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Preamble Based Channel and CFO Estimation for MIMO-OFDM Systems with Null Subcarriers

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SUMMARY In this paper, challenges regarding the provision of channel state information (CSI) and carrier frequency synchronization for orthogonal frequency division multiplexing (OFDM) systems with null subcarriers are addressed. We propose novel maximum likelihood (ML) based schemes that estimate the aggregate effects of the CFO and channel by using two successive OFDM preambles. In the presented scheme, CFO is estimated by considering the phase rotation between two consecutive received OFDM preambles. Both single input single output (SISO) as well as multiple input multiple output (MIMO) OFDM systems are considered. The mean squared errors (MSE) of the channel and CFO are used to evaluate the performance of our proposed scheme. By using two successive OFDM preambles, the estimation of channel and the estimation of CFO are decoupled, which leads to a simple estimation method. Simulation results show that the BER performance of the proposed estimators is comparable to that of known channel state information and the CFO MSE performance achieves the Cramer-Rao bound (CRB) of the fully loaded OFDM system.

key words: *OFDM, ML estimation, MIMO, CFO, CRB*

1. Introduction

The demand for high data rate transmission together with significant information capacity gain in wireless communication systems has attracted a lot of attention to the techniques such as orthogonal frequency division multiplexing (OFDM) systems. The advantages offered by combining OFDM with multiple-input multiple-output (MIMO) techniques are manifold. The most remarkable of them are robustness of OFDM systems against frequency selective fading channels, obtained by converting the channel into flat fading subchannels [1], [2] and the significant information capacity gain together with improved BER performance of the MIMO systems [3], [4].

In contrast to these appealing attributes, OFDM systems perform poorly under the influence of carrier frequency offset (CFO) [5]. Carrier frequency offset (CFO) may damage the orthogonality of subcarriers and lead to inter-carrier interference (ICI) that results to severe degradation of the system's performance [6]–[9]. Likewise, a MIMO system with N_t transmit and N_r receive antennas necessitates $N_t \times N_r$ channels to be estimated [2], while for a single input single output (SISO) system only one channel is to be estimated.

To obtain better quality of the high rate communications systems, efficient channel estimation and frequency synchronization techniques are crucial. When the OFDM-based wireless systems operates in a slow fading multiple-access environment, the use of preambles to facilitate channel estimation have been well discussed in IEEE 802.11a/g standard [10]. The short preambles in a WLAN system can be used to estimate and correct the coarse CFO. However, residual CFO always remains, producing a non negligible phase shift between consecutive OFDM blocks which can significantly deteriorate the signal detection. The accuracy of CFO estimate can be improved by the succeeding residual CFO estimation based on the long preambles.

In the literature, several techniques for ICI suppression and channel estimation have been predominantly developed for single input single output (SISO)-OFDM systems [11]–[15], and the reference therein. In [11]–[13], algorithms that employ a priori known training sequences are proposed, while in [14], the redundancy in cyclic prefix is utilized and in [15], scattered pilots and virtual carriers based frequency offset tracking algorithms are proposed. In [16], the pilot symbols has been designed to obtain the CFO estimate only by one OFDM symbol. The estimation with one OFDM symbol is spectral-efficient but its estimation range is limited compared with the methods using multiple OFDM symbols. Furthermore, schemes that utilize only one OFDM symbol requires the pilot symbols within an OFDM symbol to follow a certain framework such as repetitive slots (RS) structure (see [16], [21] and the reference therein).

If we extend the preamble based techniques to MIMO systems, the same short training symbols can be sent from only one antenna. At the receiver, the known SISO synchronization algorithm can accomplish the packet detection and frequency synchronization [17]. However, for the long preambles, in order to mitigate the effect of co-channel interference between the transmitting antennas, it is necessary for the training signals from each antenna to be orthogonal. The orthogonality of the training sequences for MIMO OFDM preambles can be established by special codes, such as Phase-Shift (PS) proposed in [2], [18]. These techniques were adopted in [17] to design long preambles for a MIMO OFDM system.

Channel and CFO estimation methods for MIMO-OFDM systems have been studied as well, e.g. in [3], [5], [7], [19], [20] and the reference therein. In [19], a technique for joint estimating the channel and CFO in a MIMO system using block type pilots is proposed. To estimate CFO, the method utilizes both grid search and Newton method which increases the complexity of the algorithm. In [7], a method that utilizes pilot symbols to estimate the resid-

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ual CFO is proposed, however the method assumes known channel state information which is not available in practical systems. Moreover, most of the existing CFO estimators in MIMO OFDM do not consider OFDM systems with with null subcarriers.

In this paper, we focus on channel estimation and synchronization of the residual carrier frequency offset (CFO) in time domain for both SISO and MIMO OFDM systems. Unlike the methods using one OFDM preamble, the maximum-likelihood (ML) based algorithm is proposed to estimate the channel and the CFO by using successive block of OFDM preambles. We derive the ML estimators that utilizes two consecutive preambles to estimate the channel as well as the CFO by considering the phase rotation between the successive OFDM blocks or frames. We show that the estimation of the channel and the estimation of CFO can be decoupled.

Similar ML techniques, that involves repetition of training symbols between two successive OFDM blocks for SISO systems is proposed in [11], [12]. In [12], a maximum-likelihood CFO estimator uses the phase difference between channel estimates of two successive OFDM blocks. Here we provide a mathematical derivation of the ML estimators that capture the aggregate effects of the channel and the CFO. Unlike [12], the proposed CFO estimators use the phase difference between two successive received OFDM blocks. Thus, CFO can be estimated without a prior knowledge of the channel. The proposed scheme utilizes a grid search within the acquisition range to obtain the suboptimal value of the CFO estimate. Simulations are provided to demonstrate the efficiency of our method.

Notations: The following notations will be used throughout this article, the frequency domain and time domain vectors will be represented by the upper and lower case letters respectively, while the superscript $(\cdot)^T$ and $(\cdot)^{\mathcal{H}}$ will denote transpose and Hermitian transpose respectively.

2. MIMO-OFDM System Model

Let us consider a MIMO-OFDM wireless system with N_t transmit and N_r receive antennas over frequency selective channels. The frequency domain representation of the kth transmitted OFDM block with N number of subcarriers at the pth transmit antenna can be written as the vector $\boldsymbol{X}_k^p = [X_{k,0}^p, X_{k,1}^p, \ldots, X_{k,N-1}^p]^T$, and the corresponding time domain signal is given by

$$\boldsymbol{x}_k^p = \frac{1}{N}\boldsymbol{F}^{\mathcal{H}}\boldsymbol{X}_k^p \tag{1}$$

where $\boldsymbol{F}$ is an $N \times N$ DFT matrix with $(m+1, n+1)$th entry $[\boldsymbol{F}]_{m,n} = e^{\frac{j2\pi mn}{N}}$ and $\boldsymbol{x}_k^p = [x_{k,0}^p, x_{k,1}^p, \ldots, x_{k,N-1}^p]^T$.

We assume that the discrete-time baseband equivalent channel between each transmit-receive antenna has FIR of maximum length L, and remains constant in at least two OFDM symbols, i.e., is quasi-static. Let us denote an $L \times 1$ vector of the time domain channel from the pth transmit antenna to the qth receive antenna as $\boldsymbol{h}^{(q,p)} = [h_0^{(q,p)}, \ldots, h_{L-1}^{(q,p)}]^T$ and its corresponding $N \times 1$ vector of frequency response as $\boldsymbol{H}^{(q,p)} = \boldsymbol{F}_L\boldsymbol{h}^{(q,p)}$ with $\boldsymbol{F}_L = [\boldsymbol{f}_0, \ldots, \boldsymbol{f}_{L-1}]$ representing the N rows and the first L columns of the DFT matrix $\boldsymbol{F}$.

The CFO between the transmitter and the receiver antenna q is normalized by the subcarrier spacing (also referred to as inter-carrier spacing), and is denoted by ϵ_q where ϵ_q is assumed to be in $(-0.5, 0.5]$. Practically, the instability of the transmit/receiver oscillators influence the maximum frequency offset. This implies that the CFO can be different at each receive antenna due to, for example having different local oscillator at each RF chain, however the model can also be used for the case of having common CFO for all receive antennas.

Assume that the insertion of a long enough cyclic prefix (CP) at the transmitter maintains the orthogonality of the subcarriers after transmission. Then, at the receiver, after discarding the cyclic prefix, the complex envelope of the baseband received signal in an OFDM block including CFO can be described as [19], [22]

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{H}_c^{(q,p)}\boldsymbol{x}_k^p + \boldsymbol{v}_k^q \tag{2}$$

where $\boldsymbol{y}_k^q$ is the kth received OFDM block and $\boldsymbol{H}_c^{(q,p)}$ is the circulant channel matrix associated with $\boldsymbol{h}^{(q,p)}$, and $\boldsymbol{D}(\epsilon_q) = \mathrm{diag}\left(1, e^{j2\pi\epsilon_q\frac{1}{N}}, \ldots, e^{j2\pi\epsilon_q\frac{N-1}{N}}\right)$, is an $N \times N$ diagonal matrix that describes the phase rotating effect by the frequency offset on each time domain OFDM symbol.

To obtain more insight, we can write (2) for the first OFDM block as

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{H}_c^{(q,p)}\boldsymbol{F}^{\mathcal{H}}\boldsymbol{X}_k^p + \boldsymbol{v}_k^q. \tag{3}$$

Equation (3) can be expressed as

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{F}^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{H}^{(q,p)})\boldsymbol{X}_k^p + \boldsymbol{v}_k^q \tag{4}$$

where $\boldsymbol{D}(\boldsymbol{H}^{(q,p)})$ is an $N \times N$ diagonal matrix of the vector $\boldsymbol{H}^{(q,p)}$. We can also represent (4) as

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{F}^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_k^p)\boldsymbol{H}^{(q,p)} + \boldsymbol{v}_k^q \tag{5}$$

where $\boldsymbol{D}(\boldsymbol{X}_k^p)$ is a diagonal matrix with vector $\boldsymbol{X}_k^p$ as its diagonal elements.

Suppose that there are some null subcarriers and let N_a be the number of active subcarriers. Then, given $\boldsymbol{X}_{k,a}^p$ as a transmitted OFDM block in frequency domain at the active subcarriers, the received signal can be expressed as

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{F}_a^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_{k,a}^p)\boldsymbol{H}_a^{(q,p)} + \boldsymbol{v}_k^q \tag{6}$$

where $\boldsymbol{H}_a^{(q,p)}$ is an $N_a \times 1$ channel coefficient vector at the active subcarriers and $\boldsymbol{F}_a$ is an $N_a \times N$ DFT sub-matrix corresponding to N_a number of active subcarriers. We can rewrite (6) as

$$\boldsymbol{y}_k^q = \sum_{p=1}^{N_t} \boldsymbol{D}(\epsilon_q)\boldsymbol{F}_a^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_{k,a}^p)\boldsymbol{F}_{L,a}\boldsymbol{h}^{(q,p)} + \boldsymbol{v}_k^q \quad (7)$$

where $\boldsymbol{F}_{L,a}$ is an $N_a \times L$ submatrix of $\boldsymbol{F}_L$ corresponding to the active subcarriers. Likewise, the next received OFDM block for the transmitted active subcarriers $\boldsymbol{X}_{k+1,a}^p$ can be expressed as

$$\boldsymbol{y}_{k+1}^q = \sum_{p=1}^{N_t} e^{j\alpha\epsilon_q}\boldsymbol{D}(\epsilon_q)\boldsymbol{F}_a^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_{k+1,a}^p)\boldsymbol{F}_{L,a}\boldsymbol{h}^{(q,p)} + \boldsymbol{v}_{k+1} \quad (8)$$

where $\alpha = \frac{2\pi(N+N_{cp})}{N}$, is one OFDM block duration including cyclic prefix of length N_{cp}.

In the following section, we will derive the maximum-likelihood (ML) estimator capable of decoupling the CFO and channel estimation from the two received successive long training symbols.

3. Decoupled ML Channel and CFO Estimation

Since the same channel and CFO estimation process is performed at each receive antenna, we only need to consider N_t transmit antennas and one receive antenna in deriving our ML estimator, that is, the system is modeled as a superposition of multiple-input single-output (MISO) systems [5], [23]. Thus, without loss of generality, we can describe the first receive antenna and omit the receive antenna index.

From the expressions derived in Sect. 2. Stacking the two consecutive received signals leads to

$$\tilde{\boldsymbol{y}}_k = \sum_{p=1}^{N_t} \boldsymbol{A}_p\boldsymbol{h}^{(p)} + \tilde{\boldsymbol{v}}_k \quad (9)$$

where $\tilde{\boldsymbol{v}}_k = [\boldsymbol{v}_k^T, \boldsymbol{v}_{k+1}^T]$ and

$$\boldsymbol{A}_p = \begin{bmatrix} \boldsymbol{D}(\epsilon)\boldsymbol{F}^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_k^p) \\ e^{j\alpha\epsilon}\boldsymbol{D}(\epsilon)\boldsymbol{F}^{\mathcal{H}}\boldsymbol{D}(\boldsymbol{X}_{k+1}^p) \end{bmatrix}\boldsymbol{F}_{L,a}.$$

Then (9) can also be represented as

$$\tilde{\boldsymbol{y}}_k = \boldsymbol{A}(\epsilon)\boldsymbol{h} + \tilde{\boldsymbol{v}}_k \quad (10)$$

where $\boldsymbol{A}(\epsilon) = [\boldsymbol{A}_1, \ldots, \boldsymbol{A}_{N_t}]$ and $\boldsymbol{h}^T = [\boldsymbol{h}^{(1)T}, \ldots, \boldsymbol{h}^{(N_t)T}]$ is channel vector of length $N_tL \times 1$.

For this model we would like to obtain the ML estimates of the channel $\boldsymbol{h}$ and the frequency offset ϵ for both SISO and MISO systems. The Cramer-Rao Bounds (CRB) of the ML estimates of a SISO system have been derived in [12] for the case that all subcarriers are active, that is, full loaded OFDM. It should also be remarked that if we do not ignore the cyclic prefixes of the OFDM signal, then we will have another model.

If the noise is i.i.d. white Gaussian, then the ML estimate of the channel $\boldsymbol{h}$ and CFO ϵ are obtained by minimizing

$$\|\tilde{\boldsymbol{y}}_k - \boldsymbol{A}(\epsilon)\boldsymbol{h}\|^2. \quad (11)$$

If $\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon) > 0$, then, for a given ϵ, the ML estimate of $\boldsymbol{h}$ is given by

$$\hat{\boldsymbol{h}} = \left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\boldsymbol{A}(\epsilon)^{\mathcal{H}}\tilde{\boldsymbol{y}}_k \quad (12)$$

and that the channel MSE is given by

$$E\{\|\hat{\boldsymbol{h}} - \boldsymbol{h}\|^2\} = \sigma_v^2 \,\mathrm{trace}\{\left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\}. \quad (13)$$

It should be remarked that, although $\mathbf{A}(\epsilon)$ depends on ϵ but $\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)$ does not depend on the value of ϵ.

Next, we minimize $\|\tilde{\boldsymbol{y}}_k - \boldsymbol{A}(\epsilon)\hat{\boldsymbol{h}}\|^2$ to obtain the ML estimate of $\hat{\epsilon}$. Substituting (12) to (11), results into

$$\begin{aligned}&\|\left(\boldsymbol{I} - \boldsymbol{A}(\epsilon)\left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\boldsymbol{A}(\epsilon)^{\mathcal{H}}\right)\tilde{\boldsymbol{y}}_k\|^2 \\ &\quad = \|\tilde{\boldsymbol{y}}_k\|^2 - \|\boldsymbol{A}(\epsilon)\left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\boldsymbol{A}(\epsilon)^{\mathcal{H}}\tilde{\boldsymbol{y}}_k\|^2 \end{aligned} \quad (14)$$

Since the first term of the R.H.S. of the equation above is constant, for the CFO estimation, we need to maximize

$$\begin{aligned}&\|\boldsymbol{A}(\epsilon)\left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\tilde{\boldsymbol{y}}_k\|^2 \\ &\quad = \tilde{\boldsymbol{y}}_k^{\mathcal{H}}\boldsymbol{A}(\epsilon)\left[\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)\right]^{-1}\boldsymbol{A}(\epsilon)^{\mathcal{H}}\tilde{\boldsymbol{y}}_k \end{aligned} \quad (15)$$

with respect to ϵ.

In the following, we will utilize some approximations to simplify the ML estimator obtained by maximizing (15) to come up with a simpler estimator. Let us normalize the total power of one OFDM preamble to one, i.e.,

$$\sum_{k\in\mathcal{K}_a} |X_{n,k}^p|^2 = 1 \quad (16)$$

where $X_{n,k}^p$ stands for the pilot symbol of the pth transmit antenna at the kth subcarrier and $\mathcal{K}_a$ denotes a set of active subcarrier in one OFDM block. Let us define $\boldsymbol{C}_p$ as

$$\begin{aligned}\boldsymbol{C}_p := \boldsymbol{A}_p^{\mathcal{H}}\boldsymbol{A}_p = \boldsymbol{F}_{L,a}^{\mathcal{H}}&\left[\boldsymbol{D}^{\mathcal{H}}(\boldsymbol{X}_{k,a}^p)\boldsymbol{D}(\boldsymbol{X}_{k,a}^p)\right. \\ &\left. + \boldsymbol{D}^{\mathcal{H}}(\boldsymbol{X}_{k+1,a}^p)\boldsymbol{D}(\boldsymbol{X}_{k+1,a}^p)\right]\boldsymbol{F}_{L,a}.\end{aligned} \quad (17)$$

Assume that $\boldsymbol{C}_p$ is positive definite, then, we have $\mathrm{trace}(\boldsymbol{C}_p)\ \mathrm{trace}(\boldsymbol{C}_p^{-1}) \geq (L+1)^2$, where the equality holds true if and only if $\boldsymbol{C}_p$ is a scaled identity matrix. If all subcarriers are active and have the same power σ_X^2, then we can obtain

$$\boldsymbol{D}^{\mathcal{H}}(\boldsymbol{X}_{k,a}^p)\boldsymbol{D}(\boldsymbol{X}_{k,a}^p) + \boldsymbol{D}^{\mathcal{H}}(\boldsymbol{X}_{k+1,a}^p)\boldsymbol{D}(\boldsymbol{X}_{k+1,a}^p) = 2\sigma_X^2\boldsymbol{I} \quad (18)$$

so that $\boldsymbol{C}_p$ can be expressed from (17) as

$$\boldsymbol{C}_p = L\boldsymbol{I}. \quad (19)$$

However, in practice, several subcarriers of OFDM blocks are null to avoid interference between adjacent bands. In this case we can not take $C_p = LI$ except for some special cases. In [24], using convex optimization, trace(C_p^{-1}) is minimized so that

$$F_a^{\mathcal{H}} D^{\mathcal{H}}(X_{m,a}^p) D(X_{m,a}^p) F_a \simeq \sigma_X^2 I, \quad m = k, k+1. \tag{20}$$

Then we can approximate C_p as

$$C_p = A_p^{\mathcal{H}} A_p \simeq 2\sigma_X^2 I. \tag{21}$$

If we consider signal from each transmit antenna separately, it follows from (10), that

$$\begin{aligned} A_p^{\mathcal{H}} \tilde{y}_k &= F_{L,a}^{\mathcal{H}} [D^{\mathcal{H}}(X_{k,a}^p) F_a D(-\epsilon) y_k \\ &\quad + e^{-j\alpha\epsilon} D^{\mathcal{H}}(X_{k+1,a}^p) F_a D(-\epsilon) y_{k+1}]. \end{aligned} \tag{22}$$

Under (21), for each transmit antenna, it suffices to maximize

$$\begin{aligned} &\|D^{\mathcal{H}}(X_{k,a}^p) F_a D(-\epsilon) y_k \\ &+ e^{-j\alpha\epsilon} D^{\mathcal{H}}(X_{k+1,a}^p) F_a D(-\epsilon) y_{k+1}\|^2 \end{aligned} \tag{23}$$

which is equivalent to

$$\|D^{\mathcal{H}}(X_{k,a}^p) F_a D(-\epsilon) y_k\|^2 \tag{24}$$

$$\begin{aligned} &+ \|e^{-j\alpha\epsilon} D^{\mathcal{H}}(X_{k+1,a}^p) F_a D(-\epsilon) y_{k+1}\|^2 \\ &+ 2Re\{e^{-j\alpha\epsilon} y_k^{\mathcal{H}} \mathcal{M} y_{k+1}\} \end{aligned} \tag{25}$$

where

$$\mathcal{M} = D(\epsilon) F_a^{\mathcal{H}} D(X_{k,a}^p) D^{\mathcal{H}}(X_{k+1,a}^p) F_a D(-\epsilon).$$

Moreover, under (21), $\|D^{\mathcal{H}}(X_{k,a}^p) F_a D(-\epsilon) y_k\|^2$ and $\|e^{-j\alpha\epsilon} D^{\mathcal{H}}(X_{k+1,a}^p) F_a D(-\epsilon) y_{k+1}\|^2$ are constant. Thus we only need to maximize

$$Re\{e^{-j\alpha\epsilon} y_k^{\mathcal{H}} \mathcal{M} y_{k+1}\} \tag{26}$$

with respect to ϵ to obtain the estimate of ϵ.

For two consecutive long preambles, if we set $X_{k+1}^p = X_k^p$, then from (20), the objective function in (26), is approximately equal to the maximization of

$$Re\{e^{-j\alpha\epsilon} y_k^{\mathcal{H}} y_{k+1}\}, \tag{27}$$

which is the same as the objective function in [11]. The solution in (27) can be expressed as [11]

$$\hat{\epsilon} = \frac{1}{\alpha} \angle \left(y_k^{\mathcal{H}} y_{k+1} \right). \tag{28}$$

It should be remarked that $e^{-j\alpha\epsilon}$ is a periodic function in ϵ with period $N/(N + N_{cp})$. Thus, the CFO estimation range by (28) is smaller than $(-0.5, 0.5]$. On the other hand, the period of $\mathcal{M}$ in (26) is 1, hence we consider our CFO estimation range to be $(-0.5, 0.5]$ which concurs with the ideal range of the residual CFO.

Since Eqs. (15), (23), and (26) are not linear in ϵ, we resort to numerical minimization. More specifically, we utilize a grid search within the acquisition range to obtain the (sub-)optimal value of the CFO estimate. We only consider the residual CFO estimation, which means that CFO is small enough for the grid search to work. Even though the residual CFO is small, if left uncorrected can result in severe BER degradation at the receiver [22]. The CFO MSE is given by

$$\eta = E\{\|\hat{\epsilon} - \epsilon\|^2\} \tag{29}$$

which is the mean squared error of the estimator. To evaluate the performance of the proposed CFO estimators, numerical simulations for a given range of normalized CFO between $(-0.5, 0.5]$ will be conducted.

Note that, for SISO-OFDM, $A(\epsilon)^{\mathcal{H}} A(\epsilon) > 0$ is easily guaranteed even if there are some null subcarriers. However, for MIMO-OFDM, we need to mitigate the effect of co-channel interference between the transmitting antennas to obtain good estimates. Thus, the challenge is not only to design $X^1, X^2, \ldots, X^{N_t}$ to meet $A(\epsilon)^{\mathcal{H}} A(\epsilon) > 0$.

If the training signals between the transmit antennas are orthogonal, then one can show that $A(\epsilon)^{\mathcal{H}} A(\epsilon) > 0$. Several techniques have been developed to ensure the orthogonality of the training sequences for MIMO-OFDM (see [2], [17], [18], [23] and the references therein). In [17], the orthogonality of the MIMO long training preambles is established by the Phase-Shift (PS) codes proposed in [2], [18], where all the symbols have the same power, while in [25], the orthogonality of the training symbols is achieved by ensuring that, the training symbols of one antenna are disjoint from the training symbols of any other antenna in the frequency domain and power of each training symbols is obtained by minimizing the channel MSE $E\{\|\hat{h} - h\|^2\} = \sigma_v^2 \,\mathrm{trace}\{[A(\epsilon)^{\mathcal{H}} A(\epsilon)]^{-1}\}$ with respect to the total power of symbols in a preamble. Design of disjoint training set is also present in [23] under the assumption that all subcarriers in an OFDM block are used as pilot tones. However, in most realistic case some of the subcarriers at the edge of the spectrum are nulled to avoid interference between adjacent OFDM blocks. This limit the adoption of the design technique in [23] in practical systems.

Figure 1 shows the training symbols designed by the

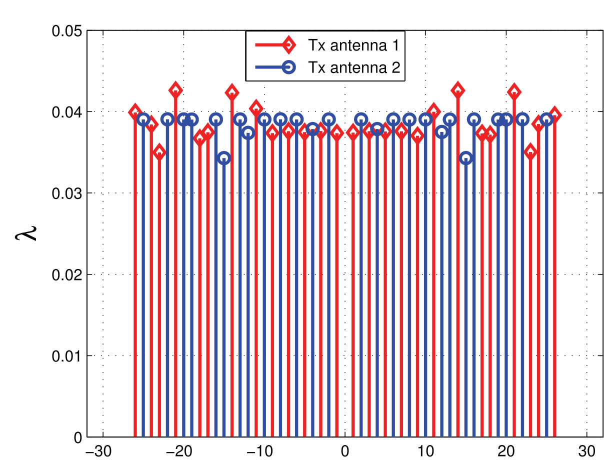

Fig. 1 Disjoint training signals for 2-transmitter.

algorithm proposed in [25] for two transmit antennas. The total power of the training signals for each antenna is normalized to one. Thus, we can easily distribute power to the training symbols for any given power per OFDM block by multiplying the total power with the normalized power.

4. Simulation Results

In this section, we conduct computer simulations to demonstrate the effectiveness of our proposed schemes. The efficacy of the proposed ML estimators are evaluated by the mean squared error (MSE) as well as bit error rate (BER) performances. The parameters of the transmitted OFDM signal studied in our design examples are as in the IEEE 802.11a, standard in [10, p.600], where an OFDM transmission frame with $N = 64$ is considered. Out of 64 subcarriers, 52 subcarriers are used as data subcarriers. Of the remaining 12 subcarriers, 6 are null in the lower frequency guard band while 5 are nulled in the upper frequency guard band and one is the central DC null subcarrier. Of the 52 used subcarriers, 4 are allocated as pilot subcarriers, while the remaining 48 are used for data transmission.

For the long preamble, all 52 active subcarriers are used as training symbols. For single transmit antenna, we adopt a standard long preamble sequence in [10], which allocate equal power to all active subcarriers. Simulation results (which are not shown here) verify that the performance of the designed preamble in [24] is comparable to that the standard preamble [10]. For MIMO case, to ensure orthogonality to multiple transmit antennas, we resort to the disjoint pilot sequence depicted in Fig. 1 as well as training sequences presented in [17] where equal power is allocated to all active subcarriers and the orthogonality of the designed preamble is obtained by adopting the special phase shift codes proposed in [2], [18].

The channel length $L = 8$ is considered and the channel coefficients are modeled as i.i.d. complex valued Gaussian random variables with zero mean and an exponential power delay profile given by the vector $\boldsymbol{\rho} = [\rho_0 \dots \rho_{L-1}]$ where $\rho_l = Ce^{-l/2}$, and C is a constant selected so that $\sum_{l=0}^{L-1} \rho_l = 1$.

Simulation conditions are summarized in Table 1.

Figure 2 depicts the mean squared error versus normalized CFO ϵ for the estimators in (15), (23), (26), and (28), which we refer to it as Estimator1, Estimator2, Estimator3 and Moose respectively. The signal to noise ratio (SNR) is set to 10 dB.

From the plots it is clear that Estimator1, Estimator2 and Estimator3 perform equally well for a wide range the residual CFO and their differences are almost negligible. However, at low signal to noise ratio (SNR), Estimator1 outperforms other estimators by a small margin. This is due to the fact that, Estimator2, Estimator3 and Moose estimator utilize some approximation which reduces the computational complexity as compared to Estimator1, but these approximations may alter the accurate of the estimations especially at low SNR. The complexity of the Moose estimator is much low as compared to the proposed estimators. However, from Fig. 2 it is clear that, the Moose estimator proposed in [11] does a poor job of estimating CFO for some normalized CFO values close to the integral CFO. This is because the estimator utilizes more approximation as compared to Estimator2 and Estimator3.

Table 1 Simulation conditions.

OFDM parameters	IEEE 802.11a
Number of (subcarrier) DFT point	$N = 64$
Length of cyclic prefix	$N_{cp} = 16$
Number of active subcarriers	$N_a = 52$
Number of antennas (N_t, N_r)	(2, 1)
Residual CFO range	$(-0.5\ 0.5]$
Modulation schemes	16-PSK, 64-PSK
Channel model	Rayleigh fading channel
Channel length	$L = 8$
channel delay profile	Exponential
Signal to noise ratio (SNR) range	[0 20] dB

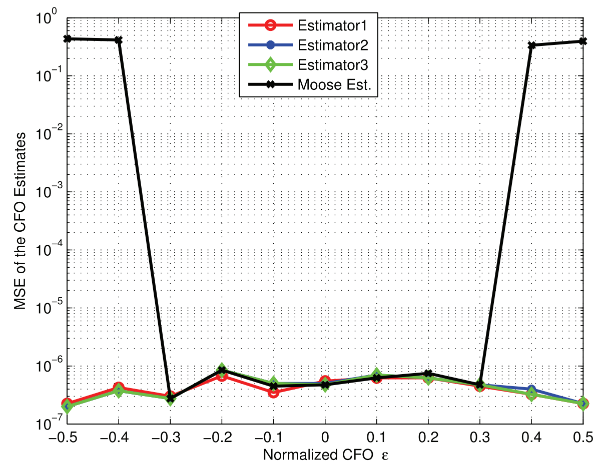

Fig. 2 Comparison of MSE performance of the estimators for different CFO's.

The results in Fig. 2 show that for some residual CFO values the MSE of Moose estimator is as good as our proposed Estimators. However, it is desirable to have an estimator capable of correcting the residual CFO within the range, that is, $(-0.5, 0.5]$, this makes our proposed designs superior over the Moose estimator in [11].

Next, we provide simulation results showing the CFO MSE and its comparison with the CRB for the full loaded OFDM system derived in [12]. The CRB allows to get an insight into the theoretical performance limit of the estimators. Figure 3 shows the mean CFO MSE for the proposed ML estimators, the Moose and the CRB for $\epsilon = 0.2$. Note that for fare comparison, we select the normalized CFO value of $\epsilon = 0.2$ since it is within a range where the MSE performance of all estimators are almost similar (see Fig. 2). At low SNR, Estimator1 outperforms Estimator2, Estimator3 and Moose estimator that utilizes some approximations. Moose estimator does a poor job of estimating CFO at low SNR than other estimators. However, at high SNR the performance of the proposed estimators as well as the Moose estimator achieves the CRB of the full loaded OFDM. Esti-

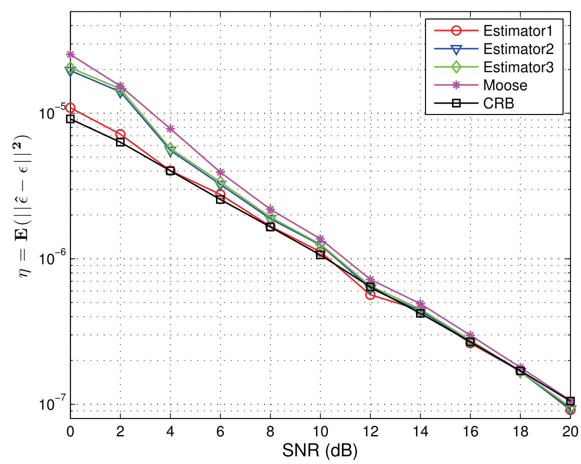

Fig. 3 Comparison of the CFO MSE performance of the estimator and the CRB of the full loaded OFDM.

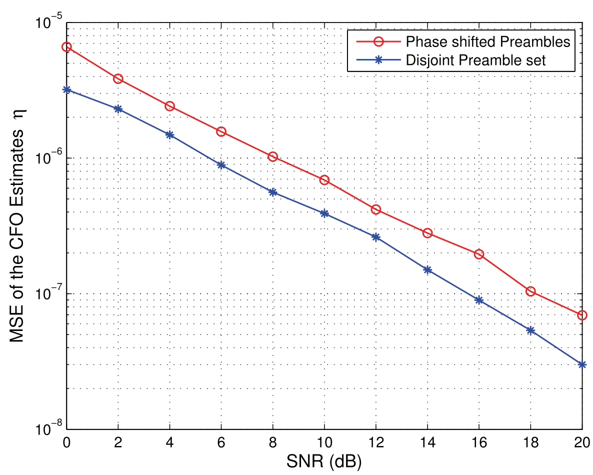

Fig. 4 Performance of the MSE of the CFO estimator for different SNRs.

mator1 performs efficiently, even for low values of the SNR. This demonstrates its superior performance over other estimators that utilizes some approximation. The accuracy of Estimator1 is not altered by the value of the residual CFO, that is the estimator is capable of estimating the residual CFO within the range with the same accuracy.

Figure 4 shows the mean squared error of the CFO estimator $\boldsymbol{\eta} = E\{||\hat{\epsilon} - \epsilon||^2\}$ vs signal to noise ratio (SNR) for the phase shifted long preambles and the disjoint training symbols for $N_t = 2$. From the plots it is clear that the disjoint preamble set outperforms the phase shifted preambles for different signal to noise ratios (SNRs).

For a given power per OFDM block, the disjoint set outperforms the phase shifted preambles by a small margin. This may be due to the fact that the minimization of the channel MSE improves the condition number of the matrix $\boldsymbol{A}(\epsilon)^{\mathcal{H}}\boldsymbol{A}(\epsilon)$. However, simulation results (which are not shown here) verified that there is no significant difference in bit error rate (BER) performance between the disjoint pilot set and the phase shifted preambles.

Next, we demonstrate the performance of our channel

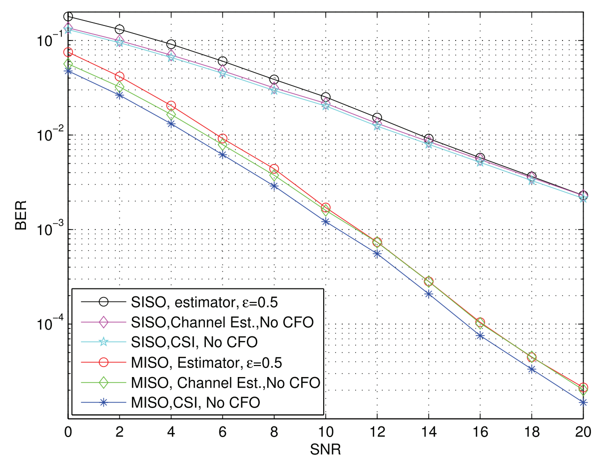

Fig. 5 Comparison of the BER performances for 16-PSK.

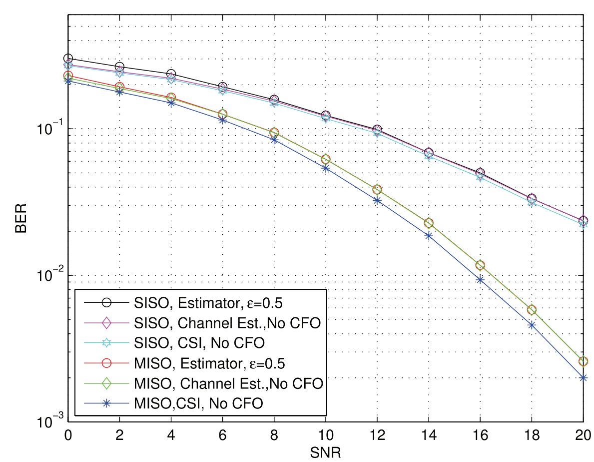

Fig. 6 Comparison of BER performances for 64-PSK.

and CFO estimator by considering the BER performance of both SISO and Alamouti STBC with two transmit antennas and one receive antenna (MISO). It is well known that, to obtain better BER performance, proper compensation of carrier frequency and accurate channel estimates are of primary important. In the BER analysis we adopt Estimator1 due to its superior performance over the others.

The results in Fig. 5 and Fig. 6 depict the BER performance of our channel and CFO estimator together with the results of the known channel state information and the estimated channel without any residual CFO component. From the results it is clear that the BER performance of the estimated channel is comparable to that of the known channel state information for CFO free case. This demonstrates the efficiency of our channel estimator. Also the performance of our combined channel and CFO estimator gives nearly the same BER performance as the estimated channel without any residual CFO for both 16-PSK and 64-PSK. The proposed approach maintains the bit error rate (BER) within 1 dB of the value obtained from the CFO-free system for both SISO and MISO case. Note that, in the BER simulations we consider $\epsilon = 0.5$ because it is close to the integral CFO. Some algorithms for estimating and correcting the residual CFO does not perform well at CFO values close to the integral value. Any value of the residual CFO within the range i.e., (−0.5 0.5] can be used for demonstration.

5. Conclusion

In this paper we addressed the problem of channel and CFO estimation for both SISO and MIMO-OFDM systems with null subcarriers. Through numerical simulations, we have verified that the the proposed scheme can be used to efficiently estimate the channel as well as carrier frequency offset in OFDM systems. Simulation results show the MSE of the proposed ML estimator is comparable to the derived CRB for the full loaded OFDM. The channel and CFO estimators provide reasonable BER performance in comparison with the known channel state information. The main contribution of this article is the derivation of the MLE estimators with significant CFO and channel estimates.

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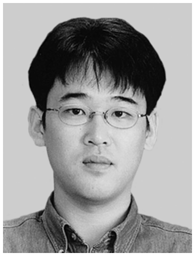

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