Doctoral Dissertation

Frequency Domain Causality Test and Its Applications

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Graduate School of Social Sciences
Hiroshima University

Yanfeng Wei

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Advisor: Professor Hiroshi Yamada

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Introduction

In the last decades, the study of causal relationships among a set of stationary variables has been one of the most important topics in the economic literature. A variety of causality tests including Sims (1972), Pierce and Haugh (1977), Geweke (1982), Boudjellaba, Dufour and Roy (1992), Gfillenzoni (1996) and Hidalgo (2005) have been developed, inspired by the causality definition originally proposed by Granger (1969). These causality tests are mainly constructed in the time domain, though a spectral approach is also recommended by Granger (1969).

Geweke (1982) awared that the causal relationships between two variables can be quite different across the spectra and proposed a frequency domain, directional measure of linear feedback between two variables in the context of vector autoregressive (VAR) models. Hosoya (1991) also introduced a similar way to measure the interdependency among the variables in the frequency domain. This measure can be viewed as an extension of the Geweke (1982) measure. However, further establishing a causality test statistic under the frequency domain is proved to be complicated due to nonlinearities. Fortunately, this difficulty was recently solved by Breitung and Candelon (2006). They proposed a simple test for causality under the frequency domain. Their test is based on a set of linear restrictions on the parameters of the VAR model, which is as simple as the conventional Granger causality tests. In this thesis, we primarily focus on the frequency domain causality test and its empirical applications.

The organization of this thesis is as follows. In Chapter 1, we attempt to clarify whether the frequency domain causality test really suffers from strictly low power at the frequencies close to 0 or π. In the original paper of Breitung and Candelon (2006), they document that their test severely suffers from extremely low power when the non-causality hypothesis is tested at a frequency close to 0 or π. Because the causalities at the low and high frequencies are corresponding to the long- and short-run causal relations, the test will be less attractive to the economists if the power properties of the test reported by Breitung and Candelon (2006) are true. In this Chapter, we introduce two data generating processes (DGPs) and construct two Wald test statistics which asymptotically have a noncentral chi-squared distribution. The theoretical analysis of the two Wald test statistics indicates that this frequency domain causality test does not suffer from low power at frequencies close to 0 or π. We also conduct Monte Carlo simulation experiments to investigate the finite sample properties of the two Wald test statistics, which further confirm our analysis results related to the power of the test.

In Chapter 2, using the frequency domain causality test of Breitung and Candelon (2006) and the frequency dependent regression method proposed by Ashley and Verbrugge (2009), we investigate the dynamic relationships between oil prices and the Japanese economy. Several researchers have already analyzed the oil price–macroeconomy relationship for Japan (Hutchison, 1993; Mork *et al.*, 1994; Lee *et al.*, 2001; Cunado and Gracia, 2005; Zhang, 2008). However, these analyses are mainly concentrated on the time domain and thus fails to uncover the relationships between oil prices and the selected macroeconomic variables at different frequencies. This can be overcome by the frequency domain causality test and the frequency dependent regression method. In this Chapter, the frequency dependent regression analysis indicates that nonlinear relationships exist between oil prices and the variables such as industrial production and consumer price index at the low frequencies, while the nonlinear associations at the high frequencies are merely detected between oil prices and unemployment rates. The results of the frequency domain causality tests suggest that oil prices have significant predictive power for industrial production, consumer price index and unemployment rates at the low frequencies. In addition, oil prices can predict industrial production and unemployment rates at some high frequencies.

In Chapter 3, we evaluate the composite leading indicator component series for Japan. To forecast the business climate of Japan, the Organization for Economic Cooperation and Development (OECD) and the Japanese government independently provide composite leading indicator (CLI) series. Both sets of CLI are calculated using component series. In this Chapter, several methods are employed to assess these component series. Specifically, we first use the Hodrick and Prescott (1997) (HP) high-pass filter to remove the long-term trend from the series. We then apply the frequency domain causality test to analyze the causality running from each component series to the reference series. Finally, we assess the forecasting performance of the component series using the method of Bry and Boschan (1971). We utilize the OECD Cyclical Analysis and Composite Indicators System (CACIS) software to this purpose.

In Chapter 4, we employ the frequency domain causality test to investigate whether commodity prices are useful in formulating monetary policy. There is by now substantial evidence in the literature that commodity prices can signal the future movements in the economy and thus are useful in setting monetary policy (Cody and Mills, 1991; Awokuse and Yang, 2003; Bhar and Hamori, 2008). However, the literature does not clarify if this relationship is stable over time. In this Chapter, we apply the Toda and Yamamoto (1995) procedure to establish standard inference for the frequency domain causality test. Using the monthly U.S. data from January 1957 to December 2011, we find the frequency domain causal relationship between non-oil commodity prices and economic activities (*e.g.*, consumer prices and industrial production) has changed dramatically over time. Our results indicate that the non-oil commodity prices are useful in setting monetary policy in the 1970s and the beginning of 1980s, but the usefulness has disappeared completely since the early 1980s. In contrast, the oil price or the commodity price index including oil price can still be used as an informational variable for managing monetary policy after the early 1980s.

Finally, in Chapter 5, we apply the frequency domain causality test to analyze the predictive power of commodity prices and manufactured goods prices for inflation of Japan. Commodity prices are generally considered to have more predictive power for inflation than do the manufactured goods prices (Bordo, 1980; Garner, 1989; Cody and Mills, 1991). However, some recent studies find that the predictive power of commodity prices for inflation has significantly decreased since the mid-1980s (Herrera and Pesavento, 2009; Verheyen, 2010). In this case, using the monthly Japanese data from January 1970 to December 2011, this Chapter attempts to analyze the predictive power of commodity prices and manufactured goods prices for inflation. Because the Japanese economy suffers from structural changes in the early 1990s (Sato, 2002; Fang and Miller, 2009; Yamada and Jin, 2012), we split the full sample into the two sub-periods 1970M1-1990M12 and 1991M1-2011M12. Like in Chapter4, we employ the Toda and Yamamoto (1995) procedure to establish standard inference for the test. By testing the causality at various frequencies, we find that in recent years commodity prices can only predict the long-term fluctuations of inflation, while manufactured goods prices are significant in forecasting inflation in both the short- and long-term periods.

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Chapter 1

Some Theoretical and Simulation Results on the Frequency Domain Causality Test

1.1 Introduction

Breitung and Candelon (2006) recently proposed a statistical testing procedure for the noncausality hypothesis at a given frequency (ω_0). This procedure has steadily been gaining attention among macroeconometricians. This is partly because the procedure enables us to see how the causal relations between macroeconomic variables vary according to frequency, and partly because, as in the case of the conventional Granger noncausality test, their test is based on a set of linear restrictions on the coefficients of the vector autoregressive (VAR) model. Examples of papers applying their procedure include Assenmacher-Wesche and Gerlach (2008), Bodart and Candelon (2009), Gronwald (2009), Schreiber (2009), Mermod *et al.* (2010), and Ciner (2011a, 2011b).

Breitung and Candelon (2006) reported both theoretical and simulation results on the power properties of their test. To evaluate the properties theoretically, they employed a bivariate stationary VAR(3) model as follows:

$$x_t = \alpha y_{t-1} - 2\alpha \cos\left(\omega_0 + \frac{c}{\sqrt{T}}\right) y_{t-2} + \alpha y_{t-3} + u_t, \quad y_t = v_t, \quad t = 1, \ldots, T.$$

In this model, as explained in Sections 1.2 and 1.3 of this paper, y does not cause x at frequency ω_0 when $c = 0$. They showed that when ω_0 is close to 0 or π, the power of the test is close to size even when $c \neq 0$.

Does this result indicate that the test of Breitung and Candelon (2006) is useless when ω_0 is close to 0 or π? If it does, then the test procedure is less attractive for use by macroeconometricians. In this paper, we try to provide both theoretical and simulation evidence to answer this question. We argue that the result stated above depends on the model treated and we show, with slightly different models, that the test does not necessarily suffer from low power at such frequencies. The conclusion of this paper is that the test of Breitung and Candelon (2006) is still useful even at such frequencies.

The structure of this paper is as follows. In Section 1.2, we briefly review the test proposed in Breitung and Candelon (2006). In Section 1.3, we show some theoretical results on the test and in Section 1.4 we report some results obtained from Monte Carlo simulation experiments. Section 1.5 concludes this paper.

The notation that we use in this paper is as follows. We use $\xrightarrow{d}$ and $\xrightarrow{p}$ to signify convergence in distribution and convergence in probability, respectively. L is the lag operator and $\chi^2_n(m)$ represents noncentral χ^2 distribution with n degrees of freedom and noncentrality parameter m. iid denotes independent and identically distributed. $\boldsymbol{I}_m$ is the identity matrix of size m and $\boldsymbol{Q}_{\boldsymbol{X}} = \boldsymbol{I}_T - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$.

1.2 Frequency domain causality test

In this section, we briefly review the statistical testing procedure proposed in Breitung and Candelon (2006). Let us suppose that a bivariate time series $[x_t, y_t]'$ is generated by the following stationary VAR(p) model:[1]

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}, \quad t = 1, \ldots, T, \tag{1.1}$$

where $\theta_{ij}(L) = \theta_{ij,1}L^0 + \cdots + \theta_{ij,p}L^{p-1}$ for $i, j = 1, 2$, and $[u_t, v_t]' \sim \text{iid}(\mathbf{0}, \boldsymbol{\Sigma})$. Here, $\boldsymbol{\Sigma}$ is positive-definite and let $\boldsymbol{G}$ be the lower triangular matrix of the Cholesky decomposition $\boldsymbol{G}'\boldsymbol{G} = \boldsymbol{\Sigma}^{-1}$. In addition, $[\varepsilon_t, \eta_t]'$ is defined as $[\varepsilon_t, \eta_t]' = \boldsymbol{G}[u_t, v_t]'$ and $\psi_{ij}(L)$ for $i, j = 1, 2$ are defined accordingly.

[1]Breitung and Candelon (2006) proposed the frequency domain causality test not only within a stationary VAR framework but also within a cointegrated VAR framework. However, we deal only with a stationary VAR model, because their results on the power of their test are based on a stationary VAR model.

Then, the population spectrum of x, denoted by $f_x(\omega)$, can be expressed as

$$f_x(\omega) = \frac{1}{2\pi}(|\psi_{11}(e^{-\mathrm{i}\omega})|^2 + |\psi_{12}(e^{-\mathrm{i}\omega})|^2).$$

The measure of causality suggested by Geweke (1982) and Hosoya (1991) is defined as

$$M_{y\to x}(\omega_0) = \log\left[\frac{2\pi f_x(\omega_0)}{|\psi_{11}(e^{-\mathrm{i}\omega_0})|^2}\right] = \log\left[1 + \frac{|\psi_{12}(e^{-\mathrm{i}\omega_0})|^2}{|\psi_{11}(e^{-\mathrm{i}\omega_0})|^2}\right]. \tag{1.2}$$

When $M_{y\to x}(\omega_0) = 0$, there is no causality from y to x at frequency ω_0.

Let us denote $\boldsymbol{\beta} = [\theta_{12,1}, \ldots, \theta_{12,p}]'$. Breitung and Candelon (2006) showed that the hypothesis expressed by $M_{y\to x}(\omega_0) = 0$ is equivalent to the hypothesis expressed by a linear restriction on the VAR coefficients $H_0 : \boldsymbol{R}\boldsymbol{\beta} = \boldsymbol{0}$, where

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega_0) & \cdots & \cos(p\omega_0) \\ \sin(\omega_0) & \cdots & \sin(p\omega_0) \end{bmatrix}.$$

Similarly to the conventional Granger causality test, this hypothesis, $H_0 : \boldsymbol{R}\boldsymbol{\beta} = \boldsymbol{0}$, can be tested using a Wald test statistic such as

$$W = (\boldsymbol{R}\hat{\boldsymbol{\beta}})'[\hat{\sigma}_u^2 \boldsymbol{R}(\boldsymbol{Z}_2'\boldsymbol{Q}_{\boldsymbol{Z}_1}\boldsymbol{Z}_2)^{-1}\boldsymbol{R}']^{-1}\boldsymbol{R}\hat{\boldsymbol{\beta}}, \tag{1.3}$$

where $\hat{\boldsymbol{\beta}} = (\boldsymbol{Z}_2'\boldsymbol{Q}_{\boldsymbol{Z}_1}\boldsymbol{Z}_2)^{-1}\boldsymbol{Z}_2'\boldsymbol{Q}_{\boldsymbol{Z}_1}\boldsymbol{x}$ and $\hat{\sigma}_u^2 = \hat{\boldsymbol{u}}'\hat{\boldsymbol{u}}/(T-p)$ with $\hat{\boldsymbol{u}} = \boldsymbol{Q}_{\boldsymbol{Z}_1}\boldsymbol{x} - \boldsymbol{Q}_{\boldsymbol{Z}_1}\boldsymbol{Z}_2\hat{\boldsymbol{\beta}}$. Here, $\boldsymbol{x} = [x_{p+1}, \ldots, x_T]'$, $\boldsymbol{Z}_i = [\boldsymbol{z}_{i,p+1}, \ldots, \boldsymbol{z}_{i,T}]'$ for $i = 1, 2$ with $\boldsymbol{z}_{1,t}' = [x_{t-1}, \ldots, x_{t-p}]$ and $\boldsymbol{z}_{2,t}' = [y_{t-1}, \ldots, y_{t-p}]$. When $H_0 : \boldsymbol{R}\boldsymbol{\beta} = \boldsymbol{0}$ is true, this Wald test statistic is asymptotically distributed as $\chi_2^2(0)$ for $\omega_0 \in (0, \pi)$.

1.3 Power of the test

To evaluate the power properties of the test, for simplicity, Breitung and Candelon (2006) studied the following statistic, W^*, instead of W given in Eq. (1.3):

$$W^* = (\boldsymbol{R}\tilde{\boldsymbol{\beta}})'[\tilde{\sigma}_u^2 \boldsymbol{R}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{R}']^{-1}\boldsymbol{R}\tilde{\boldsymbol{\beta}}, \tag{1.4}$$

where $\tilde{\boldsymbol{\beta}} = (\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{Z}_2'\boldsymbol{x}$ and $\tilde{\sigma}_u^2 = \tilde{\boldsymbol{u}}'\tilde{\boldsymbol{u}}/(T-p)$ with $\tilde{\boldsymbol{u}} = \boldsymbol{x} - \boldsymbol{Z}_2\tilde{\boldsymbol{\beta}}$.

When $p = 3$, $\boldsymbol{\beta} \neq \mathbf{0}$ such that $\boldsymbol{R\beta} = \mathbf{0}$ is given as[2]

$$\boldsymbol{\beta} = \alpha[1, -2\cos(\omega_0), 1]', \tag{1.5}$$

where $\alpha \neq 0$. Accordingly, if

$$\theta_{12}(L) = \alpha[1 - 2\cos(\omega_0)L + L^2], \tag{1.6}$$

there is no causality from y to x at frequency ω_0.

Breitung and Candelon (2006) employed

$$\theta_{12}(L) = \alpha \left[1 - 2\cos\left(\omega_0 + \frac{c}{\sqrt{T}}\right) L + L^2\right] \tag{1.7}$$

and obtained the following proposition.

Proposition 1. *Suppose that in Eq.* (1.1) $\boldsymbol{\Sigma} = diag(\sigma_u^2, \sigma_v^2)$, $\theta_{11}(L) = \theta_{21}(L) = \theta_{22}(L) = 0$ *and* $\theta_{12}(L)$ *is given as in Eq.* (1.7). *Under these conditions,* $W^* \xrightarrow{d} \chi_2^2(\lambda^2)$ *for* $\omega_0 \in (0, \pi)$, *where*

$$\lambda = \frac{\sigma_v[2c\alpha\sin(\omega_0)]}{\sigma_u\sqrt{1 + 2\cos(\omega_0)^2}}. \tag{1.8}$$

Proof: See Breitung and Candelon (2006, pp. 370–371).

Because the noncentrality parameter λ^2 converges to 0 when ω_0 converges to 0 or π, Breitung and Candelon (2006) pointed out that when ω_0 is close to 0 or π, the power of the test is close to size, which indicates the test severely suffers from quite low power at such frequencies at least for this model.

Does this result indicate that this testing procedure is always useless when the hypothesis is no causality at a frequency close to 0 or π? To answer this question, we examine a slightly different model:

$$\theta_{12}(L) = \alpha \left[1 - 2\left(\cos(\omega_0) + \frac{c}{\sqrt{T}}\right) L + L^2\right]. \tag{1.9}$$

Although these two $\theta_{12}(L)$s, which are given by Eq. (1.7) and Eq. (1.9), are very similar, they are quite different in the following point. When ω_0 is close to 0 or π and $c/\sqrt{T}$ is

[2] See Appendix for details.

small, $\cos(\omega_0 + c/\sqrt{T})$ in Eq. (1.7) is nearly equal to $\cos(\omega_0)$[3], then we can see

$$\alpha\left[1 - 2\cos\left(\omega_0 + \frac{c}{\sqrt{T}}\right)L + L^2\right] \approx \alpha[1 - 2\cos(\omega_0)L + L^2].$$

In contrast, in Eq. (1.9), for $c \neq 0$, obviously,

$$\alpha\left[1 - 2\left(\cos(\omega_0) + \frac{c}{\sqrt{T}}\right)L + L^2\right] \neq \alpha[1 - 2\cos(\omega_0)L + L^2].$$

Hence, we can expect that with Eq. (1.9), the test does not suffer severely from quite low power even when ω_0 is close to 0 or π.

As expected, with Eq. (1.9), we obtained the following proposition.

Proposition 2. *Suppose that in Eq.* (1.1) $\mathbf{\Sigma} = diag(\sigma_u^2, \sigma_v^2)$, $\theta_{11}(L) = \theta_{21}(L) = \theta_{22}(L) = 0$ *and* $\theta_{12}(L)$ *is given as in Eq.* (1.9). *Under these conditions,* $W^* \xrightarrow{d} \chi_2^2(\delta^2)$ *for* $\omega_0 \in (0, \pi)$, *where*

$$\delta = -\frac{\sigma_v(2c\alpha)}{\sigma_u\sqrt{1 + 2\cos(\omega_0)^2}}. \tag{1.10}$$

Proof: See Appendix.

It is noteworthy that the noncentrality parameter, here denoted δ^2, no longer depends on $\sin(\omega_0)$. This means that in this model the power of the Wald test statistic is not close to size even when ω_0 is close to 0 or π. To gain further insights on the power properties of the test, we also tried another slightly different model:

$$\theta_{12}(L) = \alpha\left[\left(1 + \frac{c}{\sqrt{T}}\right) - 2\cos(\omega_0)L + \left(1 + \frac{c}{\sqrt{T}}\right)L^2\right]. \tag{1.11}$$

This model is interesting because when $\omega_0 = \pi/2$,

$$\alpha\left[\left(1 + \frac{c}{\sqrt{T}}\right) - 2\cos(\omega_0)L + \left(1 + \frac{c}{\sqrt{T}}\right)L^2\right] = \alpha^*[1 - 2\cos(\omega_0)L + L^2], \tag{1.12}$$

where $\alpha^* = \alpha(1 + c/\sqrt{T})$. That is, when $\theta_{12}(L)$ is given as in Eq. (1.11), y does not cause x at $\omega_0 = \pi/2$ even when $c \neq 0$.

The asymptotics of the test statistic, W, under Eq. (1.11) are as follows.

Proposition 3. *Suppose that in Eq.* (1.1) $\mathbf{\Sigma} = diag(\sigma_u^2, \sigma_v^2)$, $\theta_{11}(L) = \theta_{21}(L) = \theta_{22}(L) = 0$

[3]When $c/\sqrt{T}$ is small, $\cos(\omega_0 + c/\sqrt{T}) \approx \cos(\omega_0) - (c/\sqrt{T})\sin(\omega_0)$ and, in addition, when ω_0 is close to 0 or π, $\sin(\omega_0) \approx 0$. Then, when ω_0 is close to 0 or π and $c/\sqrt{T}$ is small, we see $\cos(\omega_0 + c/\sqrt{T}) \approx \cos(\omega_0)$.

and $\theta_{12}(L)$ *is given as in Eq.* (1.11)*. Under these conditions,* $W^* \xrightarrow{d} \chi^2_2(\gamma^2)$ *for* $\omega_0 \in (0, \pi)$*, where*

$$\gamma = \frac{\sigma_v[2c\alpha\cos(\omega_0)]}{\sigma_u\sqrt{1 + 2\cos(\omega_0)^2}}. \tag{1.13}$$

Proof: Proof of this proposition is omitted because it is entirely analogous to the proof of Proposition 2.

Note that $\sin(\omega_0)$, which appeared in the noncentrality parameter λ^2, is replaced with $\cos(\omega_0)$ in the noncentrality parameter γ^2. Then, as expected, we can see that the noncentrality parameter γ^2 is 0 when $\omega_0 = \pi/2$ and at the frequencies around $\pi/2$, the noncentrality parameter γ^2 is relatively small. We further note that the power of the test is high when ω_0 is close to 0 or π.

Here, we would like to give remarks, which could be helpful to get a deeper understanding on the relations among the propositions stated above[4]. Because $\cos(\omega_0 + c/\sqrt{T}) \approx \cos(\omega_0) - c\sin(\omega_0)/\sqrt{T}$, $\theta_{12}(L)$ in Eq. (1.7) can be represented as

$$\begin{aligned}\theta_{12}(L) &= \alpha\left[1 - 2\cos\left(\omega_0 + \frac{c}{\sqrt{T}}\right)L + L^2\right] \\ &\approx \alpha\left[1 - 2\left(\cos(\omega_0) + \frac{c^\dagger}{\sqrt{T}}\right)L + L^2\right],\end{aligned} \tag{1.14}$$

where $c^\dagger = -c\sin(\omega_0)$. From this representation, we can expect the results stated in Proposition 2 from those of Proposition 1. That is to say, by replacing $c\sin(\omega_0)$ in Eq. (1.8) with $-c^\dagger$, we obtain the noncentrality parameter given in Eq. (1.10).

Similarly, we can expect the results of Proposition 3 from those of Proposition 1 as follows. Let $c^\ddagger = c\sin(\omega_0)/\cos(\omega_0)$. Then, using the relation given by $(1 - c^\ddagger/\sqrt{T})^{-1} \approx (1 + c^\ddagger/\sqrt{T})$, $\theta_{12}(L)$ in Eq. (1.7) can also be expressed with $c^\ddagger$ as

$$\begin{aligned}\theta_{12}(L) &= \alpha\left[1 - 2\cos\left(\omega_0 + \frac{c}{\sqrt{T}}\right)L + L^2\right] \\ &\approx \alpha\left(1 - \frac{c^\ddagger}{\sqrt{T}}\right)\left[\left(1 + \frac{c^\ddagger}{\sqrt{T}}\right) - 2\cos(\omega_0)L + \left(1 + \frac{c^\ddagger}{\sqrt{T}}\right)L^2\right].\end{aligned} \tag{1.15}$$

By replacing $c\sin(\omega_0)$ in Eq. (1.8) with $c^\ddagger\cos(\omega_0)$, we obtain the noncentrality parameter given in Eq. (1.13).

[4]These are pointed out by one of the anonymous referees. We appreciate these valuable comments.

1.4 Monte Carlo simulation experiments

In this section, we conduct Monte Carlo simulation experiments to investigate the finite sample properties of the test statistic, W, given in Eq. (1.3). We generate data using the following data-generating process (DGP):

$$x_t = \theta_{12}(L)y_{t-1} + u_t, \quad y_t = v_t, \quad [u_t, v_t]' \sim \text{iid}N(\mathbf{0}, \boldsymbol{I}_2), \quad t = 1, \ldots, T, \tag{1.16}$$

where

$$\text{DGP1: } \theta_{12}(L) = 1 - 2\cos(\omega_0 + c^*)L + L^2, \tag{1.17}$$

$$\text{DGP2: } \theta_{12}(L) = 1 - 2[\cos(\omega_0) + c^*]L + L^2. \tag{1.18}$$

We set the values of ω_0, c and T as[5]

$$\omega_0 = \frac{\pi}{8}, \frac{2\pi}{8}, \cdots, \frac{7\pi}{8}, \; c^* = -0.3, -0.2, \ldots, 0.3, \text{ and } T = 50, 100, 200.$$

For simulation experiments, we calculated rejection frequencies based on 30,000 replications. All computations were performed by Matlab with `randn` function. Nominal size was set equal to 0.05 and we assumed that the lag order of the VAR model is known.[6]

Before explaining the simulation results, we briefly examine the properties of the two DGPs above. In these two DGPs, note that $\psi_{ij}(L)$ $(i, j = 1, 2)$ of Eq. (1.1) are respectively expressed as

$$\psi_{11}(L) = 1, \quad \psi_{12}(L) = \theta_{12}(L)L, \quad \psi_{21}(L) = 0, \quad \psi_{22}(L) = 1,$$

and then $[x_t, y_t]'$ follows a bivariate vector moving-average model of order 3. To interpret the simulation results, the values of the Geweke measure, given by Eq. (1.2), are useful. For DGP1, $\psi_{11}(e^{-\mathrm{i}\omega_0}) = 1$ and because when c^* is small $\cos(\omega_0 + c^*) \approx \cos(\omega_0) - c^* \sin(\omega_0)$, then $\psi_{12}(e^{-\mathrm{i}\omega_0}) \approx 2c^* \sin(\omega_0)e^{-\mathrm{i}2\omega_0}$. Accordingly, we can see that the Geweke measure for

[5]Because $\pi/8 \approx 0.4$, $0 < (\omega_0 + c^*) < \pi$ in this setting.

[6]This assumption is convenient for our purpose, but practically the lag order selection for VAR model is important. Lemmens *et al.* (2008) reported some interesting simulation results assuming the lag order is unknown.

DGP1 is

$$M_{y\to x}(\omega_0) \approx \log[1+4c^{*2}\sin(\omega_0)^2]. \tag{1.19}$$

Similarly, we can see that the measure for DGP2 is

$$M_{y\to x}(\omega_0) = \log(1+4c^{*2}). \tag{1.20}$$

Because $\omega_0 \in (0,\pi)$, in both DGPs, the null of noncausality at frequency ω_0 is true only when $c^* = 0$. However, it should be noted that for DGP1, as depicted in Panel A of Figure 1.1, when ω_0 is close to 0 or π, the Geweke measure is close to 0. Accordingly, it can be expected that when ω_0 is close to 0 or π, the rejection frequencies are quite low. On the other hand, for DGP2 the measure no longer depends on ω_0 (see Panel C of Figure 1.1). Together with the noncentrality parameter, δ^2, given in Eq. (1.10), we can expect that the rejection frequencies do not heavily depend on the values of ω_0. (Note that because δ^2 is relatively small when ω_0 is close to 0 or π, the rejection frequencies could be relatively small for such frequencies.)

Tables 1.1 and 1.2 set out the rejection frequencies of the test. From these tables, we can observe that, as expected, for the case of DGP1, (i) when ω_0 is close to 0 or π, the rejection frequencies are relatively low, whereas (ii) when ω_0 equals $\pi/2$, the corresponding rejection frequencies are relatively high. In addition, it is notable that (iii) when $c^* < 0$ ($c^* > 0$), the rejection frequencies corresponding to $\omega_0 = \pi/8$ ($\omega_0 = 7\pi/8$) are lower than those corresponding to $\omega_0 = 7\pi/8$ ($\omega_0 = \pi/8$). For the case of DGP2, (iv) we can observe that the rejection frequencies do not heavily depend on the values of ω_0. Furthermore, for both DGPs, we can observe that, as reported in Breitung and Candelon (2006), (v) the power (the size distortion) of the test increases (decreases) as the sample size increases and (vi) the test has reasonable size properties even in the small samples.

Here, we explain our conjecture on the results (iii) stated above by setting $c^* = -0.3$. Under this setting, for example, because we can see that

$$\cos(\pi/8 - 0.3) - \cos(\pi/8) = 0.072 < \cos(7\pi/8 - 0.3) - \cos(7\pi/8) = 0.154,$$

the case of $\omega_0 = \pi/8$ is closer to the null of noncausality than the case of $\omega_0 = 7\pi/8$. We think that this conjecture is also valid for interpreting the results shown in Breitung and

Candelon (2006, Figure 1) and Bodart and Candelon (2009, Figure 1).

Finally, it is interesting to try another DGP:

$$\text{DGP3} : \theta_{12}(L) = (1 + c^*) - 2\cos(\omega_0)L + (1 + c^*)L^2. \tag{1.21}$$

Note that in this case, the Geweke measure is

$$M_{y\to x}(\omega_0) = \log[1 + 4c^{*2}\cos(\omega_0)^2]. \tag{1.22}$$

As shown in Panel F of Figure 1.1, when $\omega_0 = \pi/2$, $M_{y\to x}(\omega_0) = 0$ even when $c^* \neq 0$ and, in addition, from Panel E of Figure 1.1, we can see that the values of the Geweke measure corresponding to ω_0 that are around $\pi/2$ are close to 0. From these, we expect that the rejection frequencies in the case of $\omega_0 = \pi/2$ are close to the nominal size and those in the cases of $\omega_0 = 3\pi/8$, $5\pi/8$ are relatively low. From our simulation results shown in Table 1.3, we obtained the expected results.

1.5 Conclusion

Our concern in this paper is to assess whether or not the frequency domain causality test of Breitung and Candelon (2006) is useful when the null hypothesis is noncausality at a frequency close to 0 or π. Through both theoretical and simulation evaluations, we found that the test does not necessarily suffer from low power at such frequencies. In addition, we obtained results indicating the size stability of the test. From these results, we conclude that the test is still useful even at such frequencies.

Appendix

Proof of Eq. (1.5)

When $p = 3$, $\boldsymbol{R\beta} = \mathbf{0}$ is

$$\begin{bmatrix} \cos(\omega_0) & \cos(2\omega_0) & \cos(3\omega_0) \\ \sin(\omega_0) & \sin(2\omega_0) & \sin(3\omega_0) \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let $\beta_3 = \alpha$; then, the system of linear equations above can be represented as

$$\cos(\omega_0)\beta_1 + \cos(2\omega_0)\beta_2 = -\alpha\cos(3\omega_0),$$
$$\sin(\omega_0)\beta_1 + \sin(2\omega_0)\beta_2 = -\alpha\sin(3\omega_0).$$

By using formulas such as

$$\cos(2\omega_0) = 2\cos(\omega_0)^2 - 1, \quad \sin(2\omega_0) = 2\sin(\omega_0)\cos(\omega_0), \tag{A.1}$$
$$\cos(3\omega_0) = 4\cos(\omega_0)^3 - 3\cos(\omega_0), \quad \sin(3\omega_0) = 3\sin(\omega_0) - 4\sin(\omega_0)^3, \tag{A.2}$$

we can see $\beta_1 = \alpha$ and $\beta_2 = -2\alpha\cos(\omega_0)$.

Proof of Proposition 2

Let us define the following two matrices:

$$\boldsymbol{S} = \begin{bmatrix} \cos(\omega_0) & 1 & \cos(\omega_0) \\ -\sin(\omega_0) & 0 & \sin(\omega_0) \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} \cos(2\omega_0) & [\cos(2\omega_0)\cos(\omega_0) - \cos(\omega_0)]/\sin(\omega_0) \\ \sin(2\omega_0) & [\sin(2\omega_0)\cos(\omega_0) - \sin(\omega_0)]/\sin(\omega_0) \end{bmatrix}.$$

Then, by using the formulas Eqs. (A.1) and (A.2), we can see that $|\boldsymbol{\Gamma}| = 1$ and $\boldsymbol{R} = \boldsymbol{\Gamma S}$. Accordingly, the Wald test statistic W^* can be expressed with $\boldsymbol{S}$ as

$$\begin{aligned} W^* &= (\boldsymbol{R}\tilde{\boldsymbol{\beta}})'[\tilde{\sigma}_u^2\boldsymbol{R}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{R}']^{-1}\boldsymbol{R}\tilde{\boldsymbol{\beta}} = (\boldsymbol{\Gamma S}\tilde{\boldsymbol{\beta}})'[\tilde{\sigma}_u^2\boldsymbol{\Gamma S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}(\boldsymbol{\Gamma S})']^{-1}\boldsymbol{\Gamma S}\tilde{\boldsymbol{\beta}} \\ &= (\boldsymbol{S}\tilde{\boldsymbol{\beta}})'[\tilde{\sigma}_u^2\boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1}\boldsymbol{S}\tilde{\boldsymbol{\beta}}. \end{aligned}$$

Define $\boldsymbol{\xi} = [-2c\alpha/\sqrt{T}, 0]'$, and by the definitions of $\boldsymbol{S}$, $\boldsymbol{\beta}$ and $\boldsymbol{\xi}$, we have $\boldsymbol{S}\boldsymbol{\beta} - \boldsymbol{\xi} = \mathbf{0}$. Then, W^* can be expressed as

$$W^* = (\boldsymbol{S}\tilde{\boldsymbol{\beta}} - \boldsymbol{S}\boldsymbol{\beta} + \boldsymbol{\xi})'[\tilde{\sigma}_u^2 \boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1}(\boldsymbol{S}\tilde{\boldsymbol{\beta}} - \boldsymbol{S}\boldsymbol{\beta} + \boldsymbol{\xi}).$$

Let

$$\boldsymbol{\eta}_1 = [\tilde{\sigma}_u^2 \boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1/2}\boldsymbol{S}(\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}), \quad \boldsymbol{\eta}_2 = [\tilde{\sigma}_u^2 \boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1/2}\boldsymbol{\xi},$$

where $[\tilde{\sigma}_u^2 \boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1/2}$ denotes the square half matrix of $[\tilde{\sigma}_u^2 \boldsymbol{S}(\boldsymbol{Z}_2'\boldsymbol{Z}_2)^{-1}\boldsymbol{S}']^{-1}$. Then, we can see $\boldsymbol{\eta}_1 \xrightarrow{d} N(\mathbf{0}, \boldsymbol{I}_2)$, and from $\boldsymbol{Z}_2'\boldsymbol{Z}_2/T \xrightarrow{p} \sigma_v^2 \boldsymbol{I}_3$, we can also see that

$$\boldsymbol{\eta}_2 = \left[\tilde{\sigma}_u^2 \boldsymbol{S}\left(\frac{1}{T}\boldsymbol{Z}_2'\boldsymbol{Z}_2\right)^{-1}\boldsymbol{S}'\right]^{-1/2}\sqrt{T}\boldsymbol{\xi} \xrightarrow{p} \begin{bmatrix} \delta \\ 0 \end{bmatrix},$$

where

$$\delta = -\frac{\sigma_v(2c\alpha)}{\sigma_u\sqrt{1 + 2\cos(\omega_0)^2}}.$$

Combining these results, we obtain

$$\boldsymbol{\eta}_1 + \boldsymbol{\eta}_2 \xrightarrow{d} N([\delta, 0]', \boldsymbol{I}_2).$$

Hence, $W^* = (\boldsymbol{\eta}_1 + \boldsymbol{\eta}_2)'(\boldsymbol{\eta}_1 + \boldsymbol{\eta}_2)$ is asymptotically distributed as $\chi_2^2(\delta^2)$.

Table 1.1: Rejection frequencies (DGP1)

		ω_0						
T	c^*	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$
50	-0.3	0.11	0.36	0.69	0.83	0.75	0.52	0.23
	-0.2	0.10	0.22	0.42	0.52	0.43	0.27	0.14
	-0.1	0.09	0.12	0.17	0.19	0.17	0.12	0.09
	0.0	0.09	0.09	0.08	0.08	0.08	0.08	0.08
	0.1	0.09	0.13	0.17	0.19	0.17	0.12	0.09
	0.2	0.14	0.27	0.43	0.51	0.42	0.23	0.11
	0.3	0.22	0.51	0.75	0.82	0.70	0.35	0.11
100	-0.3	0.12	0.57	0.93	0.99	0.97	0.80	0.36
	-0.2	0.10	0.34	0.68	0.80	0.71	0.44	0.17
	-0.1	0.08	0.14	0.23	0.29	0.24	0.15	0.08
	0.0	0.07	0.06	0.06	0.06	0.07	0.07	0.07
	0.1	0.09	0.15	0.24	0.29	0.24	0.14	0.08
	0.2	0.17	0.44	0.72	0.80	0.67	0.34	0.10
	0.3	0.35	0.80	0.97	0.99	0.93	0.58	0.12
200	-0.3	0.18	0.86	1.00	1.00	1.00	0.98	0.61
	-0.2	0.13	0.57	0.93	0.98	0.96	0.73	0.27
	-0.1	0.08	0.20	0.41	0.50	0.41	0.22	0.10
	0.0	0.06	0.06	0.05	0.06	0.05	0.06	0.06
	0.1	0.10	0.22	0.41	0.50	0.40	0.20	0.08
	0.2	0.27	0.73	0.96	0.98	0.93	0.58	0.14
	0.3	0.61	0.98	1.00	1.00	1.00	0.86	0.18

Note: Rejection frequencies of $30,000$ replications based on the model Eqs. (1.16) and (1.17). The nominal size is 0.05 and the lag order of the VAR model is assumed to be known.

Table 1.2: Rejection frequencies (DGP2)

		ω_0						
T	c^*	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$
50	-0.3	0.59	0.69	0.78	0.84	0.82	0.72	0.58
	-0.2	0.33	0.39	0.46	0.51	0.50	0.43	0.33
	-0.1	0.14	0.16	0.18	0.19	0.19	0.17	0.15
	0.0	0.08	0.08	0.08	0.08	0.08	0.08	0.08
	0.1	0.14	0.17	0.19	0.19	0.18	0.16	0.14
	0.2	0.33	0.43	0.50	0.51	0.46	0.39	0.33
	0.3	0.59	0.72	0.82	0.83	0.78	0.69	0.59
100	-0.3	0.86	0.94	0.98	0.99	0.98	0.94	0.87
	-0.2	0.54	0.64	0.75	0.80	0.78	0.68	0.54
	-0.1	0.18	0.22	0.26	0.29	0.28	0.23	0.18
	0.0	0.06	0.07	0.06	0.06	0.07	0.06	0.07
	0.1	0.18	0.23	0.28	0.29	0.27	0.22	0.18
	0.2	0.54	0.68	0.78	0.81	0.75	0.64	0.54
	0.3	0.86	0.95	0.98	0.99	0.98	0.93	0.86
200	-0.3	0.99	1.00	1.00	1.00	1.00	1.00	0.99
	-0.2	0.84	0.92	0.97	0.98	0.97	0.93	0.83
	-0.1	0.30	0.37	0.46	0.51	0.49	0.40	0.30
	0.0	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	0.1	0.30	0.39	0.48	0.50	0.46	0.37	0.30
	0.2	0.84	0.93	0.97	0.98	0.97	0.91	0.84
	0.3	0.99	1.00	1.00	1.00	1.00	1.00	0.99

Note: Rejection frequencies of $30,000$ replications based on the model Eqs. (1.16) and (1.18). The nominal size is 0.05 and the lag order of the VAR model is assumed to be known.

Table 1.3: Rejection frequencies (DGP3)

		ω_0						
T	c^*	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$
50	-0.3	0.53	0.47	0.25	0.08	0.25	0.46	0.52
	-0.2	0.29	0.26	0.15	0.08	0.15	0.27	0.29
	-0.1	0.13	0.13	0.10	0.08	0.10	0.13	0.13
	0.0	0.08	0.08	0.08	0.08	0.08	0.09	0.08
	0.1	0.13	0.12	0.09	0.08	0.10	0.12	0.13
	0.2	0.29	0.23	0.14	0.08	0.13	0.23	0.28
	0.3	0.52	0.41	0.20	0.08	0.20	0.42	0.52
100	-0.3	0.81	0.74	0.38	0.06	0.39	0.73	0.81
	-0.2	0.48	0.41	0.20	0.06	0.20	0.41	0.47
	-0.1	0.17	0.15	0.09	0.06	0.09	0.15	0.17
	0.0	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	0.1	0.17	0.14	0.09	0.06	0.09	0.14	0.17
	0.2	0.47	0.37	0.17	0.07	0.17	0.37	0.47
	0.3	0.80	0.67	0.30	0.07	0.30	0.67	0.80
200	-0.3	0.98	0.96	0.65	0.06	0.65	0.96	0.98
	-0.2	0.77	0.68	0.33	0.06	0.33	0.69	0.78
	-0.1	0.26	0.23	0.12	0.06	0.12	0.22	0.27
	0.0	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	0.1	0.27	0.21	0.11	0.06	0.11	0.20	0.26
	0.2	0.77	0.63	0.28	0.06	0.28	0.63	0.77
	0.3	0.98	0.93	0.54	0.06	0.54	0.93	0.98

Note: Rejection frequencies of $30,000$ replications based on the model Eqs. (1.16) and (1.21). The nominal size is 0.05 and the lag order of the VAR model is assumed to be known.

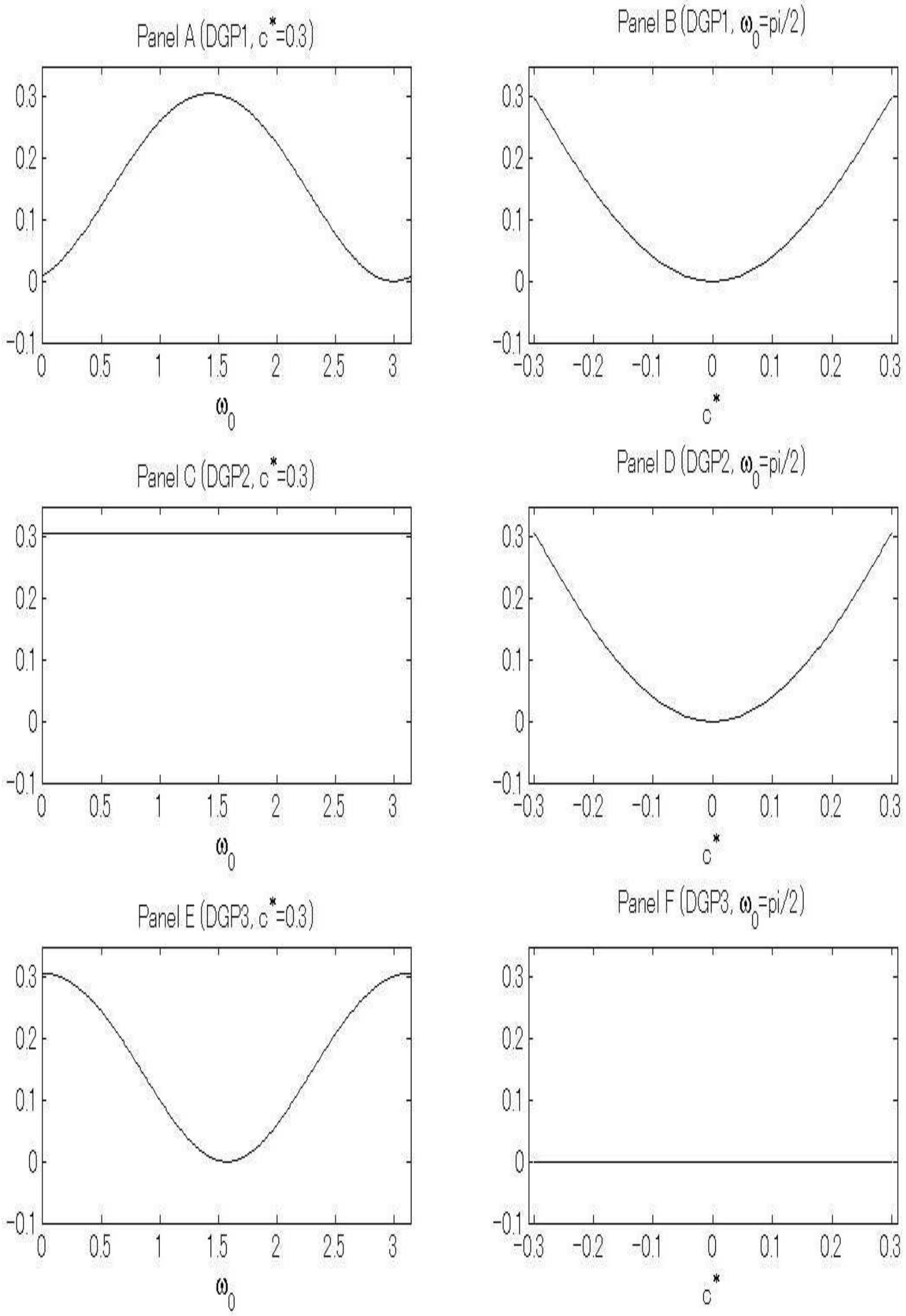

Figure 1.1: Geweke measures for DGP1, DGP2, and DGP3

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Chapter 2

The Dynamic Relationships between Oil Prices and the Japanese Economy: A Frequency Domain Analysis

2.1 Introduction

There is substantial empirical evidence that suggests oil price fluctuations may be closely related to variability of economic activities. This is particularly obvious for the 1970s when economic performance in the United States and elsewhere declined after the oil price shocks emanating from OPEC. Consequently, although a large body of empirical research has been devoted to the impacts of oil price fluctuations on macroeconomic performance, it has not reached a consensus. For instance, Hamilton (1983) and Mork (1989) find that oil prices are significantly related to the performance of the US economy, while Hooker (1996) argues strongly that oil prices failed to Granger-cause many US macroeconomic variables in data after 1973, and this further stimulated many economists' interest in analyzing the oil price–economy relationship for different countries.

The main objective of this study is to examine the dynamic relationships between oil prices and the Japanese economy. Oil is of particular importance to Japan's economy since it is one of the world's largest net oil importers. Accordingly, numerous articles have already addressed the oil price–macroeconomy relationship for Japan. For example, Hutchison (1993) documents that Japan has become better insulated from oil price shocks since the late 1970s. Mork *et al.* (1994) find the oil price–macroeconomy relationship for Japan is asymmetric. Lee *et al.* (2001) detect that oil price shocks are statistically significant in explaining changes in the level of Japan's economic activity and have a remarkable influence on its monetary policy. Cunado and Gracia (2005) illustrate that oil prices have short-term nonlinear relationships with the Japanese economy. Zhang (2008) also finds that there may exist certain nonlinear relationships between oil price shocks and Japan's economic growth.

However, the papers mentioned above are mainly concentrated on the time domain, and the conventional Granger causality tests based on vector autoregressive models (VARs) are often employed to investigate the causal relationships between oil prices and macroeconomic variables. Because the linkages between oil prices and macroeconomic variables may vary across the frequency bands, a time domain analysis may fail to fully capture such links. Moreover, researchers including Granger (1969), Geweke (1982), Hosoya (1991) and Granger and Lin (1995) argue that the extent and direction of causality can differ across the frequency domain. Consequently, unlike such previous papers, the present research adopts some frequency domain statistical methods to study the dynamic linkages between oil prices and the Japanese economy.

Specifically, we first apply the frequency dependent regression method proposed by Ashley and Verbrugge (2009) to investigate the nonlinear relationships between oil prices and Japan's economy. This procedure also enables us to analyze the associations at low and high frequencies, and can provide some additional insights into the oil price–macroeconomy relationship for Japan. Second, we conduct the frequency domain causality tests introduced by Breitung and Candelon (2006) to evaluate the dynamic causal effects from oil prices on the Japanese economy. We employ this test for two reasons. (i) This test allows us to decompose the full causal relationships of the variables into different frequencies, so we can assess the significance of the Granger causalities at some specific frequencies. (ii) Like the conventional Granger causality test, this test is based on a set of linear restrictions of coefficients of the VAR model and so is relatively easy to carry out. Examples of research applying this causality test include Assenmacher-Wesche *et al.* (2008), Bodart and Candelon (2009), Gronwald (2009), Schreiber (2009) and Ciner (2011a, 2011b).

The remainder of this paper is organized as follows. In Section 2.2, we briefly introduce the methods utilized in this study. In Section 2.3, we present our data including the results of the unit root and cointegration tests. In Section 2.4, we document the empirical analysis results and Section 2.5 concludes the paper.

2.2 Methodology

2.2.1 Frequency dependent regression model

The frequency domain regression analysis was first proposed by Hannan (1963) and was further developed by Duncan and Jones (1966), Engle (1974, 1978), Harvey (1978), Tan and Ashley (1999) and Ashley and Verbrugge (2009). To illustrate the method introduced by Ashley and Verbrugge (2009), let us first consider the ordinary multiple regression model

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{2.1}$$

where $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2\boldsymbol{I})$, $\boldsymbol{Y}$ is $T \times 1$ and $\boldsymbol{X}$ is $T \times K$. This model can be transformed into frequency domain via premultiplication of a $T \times T$ orthogonal matrix $\boldsymbol{A}$, whose (s,t)th element is given by

$$a_{s,t} = \begin{cases} \sqrt{\frac{1}{T}}, & \text{if} \quad s{=}1; \\ \sqrt{\frac{2}{T}}\cos[\frac{\pi s(t-1)}{T}], & \text{if} \quad s{=}2,\ 4,\ 6,\ \ldots,\ (T\text{-}2) \text{ or } (T\text{-}1); \\ \sqrt{\frac{2}{T}}\sin[\frac{\pi(s-1)(t-1)}{T}], & \text{if} \quad s{=}3,\ 5,\ 7,\ \ldots,\ (T\text{-}1) \text{ or } T; \\ \sqrt{\frac{1}{T}}(-1)^{t+1}, & \text{if} \quad s{=}T \text{ when } T \text{ is even.} \end{cases}$$

Premultiplying Eq. (2.1) by $\boldsymbol{A}$ yields

$$\begin{aligned} \boldsymbol{AY} &= \boldsymbol{AX}\boldsymbol{\beta} + \boldsymbol{A}\boldsymbol{\varepsilon}, \\ \boldsymbol{Y}^* &= \boldsymbol{X}^*\boldsymbol{\beta} + \boldsymbol{\varepsilon}^*, \end{aligned} \tag{2.2}$$

where $\boldsymbol{Y}^*$, $\boldsymbol{X}^*$ and $\boldsymbol{\varepsilon}^*$ are defined accordingly, and $\boldsymbol{\varepsilon}^* \sim N(0, \sigma^2\boldsymbol{I})$ because $\boldsymbol{A}$ is an orthogonal matrix. Now, for example, if T is even, the T components of $\boldsymbol{Y}^*$ and $\boldsymbol{\varepsilon}^*$ and T rows of $\boldsymbol{X}^*$ correspond to $(1 + T/2)$ distinct frequencies, $\{0, 2\pi/T, 4\pi/T, \ldots, 2\pi(T/2 - 1)/T, \pi\}$ rather than to T time periods[1]. Consequently, detecting that the jth component (β_j) of $\boldsymbol{\beta}$ "depends on frequency" is equivalent to detecting that β_j is unstable across the T "observations" in the model for $\boldsymbol{Y}^*$. The constancy of β_j across the T "observations" in Eq. (2.2) can be examined using the parameter instability test given in Ashley (1984). In this approach, the T "frequency components" of $\boldsymbol{X}_j^*$ (jth column of $\boldsymbol{X}^*$) are partitioned

[1]If T is odd, the T components of $\boldsymbol{Y}^*$ and $\boldsymbol{\varepsilon}^*$ and T rows of $\boldsymbol{X}^*$ correspond to $(T+1)/2$ distinct frequencies, $\{0, 2\pi/T, 4\pi/T, \ldots, 2\pi((T-1)/2 - 1)/T, \pi(T-1)/T\}$.

into M "frequency bands" and dummy variables $\boldsymbol{D}^{*1}, \ldots, \boldsymbol{D}^{*M}$ are defined corresponding to each band. The sth component of dummy variable $\boldsymbol{D}^{*m}$, denoted by D_s^{*m}, would equal $X_{s,j}^*$ (sth component of $\boldsymbol{X}_j^*$) for values of s in the mth frequency band and would equal zero for values of s outside this band. Consequently, the frequency domain regression Eq. (2.2) can be rewritten as

$$\boldsymbol{Y}^* = \boldsymbol{X}_{[j]}^* \boldsymbol{\beta}_{[j]} + \sum_{m=1}^{M} \beta_{j,m} \boldsymbol{D}^{*m} + \boldsymbol{\varepsilon}^*, \tag{2.3}$$

where $\boldsymbol{X}_{[j]}^*$ denotes the $\boldsymbol{X}^*$ with its jth column omitted and $\boldsymbol{\beta}_{[j]}$ denotes the $\boldsymbol{\beta}$ vector with its jth component deleted. Meanwhile, we should note that $\sum_{m=1}^{M} \boldsymbol{D}^{*m} = \boldsymbol{X}_j^*$. Therefore, the frequency dependent coefficients $\beta_{j,1}, \ldots, \beta_{j,M}$ can be estimated and the significance of these parameters can be tested.

By now, however, it is more convenient to transform Eq. (2.3) back into the time domain through premultiplying it by $\boldsymbol{A}^{-1}$. Because the inverse matrix of $\boldsymbol{A}$ is equivalent to its transpose, we can obtain the equation

$$\begin{aligned} \boldsymbol{A}^T \boldsymbol{Y}^* &= \boldsymbol{A}^T \boldsymbol{X}_{[j]}^* \boldsymbol{\beta}_{[j]} + \sum_{m=1}^{M} \beta_{j,m} \boldsymbol{A}^T \boldsymbol{D}^{*m} + \boldsymbol{A}^T \boldsymbol{\varepsilon}^*, \\ \boldsymbol{Y} &= \boldsymbol{X}_{[j]} \boldsymbol{\beta}_{[j]} + \sum_{m=1}^{M} \beta_{j,m} \boldsymbol{A}^T \boldsymbol{D}^{*m} + \boldsymbol{\varepsilon}, \end{aligned} \tag{2.4}$$

where $\boldsymbol{X}_{[j]}$ denotes the original matrix $\boldsymbol{X}$ with its jth column deleted. Thus, Eq. (2.4) is similar to Eq. (2.1) except that now there are M new regressors $\boldsymbol{A}^T\boldsymbol{D}^{*1}, \ldots, \boldsymbol{A}^T\boldsymbol{D}^{*M}$ replacing $\boldsymbol{X}_j$, the jth column of $\boldsymbol{X}$. Here, we should note again that $\sum_{m=1}^{M} \boldsymbol{A}^T\boldsymbol{D}^{*m} = \boldsymbol{X}_j$ [2]. Each new regressor $\boldsymbol{A}^T\boldsymbol{D}^{*m}$ can be interpreted as the result of applying a simple bandpass filter on $\boldsymbol{X}_j$. Ashley and Verbrugge (2009) argue that this will make the tth component of $\boldsymbol{A}^T\boldsymbol{D}^{*m}$ depend on all T values of $\boldsymbol{X}_j$, including the future values $X_{t+1,j}, \ldots, X_{T,j}$. Consequently, the least square estimates of $\beta_{j,1}, \ldots, \beta_{j,M}$ will be inconsistent if there exists feedback between $\boldsymbol{Y}$ and $\boldsymbol{X}_j$.

To solve the above problem, Ashley and Verbrugge (2009) suggest partitioning $\boldsymbol{X}_j$ into frequency components through one-sided filters based on a moving window. The last observation in each of these frequency components is retained for each window. This method not only resolves the estimation problem but also eliminates the need to decide a

[2] $\sum_{m=1}^{M} \boldsymbol{A}^T\boldsymbol{D}^{*m} = \boldsymbol{A}^T \sum_{m=1}^{M} \boldsymbol{D}^{*m} = \boldsymbol{A}^T\boldsymbol{X}_j^* = \boldsymbol{A}^T\boldsymbol{A}\boldsymbol{X}_j = \boldsymbol{X}_j$

value for M. Their approach is adopted in this paper.

2.2.2 Frequency domain causality test

The frequency domain causality test developed by Breitung and Candelon (2006) is based on the framework of Geweke (1982) and Hosoya (1991). To explain this causality measure, let us consider a two-dimensional time series vector $\boldsymbol{z}_t = [x_t, y_t]'$ with T observations. In the present paper, x_t will be one of the Japanese macroeconomic variables, and y_t will be oil prices. It is assumed that $\boldsymbol{z}_t$ has a finite-order VAR representation of the form

$$\boldsymbol{\Theta}(L)\boldsymbol{z}_t = \boldsymbol{\varepsilon}_t, \tag{2.5}$$

where $\boldsymbol{\Theta}(L) = \boldsymbol{I} - \boldsymbol{\Theta}_1 L - \cdots - \boldsymbol{\Theta}_p L^p$ is a 2×2 lag polynomial with $L^k \boldsymbol{z}_t = \boldsymbol{z}_{t-k}$. We also assume that the error vector $\boldsymbol{\varepsilon}_t$ is white noise with zero mean and positive definite covariance matrix $\boldsymbol{\Sigma}$. Furthermore, we let $\boldsymbol{G}$ be the lower triangular matrix of the Cholesky decomposition $\boldsymbol{G}'\boldsymbol{G} = \boldsymbol{\Sigma}^{-1}$, such that $E(\boldsymbol{\eta}_t \boldsymbol{\eta}_t') = \boldsymbol{I}$ and $\boldsymbol{\eta}_t = \boldsymbol{G}\boldsymbol{\varepsilon}_t$. If the system is assumed to be stationary, the moving average (MA) representation can be derived as

$$\begin{aligned} \boldsymbol{z}_t = \boldsymbol{\Phi}(L)\boldsymbol{\varepsilon}_t &= \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \\ = \boldsymbol{\Psi}(L)\boldsymbol{\eta}_t &= \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}, \end{aligned} \tag{2.6}$$

where $\boldsymbol{\Phi}(L) = \boldsymbol{\Theta}(L)^{-1}$ and $\boldsymbol{\Psi}(L) = \boldsymbol{\Phi}(L)\boldsymbol{G}^{-1}$. Based on this representation, the spectral density of x_t can be written as

$$f_x(\omega) = \frac{1}{2\pi}\{|\Psi_{11}(e^{-\mathrm{i}\omega})|^2 + |\Psi_{12}(e^{-\mathrm{i}\omega})|^2\}.$$

The measure of causality defined by Geweke (1982) and Hosoya (1991) can be expressed as

$$M_{y \to x}(\omega) = \log\left[\frac{2\pi f_x(\omega)}{|\Psi_{11}(e^{-\mathrm{i}\omega})|^2}\right] \tag{2.7}$$

$$= \log\left[1 + \frac{|\Psi_{12}(e^{-\mathrm{i}\omega})|^2}{|\Psi_{11}(e^{-\mathrm{i}\omega})|^2}\right]. \tag{2.8}$$

Within this framework, if $M_{y\to x}(\omega)=0$, we say that y does not cause x at frequency ω.

Breitung and Candelon (2006) try to test the hypothesis that y does not cause x at frequency ω through considering the null hypothesis [3]

$$M_{y\to x}(\omega)=0. \tag{2.9}$$

It follows that $M_{y\to x}(\omega)=0$ if $|\Psi_{12}(e^{-\mathrm{i}\omega})|=0$ in Eq. (2.8). Meanwhile, using $\boldsymbol{\Psi}(L)=\boldsymbol{\Theta}(L)^{-1}\boldsymbol{G}^{-1}$, we obtain

$$\Psi_{12}(L)=-\frac{g_{22}\Theta_{12}(L)}{|\boldsymbol{\Theta}(L)|},$$

where g_{22} is the lower diagonal element of $\boldsymbol{G}^{-1}$ and $|\boldsymbol{\Theta}(L)|$ is the determinant of the matrix $\boldsymbol{\Theta}(L)$. Since g_{22} is positive [4], $|\Psi_{12}(e^{-i\omega})|=0$ is equivalent to

$$|\Theta_{12}(e^{-i\omega})|=\left|\sum_{k=1}^{p}\theta_{12,k}\cos{(k\omega)}-\sum_{k=1}^{p}\theta_{12,k}\sin{(k\omega)}i\right|=0,$$

where $\theta_{12,k}$ is the $(1,2)$-element of $\boldsymbol{\Theta}_k$. Therefore, a necessary and sufficient set of conditions for $|\Psi_{12}(e^{-i\omega})|=0$ or $M_{y\to x}(\omega)=0$ is

$$\sum_{k=1}^{p}\theta_{12,k}\cos{(k\omega)}=0, \tag{2.10}$$
$$\sum_{k=1}^{p}\theta_{12,k}\sin{(k\omega)}=0, \tag{2.11}$$

and y does not cause x at frequency ω under this set of restrictions.

The Breitung and Candelon (2006) approach is just based on the linear restrictions Eq. (2.10) and Eq. (2.11). To simplify the notation, let $\alpha_j=\theta_{11,j}$ and $\beta_j=\theta_{12,j}$, then the VAR equation for x_t can be written as

$$x_t=\alpha_1x_{t-1}+\cdots+\alpha_px_{t-p}+\beta_1y_{t-1}+\cdots+\beta_py_{t-p}+\varepsilon_{1t}. \tag{2.12}$$

Accordingly, the null hypothesis $M_{y\to x}(\omega)=0$ is equivalent to the linear restriction

$$H_0:\boldsymbol{R\beta}=\boldsymbol{0}, \tag{2.13}$$

[3]In this paper, we only consider a bivariate framework although a higher dimensional system was introduced in the original paper.

[4]This is because of the assumption that $\boldsymbol{\Sigma}$ is a positive definite matrix.

where $\boldsymbol{\beta}$=$[\beta_1, \beta_2, \ldots, \beta_p]'$ and

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cdots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \cdots & \sin(p\omega) \end{bmatrix}.$$

In the above interpretation, we neglect any deterministic terms in the VAR model. If there is a constant vector $\boldsymbol{\mu} = [\mu_1, \mu_2]'$ in Eq. (4.3), the Wald test statistic for Eq. (2.13) can be expressed as

$$W = (\boldsymbol{R}\hat{\boldsymbol{\beta}})' \left[\hat{\sigma}^2 \boldsymbol{R} \left(\boldsymbol{Z}_2'\boldsymbol{Q}\boldsymbol{Z}_2\right)^{-1} \boldsymbol{R}'\right]^{-1} \boldsymbol{R}\hat{\boldsymbol{\beta}}, \tag{2.14}$$

where

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{Z}_2'\boldsymbol{Q}\boldsymbol{Z}_2\right)^{-1} \boldsymbol{Z}_2'\boldsymbol{Q}\boldsymbol{x}, \quad \hat{\sigma}^2 = \frac{1}{T-p}\left(\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{Q}\boldsymbol{Z}_2\hat{\boldsymbol{\beta}}\right)' \left(\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{Q}\boldsymbol{Z}_2\hat{\boldsymbol{\beta}}\right),$$

and

$$\boldsymbol{Q} = \boldsymbol{Q}_\iota - \boldsymbol{Q}_\iota\boldsymbol{Z}_1(\boldsymbol{Z}_1'\boldsymbol{Q}_\iota\boldsymbol{Z}_1)^{-1}\boldsymbol{Z}_1'\boldsymbol{Q}_\iota, \quad \boldsymbol{Q}_\iota = \boldsymbol{I}_{T-p} - \boldsymbol{\iota}(\boldsymbol{\iota}'\boldsymbol{\iota})^{-1}\boldsymbol{\iota}'.$$

Here, $\boldsymbol{\iota}$ is the $(T-p) \times 1$ vector of ones. Meanwhile, $\boldsymbol{x} = [x_{p+1}, \ldots, x_T]'$, and $\boldsymbol{Z}_i = [\boldsymbol{z}_{i,p+1}, \ldots, \boldsymbol{z}_{i,T}]'$ for $i = 1, 2$ with $\boldsymbol{z}_{1,t}' = [x_{t-1}, \ldots, x_{t-p}]$, $\boldsymbol{z}_{2,t}' = [y_{t-1}, \ldots, y_{t-p}]$.

Like the conventional causality test, the Wald test statistic W based on the above linear restriction is asymptotically distributed as $\chi^2(2)$ for $\omega \in (0, \pi)$. To evaluate the significance of the variables' causal relationships, the Wald test statistic is compared with the 5% critical value (5.99) of a chi-square distribution with 2 degrees of freedom throughout of this paper.

2.3 Data information

A total of four data series, which include oil prices (OIL) and three Japanese macroeconomic variables, namely industrial production index (IPI), consumer price index (CPI), and unemployment rates (UR), will be employed in this paper to study the relationships between oil prices and the Japanese economy [5]. The data are obtained from THOMSON Datastream Database and the frequency of the series is monthly. The period covered is between January 1979 and December 2010, with a total of 384 observations.

[5]The oil prices are calculated by transforming the UK average Brent oil price into Japanese yen.

As the statistical methods require stationary data, unit root tests have been implemented for all of the variables. Table 2.1 presents the results of the unit root tests corresponding to the Augmented Dickey–Fuller and Phillips–Perron procedures. It is evident that all the variables are integrated of order one ($I(1)$). Hence, cointegration tests are carried out between oil prices and each of the other three variables. Both the max eigenvalue and trace tests indicate no cointegration at the 0.05 significance level (Table 2.2). All the cointegration tests are conducted by using a constant and a trend in the VAR representation [6]. Therefore, in this article, OIL, IPI and UR are taken in first differences of logarithms, while CPI is only taken in first difference.

2.4 Empirical analysis results

2.4.1 Frequency dependent regression analysis

In the empirical analysis, researchers usually conduct the following conventional time series regression model to explore the relationships between two variables

$$y_t = \alpha + \beta x_t + u_t, \tag{2.15}$$

where u_t is the disturbance with zero mean and constant variance. In the present paper, y_t will be one of the three variables including IPI, CPI and UR, while x_t will be OIL. The above model implies that the relationship between x_t and y_t is linear and the coefficient β is invariable. In practice, it may be not possible to match such assumptions. In fact, the ordinary least square (OLS) estimators of β are 0.281130 and –0.009932 for OIL/CPI and OIL/UR relationships, and the corresponding p-values are 0.1774 and 0.7331, respectively, computed with White robust standard errors. Accordingly, one may conclude that the OIL/CPI and OIL/UR relationships are insignificant for Japan. However, such conclusions may be attributable to the facts that β varies across the frequency bands or the relationships are nonlinear. Therefore, we employ the frequency dependent regression method proposed by Ashley and Verbrugge (2009) to investigate the relationships between OIL and each of the other three variables. This methodology allows us to analyze the variables' nonlinear relationships at some prespecified frequencies.

We employ a moving window 48 periods in length to decompose the series into 31

[6] The hypothesis of no cointegration is also accepted when only a constant is included in the VAR model.

possible frequency components, with the lowest nonzero frequency component $2\pi/60$ corresponding to 60 months [7]. The selected parameter estimates for low and high frequencies are listed in Table 2.3. The corresponding p-values are also calculated based on the White robust standard errors. It is clearly evident that the frequency dependent regression analysis can shed further light on the variables' relationships because the parameter estimates are statistically significant at several frequencies for the OIL/CPI and OIL/UR relationships. Specifically, for the OIL/IPI relationships, the estimates of the parameters are statistically significant at the low frequencies, corresponding to the periods between 15 months and 60 months. Similarly, noteworthy relationships between OIL and CPI are also detected at the low frequencies (30 months and 60 months). In contrast, the linkages between OIL and UR are mainly concentrated in the high frequencies (2 months and 6 months). In addition, there may also exist certain associations between OIL and UR at the low frequencies since the coefficient's p-value is only 0.0688 at $\pi/30$ (60 months).

Overall, the frequency dependent regression analysis indicates that oil price fluctuations are related to the Japanese economy, especially at the low frequencies. Moreover, as mentioned in Ashley and Verbrugge (2009), such relationships are dynamically nonlinear, which is consistent with the analysis in Cunado and Gracia (2005) and Zhang (2008).

2.4.2 Frequency domain causality analysis

The above analysis shows that oil prices have nonlinear linkages with the Japanese economy at certain frequencies. To understand the oil price–economy relationship in greater depth, this subsection evaluates the dynamic causal effects from oil prices on the other three variables, using the frequency domain causality tests within the context of bivariate VAR models. The lag lengths of the VAR models are determined by the Akaike Information Criterion with a maximum lag length being equal to 12.

The results of the frequency domain causality tests are presented in Table 2.4. First, we see that OIL has significant causal effects on IPI, CPI and UR at the low frequency $\omega = 0.1$ (63 months). This also suggests that oil prices have connections with the Japanese economy at the low frequency, which is in line with the frequency dependent regression analysis. Second, Granger causalities from OIL to IPI are also detected at frequencies

[7] As suggested by Ashley and Verbrugge (2009), the 48 months in each window are augmented with 12 projected values, so there are 31 possible frequencies given by $0, 2\pi{*}1/60, 2\pi{*}2/60, \ldots, 2\pi{*}29/60, 2\pi{*}30/60$, and the corresponding periods for the nonzero frequencies are given by $60, 30, \ldots, 60/29, 2$.

$\omega = 0.5, 1, 1.5$, and this can be interpreted as short- and medium-term causal effects from oil prices to industrial production. Third, the hypothesis of no Granger causality from OIL to UR is also rejected at the high frequencies $\omega = 2, 2.5$, which implies that there exist short-term causal relationships from oil prices to unemployment rates. This is again similar to our previous analysis of the OIL/UR relationships.

On the whole, frequency domain causality analysis suggests that oil prices have remarkable predictive power for Japanese economic activities, which is in line with the findings in Lee *et al.* (2001). However, by decomposing the causality into different frequencies, our research provides a much deeper understanding of such predictability.

2.5 Conclusions

This paper investigates the relationships between oil prices and the performance of Japan's economy, using some frequency domain statistical methods, and data for the period from January 1979 to December 2010. By decomposing the relationships into certain specific frequencies, we offer some new findings on the oil price–economy relationship for Japan. Such findings may be undetectable in the conventional time domain analysis.

Our main research results can be summarized as follows. (i) There exist nonlinear relationships between oil prices and the variables such as industrial production and consumer price index at the low frequencies. (ii) The nonlinear linkages between oil prices and unemployment rates are mainly detected at the high frequencies. (iii) Oil prices contain useful information for forecasting industrial production, consumer price index and unemployment rates at the low frequencies. (iv) Moreover, oil prices can forecast the industrial production and unemployment rates at certain high frequencies.

The findings imply that oil prices tend to be related to the Japanese economy at the low frequencies. This may be responsible for the phenomena nowadays that (i) oil price shocks appear to have no immediate impacts on Japan's economy, and (ii) individuals may rarely feel any instant consequences of oil price shocks. Nevertheless, our findings are useful for the Japanese government and central bank and suggest that policy makers should pay more attention to the long-term effects of oil price shocks on Japan's economy.

Finally, although we find oil prices are correlated with the selected macroeconomic variables at various frequencies, we should note that the factors such as the exchange rate changes and money growth also exert certain effects on these variables. For example, Assenmacher-Wesche *et al.* (2008) argue that money growth is linked with the consumer price index of Japan at the low frequencies. In such a case, the frequency domain causality tests can be conducted in a three dimensional system by extending the bivariate VAR model to include money growth. Furthermore, as demonstrated by Hosoya (2001) and Breitung and Candelon (2006), the partial causality running from oil prices to consumer price index can be examined at various frequencies by eliminating the effect of money growth. Therefore, the analysis in the paper can be improved in the future, and we consider that including the third variable in the VAR models and analyzing the partial causality of oil prices in the frequency domain would be an interesting research topic.

Table 2.1: Unit root tests

Variable	ADF (level)	ADF (first difference)	PP (level)	PP (first difference)
OIL	0.3534	0.0000	0.5326	0.0000
IPI	0.1495	0.0000	0.3418	0.0000
CPI	0.0698	0.0000	0.0931	0.0000
UR	0.2081	0.0000	0.2075	0.0000

Note: This table presents the p-values corresponding to the unit root test statistics. OIL, IPI and UR have been taken in logs. For OIL, an intercept is included in the test equation. For the other variables, a linear trend and an intercept are included in the test equation. ADF: Augmented Dickey–Fuller; PP: Phillips–Perron.

Table 2.2: Unrestricted cointegration rank tests

H_0	Statistic (Max Eigenvalue)	Critical Value (0.05)	Statistic (Trace)	Critical Value (0.05)
OIL/IPI				
r=0	19.31773	19.38704	24.56362	25.87211
$r \leq 1$	5.245885	12.51798	5.245885	12.51798
OIL/CPI				
r=0	11.13152	19.38704	15.50651	25.87211
$r \leq 1$	4.374991	12.51798	4.374991	12.51798
OIL/UR				
r=0	5.677208	19.38704	8.747447	25.87211
$r \leq 1$	3.070239	12.51798	3.070239	12.51798

Note: OIL, IPI and UR have been taken in logs. r is the cointegration rank and the critical value corresponds to a 0.05 significance level.

Table 2.3: Frequency dependence regression

	OIL/IPI		OIL/CPI		OIL/UR	
Frequency	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
2 months (π)	–0.0378	0.7127	–0.7802	0.6851	0.8157	0.0043
6 months ($\frac{\pi}{3}$)	–0.0779	0.1226	–0.2416	0.8379	0.3709	0.0382
12 months ($\frac{\pi}{6}$)	0.0424	0.3599	–0.3732	0.6650	–0.1371	0.2311
15 months ($\frac{2\pi}{15}$)	0.1319	0.0244	–1.7235	0.1839	–0.0486	0.7596
30 months ($\frac{\pi}{15}$)	0.1898	0.0049	2.3374	0.0106	–0.1294	0.3843
60 months ($\frac{\pi}{30}$)	0.2033	0.0048	4.9970	0.0000	–0.3069	0.0688

Note: This table presents the results of the frequency dependence regression between OIL and the respective variable, using the method of detecting frequency dependence in regression model coefficients proposed by Ashley and Verbrugge (2009).

Table 2.4: Frequency domain causality tests

	$\omega = 0.1$	$\omega = 0.5$	$\omega = 1$	$\omega = 1.5$	$\omega = 2$	$\omega = 2.5$	$\omega = 3$
OIL $\rightarrow$ IPI	32.00*	32.11*	28.39*	9.84*	1.66	2.97	4.03
OIL $\rightarrow$ CPI	24.68*	1.48	0.16	1.74	2.26	0.23	3.85
OIL $\rightarrow$ UR	10.69*	0.74	0.75	2.81	7.22*	7.15*	1.75

Note: This table presents the Breitung–Candelon frequency domain causality measure from OIL to the respective variable. The Wald test statistics are derived from bivariate VAR models. The VAR model corresponding to IPI is estimated with three lags, and the other two models are estimated with 12 lags. These lags are determined by the Akaike Information Criterion.

* = significant at 5% level.

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Chapter 3

An Evaluation of the Composite Leading Indicator Component Series for Japan

3.1 Introduction

To forecast the business climate of Japan, the Organization for Economic Cooperation and Development (OECD) and the Japanese government independently provide composite leading indicator (CLI) series. Both sets of CLI are calculated using component series. More precisely, as shown in Tables 3.1 and 3.2, the OECD CLI series comprises eight component series, whereas the Japanese CLI series is based on 11 component series, where the intersection of the two component sets is not empty. As a means of improving the forecasting performance of both CLI series, an evaluation of each of their component series may be useful. For example, Yamada *et al.* (2007) evaluated the performance of the Japanese government CLI component series. After extracting the business cycle frequency component using the band-pass filter proposed by Baxter and King (1999), they then clarified empirically whether the leading composite index and its component series truly lead the business cycle turning point dates officially determined by the Japanese government.

In this paper, we revisit this important topic and evaluate the component series, not only for the CLI series prepared by the Japanese government but also for the alternative CLI series constructed by the OECD.[1] Specifically, we first assess the component series through the frequency domain causality test proposed by Breitung and Candelon (2006).[2]

[1] Yamada *et al.* (2010) also compared the performance of both of these CLI series.

[2] Examples of papers applying the frequency domain causality test of Breitung and Candelon (2006) include Assenmacher-Wesche and Gerlach (2008), Bodart and Candelon (2009), Gronwald (2009), Mermod *et al.* (2010), and Ciner (2011a, 2011b).

We apply this procedure because it enables us to see how the causal relations vary according to frequency. Note that as with the conventional Granger causality test, their test is based on a set of linear restrictions of the coefficients of the vector autoregressive (VAR) model and so is relatively easy to implement. Besides, in the frequency domain causality test we use the series whose long-term trend is removed by the Hodrick and Prescott (1997) (HP) high-pass filter. We then evaluate the forecasting performance of the component series using the method of Bry and Boschan (1971). We utilize the OECD Cyclical Analysis and Composite Indicators System (CACIS) software to this purpose. In this procedure, the HP band-pass filter is employed to extract the business cycle frequency component from the original series.

The remainder of the paper is organized as follows. Section 3.2 introduces the methodologies used in the paper. Section 3.3 reports the empirical results. Section 3.4 concludes the paper.

3.2 Methodology

In this section, we explain the methodology we apply in the paper. We firstly explain the HP filters used by the OECD. We then briefly review the frequency domain causality test proposed by Breitung and Candelon (2006).

3.2.1 The HP filters

Since December 2008, the OECD has applied the HP band-pass filter when extracting the business cycle frequency component from the original series.[3] More specifically, to extract the business cycle frequency component, the OECD first uses the HP high-pass filter to remove the long-term trend from the series and then employs the HP low-pass filter to remove the high frequency noise.

Let $\boldsymbol{z} = [z_1, \ldots, z_T]'$ and define the $\boldsymbol{D}$ matrix such as $\boldsymbol{Dz} = [\Delta^2 z_3, \ldots, \Delta^2 z_T]'$, where $\Delta^2 z_t = (z_t - z_{t-1}) - (z_{t-1} - z_{t-2})$ for $t = 3, \ldots, T$. Here, the $\boldsymbol{D}$ matrix can be explicitly

[3]See OECD (2012).

presented as:[4]

$$\boldsymbol{D} = \begin{bmatrix} 1 & -2 & 1 & & \text{\Large 0} \\ & \ddots & \ddots & \ddots & \\ \text{\Large 0} & & 1 & -2 & 1 \end{bmatrix} \in \mathbb{R}^{T-2\times T}.$$

Then the HP high-pass and low-pass filters can be respectively expressed as:

$$\boldsymbol{b}^* = \left[\boldsymbol{I}_T - (\boldsymbol{I}_T + \lambda_1 \boldsymbol{D}'\boldsymbol{D})^{-1}\right] \boldsymbol{z}, \tag{3.1}$$

$$\boldsymbol{b} = \left(\boldsymbol{I}_T + \lambda_2 \boldsymbol{D}'\boldsymbol{D}\right)^{-1} \boldsymbol{b}^*, \tag{3.2}$$

where $\boldsymbol{I}_T$ is the T-dimensional identity matrix, $\lambda_1 = 133107.94$, and $\lambda_2 = 13.93$. Here, $\lambda_1 = 133107.94$ and $\lambda_2 = 13.93$ are, respectively, corresponding to the cut-off frequencies $\omega_1 = 0.05236$ and $\omega_2 = 0.52358$. Accordingly, the HP band-pass filter applied by the OECD can be written as follows:[5]

$$\boldsymbol{b} = \left(\boldsymbol{I}_T + \lambda_2 \boldsymbol{D}'\boldsymbol{D}\right)^{-1} \left[\boldsymbol{I}_T - (\boldsymbol{I}_T + \lambda_1 \boldsymbol{D}'\boldsymbol{D})^{-1}\right] \boldsymbol{z}, \tag{3.3}$$

By applying this filter, the business cycle frequency component with periodicity from 12 months to 120 months can be extracted.

3.2.2 Frequency domain causality test

We next briefly review the frequency domain causality test in Breitung and Candelon (2006).[6] Let us consider the following stationary bivariate VAR model of order p:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \theta_{11,1} & \theta_{12,1} \\ \theta_{21,1} & \theta_{22,1} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \theta_{11,p} & \theta_{12,p} \\ \theta_{21,p} & \theta_{22,p} \end{bmatrix} \begin{bmatrix} x_{t-p} \\ y_{t-p} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}, \tag{3.4}$$

where $t = 1, \ldots, T$, and $[u_t, v_t]' \sim \text{iid}(\mathbf{0}, \boldsymbol{\Sigma})$ with $\boldsymbol{\Sigma}$ being positive-definite.

Letting $\boldsymbol{\beta} = [\theta_{12,1}, \ldots, \theta_{12,p}]'$, the null hypothesis of no causality from y to x at fre-

[4] Matlab/GNU Octave code for constructing $\boldsymbol{D}$ is `D = diff(eye(T), 2)`.

[5] See Yamada (2011) for an alternative HP band-pass filter.

[6] Yamada and Wei (2012) provide some theoretical and simulation results concerning the power properties of the test.

quency ω_0 can be expressed by a linear restriction on the VAR coefficients as:

$$H_0:\ \boldsymbol{R\beta} = \boldsymbol{0},$$

where

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega_0) & \cos(2\omega_0) & \cdots & \cos(p\omega_0) \\ \sin(\omega_0) & \sin(2\omega_0) & \cdots & \sin(p\omega_0) \end{bmatrix} \in \mathbb{R}^{2\times p}.$$

Note that where $\boldsymbol{R} = \boldsymbol{I}_p$, the above hypothesis corresponds to the null hypothesis of the conventional Granger causality test.

Let $\boldsymbol{x} = [x_{p+1}, \dots, x_T]'$, and $\boldsymbol{Z}_i = [\boldsymbol{z}_{i,p+1}, \dots, \boldsymbol{z}_{i,T}]'$ for $i = 1, 2$ with $\boldsymbol{z}'_{1,t} = [x_{t-1}, \dots, x_{t-p}]$, $\boldsymbol{z}'_{2,t} = [y_{t-1}, \dots, y_{t-p}]$. Similarly to the conventional Granger causality test, this null hypothesis can be tested using the following Wald test statistic:

$$W = (\boldsymbol{R}\hat{\boldsymbol{\beta}})' \left[\hat{\sigma}^2 \boldsymbol{R}(\boldsymbol{Z}'_2\boldsymbol{Q}\boldsymbol{Z}_2)^{-1}\boldsymbol{R}'\right]^{-1} \boldsymbol{R}\hat{\boldsymbol{\beta}}, \tag{3.5}$$

where

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{Z}'_2\boldsymbol{Q}\boldsymbol{Z}_2)^{-1}\boldsymbol{Z}'_2\boldsymbol{Q}\boldsymbol{x}, \quad \hat{\sigma}^2 = \frac{1}{T-p}(\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{Q}\boldsymbol{Z}_2\hat{\boldsymbol{\beta}})'(\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{Q}\boldsymbol{Z}_2\hat{\boldsymbol{\beta}}),$$

and

$$\boldsymbol{Q} = \boldsymbol{Q}_\iota - \boldsymbol{Q}_\iota\boldsymbol{Z}_1(\boldsymbol{Z}'_1\boldsymbol{Q}_\iota\boldsymbol{Z}_1)^{-1}\boldsymbol{Z}'_1\boldsymbol{Q}_\iota, \quad \boldsymbol{Q}_\iota = \boldsymbol{I}_{T-p} - \frac{1}{T-p}\boldsymbol{\iota}\boldsymbol{\iota}'.$$

Here, $\boldsymbol{\iota}$ is the $(T-p)\times 1$ vector of ones. As with the conventional causality test, the Wald test statistic, W, is asymptotically distributed as $\chi^2(2)$ for $\omega_0 \in (0, \pi)$.

3.3 Empirical results

3.3.1 Data

Tables 3.1 and 3.2 list the series analyzed in the paper. The first row in both tables corresponds to the reference series while the remaining rows correspond to the CLI component series. Table 3.1 provides the series related to the OECD CLI. We obtain these data from the Thomson Datastream database. The final column in Table 3.1 indicates the series codes. Table 3.2 provides the comparable series from the Japanese government's CLI. We obtain these data from the website of the Japan Cabinet Office.[7]

3.3.2 Results of the frequency domain causality test

Letting x and y in Eq. (3.4) denote the Industrial Production Index (IPI) series and the CLI component series, respectively, we apply the frequency domain causality test. Because the frequency domain causality tests require the series containing certain high frequency noise, we only apply the HP high-pass filter to remove the long-term trend from the series.[8] Note that because the HP high-pass filter is used to remove the long-term trend from the x and y series, we may consider that the variables are stationary. Meanwhile, since the long-term trend corresponding to the frequencies $\omega_0 \in (0, 0.05)$ has been removed by the HP high-pass filter, we apply the test for $\omega_0 \in (0.05, \pi)$. To apply the test, we need to select the lag length, p. We select the value of p using various information criteria, including the Akaike information criterion (AIC), the Hannan–Quinn information criterion (HQC), and the Schwarz information criterion (SC) by setting the maximal lag length as 12.[9]

Figures 3.1, 3.2, and 3.3 provide the results of the frequency domain causality tests. Figure 3.1 depicts the OECD CLI while Figures 3.2, and 3.3 correspond to the Japanese government CLI. The horizontal line in each panel provides the 95% values of the $\chi^2(2)$ distribution. Using these figures, we can observe that (i) the five component series, OJ3, OJ5, OJ7, JG5 and JG8, do not cause the reference series at all frequencies. Note that OJ7 and JG8 are actually the same series. Besides, the series OJ5 is similar to the series JG5. It is noteworthy that both OJ3 (Ratio of loans to deposits) and OJ7/JG8 (Spread of interest rates) are monetary series. The poor performances of these series may result from: (a) the zero interest rate and (b) the quantitative easing policies pursued by the Bank of Japan. We can also see that (ii) all other component series exhibit significant causalities, at least at some frequencies. In addition, we can observe that (iii) most component series exhibit significant causality at the low frequencies. This indicates that these component series have long-term causal effects on the reference series.

[7] `http://www.esri.cao.go.jp/en/stat/di/di-e.html`

[8] Before applying the HP high-pass filter, the outliers of the series are identified and replaced through the CACIS software. This is because Bodart and Candelon (2009) find that the size of the frequency domain causality test is severely affected by the presence of one or two large outliers in the series.

[9] It is well known that the HQC and SC select the lag length consistently, unlike the AIC. See, for example, Lütkepohl (1993). However, we also employ the AIC because it is commonly used.

3.3.3 Analysis with the CACIS software

In addition to the frequency domain causality analysis, we undertake our evaluations using the CACIS software. We use this software because: (i) it enables us to detect the outliers of the series and then replace them with appropriate values, (ii) it provides a tool for applying the (simplified) Bry–Boschan method (Bry and Boschan, 1971), with which we can detect the business cycle turning points of the reference and component series, and (iii) we can evaluate the cross correlations between the reference series and the component series.

Tables 3.3 and 3.4 tabulate the results. For each component series, the CACIS software provides the following:[10] (i) "Targeted: number of the reference series' turning points in the range overlapping with the range covered by the series analyzed"; (ii) "Missed: number of turning points present in the reference series but not in the series of interest"; (iii) "Extra: number of turning points detected in the series but not present in the reference series"; (iv) "Mean Lead: the average lead time of the matched turning points"; (v) "St. Dev.: the standard deviation of the leads and lags of the corresponding series." The CACIS software also provides: (vi) "Peak Lead: the position on the cross-correlation function of the highest value" and (vii) "the value of the cross-correlation function at this point", which is labeled with Corr. Coeff. in Tables 3.3 and 3.4.

To best observe the results, the following information in the manual (p. 31) is also useful:

- "Ideally, potential component series should have a mean lead greater than 2 and a correlation at peak greater than 0.5 (with a peak lead equal or greater than 2)."
- "Indeed, while having extra turning points in some component series might not be a serious concern (since, in principle they should cancel each other out in the aggregate), missing some peaks or some troughs is much more problematic, since signals not captured in the components will not likely be part of the aggregate."

Our first concern is the performance of the component series, OJ3, OJ5, OJ7/JG8 and JG5. From Tables 3.3 and 3.4, we can see that (i) the peak correlation (Corr. Coeff. in the tables) for OJ7/JG8 (Interest rate spread) is only 0.16 and that it missed the targeted turning points on 50% of occasions, and that (ii) the peak correlations of OJ5 (Construc-

[10] From p. 23 of the CACIS software manual.

tion: dwellings started) and JG5 (Total the floor area of new housing construction started) are about 0.4, and both of them missed the targeted turning points on more than 50% of occasions, and that (iii) the peak correlation of OJ3 (Ratio of loans to deposits) is also quite low. We can also see that (iv) OJ2 (Ratio of imports to exports) missed the targeted turning points on more than 50% of occasions and its peak correlation is less than 0.5, and that (v) the mean and peak lead for the JG7 (Nikkei commodity price index (42 items)) are both negative. Other than these, we can also see that (vi) JG6 (Consumer confidence index) and JG10 (Index of investment climate (manufacturing)) performed fairly well. Because neither of these components are included in the OECD CLI, they may be potential candidates for future inclusion.

3.4 Summary and conclusion

To forecast the business climate of Japan, the OECD and the Japanese government independently provide composite CLI series. As a means of possibly improving the forecasting performance of these alternative series, we evaluated the performance of each of their component series by applying the frequency domain causality test of Breitung and Candelon (2006) and the CACIS software.

Our main empirical findings are summarized as follows: (i) The forecasting performances of the OJ3 (Ratio of loans to deposits), OJ5 (Construction: dwellings started), OJ7/JG8 (Spread of interest rates) and JG5 (Total floor area of new housing construction started) component series are quite poor. The poor performances of OJ3 and OJ7/JG8 may be attributable to (a) the zero interest rate and (b) quantitative easing policies pursued by the Bank of Japan; (ii) The frequency domain causality tests show that all component series, except for OJ3, OJ5, OJ7/JG8 and JG5, exhibit significant causalities at least at some frequencies; (iii) The results from the CACIS analysis indicate that two other component series OJ2 (Ratio of imports to exports) and JG7 (Nikkei commodity price index (42 items)) also do not perform well; (iv) The CACIS analysis also shows that JG6 (Consumer confidence index) and JG10 (Index of investment climate (manufacturing)) perform fairly well, and because these are not currently included in the OECD CLI, they may be potential candidates as component series.

Table 3.1: Reference and component series in the OECD CLI series

	Series name	Sample period	Series code
(IPI)	Industrial production index	Jan 1975–Sep 2011	`JPOPRI35G`
(OJ1)	Inventories to shipments ratio (mining and manufacturing)	Jan 1975–Sep 2011	`JPOL2088E`
(OJ2)	Ratio of imports to exports	Jan 1975–Sep 2011	`JPOL2064E`
(OJ3)	Ratio of loans to deposits	Jan 1975–Sep 2011	`JPOL2010Q`
(OJ4)	Monthly overtime hours (manufacturing)	Jan 1975–Sep 2011	`JPOL2050G`
(OJ5)	Construction: Dwellings started	Jan 1975–Sep 2011	`JPOL2034G`
(OJ6)	Share price index (TOPIX) Tokyo	Jan 1975–Sep 2011	`JPOL2084E`
(OJ7)	Spread of interest rates	Jan 1975–Sep 2011	`JPOL2056R`
(OJ8)	Small business survey: Sales tendency	Jan 1985–Sep 2011	`JPOL2030Q`

Table 3.2: Reference and component series of the Japanese government CLI series

	Series name	Sample period
(IPI)	Industrial production index	Jan 1975–Sep 2011
(JG1)	Index of producer's inventory ratio of finished goods (final demand goods)	Jan 1975–Sep 2011
(JG2)	Index of producer's inventory ratio of finished goods (producer goods for mining and manufacturing)	Jan 1975–Sep 2011
(JG3)	New job offers (excluding new school graduates)	Jan 1975–Sep 2011
(JG4)	New orders for machinery (excluding volatile orders)	Jan 1975–Sep 2011
(JG5)	Total floor area of new housing construction started	Jan 1975–Sep 2011
(JG6)	Consumer confidence index	Jan 1975–Sep 2011
(JG7)	Nikkei commodity price index (42 items)	Jan 1975–Sep 2011
(JG8)	Interest rate spread	Jan 1975–Sep 2011
(JG9)	Stock prices (TOPIX)	Jan 1975–Sep 2011
(JG10)	Index of investment climate (manufacturing)	Jan 1975–Sep 2011
(JG11)	Sales forecast D.I. of small businesses	Jan 1975–Sep 2011

Table 3.3: Analysis with the CACIS software (OECD CLI series)

Series	Turning points detection					Peak Lead	Corr. Coeff.
	Targeted	Missed (%)	Extra (%)	Mean Lead	St. Dev.		
OJ1	15	13.3	40.0	5.1	3.9	3	0.81
OJ2	16	56.3	50.0	12.7	4.1	15	0.45
OJ3	15	33.3	40.0	5.5	5.8	3	0.35
OJ4	15	6.7	13.3	0.1	1.9	0	0.92
OJ5	15	53.3	66.7	7.7	5.8	18	0.41
OJ6	16	31.3	37.5	6.9	6.4	4	0.58
OJ7	15	53.3	60.0	7.1	6.2	24	0.16
OJ8	12	8.3	16.7	4.7	5.1	3	0.80

Table 3.4: Analysis with the CACIS software (Japanese government CLI series)

	Turning points detection						
Series	Targeted	Missed (%)	Extra (%)	Mean Lead	St. Dev.	Peak Lead	Corr. Coeff.
JG1	15	13.3	46.7	6.0	5.0	2	0.76
JG2	15	6.7	46.7	4.3	4.0	3	0.78
JG3	15	40.0	26.7	1.6	2.3	0	0.73
JG4	15	33.3	20.0	0.5	3.2	−1	0.82
JG5	16	50.0	56.3	5.5	6.3	17	0.37
JG6	16	18.8	43.8	2.8	4.6	6	0.74
JG7	15	6.7	6.7	−0.4	3.3	−2	0.76
JG8	15	53.3	60.0	7.1	6.2	24	0.16
JG9	16	31.3	37.5	6.9	6.4	4	0.58
JG10	15	26.7	53.3	0.8	3.5	1	0.77
JG11	15	13.3	60.0	6.1	3.5	4	0.79

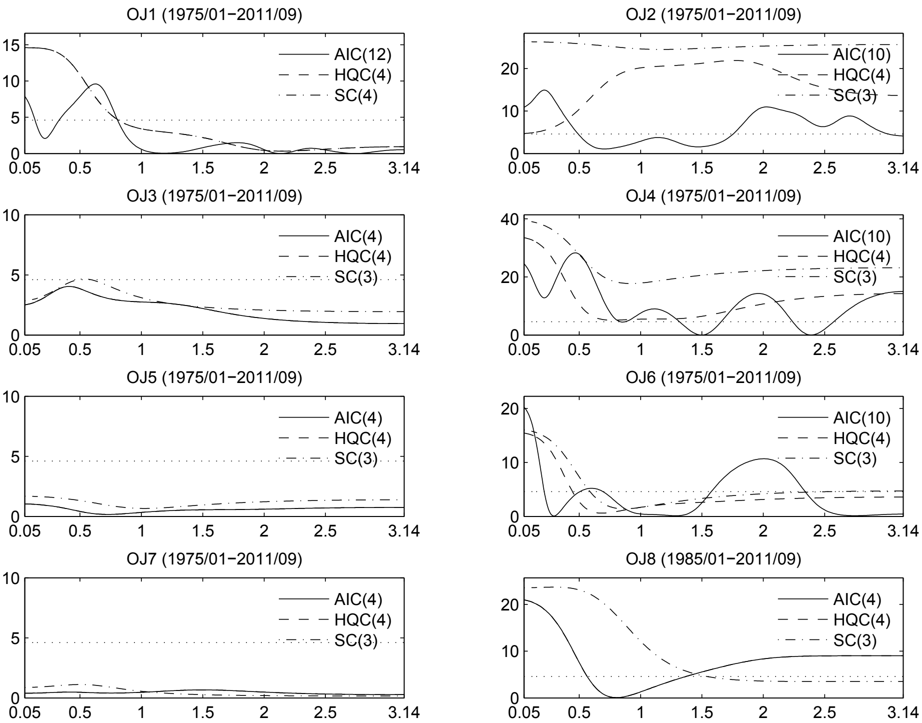

Figure 3.1: Frequency domain causality tests of the component series of the OECD CLI series

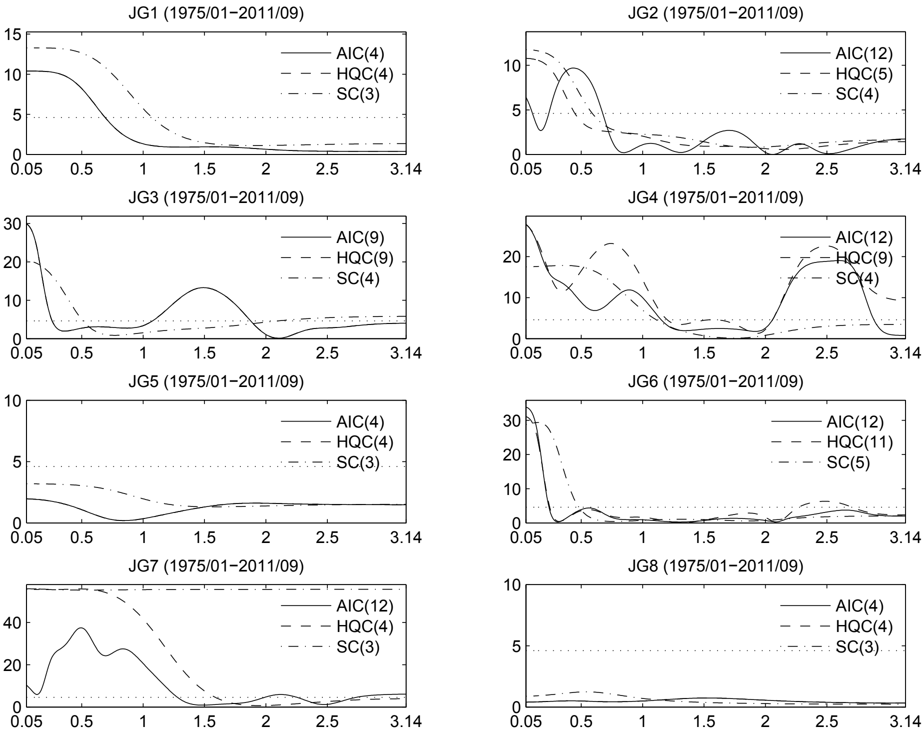

Figure 3.2: Frequency domain causality tests of the component series of the Japanese government CLI series

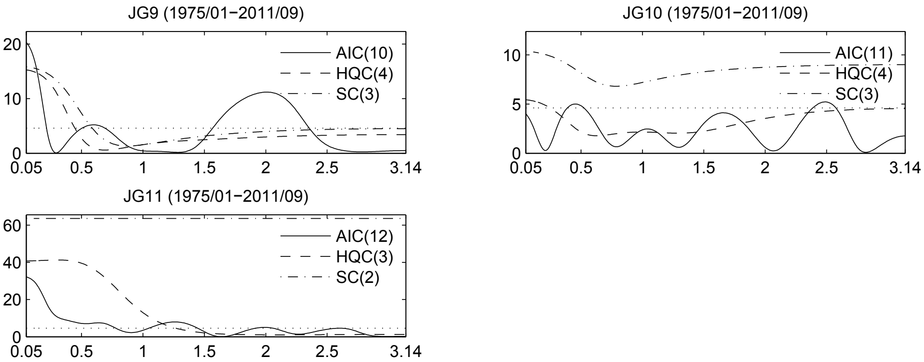

Figure 3.3: Frequency domain causality tests of the component series of the Japanese government CLI series

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Chapter 4

The Informational Role of Commodity Prices in Formulating Monetary Policy: A Reexamination under the Frequency Domain

4.1 Introduction

The recent dramatic increases in commodity prices have stimulated the research on the relationship between commodity prices and monetary policy. There is by now substantial evidence in the literature that commodity prices can signal the future movements in the economy and thus are useful in formulating monetary policy.[1] However, the literature does not clarify if this relationship is stable over time. In this paper, based on the monthly U.S. data from January 1957 to December 2011, we attempt to analyze whether commodity prices contain useful information for managing monetary policy at various time periods. This analysis is rather important because it might provide policymakers with additional insights on the informational role of commodity prices in setting monetary policy.

The usefulness of commodity prices in formulating monetary policy can be assessed by examining the predictive power of commodity prices for a set of macroeconomic and monetary variables. There are several methodologies that allow us to achieve this. In the earlier studies, the standard Granger causality test based on the vector autoregressive (VAR) model enjoyed considerable popularity (Garner, 1989; Cody and Mills, 1991). This test requires the VAR system to be stationary. Therefore, it is necessary to carry out the unit root and cointegration tests to correctly specify the VAR model. However, as pointed

[1] See, for example, Cody and Mills (1991), Awokuse and Yang (2003) and Bhar and Hamori (2008).

out by Mavrotas and Kelly (2001), both of these two tests are not very reliable in some situations, and thus incorrect inferences could be made about the issue of causality. To avoid this, the recent studies employed the Toda and Yamamoto (1995) methodology to reexamine the causality between commodity prices and macroeconomic variables (Awokuse and Yang, 2003; Bhar and Hamori, 2008). This methodology does not require precise knowledge of the order of integration of the variables and the order of cointegration among the series. Nevertheless, there exists a drawback in this methodology. That is, it can not further detect the causality in the frequency domain. In fact, several researchers have already documented that the extent and direction of causality can differ remarkably across the frequency domain (Granger, 1969; Geweke, 1982; Hosoya, 1991). Consequently, the research based on this methodology may fail to achieve a comprehensive understanding on the relationship between commodity prices and macroeconomic variables.

In contrast to the aforementioned studies, we thus apply the frequency domain causality test proposed by Breitung and Candelon (2006) to evaluate the predictive power of commodity prices for the selected variables. This test enables us to investigate the causality at various frequencies. In addition, we employ the Toda and Yamamoto (1995) procedure to establish standard inference for this test. Then, we can overcome the problems mentioned above and obtain relatively robust results. Several researchers have already applied this frequency domain causality test (Assenmacher-Wesche *et al.*, 2008; Bodart and Candelon, 2009; Gronwald, 2009; Ciner, 2011a, 2011b), but we may be the first to employ the Toda and Yamamoto (1995) procedure to establish standard inference for this test.

The remainder of this paper is organized as follows. In Section 4.2, we present the data set. In Section 4.3, we briefly introduce the method utilized in this study. In Section 4.4, we document the empirical analysis results and Section 4.5 concludes the paper.

4.2 Data

Following Cody and Mills (1991) and Awokuse and Yang (2003), the monthly US data used in this research are the money stock (M2), the interest rate on federal funds (FF), the consumer price index (CPI), the index of industrial production (IP) and the Commodity Research Bureau's price index for all commodities (CRB). The CRB index measures spot market prices of 22 sensitive basic commodities and usually serves as an early indicator of impending changes in economic activity. Because the 22 commodities do not include the energy materials such as crude oil and natural gas, the UK average brent oil price index (OIL) and the Reuters continuous commodity index (CCI) are also investigated in the research. The Reuters CCI is a futures index and its former name is Reuters CRB index.[2] This index has undergone several revisions. From 1983, the crude oil and heating oil are included in the CCI, and the natural gas is also added in the last revision in 1995. In this paper, all of the data are obtained from Thomson Datastream Database. The period covered is between January 1957 and December 2011. All variables with the exception of federal fund rate are transformed by natural logarithms.

4.3 Methodology

In this section, we briefly illustrate the frequency domain causality test based on a five-dimensional VAR model. First, let us consider a five-dimensional vector of time series $\boldsymbol{x}_t = [x_{1t}, x_{2t}, x_{3t}, x_{4t}, x_{5t}]'$ observed at $t = 1, \ldots, T$. It is supposed that $\boldsymbol{x}_t$ has a finite-order VAR representation of the form

$$\boldsymbol{x}_t = \boldsymbol{\mu} + \boldsymbol{\Theta}_1 \boldsymbol{x}_{t-1} + \boldsymbol{\Theta}_2 \boldsymbol{x}_{t-2} + \cdots + \boldsymbol{\Theta}_k \boldsymbol{x}_{t-k} + \boldsymbol{\varepsilon}_t, \tag{4.1}$$

where $\boldsymbol{\mu}$ is the constant term; $\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \ldots, \boldsymbol{\Theta}_k$ are coefficient matrices; and the error vector $\boldsymbol{\varepsilon}_t$ is white noise with zero mean and positive definite covariance matrix.

Let $\theta_{15,i}$ denote the (1,5)-element of $\boldsymbol{\Theta}_i$ for $i = 1, \ldots, k$, $\boldsymbol{\beta} = [\theta_{15,1}, \theta_{15,2}, \ldots, \theta_{15,k}]'$ and

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cdots & \cos(k\omega) \\ \sin(\omega) & \sin(2\omega) & \cdots & \sin(k\omega) \end{bmatrix}.$$

Then, according to Breitung and Candelon (2006), if the system is stationary, the null

[2]There is another popular commodity price index named Reuters/Jefferies CRB Index. Both the CCI and Reuters/Jefferies CRB index stem from the old Reuters CRB index. In 2005, the Reuters CRB index was revised (the tenth revision) greatly, and has been named Reuters/Jefferies CRB Index since this revision. Meanwhile, the old Reuters CRB index based on the ninth revision is still provided, and named as continuous commodity index (CCI). Thus, before 2005, the CCI is the same as the Reuters/Jefferies CRB index. In the present paper, we use the CCI because its composition does not change too much in recent years. For more details, please see `http://www.crbtrader.com/crbindex/symbols.asp`

hypothesis that x_{5t} does not cause x_{1t} at frequency ω can be expressed as

$$H_0 : \boldsymbol{R\beta} = \boldsymbol{0}. \tag{4.2}$$

The ordinary χ^2 test statistic for Eq. (4.2) is asymptotically distributed as $\chi^2(2)$ for $\omega \in (0, \pi)$.

Here, we assume that the order of integration of the series is unknown and the above VAR model is fitted in levels. In this case, as suggested by Breitung and Candelon (2006), the Toda and Yamamoto (1995) procedure can be applied. First, we artificially augment the correct level VAR order k, by the maximal integration order d of the series. Then, let us consider the level VAR model of order $(k + d)$:

$$\boldsymbol{x}_t = \boldsymbol{\mu}^* + \boldsymbol{\Theta}_1^* \boldsymbol{x}_{t-1} + \boldsymbol{\Theta}_2^* \boldsymbol{x}_{t-2} + \cdots + \boldsymbol{\Theta}_k^* \boldsymbol{x}_{t-k} + \boldsymbol{\Theta}_{k+1}^* \boldsymbol{x}_{t-k-1} + \cdots + \boldsymbol{\Theta}_{k+d}^* \boldsymbol{x}_{t-k-d} + \boldsymbol{\varepsilon}_t^*, \tag{4.3}$$

where $\boldsymbol{\mu}^*$ is the constant term; $\boldsymbol{\Theta}_1^*, \boldsymbol{\Theta}_2^*, \ldots, \boldsymbol{\Theta}_{k+d}^*$ are coefficient matrices; and $\boldsymbol{\varepsilon}_t^*$ is the error vector.

Let $\theta_{15,i}^*$ denote the (1,5)-element of $\boldsymbol{\Theta}_i^*$ for $i = 1, \ldots, k$, $\boldsymbol{\beta}^* = [\theta_{15,1}^*, \theta_{15,2}^*, \ldots, \theta_{15,k}^*]'$. Then, we can test the null hypothesis that x_{5t} does not cause x_{1t} at frequency ω by testing the linear restrictions on the (1,5)-elements of the first k coefficient matrices in Eq. (4.3), which can be expressed as

$$H_0 : \boldsymbol{R\beta}^* = \boldsymbol{0}. \tag{4.4}$$

Note that if $\boldsymbol{R} = \boldsymbol{I}_k$, the above null hypothesis corresponds to the hypothesis of the conventional Granger causality test based on the Toda and Yamamoto (1995) procedure.

Let $\boldsymbol{S}_1 = [1, 0, 0, 0, 0]'$, $\boldsymbol{S}_2 = [0, 0, 0, 0, 1]'$ and $\boldsymbol{S} = \boldsymbol{I}_k \otimes \boldsymbol{S}_2'$, then the Wald test statistic W for Eq. (4.4) can be written as

$$W = (T-k-d)\{\boldsymbol{R}(\boldsymbol{S} \otimes \boldsymbol{S}_1')\mathrm{vec}(\hat{\boldsymbol{\Theta}}^*)\}'[\boldsymbol{R}(\boldsymbol{S} \otimes \boldsymbol{S}_1')\hat{\boldsymbol{\Sigma}}(\boldsymbol{R}(\boldsymbol{S} \otimes \boldsymbol{S}_1'))']^{-1}\{\boldsymbol{R}(\boldsymbol{S} \otimes \boldsymbol{S}_1')\mathrm{vec}(\hat{\boldsymbol{\Theta}}^*)\}, \tag{4.5}$$

where $\hat{\boldsymbol{\Theta}}^*$ is the ordinary least squares estimator of $\boldsymbol{\Theta}^* = [\boldsymbol{\Theta}_1^*, \ldots, \boldsymbol{\Theta}_k^*]$ and $\hat{\boldsymbol{\Sigma}}$ is the consistent estimator of the covariance matrix of $\sqrt{T-k-d}\ \mathrm{vec}(\hat{\boldsymbol{\Theta}}^* - \boldsymbol{\Theta}^*)$ based on Eq. (4.3).

The Wald test statistic W is asymptotically distributed as $\chi^2(2)$ for $\omega \in (0, \pi)$. To evaluate the significance of the causal relationship, the Wald test statistic is compared with the 5% critical value (5.99) of a chi-square distribution with two degrees of freedom in this study.

4.4 Empirical results

In this section, the predictive power of commodity prices for the macroeconomic and monetary variables is evaluated in the frequency range $\omega \in (0, \pi)$. We choose $k = 4$ and $d = 1$ to specify the five dimensional VAR model. Below, we will show that the empirical results are robust to other alternative values of k and d.

4.4.1 Full sample results and sample split

Using the monthly U.S. data from January 1959 to December 1987, Cody and Mills (1991) find commodity prices, which are represented by the CRB index, can provide useful information for formulating monetary policy. Awokuse and Yang (2003) obtain similar results for a more recent sample period from January 1975 to December 2001. For comparison, we first assess if the CRB index is useful in setting monetary policy over the full sample period 1957M1-2011M12. Then we split the full sample into two sub-periods and check if the relationship is stable between the two sub-periods.

The results of the frequency domain causality tests for the full sample period are shown in Figure 4.1. It turns out that the CRB index is significant in forecasting federal fund rate, CPI and industrial production at the frequencies less than 0.5, 1.58 and 0.31 respectively, corresponding to the cycle lengths longer than about 12.5 months, 4 months and 20 months. From these empirical results, we may conclude that commodity prices are useful in managing monetary policy, and this is consistent with the findings of Cody and Mills (1991) and Awokuse and Yang (2003). However, we should note that the full sample analysis fails to illustrate if the above relationship is valid during different sub-periods of the full sample. Therefore, we split the full sample into the two sub-periods 1957M1-1983M12 and 1984M1-2011M12, and check if the CRB index is useful in managing monetary policy in these two sub-periods. There are two reasons for us choosing these two sub-periods. First, the volatility of inflation and output in the U.S. economy has been relatively low and stable since 1984 (McConnell and Perez-Quiros, 2000; Kahn *et al.*, 2002),[3] and the relationship between CRB index and monetary policy might be different in the high and low volatility periods of the U.S. economic activity (before and after 1984). Second, the predictive power of commodity prices for CPI has decreased significantly since about the mid-1980s (Furlong and Ingenito, 1996; Verheyen, 2010). In particular, using the CRB index and its sub-index for raw materials, Furlong and Ingenito (1996) argue that there has been no conventional Granger causality running from commodity prices to CPI since 1984.

Figures 4.2 and 4.3 present the results of the frequency domain causality tests for the respective sub-periods. We notice that the empirical results differ substantially between the two sub-periods. In the first sub-period, the predictive power of CRB index for M2, federal fund rate and CPI is found at frequencies less than 0.21, 0.65 and 1.73 respectively, corresponding to the cycle lengths longer than about 30 months, 9.67 months and 6.63 months. In contrast, in the second sub-period, the predictive content of CRB index for M2, federal fund rate and CPI has completely disappeared. Thus, unlike the full sample analysis, the results based on the sample split indicate that the CRB index is useful in setting monetary policy only in the first sub-period 1957M1-1983M12, and in the second sub-period 1984M1-2011M12 the CRB index can not be used as an informational variable for formulating monetary policy. Besides, we further confirm the results of Furlong and Ingenito (1996) in the frequency domain, that is, the CRB index does not Granger cause CPI after 1984.

Finally, it is of interest to compare the empirical results among the three periods. We notice that the causality measure from CRB index to federal fund rate and CPI in the full sample period is qualitatively similar to the results obtained in the first sub-period. Combing the results that the CRB index does not cause federal fund rate and CPI in the second sub-period, we consider that the predictive power of the CRB index for federal fund rate and CPI found in the full sample period may be attributed to the corresponding predictive power detected in the first sub-period. This may explain why the CRB index can predict federal fund rate and CPI in both the full sample period and the first sub-period, but not in the second sub-period. These findings may be undetectable through conventional causality tests under the time domain.

[3] The volatility of the U.S. economic activity has declined substantially since the mid-1980s, and this phenomenon is typically referred to as the "Great Moderation" (Stock and Watson, 2002).

4.4.2 Rolling sample results and robustness

To shed further light on the predictive power of CRB index for the macroeconomic and monetary variables, we roll the sample and conduct the frequency domain causality tests with a window of 25 years each time. The first sample is 1957M1-1981M12, and the last refers to 1987M1-2011M12. The results are presented in Figure 4.4. The date indicated on the coordinate axis is the beginning year of the window. It shows that significant predictive power of CRB index for federal fund rate, CPI and industrial production is found at the medium and low frequencies in the 1970s and early 1980s, but the predictability has disappeared since the beginning of the 1980s. In addition, although the predictive content of CRB index for M2 is detected at certain low frequencies in recent time, this predictability is relatively weak. Therefore, in line with the results based on the sample split, the rolling sample analysis also illustrates that the CRB index has contained no useful information for setting monetary policy since the early 1980s.

We do a number of robustness checks on our rolling sample analysis results corresponding the CRB index. We firstly assume the maximal order of integration of the series d=2, so the level VAR model is specified with k=4 and d=2. Secondly, we extend the number of lags and consider the case for k=6 and d=1. Thirdly, we assume that the series is first difference stationary and there exists no cointegration among them, so the null hypothesis can be tested in a stationary system of the first differences of the series with lag length k=4.[4] Finally, following Garner (1989), we replace the federal fund rate with the treasury bill rate in the level VAR model with k=4 and d=1. As the above analysis, we roll the sample and conduct frequency domain causality tests for these four cases. For simplicity, we only report the results corresponding to the predictive power of CRB index for CPI, which are presented in Figure 4.5.[5] We can observe that these empirical results are quite similar to the corresponding results for k=4 and d=1, both qualitatively and quantitatively.[6]

It is worthwhile to mention that the CRB index does not measure the prices of the

[4]Here we do not consider the cointegration case, but we should note that the frequency domain causality test can be conducted within a cointegration framework, please see Breitung and Candelon (2006) for more details.

[5]The results corresponding to the predictive power of CRB index for the other three variables are available from the author on request.

[6]We also replace the CPI with producer price index (PPI) in the level VAR model with k=4 and d=1, and investigate if the CRB index Granger causes PPI under the frequency domain. The findings are quite similar to results corresponding to CPI.

energy materials such as crude oil. Therefore, we further investigate if the oil price and continuous commodity index (CCI) are useful for setting monetary policy during different sub-periods. As we mentioned previously, the crude oil has been included in CCI since 1983. Similarly, we roll the sample and conduct the frequency domain causality tests in the level VAR models with k=4 and d=1. The empirical results corresponding to the two variables are shown in Figures 4.6 and 4.7. We can observe that, unlike the CRB index, both the oil price and CCI are significant in forecasting CPI after the early 1980s. More specifically, we find that, after the early 1980s, oil price can predict the short-term fluctuations of CPI while CCI can even forecast both the short- and long-term changes of CPI. Thus, since the early 1980s, although the CRB index which can be viewed as the non-oil commodity prices has lost its usefulness in formulating monetary policy, the oil price or the commodity price index including the oil price is still helpful for setting the monetary policy.[7]

4.5 Conclusions

Using the frequency domain causality tests, we have evaluated the informational role of commodity prices in formulating monetary policy over the period from January 1957 to December 2011. Following Cody and Mills (1991) and Awokuse and Yang (2003), we let the CRB index denote commodity prices. The full sample analysis indicates that the CRB index can provide valuable information for managing monetary policy, which is consistent with the results of Cody and Mills (1991) and Awokuse and Yang (2003). However, further investigation based on the sample split and rolling sample shows that the CRB index is useful in setting monetary policy in the 1970s and the beginning of the 1980s, but this usefulness has disappeared since the early 1980s. Because the CRB index does not include the price of crude oil, we also investigate if the oil price and CCI are helpful for setting monetary policy. We find that, after the early 1980s, both the oil price and CCI are able to predict the changes of CPI. Therefore, in this paper we argue that the non-oil commodity prices which are represented by the CRB index have lost its usefulness for formulating monetary policy since the early 1980s, but the oil price or the commodity price index including oil price can still be used as an informational variable for managing monetary

[7] Here, we want to mention that even though the oil price is useful for formulating monetary policy, the best monetary policy response to oil price fluctuations may depend on the reasons of the oil price changes, please see Bodenstein *et al.* (2012) for more details.

policy in recent years.

Our analysis results have several important implications for the previous research. (i) If the researchers let the CRB index denote commodity prices and do not split the sample, they may find commodity prices are useful for predicting CPI during the full sample period. (Cody and Mills, 1991; Awokuse and Yang, 2003). (ii) If the researchers let the CRB index denote commodity prices and split the sample into before and after the early 1980s, they may find commodity prices are useless for forecasting CPI after the beginning of 1980s (Furlong and Ingenito, 1996; Verheyen, 2010). (iii) The predictive power of the CRB index for CPI detected in the full sample period may be attributed to the predictive power that exists only before the early 1980s.

Interpreting the informational role of oil price into the frequency domain yields some additional insights for the policymakers. Before the early 1980s, oil price can predict CPI at both the low and high frequencies. After the early 1980s, however, the predictive power of oil price for CPI is only detected at the high frequencies. Consequently, in recent years oil price can only be used to forecast the short-term fluctuations of CPI. This finding is pretty in line with Herrera and Pesavento (2009), in which a relatively small and short-lived response of inflation to oil price shock has been detected since the early 1980s.

One issue we have not investigated in this research is the reason why the non-oil commodity prices have lost its usefulness in managing monetary policy since the early 1980s. Similarly to Clarida *et al.* (2000) and Herrera and Pesavento (2009), we argue that the more efficient monetary policy during the Volcker-Greenspan-Bernanke era may insulate the U.S. economy from the fundamental price shocks of the non-oil commodity, which may make the non-oil commodity prices contain no helpful information for formulating monetary policy after the early 1980s. Accordingly, in the future it might be interesting to examine the role of the monetary authorities on the relationship between non-oil commodity prices and macroeconomic variables, and verify if our results are valid when controlling for monetary policy.

Generalizing our findings to other economies should be cautious because the relationship between commodity price and monetary policy may be related to (i) whether the country is a net importer or exporter of the commodity, and (ii) the attitude of the country's central bank to price stability. Besides, we should note that some other factors may also be linked with these selected variables. For example, the exchange rate changes may have certain connections with CPI. In such a case, as suggested by Hosoya (2001) and Breitung and Candelon (2006), the influence of the exchange rate can be eliminated, and the partial causality running from commodity prices to CPI can be investigated in a three dimensional system including commodity prices, exchange rate and CPI. Therefore, our research can be improved in the future, and we consider that analyzing the partial causality running from commodity prices to the macroeconomic and monetary variables would be a very interesting research topic.

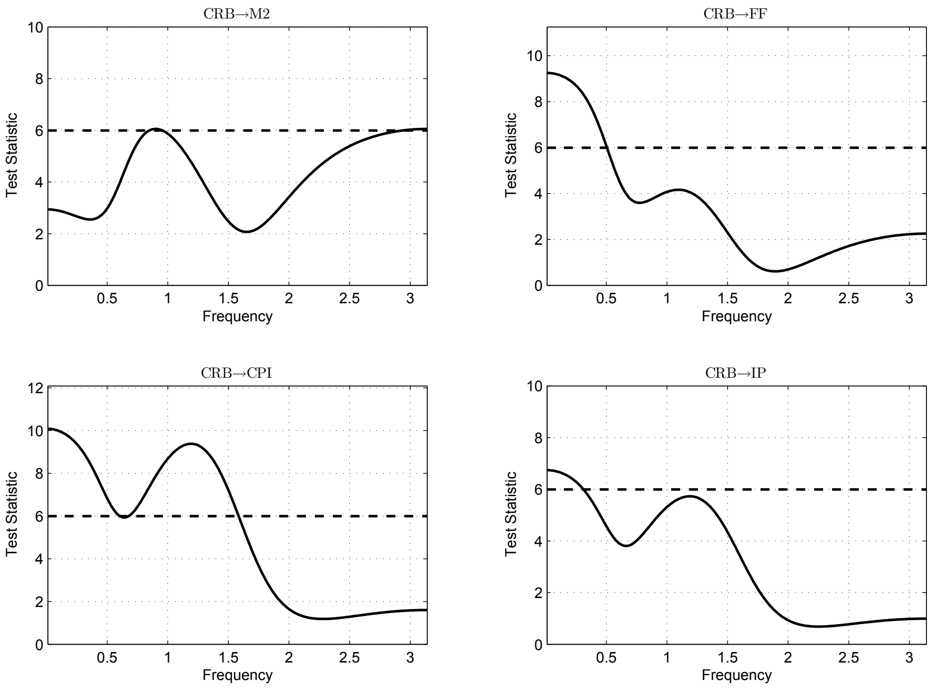

Figure 4.1: The causality measure from CRB to the respective variables: 1957M1-2011M12. The solid lines depict the Wald test statistics. The horizonal line represents the 5% critical value (5.99).

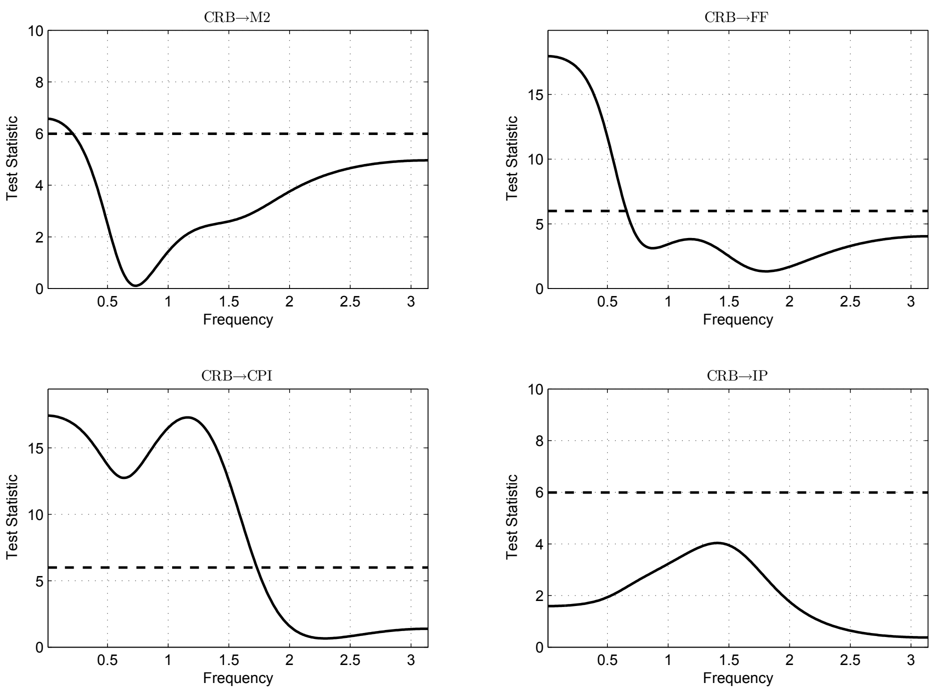

Figure 4.2: The causality measure from CRB to the respective variables: 1957M1-1983M12. The solid lines depict the Wald test statistics. The horizonal line represents the 5% critical value (5.99).

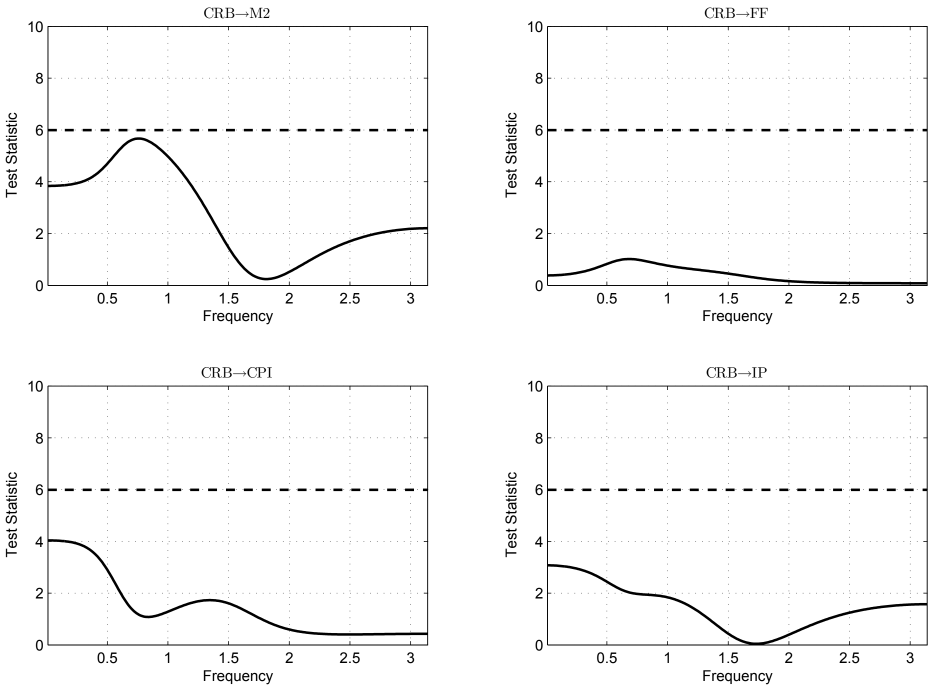

Figure 4.3: The causality measure from CRB to the respective variables: 1984M1-2011M12. The solid lines depict the Wald test statistics. The horizonal line represents the 5% critical value (5.99).

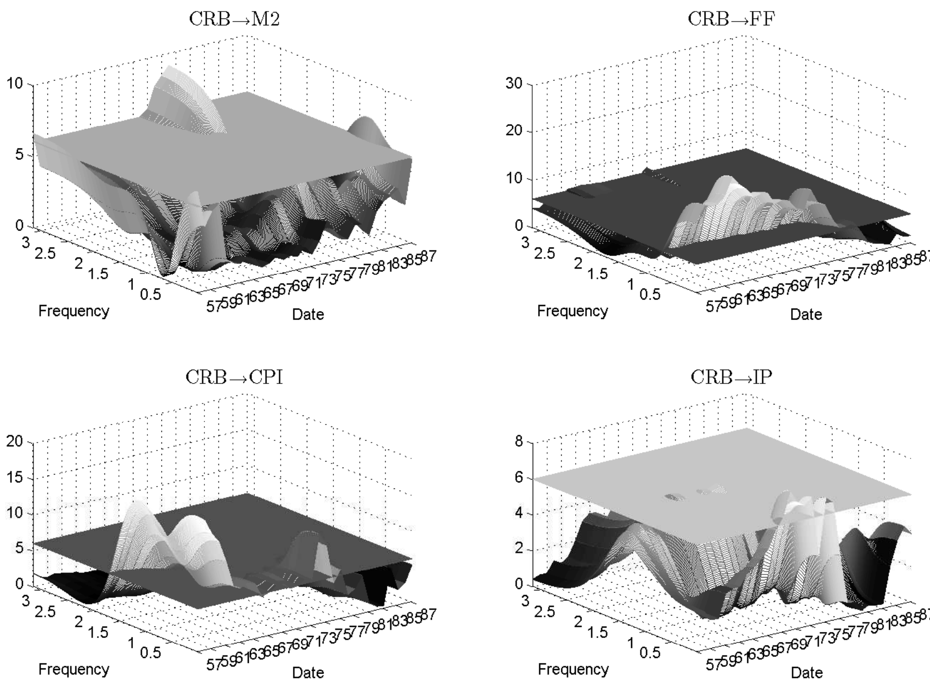

Figure 4.4: The causality measure from CRB to the respective variables: rolling samples. The horizonal plane represents the 5% critical value (5.99).

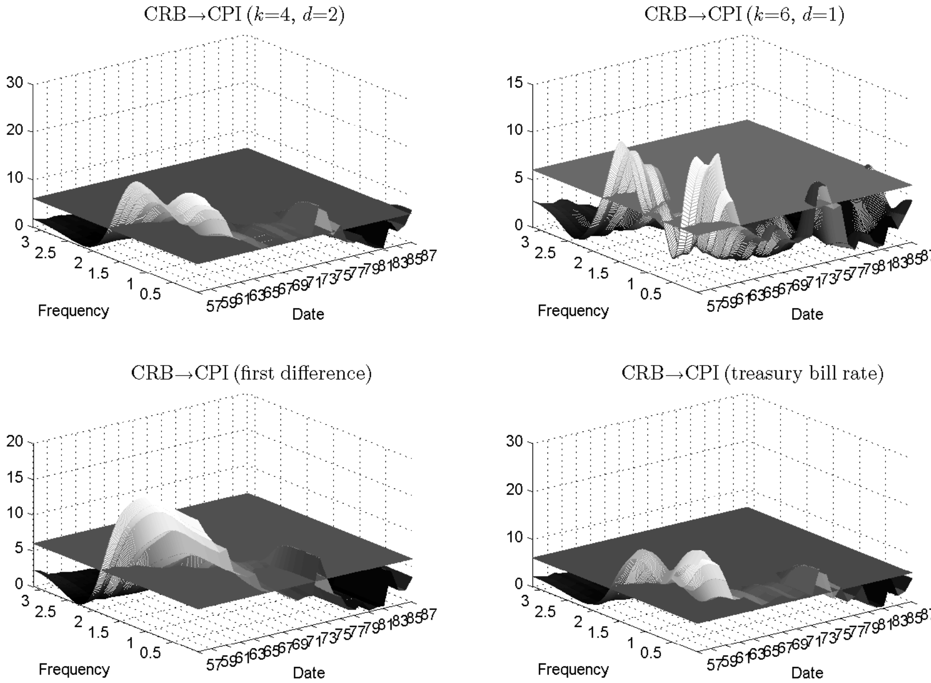

Figure 4.5: The causality measure from CRB to CPI: rolling samples and different specifications of the VAR models. The horizonal plane represents the 5% critical value (5.99).

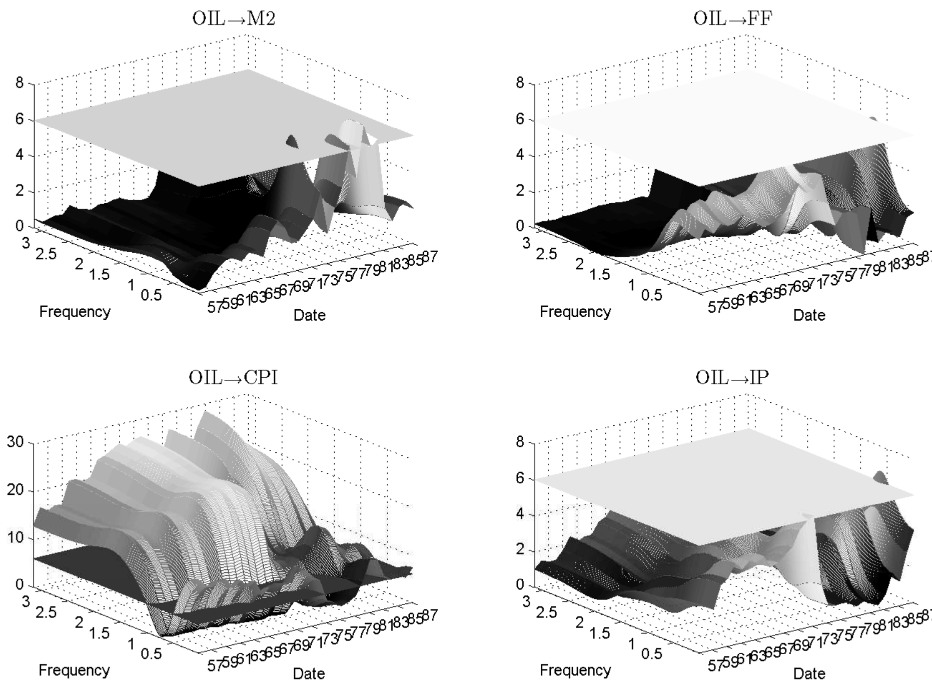

Figure 4.6: The causality measure from OIL to the respective variables: rolling samples The horizonal plane represents the 5% critical value (5.99).

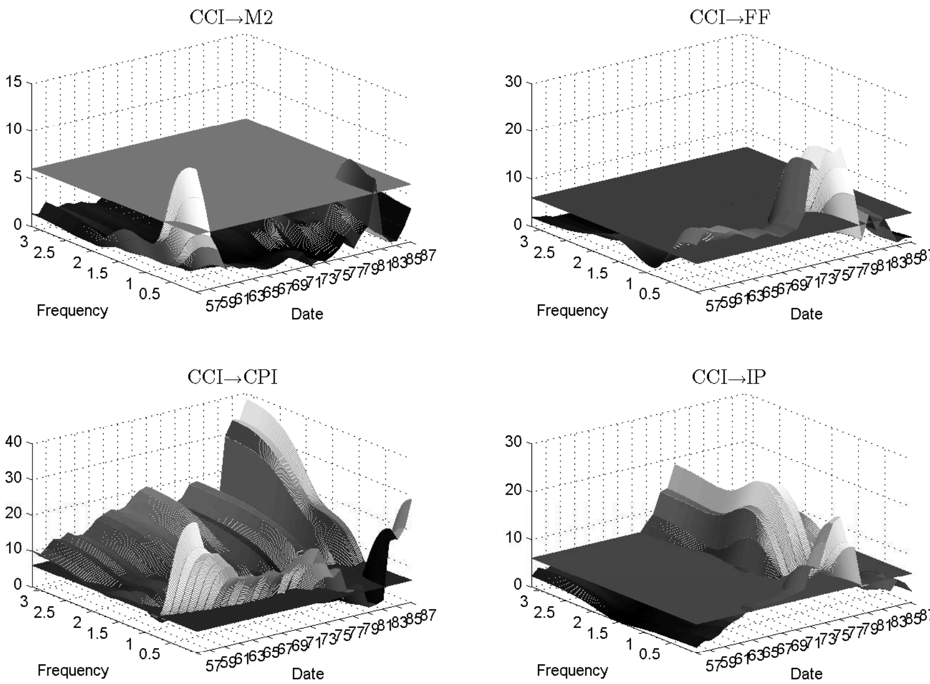

Figure 4.7: The causality measure from CCI to the respective variables: rolling samples The horizonal plane represents the 5% critical value (5.99).

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Chapter 5

Commodity Prices, Manufactured Goods Prices and Inflation: Evidence From Japan

5.1 Introduction

Commodity prices are generally considered to have more predictive power for inflation than do the manufactured goods prices. There are at least two reasons for this argument. First, since commodities are traded in continuous auction market, commodity prices react demand pressures or supply shocks more quickly than do the manufactured goods prices (Garner, 1989; Cody and Mills, 1991). Second, traders tend to sign short-term contracts for primary commodities and long-term contracts for manufactured goods, so commodity prices respond more rapidly than manufactured goods prices to economic fluctuations (Bordo, 1980).

However, some recent studies find that the predictive power of commodity prices for inflation has significantly decreased since the mid-1980s (Herrera and Pesavento, 2009; Verheyen, 2010). In this case, using the monthly Japanese data from January 1970 to December 2011, this article attempts to analyze the predictive power of commodity prices and manufactured goods prices for inflation. Because the Japanese economy suffers from structural changes in the early 1990s (Sato, 2002; Fang and Miller, 2009; Yamada and Jin, 2012), we split the full sample into the two sub-periods 1970M1-1990M12 and 1991M1-2011M12. Then we investigate the differences of the predictive power of commodity prices and manufactured goods prices for inflation in the two subperiods. This analysis is rather important because it might provide policymakers with additional insights on the informational roles of commodity prices and manufactured goods prices for inflation.

The method employed in this paper is the frequency domain causality test proposed by Breitung and Candelon (2006). Unlike the conventional Granger causality test, this test allows us to statistically test the causality at various frequencies. Consequently, using this test, we can easily find the differences of the predictive power of commodity prices and manufactured goods prices for inflation during the two sub-periods. We choose consumer price index (CPI) as a measure of inflation. Since the previous research fails to reach a consensus on its integration order (Cody and Mills, 1991; Beechey and Österholm, 2008; Verheyen, 2010), we employ the Toda and Yamamoto (1995) procedure to establish standard inference for the test, which makes us skip the unit root tests of the variables, including CPI. We believe that it makes us obtain the robust empirical results.

The remainder of this paper is organized as follows. Section 5.2 introduces the data and empirical methodology. Section 5.3 presents the empirical results and Section 5.4 concludes the paper.

5.2 Data and empirical methodology

As mentioned previously, we choose CPI to measure inflation, and we select NIKKEI commodity price index (NCP) and domestic corporate goods price index (DCGP) to represent commodity prices and manufactured goods prices respectively. The data are obtained from Thomson Datastream Database. The period covered is between January 1970 and December 2011. As mentioned above, we split the full sample into the two sub-periods 1970M1-1990M12 and 1991M1-2011M12, with 252 monthly observations in each sub-period.

The method applied in this research is the frequency domain causality test whose inference is established through the Toda and Yamamoto (1995) procedure. To illustrate this method, let us consider a two dimensional vector of time series $\boldsymbol{z}_t = [x_t, y_t]'$ observed at $t = 1, \ldots, T$. In the present paper, x_t will be CPI and y_t will be NCP or DCGP. We assume that k and d are the correct vector autoregressive (VAR) order and maximal integration order of the series. To apply the Toda and Yamamoto (1995) procedure, we artificially augment the correct VAR order k by the maximal integration order d. Then, let us consider the level VAR model of order $(k + d)$:

$$\boldsymbol{z}_t = \boldsymbol{\mu} + \boldsymbol{\Theta}_1 \boldsymbol{z}_{t-1} + \boldsymbol{\Theta}_2 \boldsymbol{z}_{t-2} + \cdots + \boldsymbol{\Theta}_k \boldsymbol{z}_{t-k} + \boldsymbol{\Theta}_{k+1} \boldsymbol{z}_{t-k-1} + \cdots + \boldsymbol{\Theta}_{k+d} \boldsymbol{z}_{t-k-d} + \boldsymbol{\varepsilon}_t, \quad (5.1)$$

where $\boldsymbol{\mu}$ is the constant term; $\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \ldots, \boldsymbol{\Theta}_{k+d}$ are coefficient matrices; and $\boldsymbol{\varepsilon}_t$ is the error vector.

Let $\theta_{12,i}$ denote the (1,2)-element of $\boldsymbol{\Theta}_i$ for $i = 1, \ldots, k$, $\boldsymbol{\beta} = [\theta_{12,1}, \theta_{12,2}, \ldots, \theta_{12,k}]'$ and

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cdots & \cos(k\omega) \\ \sin(\omega) & \sin(2\omega) & \cdots & \sin(k\omega) \end{bmatrix}.$$

Then, we can test the null hypothesis that y_t does not cause x_t at frequency ω by testing the linear restrictions on the (1,2)-elements of the first k coefficient matrices in Eq. (5.1), which can be expressed as

$$H_0 : \boldsymbol{R\beta} = \boldsymbol{0}. \tag{5.2}$$

Note that if $\boldsymbol{R} = \boldsymbol{I}_k$, the above null hypothesis corresponds to the hypothesis of the conventional Granger causality test based on the Toda and Yamamoto (1995) procedure, and the Wald test statistic can be calculated the same as below.

Let $\boldsymbol{S}_1 = [1, 0]'$, $\boldsymbol{S}_2 = [0, 1]'$ and $\boldsymbol{S} = \boldsymbol{I}_k \otimes \boldsymbol{S}_2'$, then the Wald test statistic W for Eq. (5.2) can be written as

$$W = (T-k-d)\{\boldsymbol{R}(\boldsymbol{S}\otimes\boldsymbol{S}_1')\mathrm{vec}(\hat{\boldsymbol{\Theta}})\}'[\boldsymbol{R}(\boldsymbol{S}\otimes\boldsymbol{S}_1')\hat{\boldsymbol{\Sigma}}(\boldsymbol{R}(\boldsymbol{S}\otimes\boldsymbol{S}_1'))']^{-1}\{\boldsymbol{R}(\boldsymbol{S}\otimes\boldsymbol{S}_1')\mathrm{vec}(\hat{\boldsymbol{\Theta}})\}, \tag{5.3}$$

where $\hat{\boldsymbol{\Theta}}$ is the ordinary least squares estimator of $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1, \ldots, \boldsymbol{\Theta}_k]$ and $\hat{\boldsymbol{\Sigma}}$ is the consistent estimator of the covariance matrix of $\sqrt{T-k-d}\ \mathrm{vec}(\hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta})$ based on Eq. (5.1).

The Wald test statistic W is asymptotically distributed as $\chi^2(2)$ for $\omega \in (0, \pi)$. To evaluate the significance of the causal relationship, the Wald test statistic is compared with the 5% critical value (5.99) of a chi-square distribution with two degrees of freedom in this study.

5.3 Empirical results

We employ Schwarz Information Criterion (SIC) to decide the correct VAR order k. For the value of d, because the order of integration of these macroeconomic variables is at most two, both $d = 1$ and $d = 2$ are considered.

Figures 5.1 and 5.2 present the results of the frequency domain causality tests for the respective sub-periods. As we can see the causality measure for $d = 1$ is qualitatively the same as the causality measure for $d = 2$, so we mainly focus on the results for $d = 1$. We notice that the empirical results differ substantially between the two sub-periods. In the first sub-period, the causality running from NCP and DCGP to CPI is detected in the frequency range $\omega \in (0, \pi)$, which indicates that NCP and DCGP can predict inflation in both the short- and long-term periods. However, in the second sub-period, the causality running from NCP to CPI is merely detected at frequencies less than 0.59, and the causality running from DCGP to CPI is found for frequencies below 0.69, between 0.78 and 1.38 and above 2.31. Consequently, both NCP and DCGP fail to forecast CPI at certain frequencies during the second sub-period. In particular, we notice that NCP fails to Granger cause CPI at the high frequencies. This implies that NCP can not forecast the short-term changes of inflation. Thus, in the second sub-period, NCP can only predict the long-term fluctuations of inflation, while DCGP is still significant in forecasting inflation in both the short- and long-term periods.

5.4 Conclusions

Using the monthly Japanese data from January 1970 to December 2011, we investigate the predictive power of commodity prices and manufactured goods prices for inflation. Because the Japanese economy suffers from some structural changes in the beginning of the 1990s, we split the full sample into the two sub-periods 1970M1-1990M12 and 1991M1-2011M12. In the first sub-period 1970M1-1990M12, we find commodity prices and manufactured goods prices Granger cause CPI in the frequency range $\omega \in (0, \pi)$. This indicates that commodity prices and manufactured goods prices can predict the short- and long-term fluctuations of inflation. In the second sub-period 1991M1-2011M12, however, the causality running from commodity prices and manufactured goods prices to CPI is merely detected at some small ranges of frequencies. In this sub-period, manufactured goods prices are still significant in forecasting inflation in both the short- and long-run periods, while commodity prices can only predict the long-run fluctuations of inflation. Thus, in recent years it is inappropriate to use the commodity prices to predict the short-term changes of inflation in Japan.

Before closing, we should note that the factors such as the exchange rate changes and money growth also have certain effects on inflation. For example, Assenmacher-Wesche *et* *al.* (2008) find that money growth can predict the inflation of Japan at the low frequencies. In this case, using the Hosoya (2001) procedure, we can eliminate the effects of money growth and test the partial causality running from commodity prices and manufactured goods prices to inflation at various frequencies, which would be a very interesting research topic in the future.

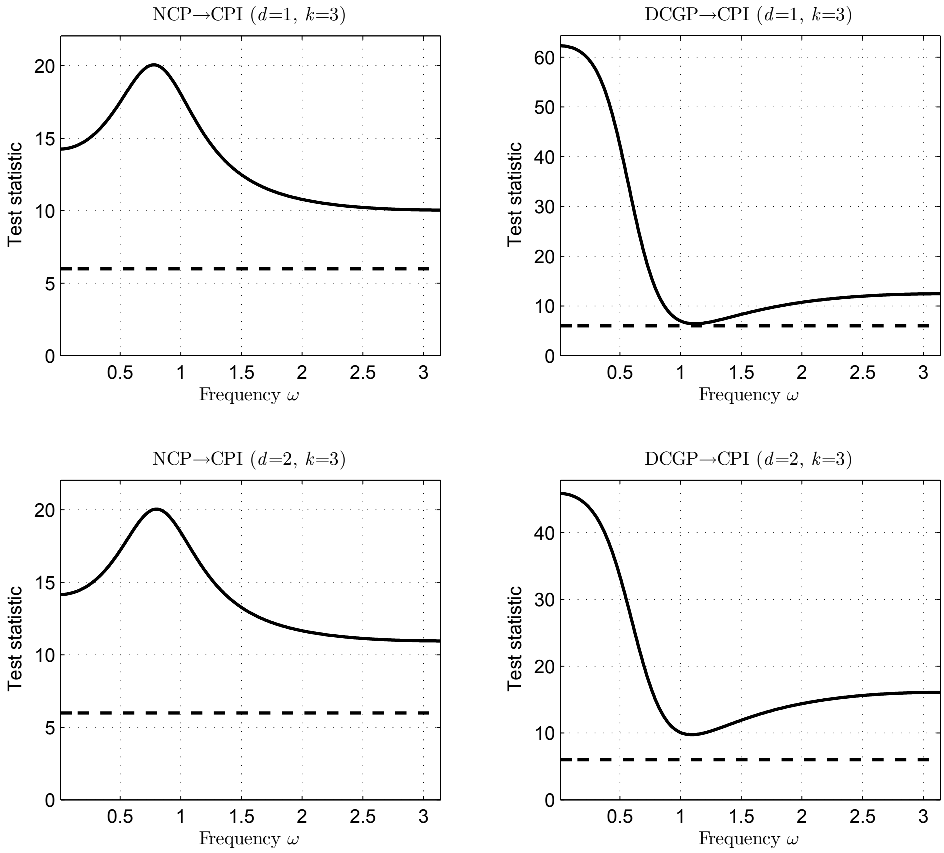

Figure 5.1: The results of the frequency domain causality tests for the first sub-period 1970M1-1990M12. Two top panels: causality measure from NCP and DCGP to CPI for d=1. Two bottom panels: causality measure from NCP and DCGP to CPI for d=2. The solid lines depict the Wald test statistics. The horizonal line represents the 5% critical value (5.99).

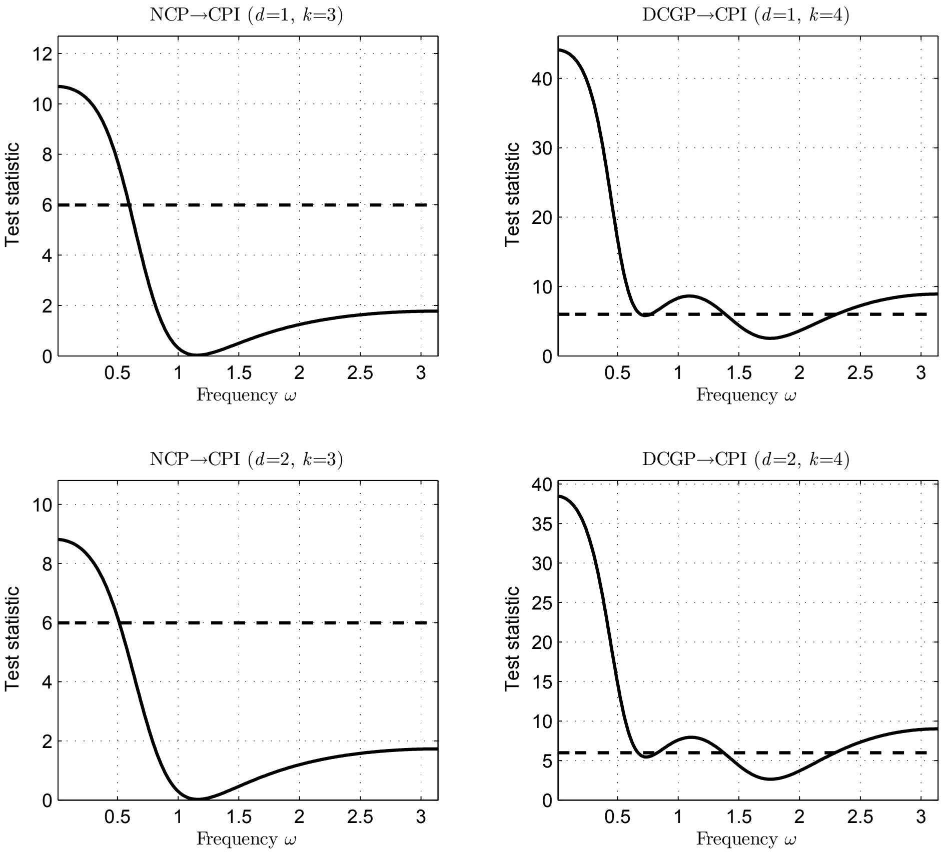

Figure 5.2: The results of the frequency domain causality tests for the second sub-period 1991M1-2011M12. Two top panels: causality measure from NCP and DCGP to CPI for d=1. Two bottom panels: causality measure from NCP and DCGP to CPI for d=2. The solid lines depict the Wald test statistics. The horizonal line represents the 5% critical value (5.99).

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