports to cover the unit of statistics and probability might be considered as evidence that teachers attach little importance to the teaching of such contents, considering them as expendable contents rather than as literacy elements desirable to be attained by every citizen. Moreover, most of the surveyed teachers admitted the lack of training to work with these topics. Salcedo finishes his article making a call for research in the area of statistics education in Venezuela at all levels, in order to improve the quality of teaching and learning of statistics at all levels.

Santamaría and Sanoja (2013) interviewed, observed the lessons of, and surveyed eight Grade 8 mathematics teachers at the "Priest Manuel Arocha" Bolivarian Secondary School in town of Tinaquillo, Cojedes State. From such study, they found out empirical evidence that, despite statistics being a fundamental strand in the Venezuelan mathematics curriculum, in practice statistics is a neglected topic by teachers, who do not adequately address statistical topics at classroom; lack of stimulating methodological strategies; overlook the statistical topics, basically because they are at the end of the textbooks; teach in a traditional way, giving priority to algebraic contents; and acknowledge to be unaware of the statistical topics present in the mathematics curriculum. Despite all this, the surveyed teachers seems to realize that teaching statistics in a daily life context makes students to understand better both the statistical concepts and their own reality, so teachers acknowledge their own need to get training about statistics and its appropriate way of teaching.

The brevity of research on Venezuelan mathematics teachers' professional competence to teach statistics at secondary school level suggests that this is an area open to investigation in further studies, and highlights the need of a more solid foundation, providing in that way motivation for the present study. Therefore, on the basis of formulations and prior research findings reported in this chapter, the present study attempts to fill this void in research by theoretically and empirically examining this issue, with the purpose of shedding light on the characteristics of the knowledge base of SKT, conceptions of variability, and statistics-related beliefs held by Venezuelan in-service secondary school mathematics teachers.

CHAPTER 3:

Literature Review

In this section, the author examines research relevant to the present study, in particular, on the ideas of statistical literacy, variability, mathematics teachers' professional competencies, and teachers' statistical knowledge for teaching. The definition of statistical literacy, its characteristics as well as its societal and educational importance for all individuals in today's knowledge-based society are briefly discussed. Several aspects related to the concept of variability, such as its definition, how it is measured or estimated, and how it is conceptualized by students and teachers according to previous researches, are also discussed. During the discussion of mathematics teachers' professional competencies, it is presented a brief overview of research on the definition of this construct, as well as on the constructs of *subject matter knowledge* (SMK), *pedagogical content knowledge* (PCK), *mathematical knowledge for teaching* (MKT), and teachers' beliefs. In addition, the author discusses the construct of statistical knowledge for teaching; particularly the models proposed to date that employ the constructs of SMK, PCK and MKT. Finally, specific gaps in statistics education research literature, along with areas in need of further attention in research, are considered.

3.1 Statistical Literacy

Moore (1998a) asked "What statistical ideas will educated people who are not specialists require in the twenty-first century? The answer of this question is the concern of statistical literacy. A review of the statistics education literature shows that many statistics educators, researchers, national councils and education boards had listed the basic requirements, or learning objectives, which must be satisfied for someone who is statistically "literate." For example, Watson (1997) identifies three stages as components of the "ultimate aim" of development of statistical literacy:

1. the basic understanding of statistical terminology,
2. the understanding of statistical language and concepts embedded in a context of wider social discussion, and
3. the development of a questioning attitude which can apply more sophisticated concepts to contradict claims that are made without proper statistical foundation.

For Moore (1998a, 2001), statistical literacy involves the application of the following "big ideas:

- The omnipresence of variation,
- Conclusions are uncertain,
- Avoid inference from short-run irregularity,
- Avoid inference from coincidence,
- Beware the lurking variable,
- Association is not causation,
- Where did the data come from? and

- Observation versus experiment.

Also Gal (2000) has identified characteristics of a scientific study that a consumer of information should be able to discuss at a basic level:

- the type of study used,
- the sample that was selected,
- the measurements that were made,
- the statistics that were generated from the data,
- the graphs (visual displays) that were generated from the data,
- any probability statements that were made based on the data,
- claims that were made based on the data,
- the amount of information that was provided to the consumer, and
- the limitations of the study.

Utts (2003) provides seven key statistical topics that statistics students should encounter and have been found “to be commonly misunderstood by citizens, including the journalists who present the statistical studies to the public” (p. 74). These are the following:

1. understanding when a cause and effect relationship exists,
2. the difference between statistical significance and practical significance,

3. the difference between not finding an effect and the power of the study,
4. bias that can occur in surveys,
5. understanding that coincidences are not so coincidental,
6. understanding that conditional probability and its inverse are not equivalent, and
7. knowing that normal is not equivalent to average.

Note that a discussion of each of the items listed above by these authors begins by understanding the terminology and identifying each characteristic within the context of the problem. At the next level, the individual would be asked to describe the results of study by interpreting the results. Students may be asked to produce data on a similar study. Then they might be asked to evaluate the study (which involves critical thinking, as well as questioning the study at every phase). Finally, the student may be asked to communicate this information to peers. Some of these tasks require basic statistical literacy, and others require higher order knowledge skills, such as statistical reasoning and thinking.

Additional lists of requirements or learning outcomes for statistical literacy had been provided by other researchers (e.g., Cobb, 1992; Moore, 1998a, 1998b; Garfield, 1999). Each list seems to include two different types of learning outcomes for our students: having a basic foundational understanding of statistical terms, ideas, and techniques, and being able to function as an educated member of society in this age of information.

As can be noticed in some of the perspectives of statistical literacy that different authors points out, all of them are related to kinds of statistical skills which are needed by people in everyday life (e.g., Evans, 1992). Hence, the definition given by Gal (2004) nicely summarizes, in the authors' opinion, the ideas about statistical literacy that were previously stated:

> "the term *statistical literacy* refers broadly to two interrelated components, primarily (a) people's ability to *interpret and critically evaluate* statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to *discuss or communicate* their reactions to such statistical information, such as their understanding of the meaning of the information, their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions. " (ibid., p. 49, emphasis in original)

Gal (2002, 2004) also emphasizes that the skills related to statistical literacy are based simultaneously on the interaction between a dispositional component (as personal experiences and beliefs) and a knowledge component (as statistical, mathematical, and context knowledge), as well as he highlighted the need for statistical literacy for all citizens who interpret statistics in various everyday situations. For example, he suggests that when people read statistics from media they have to make inferences, quite often in the presence of irrelevant or distracting information, and perhaps they also have to apply mathematical operations to data contained in graphs. Figure 1 illustrates Gal's perspective of statistical literacy.

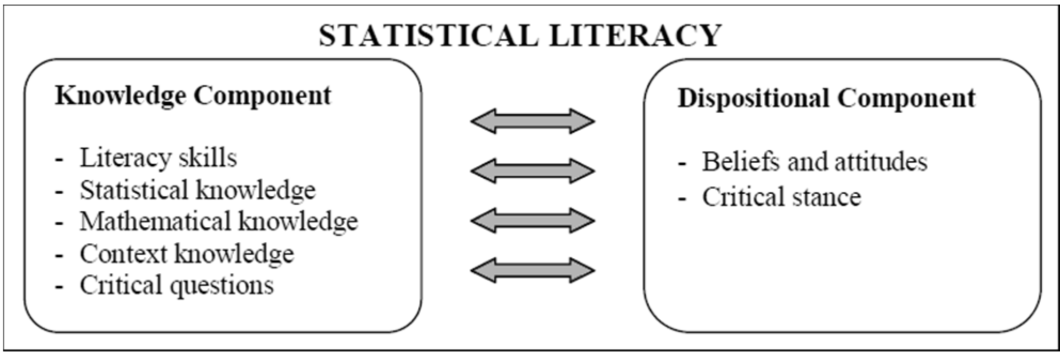


Figure 1: A framework for statistical literacy, according to Gal (2002, 2004)

Figure 1 represents two ranges of elements which when combined can enable readers to understand statistical messages. On one side of the diagram there are *knowledge elements* which involve *cognitive* components of the statistical literacy (e.g., rational understanding of the data such as knowing how to decode and make calculations about it). On the other side *dispositional elements* are presented which comprise a range of 'non-cognitive' aspects (e.g., a person who interprets a graph can have knowledge, experiences and beliefs which might differentiate his/her interpretation of the graph). According to Gal, statistical literacy is based on the interaction of the components which comprise each range of elements. Gal's statistical literacy model underlines that the academic or formal schooling background is not the only determinant of use of statistical skills, as it was discussed in other studies (e.g., François, Monteiro, & Vanhoof, 2008). To develop statistical literacy, it may be necessary to work with learners in ways that go beyond instructional methods currently in use. To implement all knowledge bases supporting statistical literacy, topics and skills that are normally not stressed at school may have to be addressed (Gal, 2004).

It is becoming more widely recognized among the mathematics education community that in today's knowledge-based society, no student should leave high school without engaging in the study of statistics. Statistical literacy is essential for all students, regardless of what occupation they may choose to pursue (Gal & Garfield, 1997). Statistics and statistical literacy play a key role in shaping policy in a democratic society (Wallman, 1993). Several professional organizations had recognized the key role statistics play in our modern society; for example, in 2000 the National Council of Teachers of Mathematics recommended the promotion of teaching and learning of statistical topics, concepts and procedures across all the grades, so that "by the end of high school students have a sound knowledge of elementary statistics" (National Council of Teachers of Mathematics [NCTM], 2000, p. 48).

The need for comprehensive statistical education at all grade levels and modernizing statistics education has been recognized, in the last two decades, in several countries around the world, like the United States, Australia, the United Kingdom, New Zealand, and Japan, among others (e.g., NCTM, 1989, 2000; American Statistical Association [ASA], 1991; American Association for the Advancement of Science [AAAS], 1993; Australian Education Council, 1994; School Curriculum and Assessment Authority & Curriculum and Assessment Authority for Wales, 1996; United Kingdom Department for Education and Employment, 1999; Ministry of Education of New Zealand, 2007; Ministry of Education, Culture, Sports, Science and Technology of Japan, 2008a, 2008b, 2009). The common thread those reform efforts in statistics education has been the emphasis on statistical literacy and thinking (Cobb, 1992; Snee, 1993; Garfield, Hogg, Schau, & Whittinghill, 2002; Mathematical Association of America [MAA], 2004). Instructors of introductory level courses want their students to understand statistical terms, symbols, graphs, and fundamental ideas, which the Guidelines for Assessment and Instruction in Statistics Education reports' authors consider to be statistical literacy. Along with statistical literacy, students in those courses should be able to understand the omnipresence of variability in statistics, and the quantification and explanation of variability (Guidelines for Assessment and Instruction in Statistics Education [GAISE], 2005; Franklin et al., 2007). Therefore, in the late reforms of all the mathematics curricula in those countries, the lack of statistics and the overemphasis on measures of central tendency in such curricula pointed out by several researches (e.g., Shaughnessy, 1992; Shaughnessy, Watson, Moritz, & Reading, 1999) have been partially displaced by the incorporation of statistical objects in which variability can arise, such as data sets, samples, probabilistic experiments, statistical graphs, and distributions. This is in concordance with the general agreement in the research community, who report that the reform movement in statistic education would be more successful in achieving its objectives if it put more emphasis on helping students to build solid intuitions about variation and variability, as well as on its relevance to statistics (e.g., Shaughnessy, 1992; Ballman, 1997.)

3.2 Variability

Some recent research has attempted to be more specific about the difference between variation and variability, since many research studies use these terms interchangeably, and assume them as having self-evident definitions. Reading and Shaughnessy (2004) distinguish the difference between the two terms as follows:

> "The term variability will be taken to mean the [varying] characteristic of the entity that is observable, and the term variation to mean the describing or measuring of that characteristic. Consequently, the following discourse, relating to "reasoning about variation," will deal with the cognitive processes involved in describing the observed phenomenon in situations that exhibit variability, or the propensity for change" (p. 202).

Hence, variation is a *measurement* that describes the dispersion of data points; that is, how data deviates, while variability is "the quality of nonuniformity of a class of entities" (Hopp & Spearman, 2001, p. 249); that is, the *description* of how much variation is present in the data, how spread out the data is.

However, reasoning about variation and reasoning about variability are not always differentiated this way in the literature. So, *think about variability* includes several things such as *reasoning about measures of variation*, how they are used as a tool and why they are used in certain contexts (Makar & Confrey, 2005).

In statistics and statistical problem solving, the importance of the role of variability has been recognized and documented by statistics educators and researchers around the world, primarily because all the statistical objects—such as distributions, data sets, samples and statistical graphs, among others—vary and/or represents variability. For example, David S. Moore, the renowned statistician and former president of the International Association for Statistical Education (IASE) and the American Statistical Association (ASA), describes statistical thinking as recognizing the omnipresence of variability and considering appropriate ways to quantify and model the variability of data, and opines that the discipline of statistics arises from the need to deal with that omnipresence of variability in data (Moore, 1990). Cobb (1991), Moore (1992, 1997) and Snee (1993) reported that the idea of variability and its associated measures—such as standard deviation—are some of the core concepts presented in an introductory course in statistics; Pfannkuch (1997) thinks that variation is a critical issue throughout the statistical inquiry process, from posing a question to drawing conclusions; Wild and Pfannkuch (1999) considered variation and variability as the heart of their model of statistical thinking; according to Shaughnessy and Ciancetta (2001), variation and variability is the foundation of statistical thinking, and the very reason for the existence of the discipline of statistics; whereas Watson & Kelly (2002, p. 1) considered that "Variation is at the heart of all statistical investigation. If there were no variation in data sets, there would be no need for statistics". Some of the studies focused on thinking and reasoning about variability had recognized the importance of students' conceptual understanding of variability as crucial to their development of increasingly sophisticated understandings in statistics (e.g., Chance, delMas, & Garfield, 2004; Garfield & Ben-Zvi, 2005; Reading & Reid, 2006; Leavy, 2006; Franklin et al., 2007; Garfield & Ben-Zvi, 2008).

3.3 Measures of Variation

The measures of variation are such concerned with the distribution of values around the mean in data. Among them, the most commonly used are the range, interquartile range, standard deviation, and coefficient of variation.

Range: The range is simply the difference between the highest and lowest value in the sample. The range has as advantage that is easy to calculate, it is easily understood by general audiences, and it can provide a very quick and general idea of dispersion. Unfortunately, it is particularly sensitive to the influence of outliers, lacks sensitivity to varying values between those extremes, and does not inform about the scores between the end points.

Interquartile range: The interquartile range is the distance between the first quartile (the point in the distribution that 25% of the sample is below) and the third quartile (the point in the distribution that 75% of the sample is below).

The interquartile range has as advantage that is not sensitive to extreme scores, it is the only reasonable measure of variability with open-ended distributions, it can be appropriately applied with ordinal variables, unlike the standard deviation, and should be used with highly skewed distributions. Nevertheless, even though it is more stable and informative than the range, the interquartile range is a terminal statistic (that is, a statistic which usefulness in advanced descriptive and inferential procedures is very limited, so can't be used for further calculations and there is little else that can be done with additional analyses with this statistic), as well as it is unfamiliar to most people.

Standard deviation: The standard deviation can be thought of as the average distance that values are from the mean of the distribution. The expressions to calculate the standard deviation for ungrouped (left) and grouped (right) data are given by the following formulas:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} \qquad s = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{n}}$$

where n is the total number of objects or data points, x_i is the i-th object or data point, and f_i is the frequency that is associated with x_i.

In the case of a normal distribution, when you know the standard deviation, it always turns out that approximately 68.26%, 95.46%, and 99.74% of the values lie within 1, 2, and 3 standard deviations around the mean, respectively.

The standard deviation has as advantage that is quite resistant to sampling variability, as well as is mathematically tractable. Nevertheless, it should not be used with highly skewed distributions, it is not a good index of variability with a few very extreme scores, and cannot be used with open-ended distributions.

Coefficient of variation: The coefficient of variation is a dimensionless number that quantifies the degree of variability relative to the mean. The typical sample estimate is given as the ratio of the sample standard deviation to the mean. Sometimes, this result is multiplied by 100 so that the ratio is expressed in terms of a percentage.

The coefficient of variation is useful when comparing the standard deviations of two variables with different units of measure. Nevertheless, when the mean of a variable is zero, the coefficient of variation cannot be calculated. Even if the mean of a variable is not zero, but the variable contains both positive and negative values and the mean is close to zero, then the coefficient of variation can be misleading. The coefficient of variation can be considered as a reasonable measure if the variable contains only positive values.

3.4 Conceptions of Variability

After reviewing and carrying out himself several studies related to how students think about, understand, and acknowledge variation and variability in the presence of a number of statistical objects that can vary—such as data, samples and distributions—, Shaughnessy (2007) identified eight ways in which students conceptualize variability, which depend on the statistical context and the students' own preferred strategies while working on statistical tasks involving variability.

The framework proposed by Shaughnessy (2007) provides a comprehensive structure for looking at how people reason about variation and variability in several statistical contexts. This framework is also useful to provide insight into how teachers think about variation and variability (González & Isoda, 2010; González, 2011; Isoda & González, 2012), and it may provide ground to develop a better teaching practice about those concepts. First, teachers frequently possess similar reasoning to their students (Hammerman & Rubin, 2004; McClain, 2002; González, 2011; Isoda & González, 2012). Second, understanding students' conceptions of variation and variability may help teachers to plan instruction (Makar & Confrey, 2005).

The conceptions identified by Shaughnessy (2007) are the following:

1. **Variability in *particular values*, including extremes or outliers**: In this conception, students focus their attention on particular data values in a graph or in a data set (Konold & Pollatsek, 2002; Shaughnessy, 2007). For example, when describing the graph of the size of the families of the students in a classroom (Figure 2), people who focus their attention in the mode (6 persons) or the outliers (18 persons) use this conception.

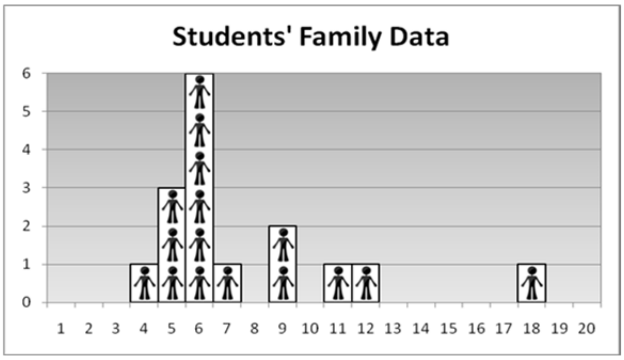


Figure 2: Stacked dotplot representing the students' family size in an American Grade 4 class (adapted from Konold, Higgins, Russell, & Khalil, 2004)

2. **Variability as *change over time***: In this conception, by using time as independent variable in graphs, students need to look for causes of variability in the dependent variable. This conception involves multivariate situations and may be a good starting point to introduce covariation (Nemirovsky, 1996; Moritz, 2004b; Shaughnessy, 2007).

When analyzing data, the role of a student or a statistician is to be a "data detective", to uncover the stories that are hidden in the data, to note the important signals in the data variability

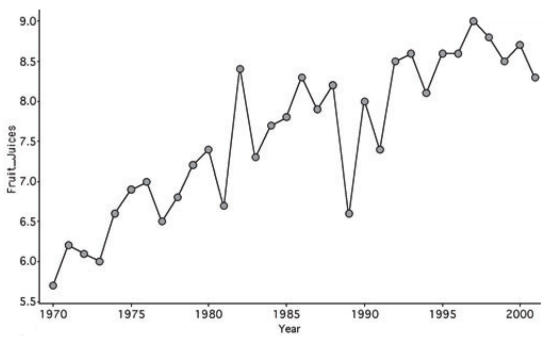


Figure 3: Fruit juice consumption in the United States, gallons per capita, over time

(Shaughnessy, 2008). Such signals are particularly evident in data that are collected over time, such as the data on annual consumption of fruit juice collected by the U.S. Department of Agriculture, shown in Figure 3 (Shaughnessy, 2008). When asked what the overall pattern in the data was, and why the data might be varying, people use this conception about variability.

3. **Variability as *whole range* – the spread of all possible values**: This conception involves the spread of an entire data set or distribution and is closely related to the concept of sample space in probability. In this conception students have begun to move away from seeing data only as individual values that vary, to recognizing that entire samples of data can also vary (Shaughnessy, Watson, Moritz, & Reading, 1999; Shaughnessy, 2007).

For example, in the task known as "The Dice Problem" (Konold & Kazak, 2008), students were asked to select from the alternatives shown in the Figure 4 the distribution we would most likely get if we rolled two dice 1000 times and plotted the sums, 2-12.

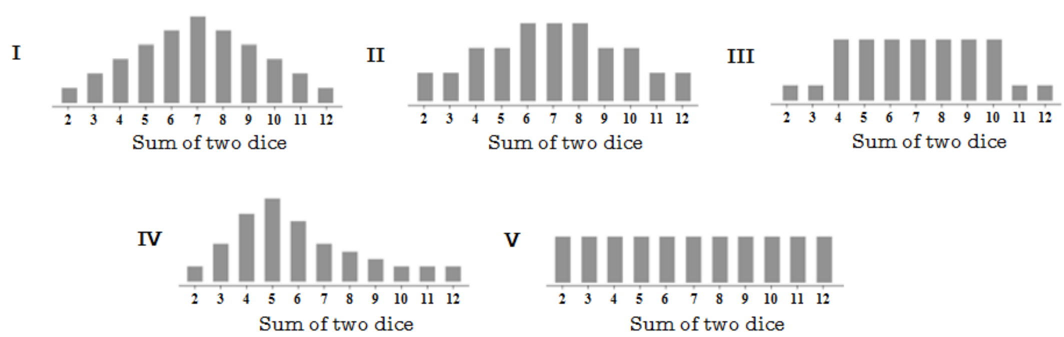


Figure 4: Five possible distributions for the sum of two dice (Konold & Kazak, 2008)

In order to choose the right option, people have to use this conception. People have to build an “expected distribution” by generating the elements in the sample space, arranging them as a distribution, and then see how well this “expected distribution” fits any of the given distributions.

4. **Variability as the *likely range of a sample***: This conception can lead to statistical tools for representing variability within or across samples such as box plots or frequency distributions. It can also lead to the concept of a sampling distribution when applied to the likely range of a distribution of means or other sample statistics. This conception of variability requires the concept of relative frequency and thus relies on proportional reasoning (Shaughnessy, Ciancetta， & Canada, 2003; Saldanha & Thompson, 2003; Reading & Shaughnessy, 2004; Shaughnessy, 2007).

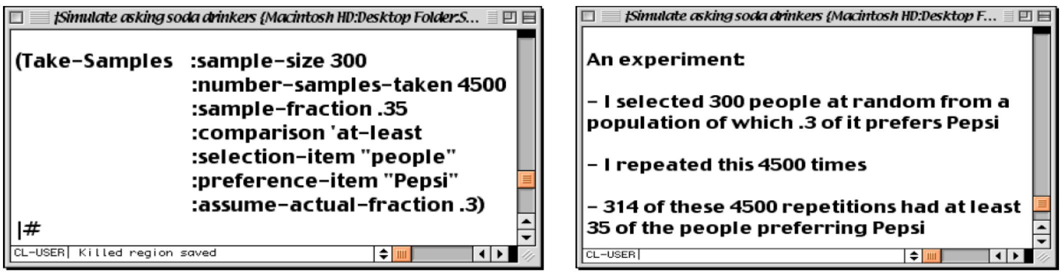


Figure 5: Part of an instructional activity to simulate multiple drawings of random samples from a population (Saldanha & Thompson, 2002)

As example, let us analyze the instructional activity posed by Saldanha & Thompson (2002, 2003), which is shown in Figure 5. This instructional activity was designed to help students make sense of computer simulations of drawing many random samples from a population. Simulation input (left) and output (right) windows were displayed in the classroom and the instructor posed questions designed to orchestrate reflective discussions about the simulations, for the purpose of answering a question like "what fraction of the time would you expect results like these?". Then, students are intended to perform repeatedly sampling from a population with known parameters through computer simulation, record a statistic, and track the accumulation of statistics as they distribute themselves along a range of possibilities; hence student doing so are using this conception of variability. So, judgments about sampling outcomes can be made on the basis of relative frequency patterns that emerge in collections of outcomes of similar samples. These themes were intended to support students' developing a distributional interpretation of sampling and likelihood.

5. **Variability as *distance or difference from some fixed point***: This conception involves an actual or a visual measurement, either from an endpoint value (as in a geometric distribution) or from some measure of center (usually the mean or median.) Here students are predominantly concerned with the variability of one data point at a time from a center rather than the variability of an entire distribution of data from a center (Moritz, 2004a; Shaughnessy, 2007).

As an example, let's see the following problem. In order to compare two samples of equal size of French fries of W Burger and M Burger, firstly we sort the fries by length in ascendant order. Secondly, we draw a line representing the mean for the fries of each company, resulting in the following graph:

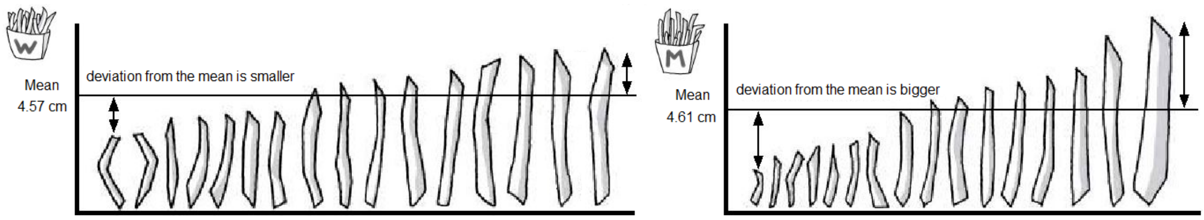


Figure 6: Comparing two samples of French fries from W Burger and M Burger (adapted from Kōgō & Tominaga, 2007)

In this case, the difference between means is only of 0.04 cm, so comparing means is not statistically significant. Moreover, we can compare, one by one, the deviation from the mean of the sorted fries, in order to determine in which case the potatoes' length has more variability. Doing so, we are using this conception of variability.

6. **Variability as the *sum of residuals***: This conception of variability involves the calculation of residuals from some fixed value in order to measure the total variation of an entire distribution of data, and provides the foundation for such concepts as standard deviation and regression analysis (Petrosino, Lehrer, & Schauble., 2003; Shaughnessy, 2007).

As an example of this conception, we can use again the problem of comparing two samples of French fries from the companies W Burger and M Burger. Suppose that students gathered data on the length of French fries for each company, and then they were asked to decide which company has the larger fries. As can be noticed in Figure 6, the mean for both types of French fries is not satisfactory as a comparison basis, but we detected that the data of one type of fries is more inconsistent (i.e., heterogeneous in length) than the other. In order to measure the inconsistency (i.e., the variability), students can compare the variation for each type of French fries by computing the absolute value of the difference between each data item and the mean, in order to get the distance of each item from the mean, and the sum of these distances would measure the total spread around the mean. Therefore, dividing the total absolute deviation by the number of data items, this will give an average absolute deviation from the mean.

$$\textit{Average Absolute Deviation} = \frac{\Sigma\left|X - \overline{X}\right|}{N}$$

This average absolute deviation gives the average distance of any data item from the mean and thus is a good measure of variation of the entire distribution of the data from W Burger and B Burger, more convenient that comparing means.

7. **Variation as *covariation or association***: This conception of variability involves the interaction of several variables, and how changes in one may correspond to (though not necessarily cause) changes in another (Batanero, Estepa, Godino, & Green, 1996; Moritz, 2004b; Shaughnessy, 2007).

An example of this conception is the following task, posed by Moritz (2004b). Students are asked to interpret a scatter graph (see Figure 7). To express appropriate acknowledgement of variation and correspondence in this task, responses needed to refer to both variables, indicate appropriate association direction, identify “noise” and “number of people”, and make appropriate use of comparative values such as “less” or “more.” Also, it is needed that students mention the imperfect nature of the covariation: “In most cases the higher the amount of noise the lower the amount of people with the exception of E.”

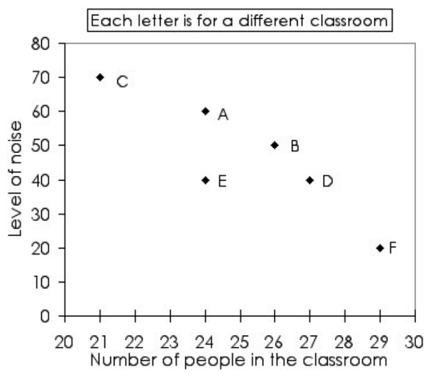
Some students were doing a project on noise.

They visited 6 different classrooms.

They measured the level of noise in the class with a sound meter.

They counted the number of people in the class.

They used the numbers to draw this graph.

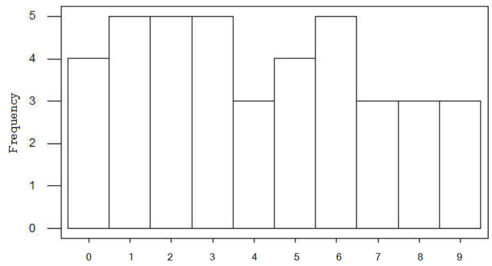


Pretend you are talking to someone who cannot see the graph. Write a sentence to tell them what the graph shows. “The graph shows...”

Figure 7: Task to assess verbal and numerical graph interpretation (Moritz, 2004)

8. **Variation as *distribution***: From this conception of variability, conceiving center, spread, and skewness as characteristics of a distribution, as well as considering theoretical probability distributions emerge to help make decisions about distributions of data, or about sampling distributions. This conception commonly arises when the variation between or among a set of distributions is compared (Bakker & Gravemeijer, 2004; Shaughnessy, Ciancetta, Best & Noll, 2005; Shaughnessy, 2007). As an example of this conception, let’s consider the problem posed by Figure 8.

The following histogram represents random numbers between 0 and 9 generated by a computer. Match this histogram to the correct summary statistics.



(a)	Mean: 6.1	Median: 3	Standard deviation: 8.91
(b)	Mean: 4.1	Median: 4	Standard deviation: 2.81
(c)	Mean: 6.8	Median: 6	Standard deviation: 4.22

Figure 8: "Matching histogram to summary statistics" activity

The variable in consideration (random numbers between 0 and 9) is a typical example of a uniform distribution. So, it will be very useful to consider the underlying theoretical probability distribution in this problem, in order to answer correctly. A uniform distribution is symmetric, so we expect that the mean and the median are the same or almost the same, and hence we discard option (a). Also, knowing that the standard deviation of a uniform distribution is given by the range divided by $\sqrt{12}$, we can identify (b) as the correct answer. We can get the same answer by translating the data in the histogram into a frequency distribution table, and then calculate the summary statistics for grouped data, but it is a bothersome calculation. Instead, considering the theoretical probability distribution of this case, as well as some characteristics of a distribution such as symmetry and skewness, is an easier way to find the right answer for this kind of problems.

3.5 Mathematics teachers' conceptions of variability

Conceptions—defined by Sfard (1991, p. 3) as the set of internal representations and the corresponding associations that a mathematical concept evokes in the individual—can be seen as lenses through which people perceive and interpret phenomena (Pratt, 1992). Hence, it is not at all surprising that people act and react to specific phenomena influenced by their conceptions (Könings, Brand-Gruwel & van Merriënboer, 2005). In particular, teachers' conceptions about the subject matter have been proved to influence students' approaches to learning and teachers' approaches to teaching (Trigwell, Prosser & Waterhouse, 1999).

Only in recent years some research works had been done on in-service teachers' conceptions of variability (e.g., Makar & Confrey, 2004; Peters, 2009; González & Isoda, 2010, González, 2011; Isoda & González, 2012). Makar and Confrey (2004), who examined four American secondary mathematics teachers in order to carry out a research on teachers' statistical reasoning when comparing two data groups, found that the conceptions of variability held by those teachers affected the way how they described and chose to compare the characteristics of the each distribution, and how they interpreted variability within a group, as well as between groups.

Peters (2009) examined 16 American high school Advanced Placement statistics teachers' conceptions of variability. From the identified conceptions of variability, Peters classified those teachers into three types: (a) those teachers focused on the design of the data and seeing variability as something that needs to be controlled; (b) those teachers focused on the data analysis and seeing variability as something that needs to be explored; and (c) those teachers focused on inference, seeing variability as something that needs to be modeled and expected. Additionally, Peters found that only five out of the 16 teachers in the study showed connected reasoning across the three types of conceptions.

González (2010, 2011) and Isoda and González (2012) analyzed the answers given by 99, 65, and 78 Japanese in-service elementary, middle, and high school teachers, respectively, in response to a survey with questions involving data variability in several contexts, in order to explore, identify and characterize the conceptions of variability held by the teachers, and to provide insights into their content knowledge about several statistical contents related to the interpretation of variability in different situations. From the results, the researchers found that surveyed teachers exhibited the same eight types of conceptions of variability identified by recent research efforts in the case of students (Shaughnessy, 2007, pp. 984-985). Moreover, the researchers found that lower level teacher responses—i.e., those which evidence a weak statistical CCK—might be concerned with holding poor conceptions of variability—such as thinking of variability focusing in particular data values, such as outliers, extremes, or only with measures of central tendency—, whereas higher level teacher responses—i.e., those which evidence a strong statistical CCK—might be concerned with holding rich conceptions of variability—such as thinking of variability with an aggregate view of data and distribution, or by focusing in both measures of central tendency and extremes in data while thinking of variability. Also, particular difficulties with variability-related concepts were spotted in the surveyed teachers at all levels, being one of them the interpretation of histograms and the translation of a histogram into other statistical objects. Teachers who exhibited this kind of difficulties often exhibited evidence of being also holding poor conceptions of variability.

3.6 Mathematics teachers' professional competencies

After reviewing several definitions of the multi-faceted term 'competence' found in the literature, Carblis (2008) realized that many of such definitions incorporated the following five key terms:

- *the idea of capacity (or capability)*, which refers to the repertoire of skills and knowledge embedded in the competent person (cf. DeSeCo Project Report, 2002, p.7; Carblis, 2008, p.23),

- *the actions of selection and application*: which are related to the ability of a person to determine which knowledge and skill(s) to select from his/her repertoire to apply in order to achieve particular outcomes (cf. Carblis, 2008, pp.23-24),

- *the notion of an attribute*: which refers to the skills and knowledge brought by an individual as an input to any work situation (cf. Carblis, 2008, p.24),

- *the specification of performance*: which is related to the actions, behaviors, or sets of actions and behaviors that may be related to defined standards and contexts (cf. Carblis, 2008, p.24), and

- *the condition of intent or purposiveness*: this requires that, for competency to be demonstrated, purposive action must be undertaken. Therefore, instinctive or reflexive responses or unconscious behaviors cannot be seen to contribute to the evidence of competence (cf. Carblis, 2008, pp.24-25).

Then, Carblis (2008) presented the following definition of ‘competence’, looking to engage all the aforementioned five key terms: “Competence or competency is the *capacity* or *capability* that underpins the *selection and application* of *attributes* through which *specified performance* is achieved by means of *purposive actions*” (ibid., p.29, emphasis in the original).

A number of educational researchers (e.g., Kautto-Koivula, 1996; Carblis, 2008; Lindmeier, 2011; Döhrmann et al., 2012; Tatto et al., 2012) agree with the opinion that professional competencies involve at least two main domains: (1) proficiencies specific to the profession, discipline or organization (which include the discipline-specific knowledge base and technical skills considered essential in the profession, and the ability to solve the type of problems encountered within the profession), and (2) general characteristics of the competent individual (among which are beliefs, conceptions, motivation, attitudes and values). This vision is well-summarized by Weinert (2001), according to whom professional competencies can be divided into cognitive and affective-motivational facets.

Under this view, both the key role played by 'knowledge', 'beliefs' and 'conceptions' in professional competencies, and the connection between 'competence', 'knowledge', 'beliefs' and 'conceptions', are very clear, since the latter three attributes can be inferred from the competent performance, and facilitate the individual's development and maintenance of professional competence. Since this separation of teachers' professional competencies in cognitive and affective-motivational facets has the advantage that, as a result, their interplay could be investigated (Lindmeier, 2011, pp. 34-35; Blömeke & Delaney, 2012, p.227), this separation is very common in the literature. An example worthy to be mentioned here is the conceptual model of mathematics teachers' professional competencies developed by the international teacher education study Teacher Education and Development Study in Mathematics (TEDS-M) (see Figure 9).

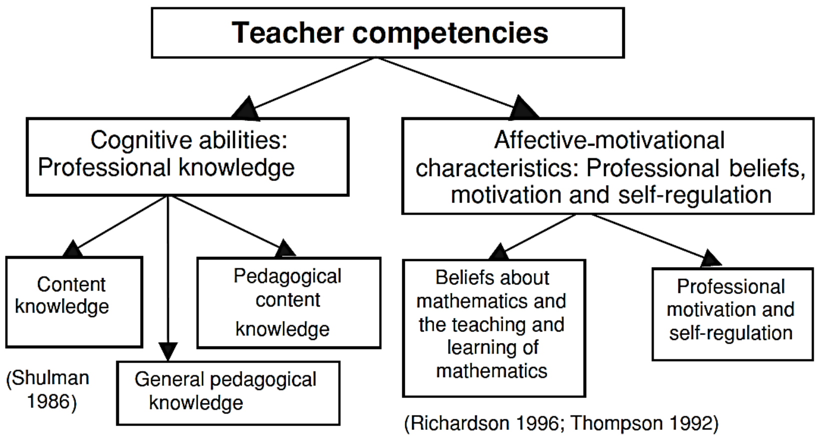


Figure 9: Conceptual model of teachers' professional competencies, according to Döhrmann et al. (2012)

In the context of the present study, the author takes the standpoint of researchers such as Döhrmann et al. (2012), Tatto et al. (2012), Kaiser & Blömeke (2013) and Goos (2013), who consider professional competencies as divided into teachers' professional knowledge in the cognitive facet, and professional beliefs in the affective-motivational facet.

3.7 Mathematics Teachers’ Professional Knowledge: The MKT Model

Several conceptualizations of teachers' mathematical knowledge entailed in teaching have been developed (for a thorough discussion on this issue, see Petrou & Goulding, 2011). Among those conceptualizations, the one developed by Deborah Ball and her team at University of Michigan not only has led to a broad consensus among the mathematics education community, but also has clarified the distinction between SMK and PCK by refining their previous conceptualizations in the literature, and has made significant progress in identifying the relationship between teacher knowledge and student achievement in mathematics.

Ball et al. (2008) developed their practice-based theory of mathematical knowledge for teaching—henceforth MKT—based on the notion of pedagogical content knowledge by Shulman (1986), and focusing on both what teachers do as they teach mathematics, and what knowledge and skills teachers need in order to be able to teach mathematics effectively. Ball and her colleagues developed this framework by examining ways in which the ideas proposed by Shulman (1986) could be operationalized in the particular case of mathematics education. As in Shulman (1986), Ball et al. (2008) proposed a model characterized by a clear emphasis in its conception of reflective practice on the content to be taught, placing the emphasis on the intellectual basis for teaching and on the transformation of SMK by teachers, instead on the traditionally emphasized subject matter content knowledge (Zeichner, 1994; Petrou & Goulding, 2011).

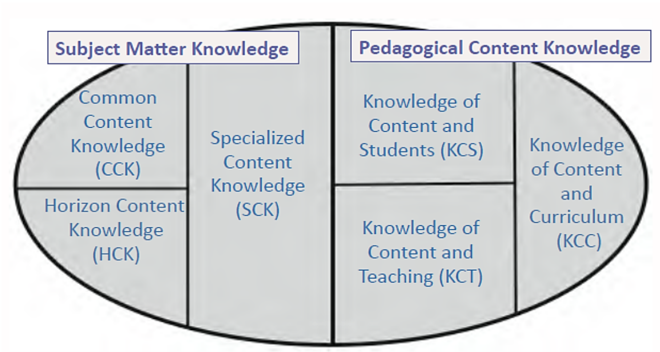


Figure 10: Domains of MKT (Ball et al., 2008)

This model describes MKT as being made up of two domains—SMK and PCK—, each of them structured in a tripartite form (Figure 10). Moreover, this model clarified the distinction between SMK and PCK, and refined their previous conceptualizations in the literature.

According to Ball et al. (2008, pp. 400-401, 404), SMK can be divided into *common content knowledge* (CCK), *specialized content knowledge* (SCK), and *horizon content knowledge* (HCK). These cognitive constructs are briefly elaborated below:

- *Common Content Knowledge (CCK)*: this construct refers to the mathematical knowledge and skills expected of any well-educated adult, which are commonly used in any setting, not necessarily the one of teaching. CCK is the "mathematics knowledge and skill used in settings other than teaching … [and thus are] not special to the work of teaching" (Ball et al., 2008). For example, the mathematics teacher, like every well-educated adult in other professions, needs to be able to use terms and notation correctly, to compute mathematical operations—such as 35 × 25—accurately, to identify the power of 10 that is equal to 1, to name a number that lies between 1.1 and 1.11, to recognize that the square is a special type of rectangle or that the diagonals of a parallelogram are not always perpendicular (Hill & Ball, 2004; Ball et al., 2008). Despite being a necessary ingredient of teachers' knowledge, CCK is considered by many researchers not sufficient for the work of teaching (Shulman, 1986, 1987, Ball et al., 2008; Döhrmann et al., 2012; Tatto et al., 2012; Kaiser & Blömeke, 2013).

- *Specialized Content Knowledge (SCK)*: this is the mathematical content knowledge specific to the work of teaching and needed in its practice, but not in the practice of other professions. This knowledge allows teachers to appraise students' methods of solving problems, assess novel approaches that students propose, and determine whether these approaches are generalizable to other problems. It also supports teachers in identifying patterns of student error, explicating why certain algorithms work or make sense, explaining a mathematical idea by selecting appropriate examples and representations, linking representations and models to their underlying meaning, evaluating students' explanations and justifications, and choosing and developing workable definitions (Hill & Ball, 2004; Ball et al., 2008). For example, although a well-educated adult is expected to be able to divide two fractions, such individual is not expected to know if alternative algorithms are suitable for dividing fractions. However, a competent mathematics teacher could benefit from possessing such knowledge.

- *Horizon Content Knowledge (HCK)*: This construct is an awareness of the connections between both the present learner experience and instructional content with the key mathematical practices and major disciplinary ideas and structures that lie ahead, on the mathematical horizon. It engages those aspects of the mathematics that, while perhaps not contained in the curriculum, are nonetheless useful to pupils' present learning, that illuminate and confer a comprehensible sense of the larger significance of what may be only partially revealed in the mathematics of the moment (Ball & Bass, 2009). Then, HCK comprises the larger perspective on mathematical ideas and practices that orient teachers to be sensitive to connections, and provides a broad view of mathematical ideas and practices. This may include the capacity to see 'backwards,' to how earlier encounters inform more complex ones, as well as how current ones will shape and interact with later ones. Moreover, it is also worthwhile to notice that HCK does not create an imperative to act in any particular mathematical direction, contrarily to CCK and SCK.

According to Ball & Bass (2009), HCK is constituted by the following four elements: (1) a sense of the mathematical environment surrounding the current "location" in instruction (e.g., factorization and modular arithmetic when teaching odd and even numbers); (2) major disciplinary ideas and structures (e.g., number systems and equations); (3) key mathematical practices (e.g., choosing representations, questioning and proving); and (4) core mathematical values and sensibilities (e.g., precision, consistency, connections, parsimony).

Furthermore, Ball et al. (2008, pp. 392, 402) presented a refined division of PCK, comprised of *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* (KCC). These cognitive constructs are briefly elaborated below:

- *Knowledge of Content and Students (KCS)*: this construct represents the teacher's amalgamated knowledge on how students come to understand mathematics. KCS intertwines knowledge of mathematical notions with knowledge of how students think or come to understand these ideas. This type of knowledge renders teachers capable of anticipating plausible student thinking trajectories, predicting student difficulties when engaged with specific mathematical ideas or processes, and hearing and interpreting students' thinking (Ball et al., 2008). Furthermore, being aware of alternative conceptions—or "misconceptions"—students are likely to hold about a topic, as well as specific difficulties they may have with learning particular content, is part of KCS. Evidently, to efficiently engage in these activities, teachers need not only understand the content, but also be familiar with students' mathematical thinking and common student misconceptions and errors. For example, knowing that students often consider 5.8 smaller than 5.67, because in the "world of whole numbers" 67 is larger than 8, helps the teacher easily recognize such errors in students' work, understand their source, and design suitable interventions.

- *Knowledge of Content and Teaching (KCT)*: this construct refers to the knowledge on how to carry out the design of instruction in order to develop mathematical understanding in students. Then, KCT aids teachers in selecting and sequencing examples to gradually lead students to develop certain mathematical ideas or in considering the relative strengths and limitations of available representations and models. For example, being aware of the different models of subtraction—e.g., take-away and comparison—and division—i.e., partitive and measurement—, and more in order to know the limitations and the affordances of each, equips teachers with the roadmap necessary for structuring teaching in ways that support learning of such mathematical ideas.

- *Knowledge of Content and Curriculum (KCC)*: this construct refers to the knowledge of "the full range of programs designed for the teaching of particular subjects and topics at a given level" (Shulman, 1986, p. 10); that is, to the knowledge that teachers have on how specific topics and concepts are offered in school curricula at a particular grade level, along with an understanding of the grade-wise relationships among them. Furthermore, KCS refers to both the goals and objectives for learning, as well as actual curricular materials to support students in meeting these goals and objectives. An example of KCC would be knowledge about what "topics and issues that have been and will be taught in the same subject area in the proceeding and later years" (Ball et al., 2008, p. 391).

Despite all the praises bestowed upon this framework, some researchers (e.g., Petrou & Goulding, 2011, p.16) highlight that this model acknowledges the role of neither beliefs nor conceptions about the subject matter in teachers' practices. This feature could be a drawback for the MKT model, since it is well documented in the literature that teachers' beliefs and conceptions are important factors affecting the work of teaching (cf. Philipp, 2007; Sivunen & Pehkonen, 2009).

3.8 Mathematics teachers' beliefs regarding statistics

In the literature, beliefs—defined by Philipp (2007) as "psychologically held understandings, premises, or prepositions about the world that are thought to be true" (p. 259)—are regarded as lenses through which we view the world. Several researchers have investigated the relation between teachers' beliefs about subject matter, teaching and learning, and teaching practice. For example, Jones and Carter (2007) highlight the major role that teachers' beliefs play in shaping their instructional practices; Keys and Bryan (2001) say that every aspect of teaching, including the instructional method, the course content, and the assessments, is influenced by teachers' beliefs, which is in tune with Pajares (1992), who asserted that teachers' beliefs influence their perceptions and judgments, and therefore they affect teachers' behavior in the classrooms. The researches carried out by several researchers, such as Korthagen and Kessels (1999) and Hancock and Gallard (2004), show that beliefs influence teachers' actions in the classroom. Finson, Thomas and Pedersen (2006) pointed out the relation between teachers' beliefs and teaching styles, agreeing in that way with Nespor (1987), who noted that, compared to knowledge, beliefs are stronger predictors of teaching behavior. Magnusson, Krajcik and Borko (1999) pointed out that, similarly to knowledge, teachers' beliefs serve as important resources, as well as constraints, in classroom teaching. Pepin (1999) acknowledged the relation between factors such as teachers' practice and their interpretations of the curriculum, their beliefs, and their understanding of students' learning processes.

Despite of this large body of research about teachers' beliefs about subject matter, only a few studies have addressed the particular case of statistics education, regardless of the calls for more research in this area made by Gal and Ginsburg (1994), Gal, Ginsburg and Schau (1997), and Batanero, Garfield, Ottaviani and Truran (2000), as well as the wake-up call provided by Shaughnessy (2007) and Pierce & Chick (2011). Among the few studies on mathematics teachers' beliefs about statistics, its teaching and learning, the one by Gal (2004) acknowledged the important role of beliefs in statistical literacy, by identifying them as a fundamental part of the dispositional elements that comprise it.

Eichler's (2011) half-year observation of four German teachers' classroom practices provides strong evidence that mathematics teachers pursue their central beliefs when teaching statistics. Also, Eichler found a direct impact on the students' beliefs about the relevance of statistics from teachers' beliefs about putting emphases on real problems, real data sets and the role of context.

Pierce and Chick (2011) highlight the influence of teachers' beliefs on what aspects of statistics should be taught in schools and how, as well as on both the teaching/learning process and their students' relationship with statistics beyond the classroom, coinciding with the opinion of Estrada and Batanero (2008) in the last assertion.

The results of all these researches are evidence of the vital role of mathematics teachers' beliefs in every aspect of teaching statistical contents, including the instructional method and teaching approach, the course content, and the assessments.

3.9 Mathematics teachers' statistical knowledge for teaching

Mathematics and statistics share some common grounds, and there is considerable overlapping and cooperation between the two disciplines—e.g., dealing with statistical concepts such as chance and data often demands making decisions by using mathematical reasoning skills, like those related to proportional reasoning (Watson & Nathan, 2010). Moreover, mathematics and statistics overlap not only content-wise and skill-wise, but also curriculum-wise, since statistics is typically offered within the school mathematics curricula (Hand, 1998; Moore & Cobb, 2000). Therefore, it is by no means surprising that the majority of the few conceptualizations of SKT reported in the literature to date have employed as a basis a framework for MKT. In this section, an overview of the conceptualizations of SKT developed by Groth (2007), Noll (2011) and Burgess (2011)—all of them based on the framework for MKT developed by Ball et al. (2008)—is presented.

Groth (2007) developed a hypothetical framework to explain the SKT required for teaching statistics at high school level (Figure 11), borrowing and focusing on the constructs of CCK and SCK described by Ball et al. (2008), and merging and adapting them with the framework for statistical problem solving given in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin et al., 2007)—i.e., formulating questions, collecting data, analyzing data, and interpreting results—, in order to make distinctions between the mathematical and nonmathematical knowledge needed for teaching statistics, characterize the work of teaching statistics, and differentiate it from the one of teaching mathematics. Key features in this framework are the exemplification of common and specialized knowledge needed to support students' learning regarding each of the components of the GAISE framework, and the distinction of such cognitive constructs as mathematical and non-mathematical in nature. In his model, Groth acknowledges that some aspects of the common and specialized knowledge entailed by the teaching of statistics require a growing research base, particularly the specialized knowledge related to nonmathematical knowledge, which encompasses the pedagogical activities that take place in the classroom. Since this model is not based on any empirical research studies, Groth himself calls for more investigation on SKT—especially of the empirical kind—, arguing the necessity of such studies, the distinctive differences between the disciplines of mathematics and statistics, and the growing statistics education movement that has yet to address this topic.

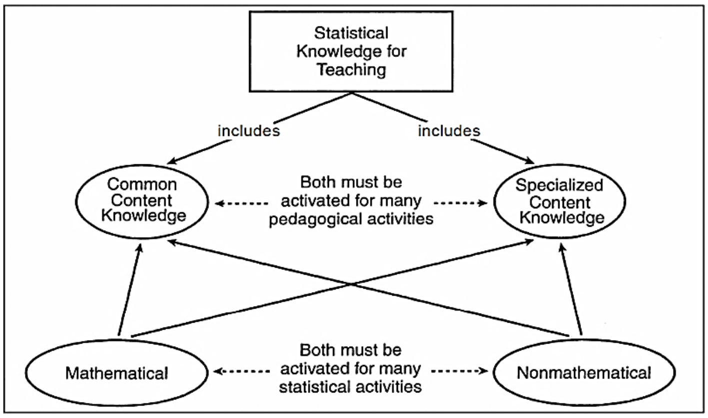


Figure 11: Framework of SKT (Groth, 2007, p.429)

Noll (2011) investigated the SKT held by a volunteer sample of 68 American graduate teaching assistants' (TAs) at 18 universities across the United States, using a task-based survey and a series of semi-structured interviews, and focusing on TAs' knowledge about data distributions and empirical sampling distributions, as well as in their knowledge of student thinking about sampling concepts. In order to develop a framework for SKT useful for describing the type and quality of knowledge that introductory college statistics TAs should have, Noll selected three of the components of MKT described in the conceptualization proposed by Ball et al. (2008) (i.e., CCK, SCK and KCS). Key features in this framework are the interpretations of CCK and SCK as statistical literacy and statistical thinking, respectively. The findings from Noll's research indicate that these TAs, despite their considerable knowledge of theoretical probability distributions, had a limited SKT in all the three components of her framework, since such TAs experienced tensions when attempting to quantify expected statistical variability in an empirical sampling situation, had difficulty teaching certain topics—particularly conceptual ideas of variability—, and seemed lost when it came to making sense of students' work and interpretation about variability and other sampling-related concepts.

In order to examine, through a classroom-based approach, the knowledge that elementary school teachers need to successfully implement the teaching of statistics through projects and investigations, Burgess (2011) developed a two-dimensional framework (Figure 12) comprised of four of the knowledge components described by Ball et al. (2008)—i.e., CCK, SCK, KCS and KCT—, and six out of eight components of Wild and Pfannkuch's (1999) model for statistical thinking in empirical enquiry—i.e., transnumeration, variation, reasoning with models, and integration of statistical and contextual knowledge as fundamental types of statistical thinking in statistical problem solving, along with teachers' general thinking about two components in the statistics research process: the Problem-Plan-Data-Analysis-Conclusions investigative cycle, and the Generate-Seek-Interpret-Criticize-Judge interrogative cycle. By using this model to analyze and report the teacher knowledge and classroom practice of different samples of elementary school teachers, Burgess identified the different types of knowledge that were either needed and used, or needed but not used, in the context of teaching experiences based on statistical investigations, finding that all aspects of knowledge included in his proposed model were needed in the classroom. Burgess also demonstrated how teachers' SKT could be usefully benchmarked using the investigative cycle, found substantial differences between the practices of the studied teachers in their ability to take advantage of the learning opportunities of a task given by the researcher, and described how lack of appropriate statistical knowledge created missed opportunities in relation to the teaching and learning of statistics.

		Statistical Knowledge for Teaching			
		Subject Matter Knowledge		Pedagogical Content Knowledge	
		Common Content Knowledge (CCK)	Specialized Content Knowledge (SCK)	Knowledge of Content and Students (KCS)	Knowledge of Content and Teaching (KCT)
Types of Thinking	Need for data				
	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					
Dispositions					

Figure 12: Framework for SKT to teach statistics through investigations (Burgess, 2011, p.264)

From this overview, it is clear that none of these previous efforts to conceptualize SKT have presented a framework considering all the six knowledge components proposed by Ball and her colleagues as necessary for teachers to be able to teach mathematics effectively, which results in a blurry picture of the preparedness of our mathematics teachers to teach statistical contents at any school level.

3.10 Research gaps and the present study

Several studies in the field of mathematics education (e.g., Ball & Bass, 2000; Cobb, 2000) have shown that teachers' knowledge is connected to what and how students learn and depends on the context in which it is used. Nevertheless, despite the crucial role that teacher knowledge plays in, among others, shaping the enacted curriculum, providing quality instruction, and developing students' knowledge base, attitudes and beliefs about the subject matter, the literature on teacher knowledge has strongly indicated that there are deficits in the subject matter knowledge that teachers teach (e.g., Fennema & Franke, 1992; Ma, 1999; González, 2011; Isoda & González, 2012). Moreover, the research efforts carried out by researchers like Shulman (1986, 1987) and Ball and her colleagues (e.g., Ball et al., 2008) have made significant strides in helping people in the mathematics education community understand that teachers' subject matter knowledge (SMK) is just one domain, one dimension of knowledge that teachers need to know and master. The importance of pedagogical content knowledge (PCK) for effective mathematics teaching and learning, as well as a fundamental criterion for effective teacher education, has been highlighted by several researchers (e.g., Ball et al., 2008; Döhrmann et al., 2012). Moreover, a number of researchers share the opinion that "successful teaching depends on professional knowledge and teacher beliefs" (e.g., Döhrmann et al., 2012, p. 327).

Nevertheless, in the case of statistics education, scarce studies can be found in the literature focused on the SMK and PCK entailed by teaching variability-related contents to help students achieve the aims of statistics education (cf. Shaughnessy, 2007), as well as on the conceptions of variability that teachers hold (cf. Batanero, Garfield, Ottaviani & Truran, 2001; Canada, 2006a, 2006b; González, 2011; Isoda & González, 2012), and the beliefs held by in-service teachers on statistics teaching and learning of such contents (cf. Pierce & Chick, 2011; Eichler, 2008, 2011). Hence, it is by no means surprising the urgent call for increasing research on these areas made by a number of concerned researchers, particularly for studies on teachers' professional knowledge and practices while teaching variability (e.g., Sánchez, da Silva & Coutinho, 2011, p.219), as well as for teachers' beliefs on statistics itself and on what aspects of statistics should be taught in schools and how (e.g., Pierce & Chick, 2011, p.160). These numerous calls for research that have been recently made, as well as the paucity of research in the aforementioned issues acknowledged by several researchers and statistics educators, provide clear evidence of the need and importance of conducting research on the aforementioned issues, which would provide insights into the level of professional knowledge that our secondary school teachers have to teach several statistical contents in which variability can be appreciated. This is particularly true in the case of Venezuela, country in which the few reported researches on statistics education to date have been centered on the statistical contents in the school curriculum (e.g., Salcedo, 2006) or on students' knowledge about statistics and probability (e.g., León, 2011), with no studies reported, to the knowledge of the present author, on teachers' professional knowledge, conceptions of variability and beliefs related to the teaching of statistics at any school level. In addition, from the review of literature on statistical knowledge for teaching made in this chapter, it is clear that none of these previous MKT-based efforts to conceptualize SKT have presented a framework considering all the six knowledge components proposed by Ball and her colleagues as necessary for teachers to be able to teach mathematics effectively, which results in a blurry picture of the preparedness of our mathematics teachers to teach statistical contents at any school level. One of the purposes of the current study is to remedy this situation and to provide researchers, teachers and students with a more comprehensive framework for SKT, as well as with a survey questionnaire that could provide a clear snapshot of the current state of the knowledge base of SKT held by mathematics teachers.

CHAPTER 4:

Framework, Research Design, Methodology and Sample Characteristics

4.1 Conceptualizing "Statistical Knowledge for Teaching"

To develop a conceptual framework for the current study, a literature review relating to mathematics teachers' knowledge was undertaken, particularly targeting previous researches that have set out to explore the cognitive demands of teaching mathematics and statistics; that is, the knowledge that can support teachers in their work—e.g., Shulman, 1986, 1987; Wilson, Shulman & Richert, 1987; Grossmann, 1990; Borko et al., 1992; Lampert & Ball, 1998; Ma, 1999; Ball & Bass, 2000, 2003a, 2003b, 2009; Ball et al., 2001; Gal, 2002, 2004; delMas, 2002; An, Kulm & Wu, 2004; Ball et al., 2005; Groth, 2007; Hill et al., 2007; Ferrini-Mundi, Floden, McCrory, Burrill & Sandow, 2007; Garfield & Ben-Zvi, 2008; Petrou & Golding, 2011; Noll, 2011; Burgess, 2011; Godino et al., 2011; Callingham & Watson, 2011; Harradine, Batanero & Rossman, 2011; Reading & Canada, 2011; González, Espinel & Ainley, 2011; Eichler, 2011; Pierce & Chick, 2011. On the basis of the aforementioned literature review, several components that were thought to be potential indicators of mathematics teachers' professional competencies for teaching statistics were identified and considered for analysis. Then, paying particular attention to the disciplinary demands of teaching statistics as a mathematics teacher, the following conjectures were raised as a result of such analysis:

(a) *A model of SKT should be closely tied to a model of MKT*: On the basis that the statistical contents studied at school are often taught as part of the mathematics curriculum by mathematics teachers, as well as due to the common grounds shared by mathematics and statistics, it is anticipated that a model of SKT should be closely tied to a model of MKT. Consequently, in this research is argued that all the six constructs necessary for having a solid MKT identified by Ball et al. (2008) in their framework would serve as a useful starting point to hypothesize what knowledge might be needed for teaching statistics effectively at secondary school. Therefore, in the present study it is assumed that, in order to effectively teach statistics at secondary school level, mathematics teachers should have a solid knowledge about the statistical ideas included in the mathematics curriculum and ability to perform tasks related to statistical literacy over such ideas—i.e., *common content knowledge*—; ability to determine the accuracy of common and non-standard solutions that could be given by students when solving statistical problems—i.e., *specialized content knowledge*—; understanding of the broader set of statistical ideas to which a particular concept connects—i.e., *horizon content knowledge*—; knowledge about how students think about the statistical ideas being taught in the mathematics curriculum—i.e., *knowledge of content and students*—; knowledge on how the statistical ideas they have to teach at a particular grade level are developed throughout the mathematics curriculum—i.e., *knowledge of content and curriculum*—; and capacity to plan and execute meaningful teaching of statistical ideas in the light of the previous cognitive traits—i.e., *knowledge of content and teaching*.

(b) *The used MKT model has to be adapted in order to account for specific requirements of teaching statistics*: Although mathematics and statistics share some common grounds, the two disciplines are different in several ways. Basically, mathematics and statistics are different in the ways that they use numbers. Mathematics has a deterministic nature, mostly dealing with numbers, their operations, generalizations and abstractions, in which problems have a single mathematical solution the most of the time. For statistics, which is stochastic in nature—i.e., deals with phenomena that have a variable nature, for which one could only specify either outcomes within certain operational limits or a probability distribution of possible outcomes, rather than fix on any particular outcome as certain (Davis, 1987; Good & Hardin, 2012)—, numbers are varying data embedded within a context (Cobb & Moore, 1997; Moore, 1998b; Hand, 1998; Wild & Pfannkuch, 1999; Pfannkuch & Wild, 2000; delMas, 2004; Begg et al., 2004; Gattuso & Ottaviani, 2011), and problems can have multiple answers, often with no right or wrong ones. Moreover, statistical literacy, statistical problem solving and decision-making depend on knowledge about the context and understanding, explaining, and quantifying the variability in data (Moore & Cobb, 1997; Gal, 2004; Franklin et al., 2007). Then, when doing statistics, one must know the nature of data, and where and how they are produced, in order to proceed with the analysis and to draw some conclusions. Mathematics, on the contrary, may rely on context for motivation in the classroom, or as a source of research problems, but its main goal is abstracting, finding patterns and generalizing while putting aside the context, in order to grasp the model or the structure behind the numbers (Moore &

Cobb, 1997; Gattuso & Ottaviani, 2011). Therefore, having in mind that statistics is concerned with “reasoning about varying data and uncertainty” whereas mathematics is concerned with “reasoning with certainty” (Begg et al., 2004), it seems necessary to revise the MKT framework developed by Ball et al. (2008), in order to take into account these differences between mathematics and statistics—which are not accounted for in its current form—, and then meet particular requirements specific to the teaching of statistics.

In the case of the conceptualization of MKT developed by Ball and her colleagues—the one used in this study—, CCK is defined as the mathematics knowledge that any well-educated adult—i.e., a person who has acquired the skills, attitudes, competencies and cultural traits that one would expect from anyone who went through compulsory education—should have acquired after finishing school education (Ball, Hill, & Bass, 2005), common to a wide variety of settings—i.e., the mathematical knowledge used in all mathematically intensive professions—and, therefore, not unique to teaching (Ball et al. 2008; Charalambous, 2010). CCK mostly consists in “simply calculating an answer or, more generally, correctly solving mathematics problems” (Ball et al., 2008, p.399). In the case of statistics education, the ability to correctly solve statistical problems is not enough to be regarded as having a minimum of statistical literacy skills. In addition to be able to correctly solve statistical problems, the acquisition of basic skills related to statistical literacy—e.g., identifying examples or instances of a statistical concept; describing graphs, distributions, and relationships; rephrasing or translating statistical findings, interpreting the results of a statistical procedure, and acknowledging and measuring variability (delMas, 2002)—is also important, since it is regarded as a main goal of both statistics education and mathematics curricula at all educational levels (e.g., Gal, 2004; Pfannkuch & Ben-Zvi, 2011), for which it is also expected from any well-educated adult after finishing compulsory education. For this reason, in the framework proposed here, CCK will be seen as statistical literacy, since the acquisition of its associated skills is expected from any individual after completing compulsory schooling. The rest of knowledge components in this framework are defined in the same way as in the model of MKT by Ball et al. (2008), but rephrased in some cases to meet the requirements of teaching statistics.

(c) *In order to conceptualize SKT, teachers' beliefs about statistics, its teaching and learning must be considered*: In the case of mathematics teachers who teach statistics at school, the relationship between beliefs and teachers' classroom practice has been well articulated in the literature by some researchers (e.g., Gal, Ginsburg & Schau, 1997; Pierce & Chick, 2008; Eichler, 2008). For example, teachers who believe in "group work" and "class discussion" appear to recognize that an interactive approach would best serve the purpose of engaging the students with statistical ideas (Chick & Pierce, 2008), while teachers who espoused the importance of real statistical problems or a theoretical foundation for statistics, actually used problems embedded in a real-life context or more routine tasks and traditional methods, respectively, to develop statistical methods in class (Eichler, 2008). Moreover, beliefs are identified by Gal (2004) as one of the dispositional elements of statistical literacy—i.e., attitudinal aspects related to individuals' ability to discuss or communicate their reactions to statistical information, their opinions about the implications of this information, or their concern regarding the acceptability of given conclusions" (Gal, 2004, p. 49). Since statistical literacy is regarded as CCK in the present conceptualization of SKT, in the present study teachers' beliefs about statistics, its teaching and learning, are going to be regarded as a fundamental factor that influence a teacher's statistical literacy and hence SKT, attempting in that way to obtain a much richer and broader picture of the competencies needed to teach statistics efficiently, as well as to overcome a common drawback in all the MKT-based frameworks of SKT reviewed previously.

(d) *Tasks designed to elicit teachers' conceptions of variability would be helpful to provide indicators to measure SKT as defined in this study*: Conceptions are defined as the whole cluster of internal representations and associations evoked by a particular concept (Sfard, 1991, p.3); that is, not necessarily the "official" form of such concept—what Furinghetti & Pehkonen (2002) call *objective knowledge*—, but rather the concept's personal and private counterpart in the mind of the actor who gives meaning to such notion—regarded in the literature as *subjective knowledge* (Furinghetti & Pehkonen, 2002)—, based on personal experiences and understanding. Therefore, conceptions can be considered as a "picture" held by an individual of a certain concept, which might be, or asymptotically get closer to, the “official” concept that pertain to *objective knowledge* (Bereiter & Scardamalia, 1996; Pehkonen & Pietilä, 2003). Thus, how close a teacher’s conception of a particular concept is to its formal and academically accepted version plays an important role in teacher’s effectiveness, as the primary mediator between the subject matter and the learners (Thompson, 1984). In the case of mathematics teachers, as well as teachers in many other disciplines, conceptions about the subject matter have been proved to influence their own approaches to teaching, and consequently their students’ approaches to learning (e.g., Carpenter, Fennema & Peterson, 1986; Trigwell, Prosser & Waterhouse, 1999). In the case of statistics education, the few studies reported in the literature point out the relation between the conceptions of variability held by teachers, their subject matter knowledge in statistics, and their perception of data as an aggregate, as individual data values, or as a small group of individual data values (Peters, 2009; González, 2011; Isoda & González, 2012). Moreover, the work carried out by González (2011) and Isoda and González (2012) provides empirical evidence that the use of tasks addressing variability and variability-related concepts is an effective method for eliciting, identifying, describing and assessing teacher’s conceptions of variability, as well as their statistical subject matter knowledge. On the basis of these facts, and because conceptions represent knowledge and beliefs working in tandem (Knuth, 2002), gaining insight into the teachers’ conceptions of variability is regarded as necessary in the model for SKT proposed in the present study.

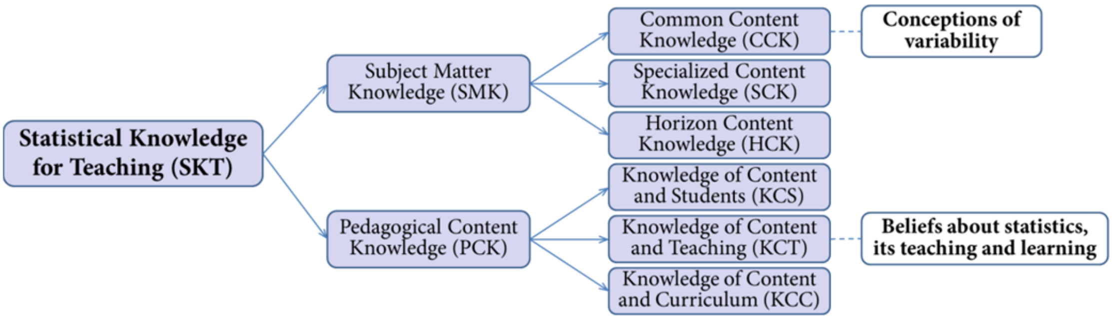


Figure 13: Proposed conceptual model of teachers' competencies to teach secondary school statistics

The eight components identified by the aforementioned four conjectures, and associated to mathematics teachers' professional competencies to teach efficiently statistics at school level, are summarized in the framework depicted in Figure 13. As can be appreciated in Figure 13, the cognitive facet of the conceptualization of SKT proposed here—depicted by gray-colored boxes—is six-fold, comprised of all the six subdomains of professional knowledge identified by Ball et al. (2008) in their model of MKT, with CCK understood as statistical literacy, in order to meet the case of teaching statistics. The affective facet of the model proposed here—depicted by white-colored boxes—is comprised of two components: teachers' beliefs about statistics, its teaching and learning; and teachers' conceptions of variability, since both beliefs and conceptions—the latter often explained in the literature as "conscious beliefs" (Sivunen & Pehkonen, 2009)—, have been regarded by a number of studies as factors influencing every aspect of teaching, including the instructional method and the course content (cf. Philipp, 2007). Despite the evident connection between conceptions of a particular concept and its "official" form, conceptions are going to be regarded in the present study as pertaining to the affective domain, due to them being the personal interpretation of the concept in issue, based on the individual's personal experiences and understanding (Sfard, 1991; Furinghetti & Pehkonen, 2002; Pehkonen & Pietilä, 2003; Sivunen & Pehkonen. 2009).

Evidence of each of the cognitive components identified by the proposed framework could be elicited through either open or closed questions, as it is suggested by the literature of mathematics teachers' knowledge (e.g., Ball et al., 2008; Manizade & Mason, 2011; Döhrmann et al., 2012; Tatto et al., 2012). Regarding the affective facet, mathematics teachers' conceptions of variability can be made explicit by answering tasks in which knowledge and understanding of variability-related ideas, as well as the ability to connect and represent them, are required (cf. González, 2011; Isoda & González, 2012). Furthermore, mathematics teachers' beliefs about statistics, its teaching and learning, will emerge from examining the features of the lesson plans that teachers produce—such as the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson (e.g., Eichler, 2008, 2011; Pierce & Chick, 2011).

Based on the aforementioned four conjectures, and following the identification of the eight dimensions of professional competencies for teaching variability-related contents depicted in Figure 13—namely the six cognitive subdomains of SKT, teachers' beliefs about statistics teaching and learning, and teachers' conceptions of variability—, twelve indicators associated to the cognitive facet of such competencies were selected from the literature, in order to provide a comprehensive framework for conceptualizing SKT (see Table 8).

Table 8: Set of indicators proposed to assess SKT

A: INDICATORS RELATED TO CCK (STATISTICAL LITERACY):

1. Is the teacher able to give an appropriate and correct answer to the given task?
2. Does the teacher consistently acknowledge variability and correctly interpret its meaning when answering the given task?

B: INDICATORS RELATED TO SCK:

1. Does the teacher show evidence of ability to determine the accuracy of common and non-standard arguments, methods and solutions that could be proposed to the given task by students (especially while recognizing whether a student's answer is right or not)?
2. Does the teacher show evidence of ability to analyze right and wrong solutions that could be given by students to the present task, by providing explanations about what reasoning and/or mathematical/statistical steps likely produced such responses, and why, in a clear, accurate and appropriate way?

C: INDICATORS RELATED TO HCK:

1. Does the teacher show evidence of having ability to identify whether a student response is interesting or significant, mathematically or statistically?
2. Is the teacher able to identify the significant notions, practices or values related to the statistical ideas involved in the given task?

D: INDICATORS RELATED TO KCS:

1. Is the teacher able to anticipate students' common responses and difficulties on the given task?
2. Does the teacher show evidence of knowing the most likely reasons for students' common responses and difficulties in relation to the statistical concepts involved in the given task?

E: INDICATORS RELATED TO KCT:

1. In design of teaching, does the teacher show evidence of knowing what tasks, activities and strategies could be used to set up a productive whole-class discussion aimed at developing students' understanding of the key statistical concepts involved in the given task, instead of focusing just in computation methods or general calculation techniques?
2. Does the teacher show evidence of knowing how to sequence such tasks, activities and strategies, in order to develop students' understanding of the key statistical concepts involved in the given task?

F: INDICATORS RELATED TO KCC:

1. Does the teacher show evidence of knowing at what grade levels and content areas students are typically taught about the statistical concepts involved in the given task?
2. Does the designed lesson (or series of lessons) show evidence of teacher's knowledge and support of the educational goals and intentions of the official curriculum documents in relation to the teaching of the statistical contents present in the given task, as well as statistics in general?

The cognitive indicators in Table 8 were selected after a review of prior empirical studies—in particular, those developed by Deborah Ball and her team at University of Michigan (e.g., Ball et al., 2008; Hill, Ball & Schilling, 2008; Ball & Bass, 2009)—, which revealed several key areas of concern regarding mathematics teachers' professional knowledge in the case of statistics education.

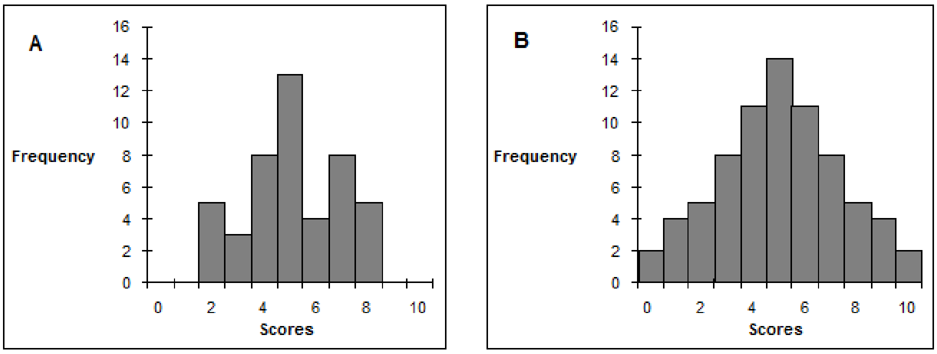
4.2 The survey instrument

One of the primary purposes of this study was to achieve a rich and detailed understanding of secondary school mathematics teachers' SKT from the viewpoint of variability. This goal implied building a conceptual model of teachers' professional knowledge about variability-related concepts; that is, suggesting a framework for the examination of the professional knowledge that teachers need to be well-developed in order to achieve effective teaching, which was introduced in the previous section. The second primary purpose of this study was to investigate Venezuelan secondary school mathematics teachers' knowledge base of SKT, which implies to use a data collection method, through which it would be possible to check whether or not teachers show evidence of the indicators identified in Table 8. In order to achieve this goal, a pen-and-paper survey instrument was designed, based on the conceptualization of SKT previously outlined. This instrument is comprised of one task addressing—through comparing the histograms of two distributions—many variability-related ideas present in the Venezuelan secondary school mathematics curriculum, and seven SKT-related questions aiming to elicit and gather information about each one of the eight components of teachers' professional competencies to teach variability-related contents identified by this study. Each question was developed based on previous studies with similar aims reported in the literature (e.g., Meletiou & Lee, 2003; Ball et al., 2008; Manizade & Mason, 2011; Isoda & González, 2012), which were adapted to reflect the context of the selected task and the specific objectives of the present conceptual framework proposed by this study.

4.3 Development process of the survey instrument

Using the statistical contents present in the Venezuelan secondary school mathematics curriculum as a filter, the task entitled "Choosing the distribution with more variability" task—originally developed by Garfield, delMas and Chance (1999)—was chosen for the item posed in the survey instrument (Figure 14). The fact that most of the statistical contents in the Venezuelan secondary school mathematics curriculum are ideas related to descriptive statistics—in particular with the handling of grouped data, frequency distribution tables and histograms—was crucial in selecting such task. In fact, two of the methodological suggestions appearing throughout the teaching guide for the Basic cycle of the secondary school are to "Organize data in frequency distribution tables" and "Analyze data represented in absolute frequency histograms" (ME, 1987, pp. 55–56). In the case of the Diversified cycle of secondary education, the Venezuelan mathematics curriculum states as one of the goals of the unit "Probability, statistics and combinatorics" that "The student will be able to describe, by the means of characteristic values, distributions of probability associated to experimental data" (CENAMEC, 1991, pp.35–36), being these "characteristic values" the measures of central tendency, measures of dispersion—specifically the range, standard deviation and variance—, quartiles, deciles and percentiles. Moreover, the study of histograms, frequency distributions, measures of central tendency and measures of variation, is revisited in a spiral way throughout the subject of mathematics during secondary school in Venezuela.

Look at the histogram of the following two distributions:



Which of the two distributions you think has more variability?

(a) Distribution A (b) Distribution B

Why do you think this?

Figure 14: Original version of the "Choosing distribution with more variability" task (Garfield et al., 1999)

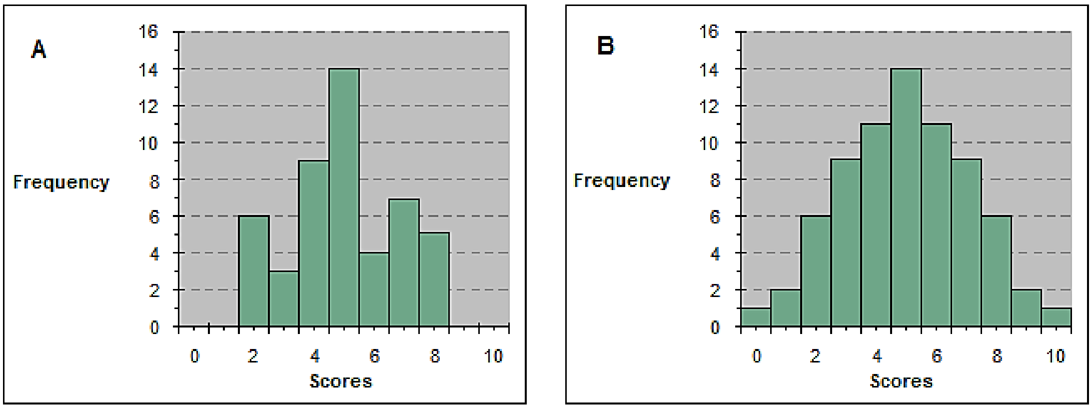
Other reasons for the selection of this task are listed below:

- *The chosen task requires comparing distributions with different sample sizes*: the mistake of thinking that group comparison can be done only if both group sizes are equal is a common misconception that has been reported in the literature (e.g., Tempelaar, 2007; Wang, Wang & Chen, 2009). It is anticipated that, through using the chosen task, it would be possible to determine whether a respondent holds such misconception or not.

- *The chosen task requires comparing a histogram having irregular bar heights with one showing a normal-like, symmetrical distribution*: when comparing two histograms to say which data set has higher variability or spread, some researchers have reported that many students—and even in-service mathematics teachers—have indicated that the histogram with smaller range but irregular bar heights—such as that shown in Figure 14 as "Distribution A"—has more variability than a histogram showing a normal or symmetrical distribution with a larger range—such as the one shown in Figure 14 as "Distribution B"—(e.g., Garfield et al., 1999; Meletiou & Lee, 2003, 2005; Stylianou & Meletiou, 2006; Cooper & Shore, 2007; Kaplan, Fisher & Rogness, 2009; González, 2011; Isoda & González, 2012). It is anticipated that, through using the chosen task, it would be possible to determine whether a respondent thinks of variability as evenness—or lack thereof—of the heights of the bars in the vertical direction—which is a misconception—, or thinks of variability statistically, as spread in the horizontal direction or as how much the data differ from a measure of central tendency.

- *The chosen task has a lack of context or story behind the given data*: the meta-data—i.e., the story behind the data, the meaning and the names of the variables, why and how data were collected and processed, and so on (Huber, 1997)—, are just as important as the data themselves. Thus, the data cannot be fully understood without the story. The selection of this task dealing with data without a story behind was on purpose, in order to see whether the respondents were able to acknowledge the need for data within a clear context, which would provide, during teaching, opportunities for students to be "data detectives" and then develop and share their reasoning about sources of variability in data (Shaughnessy & Pfannkuch, 2002; Shaughnessy, 2007, 2008).

In addition to the aforementioned reasons to choose the task developed by Garfield, delMas and Chance (1999) for the present study, it is the fact that the chosen task has been reported in the literature as an effective means of investigating both students' (e.g., Meletiou & Lee, 2003, 2005) and teachers' (González, 2011; González & Isoda, 2011; Isoda & González, 2012) statistical literacy skills and conceptions of variability in the context of histograms. For the present study, the original task by Garfield et al. (1999) was slightly modified, in order to facilitate the calculations that could be made by the respondents. The modifications to the original task by Garfield et al. (1999) (Figure 15) include the addition of horizontal gridlines to facilitate counting the frequency associated to each bar—which was a source of mistakes identified during the piloting phase—, as well as alterations in the value of the height of some bars in both histograms—in order to make both distributions to have an arithmetic mean equal to five and easy-to-get quartiles, which values can be determined directly from the frequency distribution tables. In the present research, the distributions were purposely made with the same arithmetic mean. The purpose behind this was to prevent respondents from making decisions based on a comparison of means, which is a common mistake made by students in this type of tasks (cf. Shaughnessy, Ciancetta, Best & Canada, 2004).



Distribution A					
Lower class limit	Upper class limit	Class mark	Frequency	Cumulative Frequency	Cumulative Relative Frequency
1.5	2.5	2	6	6	0.125
2.5	3.5	3	3	9	0.188
3.5	4.5	4	9	18	0.375
4.5	5.5	5	14	32	0.667
5.5	6.5	6	4	36	0.750
6.5	7.5	7	7	43	0.896
7.5	8.5	8	5	**48**	1.000

75% of the data is below 6.5. Hence, $Q_{3A} = 6.5$

Distribution B					
Lower class limit	Upper class limit	Class mark	Frequency	Cumulative Frequency	Cumulative Relative Frequency
-0.5	0.5	0	1	1	0.014
0.5	1.5	1	2	3	0.042
1.5	2.5	2	6	9	0.125
2.5	3.5	3	9	18	0.250
3.5	4.5	4	11	29	0.403
4.5	5.5	5	14	43	0.597
5.5	6.5	6	11	54	0.750
6.5	7.5	7	9	63	0.875
7.5	8.5	8	6	69	0.958
8.5	9.5	9	2	71	0.986
9.5	10.5	10	1	**72**	1.000

25% of the data is below 3.5. Hence, $Q_{1B} = 3.5$

75% of the data is below 6.5. Hence, $Q_{3B} = 6.5$

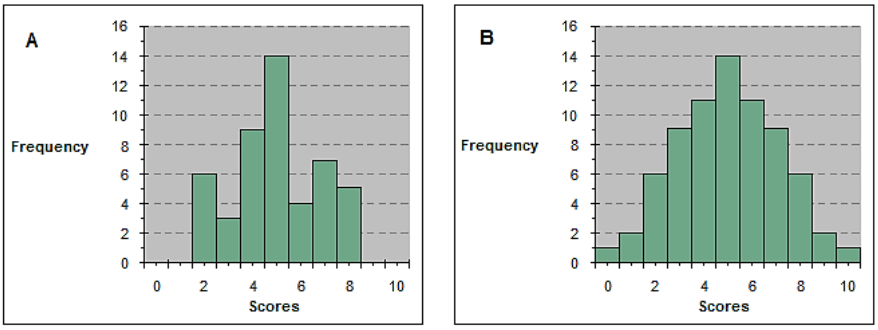
Figure 15: Calculation of the quartiles in the modified version of the "Choosing the distribution with more variability" task

After the aforementioned modifications, the task was enriched with questions aiming to elicit all the facets of teachers' professional competencies to teach variability-related contents previously identified by this study (Figure 16).

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(a) Answer this task in as many different ways as you can. Please, be sure to show every step of your solution process.

(b) What are the important ideas that might be used to answer this task?

(c) Suppose that, after posing this task to your students, three of them come up with the following answers:

STUDENT 1: "Distribution A has more variability because it's not symmetrical."

STUDENT 2: "Distribution A ranges from 3 to 14, while Distribution B ranges from 1 to 14. Then, Distribution B has more variability."

STUDENT 3: "The bars in Distribution A are clumped closer to the central bar than they are in Distribution B. Then, Distribution B has more variability."

Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer.

(d) Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), and difficulties you would expect from them? Briefly explain why you think so. (Regarding to the most likely answers that you might get from the students, please do not include those mentioned in part (c).)

(e) Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects. (Please, focus your response on whether there is a significant mathematical/statistical insight in the student's answer, and whether there are forthcoming contents in future classroom subjects connected to the notions being said or implied in such answer.)

(f) Briefly describe how the important ideas involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling.

(g) Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the setting of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order.

Figure 16: Final survey item: "Choosing the distribution with more variability" task

Besides Question (a), each of the questions included in Item 1 were developed from previous studies reported and validated in English on mathematics teachers' knowledge (e.g., Ball et al., 2008; Li & Huang, 2008; Watson, Callingham & Donne, 2008; Ball & Bass, 2009; Brakoniecki, 2009; Watson, Callingham & Nathan, 2009; Kazemi et al., 2010; Wassong & Biehler, 2010; Ball, 2011; Manizade & Mason, 2011; Beswick, Callingham & Watson, 2012; Döhrmann et al., 2012; Qinqiong & Stephens, 2013). A brief explanation on how each of the seven questions included in Item 1 was developed follows:

- *Question (a)*: the task chosen for this study can be answered in many ways: by comparing the ranges, the variances, the standard deviations, the interquartile ranges, or the mean absolute deviations from the mean; by transforming the given histograms into boxplots or ogives and comparing them; and even by comparing naked-eye estimations of the degree of data concentration around the modal class in each histogram. All these ways to solve the given task are expected to be taught during secondary school mathematics in Venezuela, as well as in other countries around the world (cf. Isoda & González, 2012). Nevertheless, in a previous study with Japanese in-service mathematics teachers (cf. González, 2011; Isoda & González, 2012), when teachers were prompted to answer the original version of this task—i.e., when teachers were asked "Which of the two distributions you think has more variability? Why do you think this?"—, 27 out of the 39 senior high school teachers who answered correctly justified choosing Distribution B based only on its larger range, with 10 teachers giving as an only reason the concentration or dispersion of data around the mean, and only 3 teachers mentioning a measure of variation—specifically, the variance. In the case of junior high school, 23 out of the 26 teachers who answered correctly justified choosing Distribution B based only on its larger range, with only 1 teacher basing his choice on the concentration or dispersion of data around the mean, and only 2 teachers mentioning a measure of variation—specifically, the standard deviation. Therefore, since it seems that teachers, when prompted to answer the chosen task in its original version, tend to answer by providing only one reason, being such reason the difference in range—which, although correct, is considered as a "lower level response" (Shaughnessy, 2008) or as an evidence of "simple recognition of variation" (Watson & Callingham, 2003). Therefore, instead of simply asking "Which of the two distributions you think has more variability? Why do you think this?"—which might conduct to answer tendencies like the ones reported by González (2011) and Isoda and González (2012)—, the author decided that the first question of Item 1 should be "Answer this task in as many different ways as you can", trying in that way to elicit from teachers as many different reasons and justifications for their chosen response as possible. In that way, it is anticipated that a deeper insight into respondents' CCK and conceptions of variability may emerge.

– *Question (b)*: In order to elicit mathematics teachers' HCK in relation with particular ideas dealt with in a given task, previous researches on teachers' knowledge posed questions such as “Are there mathematically significant notions that underlie division of fractions?” (Ball, 2011); “What are the big statistical ideas in this problem?” (Watson et al., 2009); “Is this mathematically interesting?” and “Is this an important mathematical insight?” (Ball & Bass, 2009). Based on this, the author developed Question (b) as “What are the important ideas that might be used to answer this task?”, as an attempt to elicit from teachers as many statistical and mathematical ideas as possible.

- *Question (c)*: scripted answers from hypothetical students to a given task were regularly found in previous survey researches on teacher knowledge, with the purpose of eliciting mathematics teachers’ SCK through questions such as “Choose one of the inappropriate responses” (Beswick et al., 2012); “Ms. Wilson is not sure that both of their explanations are correct. What do you think? Why?” (Manizade & Mason, 2011); “Which of these is a mathematically accurate definition of ‘rectangle’?”, “Which of these can be used to represent $\frac{5}{6}\div\frac{1}{3}$?”, “If so, why does it work?” (Ball, 2011); “What reasoning could produce each of these answers?”, “What mathematical steps likely produced each incorrect answer?”(Ball et al., 2009); “For each [answer]: Could this represent $\frac{3}{4}$? If yes, explain how it could represent $\frac{3}{4}$. If no, explain why it could not represent $\frac{3}{4}$.” (Kazemi et al., 2010). Based on such questions, the author developed Question (c) as follows: “Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer". This question deals with three scripted answers to the given task provided by fictitious students (Figure 16). The answers given by Student 1 and 2 are examples of common misconceptions related to the estimation of variability that students frequently exhibit when comparing histograms (cf. Garfield et al., 1999; Meletiou & Lee, 2003, 2005; Cooper & Shore, 2007; Kaplan et al., 2009; González, 2011; Isoda & González, 2012). The response given by Student 3 exemplifies a right interpretation of the variability in the given histograms, done by a naked-eye estimation and comparison of the degree of data clustering around the modal class in both graphs.

- *Question (d)*: besides possessing the competencies assessed in the previous questions, teachers must also be able to anticipate what students are likely to think and what they will find confusing when dealing with a particular problem. This kind of knowledge—regarded in the literature as KCS (cf. Ball et al., 2008)—includes teachers' ability to anticipate what students are likely to do with a particular task or problem, and what they will find easy or difficult in it (Ball et al., 2008, p.401). In previous survey researches on teacher knowledge reported in the literature, KCS has been elicited through questions such as "What difficulties and/or common misconceptions related to this topic might the students have as they solve these problems? Explain your answers" (Manizade & Mason, 2011); "What responses would you expect from your students? Write down some appropriate and inappropriate responses" (Beswick et al., 2012); "Please can you give an example of an appropriate response and an inappropriate response that your students might give. Can you explain why it is appropriate/inappropriate?" (Watson et al., 2009); "What kind of responses would you expect from your students? Write down some appropriate and inappropriate responses" (Watson et al., 2008); "Which of the following strategies would you expect to see some elementary school students using to find the answer of 8 × 8?", "What are the most likely reason(s) for this student confusion?", "Which of the following is the most likely reason for these incorrect answers", "Which version of the … problem below is likely to be the most challenging for students?" (LMT Project, 2008); and "What do students typically have difficulty with in learning about rectangles, and why?" (MTLT Project, 2011). With this in mind, the author developed Question (d) as "Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), and difficulties you would expect from them? Briefly explain why you think so", as an attempt to elicit teachers' KCS regarding the chosen task for this study.

- *Question (e)*: another aspect of teachers' HCK is the ability to identify interesting association relationships among a particular idea, practice, student's answer or classroom value with larger mathematical ideas, structures, and principles, even across great expanses of regular curricular sequence (Ball & Bass, 2009). In previous researches, it has been reported that this attribute of HCK could be elicited through questions such as “Is something being said or implied that could have mathematically problematic consequences later?”, “Is this an important mathematical insight?” (Ball & Bass, 2009); and “Is this mathematically significant or interesting?” (Ball, 2011). With such questions in mind, the author developed Question (e) as follows: “Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects”.

- *Question (f)*: the literature on teacher knowledge regards as very important having teachers’ awareness on how the instructional contents being taught at a certain grade are developed during school education. This cognitive trait is a characteristic of KCC (Ball et al., 2008). In previous researches, this aspect of KCC was elicited through questions such as “Describe briefly how the key mathematical ideas or critical points presented in [this question] are addressed in official curriculum documents (e.g. VELS) for Year 6 or Year 7 depending on your school” (Qinqiong & Stephens, 2013); “Where would you place [this problem] in your school’s curriculum sequence?” (Watson et al., 2009); “At what grade level are students first introduced to rectangles?” (MTLT Project, 2011); “At what grade level are students typically taught to divide fractions?”, “How is the division of fractions related to division of whole numbers in the school curriculum?”, and “What are the models for fractions and for division with which students would be familiar?” (Ball, 2011). Based on such questions, the author developed Question (f) as follows: "Briefly describe how the important ideas involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling".

- *Question (g)*: in order to collect data on the six cognitive traits identified by Ball and her colleagues, it is necessary to develop a question that can assess teachers' skills related to KCT (Ball et al., 2008), the only kind of knowledge that has not yet been elicited through the questionnaire. In previous researches on teacher knowledge, this cognitive trait was elicited through questions such as "How would you explain/model/demonstrate this item to someone who did not understand?" (Mohr, 2006); "What opportunities would this problem provide for you teaching?", "A student gave this answer. How would you move this student's understanding forward?" (Watson et al., 2009); "How would/could you use this item in the classroom? For example, choose one of the inappropriate responses and explain how would you intervene" (Beswick et al., 2012); "How would/could you use this item in the classroom? For example, how would you intervene to address the inappropriate responses?" (Watson et al., 2008); "What instructional strategies and/or tasks would you use during the next instructional period? Why?", "Which of these activities would be appropriate to include? Why, or why not? In what order should these activities be presented? Explain why you chose to put the activities in this particular order" (Manizade & Mason, 2011); "How would you sequence these figures to discuss the concept of a rectangle? What task would you create using these figures (or others) to set up a productive discussion aimed at developing a definition? In a whole-class discussion, which one would be good to discuss first?" (MTLT Project, 2011); and "What sequence of problems would you use to begin work on division of fractions? In a whole-class discussion, what solution methods would you want presented, and in what order?" (Ball, 2011). Based on such questions, the author developed Question (g) as follows: "Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the setting of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order". By posing Question (g) in this way, two important aspects of KCT are explicitly accounted for: on one hand, mathematical knowledge of tasks and examples to start with and use to take students deeper into the content to be taught; on the other hand, knowledge on the design of instruction, in particular, on how to sequence particular content, tasks and examples, in order to achieve an effective instruction (Ball et al., 2008, p.401).

In addition to eliciting information on respondents' KCT-related skills, it is anticipated that Question (g) will also gather evidence on whether or not statistics-related curriculum goals are known and supported by teachers in their planned lessons, which is an aspect of teachers' KCC (Ball et al., 2008, p.395).

Finally, as reported by previous researches in the field of statistics education (cf. Eichler, 2008, 2011; Pierce & Chick, 2011), teachers' beliefs about statistics teaching and learning could be identified through examining the features of the lesson plans prepared by them. Therefore, it is anticipated that teachers' answers to Question (g) will also provide valuable information about their statistics-related beliefs.

Because all the original previous survey researches consulted to develop Item 1 were reported in the literature in English, Questions (a) to (g) were also stated in English at the beginning. The reason of this was that, after developing the survey questions and before translating them into Spanish, a first English version of the instrument was examined by two Japanese mathematics educators, fluent in both English and Japanese and with previous experience in designing survey instruments as well, in order to establish the item's content validity—i.e., in order to determine, based on their expert judgments, whether the content of the questions in Item 1 was consistent with what was being intended to be assessed. After confirming the validity of the proposed item, the resulting instrument and the indicator associated to it were presented to specialists in the area of teacher knowledge and statistics education, via email and personal meetings during an international conference in July of 2012. In particular, confirmation and feedback received from four members of the research group of Deborah Ball, who recognized and verified one-to-one correspondence between the developed questions and the proposed indicators, were very valuable to the present study, since the sixfold framework for mathematics teachers' knowledge for teaching proposed by such team (e.g., Ball et al., 2008) is used as a cornerstone in this research.

During this phase, a few changes were made—regarding to wording—, but no significant content or structure changes were recommended by any of the specialists consulted, either in Japan or abroad. After this developmental stage of the survey instrument, the content validity of the questionnaire as a whole increased (Litwin, 1995; Manizade & Mason, 2011).

After finishing the development of the survey instrument in English, the translation of the whole questionnaire into Spanish was carefully undertaken by the author. Special care was taken in the translation process in order to preserve the original meaning of the English items to the maximum, while trying to get as close as possible to our respondents' point of view and the way they describe the world in their mother tongue, as questionnaire design specialists often recommend (cf. Harvatopoulos et al., 1992, p.53; Oppenheim, 1992, p.122; Litwin, 1995; Cea d'Ancona, 1996, p.263). To guarantee the appropriateness of the wording and technical accuracy, three Venezuelan mathematics educators—two of them professors of statistics and probability at undergraduate and graduate level—carried out a back-translation of a first draft of the Spanish version of the instrument, and discrepancies were discussed until agreement was obtained. Their comments or doubts about the content of different questions were taken into account while preparing the second and final version of the Spanish instrument. After that, these mathematics educators piloted the resulting version of the survey instrument. The pilot responses allowed the author to see whether the participants would be able to understand correctly the questions, as well as to determine if the question responses would be analyzable. The responses given by the mathematics educators during the pilot stage showed good consistency among them. This is the foundation for internal consistency reliability, which is, in turn, the foundation for validity (DeVellis, 2003). So, through these stages of instrument development, peer review and pilot study were established content validity, criterion-related validity, construct validity (Creswell, 2005; Utts, 2005). Moreover, from the piloting, these mathematics educators agreed that a Venezuelan secondary school mathematics teacher should require around one hour in order to complete the survey.

A mapping between the components of SKT that would be elicited by each question in Item 1, as well as the indicators associated to each cognitive aspect considered by this framework, is shown in Table 9.

Table 9: Knowledge components of SKT elicited by each of the questions posed in Item 1

Elicited knowledge component of SKT	Related indicator of SKT	Question
Common Content Knowledge (CCK, as Statistical Literacy)	A1	(a)
	A2	(a)
Specialized Content Knowledge (SCK)	B1	(c)
	B2	(c)
Horizon Content Knowledge (HCK)	C1	(e)
	C2	(b)
Knowledge of Content and Students (KCS)	D1	(d)
	D2	(d)
Knowledge of Content and Teaching (KCT)	E1	(g)
	E2	(g)
Knowledge of Content and Curriculum (KCC)	F1	(f)
	F2	(g)

In regard to the traits in the affective facet of the conceptual model for SKT proposed here—i.e., teachers' conceptions of variability and beliefs about teaching and learning of statistics—, it is anticipated that teachers' answers to Question (a) will provide enough information about how the respondents conceptualize variability, based on the fact that teachers' conceptions of variability can be made explicit by answering tasks in which knowledge and understanding of variability-related ideas, as well as the ability to connect and represent them, are required (González, 2011; Isoda & González, 2012). These conceptions of variability that might be distinguished in teachers' answers will be classified, at first, using the eight-type categorization proposed by Shaughnessy (2007, pp. 984–985). In the case of teachers' beliefs about statistics teaching and learning, the limited research on this topic (e.g., Eichler, 2008, 2011; Pierce & Chick, 2008, 2011) suggests that they could be identified through examining the features of the lesson plans that teachers produce, such as the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson. What teachers planned to do—which is related to the construct KCT, and hence with answers to Question (g)—will be analyzed using the four categories reflecting on teachers' beliefs developed by Eichler (2011)—i.e., *traditionalists*, *application preparers*, *everyday life preparers*, and *structuralists*—, which will provide valuable information on the beliefs about the nature of statistics, as well as about the teaching and learning of statistics, held by the surveyed teachers.

4.4 Participants and survey implementation

The survey was carried out anonymously and an informed consent form was obtained from all volunteer participants in the present study. The period of data collection was from July to September 2012. The survey was implemented with the support of a Venezuelan mathematics educator with previous contact with all the institutions in which the participants were working, due to her role as responsible of the supervision of teaching practice in one Faculty of Education at a private university in Caracas, Venezuela. Then, teachers participating in this study were contacted previous agreement with their respective institutions via the principal and the person responsible for the distribution and gathering of the survey instruments, who was present at the moment that teachers filled in the questionnaires.

The data from the collected questionnaires were carefully examined to determine whether the completed response forms were usable for scoring and verification. From the 60 initially collected questionnaires, 7 were excluded from the present study, since the respondents just provided demographic information, but did not answer any of the seven questions of the developed item. Therefore, the answers provided by 53 in-service secondary school mathematics teachers, working in the metropolitan area of Caracas, Venezuela, were examined in this study. The survey participants comprise a convenience sample in which participation was voluntary and anonymous. The age of the participants ranged from 21 to 71 years-old, with an average of 42.6 years-old. The classroom experience of the participants ranged from 0 to 45 years, with an average of 16.4 years. Thirty-three of the participants (62.3%) were men, while twenty (37.7%) were women.

Among the participant teachers, 19 were working at lower high school level—11 (57.9%) men and 8 (42.1%) women, with age ranging from 21 to 56 years-old, with an average of 40.2 years-old, with a classroom experience ranging from 2 to 34 years, with an average of 14.9 years—, 15 at upper high school level—9 (60.0%) men and 6 (40.0%) women, with age ranging from 27 to 71 years-old, with an average of 46.3 years-old, with a classroom experience ranging from 5 to 45 years, with an average of 19.6 years—, and 19 at both lower and upper high school levels—13 (68.4%) men and 6 (31.6%) women, with age ranging from 23 to 65 years-old, with an average of 42.3 years-old, with a classroom experience ranging from 0 to 42 years, with an average of 15.3 years.

All this information is summarized in the following table:

Table 10: Summary table of participants in this study

	Number of teachers	Gender		Average Age	Average years of experience
		Men	Women		
Lower High School	19 (35.8%)	11 (20.8%)	8 (15.1%)	40.2	14.9
Upper High School	15 (28.4%)	9 (17.0%)	6 (11.3%)	46.3	19.6
Both Levels	19 (35.8%)	13 (24.5%)	6 (11.3%)	42.3	15.3
Total	53 (100%)	33 (62.3%)	20 (37.7%)	42.6	16.4

CHAPTER 5:

Results, Findings and Discussion

The results presented in this section were obtained by undertaking a qualitative analysis of the collected answers given by the teachers in this study, focused on verifying whether the twelve SKT-related indicators previously depicted in Table 8 were observed in such answers. In general, this qualitative analysis provided a comprehensive picture of the current state of the surveyed Venezuelan secondary school mathematics teachers' knowledge base on SKT, conceptions of variability, and beliefs about statistics teaching and learning. Based on the results and findings obtained from this study, a discussion on the eight dimensions of professional competencies for teaching variability-related contents identified here will be presented, followed by a data-based discussion of how such dimensions seem to be related.

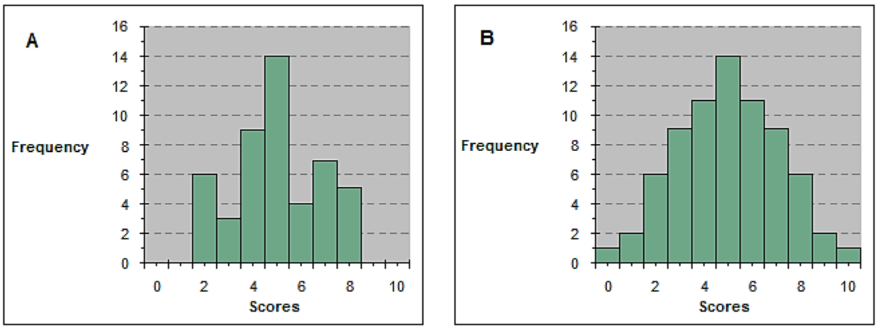
5.1. Conceptual analysis and framework for the task posed in Item 1

The task posed by Item 1 (based on the one developed by Garfield, delMas & Chance, 1999) was chosen, among other things, aiming at examining participant teachers' statistical literacy and conceptions of variability by emphasizing comparison of distributions, statistical setting which is as "a fruitful arena for expanding teachers' understanding of distribution and conceptions of variability" (Makar & Confrey, 2004, p.371).

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(a) Answer this task in as many different ways as you can. Please, be sure to show every step of your solution process.

Figure 17: Question (a) used in the present study

Question (a)—see Figure 17—prompts teachers to provide as many appropriate answers to the given task as possible. In order to do that, teachers are expected to carry out the following types of cognitive processes:

- Identify variability, as well as different aspects of the given data sets.
- Describe and compare data sets.
- Translate the given histograms into a mathematical symbolic (e.g., a frequency distribution table) or graphical (e.g., a boxplot or a frequency polygon) representation.

– Interpret data through the given histograms or any equivalent representation, in order to describe and analyze it.

– Measure variability through the computation of measures of variation (e.g., mean absolute deviation, variance, and standard deviation), and interpret appropriately such statistics.

– Understand the relation among measures of central tendency (e.g., mean and median) and measures of variation.

– Understand the meaning of fundamental statistical concepts that could be used to answer the posed task.

As several statistics educators point out (e.g., delMas, 2002; Shaughnessy, 2007; Garfield & Ben-Zvi, 2008), all the aforementioned cognitive processes are skills related to statistical literacy, and individuals who appropriately engage in such processes demonstrate being statistically literate. Moreover, through the answer provided to Question (a), it would be possible not only to gain insight into teachers' content knowledge about several statistical ideas related to the interpretation of variability in the given situation, but also to identify teachers' conceptions of variability in the particular context of histograms and comparing distributions (cf. González, 2011; González & Isoda, 2011; Isoda & González, 2012). Then, teachers' responses to Question (a) are expected to provide enough information to answer the following three questions, which were generated to assess the indicators of *common content knowledge*—which will be regarded here as statistical literacy—used in the current study:

- **Is the teacher able to give an appropriate and correct answer to the given task? (Indicator A-1):**

  In order to answer this question appropriately, teachers must engage in one or more of the aforementioned cognitive processes. For example, a teacher may translate the given histograms into their corresponding frequency distribution tables, compute measures of variation in order to measure the variability in the data, and then describe and compare both data sets through the interpretation of the measures of variation calculated by him/her, in order to figure out which of the two histograms has more variability.

- **Does the teacher consistently identify and acknowledge variability and correctly interpret its meaning in the context of the given task? (Indicator A-2):**

  Question (a) asks teachers to answer the given task in as many different ways as possible. Then, it is anticipated that all of them will show evidence of null, simple or sophisticated acknowledgment of the variability in the given data—e.g., responses concerned only with extremes and the interpretation of the range (simple acknowledgement of variability), or responses mentioning both middles and extremes in data, or even pointing out deviations of data from some fixed value, such as the mean or median (sophisticated acknowledgement of variability)—, whereas some teachers are expected to answer from the perspective of misconceptions—i.e., judging variability by putting attention on the fluctuation of the bars, the symmetry of the distributions, the sample size, or the number of bars—, which is an incorrect interpretation and acknowledgment of the variability. So, while assessing Question (a), attention will be paid to how consistent and appropriate is the acknowledgement of variability in each of the methods used by the respondent.

- **What are the different ways in which teachers conceptualize variability when dealing with a task involving histograms and comparison of distributions?**

According to González (2011), González and Isoda (2011), and Isoda and González (2012), the answers given by the surveyed teachers to the posed task will provide evidence of the conceptions of variability held by them; that is, evidence on how teachers describe and conceptualize variability in the given setting. Using the categorization proposed by Shaughnessy (2007) as starting point, the following four conceptions of variability are expected to be identified from teachers' answers (González, 2011; González & Isoda, 2011; Isoda & González, 2012):

- *Variability in particular values, including extremes or outliers*: people holding this conception focus their attention on particular data value in a graph or a data set.

- *Variability as distance or difference from some fixed point*: people holding this conception think of variability as an actual or a visual measurement of the distance of each or some elements of a data set either from an endpoint value or from some measure of center.

- *Variability as the sum of residuals*: people holding this conception think of variability as the measure of the total variation of an entire distribution of data via the calculation of residuals from some fixed value.

- *Variation as distribution*: people holding this conception are able to consider many theoretical features of a distribution simultaneously when variability between or among a set of distributions is compared; that is, they perceive data as an aggregate.

Thus, this task will provide teachers with opportunities to represent and examine many variability-related ideas, through which it will be possible to investigate how participants acknowledge and describe variability in this particular setting, and even whether teachers think about data as an aggregate—i.e., as an emergent entity (namely distribution) that has characteristics not visible in any of the individual elements in the aggregate (Mokros & Russell, 1995; Konold & Pollatsek, 2002).

Based on the previous researches with this task or similar ones reported in the literature (e.g., Garfield, delMas & Chance, 1999; Meletiou & Lee, 2003, 2005; Cooper & Shore, 2007) and refining the categorization proposed by González (2011), Isoda and González (2012) and González (2013), among the answers that respondents could provide to this task, the following ones stand out:

(1) *Distribution A, giving no reason, just guessing, by arguing intuitive ideas, or based on a mistaken calculation*: in this type of answer, the teacher not only mistakenly chooses Distribution A as the one with more variability without any justification, or supporting this choice on an idiosyncratic argument—such as "I think so", "I suppose so", "it is obvious", "it is evident"—or a misinterpreted calculation.

(2) *Distribution A, based on a misinterpretation related to symmetry and/or a poor fit to a normal distribution*: in the answers falling into this category, teachers show evidence of thinking of variability in terms of symmetry or degree of fit to a normal distribution—or lack thereof—, which are two misconceptions that disregard the connection between measures of central tendency and the variability of data dispersed around a center, and consider exclusively the aforementioned visual features of the histograms. Teachers in this category will be considered to be holding the conception labeled in this study as "Variability as visual cues in the graph", since this way of thinking about variability is not accounted for by Shaughnessy's (2007) framework (Shaughnessy, 2013, personal communication, July 19, 2013).

(3) *Distribution A, based on arguments related to differences in the heights of the bars*: this answer, which is very common in this kind of tasks—cf. Meletiou & Lee, 2003, 2005; Cooper & Shore, 2007; González, 2011; Isoda & González, 2012, González, 2013—, typically express the misconception that the histogram with more varied values—or less pattern—in the heights of the bars has a greater variability of its data set. Teachers with this misconception tend to think of symmetrical or quasi-normal distributions, as well as histograms in which the heights its bars were basically flat, as having less variability than its asymmetrical counterparts. Teachers in this category also hold the conception "Variability as visual cues in the graph", since they disregard the actual data values, their spreads, and the measures of distance or difference, and mistakenly acknowledge variability as unevenness in the frequencies of a histogram, symmetry, or closeness (or lack thereof) of fit to a normal distribution.

(4) *Distribution B, giving no reason, just guessing, by arguing intuitive ideas, or by misinterpretation*: there is the possibility of a teacher correctly selecting Distribution B as the one with more variability, but providing an idiosyncratic argument to support this choice, or by giving an argument based on the misconception of thinking of variability in terms of the largest span in the vertical axis—i.e., judging the variability of the data displayed in a histogram by the largest difference in height of its bars—, instead of looking at the horizontal spread of data around a measure of central tendency. This is a common mistake made by many students, and even by some of their teachers, at any school level. Then, teachers in this category have the tendency to

incorrectly think that a histogram with narrow tails and a high peak has greater variability than one with bars of more similar heights. Teachers providing answers of this kind seem to hold the conception named by Shaughnessy (2007, p.984) as "Variability in particular values, including extremes or outliers", since while regarded variability as the largest span in frequency, these teachers focus their attention on particular data values in the graphs.

(5) *Distribution B, based on arguments related to simple recognition of variability*: teachers in this category are those who not only choose the right distribution, but also provide an argument in which is simple recognition of variability is evidenced. Focusing on the range of the data, or giving answers concerned only with extremes in data without connecting them with a measure of central tendency, is what is meant by "simple recognition of variability". Since teachers falling into this category focus their attention on particular data values in the histogram, they seem to hold the conception named by Shaughnessy (2007, p.984) as "Variability in particular values, including extremes or outliers".

(6) *Distribution B, based on arguments related to sophisticated recognition of variability*: teachers in this category are those who not only choose the right distribution, but also provide an argument in which is sophisticated recognition of variability is evidenced. Such "sophisticated recognition of variability is evidenced" is understood here as responses mentioning both middles and extremes in data, discussing the connections

between middles in data and the variability of data dispersed around a middle, or pointing out deviations of data from some fixed value, such as the mean or median. Depending on the case, teachers providing these kind of answers might hold the conceptions of variability identified by Shaughnessy (2007, pp.984–985) as "Variability as distance or difference from some fixed point", "Variability as the sum of residuals", or "Variation as distribution".

5.2 Results and findings regarding Question (a)

Table 11: Results obtained from participants' answers to Question (a) – Frequency and percentage

		Frequency (%)			
		Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
Category	A0:No response.	2 (10.5)	2 (13.2)	1 (5.3)	5 (9.4)
	A1:Distribution A, giving no reason, just guessing, by arguing intuitive ideas, or based on a mistaken calculation.	1 (5.3)	1 (6.7)	1 (5.3)	3 (5.7)
	A2:Distribution A, based on a misinterpretation related to symmetry and/or a poor fit to a normal distribution.	1 (5.3)	1 (6.7)	3 (15.8)	5 (9.4)
	A3:Distribution A, based on arguments related to differences in the heights of the bars (e.g., "Distribution A is bumpier"; "the number of different heights in A is higher than in B").	3 (15.8)	1 (6.7)	5 (26.3)	9 (17.0)
	A4:Distribution B, giving no reason, just guessing, by arguing intuitive ideas, or by misinterpretation (e.g., "B has a larger span in frequency than A", "B because is symmetrical").	4 (21.1)	1 (6.7)	4 (21.0)	9 (17.0)
	A5:Distribution B, based on arguments related to simple recognition of variability (i.e., answers concerned only with extremes or the ranges of each distribution; e.g., "because it's more spread out").	2 (10.5)	1 (6.7)	2 (10.5)	5 (9.4)
	A6:Distribution B, based on arguments related to sophisticated recognition of variability (i.e., answers connecting both middles and extremes; e.g. "because the scores differ more from the center").	6 (31.5)	8 (53.3)	3 (15.8)	17 (32.1)

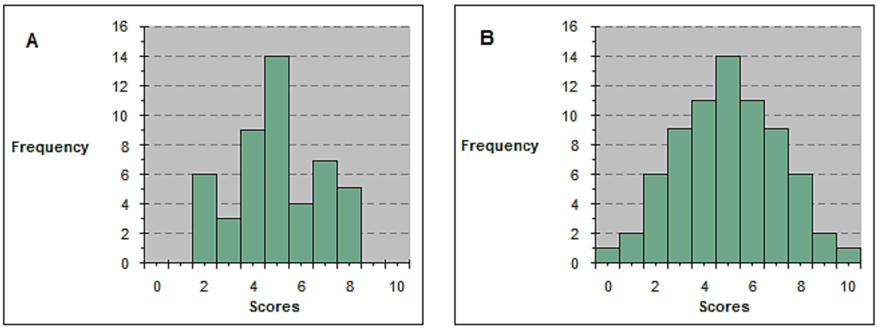
On the basis of the conceptual analysis and framework for the "Choosing the distribution with more variability" task previously described, an interpretation of teachers' answers was carried out, which is summarized in Table 11.

5.3 Results and findings regarding Question (b)

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(b) What are the important ideas that might be used to answer this task?

Figure 18: Question (b) used in the present study

Question (b)—see Figure 18—was posed in order to examine teachers' ability to anticipate and make connections with major disciplinary ideas and structures related to the concepts involved with the solving of the posed task, as well as to investigate teachers' ability to build bridges between the cognitive demands of this task and fundamental ideas, practices, values and sensibilities of the discipline, which are two of the main characteristics of the so-called *horizon content knowledge* (cf. Ball & Bass, 2009). Evidence of these skills is accounted for by Indicator C-2.

Forty-nine teachers—92.5% overall; 17 working at lower high school, 14 at upper high school, an 18 at both levels—answered to this question. A “bottom up” approach to coding (Coffey & Atkinson, 1996) was initially used by the researcher to analyze the collected data. This grounded form of analysis ensures that the themes or categories extracted were, in fact, grounded in the data and hence reflected the participants’ own knowledge base of HCK. The author reviewed all the given answers to Question (b) and identified answers that occurred frequently in the data. Such answers appearing to contain similar content were initially given the same code by the researcher, and each code was further analyzed to find true meanings within their text. A process of reduction and clustering of categories, which were refitted and refined, followed (Heath & Cowey, 2004). Summary groupings (or 'clusters') of themes that share common meaning emerged from this process were finally condensed into twelve theme clusters. A full description of the clusters that were identified from the collected data follows:

B1 – Answers mentioning ideas for the construction of frequency distribution tables: this cluster includes all the answers in which appeared key terms such as “frequency distribution table”, “frequency”, “frequency distribution”, “absolute frequency”, “relative frequency”, “cumulative frequency”, “data values”, “class marks”, “class intervals”, and “class size”.

B2 – Answers mentioning measures of central tendency: this cluster includes all the answers referring explicitly to either “measures of central tendency” or key terms such as “mean”, “median”, “mode”, “mathematical expectation” and “expected value”.

B3 – Answers mentioning measures of variation: this cluster includes all the answers referring explicitly to either “measures of variations” or key terms such as “variance”, “standard deviation”, “residuals”, “interquartile range”, “semi-quartile range”, “quartiles”, “percentiles”, “relative dispersion”, “coefficient of variation”, “mean deviation” and “kurtosis”.

B4 – Answers mentioning specific mathematical operations: this clusters groups those answers referring to key terms such as “summation”, “addition”, “subtraction”, “division” and “numerical value”.

B5 – Answers mentioning “distribution”: this cluster includes the answers referring to key terms such as “distribution” and “normal distribution”.

B6 – Answers mentioning “range” or “mid-range”: this cluster includes the answers in which the key terms “range” and “mid-range” appear.

B7 – Answers mentioning “spread” or “variability”: this cluster includes the answers referring to key terms “spread” and “variability”.

B8 – Answers mentioning “population” or “sample”: this cluster includes the answers in which the key terms “population”, “sample” and “sample size” appear.

B9 – Answers mentioning “symmetry”: this cluster includes the answers in which the key terms “symmetry” and “asymmetry” appear.

B10 – Answers mentioning “variable”: this cluster includes the answers in which key terms such as “variable”, “continuous variable”, “discrete variable”, “continuity” and “discontinuity” appear.

B11 – Answers mentioning statistical graphs: this cluster groups the answers referring to key terms such as “graphs”, “histogram” and “bar graphs”.

B12 – Answers mentioning data handling methods: this cluster groups the answers referring to key terms like “data collection”, “data grouping”, “data analysis” and “data inference”.

After building up the aforementioned clusters, the information gathered from teachers’ answers can be summarized as follows:

Table 12: Results obtained from participants' answers to Question (b) – Frequency and percentage

		Frequency (%)			
		Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
Category	B0: No response.	1 (5.3)	1 (6.7)	2 (10.5)	4 (7.5)
	B1: Answers mentioning ideas for the construction of frequency distribution tables.	11 (57.9)	8 (53.3)	7 (36.8)	26 (49.1)
	B2: Answers mentioning measures of central tendency.	9 (47.4)	13 (86.7)	14 (73.7)	36 (67.9)
	B3: Answers mentioning measures of variation.	6 (31.5)	11 (73.3)	7 (36.8)	24 (45.3)
	B4: Answers mentioning specific mathematical operations.	0 (0.0)	2 (13.2)	0 (0.0)	2 (3.8)
	B5: Answers mentioning "distribution".	4 (21.0)	1 (6.7)	2 (10.5)	7 (13.2)
	B6: Answers mentioning "range" or "mid-range".	2 (10.5)	2 (13.2)	4 (21.0)	8 (15.1)
	B7: Answers mentioning "spread" or "variability".	5 (26.3)	4 (26.7)	6 (31.5)	15 (28.3)
	B8: Answers mentioning "population" or "sample".	0 (0.0)	0 (0.0)	4 (21.0)	4 (7.5)
	B9: Answers mentioning "symmetry".	1 (5.3)	0 (0.0)	2 (10.5)	3 (5.7)
	B10:Answers mentioning "variable".	1 (5.3)	1 (6.7)	2 (10.5)	4 (7.5)
	B11:Answers mentioning statistical graphs.	3 (15.8)	2 (13.2)	3 (15.8)	8 (15.1)
	B12:Answers mentioning data handling methods.	3 (15.8)	2 (13.2)	1 (5.3)	6 (11.3)

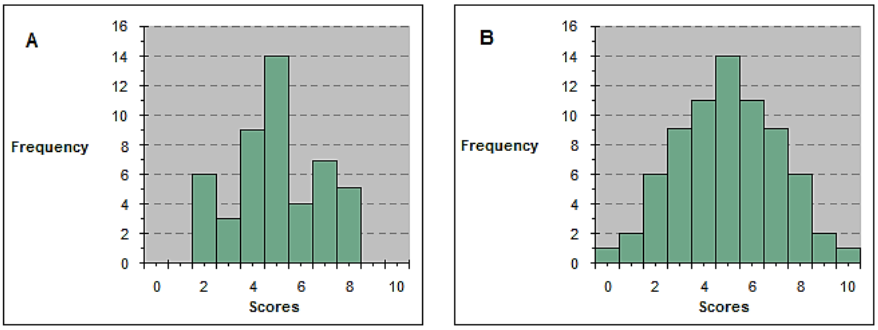
Participants were allowed to write down as many statistical ideas as they could identify to be associated with answering the posed task. Thus, a single response to Question (b) could be coded into one or more categories, and hence the percentages within each group of teachers sum to more than 100%.

5.4 Results and findings regarding Question (c)

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(c) Suppose that, after posing this task to your students, three of them come up with the following answers:

STUDENT 1: "Distribution A has more variability because it's not symmetrical."

STUDENT 2: "Distribution A ranges from 3 to 14, while Distribution B ranges from 1 to 14. Then, Distribution B has more variability."

STUDENT 3: "The bars in Distribution A are clumped closer to the central bar than they are in Distribution B. Then, Distribution B has more variability."

Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer.

Figure 19: Question (c) used in the present study

When teachers are examining the work done by students in their absence—e.g., when marking homework papers—, they are required to have some purely mathematical way of determining the accuracy or inaccuracy of students' answers and judgments, based just on the provided numerical data or written explanations, as well as to be able to explain how the students arrived at their solutions, by providing the likely reasoning that led them to their answers. These abilities exemplify a pair of important aspects of the cognitive construct known as *specialized content knowledge* (SCK) (Ball et al., 2008; Ball & Bass, 2009), a kind of mathematical knowledge specific to the work of teaching, difficult to be articulated by other mathematically trained professionals who do not teach children. Therefore, by answering Question (c)—see Figure 19—, it is anticipated that teachers will provide evidence of their ability to determine the accuracy of common and non-standard arguments given by students—skill that is accounted for by Indicator B-1—, as well as of their ability to analyze right and wrong solutions given by students by means of providing appropriate explanations about what reasoning is likely behind such solutions, and why—skill that is accounted for by Indicator B-2.

Question (c) was, then, posed in order to elicit respondents' SCK in relation to the given task, by dealing with the answers to such task provided by three fictitious students. The answers given by Student 1 and 2 are examples of common misconceptions related to the estimation of variability that students frequently exhibit when comparing histograms. In the case of the answer given by Student 1, it reflects a misconception about the relation of variability and symmetry, in which histograms with bars distributed in a symmetrical pattern have a greater variability than those that do not. In the case of the answer provided by Student 2, it reflects a misconception in which variability is measured in terms of the largest span in the vertical axis—i.e., judging the variability of the data displayed in a histogram by the largest difference in height of its bars—, instead of considering spreads of data values or measures of distance or difference from a center. In contrast to these answers, the one given by Student 3 exemplifies an appropriate visual estimation and interpretation of the variability in the given histograms, a right approach to estimate variability without making calculations or engaging in the process of transnumeration by assessing the degree of data clustering around the modal class.

Twenty-eight teachers—52.8% overall; 13 working at lower high school, 7 at upper high school, an 9 at both levels—completely answered to this question; that is, 28 teachers provided responses indicating whether the answers given by the three students were correct or not and why, as well as comments about the most likely reasoning behind such students' answers (see Table 13). From the remaining 25 participants, eight did not answer this question, while seventeen teachers—32.1% overall; 5 working at lower high school, 6 at upper high school, an 6 at both levels—partially answered to this question; that is, 17 teachers failed to provide assessment on the correctness or incorrectness of all three students' responses, or comment about the most likely reasoning behind such students' answers (see Table 14).

Table 13: Results obtained from participants who fully answered Question (c) – Frequency and percentage

		Frequency (%)								
		Student 1			Student 2			Student 3		
		L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)
Teachers' Assessment	Correct	2	1	3	5	3	4	6	5	6
	Incorrect	11	5	6	8	3	5	7	1	3
	Total	28 (52.8%)			28 (52.8%)			28 (52.8%)		

Table 14: Results obtained from participants who partially answered Question (c) – Frequency and percentage

		Frequency (%)								
		Student 1			Student 2			Student 3		
		L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)
Teachers' Assessment	Correct	1	1	3	1	2	3	2	4	2
	Incorrect	4	5	3	3	4	3	2	2	3
	Total	17 (32.1%)			16 (30.2%)			15 (28.3%)		

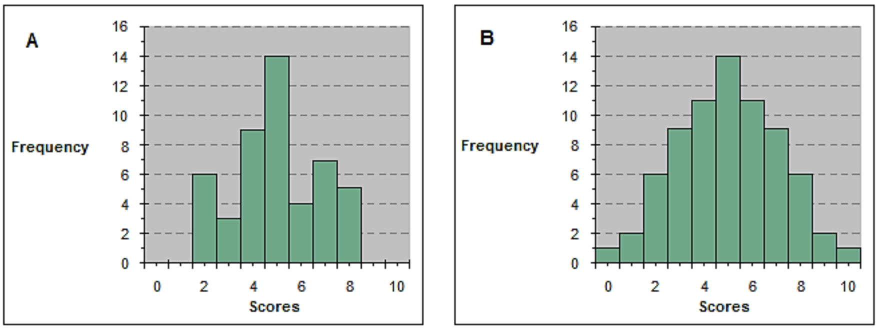
5.5 Results and findings regarding Question (d)

This question—see Figure 20—was posed with the purpose of finding out whether teachers know about students' likely responses—both correct and incorrect ones—and potential difficulties when solving the given task, as well as the reasons behind such responses and difficulties. By focusing on these aspects, this question intends to gain insight into teachers' *knowledge of content and students*, since the ability to anticipate students' common responses and difficulties on a particular task is accounted for by Indicator D-1, and the ability to provide the most likely reasons for such students' common responses and difficulties is accounted for by Indicator D-2.

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(d) Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), and difficulties you would expect from them? Briefly explain why you think so. (Regarding to the most likely answers that you might get from the students, please do not include those mentioned in part (c).)

Figure 20: Question (d) used in the present study

Question (d) was fully answered by 24 teachers—45.3% overall; 8 working at lower high school, 7 at upper high school, an 9 at both levels—; that is, 24 teachers provided responses indicating both students' likely common answers—as well as the accuracy or inaccuracy of such responses—and potential difficulties on the given task. Moreover, 19 teachers—35.8% overall; 7 working at lower high school, 6 at upper high school, an 6 at both levels—partially answered to this question; that is, 19 teachers failed to provide students' likely common answers to the given task, assessment on the correctness or incorrectness of such answers, or students' potential difficulties on the given task.

Participants were allowed to write down as many students' likely answers and potential difficulties to the posed task as they could. Thus, a single response to Question

(d) could provide different kind of possible students' answers to the posed task, which must be coded into categories, in order to make them more manageable for analysis and discussion. Therefore, a "bottom up" approach to coding (Coffey & Atkinson, 1996), followed by a process of reduction and clustering of the identified categories, which were refitted and refined (Heath & Cowey, 2004). A full description of the clusters of answers that were identified from the collected data follows:

Table 15: Clusters of answers obtained from participants who provided most likely responses from students to the given task

Most likely responses from students identified by teachers	
Choosing Distribution A as the one with more variability	**Choosing Distribution B as the one with more variability**
• Distribution A because is more clustered. • Distribution A because has less data values/graphical content. • Distribution A because is more bumpy/asymmetric. • Distribution A because has heavier tails. • Distribution A because of miscalculation.	• Distribution B because is more spread out/has a larger range. • Distribution B because has more bumps/different bar heights. • Distribution B because of measures of variation/being more clustered around a middle. • Distribution B because is symmetric/normal-like. • Distribution B because has more data values/bars. • Distribution B because has a larger span in frequency. • Distribution B because has a larger area.

In addition to the answers identified above, some teachers thought of "Neither of them"—3 teachers—as well as "I don't know"—2 teachers—as most likely answers from the students, which were considered as appropriate answers by the researcher. The most likely responses from students more frequently identified by teachers who fully answered Question (d) were "Distribution A because is more bumpy/asymmetric"—by 12 teachers

(22.6% overall)—; “Distribution B because has more data values/bars”—by 4 teachers (7.5% overall)—; “Distribution B because of measures of variation/being more clustered around a middle”—by 4 teachers (7.5% overall)—; and “Distribution B because is more spread out/has a larger range”—by 4 teachers (7.5% overall). Besides these likely responses from students identified by the respondents, neither of the remaining ones has a frequency of occurrence of more than 3 times.

Among these 19 teachers who partially answered Question (d), 15 (28.3% overall) provided responses indicating students’ likely common answers to the posed task. The most likely responses from students more frequently identified by teachers who partially answered Question (d) were “Distribution A because is more bumpy/asymmetric”—by 9 teachers (17.0% overall)—; and “I don’t know”—by 2 teachers (3.8% overall). Besides these likely responses from students identified by the respondents, neither of the remaining ones has a frequency of occurrence of more than 1 time.

A similar process to the one carried out with the most likely responses from students identified by teachers was carried out to sort out the most likely difficulties for students pointed out by the participants. The result is shown in Table 16.

Table 16: Clusters of answers identified from participants who provided most likely difficulties for students in the given task

Most likely difficulties for students identified by teachers
• Lack of knowledge/understanding about the concepts/data handling/application. • Limited handling/lack of knowledge of basic calculations and formulas. • Low level/lack of reasoning/analysis skills. • Lack of familiarity with this kind of tasks. • Interpretation of data and graphs. • Construction of frequency distribution tables. • Confusing "variability" with "symmetry". • Confusing "variability" with "clustering". • Lack of interest. • Construction of graphs and histograms. • Translation of graphs into frequency distribution tables.

The most likely difficulties from students more frequently identified by teachers who fully answered Question (d) were "Lack of knowledge/understanding about the concepts/data handling/application"—by 10 teachers (18.9% overall)—; and "Limited handling/lack of knowledge of basic calculations and formulas"—by 9 teachers (17.0% overall). Besides these likely difficulties that students might face identified by the respondents, neither of the remaining ones has a frequency of occurrence of more than 3 times.

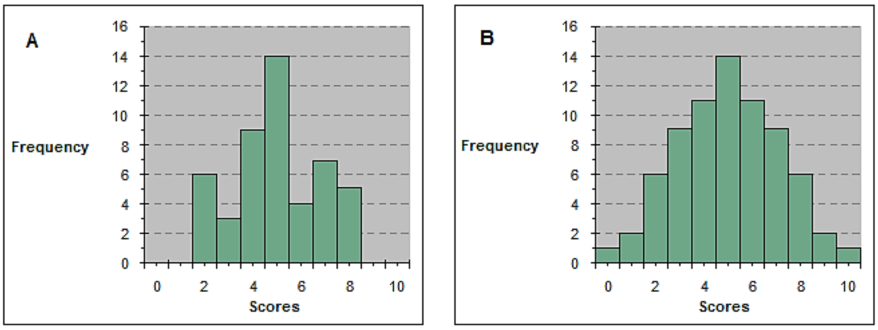
Among these 19 teachers who partially answered Question (d), 14 (26.4% overall) provided responses indicating the most likely difficulties that students could experience when trying to solve the posed task. The most likely difficulties from students more frequently identified by teachers who partially answered Question (d) were "Interpretation of data and graphs"—by 5 teachers (9.4% overall)—; and "Limited handling/lack of knowledge of basic calculations and formulas"—by 5 teachers (9.4% overall). Besides these likely difficulties students may encounter identified by the respondents, neither of the remaining ones has a frequency of occurrence of more than 2 times.

5.6 Results and findings regarding Question (e)

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(e) Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects. (Please, focus your response on whether there is a significant mathematical/statistical insight in the student's answer, and whether there are forthcoming contents in future classroom subjects connected to the notions being said or implied in such answer.)

Figure 21: Question (e) used in the present study

According to Ball and Bass (2009), *horizon content knowledge* might guide teaching responsibilities and acts such as making judgments about mathematical importance, hearing mathematical significance in what students say, and catching mathematical distortions or possible precursors to future mathematical confusions or misrepresentations. The intention of Question (e)—see Figure 21—is to examine this aspect of teachers' professional knowledge, since teachers' answers are anticipated to provide information about their ability to identify whether a student response is interesting or significant, mathematically or statistically speaking—skill that is accounted for by Indicator C-1.

Forty-four teachers—83.0% overall, 17 working at lower high school, 13 at upper high school, and 14 at both levels—answered to this question. These teachers provided a series of answers regarding whether any of the answers given by the fictitious students were mathematically/statistically interesting or significant, ranging from “Neither of the answers were interesting or significant” to “All three answers were interesting or significant”. The following table shows the combination of answers given by the respondents to Question (e) about whether the responses given by the fictitious students in Question (c) were significant or not:

Table 17: Results obtained from participants’ answers to Question (e) – Frequency and percentage

<table>
<tr><th colspan="2" rowspan="2"></th><th colspan="4">Frequency (%)</th></tr>
<tr><th>Lower High School (19 teachers)</th><th>Upper High School (15 teachers)</th><th>Both Levels (19 teachers)</th><th>Total (53 teachers)</th></tr>
<tr><th rowspan="8">Student Answer(s) found Significant/Interesting</th><td>Neither of the students’ responses.</td><td>2 (10.5)</td><td>3 (20.0)</td><td>4 (21.1)</td><td>9 (17.0)</td></tr>
<tr><td>Only Student 1’s response.</td><td>3 (15.8)</td><td>0 (0.0)</td><td>3 (15.8)</td><td>6 (11.3)</td></tr>
<tr><td>Only Student 2’s response.</td><td>0 (0.0)</td><td>2 (13.2)</td><td>2 (10.5)</td><td>4 (7.5)</td></tr>
<tr><td>Only Student 3’s response.</td><td>5 (26.3)</td><td>3 (20.0)</td><td>0 (0.0)</td><td>8 (15.1)</td></tr>
<tr><td>Student 1’s and Student 2’s responses.</td><td>0 (0.0)</td><td>0 (0.0)</td><td>0 (0.0)</td><td>0 (0.0)</td></tr>
<tr><td>Student 1’s and Student 3’s responses.</td><td>0 (0.0)</td><td>0 (0.0)</td><td>0 (0.0)</td><td>0 (0.0)</td></tr>
<tr><td>Student 2’s and Student 3’s responses.</td><td>1 (5.3)</td><td>1 (6.7)</td><td>0 (0.0)</td><td>2 (3.7)</td></tr>
<tr><td>All the three students’ responses.</td><td>6 (31.6)</td><td>4 (26.7)</td><td>5 (26.3)</td><td>15 (28.3)</td></tr>
</table>

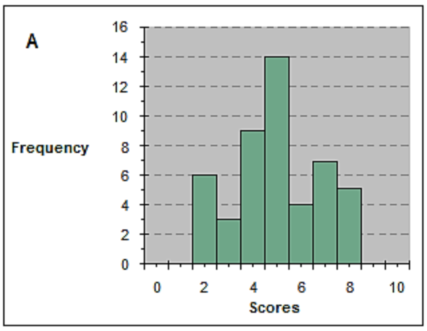
5.7 Results and findings regarding Question (f)

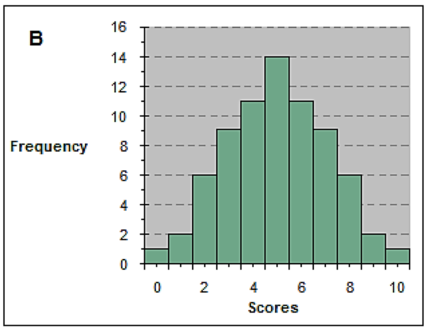
Teachers must know at which grade levels and/or content areas particular topics in their field of expertise are typically taught. This is one characteristic of the so-called *knowledge of content and curriculum*, being one of its aspects knowledge about "topics and issues that have been and will be taught in the same subject area in the preceding and later years" (Ball et al., 2008, p. 391). Question (f)—see Figure 22—seeks to elicit evidence of the KCC held by the surveyed teachers, in relation to the statistical ideas present in the task posed in Item 1, since teachers' answers are anticipated to provide information about their knowledge about how the statistical ideas included in the Venezuelan secondary school mathematics curriculum are developed throughout the compulsory education—knowledge that is accounted for by Indicator F-1.

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:





Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(f) Briefly describe how the important ideas involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling.

Figure 22: Question (f) used in the present study

Forty-five teachers—84.9% overall, 17 working at lower high school, 13 at upper high school, and 15 at both levels—answered to this question. The answers provided by these teachers were analyzed using a “bottom up” approach to coding (Coffey & Atkinson, 1996), in order to ensure that the themes or categories extracted were, in fact, grounded in the data and hence reflected the participants’ own knowledge base of KCC. All the given answers to Question (f) were reviewed, and answers that occurred frequently in the data were identified. Such answers appearing to contain similar content were initially given the same code by the researcher, and each code was further analyzed to find true meanings within their text. A process of reduction and clustering of categories, which were refitted and refined, followed (Heath & Cowey, 2004). Summary groupings (or 'clusters') of themes that share common meaning emerged from this process were finally condensed into five theme clusters. A full description of the clusters that were identified during the coding process, as well as the breakdown of the collected data, is presented in the following table:

Table 18: Results obtained from participants’ answers to Question (f) – Frequency and percentage

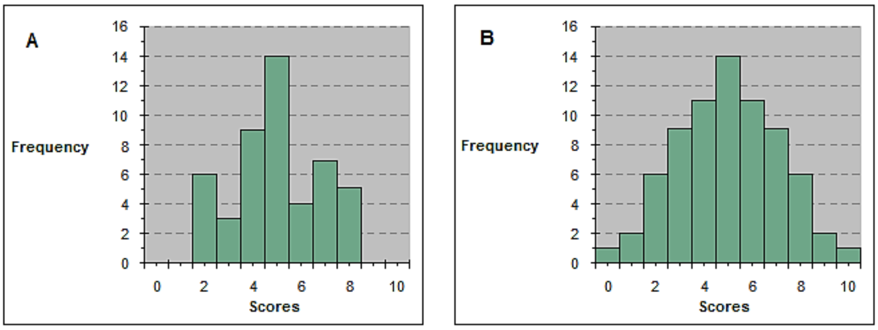
		Frequency (%)			
		Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
Category	F0: No response.	2 (10.5)	2 (13.$\overline{3}$)	4 (21.0)	8 (15.1)
	F1: I don’t know / I’m not familiar with the content/ I don’t teach statistics.	8 (42.1)	6 (40.0)	6 (31.6)	20 (37.7)
	F2: General answer, without specification of grade or grade level.	4 (21.0)	0 (0.0)	3 (15.8)	7 (13.2)
	F3: General answer, with specification of grade or grade level.	1 (5.3)	2 (13.$\overline{3}$)	5 (26.3)	8 (15.1)
	F4: Mention of specific topics, without specifying the grade or grade level.	1 (5.3)	2 (13.$\overline{3}$)	1 (5.3)	4 (7.6)
	F5: Mention of specific topics, specifying the grade or grade level.	3 (15.8)	3 (20.0)	0 (0.0)	6 (11.3)

5.8 Results and findings regarding Question (g)

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:



Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(g) Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the setting of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order.

Figure 23: Question (g) used in the present study

All the collected answers to Question (g)—see Figure 23—were examined for evidences in three specific dimensions. The first dimension was curricular features of the planned lessons. Planned lessons were specifically examined for evidence of knowledge, understanding and support of the objectives and methodological guidelines provided by official curriculum documents in Venezuela in relation to the teaching of statistics at secondary level. These knowledge and skills are features of the cognitive domain *knowledge of content and curriculum* (KCC).

The second dimension was knowledge about different instructionally viable approaches for teaching statistical ideas from the viewpoint of variability, as well as knowledge on how to deploy them effectively in the classroom. These knowledge and skills are features of the cognitive domain *knowledge of content and teaching* (KCT).

The third dimension was beliefs about the nature of statistics and about teaching and learning statistics hold by the surveyed teachers. Beliefs are identified by Gal (2004) as one of the dispositional elements of statistical literacy, and comprise the affective-motivational facet of the framework for SKT proposed in the present study.

Forty-four teachers—83.0% overall, 17 working at lower high school, 13 at upper high school, and 14 at both levels—answered to Question (g). However, due to the multidimensional and complex assessment that should be performed to analyze the collected data from Question (g), results and findings together will be presented and discussed in detail at the section “Discussion of results”.

5.9 Discussion of results

5.9.1 Discussion of results regarding Question (a)

Even though all the numerical evidence that can be deduced from the given histograms (see Table 19), only 41.5% of the surveyed teachers (22/53) gave a correct response to the posed task; that is, only 41.5% of the surveyed teachers chose Distribution B as the one with more variability, and supported their selection on arguments based on simple or sophisticated recognition of variability—i.e., teachers' answers falling into categories A5 and A6, which represent those who chose Distribution B based on arguments related to simple or sophisticated recognition of variability, respectively. Among these 22 teachers, 5 of them (9.4% overall) supported their answer with only the calculation and/or interpretation of the range—i.e., teachers' answers falling into category A5. Those teachers whose answers were exclusively based on the range are the ones who do not exhibit an aggregate view of data and distribution, since they are rather concerned with the variability of just the endpoints of the data set without considering a measure of central tendency, which could be interpreted as a very simple acknowledgment of variability. The remaining 17 teachers in this group—i.e., the ones falling into category A6—were those whose answers mention both middles and extremes in data, discuss the connections between middles in data and the variability of data dispersed around a measure of central tendency, or even point out deviations of data from some fixed value, such as the mean or median, could be placed at an even higher level, since they provide evidence of holding an aggregate view of data and distribution and evidenced a sophisticated recognition of variability.

Table 19: Comparison of some measures of variation related to Distributions A and B

Measure of Variation	Distribution A	Distribution B	Variability
Range	6 (discrete) 7 (continuous)	10 (discrete) 11 (continuous)	A < B
Variance	3.2	4.5	A < B
Standard deviation	1.8	2.1	A < B
Interquartile range	2.7	3	A < B
Mean deviation	1.4	1.7	A < B

Within the group of teachers falling into category A6—i.e., teachers' who chose Distribution B based on arguments related to sophisticated recognition of variability—, 11 out of 17 (20.8% overall) provided more than one answering approach to the given task. Moreover, among the 17 teachers falling into category A6, 9 of them (17.0% overall) created frequency distribution tables in order to calculate measures of variation; 9 teachers (17.0% overall) supported their answer with a naked eye description, comparison and interpretation of the data clustering around the mean; 7 of them (13.2% overall) calculated and compared ranges as well as at least one measure of variation; 5 teachers (9.4% overall) calculated and compared the mean absolute deviations; 5 teachers (9.4% overall) calculated and compared the standard deviations; 3 teachers (5.7% overall) calculated and compared the variances; and 1 teacher (1.9% overall) calculated and compared the residuals from the mean .

It is also noticeable that the group of teachers working only at upper high school level was the one with the highest proportion of correct answers in Question (a), with a 60.0% (9/15) of responders within that group doing so, being the teachers working at both levels the group with the lowest proportion of correct answers in Question (a), with a 26.3% (5/19) of responders within that group doing so.

Within the teachers in category A6, there were 4 of them (7.5% overall) whom provided both correct and incorrect answering approaches to the given task. From those teachers, 3 (5.7% overall) made a comparison of measures of central tendency, whereas 1 teacher (1.9% overall) calculated and compared the maximum spans in frequency in both histograms, being neither of both methods a proper way to determine which distribution has more variability. Among those teachers, 2 of them calculated the median in an improper way, since in the case of grouped data there is a special formula to calculate such statistics, which these teachers did not use.

It is worthwhile to highlight that 7 teachers (13.2% overall) whose answers fell into category A6 committed errors in the calculations of measures of variation made to support their answers. Three of these teachers (5.7% overall) made mistakes in the calculation of the ranges; 3 (5.7% overall) made mistakes in the calculation of the standard deviations; 2 (3.8% overall) made mistakes in the calculation of the variances; 1 (1.9% overall) made mistake in the calculation of the residuals from the mean; and 1 (1.9% overall) made mistake in the calculation of the mean absolute deviations. Nevertheless, despite all these errors, the arguments and interpretations made by these teachers were consistent with the numerical results obtained by them. A similar phenomenon was appreciated in a similar research carried out with Japanese secondary school mathematics teachers (cf. González, 2012, 2013a, 2013b).

Under a third of the surveyed teachers (17/53) argued incorrectly that Distribution A is the one with more variability. These teachers are those whose answers fell into categories A1—respondents basing their choice on a mistaken calculation or without reason—, A2—respondents basing their choice on a misinterpretation related to symmetry and/or a poor fit to a normal distribution—, and A3—respondents basing their choice on arguments related to differences in the heights of the bars. Among these teachers, 9 of them—7.0% overall, those teachers falling into category A3—supported their answer by giving arguments on differences in the heights of the bars, and by interpreting histograms by looking at the vertical axes when comparing the variability of the given histograms (e.g., providing answers like “Distribution A has more variability because it’s bumpier”; “Distribution A has more variability because it has more different heights than Distribution B”; “Distribution A has more variability because the heights of its bars fluctuate a lot”). It is noticeable that the group of teachers working at both lower and upper high school levels was the one with the highest proportion of answers in this category, with a 26.3% (5/19) of responders within that group doing so.

There were 5 teachers (9.4% overall) falling into category A2, which accounts for those people who show evidence of thinking of variability in terms of symmetry or degree of fit to a normal distribution, a common misconception normally exhibited by students in this type of tasks. Again, the group of teachers working at both lower and upper high school levels was the one with the highest proportion of answers in this category, with a 15.8% (3/19) of responders within that group doing so.

It is worthwhile to highlight that, even among those teachers who correctly chose Distribution B, some argued a wrong justification—i.e., answers falling into category A4. So, we can see from the results in Table 11 that 17.0% (9/53) of the surveyed teachers argued that Distribution B has more variability because “it’s close to a normal distribution, while Distribution A is like a truncated normal distribution, for that reason variability of Distribution A is smaller”, or “the difference in frequency between the highest bar and lowest bar of the histogram in Distribution B is bigger than in Distribution A”. Teachers who gave a response like the latter seem to judge the variability of the data displayed in a histogram by the fluctuation of the heights of its bars, in a similar way that people who chose Distribution A because “it’s bumpier” or “it’s shaky”—i.e., similarly to teachers falling into category A3. In that sense, teachers whose answers fell into category A4 may tend to incorrectly think that a histogram with narrow tails and a high peak has greater variability than a histogram with bars of more similar heights. These findings are similar to those reported in the statistics education literature, which point out that both students and mathematics teachers—the latter in a lesser degree—tend to compare values on the vertical axis, and to conclude that the variable which has “more varied values on Y”, “less pattern on Y” or “is more random on Y” has a larger variability, a common misconception in this kind of problems (Garfield et al., 1999; Meletiou, 2000; Meletiou & Lee, 2002, 2003, 2005; Cooper & Shore, 2007; González, 2011; González & Isoda, 2011; Isoda & González, 2012).

Finally, it is noticeable that none of the surveyed teachers used a graphical approach—such as a cumulative frequency polygon—or the interquartile range to answer Question (a), even though these topics must be covered in the mathematics curriculum at Grade 9 and 11, respectively.

5.9.1.1 Regarding the fulfillment of Indicator A-1

In order to fully meet this indicator, teachers must show evidence of being able to give an appropriate and correct answer to the given task. Therefore, let us examine the answers of the 22 surveyed teachers chose Distribution B as the one with more variability, and supported their selection on arguments based on simple or sophisticated recognition of variability—i.e., teachers' answers falling into categories A5 and A6. In the group of teachers whose answers fell within the category A6, there were 4 of them (7.5% overall) who provided both correct and incorrect answering approaches to the given task. Moreover, 7 teachers within category A6—13.2% overall, with two of them in the group of teachers who mixed correct and incorrect answering approaches—committed errors in the calculations of measures of variation made to support their answers. These teachers cannot be considered as satisfactorily meeting Indicator A-1. Therefore, taking away these 9 teachers from the collective who provided a correct answer while supporting it on arguments based on simple or sophisticated recognition of variability, we have that just 22 – 9 = 13 teachers—24.5% overall%, 4 working at lower high school, 6 at upper high school, an 3 at both levels—appear to have fully met the assessment criteria of Indicator A-1.

5.9.1.2 Regarding the fulfillment of Indicator A-2

In order to fully meet this indicator, teachers must show evidence of being able to consistently identify and acknowledge variability and correctly interpret its meaning in the setting of the given task. Since appropriate acknowledgement of variability is required to satisfy the assessment criteria of this indicator, we need to analyze the answers given by all the teachers whose answers fell into categories A5 and A6—i.e., the answers given by those respondents who chose Distribution B as the one with more variability, and supported their selection on arguments based on simple or sophisticated recognition of variability. It is worthy to bring back to mind the fact that, despite 7 teachers (13.2% overall) whose answers fell into category A6 committing errors in the calculations of measures of variation made to support their answers, the arguments and interpretations made by them were consistent with the numerical results obtained by them. So, they demonstrated a sophisticated recognition of variability and correctly interpret its meaning, despite the computation errors in which they incurred. On the contrary, the 4 teachers (7.5% overall) belonging to category A6 who provided both correct and incorrect answering approaches to the given task were not consistent in the interpretation of the meaning of variability in the given task. Thus, 22 – 4 = 18 teachers—34.0% overall, 7 working at lower high school, 7 at upper high school, an 4 at both levels—appear to have fully met the assessment criteria of Indicator A-2.

5.9.1.3 Regarding conceptions of variability held by the surveyed teachers

According to previous researches (González, 2011; González & Isoda, 2011;

Isoda & González, 2012), the answers given by the surveyed teachers to the posed task will provide evidence of the conceptions of variability held by them. In the case of the task selected for this study, four of the conceptions identified by Shaughnessy (2007)—"Variability in particular values, including extremes or outliers", "Variability as distance or difference from some fixed point", "Variability as the sum of residuals" and "Variation as distribution"—, as well as one conception that is unaccounted for by his framework—which has been labeled as "Variability as visual cues in the graph" in the current study—might be identified from teachers' answers to Question (a). Each of these conceptions is linked to one or more particular category in Table 11.

Teachers whose answers focus on individual data values, such as the extremes of the distribution to calculate the range, typify the conception type of *Variability in particular values*. Those who answered Question (a) by only calculating the range of the given distributions—i.e., those answers falling into category A5—are a good example of teachers holding this conception. Also, teachers who focused on the minimum and maximum values in frequency, regarding variability as the largest span in frequency—answers that are accounted for into category A4—, held this type of conception. In the present study, 5 teachers (9.4% overall) seemed to be harboring the conception that only the range accounts for the variability—i.e., those falling into category A5—, while 5 out of 9 (9.4% overall) of the teachers falling into category A4 seemed to be harboring the conception that variability in a histogram is measured by the largest span in frequency. Therefore, 10 teachers (18.8% overall) in the present study seem to hold the conception that Shaughnessy (2007) labeled as *Variability in particular values*.

Teachers who seem to think of variability as either a visual measurement of the distance of each or some elements of a dataset from some measure of center—e.g., those who answered that Distribution B has more variability because is the one with the bars clumped closer to the mean—, or an actual measurement from the endpoints to some measure of center—e.g., those who answered that Distribution B has more variability because it deviates ±5 units from the mean, while Distribution A just deviates ±3 units—, show evidence of holding the conception known as *Variability as distance or difference from some fixed point*. In the present study, answers evidencing this conception of variability are accounted for into category A6. Thus, the 9 teachers (17.0% overall) who chose Distribution B and supported their answer with a naked eye description, comparison and interpretation of the data clustering around the mean, seem to be holding this kind of conception.

Teachers who provided a measure of the variability of the given distributions through what Petrosino, Lehrer and Schauble (2003) called "difference scores" (p.155)—i.e., deviation-based metrics such as the mean absolute deviation, sum of residuals or averages of the absolute value differences from a measure of center (Petrosino et al., 2003; Lehrer & Kim, 2009)—, seem to be thinking of variability as a collective amount that a distribution is "off" from some measure of central tendency. These "difference scores" maintain emphasis on the case-to-aggregate relation, as opposed to other indicators of spread, which tend to obscure this relationship (Petrosino et al., 2003, p.155). People with this kind of thinking show evidence of harboring the conception labeled by Shaughnessy (2007) as *Variability as the sum of residuals*. Teachers whose answers show evidence of this conception are accounted for into category A6. Then, the 5 respondents (9.4% overall) who calculated and compared the mean absolute deviations, as well as the only teacher (1.9% overall) who calculated and compared the residuals from the mean, seem to be harboring this conception of variability. Therefore, 6 teachers (11.3% overall) in the present study seem to hold the conception that Shaughnessy (2007) labeled as *Variability as the sum of residuals*.

Teachers whose answers involve transnumeration (i.e., changing representations of data to increase understanding and identify different aspects) and use of theoretical properties of the histograms to calculate numerically the measures of central tendency and variation associated to each distribution in order to make their decision—answers accounted for into category A6—, show evidence of holding the conception known as *Variation as distribution*. Then, the teachers who created frequency distribution tables from the histograms in order to calculate measures of variation—i.e., those subjects who changed from a graphic to tabular representation, in order to then change such tabular representation into a computational/algorithmic representation—, as well as those who calculated and compared the standard deviations and variances, seem to be holding this conception of variability. Therefore, 10 teachers (18.8% overall) in the present study seem to hold the conception that Shaughnessy (2007) labeled as *Variation as distribution*.

Teachers whose responses revealed misconceptions such as thinking of variability in terms of symmetry or degree of fit—or lack thereof—to a normal distribution—answers accounted for into categories A2 and A4—, or thinking of symmetrical or quasi-normal distributions as having less—answers accounted for into category A3—or more—answers accounted for into category A4—variability than its asymmetrical counterparts, show evidence of holding the conception that we will label here as *Variability as visual cues in the graph*. Then, the 5 teachers (9.4% overall) whose answers fell into category A2, the 9 teachers (17.0% overall) whose answers fell into category A3, as well as the 3 out of 9 of the teachers (5.7% overall) whose answers fell into category A4, seem to be holding this conception of variability. Therefore, 17 teachers (32.1% overall) in the present study seem to hold the so-called conception *Variability as visual cues in the graph*.

Unlike teachers either giving an idiosyncratic answer or holding the conceptions known as *Variability in particular values* and *Variability as visual cues in the graph*, the rest of the surveyed teachers seem to exhibit an aggregate view of data and distribution, since they seem to be predominantly concerned with the variability of an entire data distribution from a center while considering several theoretical features of the given data (cf. Shaughnessy, 2007, p.985).

5.9.2 Discussion of results regarding Question (b)

Based on an adaptation of Heymann's (2003) criteria for fundamental ideas in mathematics and Heitele's (1975) criteria for fundamental ideas in stochastics, Burrill and Biehler (2011) suggest that fundamental concepts in statistics should meet the following four criteria: (1) share some commonality within the different perceptions or ways of thinking about teaching statistics; (2) be able to connect the discipline to other experiences in the world and to aspects of culture; (3) illustrate the structure of the discipline perhaps clarifying specific characteristics and features important in the discipline, and (4) allow for deepened understanding across time as students mature in their knowledge of statistics (pp.60-63). Using these four criteria, Burrill and Biehler (2011) identified the following seven concepts as fundamental ideas in statistics, critical for teachers to know and convey in their instruction (pp.62-63):

1. *Data*: including types of data, ways of collecting data, measurement, respecting that data are numbers with a context.

2. *Variation*: identifying and measuring variability to predict, explain, or control.

3. *Distribution*: including notions of tendencies and spread, which are foundational for reasoning about statistical variables from empirical distributions, random variables from theoretical distributions, and summaries in sampling distributions.

4. *Representation*: graphical or other representations that reveal stories in the data including the notion of transnumeration.

5. *Association and modelling relations between two variables*: nature of the relationships among statistical variables for categorical and numerical data, including regression for modelling statistical associations.

6. *Probability models for data-generating processes*: modelling hypothetical structural relationships generated from theory, simulations, or large data set approximations, quantifying the variability in data including long-term stability.

7. *Sampling and inference*: the relation between samples and the population and the essence of deciding what to believe from how data are collected to drawing conclusions with some degree of certainty.

The twelve clusters of themes that emerged from the processes of “bottom up” coding (Coffey & Atkinson, 1996) and the subsequent reduction and clustering of categories (Heath & Cowey, 2004) used by the researcher to analyze, refit and refine the collected data, are going to be allocated within one of the seven fundamental ideas in statistics identified by Burrill and Biehler (2011). Those teachers providing in their answers ideas that could be paired to any of the statistical ideas identified by Burrill and Biehler (2011) are going to be classified under Husserl’s (1962, 1997) notions of *inner* and *outer horizons*. According to Husserl’s (1962, 1997), when we perceive an object, it is possible to have other perceptions of that object according to its type. When we perceive something as a house, there is a horizon of perceptions involving other sides of it. The objectual horizon is a correlate of such horizonal acts. Besides the *inner horizon* of an object (the body of contents that specify, more or less determinately, the object's unperceived features, which includes possible further properties of the object, such as the size or color of the back side of an object of vision; e.g., he size or color of the other sides of the house), there is also an *outer horizon* of this object, consisting in the body of contents which includes possible further relations of the object to other objects, such as the relation of an object of vision to objects behind it, say, objects that are not currently visible (e.g., relations of this house to the trees and the other houses surrounding it) (Husserl, 1962; Husserl, 1987; Smith, 2007, p. 434).

The process of pairing of categories from the aforementioned "bottom up" coding process with those seven fundamental ideas in statistics identified by Burrill and Biehler (2011) produced the results that are summarized in Table 20.

As it can be seen in Table 20, clusters of category B4—i.e., those answers mentioning specific mathematical operations—does not have a match to a certain fundamental statistical idea, since the ideas in this theme clusters are, in some sense, non-statistical responses focused on basic arithmetic operations. Regarding the ideas in theme cluster B9—which are focused on visual cues in the graph instead of actual data values, spreads of data values, or measures of distance or difference—despite they being not useful to deal with or solve the given task, they were paired to the fundamental ideas "Distribution" and "Probability models for data-generating processes", since "symmetry"

Table 20: Results obtained from pairing the categories that emerged from the "bottom up" coding process with the seven fundamental ideas in statistics identified by Burrill and Biehler (2011)

		Fundamental ideas in statistics (Burrill & Biehler, 2011)						
		Data	**Variation**	**Distribution**	**Representation**	**Association**	**Probability Models**	**Sampling & Inference**
Clusters of categories from the "bottom-up" coding	**B1**							
	B2							
	B3							
	B4							
	B5							
	B6							
	B7							
	B8							
	B9							
	B10							
	B11							
	B12							

and “asymmetry” are features that should be grasped in order to understand distributions like the normal (Batanero et al., 2004; Garfield & Ben-Zvi, 2008, p.149), make sense of particular statistics—like relative position of the mean, median and mode—for symmetric and asymmetric distributions (Konold & Pollatsek, 2004, p.188; Batanero et al., 2004, p.260; Garfield & Ben-Zvi, 2008, p.195), and detect outliers in data distributions (Konold & Pollatsek, 2004, p.188). Moreover, probabilistic models—such as uniform and normal—are described in terms of symmetry and symmetrical shapes—e.g., bell-shaped or rectangular—(Garfield & Ben-Zvi, 2008, p.186).

It is worthwhile to point out that, even among those answers with a match to a certain fundamental statistical idea of those identified by Burril and Biehler (2011), there are keywords provided by 10 teachers (18.9% overall) that are not useful—from the standpoint of the researcher—to deal with or solve the given task; that is, keywords that are unproductive in regard to the interpretation of variability. These keywords are “normal distribution”—answer provided by 3 teachers whose responses fell into theme cluster B5—; “symmetry” and “asymmetry”—answers provided by 3 teachers whose responses fell into theme cluster B9—; “bar graph”—answer provided by 2 teachers whose responses fell into theme cluster B11—; and “population” and “sample”—answers provided by 3 teachers whose responses fell into theme cluster B8. From these 10 teachers, only one (1.9% overall) just provided an unproductive keyword as answer, while the other nine also provided keywords useful to the solving of the posed task. Moreover, those respondents who provided the terms “normal distribution” and “bar graph” as answers, as well as all the teachers whose responses fell into theme cluster B9—8 teachers in total (15.1% overall)—, might be harboring statistical misconceptions regarding the given task. In the case of the 6 teachers (11.3% overall) mentioning the keywords “normal distribution”, “symmetry” and “asymmetry”, they seem to be thinking of variability in terms of symmetry or degree of fit—or lack thereof—to a normal distribution, misconceptions that have been reported in the statistics education literature by several investigators (Garfield et al., 1999; Meletiou, 2000; Meletiou & Lee, 2002, 2003, 2005; Cooper & Shore, 2007; González, 2011; González & Isoda, 2011; Isoda & González, 2012). People holding these misconceptions tend to disregard the connection between measures of central tendency and the variability of data dispersed around a center, and focus their attention exclusively on visual features of the distributions. Thus, these teachers seem to hold the so-called conception *Variability as visual cues in the graph*. In fact, analyzing the answers provided by these teachers in Question (a), 4 out of 6 (7.5% overall) showed evidence of holding the conception *Variability as visual cues in the graph*.

In the case of the 2 teachers (3.8% overall) mentioning the keyword “bar graph”, they seem to be confusing bar graphs and histograms, which means that they do not appreciate the data reduction involved in the latter, in moving from raw data to frequencies of occurrence of different values or groups of values. This confusion might be the reason because of which these teachers did not provided any answer to Question (a). Confusing bar graphs and histograms is a misinterpretation that has been reported in the literature in the case of school students (e.g., Cohen, Smith, Chechile, Burns & Tsai, 1996; Bright &

Friel, 1998; Bakker et al., 2002; Bakker et al., 2005; delMas, Garfield & Ooms, 2005; Cooper & Shore, 2008), university students (e.g., Meletiou, 2000; Meletiou & Lee, 2002, 2003; Meletiou & Stylianou, 2003), prospective teachers (Espinel, Bruno & Plasencia, 2008), in-service school mathematics teachers (Tiefenbruck, 2007; González, 2011; Isoda & González, 2012), and even professionals in disciplines outside of education, such as archaeologists (Banning, 2000), medical researchers (Kelly, Sloane & Whittaker, 1997), and authors in wildlife journals (Tacha, Warde & Burnham, 1982).

Back to Husserl's interpretation of horizon, an object's inner horizon is composed of specific features of the object itself and includes the attributes of the object that lie in the periphery of our focus; that is, the particular features of the object are those that encompass its inner horizon (Zazkis & Mamolo, 2011). Therefore, since the focus of the posed task is to compare the variability of a pair of histograms in order to determine which of them has a larger one, then attributes such as frequency, absolute frequency, class intervals, class size—reported in the theme cluster B1—; spread, variability—reported in the theme cluster B7—; sample size—reported in the theme cluster B8—; graphs and histograms—reported in the theme cluster B11—, become "out of focus" and thus exist as aspects of the inner horizon. In contrast, an object's outer horizon represents the "greater world" in which such an object exists. The outer horizon is independent of focus and consists of the generalities which are exemplified in the particular object; that is, the features that are *connected* to the object and that embed it in a greater structure are those that encompass its outer horizon (Zazkis & Mamolo, 2011). Thus, in the case of our posed task, statistical objects such as frequency distribution tables, measures of central tendency and variation, as well as data handling methods, exist as aspects of the outer horizon.

5.9.2.1 Regarding the fulfillment of Indicator C-2

In order to fully meet this indicator, teachers must show evidence of being able to build bridges between the cognitive demands of the posed task and fundamental ideas, practices, values and sensibilities of the discipline of statistics. These assessment criteria represent two of the main characteristics of the so-called *horizon content knowledge* (cf. Ball & Bass, 2009). It is worthy to remember that the ideas included in category B4 do not have a match to a certain fundamental statistical idea. Also, there were teachers that, despite providing answers which were paired to a certain fundamental statistical idea, raised ideas regarded as not useful to deal with or solve the given task, or ideas that could be considered as evidence of possible misconceptions held by the respondents. With this in mind, teachers whose answers fell into theme clusters B4—which does not have a match to a certain fundamental statistical idea—, and B9—which is comprised of statistical ideas that are not useful to deal with or solve the given task—, were excluded of consideration of meeting the assessment criteria of Indicator C-2. Similarly, those 10 teachers who provided as answers keywords considered by the researcher not useful to deal with or solve the given task—i.e., those teachers who raised keywords that are unproductive in regard to the interpretation of variability—, were also disregarded as to be satisfactorily meeting Indicator C-2. Therefore, from the 49 respondents to Question (b)—17 working at lower high school, 14 at upper high school, and 18 at both levels—, the five teachers categorized in theme clusters B4 and B9 should be disregarded, as well as all the teachers who provided keywords considered not useful to deal with or solve the given task. Since some teachers 4 teachers belong to more than one of these clusters, at the end 12 teachers must be disregarded from the initial 49 respondents. Therefore, 37 teachers who answered Question (b)—69.8% overall, 13 working at lower high school, 12 at upper high school, an 12 at both levels—appear to have fully met the assessment criteria of Indicator C-2.

5.9.3 Discussion of results regarding Question (c)

Among the 28 teachers who completely answered Question (c), only 8 teachers (15.1% overall; 4 working at lower high school, 2 at upper high school, an 2 at both levels) consistently exhibited ability to correctly judge the accuracy of the answers given by the fictitious Students 1, 2 and 3. All these teachers not only correctly judged the accuracy of the answers given by the fictitious students in Question (c), but also made appropriate observations for each case about why they thought so, and provided accurate comments about the most likely reasoning behind each student' answer. It is worthy to highlight that, from these 8 teachers, just 1 did not provided a response to Question (a) exhibiting simple or sophisticated recognition of variability—i.e., only 1 teacher did not provided an answer falling into categories A5 and A6. This teacher—who provided an answer falling into category A4—was the one who, despite correctly selecting Distribution B as the one with more variability, supported such choice by arguing that “Distribution B has more variability because is symmetric”. Thus, this teacher seems to think of variability in terms of symmetry: the greater the degree of symmetry of a histogram, the greater is its variability, which is contrary to the common understanding that the histogram with lesser pattern in the heights of the bars or the more asymmetric one has a greater variability of its data set. This way of thinking of variability expresses a misconception that might be found commonly in students and—in a lesser extent—teachers, which has been well-reported in the literature (Meletiou & Lee, 2003, 2005; Cooper & Shore, 2007; González, 2011; Isoda & González, 2012, González, 2013a, 2013b). Among the remaining 7 teachers, 5 provided answers falling into category A6, and 2 provided answers falling into category A5.

Among the 17 teachers who partially answered Question (c), all the 17 provided assessment on the correctness or incorrectness of Student 1’s response, 12 provided some explanation about why that answers was correct or incorrect, and 11 gave a comment about the most likely reasoning behind such student’ answer, with 8 of them giving their judgment about whether the answer was correct or not, why they think so, and what reasoning might have led student to produce the given answer. Regarding Student 2, 16 teachers provided assessment on the correctness or incorrectness of Student 2’s response, 11 provided some explanation about why that answers was correct or incorrect, and 7 gave a comment about the most likely reasoning behind such student’ answer, with 4 of them giving their judgment about whether the answer was correct or not, why they think so, and what reasoning might have led student to produce the given answer. Finally, regarding

Student 3, 15 teachers provided assessment on the correctness or incorrectness of Student 1's response, 12 provided some explanation about why that answers was correct or incorrect, and 6 gave a comment about the most likely reasoning behind such student' answer, with just 3 of them giving their judgment about whether the answer was correct or not, why they think so, and what reasoning might have led student to produce the given answer. Table 21 presents a breakdown of the teachers who partially answered Question (c) by school level.

Among the 17 teachers who partially answered Question (c) but provided assessment on the correctness or incorrectness of Student 1's response, 12 (22.6% overall; 4 working at lower high school, 5 at upper high school, an 3 at both levels) correctly judged the accuracy of the answers given by the fictitious Student 1; that is, 12 teachers assessed Student 1's response as incorrect. Among this group of teachers, 7 (13.2% overall; 2 working at lower high school, 3 at upper high school, and 2 at both levels) also provided an explanation about why they considered such answer as incorrect, with all these 7 teachers giving appropriate explanations. Moreover, among these 7 teachers, 6 gave comments about the most likely reasoning behind Student 1's answer, with all of them providing appropriate explanations. The 5 teachers (9.4% overall; 1 working at lower high school, 1 at upper high school, and 3 at both levels) who assessed Student 1's response as correct seem to think of variability in terms of symmetry, regarding the histogram with the greater the degree of symmetry as the one with lees variability, which is in line with the common understanding that the histogram with lesser pattern in the

Table 21: Breakdown of the teachers who partially answered Question (c) by school level – Frequency of occurrence

		Frequency (%)								
		Student 1			Student 2			Student 3		
		L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)	L.H. School (19 teachers)	U.H. School (15 teachers)	Both Levels (19 teachers)
Teachers' Answers	Provided assessment on the accuracy of student's response	5	6	6	4	6	6	4	6	5
	Provided explanation about why student's response was correct or incorrect	3	4	5	2	5	4	2	5	5
	Provided a comment about the most likely reasoning behind student's response	2	5	4	1	3	3	1	3	2
	Provided both an assessment on the accuracy of student's response and an explanation about why so	3	4	5	2	5	4	2	5	4
	Provided both an assessment on the accuracy of student's response and a comment about the most likely reasoning behind such response	2	5	4	0	3	3	0	3	1
	Provided an assessment on the accuracy of student's response, an explanation about why so, as well as a comment about the most likely reasoning behind such response	2	3	3	0	2	2	0	2	1

heights of the bars or the more asymmetric one has a greater variability of its data set, a misconception that might be found commonly in students and—in a lesser extent—teachers, which has been well-reported in the literature (Meletiou & Lee, 2003, 2005; Cooper & Shore, 2007; González, 2011; Isoda & González, 2012, González, 2013a, 2013b). In fact, by cross-referencing the answers given by these 5 teachers regarding Student 1 with their answers to Question (a), we can see that 1 teacher did not answer it, one gave an answer falling into category A1, one gave an answer falling into category A2, and 2 gave answers falling into category A3, which indicates that most of these teachers seem to be harboring the conception "Variability as visual cues in the graph", since they disregard the actual data values, their spreads, and the measures of distance or difference, and mistakenly acknowledge variability as unevenness in the frequencies of a histogram or lack of symmetry.

Among the 16 teachers who partially answered Question (c) but provided assessment on the correctness or incorrectness of Student 2's response, 10 (18.9% overall; 3 working at lower high school, 4 at upper high school, an 3 at both levels) correctly judged the accuracy of the answers given by the fictitious Student 2; that is, 10 teachers assessed Student 2's response as incorrect. Among this group of teachers, 8 (15.1% overall; 2 working at lower high school, 3 at upper high school, and 3 at both levels) also provided an explanation about why they considered such answer as incorrect, with all these 7 teachers giving appropriate explanations. The one who did not provide an appropriate explanation—a teacher working at both levels—wrote that "Variability is not so much about the distribution limits, as about the shape how the data are distributed". This explanation is in line with the choices made regarding Student 1, in which the same teacher assessed that answer as correct, as well as in Question (a), in which this teacher chose Distribution A as the one with more variability, which appears to be evidence of the conception "Variability as visual cues in the graph".

Among these 7 teachers who gave appropriate explanations about the accuracy of Student 2's response, 2 gave comments about the most likely reasoning behind Student 2's answer, with all of them providing appropriate explanations. Moreover, the 6 teachers (11.3% overall; 1 working at lower high school, 2 at upper high school, and 3 at both levels) who assessed Student 1's response as correct seem to think of variability in terms of the span in the vertical axis—i.e., judging the variability of the data displayed in a histogram by the largest difference in height of its bars—, instead of looking at the horizontal spread of data around a measure of central tendency. This is a common mistake made by many students, and even by some of their teachers, at any school level (Isoda & Gonzalez, 2012). In fact, by cross-referencing the answers given by these 6 teachers regarding Student 2 with their answers to Question (a), we can see that 3 gave answers falling into category A4, 2 gave answers falling into category A6, and 1 gave an answer falling into category A2, which indicates that most of these teachers considered Distribution B as the one with more variability, but 3 of them evidencing in Question (a) misinterpretation of how to appropriately measure variability.

Among the 15 teachers who partially answered Question (c) but provided assessment on the correctness or incorrectness of Student 3's response, 8 (15.1% overall; 2 working at lower high school, 4 at upper high school, an 2 at both levels) correctly judged the accuracy of the answers given by the fictitious Student 3; that is, 8 teachers assessed Student 3's response as correct. Among this group of teachers, 5 (9.4% overall; 1 working at lower high school, 3 at upper high school, and 1 at both levels) also provided an explanation about why they considered such answer as correct, with all of them giving appropriate explanations. Among these 5 teachers who gave appropriate explanations about the accuracy of Student 3's response, just 2 gave comments about the most likely reasoning behind Student 3's answer, with all of them providing appropriate explanations.

All the 7 teachers (13.2% overall; 2 working at lower high school, 2 at upper high school, and 3 at both levels) who assessed Student 3's response as incorrect, provided explanations about why they thought so. From such explanations, it seems that these teachers think of variability in terms of neither the degree of clustering nor dispersion in data. In fact, these teachers provided explanations such as "the definition of variability doesn't support this statement", "this student doesn't know that symmetry qualifies as variability", "this fact doesn't constitute sufficient reason to make any conclusion", and "it's like the same argument given by Student 1". This lack of understanding was evidenced by the teachers' inability to adequately choose the distribution with more variability in Question (a). Indeed, all these 7 teachers were unable to answer correctly Question (a): 2 of these teachers gave answers falling into category A4, 2 gave answers falling into category A3, 1 gave an answer falling into category A2, 1 gave an answer falling into category A1, and 1 did not answer. This indicates that these 7 teachers have problems understanding variability as the degree of clustering or deviation from an expected value or a measure of central tendency.

5.9.3.1 Regarding the fulfillment of Indicator B-1

In order to fully meet this indicator, teachers must show evidence of being able to examining the work done by three fictitious students in their absence, and correctly determine whether their answers to the given task are right or not, based just on the provided numerical data and written explanations. Since only 8 teachers (15.1% overall; 4 working at lower high school, 2 at upper high school, an 2 at both levels) consistently exhibited ability to correctly judge the accuracy of the answers given by the fictitious Students 1, 2 and 3, as well as made appropriate observations for each case about why they thought so, Indicator B-1 seems to be fully satisfied only by these eight teachers. Those teachers who mixed correct and incorrect assessments to the answers given by Students 1, 2 and 3 were not regarded as meeting the assessment criteria of this indicator.

5.9.3.2 Regarding the fulfillment of Indicator B-2

In order to fully meet this indicator, teachers must show evidence of being able to explain how students arrived at their solutions by providing the likely reasoning that led them to their answers. This explanation is done in students' absence, and is based just on the provided numerical data and written explanations. All the 8 teachers who fully answered Question (c) and appeared to fully satisfy Indicator B-1, also provided appropriate comments about the most likely reasoning behind each student' answer. Thus, Indicator B-2 seems to be fully met—again—only by these eigth teachers (15.1% overall; 4 working at lower high school, 2 at upper high school, an 2 at both levels).

5.9.4 Discussion of results regarding Question (d)

In order to assess teachers' answers to Question (d), the likely common responses suggested by them were examined simultaneously with the accuracy or inaccuracy attributed to such responses as well as the reasons pointed out by them. So, in order to consider a teacher's answer as correct, these aspects were examined together for consistency. The potential difficulties on the given task suggested by the teachers were examined for appropriateness.

Among these 24 teachers who fully answered Question (d), 13—24.5% overall; 4 working at lower high school, 5 at upper high school, and 4 at both levels—consistently provided responses indicating both students' likely common answers to the given task, right assessment of the accuracy or inaccuracy of such answers, and appropriate reasons for such assessment. It is worthy to highlight here the fact that, if the likely common responses suggested by the respondents were examined in conjunction with just their assessment of accuracy or inaccuracy, from the 24 teachers who fully answered Question (d), 17 of them—32.1% overall; 5 working at lower high school, 7 at upper high school, and 5 at both levels—would have given appropriate answers to this question. Among the 15 teachers who partially answered Question (d) and provided responses indicating students' likely common answers to the posed task, 5—9.4% overall; 1 working at lower high school, and 4 at upper high school levels—consistently provided responses indicating both students' likely common answers to the given task, right assessment of the accuracy or inaccuracy of such answers, and appropriate reasons for such assessment. This fact might be interpreted as the surveyed teachers having difficulty in the assessment of the accuracy or inaccuracy of the most likely responses from students identified by them. In fact, looking at the teachers who did not provide consistent answers regarding the likely responses that could be expected from students, all of them assessed their provided responses as being correct or incorrect when, in reality, were the opposite. As an example, all the teachers indicating that Distribution A was the one with more variability, as well as those who chose Distribution B evidencing misconceptions or misinterpretations about variability, such as "Distribution B because has more bumps/different bar heights", "Distribution B because is symmetric/normal-like", "Distribution B because has more data values/bars", "Distribution B because has a larger span in frequency", or "Distribution B because has a larger area". Moreover, some of these answers reveal conceptions of variability that were not shown in Question (a), such as thinking of variability as a sample size.

By cross-referencing the ill-assessed answers given by these 13 teachers—i.e., by the 11 teachers who fully answered Question (d) but did not provided consistent answers, as well as 2 of the teachers who partially answered Question (d) but provided a wrong assessment about the accuracy of their responses—, with their answers to Question (a), we can see that 10 teachers (18.9% overall) gave answers evidencing misconceptions or misinterpretation of variability—i.e., 10 teachers gave answers to Question (a) falling into categories A2 (3 teachers), A3 (5 teachers), and A4 (2 teachers)—, with 3 teachers evidencing simple or sophisticated recognition of variability—2 teachers gave answers to Question (a) falling into category A5, and 1 into category A6.

Regarding the possible difficulties which students may face in solving the posed task, all those provided by the 24 teachers who fully answered Question (d), as well as by the 14 teachers who partially answered Question (d) and provided responses indicating students' likely common answers to the posed task, were assessed as appropriate answers. The fact that all the teachers who provided likely difficulties that students may encounter while dealing with the posed task were able to do it correctly, it might be interpreted as teachers having easier to appropriately identify the difficulties associated to a particular problem.

5.9.4.1 Regarding the fulfillment of Indicator D-1

In order to fully meet this indicator, teachers must show evidence of ability to appropriately anticipate students' common responses and difficulties on the task posed in the survey instrument. In the present study, a teacher is going to be regarded as exhibiting such ability if he or she (1) provides in his or her responses what could be considered—from the standpoint of the researcher—students' likely common answers and potential difficulties in answering the posed task and, at the same time, (2) correctly determine the accuracy or inaccuracy of such answers. Therefore, the 17 teachers who fully answered Question (d) and provided both likely common responses and potential difficulties to the given task, as well as a correct assessment of accuracy or inaccuracy of such answers—32.1% overall; 5 working at lower high school, 7 at upper high school, and 5 at both levels—appear to have fully met Indicator D-1. It is worthy to mention here that the ability to provide the most likely reasons for such students' common responses and difficulties is accounted for by Indicator D-2.

Notice the fact that, from the 24 teachers who fully answered Question (d) at the beginning, 7—13.2% overall; 3 working at lower high school, and 4 at both levels—failed to provide a right assessment of the accuracy or inaccuracy of students' likely common answers to the posed task given by them. This could be an indicator of the difficulty of mastering this skill for the surveyed teachers. Also, could be interpreted as a signal that the teacher is harboring some statistical misconception, since 6 out of 7 of these teachers gave answers to Question (a) evidencing misconceptions or misinterpretation of variability—i.e., 6 of these teachers gave answers to Question (a) falling into categories A2 (2 teachers) and A3 (4 teachers)—, with 1 teacher providing an answer falling into category A6.

5.9.4.2 Regarding the fulfillment of Indicator D-2

In order to fully meet this indicator, teachers must show evidence of ability to provide the most likely reasons for students' most likely responses and difficulties on the task posed in the survey instrument, a cognitive skill related to the construct *knowledge of content and students* (KCS).

In order to regard a teacher as fully meeting the assessment criteria of this indicator, the teacher must have provided (1) appropriate likely common responses and difficulties expected from the students, (2) a correct assessment of the accuracy or inaccuracy attributed to such responses, and (3) appropriate reasons for such assessment. Therefore, in the present study, the 13 teachers who fully answered Question (d) and consistently provided responses indicating both students' likely common answers to the given task, right assessment of the accuracy or inaccuracy of such answers, and appropriate reasons for such assessment—24.5% overall; 4 working at lower high school, 5 at upper high school, and 4 at both levels—are those who appear to have fully satisfied Indicator D-2.

Again, in a similar fashion that occurred in the case of Indicator D-1, it is noticeable the fact that, from the 24 teachers who fully answered Question (d) at the beginning, 11—20.8% overall; 4 working at lower high school, 2 working at upper high school, and 5 at both levels—failed to provide appropriate reasons for the assessment of the accuracy or inaccuracy of students' likely common answers to the posed task given by them. This could be an indicator of the difficulty of mastering this skill for the surveyed teachers. Also, could be interpreted as a signal that the teacher is harboring some statistical misconception, since 8 out of 11 of these teachers gave answers to Question (a) evidencing misconceptions or misinterpretation of variability—i.e., 8 of these 11 teachers gave answers to Question (a) falling into categories A2 (3 teachers), A3 (4 teachers), and A4 (1 teacher)—, with 2 teachers giving answers to Question (a) falling into category A5, and 1 into category A6.

5.9.5 Discussion of results regarding Question (e)

In order to analyze the collected data, a "bottom up" approach to coding (Coffey & Atkinson, 1996) was initially used by the researcher. From this analytic approach ensures that the themes or categories extracted were, in fact, grounded in the data and hence reflected the participants' own knowledge base of HCK. The author reviewed all the given answers to Question (e) and identified answers that occurred frequently in the data. Such answers appearing to contain similar content were initially given the same code by the researcher, and each code was further analyzed to find true meanings within their text. A process of reduction and clustering of categories, which were refitted and refined, followed (Heath & Cowey, 2004). Summary groupings (or 'clusters') of themes that share common meaning emerged from this process were finally condensed into twelve theme clusters. A full description of the clusters that were identified from the collected data follows:

E0: *No response*.

E1: *Responses like "Neither of the students' answers is important nor significant"*.

E2: *Answers selecting one or more students, providing a vague or no reason*: teachers in this cluster only named one or more students, or all of them—e.g., those who gave answers like "Any student answer is important"—, but providing no reason, **or** an extremely vague, irrelevant and meaningless one, for backing up such a choice.

E3: *Answers reflecting a wrong interpretation or misconception of variability from the teacher*: teachers in this cluster provided answers validating some of the misconceptions showed by Students 1 and 2.

E4: *Answers evidencing just the identification of mathematical distortions or possible precursors to later mathematical confusion or misrepresentation*: According to Ball and Bass (2009, p.6), one of the characteristics of HCK is the awareness of seeds of misconceptions in students' comments and responses. Teachers in this cluster, evidencing misconceptions themselves, will be considered as having a weak HCK base.

E5: *Answers evidencing teacher's recognition and evaluation of mathematical opportunities*: According to Ball and Bass (2009, p.6), teachers' ability to afford teaching opportunities beyond the problem at hand, in order to develop a foundation for big ideas in the discipline, is one of the characteristics of HCK. Therefore, a teacher would need to decide whether the implications from a student argument are worth pursuing, in particular when the argument used by the student is unrelated to the learning goals of the lesson and is outside the mathematics curriculum. Nevertheless, this characteristic is not a must, since HCK does not create an imperative to act in any particular mathematical direction (Ball & Bass, 2009, p.10).

E6: *Answers pervading a sense of the mathematical environment surrounding the current "location" in instruction*: According to Ball and Bass (2009), one of the constituting elements of HCK is teachers' sense of the mathematical environment surrounding the current "location" in instruction (e.g., frequency distribution tables and measures of central tendency and variation when teaching histograms). This element of HCK might be matched with what Husserl identifies as "inner horizon", which focuses solely on the immediate present, a "horizon" that includes a core of absolute presence as well as the just-was and just-coming surrounding the immediate now (Rodemeyer, 2010).

E7: *Answers mentioning major disciplinary ideas and structures*: Another constituting element of HCK is teachers' understanding of relationships between specific advanced mathematics and specific ideas arising in the content being taught and learned in school

(Ball & Bass, 2009; Jakobsen et al., 2012). So, teachers in this cluster are those whose answers show evidence of seeing connections between students' arguments and topics that arise in later mathematics.

E8: *Answers mentioning key mathematical/statistical practices*: According to Ball and Bass (2009), awareness of key mathematical practices—such as choosing representations, questioning, argumentation, using definitions, and proving—is one of the constituting elements of HCK. Then, teachers in this cluster are those whose answers show evidence of such awareness.

E9: *Answers mentioning core mathematical values and sensibilities*: According to Ball and Bass (2009), awareness of core values and sensibilities in the mathematical field—such as precision, care with mathematical language consistency, parsimony, and coherence—is one of the constituting elements of HCK. Then, teachers in this cluster are those whose answers show evidence of such awareness.

E10: *Answers evidencing teacher making connections with other disciplines*: According to Montes et al. (2012) and Martínez et al. (2011), knowledge of connections with other disciplines—i.e., awareness about how a topic can be related to other content, outside the immediate curriculum, with different aims and not directly related to the content being taught—could be considered as a part of HCK. Thus, teachers in this cluster are those whose answers show evidence of them making connections between different areas of content, and even concepts or procedures.

After building up the aforementioned clusters, the information gathered from teachers' answers were summarized in Table 22.

Table 22: Results obtained from participants' answers to Question (e) – Frequency and percentage

		Frequency (%)			
		Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
Category	E0: No response.	2 (10.5)	2 (13.2)	5 (26.3)	9 (17.0)
	E1: Responses like "Neither of the students' answers is important nor significant".	3 (15.8)	4 (26.7)	4 (21.0)	11 (20.8)
	E2: Answers selecting one or more students, providing a vague or no reason.	0 (0.0)	3 (20.0)	2 (10.5)	5 (9.4)
	E3: Answers reflecting a wrong interpretation or misconception of variability from the teacher.	1 (5.3)	3 (20.0)	1 (5.3)	5 (9.4)
	E4: Answers evidencing just the identification of mathematical distortions or possible precursors to later mathematical confusion or misrepresentation.	4 (21.0)	1 (6.7)	2 (10.5)	7 (13.2)
	E5: Answers evidencing teacher's recognition and evaluation of mathematical opportunities.	3 (15.8)	0 (0.0)	2 (10.5)	5 (9.4)
	E6: Answers pervading a sense of the mathematical environment surrounding the current "location" in instruction.	3 (15.8)	4 (26.7)	2 (10.5)	9 (17.0)
	E7: Answers mentioning major disciplinary ideas and structures.	3 (15.8)	0 (0.0)	0 (0.0)	3 (5.7)
	E8: Answers mentioning key mathematical/statistical practices.	3 (15.8)	1 (6.7)	2 (10.5)	6 (11.3)
	E9: Answers mentioning core mathematical values and sensibilities.	0 (0.0)	0 (0.0)	1 (5.3)	1 (1.9)
	E10: Answers evidencing teacher making connections with other disciplines.	0 (0.0)	0 (0.0)	2 (10.5)	2 (3.8)

Participants were allowed to write down as many statistical ideas as they could identify to be associated with answering the posed task. Thus, a single response to Question (b) could be coded into one or more categories, and hence the percentages within each group of teachers sum to more than 100%.

In Table 22, clusters E4 to E10 describe some characteristics of the knowledge base of HCK (Ball & Bass, 2009). Therefore, from such table, it is possible to determine what features of HCK are the most exhibited by the surveyed teachers. 24 out of the 44 respondents to Question (e)—45.3% overall—provided answers falling exclusively into clusters E4 to E10. The answer clusters that were distinguished the most among teachers' answers were the E6—by 9 teachers, 17.0% overall—, the E4—by 7 teachers, 13.2% overall—, the E8—by 6 teachers, 11.3% overall—, and the E5—by 5 teachers, 9.4% overall.

Teachers whose responses to Question (e) fell into answer cluster E4 showed evidence of awareness of particular seeds of misconceptions in the comments made by the fictitious students in Question (c). The majority of such answers point out the misinterpretation made by Student 1, who appears to think of symmetry—or lack thereof—as variability.

Teachers whose responses fell into answer cluster E5 evidenced ability to identify teaching opportunities from a student argument, in order to further develop fundamental statistical ideas. These teachers seized students' comments as an opportunity to "clarify and correct students' misconceptions" and "clarify concepts involved in the task that can be easily confused by students".

Teachers whose responses fell into answer cluster E6 evidenced awareness of the mathematical environment surrounding the current "location" in instruction of the statistical ideas involved in the posed task, such as frequency distribution tables, histograms and measures of central tendency and variation. This awareness—which can be paired with Husserl's "inner horizon", because represents the core of absolute presence as well as the just-was and just-coming surrounding the immediate now—, was expressed by the respondents in answers such as "Student 1 should make associations with basic statistical concepts, like measures of central tendency and measures of variation", "Student 3 perceives concepts like mode and measures of central tendency", "Student 3 seem to know the meaning of mean and spread", and "only Student 3's answer is interesting, because is the only one who associates middles with dispersion".

Teachers whose responses fell into answer cluster E8 evidenced awareness of key mathematical and statistical practices, such as associating data clustering to variability, associating variability to data spread, identifying properties of the normal distribution and bell-shaped distributions, translate graphical information into tabular or numerical formats, and measure variability with regard to the mean or other measure of central tendency.

5.9.5.1 Regarding the fulfillment of Indicator C-1

In order to fully meet this indicator, teachers must show evidence of ability to identify whether a student response to the given task is interesting or significant, from a mathematical or statistical standpoint. This significance could be due to several reasons, which are compiled within in the answer clusters E4 to E10. Therefore, the 24 teachers who provided responses to Question (e) only falling into answer clusters E4 to E10 would be considered to meet this indicator satisfactorily; that is, 24 respondents—45.3% overall, 13 working at lower high school, 3 at upper high school, and 8 at both levels—provided evidence of being able to identify importance or significance in students' comments from the standpoint of ability to identify mathematical distortions or possible precursors to later mathematical confusion or misrepresentation, to recognize and evaluate mathematical opportunities, to demonstrate sense of the mathematical and statistical environment surrounding the current "location" in instruction, to recognize major disciplinary ideas and structures, to recognize key mathematical/statistical practices, to acknowledge core mathematical values and sensibilities, and to make connections with other disciplines. All these skills, as it was explained before, are associated to the cognitive construct labeled by Ball and Bass (2009) as *horizon content knowledge*.

5.9.6 Discussion of results regarding Question (f)

Teachers must know at which grade levels—and even in which content areas—particular topics are typically taught. This is one characteristic of the so-called *knowledge of content and curriculum* (KCC), being one of its aspects knowledge about "topics and issues that have been and will be taught in the same subject area in the preceding and later years" (Ball et al., 2008, p. 391). According to Ponte (2011, pp. 300–301), KCC is one of the main poles on which the professional knowledge required for teaching statistics stands, as well as a fundamental aspect related to lesson planning. Thus, Question (f) seeks to elicit evidence of the KCC held by the surveyed teachers, in relation to the statistical ideas present in the posed task in Item 1.

In the present study, 45 of the participants (84.9% overall) answered this question. The collected answers were categorized into the groups shown in Table 18. Teachers whose answers fell into category F1 are those who admitted either unfamiliarity with how statistical contents are addressed in official curriculum documents across the different grade levels of schooling, or not teaching any statistical content. Unfortunately, this group represents the majority of surveyed teachers, with 37.7% (20/53) of them in this category.

Category F2 collects those answers in which teachers gave general answers, without mentioning a specific grade or grade level. For example, teachers who made mention of "methods to identify trends in data" or "analysis of graphs", without pointing out which specific methods or graphs they were referring to, or at what grade level such contents are supposed to be taught. 13.2% (7/53) of the surveyed teachers' answers fell into this category. Similarly to teachers in category F2, those in category F3 gave general answers, but they made mention of a specific grade or grade level. 15.1% (8/53) of the surveyed teachers' answers fell into this category.

Category F4 consists of all those answers in which teachers made mention of specific statistical topics listed in the Venezuelan mathematics curriculum, without mentioning a specific grade or grade level. For example, teachers who specified "to determine the absolute and relative frequencies of a data set", "to create a frequency distribution table from unsorted data", or "to define and calculate the mean, mean deviation and variance", but did not mention at what grade level such contents are supposed to be taught. Just 7.6% (4/53) of the surveyed teachers' answers fell into this category. Teachers in category F5 also mentioned specific statistical topics, but contrarily to those in category F4, they pointed out at what grade or grade level such topics are supposed to be taught. Only 11.3% (6/53) of the participants in the present study fell into this category.

It was noticeable that only 3 (5.7%) of the surveyed teachers made explicit mention of embedding the study of particular statistical ideas in a daily-life context, which is vital to internalize in the students that statistics helps solve everyday problems and tasks (cf. Gattuso & Ottaviani, 2011, pp. 122–123, 129), and also recurrently stated in the Venezuelan mathematics curricula at all school levels (ME, 1987, 1997; CENAMEC, 1991). This fact might be a hint of the majority of the participants in this study being "traditionalists" (Eichler, 2011)—i.e., teachers who believe that statistics teaching should emphasize theory and students' acquisition of algorithmic skills, and are less concerned about applications.

5.9.6.1 Regarding the fulfillment of Indicator F-1

In order to fully meet this indicator, teachers must show evidence of knowledge about both the specific statistical concepts in the mathematics curriculum, and the grade levels and/or content areas at which students are typically taught such concepts. In the present study, teachers whose answers fell into the category F5 while answering Question (f) are going to be considered as the only ones satisfying these conditions. Therefore, based on the gathered data, it would be fair to say that only 6 teachers in this study—11.3% overall, 3 working at lower high school, and 3 at upper high school level—appear to have fully met Indicator F-1. However, those teachers in categories F3 and F4 showed evidence of partial fulfillment of this indicator, since they evidenced knowledge of either specific statistical topics present in the Venezuelan mathematics curriculum or the grade level in which such contents are supposed to be taught.

5.9.7 Discussion of results regarding Question (g)

5.9.7.1 Regarding Knowledge of Content and Curriculum (KCC)

According to Ponte (2011, pp. 300–301), knowledge of the curriculum—including, among others, knowledge about its purposes—is not only one of the main pillars on which the professional knowledge required for teaching statistics may be standing, but also a fundamental aspect related to lesson planning. Therefore, Question (g) was posed with the purpose of examining surveyed teachers' designed lessons, seeking for evidence of whether or not statistics-related curriculum goals are known, understood and supported by them. By means of collecting such evidence, it would be possible to determine whether or not the standards of KCC reflected by the Indicator F-2 appear to be satisfactorily met by the respondents to this question.

In order to determine whether respondents know, understand and support statistics-related curriculum goals and intentions, it is necessary to clarify what those goals and intentions are. In Venezuela, Mathematics is a compulsory subject at all grades, in which "Statistics and Probability" is one of the five strands comprising the mathematics curriculum at any school level (ME, 1972, 1987, 1997; CENAMEC, 1991). "To study basic notions of descriptive statistics", as well as "To solve problems in which basic notions of statistics and probability could be used" are overall objectives related to the statistical contents included in the current mathematics curriculum for the Basic cycle of secondary school in Venezuela (ME, 1987, 1997). In the case of the Diversified cycle of secondary school, "...to allow students to model situations of uncertainty" is the overall objective related to the teaching of statistics at that level. Moreover, from the analysis of the verbs used in the Venezuelan secondary school mathematics curricula carried out in Chapter 2, it is clear that teachers must emphasize, while teaching statistics, the application of statistical ideas, the acquisition of knowledge and skills and, to a lesser extent, the comprehension of such ideas.

In the methodological guidelines provided by the Venezuelan secondary school mathematics curriculum, teachers are encouraged to promote a student-centred lecture environment; emphasize the significance of trying to improve students' comprehension of statistical contents by relating them to students' own environmental and social context; and use concrete materials and draw upon students' experiences and interests as a starting point for teaching statistical ideas, in order to promote an active participation in classroom activities (ME, 1972, 1987, 1997; CENAMEC, 1991; Salcedo, 2006). These guidelines are very clear in the teaching guide for the Basic cycle of the secondary school, developed after the 1985 curricular reform by the Venezuelan Ministry of Education. This document explicitly mentions that teachers are expected to stimulate and strengthen children's statistical learning through practical experiences linked to knowledge about daily life, and in particular, to environmental situations interesting to the students, through which they could appreciate and value nature and natural resources. Therefore, the lesson designed by the surveyed teachers when answering Question (g) will be examined for evidence of knowledge and support of the aforementioned objectives and methodological guidelines provided by curriculum documents in regard to the teaching of statistics at secondary level, which will provide evidence of whether teachers' designed lessons successfully meet Indicator F-2.

Forty-four teachers—83.0% overall, 17 working at lower high school, 13 at upper high school, and 14 at both levels—answered to Question (g). As stated before, these answers were examined for curricular features of the planned lessons, specifically for evidence of knowledge, understanding and support of the objectives and methodological guidelines provided by official curriculum documents in Venezuela in relation to the teaching of statistics at secondary level. Knowledge and support of the overall objectives related to the teaching of statistics at the Basic cycle of secondary education—i.e., "To study basic notions of descriptive statistics" and "To solve problems in which basic notions of statistics and probability could be used"—seem to be evident in all the 31 answers of teachers working at lower secondary level. This could be due to the general way in which such objectives are formulated. In the case of the Diversified cycle of secondary education, it is worthy to mention that the only overall objective related to the teaching of statistics and probability—i.e., "...to allow students to model situations of uncertainty"—is even more general in nature, since working with different types of data representations and summaries, as well as commenting on features of data, are tasks considered essential building blocks of statistical models (MacGillivray & Pereira-Mendoza, 2011, p.116). Thus, it seems to be evident in all the 27 answers of teachers working at upper secondary level knowledge and support of this overall objective.

One clear methodological suggestion at both the Basic and Diversified cycle of secondary education in Venezuela is the study of statistical contents through practical experiences linked to knowledge about daily life—in particular, environmental situations interesting to the students—, life style of the community, region and country, and current national and international events. Therefore, statistical problems posed in the classroom most deal with school-related data, economical phenomena, population analysis, averages, production data, and so on (ME, 1972, 1987, 1997; CENAMEC, 1991). Examining teachers' answers to Question (g), it is possible to appreciate that 12 teachers working at lower high school (22.6% overall), 6 teachers working at upper high school (11.3% overall), and 8 teachers working at both levels (15.1% overall) seem to provide answers in which the designed lessons make strong connections to daily life, particularly by using relevant examples for the students. Among the daily life situations posed in the designed lessons by the teachers mentioned above were the following: TV commercials, daily fruit consumption, favorite foods among students, students' ways to commute between their current residence and school location, by-country medal tables in a particular sport at London Olympics, number of newborns, salaries for different professions, students' scores in a particular subject, favorite TV programs, favorite kind of music, work flow in a bank branch, students' height and students' age. The "practical experiences" suggested by the official curriculum documents were circumscribed to surveys inside and outside the classroom, as well as the conduction of real experiments by the students in the classroom, such as measuring their own height.

5.9.7.2 Regarding the fulfillment of Indicator F-2

In order to fully meet this indicator, teachers' planned lessons should show evidence of teachers' knowledge and support of the objectives and methodological guidelines provided by official curriculum documents in Venezuela in relation to the teaching of statistics at secondary level. Since both overall and specific objectives related to the Statistics and Probability strand emphasize the application of statistical ideas, the acquisition of knowledge and skills and, to a lesser extent, the comprehension of such ideas, all the answers seem to support such objectives. However, making strong connections to daily life, as well as carrying out practical experiences linked to knowledge about daily life, environmental situations, life style of the community, region and country, and current national and international events, are explicitly encouraged in the methodological suggestions for teachers throughout secondary school, as well as in the guidelines included in the teaching guide for the Basic cycle of the secondary school. Therefore, the gathered data was also examined for evidence of support of such suggestions and guidelines. As a result, 26 teachers—49.1% overall, 12 working at lower high school, 6 at upper high school, and 8 at both levels—were those whose planned lessons seem to be student-centred ones, carrying out and supporting overall and specific objectives of the secondary school mathematics curriculum, and hence fully meeting Indicator F-2. It is worthy to highlight the fact that teachers working in upper high school level planned lessons more teacher-centred than those working exclusively at lower secondary school. These teachers emphasized the instruction of concepts, the posing and individual solving of problems, and the assignation of exercises for practice.

5.9.7.3 Regarding Knowledge of Content and Teaching (KCT)

According to Ball et al. (2008), KCT could be evidenced through knowledge about different instructionally viable models for teaching a particular idea, as well as through knowledge on how to deploy them effectively. Question (g) was also posed with the purpose of eliciting evidence of these two indicators associated to KCT—which are identified in the present study as Indicators E-1 and E-2, respectively.

Forty-four teachers—83.0% overall, 17 working at lower high school, 13 at upper high school, and 14 at both levels—answered to Question (g).

In order to determine the presence of Indicators E-1 and E-2, in teachers' answers, a criterion-referenced assessment rubric was designed, based on the characteristics of effective classroom activities to promote students' understanding of variability compiled by Garfield and Ben-Zvi (2008), as well as in the statistical habits of mind required from teachers to teach fundamental statistical ideas listed by Burrill and Biehler (2011, p.66)—i.e., using real data, building intuitions, beginning with a graph, exploring alternate representations of data, investigating and exploring before introducing formulas, and using projects and experiments to engage students in doing statistics (see Table 23).

Table 23: Assessment rubric for evaluating the KCT dimension from teachers' responses to Question (g)

Assessment Criteria regarding Indicator E-1		
• To describe and represent variability with numerical measures when looking at one or more data sets (Garfield & Ben-Zvi, 2008, pp.207-208).	**Yes** ☐	**No** ☐
• To refer to more than one representation of the data (such as frequency distribution tables and boxplots) to lead to better interpretations (ibid., p.207).	**Yes** ☐	**No** ☐
• To promote discussion on how measures of central tendency and measures of variation are revealed in data sets and graphical representations of data (ibid., p.212).	**Yes** ☐	**No** ☐
• To pose activities involving comparisons of groups, instead of graphing, summarizing, and interpreting data for a single group (ibid., p.216).	**Yes** ☐	**No** ☐
• To promote discussion on what factors make the measures of variation to be larger or smaller (ibid., p.209).	**Yes** ☐	**No** ☐
• To promote whole-class discussions on why we need measures of variation in addition to measures of central tendency (ibid., p.214).	**Yes** ☐	**No** ☐
• To pose examples and activities involving real-world contexts or embedded in real-world situations, or to provide students with real data that make sense to them or for which they express very high interest (ibid., p.328).	**Yes** ☐	**No** ☐

Assessment Criteria regarding Indicator E-2		
• To start the lesson with the process of gathering data from students (by using questions like "in which month of the year were you born?") or by presenting them with some simple data, and then representing and interpreting such data (Garfield & Ben-Zvi, 2008, pp.135-137).	**Yes** ☐	**No** ☐
• To describe and compare the variability informally at first (e.g., by describing verbally how the data is spread out), and then formally, through measures of variation (ibid., p.208).	**Yes** ☐	**No** ☐
• After starting to examine the given or collected data, to make students discuss about possible reasons (sources) for the variability in such data (ibid., p.211).	**Yes** ☐	**No** ☐
• In a wrap-up discussion, to make students revisit what boxplots or each measure of variation tells and how these relate to measures of center and shape of distribution (ibid., p.234).	**Yes** ☐	**No** ☐

The researcher examined the lessons planned as answers to Question (g) in order to identify in them the presence of the general pedagogical features in Table 23. In that way, it would be possible to determine to which extent each of the planned lessons use effective classroom activities to promote students' understanding of variability, and sequence them in an effective fashion. Tables 24 and 25 summarize the results of the aforementioned examination to all the planned lessons provided by the teachers:

Table 24: Assessment rubric for evaluating the KCT dimension related to Indicator E-1 from teachers' responses to Question (g)

		Frequency (%)			
		Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
Assessment Criteria regarding Indicator E-1	E-1.1: To describe and represent variability with numerical measures when looking at one or more data sets.	8 (42.1)	6 (40.0)	5 (26.3)	19 (35.8)
	E-1.2: To refer to more than one representation of the data to lead to better interpretations.	10 (52.6)	7 (46.7)	8 (42.1)	25 (47.2)
	E-1.3: To promote discussion on how measures of central tendency and measures of variation are revealed in data sets and graphical representations of data.	7 (36.8)	4 (26.7)	3 (15.8)	14 (26.4)
	E-1.4: To pose activities involving comparisons of groups, instead of graphing, summarizing, and interpreting data for a single group.	5 (26.3)	3 (20.0)	2 (10.5)	10 (18.9)
	E-1.5: To promote discussion on what factors make the measures of variation to be larger or smaller.	4 (21.1)	3 (20.0)	3 (15.8)	10 (18.9)
	E-1.6: To promote whole-class discussions on why we need measures of variation in addition to measures of central tendency.	2 (10.5)	2 (13.3)	0 (0.0)	4 (7.5)
	E-1.7: To pose examples and activities involving real-world contexts or embedded in real-world situations, or to provide students with real data that make sense to them or for which they express very high interest.	12 (63.2)	6 (40.0)	8 (42.1)	26 (49.1)

Table 25: Assessment rubric for evaluating the KCT dimension related to Indicator E-2 from teachers' responses to Question (g)

Assessment Criteria regarding Indicator E-2	Frequency (%) Lower High School (19 teachers)	Upper High School (15 teachers)	Both Levels (19 teachers)	Total (53 teachers)
E-2.1: To start the lesson with the process of gathering data from students or by presenting them with some simple data, and then representing and interpreting such data.	11 (57.9)	5 (33.3)	7 (36.8)	23 (43.4)
E-2.2: To describe and compare the variability informally at first, and then formally, through measures of variation.	4 (21.1)	1 (6.7)	1 (5.3)	6 (11.3)
E-2.3: After starting to examine the given or collected data, to make students discuss about possible reasons for the variability in such data.	6 (31.6)	4 (26.7)	4 (21.1)	14 (26.4)
E-2.4: In a wrap-up discussion, to make students revisit what boxplots or each measure of variation tells and how these relate to measures of center and shape of distribution.	3 (15.8)	0 (0.0)	0 (0.0)	3 (5.7)

Some of the results shown in Tables 24 and 25 were already expected. For example, in Table 24, the fact that 26 teachers—49.1% overall, 12 working at lower high school, 6 teachers working at upper high school, and 8 teachers working at both levels—met the seventh criterion regarding Indicator E-1—i.e., the assessment criterion for posing examples and activities involving real-world contexts or real data—was expected from the previous examination for curricular features of the planned lessons. In fact, the same 26 teachers were those who provide answers in which the designed lessons make strong connections to daily life, particularly by using relevant examples for the students and providing them with real data gathering experiences, which is in accord with the methodological suggestions in the Venezuelan secondary school mathematics curriculum at all grades.

According to Ball et al (2008, p.401), how teachers sequence particular content for instruction is one of the features of the cognitive domain *knowledge of content and teaching* (KCT). By the means of examining the planned lessons looking for the assessment criteria in Table 25, the presence of these four aspects of effective instructional sequence for the teaching of variability-related ideas that facilitates student acquisition of statistical literacy skills will be determined.

In addition, an aspect worthy of attention is the fact that some misconceptions held by the surveyed teachers not only emerged in their planned lessons, but also had a strong influence in the content to be taught. For example, 2 teachers—3.8% overall, working at both secondary school levels—planned lessons in which the focus was to study symmetry and asymmetry, and to relate those concepts to the one of variability. Both teachers pointed out before, when answering Question (a), that Distribution A was the one with more variability—one of them with an answer falling into category A2, and the other into category A3—, which may suggest the strong relation among CCK—or lack thereof—and KCT, particularly the planning facet.

5.9.7.4 Regarding the fulfillment of Indicator E-1

Due to the multidimensionality of the assessment criteria related to Indicator E-1, as well as to the complex qualitative information that could be obtained from teachers' answers to Question (g), the examination to determine whether or not a teacher meets Indicator E-1 will be undertaken criterion by criterion; that is, dimension by dimension. The features of each criterion will be provided, and the implications of fulfilling each criterion will be discussed. In that way, it would be possible to determine qualitatively to which extent each of the planned lessons uses effective classroom activities to promote students' understanding of variability.

The most used activity suggested by these teachers for collecting real data was the survey—by 13 teachers (24.5% overall), 6 working at lower high school, 3 working at upper high school, and 4 working at both levels—, followed by providing students with sets of real data—by 4 teachers (7.5% overall), 3 working at lower high school, and 1 working at both levels—, and carrying out in-class experiments involving repeated measurements and random trials—by 3 teachers (5.7% overall), 2 working at lower high school, and 1 working at upper high school level.

Another interesting fact from Table 24 is having 25 teachers—47.2% overall, 10 working at lower high school, 7 working at upper high school, and 8 working at both levels—meeting the second criterion regarding Indicator E-1, related to multiple representations of data. Such multiple representations are fundamental to increase understanding about the data on study (Burril & Biehler, 2011, p.58), as well as a basic step to transnumeration—i.e., to the process of changing representations of data to identify different aspects of the same data. Among the remaining of the respondents who did not met this criterion, it was common asking students to calculate measures of central tendency or variation directly from a given problem, graph or data set, without engaging in transnumeration.

The fact that only 19 teachers—35.8% overall, 8 working at lower high school, 6 working at upper high school, and 5 working at both levels—met the first criterion regarding Indicator E-1, is also worthy to be highlighted. Many of the respondents to this question planned very shallow lessons, in which they just specified the elements "Concepts", "Examples", "Exercises", and "Practice problems" and shuffled them. Those who planned more in-detail lessons and did not met this criterion, typically posed activities asking students to create and interpret graphs without saying how, or just proposed to pose problems to students.

One of the main concerns that arise from Table 24 is the low proportion of teachers meeting the third, fourth, fifth and sixth criteria regarding Indicator E-1. Regarding criterion E-1-3, the discussion of both measures of central tendency and measures of variation is fundamental for data analysis, particularly when comparing groups, in which examining the differences in the medians, means, modes and measures of spread is a critical step (Gattuso & Ottaviani, 2011, p.128). Regarding criterion E-1-4, comparison of two distributions is one of the basic tasks in the exploratory data analysis that must be presented in teaching statistical ideas at any school level (Jacobbe & Carvalho, 2011, p.21), and these comparison must include considerations of both measures of central tendency and measures of variation in order to properly describe and acknowledge variability (Canada & Ciancetta, 2007; Reading & Canada, 2011). Regarding criterion E-1-5, lack of understanding which factors makes measures of variation smaller or larger could turn the lesson in a mere act of calculate such measures and, in the case of comparison of distribution, select the one with smaller or larger numerical value, depending of the question made. Regarding criterion E-1-6, the promotion of whole-class discussions about why measures of variation are needed in addition to measures of central tendency is crucial, since it is one of the ways to avoid misconceptions in students such as focusing merely in middles or visual cues of the distributions. Therefore, the low proportion of secondary school mathematics teachers planning lessons considering these statistical habits of mind might be interpreted as a major practical concern for teacher education in Venezuela.

5.9.7.5 Regarding the fulfillment of Indicator E-2

Again, due to the multidimensionality of the assessment criteria related to Indicator E-2, as well as to the complex qualitative information that could be obtained from teachers' answers to Question (g), the examination to determine whether or not a teacher meets Indicator E-2 will be undertaken criterion by criterion. The features of each criterion will be provided, and the implications of fulfilling each criterion will be discussed. In that way, it would be possible to determine qualitatively to which extent the instructional activities in each one of the planned lessons are effectively sequenced for a better student learning and understanding of variability.

As can be seen in Table 25, the instructional aspect that appeared the most in the lessons planned by the respondents was the one assessed by first criterion regarding Indicator E-2. 23 teachers—43.4% overall, 11 working at lower high school, 5 working at upper high school, and 7 working at both levels—met this criterion. This means that these 23 teachers posed lessons using a constructivist approach, posing activities in which students have to either engage in the process of gathering real data, or organize and interpret sets of real data provided by the teacher. Moreover, activities such as collecting sample data is one of the specific statistical practices identified by Wild and Pfannkuch (1999), together with randomization, tabulation and transnumeration, data reduction, and using statistical models. However, it is worthy to highlight that just 14 teachers—26.4% overall, 6 working at lower high school, 4 working at upper high school, and 4 working at both levels—met the third criterion regarding Indicator E-2. This means that even though just over 40% of the surveyed teachers planned lessons starting with statistical practices such as gathering real data or engaging in trasnumeration from sets of real data, just over a quarter of the surveyed teachers examined the reasons or sources of the variability in the data they were studying. According to Shaughnessy (2007, 2008) and Shaughnessy and Pfannkuch (2002), engaging students in discussions about the sources of variation in data provides them opportunities to be “data detectives” and to develop their statistical literacy skills and share their reasoning in a data-based fashion. In this way, students would be able to uncover the stories that are hidden in the data. This is fundamental for the understanding of variability, since the heart of any statistical story is usually contained in the variability in the data (Shaughnessy, 2008).

It is worthy to highlight the low proportion of planned lessons meeting the second and fourth criteria regarding Indicator E-2. Regarding the criterion E-2-2, lessons which met this criterion are those that foster the statistical habits of mind of building intuitions and exploring data before introducing formulas, which are highly expected from teachers when they teach fundamental statistical ideas (Burrill & Biehler, 2011, p.66). Moreover, deriving logical conclusions from data—whether formally or informally—is accompanied by the need to provide persuasive arguments based on data analysis, which promotes students' argumentation and statistical literacy skills (Pfannkuch & Ben-Zvi, 2011, p.329).

Regarding the criterion E-2-4, this low proportion of lessons meeting this criterion identifies a characteristic of Venezuelan mathematics lessons: the almost total absence of wrap-up discussions involving students. In the collected data, the stage "Conclusion" in a lesson is particularly characterized by teachers assigning homework or practice problems after students engaged in solving particular tasks. It is interesting the fact that the 3 teachers whose lessons met this criterion are working in lower secondary level, and none of the teachers working at upper secondary school satisfying the criterion.

5.9.7.6 Regarding teachers' statistics-related beliefs

According to Eichler (2008, 2011) and Pierce and Chick (2011), teachers' beliefs about statistics teaching and learning could be identified through examining the features of the lesson plans prepared by them. Some of these features are the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson. What teachers planned to do—which is related to the construct KCT, and hence with answers to Question (g)—will be analyzed using the four categories reflecting on teachers' beliefs developed by Eichler (2011)—i.e., traditionalists, application preparers, everyday life preparers, and structuralists—, which will provide valuable information on teachers' beliefs about the nature of statistics, as well as about the teaching and learning of statistics.

Eichler (2008, 2011) identifies four categories of teachers, depending on the beliefs they seem to hold after analyzing their lessons. *Traditionalists* are those teachers who appear to be more concerned about students gaining algorithmic skills, and less about context and applications. *Application preparers* are those teachers focused on teaching theory and algorithms wanting to promote interplay between theory and application, so students could use such theory and algorithms to solve real-world problems. *Everyday life preparers* are teachers who teach through applications, in order to develop abilities to address real stochastic problems. Finally, *structuralists* are those teachers who examine applications but as a starting point for exemplifying mathematical theory and abstract systems; that is, *structuralists* encourage students' understanding of the abstract system of mathematics or statistics in a process of abstraction, starting from applications (Eichler, 2008, 2011; Girnat & Eichler, 2011).

None of the participants in the current sample of surveyed teachers was categorized as *structuralist*, since all the respondents started their planned lessons explaining theory, providing students with an overview of the concepts to use during class, engaging students in different statistical practices, or posing problems. The lessons planned by *application preparers* and *everyday-life preparers* must consider the solving of real-world problems. The difference between both types of teachers are the way the start their lessons: *application preparers* seem to believe that the effective way to teach statistics consists in teaching theory and algorithms at first, and then apply such knowledge to solve real-world problems. On the contrary, *everyday-life preparers* seem to believe that an effective lesson must start using a realistic situation and, while examining it, statistical methods should be developed, in order to develop both students' ability to cope with real stochastic problems and students' ability to criticize decision-making processes and situations in real life. In the collected data, 26 teachers—49.1% overall, 12 working at lower high school, 6 teachers working at upper high school, and 8 teachers working at both levels—posed examples and activities involving real-world contexts or real data. Teachers who started with explanations of theory and algorithms are those that Eichler calls *application preparers*, whereas those who started from an application or the consideration of a real life situation are those that Eichler calls *everyday-life preparers*. After examining the collected data, it was determined that 8 teachers—15.1% overall, 2 working at lower high school, 2 teachers working at upper high school, and 4 teachers working at both levels—could be regarded as *application preparers*, while 18 teachers—34.0% overall, 10 working at lower high school, 4 teachers working at upper high school, and 4 teachers working at both levels—could be considered as *everyday-life preparers*. The remaining respondents to Question (g)—i.e., 18 teachers, 34.0% overall, 5 working at lower high school, 7 teachers working at upper high school, and 6 teachers working at both levels—could be regarded as *traditionalists*, teachers who seem to believe that the goal of teaching statistics is students' acquisition of algorithmic skills and procedures, disregarding context and applications. This neglect of the relevance of the role of context in statistics raises particular concern, since many statistics educators—e.g., Shaughnessy, 2007; Eichler, 2011—have regarded the role of context as one of the main aspects of teaching statistics.

CHAPTER 6:

Conclusions

In this concluding chapter will be highlighted the central findings on Venezuelan secondary school mathematics teachers' professional knowledge and affective characteristics for teaching variability-related concepts. This chapter is presented in five sections. In the first section, the author summarizes all the previous chapters, providing a compact and general view of the present study. In the second section, the author discusses the central findings of this research study, and provides answer to the research questions addressed in the current study. In third section, the author discusses the study's contributions and implications. In fourth section, the author discusses the study's limitations. In the fifth section, the author concludes the chapter with a discussion that points to potential relevant future research.

6.1 Chapter abstracts

In this dissertation, the construct of *statistical knowledge for teaching* (SKT) is explored, in particular, for the case of Venezuelan secondary school mathematics teachers. In **Chapter 1**, I provide the rationale, background, motivation and purpose for the present study, as well as describe how the study fills a critical gap in the understanding of the professional knowledge and affective-motivational characteristics required from mathematics teachers to accomplish an effective teaching and learning of statistical ideas—in particular those related to variability—at secondary school level. The inclusion and emphasis of topics related to statistics at all school levels, particularly in the secondary one, is pointed out, as well as the acquisition and development of statistical literacy skills by students as an educational goal not only in Venezuelan mathematics curriculum, but worldwide. Such skills have nowadays an increasing societal importance, since they make able to the citizen who possesses them to intelligently participate in many fields of today's knowledge-based society after the end of compulsory school. Therefore, these skills—many of which are based on a proper acknowledgement and understanding of the statistical idea of variability—must be developed particularly at secondary school level, since secondary school might be either the last exposure to statistics that many future users of statistics might have, or the stepping stone to advanced contents for those students moving on to tertiary education. It is for this reason that secondary school mathematics teachers, now more than ever, require to have specific professional knowledge, skills and habits of mind, without which the aims of the mathematics curriculum regarding statistics education cannot be achieved. Mainly from this, as well as from the paucity in research on professional knowledge for teach statistics at school level, stems the need for making research on SKT.

After formulating the Problem Statement in Chapter 1—i.e., the clarification of the current state of the professional knowledge base on statistics, conceptions of variability, and beliefs about teaching and learning of statistics, held by Venezuelan secondary school mathematics teachers—, the twofold Purpose of the Research is described, and three Research Questions to be answered are posed, in order to fulfill the purpose of the current study. Such questions are the following:

1. What is statistical knowledge for teaching, and what are the indicators that could serve to evaluate it in the case of teaching variability-related concepts?

2. On the basis of the conceptualization of SKT adopted by this research, what is the knowledge base of SKT that Venezuelan secondary school mathematics teachers have to teach variability-related concepts?

3. How do Venezuelan secondary school mathematics teachers conceptualize variability, and what beliefs about statistics, its teaching and learning, do they have?

Chapter 1 finishes describing particular aspects of the significance of the present study, such as, among others, academically contributing in a field in which there is a duly noted lack of studies focusing on professional statistical knowledge, conceptions of variability, and statistics-related beliefs of secondary school mathematics teachers from the viewpoint of variability, lack which is particularly remarkable in the case of Venezuela.

In **Chapter 2**, I provide a review of the structure of the Venezuelan educational system, an extensive description of the statistics curriculum in Venezuela at both elementary and secondary school levels, as well as a summary of the researches carried out to date on Venezuelan mathematics teachers' professional competence to teach statistics at compulsory school. Regarding the Venezuelan educational system, it is described each of the four levels in which the mainstream education system is organized: initial education, elementary school, secondary school, and higher education. Thereafter, I explain about the statistics curriculum in Venezuela, making clear the fact that statistics is taught as part of the mathematics curriculum at all levels of compulsory education, which includes secondary school. In fact, statistical contents are acknowledged in national curriculum documents as a single unit, with the strand "Statistics and Probability" being present throughout all compulsory education. In Venezuela, the study of statistics at compulsory school started in 1972, when the reform to the secondary school mathematics curriculum included, for first time, the study of topics on statistics and probability at school level, specifically at Grades 10 and 11. Before this year, the study of statistical topics was exclusive to students enrolled in higher education. The inclusion of notions of statistics and probability in elementary school occurred years later, in 1985. After the last reforms to the school mathematics curricula in Venezuela, only at Grade 10 topics on statistics are not studied. In Chapter 2, the statistical contents from Grade 1 to Grade 11, the overall and specific objectives, as well as the methodological guidelines outlined in the Venezuelan mathematics curriculum for both elementary and secondary schools are explained in detail. Also, an analysis of the verbs used in the Venezuelan secondary school mathematics curriculum, using the six hierarchical thinking levels in Benjamin Bloom's cumulative hierarchical theoretically-based revised taxonomy framework in the cognitive domain, is carried out, in order to indicate the level of critical thinking expected of students, as well as to describe the cognitive demands in the Venezuelan secondary school mathematics curriculum in relation to the teaching and learning of statistics. Chapter 2 finishes with an overview of the few research efforts on investigating Venezuelan mathematics teachers' professional knowledge and affective-motivational characteristics to teach statistics at compulsory school level, most of which have been focused on the elementary school, or draw conclusions about the current state of teaching and learning of statistics at school level in Venezuela by extrapolation from research on either individual perceptions of teacher preparation to teach statistics, or students' performance and achievement in statistics at tertiary education.

In **Chapter 3**, I provide a review of the literature and frameworks relevant to the current research. At first, a review of the literature on statistical literacy is undertaken. From such review, the characteristics as well as societal and educational importance of statistical literacy for all individuals in today's knowledge-based society are clearly identified. Thereafter, I explain about the statistical idea of variability, its definition, its importance in the field of statistics—since it is regarded as the raison d'être of the discipline—, how it is measured or estimated, and how is conceptualized by students and teachers according to previous researches, particularly using the categorization of conceptions of variability developed by Shaughnessy (2007). A review of the literature on mathematics teachers' professional competencies follows, during which it is presented a brief overview of research on the definition of this construct, as well as on the constructs of *subject matter knowledge* (SMK), *pedagogical content knowledge* (PCK), *mathematical knowledge for teaching* (MKT) —developed by Ball et al. (2008)—, and teachers' beliefs. In addition, the construct of *statistical knowledge for teaching* (SKT) is also discussed, as well as the few theoretical models of SKT proposed to date that employ the cognitive constructs of SMK, PCK and MKT to conceptualize it, which were developed by Groth (2007), Burgess (2011), and Noll (2011). From this discussion, specific gaps in statistics education research literature are identified, as well as particular areas in need of further attention in research, which the current research attempts to deal with. In particular, the one that none of those MKT-based frameworks of SKT takes into account either all the six components identified by Ball et al. (2008), the role of beliefs in teachers' professional practice, or the conceptions of variability held by the teachers, which could result in an inaccurate picture of their preparedness to teach statistical contents related to variability at any school level.

In **Chapter 4**, I provide a conceptual analysis and framework for mathematics teachers' statistical knowledge for teaching, focused on the acknowledgement and understanding of variability as a fundamental idea in statistics. In this framework, the construct of statistical literacy is an essential feature that serves to frame my discussion of SKT. Particular aspects of statistical literacy provide a picture of the knowledge base of *common content knowledge* necessary for effective teachers, who have need to know how to correctly solve the problems they pose to students and to appropriately understand the variability arising in a statistical problem. Indeed, a sound grounding in statistical literacy is a necessary condition for having a robust SKT, but it is not sufficient. I would have to add in knowledge on how statistical ideas are developed throughout the courses of study—i.e., *knowledge of content and curriculum*—, knowledge of students' thinking statistically—i.e., *knowledge of content and students*—, capacity to plan and execute meaningful teaching in the light of the previous two cognitive traits—i.e., *knowledge of content and teaching*—, understanding of the broader set of statistical ideas to which a particular concept connects—i.e., *horizon content knowledge*—, and ability to determine the accuracy of common and non-standard solutions that could be given by students when solving statistical problems—i.e., *specialized content knowledge*. These six traits, alongside teachers' beliefs and conceptions of variability, comprise the conceptual framework for SKT proposed in the present study. The identification of these eight traits from an extensive literature review is followed by the selection, also from the literature, of twelve indicators associated to each cognitive trait, in order to provide a comprehensive framework for conceptualizing SKT.

After introducing the aforementioned framework and its indicators, I also talk about how this framework can be used to assess the knowledge base of SKT held by Venezuelan secondary school mathematics teachers. I provide a discussion of my research methodology, study design, and analysis procedures. A general overview of my data collection methods and rationale for those methods is provided, alongside with a detailed discussion on how the research instrument used in this study was designed. Emphasis is put on the fact that most of the statistical contents in the Venezuelan secondary school mathematics curriculum are ideas related to descriptive statistics, reason because of which the selected task to be posed in the research instrument—"Choosing the distribution with more variability" task, originally developed by Garfield et al. (1999)—was one dealing with many ideas in such domain. Some of those ideas are histograms, frequency distributions, measures of central tendency and measures of variation, which are revisited in a spiral way throughout the subject of mathematics at secondary school in Venezuela. For the present study, the selected task was slightly modified in order to facilitate the calculations that could be made by the respondents, and was also enriched with seven questions, aiming to elicit all the eight facets of teachers' professional competencies to teach variability-related contents previously identified by the framework for SKT proposed here.

Chapter 5 shows the results of carrying out the survey instrument in a sample of 53 Venezuelan secondary school mathematics teachers, who work in the metropolitan area of Caracas, the capital city of Venezuela. Furthermore, a detailed discussion of such results on each of the seven questions accompanying the "Choosing the distribution with more variability" task is undertaken. In such discussion, I analyze the collected data in light of the model for SKT proposed in the present study, and discuss the six types of knowledge, as well as teachers' conceptions of variability and statistics-related beliefs, the surveyed teachers in this study demonstrated.

As for the analysis, all questions in the survey instrument were examined to determine whether or not each one of the twelve indicators previously identified was met. Furthermore, as a result of this analytic approach of assessment, valuable qualitative information about the cognitive traits comprising the framework for SKT proposed here was gathered. In order to carry out such analysis, and depending on the question being analyzed as well as on the assessment criteria related to meeting each indicator, different theoretical frameworks were used, many assessment rubrics were developed and implemented, and grounded form of analytic approach to coding were repeatedly undertaken. After this analytical discussion looking for evidence of whether or not each of the 12 SKT-related indicators was met, it was collected enough qualitative information describing the knowledge base of SKT held by the participants in this study; that is, this analysis provided a truly comprehensive picture of the current state of the surveyed Venezuelan secondary school mathematics teachers' knowledge base on SKT, conceptions of variability, and beliefs about statistics teaching and learning.

This current chapter, **Chapter 6**, examines and discusses the links between the broad aims of the research as derived from the *Literature Review,* the results and discussion from Chapters 3 and 4, the specific research questions, and the contribution that this thesis has made to the research field. Implications are drawn from the research and discussed, with regard to the current state of teacher knowledge for teaching statistics at secondary school from the viewpoint of variability. Also, suggestions for future research concerning, among others, practicing and pre-service teachers, teacher educators and curriculum developers, are outlined.

6.2 Answers to research questions

In this section, each one of the research questions will be addressed in relation to the conclusions that can be drawn from the present study.

6.2.1 Answer to the First Research Question: *What is statistical knowledge for teaching, and what are the indicators that could serve to evaluate it in the case of teaching variability-related concepts?*

In Chapter 4, and on the basis of an exhaustive literature review, it was proposed a conceptual framework for describing several components that were thought to be potential indicators of teachers' professional competencies for teaching statistics from the viewpoint of variability. This framework was built by fusing the constructs of statistical literacy and mathematical knowledge for teaching with current research on statistical knowledge for teaching, conceptions of variability and teachers' beliefs about statistics teaching and learning (e.g., Gal, 2002; Ball et al., 2008; Ball & Bass, 2009; Shaughnessy, 2007; Eichler, 2008, 2011; González, 2011; Isoda & González, 2012). The visual representation from this framework is shown again in Figure 24.

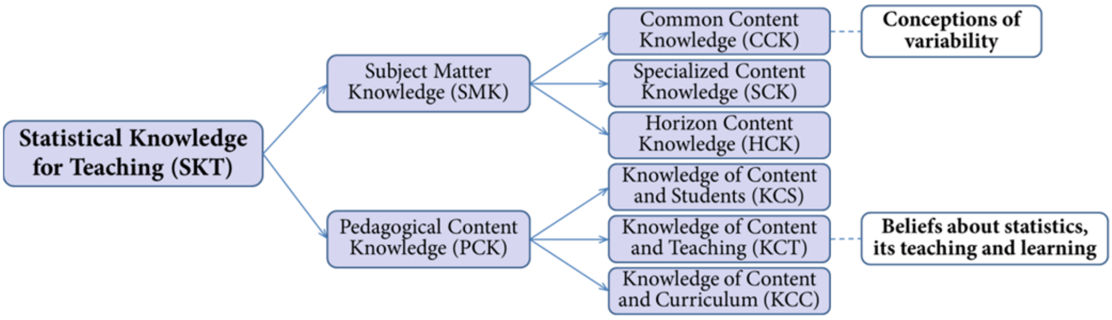


Figure 24: Proposed conceptual model of teachers' competencies to teach secondary school statistics

The model proposed here is a two-faceted one, with one cognitive and one affective facet. The cognitive facet of the framework proposed here is sixfold, comprised of all the six subdomains of professional knowledge identified by Ball et al. (2008) in their model of *mathematical knowledge for teaching*—i.e., *common content knowledge*, *specialized content knowledge*, *horizon content knowledge*, *knowledge of content and students*, *knowledge of content and teaching*, and *knowledge of content and curriculum*—, with CCK understood as statistical literacy in order to meet the case of teaching statistics. This facet is the one that represents SKT—i.e., the professional knowledge needed to carry out effectively the work of teaching statistics. The affective facet of the model proposed here is comprised of two components: teachers' beliefs about statistics, its teaching and learning; and teachers' conceptions of variability. Although the focus of this research is to examine SKT, the affective-motivational traits are considered as well, since numerous studies have highlighted the inextricable relation between teachers' knowledge, conceptions and beliefs about the subject matter (e.g., Putnam & Borko, 2000; Pajares, 1992; Knuth, 2002), as well as the important role played by teachers' conceptions and personal beliefs in the implementation of a curriculum (e.g., Duschl & Wright, 1989; Ball

& Cohen 1996; Trigwell, Prosser & Waterhouse, 1999).

In order to apply the proposed framework for SKT to exploratory analysis of empirical data, specific indicators of each component in the framework were needed to identify features of these framework components and their relations. Then, twelve indicators associated to the cognitive facet of such competencies were selected from the literature, in order to provide a comprehensive framework for conceptualizing SKT. Those indicators are shown again in Table 26.

Table 26: Set of indicators proposed to assess SKT

A: INDICATORS RELATED TO CCK (STATISTICAL LITERACY):

1. Is the teacher able to give an appropriate and correct answer to the given task?
2. Does the teacher consistently acknowledge variability and correctly interpret its meaning when answering the given task?

B: INDICATORS RELATED TO SCK:

1. Does the teacher show evidence of ability to determine the accuracy of common and non-standard arguments, methods and solutions that could be proposed to the given task by students (especially while recognizing whether a student's answer is right or not)?
2. Does the teacher show evidence of ability to analyze right and wrong solutions that could be given by students to the present task, by providing explanations about what reasoning and/or mathematical/statistical steps likely produced such responses, and why, in a clear, accurate and appropriate way?

C: INDICATORS RELATED TO HCK:

1. Does the teacher show evidence of having ability to identify whether a student response is interesting or significant, mathematically or statistically?
2. Is the teacher able to identify the significant notions, practices or values related to the statistical ideas involved in the given task?

D: INDICATORS RELATED TO KCS:

1. Is the teacher able to anticipate students' common responses and difficulties on the given task?
2. Does the teacher show evidence of knowing the most likely reasons for students' common responses and difficulties in relation to the statistical concepts involved in the given task?

E: INDICATORS RELATED TO KCT:

1. In design of teaching, does the teacher show evidence of knowing what tasks, activities and strategies could be used to set up a productive whole-class discussion aimed at developing students' understanding of the key statistical concepts involved in the given task, instead of focusing just in computation methods or general calculation techniques?
2. Does the teacher show evidence of knowing how to sequence such tasks, activities and strategies, in order to develop students' understanding of the key statistical concepts involved in the given task?

F: INDICATORS RELATED TO KCC:

1. Does the teacher show evidence of knowing at what grade levels and content areas students are typically taught about the statistical concepts involved in the given task?
2. Does the designed lesson (or series of lessons) show evidence of teacher's knowledge and support of the educational goals and intentions of the official curriculum documents in relation to the teaching of the statistical contents present in the given task, as well as statistics in general?

6.2.2 Answer to the Second Research Question: *On the basis of the conceptualization of SKT adopted by this research, what is the knowledge base of SKT that Venezuelan secondary school mathematics teachers have to teach variability-related concepts?*

The present research set out to explore Venezuelan secondary school mathematics teachers' knowledge base of SKT, which led to this research question. In an attempt to find answer to it, the researcher drew upon the eight dimensions of professional competencies for teaching variability-related contents identified by this study, the twelve indicators of SKT previously outlined, and an exhaustive review of the statistic education literature, to develop a survey instrument with the purpose of eliciting and gathering information about each one of the aforementioned eight components of teachers' professional competencies to teach variability-related contents. Each question was developed based on previous studies with similar aims reported in the literature (e.g., Meletiou & Lee, 2003; Ball et al., 2008; Manizade & Mason, 2011; Isoda & González, 2012), which were adapted to reflect the setting of the selected task and the specific objectives of the present conceptual framework proposed by this study. The survey was circumscribed to statistical ideas in the field of descriptive statistics, a restriction mainly motivated by the fact that most of the statistical contents in the Venezuelan secondary school mathematics curriculum are ideas related to such field.

The collected data from the survey and the findings presented and discussed in Chapters 5 provide a snapshot of the current state of Venezuelan secondary school mathematics teachers' knowledge base of SKT in relation to variability-related contents. After a qualitative analysis of the answers given by the participants in this study, looking for evidence of whether or not each of the 12 SKT-related indicators was met, it was collected valuable information on the knowledge base of SKT, conceptions of variability, and beliefs about statistics teaching and learning held by the participants in this study. The results and main findings regarding surveyed teachers' knowledge base on SKT are summarized to certain extent in Table 27.

Table 27: Summary of the results on surveyed teachers meeting the indicators associated to SKT proposed in this study

Elicited Knowledge Component of SKT	Indicator Associated to SKT	# of teachers meeting the indicator	% of teachers meeting the indicator
Common Content Knowledge (CCK) (as Statistical Literacy)	A-1	13	24.5%
	A-2	18	34.0%
Specialized Content Knowledge (SCK)	B-1	8	15.1%
	B-2	8	15.1%
Horizon Content Knowledge (HCK)	C-1	24	45.3%
	C-2	37	69.8%
Knowledge of Content and Students (KCS)	D-1	17	32.1%
	D-2	13	24.5%
Knowledge of Content and Teaching (KCT)	E-1	Refer to Table 24	
	E-2	Refer to Table 25	
Knowledge of Content and Curriculum (KCC)	F-1	6	11.3%
	F-2	26	49.1%

Also, significant connections among the six components within the cognitive facet, as well as among cognitive and affective components, are revealed, as well as the co-existence of simple or sophisticated recognition of variability with statistical misconceptions and algorithmic and procedural limitations. Despite the latter, it could be said, generally speaking, that teachers who showed evidence of paying attention to both middles and spread in Question (a) did quite well in the rest of the survey, unlike those teachers who evidenced holding misconceptions in their answers to the same question. A summary of the results from each question in the survey, as well as some conclusions that could be drawn from them, follows. Such conclusions were formulated on the basis of the proportion of surveyed teachers who fully met each of the twelve SKT- related indicators previously identified.

From the survey responses to Question (a), it could be said that, in the statistical setting explored, Venezuelan secondary school mathematics teachers' common content knowledge, statistical literacy skills, and conceptions of variability need to be improved, and even built in some cases. This comes from several facts: just 13 out of 53 teachers—24.5% overall—provided a correct answer without committing any calculation mistake and supported it on arguments based on simple or sophisticated recognition of variability; the low proportion of surveyed teachers considering in their answers the connections between middles in data and the variability of data dispersed around a middle—almost 19% overall in the best case—; and the high proportion of teachers who provided a wrong answer and evidenced statistical misconceptions—26 teachers either selecting the wrong distribution or choosing the right one for the wrong reasons, and 17 teachers conceptualizing variability as visual cues in the graph.

From the survey responses to Question (b), it could be said that a sizable proportion of surveyed teachers—i.e., 37 teachers, 69.8% overall—showed evidence of being able to establishing connections between the cognitive demands of the posed task and fundamental ideas, practices, values and sensibilities of the discipline of statistics; that is, evidenced a solid *horizon content knowledge*. The majority of these teachers indicated statistical ideas and practices related to the construction of frequency distribution tables and the calculation of measures of central tendency and variation.

From the survey responses to Question (c) arises a clear example of the strong relation between *common content knowledge*, high-level conceptions of variability, and *specialized content knowledge*. Only 8 of the surveyed teachers—15.1% overall—consistently exhibited ability to correctly judge the accuracy of the answers given by three fictitious students. These teachers were the only ones in the present study who correctly judged the accuracy of the answers given by the fictitious students in Question (c), but also made appropriate observations for each case about why they thought so, and provided accurate comments about the most likely reasoning behind each student' answer. Moreover, 7 out of 8 of these teachers provided right responses to Question (a) exhibiting simple or sophisticated recognition of variability.

From the survey responses to Question (d), the weak knowledge base of knowledge of content and students held by the surveyed teachers is revealed. Just 13 teachers—24.5% overall—were able to simultaneously provide appropriate likely common responses and difficulties expected from the students, a correct assessment of the accuracy or inaccuracy attributed to such responses, and appropriate reasons for such assessment. Harboring some statistical misconceptions, as well as failing to provide a right assessment of the accuracy or inaccuracy of students' likely common answers given by them, seem to be the main problems among the surveyed teachers in this question.

Survey responses to Question (e) shed light on another feature of the knowledge base of SKT held by the surveyed teachers: their ability to identify whether a student response to the given task is interesting or significant, from a mathematical or statistical standpoint, while evidencing skills associated to the cognitive construct *horizon content knowledge*. Some of those skills are identifying seeds of misconceptions or misrepresentations in the comments made by the students; recognizing mathematical opportunities; demonstrating sense of the mathematical and statistical environment surrounding the current "location" in instruction; recognizing major disciplinary ideas, structures and practices; acknowledging core mathematical and statistical values and sensibilities; and making connections with other disciplines. In the present study, only 24 respondents—45.3% overall—provided evidence of such skills.

From the survey responses to Question (f), the weak knowledge base of *knowledge of content and curriculum* held by the surveyed teachers is left exposed.

Among the 45 teachers who provided an answer to Question (f), only 6 of them—11.3% overall—evidenced knowledge about both the specific statistical concepts in the mathematics curriculum, and the grade levels and/or content areas at which students are typically taught such concepts.

From the survey responses to Question (g), it was possible to gather valuable information about the surveyed teachers' knowledge base in *knowledge of content and curriculum* (KCC) and *knowledge of content and teaching* (KCT), as well as about their beliefs on statistics teaching and learning. Regarding KCC, 26 teachers—49.1% overall—were those whose planned lessons seem to be student-centred ones, showing evidence of teachers' knowledge and support of the objectives and methodological guidelines provided by official curriculum documents in Venezuela in relation to the teaching of statistics at secondary level. Regarding KCT, on the basis of the multidimensional assessment rubric designed to examine the planned lessons provided by the surveyed teachers, it could be said, among other things, that 26 of them—49.1% overall—planned to pose examples and activities involving real-world contexts or real data; 13 teachers—24.5% overall—planned surveys inside and outside the classroom as collecting real data activities; 25 teachers—47.2% overall—planned using multiple representations of data during their lessons; only 19 teachers—35.8% overall—explicitly planned to measure variability during their lessons; just 4 teachers—7.5% overall—planned lessons promoting whole-class discussions about the need of measures of variation in addition to measures of central tendency; and 23 teachers—43.4%

overall—planned to pose lessons starting with activities in which students have to either engage in the process of gathering real data, or organize and interpret sets of real data provided by the teacher. Finally, regarding teachers' beliefs on statistics teaching and learning, it seems that the majority of the surveyed teachers believe that teaching statistics must emphasize the acquisition of algorithmic skills and procedures. On the one hand, some of those teachers seem to believe that it is possible to teach statistics is a teacher-centred way, and paying no attention to context and applications—the so-called *traditionalists*, who represent, overall, 34.0% of the surveyed teachers—; on the other hand, some of them seem to believe that the effective way to teach statistics consists in teaching theory and algorithms at first, and then apply such knowledge to solve real-world problems—the so-called *application preparers*, who also represent, overall, 34.0% of the surveyed teachers.

From the analysis of the collected data, many interrelations between the eight components of mathematics teachers' professional competencies to teach efficiently statistics identified by this study seem to be taking place. For example, from the 6 teachers who made reference to "symmetry" and "asymmetry" in Question (b), 4 failed in answering correctly Question (a)—showed evidence of holding the conception Variability as visual cues in the graph—; all failed Questions (c) and (d); and 4 designed lessons focused on "symmetry", "asymmetry" and "normality". This seems to indicate a relation between holding a particular statistical misconception—i.e., thinking of variability in terms of symmetry, or lack thereof—and the cognitive constructs CCK, SCK, HCK, KCS and

KCT. 7 out of 8 of the teachers who fully met Indicators B-1 and B-2—related to Question (c), posed to gather information on SCK—, exhibited simple or sophisticated conceptions of variability in Question (a), which implies to fully meet Indicator A-2—i.e., to be able to consistently identify and acknowledge variability and correctly interpret its meaning in the setting of the given task. Thus, this result seems to indicate a relation between holding simple or sophisticated conceptions of variability and the cognitive constructs CCK and SCK. 10 out of the 13 teachers who provided a wrong assessment about the accuracy of their responses in Question (d)—posed to gather information on KCS—, also gave answers evidencing misconceptions in Question (a). Therefore, it seems to be a relation between holding statistical misconceptions and the cognitive constructs CCK and KCS. All these relations that seem to emerge from the analysis of the collected data may be further examined by future research projects, in order to improve our understanding of such intertwinements and guide further inquiry.

It is worthy to highlight the fact that the present study represents a research effort to clarify the current situation of teaching statistics at secondary school in Venezuela, focusing of the statistical knowledge for teaching, conceptions of variability, and statistics-related beliefs held by mathematics teachers. Thus, this study is, to a certain extent, a research on the current situation of the Venezuelan secondary school mathematics curriculum at intended and implemented levels. However, from the findings of this study, it is clear some aspects of "what should be" the statistics education in Venezuela, in particular, from which implications for improving the intended and implemented secondary school mathematics curriculum could be drawn. Such implications are discussed in the section "Implications", at the end of this chapter.

6.2.3 Answer to the Third Research Question: *How do Venezuelan secondary school mathematics teachers conceptualize variability, and what beliefs about statistics, its teaching and learning, do they have?*

González (2011) and Isoda and González (2012) have shown empirically that mathematics teachers' conceptions of variability can be made explicit by solving tasks in which knowledge and understanding of variability-related ideas, as well as the ability to connect and represent them, are required. Based on this fact, it is anticipated that teachers' answers to Question (a) will provide enough information about how the respondents conceptualize variability, since such cognitive demands and skills are required to answer correctly the task posed in the survey questionnaire. These conceptions of variability that might be distinguished in teachers' answers were classified, at first, using the eight-type categorization proposed by Shaughnessy (2007, pp. 984–985). Later, since not all the responses from teachers fitted into one of those 8 descriptions of variability—in particular, those from teachers providing responses focused on visual cues in the graph—, different conceptions of variability to those identified by Shaughnessy (2007) were found.

In summary, the ways in which Venezuelan secondary school mathematics teachers seem to conceptualize variability, when dealing with a task in the particular context of histograms and comparing distributions, are the following:

Variability in particular values: This conception is characterized by the person focusing on individual data values, such as the extremes of the distribution to calculate the range. In the present study, 10 teachers—18.8% overall, 4 working at lower high school, 1 at upper high school, and 5 at both levels—seem to be holding this conception.

Variability as distance or difference from some fixed point: Teachers who seem to think of variability as either a visual measurement of the distance of each or some elements of a dataset from some measure of center, or an actual measurement from the endpoints to some measure of center, show evidence of holding this conception. In the present study, 9 teachers—17.0% overall, 4 working at lower high school, 4 at upper high school, and 1 at both levels—seem to be holding this kind of conception.

Variability as the sum of residuals: Teachers who measured variability through deviation-based metrics, such as the mean absolute deviation, sum of residuals or averages of the absolute value differences from a measure of center, show evidence of harboring this conception. In the present study, 6 teachers—11.3% overall, 1 working at lower high school, 4 at upper high school, and 1 at both levels—seem to be holding this conception.

Variation as distribution: Teachers whose answers involve transnumeration, as well as use of theoretical properties of the histograms to calculate numerically the measures of central tendency and variation, show evidence of holding this conception. Then, 10 teachers in the present study—18.8% overall, 2 working at lower high school, 6 at upper high school, and

2 at both levels—seem to hold this conception.

Variability as visual cues in the graph: Teachers whose responses revealed misconceptions such as thinking of variability in terms of symmetry or degree of fit—or lack thereof—to a normal distribution, or thinking of symmetrical or quasi-normal distributions as having less or more variability than its asymmetrical counterparts, show evidence of holding the conception. In the present study, 17 teachers—32.1% overall, 4 working at lower high school, 3 at upper high school, and 10 at both levels—seem to hold this conception.

In the case of teachers' beliefs about statistics teaching and learning, they were identified from particular features of the lesson plans that teachers produce (cf. Eichler, 2008, 2011; Pierce & Chick, 2011). Therefore, answers to Question (g) were analyzed in order to identify the four categories reflecting on teachers' beliefs developed by Eichler (2011)—i.e., *traditionalists*, *application preparers*, *everyday life preparers*, and *structuralists*. As result, none of the participants in the present study was categorized as a *structuralist*; 8 teachers—15.1% overall, 2 working at lower high school, 2 teachers working at upper high school, and 4 teachers working at both levels—could be regarded as *application preparers*; 18 teachers—34.0% overall, 10 working at lower high school, 4 teachers working at upper high school, and 4 teachers working at both levels—could be considered as *everyday-life preparers*; and 18 teachers—34.0% overall, 5 working at lower high school, 7 teachers working at upper high school, and 6 teachers working at both levels—could be regarded as *traditionalists*.

6.3 Implications

The results of the present study have wide implications for classroom implementation, teacher education and teacher training in the teaching of statistics at secondary school level in Venezuela. First and foremost, the intended curriculum might require some obvious modifications. As pointed out during the analysis of the verbs used in the entire Venezuelan mathematics curriculum for secondary school in Chapter 2—see Table 5—, Venezuelan secondary school mathematics curriculum does not seem to promote critical thinking—which is stressed in the definition of statistical literacy—, and focus on knowledge, comprehension and/or application instead of higher order thinking skills. These skills, sometimes regarded in the literature as critical thinking skills, allow students to think convergently and divergently to investigate challenges and problems, as well as to think in complex and creative ways rather than in a linear fashion (O'Tuel & Bullard, 1995; Resnick, 1987; Tishman et al., 1993; Garfield & Ben-Zvi, 2008). Furthermore, it was found that, even though there are many instructional contents related to statistics in the Diversified cycle of secondary school in Venezuela—see Table 4—, there is no overall objective related to the learning of such contents, but only to the probability-related ones.

In addition to the intended curriculum issues, Venezuelan teachers' knowledge base of *statistical knowledge for teaching* may not be adequate. More importantly, from the results obtained here one can conclude that teachers may not have the techniques, the required conceptions, the affective grounds, or a suitable instructional approach to use for effective teaching of statistics, specifically those contents where variability may arise. For this reason, an effort is needed to thoroughly teach statistics to future teachers, and to give to practicing teachers the tools—both cognitive and affective—required to teach successfully the statistical contents included in the Venezuelan secondary school mathematics curriculum, according to the recommendations and research findings reported in the specialized literature on statistics education. Such conscious effort involves, first and foremost, shifting teachers' attention to the variability instead of to measures of central tendency and the shape of the graph, fact that was corroborated in the results obtained through the implementation of the survey questionnaire. Also, it might be required to provide Venezuelan prospective and in-service mathematics teachers with a solid cognitive and pedagogical grounding in the statistical topics they must teach at school. In order to do that, teacher training programs must stress statistical contents, especially those where variability arises, with the purpose of equipping teachers with effective instructional strategies and sophisticated conceptions of variability to teach statistics effectively.

The active and directive role played by the teacher in the classroom is the most important characteristic of the direct instruction, since teacher "tells, shows, models, demonstrates, teaches the skills to be learned" (Baumann, 1988, p.714). So, teacher education programs might have be in an urgent need to inculcate in teachers awareness, skills and understanding of variability and other fundamental statistical ideas, in order to build and improve teachers' statistical literacy skills, such as data analysis strategies, data representation and critical thinking. Without such awareness, conceptions, and skills, the teaching of statistics in Venezuela at school level is unlikely to fully develop its potential, and the mathematics curriculum may not be achieved successfully.

In addition to the aforementioned implications, future research projects may further examine many of the relations between the eight components of mathematics teachers' professional competencies to teach efficiently statistics identified by this study that seem to emerge from the analysis of the collected data. Such future studies will contribute to improve our current understanding of such intertwinements, and would guide further inquiry.

6.4 Limitations

A possible limitation of the present investigation might be found in the implementation of the survey. Such implementation was limited to junior and senior high schools in the metropolitan area of Caracas, the capital city of Venezuela, and hence the obtained results and drawn conclusions may not represent the true characteristics of the whole population of Venezuelan middle and high school teachers. Another limitation was that participation was voluntary, which could have affected the total number of participants in the present research.

Other possible constraint of the study was the surveyed contents, because of the possible unfamiliarity with statistics of some respondents or possible participants. As discussed in Chapter 2, a research carried out by Salcedo (2008) over secondary school mathematics teachers working in the Venezuelan Capital District, shows that most of the participants were not teaching contents related to statistics and probability, and the few who did, tended to ask students for a written report about definitions of statistical concepts. Furthermore, most of the teachers in Salcedo's study admitted the lack of training to work with statistical topics. This likely unfamiliarity with the contents, as well as teachers' perceived lack of preparation to teach statistics, might affect survey results and participation rate, since when being asked to fill out a survey, the respondent's decision will be positive when he or she considers a survey a pleasant activity (survey enjoyment), which produces useful (survey value) and reliable (survey reliability) results and when the perceived cost of cooperation in the interview (time and cognitive efforts = survey cost) and impact on privacy (survey privacy) are minimal (Loosveldt & Storms, 2008). So, even though the impact on privacy is minimal for the current survey instrument, the survey cost would be high for some teachers, since statistics is a topic that seems to be avoided by many Venezuelan teachers, and a lot of them may have felt uncomfortable participating in this research, generating a minimal survey enjoyment and consequently avoiding taking the survey or incompletely filling it in.

Another limitation could be that, to the knowledge of the researcher, there are no previous studies in Venezuela which tried to answer the research questions posed by the present study, so the results presented here cannot be largely corroborated by triangulation with results from other research efforts done in the country, even though some results were corroborated with previous researches in other countries, but targeted to different populations. As is used here, triangulation is a process that can be used to judge and enhance the reliability of research findings by seeking a convergence of results using multiple methods, investigators, data sources, or theoretical lenses (Green, Caracelli, & Graham, 1989; Tashakkori & Teddlie, 1998). However, results in this study may be used with reasonable reliability (as done above) to explore how Venezuelan in-service secondary school mathematics teachers acknowledge variability in a setting of comparing distributions of data, what kind of beliefs towards teaching and learning of statistics they might hold, and what knowledge base of *statistical knowledge for teaching*, or lack thereof, they might have about the instruction of statistical contents related to variability in data.

The fact that this study is based on one researcher's interpretations of data collected from survey questionnaires might represent a limitation. Other interpretations may be possible.

Finally, maybe the biggest limitation of the present study was not interviewing the participants—or a sub-sample of them—after carrying out the survey. This would have allowed going deeper into the understanding of some traits identified in the framework for SKT proposed here.

6.5 Suggestions for future research

There are many directions for future research that could enable a greater understanding of Venezuelan secondary school mathematics teachers' knowledge base of *statistical knowledge for teaching*, and what features of their *statistical knowledge for teaching*, conceptions of variability and statistics-related beliefs could result in gains in student achievement. A natural next step for this research would be to organize and conduct a seminar or teaching experiment using the same task chosen for the present research, in order to study the effects of such an experience on teachers' SKT. A possible way could be a comparison study between seminar participants and non-participants and their students' achievement. By conducting this comparative study, it could be possible to determine whether the seminar or teaching experiment was effective in improving mathematics teachers' SKT, and, subsequently, their students' statistical knowledge and skills related to statistical literacy.

Even though this study has provided some valuable insights into how Venezuelan secondary school mathematics teachers acknowledge, think, describe and teach variability-related contents, we still need to learn a lot more about this area. One limitation of the study is that it was focused on Venezuelan in-service mathematics teachers in Caracas, which have been reported as having a loose grounding in statistics as well as a negative self-perception of their competence to teach statistics at secondary school level (Salcedo, 2008). Similar studies might be carried out in the future over different groups—such as pre-service teachers and elementary school mathematics teachers—in order to broaden the investigation of Venezuelan mathematics teachers' SKT at other school levels. Also, a similar study targeting in-service teachers at other cities and towns across Venezuela could be conducted.

This research also gained insight into Venezuelan in-service secondary school mathematics teachers' knowledge base of *statistical knowledge for teaching*, or lack thereof, about several intertwined variability-related ideas in the particular setting of comparing distributions of data. Future researches may use the results obtained in this study to find ways to create learning environments that facilitate deeper understanding of variability in the given setting, as well as of those statistical ideas in which surveyed teachers exhibited difficulties, lack of knowledge or misconceptions. Also, future research efforts may investigate ways that could help teachers—and even students—develop sophisticated conceptions of variability, and use them appropriately depending on the statistical context.

Finally, due to the strong correlation between beliefs and values (cf. Sjöberg & Winroth, 1986; Philipp, 2007), a future implementation of the present study might target the values held by in-service teachers in relation to the teaching and learning of statistics.

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