

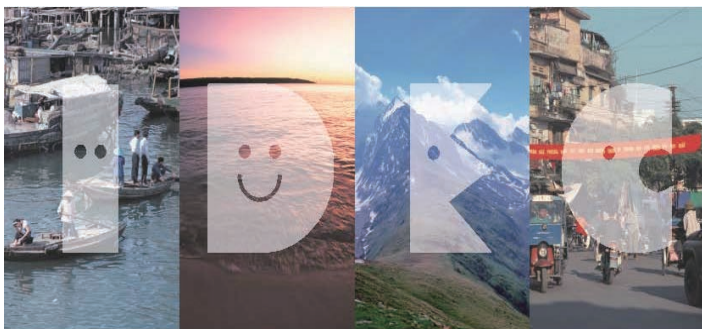
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Keisuke Kawata

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Department of Development Policy
Division of Development Science
Graduate School for International
Development and Cooperation (IDEC)
Hiroshima University
1-5-1 Kagamiyama, Higashi-Hiroshima
739-8529 JAPAN

Work hour mismatches and on-the-job search*

Keisuke Kawata[†]

Graduate School for International Development and Cooperation, Hiroshima University

Abstract

This paper explores an on-the-job search model with wage bargaining and mismatch. It considers two types of jobs and workers, and the instantaneous value of the job-worker match depends on their type. The most important assumption is that while the job type is fixed throughout its life, the worker type changes in accordance with a stochastic process. This paper shows that although the workers' turnover decision is privately efficient, this decision may be socially inefficient because of the hold-up problem.

1 Introduction

Today, work hour mismatches are one of the most important problems in labor markets. For instance, about a third of all U.S. workers say they would like to work either more or fewer hours than their currently work at the same hourly wage rate. This fact is clearly inconsistent with the standard neoclassical theory that assumes a worker can choose her (or his) working hours either directly by choosing working hours within a job or indirectly by choosing a job in a frictionless labor market.

Many empirical works suggest the importance of job-to-job transitions to eliminate these work hour mismatches because the variance of the change in hours worked is higher for movers than for stayers¹. These works also suggest that free choice of working hours within a job is unavailable, and workers must move to new jobs to

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[†]*E-mail address:* keisuke@hiroshima-u.ac.jp

¹See for example, Altonji and Paxson (1986, 1988), Kahn and Lang (1991), Dickens and Lundberg (1993), Stewart and Swaffield (1997), Euwals (2001), Martinez-Granado (2005), and Brewer and Framcescpmo (2008).

change their working hours. However, many labor economists have emphasized the importance of labor market friction which prevents workers from moving instantaneously to more suitable jobs. In fact, empirical evidence (see for example, Euwal, 2001 and Johnson, 2010) indicates that labor market frictions helps to explain the existence of work hour mismatches. Further, Reynolds and Aletraris (2006) suggested that some people reduce or eliminate mismatches in another way, that is by changing their preference of working hours.

Motivated by these empirical works, this paper provides an on-the-job search model that incorporates fluctuations in labor supply preferences, in which workers can eliminate the mismatch in two ways: (i) by moving to a suitable job by job-to-job transitions or (ii) by changing their labor supply preferences. This paper demonstrates that the fluctuations in labor supply preference is a source of inefficiency in job-to-job transitions, which will be the contribution of the current study to the field of on-the-job search literature. The efficiency of job-to-job transitions has been extensively studied by many researchers² because in the real economy, a large proportion of workers who move to new jobs do not experience unemployment (see Blanchard and Diamond, 1989).

In this paper, we develop a tractable on-the-job search model. There are two types of workers, and one type's opportunity costs of work are higher than that of the other type. There also exist two types of jobs, and one type's marginal productivity of working hours are higher than that of the other type. The instantaneous surplus of the job-worker match between a worker with low opportunity costs and a job with high marginal productivity is higher than the match between this worker and a job with low marginal productivity. Likewise, the instantaneous surplus of the match between a worker with high opportunity costs and a job with low marginal productivity is higher than the job-worker match between this worker and a job with high marginal productivity.

A key assumption of this model is that workers' types and the opportunity costs of working hours switch back and forth. A change in hours for housework is a typical example that supports this assumption. The necessary hours for housework depend on the family situation. The opportunity costs of a worker with a small child may be high because the worker must spend more time to care for the child. In general, by shocks such as childbirth and marriage, both the hours for housework and the opportunity costs of a worker are changed.

²For example, Burdett, Imai, and Wright (2004) demonstrated that the level of job-to-job transitions exceeded the optimal level because workers made excess effort in their on-the-job search. Gautier, Teulings, and Vuuren (2010) showed that on-the-job search models have additional externalities besides the well-known congestion externalities.

In this model, there are three types of equilibria: turnover equilibrium (TE), stay-in-type-L jobs equilibrium (SLE), and stay-in-type-S jobs equilibrium (SSE). These equilibria are characterized by workers' decisions on job-to-job transitions (called turnover decisions). In TE, workers try to move to a good match when the current match becomes a bad match. In SLE and SSE, each type of a worker continues to stay in her or his incumbent jobs regardless of her or his type. Moreover, in TE, mismatches are eliminated by both job-to-job transitions and changes in labor supply preferences, while in SLE and SSE, mismatches are eliminated only by changes in labor supply preferences. This paper first shows that the parameter domain of TE increases with the workers' bargaining power and job contact rates because the effectiveness of job-to-job transitions increases.

Second, this paper demonstrates that the market equilibrium may be socially inefficient even though the turnover decisions are privately efficient. Moreover, there are parameter sets under which the market equilibrium should be TE to maximize the social surplus, but is instead SLE or SSE. The reason behind the above phenomenon is the hold-up problem, which means that the turnover decisions have implications for not only incumbent and poaching jobs, but also for other vacant jobs that may poach workers in the future. In the steady state equilibrium, other vacant jobs can gain a positive value only in TE because these jobs can poach workers only in TE. However, these jobs cannot influence the turnover decisions, and workers then ignore the capital gain of these jobs. Only if the worker has full bargaining power, the market equilibrium is socially efficient because the hold-up inefficiencies disappear.

Similar to this model, in other studies (e.g., Pissarides, 1994; Cahuc, Postel-Vinay, and Robin, 2006), workers do not take into account the benefits of other vacant jobs, but turnover decisions are socially efficient. This is because in their models, the workers and the social planner always prefer to move to good matching jobs. In this model, the worker stays in a bad match rather than moving to a (temporarily) good match because the worker's value in a (seemingly) good match may be lower because of a low continuation value after the preference shocks on the new match.

Our paper is related to several studies. First, this paper addresses the issue of working hours in the frictional labor market. Pissarides (2007) and Kudou and Sasaki (2011) also addressed this issue using the job search model. However, in these studies, authors assumed ex-ante homogeneous workers and then ruled out work hour mismatches. Second, we study the efficiency of job-to-job transitions and demonstrate that the fluctuation of workers' type is a source of inefficiency. Kiyotaki and Lagos (2007) demonstrated that job-to-job transitions

were inefficient in the on-the-job search model with replacement hiring, which is ruled out in this paper. In another study with a frictionless labor market, Bertola (2004) found that the level of job-to-job transitions was below the efficient level when workers were risk-averse; this differs from the present paper, wherein workers are risk-neutral. Felli and Harris (1996) constructed a job turnover model with learning about the job-specific skills of workers and showed that turnover decisions were socially inefficient. However, they considered only the case in which wages were determined by the Bertrand wage competition game. In this paper, wages are determined by the more general bargaining game of Cahuc, Postel-Vinay and Robin (2006). Moreover, this paper considers the effect of workers' bargaining power on efficiency and demonstrates that turnover decisions are socially efficient if workers have monopolistic bargaining power.

The rest of this paper is organized as follows. Section 2 presents the basic model. Section 3 defines the market equilibrium. Section 4 addresses the social planner problem with job-to-job transitions and the policy implications. In Section 5, to check the robustness of main results, we extend the basic model to free entry and the other bargaining game. Finally, Section 6 concludes.

2 The Basic Model

We consider a continuous-time search model with on-the-job search and wage bargaining. There is a continuum of workers with a total mass of one and a large number of jobs.

At any instant, workers are either of type l or s . While type- l workers prefer to work full-time, type- s workers prefer to work part-time. Formally, a worker allocates her (or his) unit mass total time between market work, h , housework, t_i , and leisure, $1 - h - t_i$. While h is endogenously determined, t_i is exogenously determined and depends on the worker's type. We assume that $0 \leq t_l < t_s$, which means that type- s workers must spend more time on housework than type- l workers.

The instantaneous utility of a type- $i \in \{l, s\}$ worker is defined by a quasi-linear function as $w + u(1 - h - t_i)$, where w is wages, and $u(\cdot)$ is the utility of leisure, assuming that $u' > 0, u'' < 0$. Because $t_l < t_s$ for any h , $u'(1 - h - t_s) > u'(1 - h - t_l)$, which means that the marginal disutility from the working hours of type- s workers is higher than from the working hours of type- l workers. In other words, the opportunity cost of type- s workers is higher than that of type- l workers.

There are two types of jobs, full-time jobs (denoted as L) and part-time jobs (denoted as S). According to

Acemoglu (1999), a type- $j \in \{L, S\}$ job must buy capital k_j with constant running costs ρ . If the job employs one worker, it produces a flow of output $F(k_j, h)$. Further, we assume that $F_k, F_h > 0$, $F_{kk}, F_{hh} < 0$, and $F_{hk} > 0$, which means that capital and working hours are complementary in production. The instantaneous profit of a type- j filled job is also defined by $F(k_j, h) - w - \rho k_j$. Throughout this paper, we assume that $k_L > k_S$, which implies that not only the marginal productivity but also the running costs of type- L jobs are higher than those of type- S jobs.

A key assumption of this model is that while a job's type is constant throughout its life, a worker's type switches by exogenous shocks. $\pi_{ii'}$ ($i \neq i' \in \{l, s\}$) is the Poisson arrival rate of a shock by which a worker's type switches from i to i' . As shown in the next section, a suitable job type for a worker may change by this shock.

2.1 Turnover process

When an employed worker contacts a new vacant job, she (or he) decides whether to move to the poaching job after they have observed each other's type. More precisely, an on-the-job searcher can contact a type- j vacant job with an exogenous Poisson rate $p_j \in [0, \infty)$. If an employed worker contacts a new job (called the poaching job), an employment contract may be negotiated between the worker and the poaching job, and the worker decides whether to move to the job.

Assume that an employed worker who contacts a poaching job can move to the job with zero moving cost. Additionally, employed workers can search on the job with a very small search cost³, by which employed workers search on the job if and only if the expected capital gain of on-the-job search is positive.

To focus on on-the-job search, assume that the exogenous job destruction rate is zero and that the value of an unemployed worker is sufficiently low, which implies that voluntary job destruction can never occur. Under these assumptions, no worker is unemployed in the steady state. Note that the main results of this paper still hold even if the exogenous job destruction rate is positive.

Finally, note that throughout Sections 3 and 4, it is assumed that p_j is an exogenous rate, which is a standard assumption in on-the-job search literature. In Section 5, the main model is extended to an endogenous job entry model.

³Postel-Vinay and Robin (2004) discussed the case of endogenous search intensity.

2.2 Employment contracts

In negotiation, a job can offer a privately efficient employment contract, which determines the wage depending on the worker's type, denoted by w_h and w_l ; working hours, denoted by h_h and h_l ; and compensation from a worker moving via to the job at job-to-job transition, denoted by c_h and c_l . Working hours and compensation are first determined to maximize the joint value of the worker and the job. After that, wages are determined by the Nash sharing rule. A privately efficient contract is assumed in existing studies such as Garibaldi and Moen (2010) and Menzio and Shi (2011) with the wage posting model, and in Kawata and Sato (2012) with the wage bargaining model. If such a contract is available, there is no reason to refuse it from the viewpoint of both workers and jobs, from which we believe it to be acceptable.

Let an employment contract be denoted as $\Gamma = [w_h, w_l, h_h, h_l, c_h, c_l]$. Jobs and workers commit to the employment contract, which can be renegotiated by mutual agreement only. In an equilibrium, renegotiation may occur only if an employed worker contacts a vacant job.

Note that the efficient employment contract must satisfy individual rationality conditions for the worker because the value of unemployment is sufficiently low. Thus, the values of the employed worker before and after changing her (or his) type must be higher than the value of unemployment, and employed workers then do not quit to unemployment in an equilibrium.

3 Equilibrium

This section characterizes the equilibrium turnover pattern. The value of a type- i employed worker in a type- j job is denoted by $W(i, j, \Gamma)$, and of a type- j job with a type- i worker by $J(i, j, \Gamma)$. Additionally, the joint value $W(i, j, \Gamma) + J(i, j, \Gamma)$ is denoted by $T(i, j)$. It is important to note that the joint value does not depend on the employment contract Γ because working hours and compensation are determined to maximize the joint value, and the wages are the pure transfer device within a match.

3.1 Optimal working hours

Let (i, j) denote a match of a type- i worker and a type- j job. Under the private efficient employment contract, equilibrium working hours are determined to maximize the joint instantaneous value which is defined as

$F(k_j, h) + u(1 - h - t_i) - \rho k_j$. From the first-order condition, the equilibrium working hours of a type- i worker in a type- j job, denoted by h_{ij} , are given by

$$0 = F_h(k_j, h_{ij}) - u'(1 - h_{ij} - t_i).$$

The above equation means that for any $i \in \{l, s\}$ and $j \in \{L, S\}$, $h_{iL} > h_{iS}$ and $h_{lj} > h_{sj}$ because $F_{hk} > 0$ and $u'' < 0$. Thus, the working hours of type- l workers in type- L jobs are the longest, and the working hours of type- s workers in type- S jobs are the shortest.

The maximized instantaneous value, $F(k_j, h_{ij}) + u(l_{ij}) - \rho k_j$, is denoted by y_{ij} . For the order of y_{ij} , we can obtain the following lemma.

Lemma 1 $y_{lL} > y_{lS}$ and $y_{sS} > y_{sL}$ if and only if $\rho \in \Omega$ where

$$\Omega = \left(\frac{F(k_L, h_{sL}) + u(1 - h_{sL} - t_s) - F(k_S, h_{sS}) - u(1 - h_{sS} - t_s)}{k_L - k_S}, \frac{F(k_L, h_{lL}) + u(1 - h_{lL} - t_l) - F(k_S, h_{lS}) - u(1 - h_{lS} - t_l)}{k_L - k_S} \right). \quad (1)$$

Proof. See Appendix. ■

If ρ is very small, $y_{sL} > y_{sS}$, and workers always prefer to work in type- L jobs. Similarly, if ρ is very high, $y_{lS} > y_{lL}$, and workers always prefer to work in type- S jobs. Intuitively, because capital and working hours are complementary in production, $y_{lL} - y_{lS}$ is always higher than $y_{sL} - y_{sS}$. However, if the running costs of capital are very small, the instantaneous value of type- L jobs is higher than the value of type- S jobs regardless of the worker's type. Likewise, if the costs are very high, the instantaneous value of type- S jobs is always higher than the value of type- L jobs.

For some intermediate values of ρ , $y_{lL} > y_{lS}$ and $y_{sS} > y_{sL}$ can hold simultaneously, and then type- $l(s)$ workers matched with type- $S(L)$ jobs may move to type- $L(S)$ jobs. In the following analysis, to focus on the interesting case where k_L is too high for type- s workers and k_S is too small for type- l workers, we assume that (1) holds and call (l, L) and (s, S) good matches and (l, S) and (s, L) bad matches.

3.2 Optimal compensation

To maximize the joint surplus, the amount of compensation must be equal to changes in the value of the current job as $c_i = J(i, j, \Gamma)$. When a type- i worker in a type- j job with contract Γ moves to a type- j' job with contract Γ' , the worker's and incumbent job's capital changes are $W(i, j', \Gamma') - W(i, j, \Gamma) - c_i$ and $0 - J(i, j, \Gamma) + c_i$, respectively, where $c_i \in \Gamma$. Thus, from this job-to-job transition, the joint value of the worker and the incumbent job is increased if and only if $W(i, j', \Gamma') - T(i, j) > 0$.

Because the worker searches for type- j' jobs if and only if $W(i, j', \Gamma') - W(i, j, \Gamma) - c_i > 0$, the private efficient turnover decision is implemented by c_i such that $W(i, j', \Gamma') - W(i, j, \Gamma) - c_i = W(i, j', \Gamma') - T(i, j)$, which can be rewritten as $c_i = J(i, j, \Gamma)$. Intuitively, to internalize the externalities on the incumbent job, the worker must compensate the capital loss of the incumbent job from her (or his) job-to-job transition.

3.3 Bargaining

Following Cahuc, Postel-Vinay, and Robin (2006) and Lentz (2010), the new employment contracts after renegotiation are set through Nash bargaining⁴ in which the employed worker can use the employment contract with one job. Specifically, when the employed worker contacts a vacant job, she (or he) will match with the highest joint value of the two jobs and bargain over the joint value with a threat point of full value extraction with the other job. For simplicity, we assume that the value of vacant jobs is always zero⁵, which implies that the threat point of a job is always zero.

If a type- i employed worker in a type- j job moves to a type- j' job, where necessarily $T(i, j') > T(i, j)$, the employment contract Γ' is determined by following sharing rule:

$$W(i, j', \Gamma') = \beta T(i, j') + (1 - \beta) T(i, j), \quad (2)$$

where $\beta \in [0, 1]$ is the workers' bargaining power. The above equation implies that the employment contracts depend on types of new and old jobs, and the value of an employed worker before changing type can then be

⁴Cahuc, Postel-Vinay, and Robin (2006) provided the game-theoretic foundations of Nash bargaining in the paper. In their model, employment contracts are determined according to Rubinstein's (1982) alternating offers game.

⁵In Section 4, the basic model is extended to introduce the free entry condition, in which the outside value of a job is zero by the zero profits condition. Moreover, the zero threat point assumption is also justified given large firms and constant returns to scale production technology (see Lentz 2010).

denoted by $W(i, j', j) \equiv W(i, j', \Gamma')$. Thus, the capital gain of the worker is

$$W(i, j', j) - W(i, j, \Gamma) - c_i (= J(i, j, \Gamma)) = \beta (T(i, j') - T(i, j)). \quad (3)$$

From (2), it is straightforward that the value of the poaching job is

$$J(i, j', j) \equiv J(i, j', \Gamma') = (1 - \beta) (T(i, j') - T(i, j)). \quad (4)$$

The wage bargaining process leads to splitting the social capital gain, which is the sum of (3) and (4). (3) and (4) imply that β increases the worker's capital gain and decreases the capital gain of a poaching job. Moreover, an increase in β reduces the gap between the social capital gain and the worker's capital gain because the capital gain of the poaching job decreases with β .

Note that since both workers and jobs are risk-neutral, the equilibrium employment contract is not uniquely determined. Moreover, when a type- i worker moves to a type- j' job from a type- j job, while $W(i, j', j)$ is uniquely determined, the value of changing to type i' , $W(i', j', \Gamma')$, is not uniquely determined.

3.4 Turnover decisions

In this model, there are three types of equilibrium characterized by turnover decisions. The first equilibrium is called *turnover equilibrium* (TE), in which workers in bad matches move to good matches. For example, if a type- s worker originally working in a type- S job turns out to be type- l , she (or he) moves to a type- L job if she (or he) contacts a vacant type- L job. The second equilibrium is called *stay-in-type- L -jobs equilibrium* (SLE), in which workers stay in type- L jobs regardless of their type, and then all workers work in type- L jobs in the steady state. In contrast, in the final equilibrium, called *stay-in-type- S -jobs equilibrium* (SSE), workers stay in type- S jobs regardless of their type. All workers then work in type- S jobs in the steady state.

Because an employed worker in a type- j job moves to a type- j' job if and only if $T(i, j') > T(i, j)$, the following lemma can be shown.

Lemma 2 *The market equilibrium is TE if $T(l, L) > T(l, S)$ and $T(s, S) > T(s, L)$, the market equilibrium is SLE if $T(l, L) > T(l, S)$ and $T(s, S) \leq T(s, L)$, and the market equilibrium is SSE if $T(l, L) \leq T(l, S)$ and $T(s, S) > T(s, L)$.*

The intuition behind this result is simple. Because a type- i worker in a type- j job moves to a type- j' job if and only if $T(i, j') > T(i, j)$, a worker in a bad match would like to move to a good match if and only if $T(l, L) > T(l, S)$ and $T(s, S) > T(s, L)$. If $T(s, S) \leq T(s, L)$, both type- l and s workers would like to work in type- L jobs, and then, the market equilibrium is SLE. In contrast, if $T(l, L) \leq T(l, S)$, both type- l and s workers would like to work in type- S jobs, and then, the market equilibrium is SSE.

Note that there is a potential case in which $T(l, L) \leq T(l, S)$ and $T(s, S) \leq T(s, L)$. However, in equilibrium, these inequalities cannot hold simultaneously.

3.5 Joint values

To characterize the market equilibrium, we now define a form of the joint value functions. From Lemma 2, the equilibrium turnover pattern only depends on the joint values $T(i, j)$, not on the values of a worker and a job. Thus, we define only the form of joint value functions.

Using (3), the joint value with a match between a type- i worker and a type- j job is given by

$$rT(i, j) = \begin{cases} y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) + p_{j'}\beta(T(i, j') - T(i, j)) & \text{if } T(i, j') > T(i, j), \\ y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) & \text{if } T(i, j') \leq T(i, j). \end{cases} \quad (5)$$

In the first case, where $T(i, j') > T(i, j)$, a worker moves to a poaching job when she (or he) contacts the job. In the second case, where $T(i, j') \leq T(i, j)$, she (or he) does not move. In both cases, the first term of the above equation is the sum of the worker's instantaneous utility and the job's instantaneous profit, while the second term is the sum of the expected capital changes of the worker, $\pi_{ii'}(W(i', j, \Gamma) - W(i, j, \Gamma))$, and the job, $\pi_{ii'}(J(i', j, \Gamma) - J(i, j, \Gamma))$, from the changing worker type. Additionally, the final term in the first case represents the expected capital gain, which is the product of the contract rate of type- j' jobs and the worker's capital gain defined by (3).

Finally, if the worker may move to a poaching job, the joint value increases with the workers' bargaining power β . Intuitively, if the worker may move to a poaching job, an increase in β increases the worker's capital gain which is included in the joint value. Meanwhile, if the worker does not move, the joint value does not include the worker's capital gain, and then, does not depend on β .

3.6 Equilibrium turnover decisions

Formally, turnover decisions are a function of the type of worker and incumbent and poaching jobs, and the form of the function depends on $\omega_l = y_{lL} - y_{lS}$ and $\omega_s = y_{sS} - y_{sL}$. ω_i is the *good match premium* of type- i workers, respectively. The market equilibrium is then summarized as follows.

Lemma 3 *The market equilibrium is TE if $\omega_s/\omega_l \in (\pi_{sl}/(r + \pi_{ls} + p_L\beta), (r + \pi_{sl} + p_S\beta)/\pi_{ls})$, SLE if $\omega_s/\omega_l \leq \pi_{sl}/(r + \pi_{ls} + p_L\beta)$, and SSE if $\omega_s/\omega_l \geq (r + \pi_{sl} + p_S\beta)/\pi_{ls}$.*

Proof. See Appendix ■

If ω_s/ω_l is low, the market equilibrium is SLE because the good match premium of type- s workers is relatively lower than that of type- l workers. In contrast, if ω_s/ω_l is high, the market equilibrium is SSE. For some intermediate values of ω_s/ω_l , the market equilibrium is TE.

Furthermore, an increase in workers' bargaining power β increases the domain of TE. This result is most important among the comparative statics of the market equilibrium. Intuitively, an increase in β raises the joint value in TE because of the raise in the expected capital gain from job-to-job transitions. Meanwhile, in SLE or SSE, workers do not move to a new job, and the joint values are then independent of β . The contact rates p_L and p_S also increase the domain of TE because if job-to-job transitions are easy, the expected period of the bad match in TE is short.

Moreover, when p_j and β are low, mismatches are eliminated only by changes in labor supply preferences, and workers do not search on the job. When p_j and β are high, job-to-job transitions become the effective method to eliminate the mismatch, and then, workers have an incentive to start searching for good-matching jobs.

Finally, note that the frictional labor market is the key assumption of this paper. In fact, if a labor market is frictionless, which means that $p_L, p_S \rightarrow \infty$, the market equilibrium must be TE, and the results of this paper are trivial.

3.7 Flow conditions

To define the steady-state equilibrium, we define the flow conditions. First, the number of type- i workers is denoted by N_i , and the flow of type- l workers is

$$\dot{N}_l = \pi_{sl}N_s - \pi_{ls}N_l,$$

where the first term represents the inflow of workers to the pool of type- l workers, and the second term represents the outflow due to exogenous shocks. Using $N_s = 1 - N_l$, in the steady state $\dot{N}_l = 0$, the above equation yields

$$N_l = \frac{\pi_{sl}}{\pi_{sl} + \pi_{ls}}. \quad (6)$$

Naturally, the number of type- l workers increases with π_{sl} and decreases with π_{ls} . In the rest of the analysis, we assume that N_l and N_s are in the steady state.

Next, we define the flow conditions of a job match (i, j) . Let the number of workers in (i, j) be denoted by e_{ij} . $\mu_{jj'}^i \in [0, 1]$ is the probability that a type- i worker moves to a type- j' job from a type- j job conditional on contacting the type- j' job. Because the costs of on-the-job search are very small, $\mu_{SL}^l = 1 - \mu_{LS}^l$ and $\mu_{LS}^s = 1 - \mu_{SL}^s$, that is, if a type- i worker in a type- S job does not move to a type- L job, workers of the same type in type- L jobs move to a type- S job, and vice versa. According to the definition of equilibrium, $\mu_{SL}^l = \mu_{LS}^s = 1$ in TE, $\mu_{SL}^l = 1, \mu_{LS}^s = 0$ in SLE, and $\mu_{SL}^l = 0, \mu_{LS}^s = 1$ in SSE.

Using μ_{SL}^l , the flow condition of e_{lL} can be defined as

$$\dot{e}_{lL} = p_L \mu_{SL}^l e_{lS} + \pi_{sl} e_{sL} - [p_S (1 - \mu_{SL}^l) + \pi_{ls}] e_{lL}, \quad (7)$$

where $p_L \mu_{SL}^l$ is the rate of type- l workers who move to type- L jobs from type- S jobs, and $p_S (1 - \mu_{SL}^l)$ is the rate of type- l workers who move to type- S jobs from type- L jobs. The first and second terms of (7) represent the inflow of e_{lS} and e_{sL} into the pool of e_{lL} , and the final term represents the outflow of e_{lL} due to job-to-job transitions and changing type.

Similarly, using μ_{LS}^s , the flow condition of e_{sS} is given by

$$\dot{e}_{sS} = p_S \mu_{LS}^s e_{sL} + \pi_{ls} e_{lS} - [p_L (1 - \mu_{LS}^s) + \pi_{sl}] e_{sS}, \quad (8)$$

where $p_S \mu_{LS}^s$ is the rate of type- s workers in type- L jobs who move to type- S jobs, and $p_L (1 - \mu_{LS}^s)$ is the rate of type- s workers in type- S jobs who move to type- L jobs. The first and second terms then represent the inflow to the pool of e_{lL} , and the final term represents the outflow.

The flow conditions are defined by (6), (7), (8), $e_{lS} = N_l - e_{lL}$, and $e_{sL} = N_s - e_{sS}$. In the appendix, we demonstrate the steady-state e_{ij} in each equilibrium. Moreover, the above definitions and Proposition 1 characterize the steady-state market equilibrium.

Finally, note that equilibrium turnover decisions do not depend on the state variables N_i and e_{ij} . More formally, the market equilibrium in this model is block recursive, where workers' and jobs' value functions and policy functions do not depend on the distribution of workers (see Menzio and Shi 2010, 2011). The main model is tractable because the equilibrium is block recursive.

4 Social planner problem

To illustrate socially efficient turnover decisions, we solve the social planner problem in which the social planner can choose μ_{SL}^l and μ_{LS}^s to maximize the social surplus subject to the flow conditions. Because both workers and jobs are risk-neutral, the social surplus S , which is the sum of workers and job surpluses, can be written as

$$S = \int_{t=0}^{\infty} e^{-rt} \sum_{j=\{L,S\}} \sum_{i=\{l,s\}} e_{ij} y_{ij} dt, \quad (9)$$

where $e_{ij} y_{ij}$ represents the aggregate instantaneous value of (i, j) , and $\sum_{j=\{L,S\}} \sum_{i=\{l,s\}} e_{ij} y_{ij}$ represents the aggregate instantaneous value in the economy. The above social surplus is then the standard form (see Pissarides 2000)⁶.

Using the flow conditions, the social planner problem can be defined as

$$\max_{\mu_{SL}^l, \mu_{LS}^s} S, \quad \text{s.t. (7), (8), } e_{lS} = N_l - e_{lL}, \text{ and } e_{sL} = N_s - e_{sS}.$$

⁶In this model, no worker is unemployed in the steady state, and the search cost of a vacant job is assumed to be zero. Thus, the social surplus is the sum of only the instantaneous utility of employed workers and the instantaneous profit of jobs, and hence, does not include the instantaneous utility of unemployed workers and the search cost of vacant jobs.

The optimal condition of the social planner problem is summarized in the following proposition.

Lemma 4 *TE is socially efficient if $\omega_s/\omega_l \in (\pi_{sl}/(r + \pi_{ls} + p_L), (r + \pi_{sl} + p_S)/\pi_{ls})$, SLE is socially efficient if $\omega_s/\omega_l \leq \pi_{sl}/(r + \pi_{ls} + p_L)$, and SSE is socially efficient if $\omega_s/\omega_l \geq (r + \pi_{sl} + p_S)/\pi_{ls}$.*

Proof. See Appendix. ■

The socially efficient turnover decisions have similar implications to the market equilibrium for ω_s , ω_l , p_l , and p_s . However, the socially efficient turnover decisions naturally do not depend on workers' bargaining power β , and we can show the welfare implication as the following proposition.

Proposition 1 *If jobs have some bargaining power (implying $\beta < 1$), the socially efficient domain of TE is larger than the domain in the market equilibrium.*

It is easy to prove the above proposition from Lemma 3 and 4 and the fact that the domain of TE in market equilibrium increases with β . This proposition implies that job-to-job transitions in the market economy are socially inefficient. Intuitively, the problem is associated with the hold-up problem and reflects that a worker and a poaching job bargain over the capital gain from job-to-job transitions.

[Fig 1 around here]

Lemma 2 implies that turnover decisions are privately efficient, which means that a worker moves to a poaching job if and only if the joint value in the poaching job is higher than the value in the incumbent job. In general, turnover decisions have implications for both the poaching and incumbent jobs, as well as for other vacant jobs. In the wage bargaining game, both the poaching and incumbent jobs can offer employment contracts to influence turnover decisions. As a result, a worker is led to take indirect account of the externalities that she (or he) generates for both jobs, and turnover decisions are then privately efficient.

However, turnover decisions have implications not only for the incumbent and the poaching jobs but also for vacant jobs that may poach workers in the future. Figure 1 shows an example in which a type- s worker first moves to job 2 (type- S) from job 1 (type- L) (this is the first job-to-job transition), and she (or he) then moves to job 3 (type- L) after changing her (or his) type to l (this is the second job-to-job transition). In this case, if $\beta < 1$, job 3 obtains a positive capital gain from the second job-to-job transition. Moreover, job 3 cannot obtain a positive capital gain if the worker does not move from job 1 to job 2. In turnover decisions about

the move to job 2 (the first job-to-job transition), the worker ignores the capital gain of job 3 from the second job-to-job transition, but the social planner takes it into account. For this reason, the market equilibrium may be inefficient.

Only if the worker has full bargaining power ($\beta = 1$), the hold-up inefficiencies disappear because (4) means that the capital gain of the poaching jobs is zero, and the worker's capital gain is then equal to the social capital gain. In this case, the market equilibrium is socially efficient because there exist no externalities on the vacant jobs.

Note that Proposition 1 relies on the standard assumption that vacant jobs cannot commit to their employment contracts when they contact an employed worker in the future. The value of the vacant job can improve if all vacant jobs post employment contracts in which workers' instantaneous utility is high. However, vacant jobs cannot post these employment contracts and thus cannot influence turnover decisions.

4.1 Policy implication

We show that new hiring subsidies can eliminate the distortion presented by Proposition 1. We assume that when a type- j job hires a new worker, the job can receive the new hiring subsidy s_j , which depends on the job's type.

In this case, the condition of TE can be rewritten as

$$T(l, L) + s_L > T(l, S), \quad (10)$$

$$T(s, S) + s_S > T(s, L).$$

(10) implies that given $T(i, j)$, the new hiring subsidies induce job-to-job transitions. Under the hiring subsidy policy, the capital gains of the worker and the poaching job, (3) and (4), can be rewritten as

$$W(i, j', j) = \beta(T(i, j') + s_{j'}) + (1 - \beta)T(i, j) \quad (11)$$

and

$$W(i, j', j) - W(i, j, \Gamma) - J(i, j, \Gamma) = \beta(T(i, j') + s_{j'} - T(i, j)). \quad (12)$$

(11) and (12) imply that both the worker's and the job's capital gains increase with $s_{j'}$.

Using (11) and (12), (5) can be rewritten as

$$rT(i, j) = \begin{cases} y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) + p_{j'}\beta(T(i, j') + s_{j'} - T(i, j)) & \text{if } T(i, j') > T(i, j). \\ y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) & \text{if } T(i, j') \leq T(i, j). \end{cases} \quad (13)$$

Substituting (13) into (10), the condition of TE can be rewritten as

$$(r + \pi_{sl} + p_S\beta)\omega_l - \pi_{ls}\omega_s + [(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}]s_L + \pi_{ls}p_S\beta s_S > 0, \quad (14)$$

$$-\pi_{sl}\omega_l + (r + \pi_{ls} + p_L\beta)\omega_s + \pi_{sl}p_L\beta s_L + [(r + \pi_{ls} + p_L\beta)(r + \pi_{sl}) - \pi_{ls}\pi_{sl}]s_S > 0. \quad (15)$$

Because $(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl} > 0$, (14) implies that s_L as well as s_S promote job-to-job transitions of type- l workers from type- S jobs to type- L jobs. Likewise, job-to-job transitions of type- s workers from type- L jobs to type- S jobs are promoted by both s_L and s_S .

Finally, we define the socially efficient subsidies under which the domain of TE achieves the socially efficient level. Using Lemma 4, (14), and (15), we can characterize the socially efficient subsidies as follows.

Proposition 2 *The socially efficient subsidies, denoted by s_L^* and s_S^* , are given by*

$$\begin{aligned} s_L^* &= \frac{(1 - \beta)p_S [[(r + \pi_{ls} + p_L\beta)(r + \pi_{sl}) - \pi_{ls}\pi_{sl}]\omega_l - \beta\pi_{ls}p_L\omega_s]}{A} \\ s_S^* &= \frac{(1 - \beta)p_L [-\pi_{sl}\beta p_S\omega_l + [(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}]\omega_s]}{A} \end{aligned}$$

where

$$A = [(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}][(r + \pi_{ls} + p_L\beta)(r + \pi_{sl}) - \pi_{ls}\pi_{sl}] - \pi_{ls}p_S\pi_{sl}p_L\beta^2$$

Proof. See Appendix. ■

It is easily shown that $A > 0$, and an increase in the good match premium of type- l workers, ω_l , increases the efficient subsidies for type- L jobs, but decreases the subsidies for type- S jobs. Likewise, ω_s decreases s_L and increases s_S .

Intuitively, if ω_l is high, the hold-up problem of job-to-job transitions from type- S jobs to type- L jobs becomes more serious, and the hiring subsidies for type- L jobs should then be high. Additionally, because (14) implies that an increase in s_L also induces job-to-job transitions of type- s workers from type- L jobs to type- S jobs, s_S should be decreasing in ω_l . A similar intuition can apply to ω_s .

5 Other bargaining form

In Section 4, we show Proposition 1 under the wage bargaining game in which a worker's threat point in the bargaining is equal to the full trade surplus obtained from the original match. Finally, we discuss the robustness of this result to other bargaining forms offered by Postel-Vinay and Robin (2002), Kiyotaki and Lagos (2007), and Pissarides (1994).

First, we assume the Bertrand wage competition game as in Postel-Vinay and Robin (2002). In this case, if a type- i worker in a type- j job moves to a type- j' job, the value of the worker is given by

$$W(i, j', j) = T(i, j).$$

This is a well-known result of the Bertrand game. By comparing (2), the above equation implies that the outcome of the Bertrand wage competition game is a special case of the basic model, wherein jobs have monopolistic bargaining power ($\beta = 0$). From Proposition 1, there exists a gap between the domain of TE in the market economy and the domain determined by the social planner.

Next, as in Kiyotaki and Lagos (2007)⁷, nature randomly chooses either the worker or the job to make a take-it-or-leave-it wage offer. In the model of this paper, if the worker can make a take-it-or-leave-it wage offer, the value of the worker is given by

$$W(i, j', j) = T(i, j').$$

If the job can make a take-it-or-leave-it wage offer, the value of the worker is given by

$$W(i, j', j) = T(i, j).$$

⁷Their model analyzes a more general case involving a double breach, which is ruled out in the basic model.

Thus, $\beta^* \in [0, 1]$ is defined as the probability of choosing the worker, and the joint value can be defined as

$$\begin{aligned} rT(i, j) &= y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) + p_{j'}\beta^*(T(i, j') - T(i, j)) \quad \text{if } T(i, j') > T(i, J), \\ rT(i, j) &= y_{ij} + \pi_{ii'}(T(i', j) - T(i, j)) \quad \text{if } T(i, j') \leq T(i, J). \end{aligned}$$

The above values are the same as (5) if $\beta^* = \beta$. Thus, the results of the basic model are derived from the same logic.

Finally, we assume the Nash bargaining game in which a worker's threat point in the bargaining is equal to the value of unemployment, $U(i)$. According to Dolado, Jansen, and Jimeno (2009) and Pissarides (1994), the outcome of this wage bargaining is that the rent of a type- i worker matched with a type- j job, $W(i, j) - U(i)$, is the same as $\beta(T(i, j) - U(i))$. Note that in this case, the value of a worker does not depend on the previous job type.

The capital gain of a worker moving to a type- j' firm is

$$W(i, j') - W(i, j) = \beta(T(i, j) - T(i, j)),$$

which is the same form as (3). Thus, Proposition 1 still holds.

6 Discussion and Conclusions

This paper developed an on-the-job search model with wage bargaining, and a novel assumption is that the worker type is changed by shocks. This paper demonstrates the welfare implication that the levels of job-to-job transitions are socially inefficient because of the hold-up problem.

A few comments are in order. In this paper, we focus on the fluctuation of workers' types. Even if the job type is changed while the worker type is fixed, the main results of this paper hold. However, if we extend this model to incorporate both the fluctuation of job type and replacement hiring as in Kiyotaki and Lagos (2007), the results of this paper may change significantly. This is an important direction for the future research.

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Appendix

Proof of Lemma 1

By the definition of y_{ij} , it is straightforward that

$$\begin{aligned}y_{lL} - y_{lS} &= F(k_L, h_{lL}) + u(1 - h_{lL} - t_l) - [F(k_S, h_{lS}) + u(1 - h_{lS} - t_l)] - \rho(k_L - k_S), \\y_{sS} - y_{sL} &= F(k_S, h_{sS}) + u(1 - h_{sS} - t_s) - [F(k_L, h_{sL}) + u(1 - h_{sL} - t_s)] + \rho(k_L - k_S).\end{aligned}$$

Thus, $y_{lL} - y_{lS} > 0$ and $y_{sS} - y_{sL} > 0$ are hold if and only if $\rho \in \Omega$.

Next, we show that Ω is non-empty sets. Using the envelop theorem, we obtain

$$\frac{\partial^2 [F(k_j, h_{ij}) + u(1 - h_{ij} - t_i)]}{\partial k \partial h} = F_{kh}(k_j, h_{ij}) > 0. \quad (16)$$

(16) implies that $F(k_L, h_{lL}) + u(1 - h_{lL} - t_l) - [F(k_S, h_{lS}) + u(1 - h_{lS} - t_l)]$ is an increasing function of working hours, which means that

$$\begin{aligned}& F(k_L, h_{sL}) + u(1 - h_{sL} - t_s) - [F(k_S, h_{sS}) + u(1 - h_{sS} - t_s)] \\& < F(k_L, h_{lL}) + u(1 - h_{lL} - t_l) - [F(k_S, h_{lS}) + u(1 - h_{lS} - t_l)].\end{aligned}$$

Thus, Ω is non-empty sets.

Proof of Lemma 3

Conditions of TE

Joint values in TE are,

$$\begin{aligned}rT(l, L) &= y_{lL} + \pi_{ls}(T(s, L) - T(l, L)), \\rT(s, L) &= y_{sL} + \pi_{sl}(T(l, L) - T(s, L)) + p_S\beta(T(s, S) - T(s, L)), \\rT(l, S) &= y_{lS} + \pi_{ls}(T(s, S) - T(l, S)) + p_L\beta(T(l, L) - T(l, S)), \\rT(s, S) &= y_{sS} + \pi_{sl}(T(l, S) - T(s, S)).\end{aligned}$$

Combining above equations, we obtain

$$\begin{aligned}T(l, L) - T(l, S) &= \frac{(r + \pi_{sl} + p_S\beta)\omega_l - \pi_{ls}\omega_s}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}} \\T(s, S) - T(s, L) &= \frac{-\pi_{sl}\omega_l + (r + \pi_{ls} + p_L\beta)\omega_s}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}}\end{aligned}$$

Thus, the condition of TE, $T(l, L) > T(l, S)$ and $T(s, S) > T(s, L)$, can be rewritten as

$$\frac{r + \pi_{sl} + p_S\beta}{\pi_{ls}} > \frac{\omega_s}{\omega_l} > \frac{\pi_{sl}}{r + \pi_{ls} + p_L\beta}.$$

Conditions of SLE

Joint values in SLE are,

$$\begin{aligned}rT(l, L) &= y_{lL} + \pi_{ls}(T(s, L) - T(l, L)), \\rT(s, L) &= y_{sL} + \pi_{sl}(T(l, L) - T(s, L)), \\rT(l, S) &= y_{lS} + \pi_{ls}(T(s, S) - T(l, S)) + p_L\beta(T(l, L) - T(l, S)), \\rT(s, S) &= y_{sS} + \pi_{sl}(T(l, S) - T(s, S)) + p_L\beta(T(s, L) - T(s, S)).\end{aligned}$$

Combining above equations, we obtain

$$\begin{aligned} T(l, L) - T(l, S) &= \frac{(r + \pi_{sl} + p_L\beta)\omega_l - \pi_{ls}\omega_s}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_L\beta) - \pi_{ls}\pi_{sl}} \\ T(s, S) - T(s, L) &= \frac{-\pi_{sl}\omega_l + (r + \pi_{ls} + p_L\beta)\omega_s}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_L\beta) - \pi_{ls}\pi_{sl}} \end{aligned}$$

Thus, the condition of SLE, $T(l, L) > T(l, S)$ and $T(s, S) \leq T(s, L)$, can be rewritten as

$$\begin{aligned} \frac{r + \pi_{sl} + p_L\beta}{\pi_{ls}} &> \frac{\omega_s}{\omega_l}, \\ \frac{\pi_{sl}}{r + \pi_{ls} + p_L\beta} &\geq \frac{\omega_s}{\omega_l}. \end{aligned}$$

Because $(r + \pi_{sl} + p_L\beta) / \pi_{ls} > \pi_{sl} / (r + \pi_{ls} + p_L\beta)$, the condition of SLE is,

$$\frac{\pi_{sl}}{r + \pi_{ls} + p_L\beta} \geq \frac{\omega_s}{\omega_l}.$$

Conditions of SSE

The joint values in SSE are

$$\begin{aligned} rT(l, L) &= y_{lL} + \pi_{ls}(T(s, L) - T(l, L)) + p_S\beta(T(l, S) - T(l, L)), \\ rT(s, L) &= y_{sL} + \pi_{sl}(T(l, L) - T(s, L)) + p_S\beta(T(s, S) - T(s, L)), \\ rT(l, S) &= y_{lS} + \pi_{ls}(T(s, S) - T(l, S)), \\ rT(s, S) &= y_{sS} + \pi_{sl}(T(l, S) - T(s, S)). \end{aligned}$$

Combining above equations, we obtain

$$\begin{aligned} T(l, L) - T(l, S) &= \frac{(r + \pi_{sl} + p_S\beta)\omega_l - \pi_{ls}\omega_s}{(r + \pi_{ls} + p_S\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}} \\ T(s, S) - T(s, L) &= \frac{-\pi_{sl}\omega_l + (r + \pi_{ls} + p_S\beta)\omega_s}{(r + \pi_{ls} + p_S\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}} \end{aligned}$$

Thus, the condition of SSE, $T(l, L) \leq T(l, S)$ and $T(s, S) > T(s, L)$, can be rewritten as

$$\begin{aligned}\frac{\omega_s}{\omega_l} &\geq \frac{r + \pi_{sl} + p_S\beta}{\pi_{ls}}, \\ \frac{\omega_s}{\omega_l} &> \frac{\pi_{sl}}{r + \pi_{ls} + p_S\beta}.\end{aligned}$$

Because $(r + \pi_{sl} + p_L\beta) / \pi_{ls} > \pi_{sl} / (r + \pi_{ls} + p_L\beta)$, the condition of SSE is,

$$\frac{\omega_s}{\omega_l} \geq \frac{r + \pi_{sl} + p_S\beta}{\pi_{ls}}.$$

Steady state condition

Using (7) and (8), the steady-state fractions e_{lL} and e_{sS} in TE are given as follows:

$$e_{lL} = \frac{[(\pi_{sl} + p_S)p_L - \pi_{ls}\pi_{sl}]N_l + \pi_{sl}^2 N_s}{(\pi_{ls} + p_L)(\pi_{sl} + p_S) - \pi_{ls}\pi_{sl}}, \quad (\text{A.1})$$

$$e_{sS} = \frac{\pi_{ls}^2 N_l + [(\pi_{ls} + p_L)p_S - \pi_{ls}\pi_{sl}]N_s}{(\pi_{ls} + p_L)(\pi_{sl} + p_S) - \pi_{ls}\pi_{sl}}. \quad (\text{A.2})$$

Thus, the steady state conditions of workers whose strategy is TE are characterized by (6), (A.1), (A.2),

$$e_{lS} = N_l - e_{lL}, \text{ and } e_{sL} = N_s - e_{sS}.$$

In SLE, because workers in type- S jobs continue to move to type- L jobs, employed workers in type- S jobs converge to 0. In the steady state, $e_{lL} = N_l = \pi_{sl} / (\pi_{ls} + \pi_{sl})$, $e_{sL} = N_s = \pi_{ls} / (\pi_{ls} + \pi_{sl})$, and $e_{lS} = e_{sS} = 0$. Similarly, in SSE, since employed workers in type- L jobs converge to 0, then $e_{lS} = N_l = \pi_{sl} / (\pi_{ls} + \pi_{sl})$, $e_{sS} = N_s = \pi_{ls} / (\pi_{ls} + \pi_{sl})$, and $e_{lL} = e_{sL} = 0$.

Proof of Lemma 4

The present-value Hamiltonian is defined as,

$$\begin{aligned}H &= (\omega_l e_{lL} + \omega_s e_{sS}) e^{-rt} + \lambda_{lL} [p_L \mu^l (N_l - e_{lL}) + \pi_{sl} (N_s - e_{sS}) - (\pi_{ls} + (1 - \mu^l) p_S) e_{lL}] \\ &\quad + \lambda_{sS} [p_S \mu^s (N_s - e_{sS}) + \pi_{ls} (N_l - e_{lL}) - (\pi_{sl} + (1 - \mu^s) p_L) e_{sS}],\end{aligned}$$

where λ_1 and λ_2 are shadow values. Note here that the control variables are μ^i and θ_j , and the state variables are e_{lL} and e_{sS} . The optimal conditions are

$$\begin{aligned}\mu^i : \quad \mu^i = 1 &\iff \frac{\partial H}{\partial \mu^i} \geq 0, \\ e_{lL} : \quad \frac{d\lambda_{lL}}{dt} &= -\frac{\partial H}{\partial e_{lL}}, \\ e_{sS} : \quad \frac{d\lambda_{sS}}{dt} &= -\frac{\partial H}{\partial e_{sS}}.\end{aligned}$$

By solving the four equations, we obtain

$$\begin{aligned}\mu^l &= 1 \iff \lambda_{lL} \geq 0 \\ \mu^s &= 1 \iff \lambda_{sS} \geq 0 \\ 0 &= \omega_l e^{-rt} - \lambda_{lL} [r + \mu^l p_L + (1 - \mu^l) p_S + \pi_{ls}] - \lambda_{sS} \pi_{ls} \\ 0 &= \omega_s e^{-rt} - \lambda_{lL} \pi_{sl} - \lambda_{sS} [r + (1 - \mu^s) p_L + \mu^s p_S + \pi_{sl}]\end{aligned}$$

which determine μ^i , λ_{lL} , and λ_{sS} .

The optimal conditions are

$$\begin{aligned}\mu_L = 1 &\iff -\lambda_1(p_L(e_l - e_{lL}) + p_S e_{lL}) \geq 0, \\ \mu_S = 1 &\iff -\lambda_2(p_S(e_s - e_{sS}) + p_L e_{sS}) \geq 0, \\ e^{-rt} \omega_l + \lambda_1(p_L \mu_{SL}^l + \pi_{ls} + (1 - \mu_{SL}^l) p_S) + \lambda_2 \pi_{ls} - \dot{\lambda}_1 &= 0, \\ e^{-rt} \omega_s + \lambda_1 \pi_{sl} + \lambda_2(p_S \mu_{LS}^s + \pi_{sl} + (1 - \mu_{LS}^s) p_L) - \dot{\lambda}_2 &= 0.\end{aligned}$$

$\mu^l = \mu^s = 1$ is socially efficient if,

$$\begin{aligned}\lambda_{lL} &= e^{-rt} \frac{(r + \pi_{sl} + p_S) \omega_l - \pi_{ls} \omega_s}{(r + \pi_{ls} + p_L)(r + \pi_{sl} + p_S) - \pi_{ls} \pi_{sl}} > 0, \\ \lambda_{lL} &= e^{-rt} \frac{-\pi_{sl} \omega_l + (r + \pi_{ls} + p_L) \omega_s}{(r + \pi_{ls} + p_L)(r + \pi_{sl} + p_S) - \pi_{ls} \pi_{sl}} > 0.\end{aligned}$$

Above conditions can be rewritten as

$$\frac{r + \pi_{sl} + p_S}{\pi_{ls}} > \frac{\omega_s}{\omega_l} > \frac{\pi_{sl}}{r + \pi_{ls} + p_L}.$$

Proof of Proposition 2

To show Proposition 2, we first drive the condition of TE with the new hiring subsidy. (13) can be rewritten as,

$$\begin{aligned} rT(l, L) &= y_{lL} + \pi_{ls}(T(s, L) - T(l, L)), \\ rT(s, L) &= y_{sL} + \pi_{sl}(T(l, L) - T(s, L)) + p_S\beta(T(s, S) - T(s, L) + s_S), \\ rT(l, S) &= y_{lS} + \pi_{ls}(T(s, S) - T(l, S)) + p_L\beta(T(l, L) - T(l, S) + s_L), \\ rT(s, S) &= y_{sS} + \pi_{sl}(T(l, S) - T(s, S)). \end{aligned}$$

Using above functions, we obtain,

$$\begin{aligned} T(l, L) - T(l, S) &= \frac{(r + \pi_{sl} + p_S\beta)(\omega_l - p_L\beta s_L) - \pi_{ls}(\omega_s - p_S\beta s_S)}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}}, \\ T(s, S) - T(s, L) &= \frac{-\pi_{sl}(\omega_l - p_L\beta s_L) + (r + \pi_{ls} + p_L\beta)(\omega_s - p_S\beta s_S)}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}}. \end{aligned}$$

Thus, the condition of TE is,

$$\begin{aligned} T(l, L) - T(l, S) + s_l &= \frac{(r + \pi_{sl} + p_S\beta)\omega_l - \pi_{ls}\omega_s + [(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}]s_L + \pi_{ls}p_S\beta s_S}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}} > 0, \\ T(s, S) - T(s, L) + s_s &= \frac{-\pi_{sl}\omega_l + (r + \pi_{ls} + p_L\beta)\omega_s + \pi_{sl}p_L\beta s_L + [(r + \pi_{ls} + p_L\beta)(r + \pi_{sl}) - \pi_{ls}\pi_{sl}]s_S}{(r + \pi_{ls} + p_L\beta)(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}} > 0. \end{aligned}$$

The socially efficient condition in Lemma 4 can be implemented by s_j^* which are given by,

$$\begin{aligned} (r + p_S + \pi_{sl})\omega_l - \pi_{ls}\omega_s &= (r + \pi_{sl} + p_S\beta)\omega_l - \pi_{ls}\omega_s + [(r + \pi_{ls})(r + \pi_{sl} + p_S\beta) - \pi_{ls}\pi_{sl}]s_L^* + \pi_{ls}p_S\beta s_S^*, \\ -\pi_{sl}\omega_l + (r + p_L + \pi_{ls})\omega_s &= -\pi_{sl}\omega_l + (r + \pi_{ls} + p_L\beta)\omega_s + \pi_{sl}p_L\beta s_L^* + [(r + \pi_{ls} + p_L\beta)(r + \pi_{sl}) - \pi_{ls}\pi_{sl}]s_S^*. \end{aligned}$$

Combining these two conditions yields

$$\begin{aligned}
 s_L^* &= \frac{(1 - \beta) p_S \{[(r + \pi_{ls} + p_L \beta)(r + \pi_{sl}) - \pi_{ls} \pi_{sl}] \omega_l - \pi_{ls} p_L \beta \omega_s\}}{[(r + \pi_{ls})(r + \pi_{sl} + p_S \beta) - \pi_{ls} \pi_{sl}][(r + \pi_{ls} + p_L \beta)(r + \pi_{sl}) - \pi_{ls} \pi_{sl}] - \pi_{ls} p_S \pi_{sl} p_L \beta^2}, \\
 s_S^* &= \frac{(1 - \beta) p_L \{-\pi_{sl} \beta p_S \omega_l + [(r + \pi_{ls})(r + \pi_{sl} + p_S \beta) - \pi_{ls} \pi_{sl}] \omega_s\}}{[(r + \pi_{ls})(r + \pi_{sl} + p_S \beta) - \pi_{ls} \pi_{sl}][(r + \pi_{ls} + p_L \beta)(r + \pi_{sl}) - \pi_{ls} \pi_{sl}] - \pi_{ls} p_S \pi_{sl} p_L \beta^2}.
 \end{aligned}$$

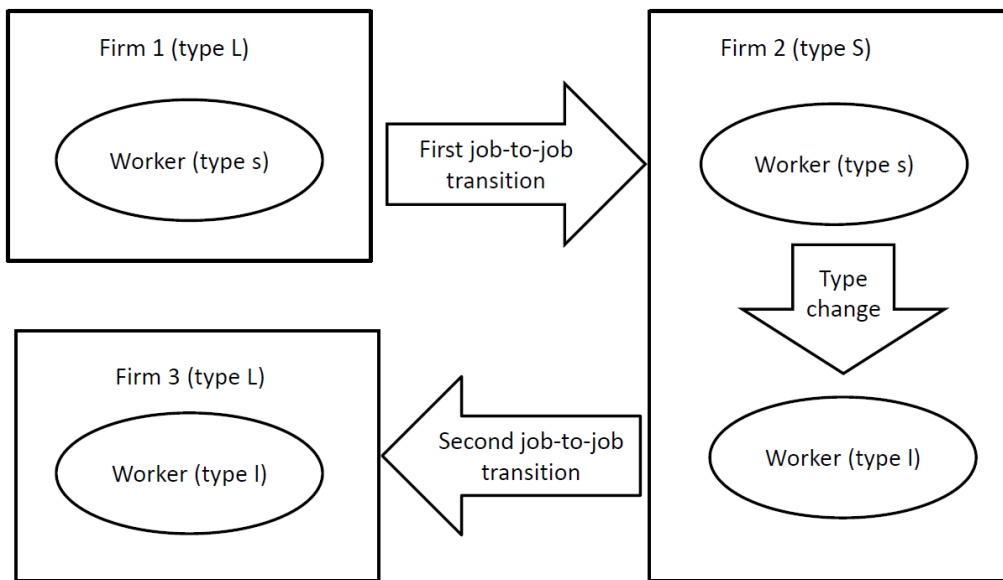


Figure 1: Example