Two Types of Discretion: Monetary Policy before 1979 and in the Greenspan Era*

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Abstract

In our neoclassical Phillips curve model, two types of discretionary equilibria can be characterized as a deflationary equilibrium and a high-inflation equilibrium and the absolute value of feedback coefficient on the lagged output gap is larger for the former. This implies that more active policy corresponding to monetary policy during the Greenspan era, brings about deflation rather than high inflation under discretion. This critical aspect contradicts the results obtained in the context of the "rule versus discretion" literature. Furthermore, the discretionary regime in the Greenspan era is unstable, and the equilibrium corresponding to the discretionary regime before 1979, is stable.

Ⅰ. Introduction

How can economists specify the difference between the discretion in the Greenspan era and activist monetary policy before 1979 at which Fed instituted a radical shift in policy? Clarida et al. [2000] and Taylor [1999] claimed that the interest rate rule adopted in the Greenspan era is characterized by an inflation coefficient greater than unity; see also Friedman [2006] and Woodford [2003, Chapter 1][1]. The responsiveness of the nominal interest rate to the inflation rate, and the inclusion of the output gap in the Taylor rule, can be seen as a reflection of activist monetary policy; and the difference in inflation coefficients implies that monetary policy in the Greenspan era was more active than the activist monetary policy before 1979. In other words, more active policies in the Greenspan era, which we call G-active monetary policies, resulted in better economic outcomes than the active policies before 1979, which we call conventionally active, or C-active monetary policies.

Given that activist monetary policy before 1979 resulted in a high-inflationary economy, it is natural to ask why G-active monetary policies resulted in lower inflation, rather than in hyper-inflation. The main purpose of this paper is to demonstrate the possibility of the prevalence of zero-inflation or deflation, when the policy maker is G-active. This critical aspect contradicts the results commonly accepted in the context of the "rule versus discretion" literature, following Barro-Gordon [1983] (hereafter BG); that is, activist policies result in high inflation, and therefore, a G-active policy maker will produce a hyperinflationary economy. To analyze the macroeconomic consequences of monetary policy regimes in a dynamic optimizing model, this paper employs a simple model, in which monetary policy takes the form of inflation-rate feedback rules, instead of interest-rate feedback rules.[2] This simple model can capture the above-mentioned aspect. The differences between the expectations formation process and the timing of moves assumed in this model, and those assumed in the previous literature, are essential for this purpose.

The fact that variants of Taylor-type rules fit the U.S. data reasonably well indicates the stability of the feedback rules chosen by the central bank. We think of this stability as the time consistency of a discretionary equilibrium. That is, the economy is in a discretionary equilibrium, in which monetary policy becomes time invariant, and actual and privately expected feedback rules coincide. We define a discretionary policy regime as one in which the central bank chooses the cost-minimizing feedback rule, given the public's expectations of feedback coefficients.

Our results show that there are two types of discretionary equilibria; a deflationary equilibrium and a high-inflationary equilibrium. The central bank chooses G-active monetary policy in deflationary types of discretionary equilibrium, and C-active in high inflationary types. In addition, we show that a G-active equilibrium, i.e., the discretionary regime in the Greenspan era, is unstable, and that a C-active equilibrium, i.e., the discretionary regime before 1979, is stable.

If the central bank were to choose a higher inflation rate in a G-active equilibrium, where the rate of inflation is lower than the optimum rate, it could decrease both the cost of inflation and output cost. It is natural, therefore, to ask, “Why doesn't the central bank raise the inflation rate in a G-active equilibrium?” To answer this question, suppose that, in the initial period, zero, the central bank facing deflation chooses a feedback rule, and that this is anticipated by the public. In the next (first) period, the central bank could adopt a surprise inflation policy, although the previous feedback rule continues to be in place, and is anticipated by the public in subsequent periods. Under the dynamic formulation of the BG model, a positive output gap with respect to the natural rate level produced by the surprise inflation policy persists for a sufficient period, and this in turn keeps both the actual and the expected inflation rates below their original path. On the one hand, the persistence of a positive output gap and the higher inflation in the first period lower the cost. On the other hand, lower inflation (deterioration of deflation) in subsequent periods increases the cost. When the marginal cost of a one-time deviation of inflation is balanced by its marginal gain, the central bank has no incentive for a surprise inflation policy, and the economy is in a G-active equilibrium.

The rest of the paper is organized as follows. Section 2 describes the basic model and explains the expectations formation process, and the relative timing of moves assumed in this paper. Section 3 shows that there are two types of discretionary equilibria; deflationary equilibrium and high-inflationary equilibrium. In addition, we show that a G-active equilibrium, i.e., the discretionary regime in the Greenspan era, is unstable, and that a C-active equilibrium, i.e., the discretionary regime before 1979, is stable. Finally, Section 4 concludes the paper.

Ⅱ. The Model

The model in this paper is essentially identical with that of Dittmar, Gavin, and Kydland [1999] (hereafter DGK), except for the following features. First, in our model the public's expectations formation is based on the perceived decision rule of the central bank, as is the case in a linear-quadratic dynamic game. Second, the timing of moves is changed relative to the DGK model, such that the central bank considers the public's expectations formation (not the expected inflation rate) when choosing a feedback rule.

The Central Bank’s Intertemporal Loss Function

Let y_τ and π_τ denote output deviation from its natural rate (in logarithm) and the rate of inflation in period τ, respectively. For an optimal level of the output gap $\bar{y} > 0$, that of the inflation rate π^*, arbitrary weight $0 \leq \lambda \leq 1$, and a discount factor $0 < \beta < 1$, the objective of monetary policy is to minimize the expected value of the loss criterion

$$(1) \quad L = \sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t \left\{ \lambda (y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2 \right\},$$

where $\bar{y}$ does not indicate the natural rate of output, as discussed in BG (BG, p.591).

The Short-Run Phillips Curve

The central bank is constrained by the aggregate supply relation of the form

$$(2) \quad y_\tau = \rho y_{\tau-1} + \alpha(\pi_\tau - \pi_\tau^e) + \eta_\tau,$$

where η_τ is an iid technology shock with mean zero and constant variance σ_η^2, ρ and α are positive constants, π_τ^e denotes the public's expectation of π_τ in period τ, conditional on the information available in period τ.[3] Neither the central bank nor the public can observe current realizations of η_τ. The inclusion of the lagged value of output deviation is justified because of wage contract, menu cost, etc., (DGK, p.24).

Expectations Formation of the Public

It is reasonable for the public to form an expectation about the central bank's *feedback rule*. More formally, the public expects inflation to be determined by the linear function

(3) $\pi_\tau = A_1^e + A_2^e y_{\tau-1}$,

where A_1^e, A_2^e are the feedback coefficients anticipated by the public. Here, we assume the Markov property, and characterize the public's perceived decision rule of the central bank as depending only on the value of the state variable, y_{t-1}, and not the entire history.

Consider an economy where it is known that the central bank's policy choice is a function of readily observable macro variables. Under such a policy choice, the public expects the function or strategy of the central bank, rather than a particular value of the policy. In the context of this model, Eq. (3) expresses the public's perception of the central bank's decision as a function of the lagged output gap. The public's expectation of π_τ in period τ, conditional on the information available in period τ, π_τ^e, can be written as

(4) $\pi_\tau^e = A_1^e + A_2^e y_{\tau-1}$.

The Central Bank's Reaction Function

We now obtain the Phillips curve (Lucas supply function) constraint consistent with the public's expectation. Substituting (3) into (2) gives

(5) $y_\tau = -\alpha A_1^e + (\rho - \alpha A_2^e) y_{\tau-1} + \alpha \pi_\tau + \eta_\tau$.

Given the linear structural equation, (5), and the quadratic loss function of our model, (1), the central bank's reaction function (authority's strategy) takes the form

(6) $\pi_\tau = A_1 + A_2 y_{\tau-1}$.

Clearly, a set of *feedback coefficients* (A_1, A_2) identifies the central bank's strategy space. Without the Markov property of the public's perception of the central bank's decision rule, (3), the central bank's reaction function does not always take the form of (6), even when its loss function is quadratic. Note that the central bank's reaction function depends on the public's expectations formation, as well as the loss function (1) and the constraint (2).

The Timing of Moves

Here, we discuss the interaction between the public's expectations formation and the central bank's choice of a feedback rule. The central bank's strategy space is a set of feedback rules of the form (6) (or equivalently, that of *feedback coefficients* (A_1, A_2)), and its loss function is defined by (1). The public's strategy space is a set of functions of the form (3); or, equivalently, the set of (A_1^e, A_2^e), but its loss function has not been specified explicitly.

The last element of the model is the timing of moves. The difference between the timing of moves assumed here and that assumed in DGK is essential. This model assumes that the central bank chooses its feedback rule *after* the public expects the inflation rate rule. In other words, the central bank considers the public's expectations formation when choosing a feedback rule. In a discretionary regime, the central bank chooses the optimal path for inflation, subject to the constraint (5).

Ⅲ. Discretionary Equilibria under G-Active and C-Active Monetary Policies

The Central Bank's Best Response

In a discretionary regime, the central bank chooses the optimum feedback rule, given the public's expectations of feedback coefficients. The optimization problem is represented by a Lagrangian of the following form:

(7) $$\Lambda = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\{\lambda(y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2\} - \mu_\tau \{y_\tau + \alpha A_1^e - (\rho - \alpha A_2^e) y_{\tau-1} - \alpha \pi_\tau - \eta_\tau\}].$$

We differentiate the Lagrangian with respect to y_τ and π_τ to obtain the first-order conditions

(8) $2\lambda(y_\tau - \bar{y}) - \mu_\tau + \beta(\rho - \alpha A_2^e) E_\tau \mu_{\tau+1} = 0$,

(9) $2(1-\lambda)(\pi_\tau - \pi^*) + \alpha \mu_\tau = 0$,

for each time τ. Conditions (8) and (9) imply the stochastic Euler equation

(10) $$\lambda(y_\tau - \bar{y}) + \frac{(1-\lambda)(\pi_\tau - \pi^*)}{\alpha} - \frac{\beta(1-\lambda)(\rho - \alpha A_2^e)}{\alpha} E_\tau(\pi_{\tau+1} - \pi^*) = 0.$$

Substituting (5) and (6) for time $\tau+1$ into (10), and equating coefficients, we obtain values for A_1 and A_2 in terms of A_1^e, A_2^e, and the parameters of the model. More concretely, setting the coefficient of $y_{\tau-1}$ and the constant term to zero results in

$$(11)\quad A_2^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho - \alpha A_2^e)^2}{\alpha\beta(1-\lambda)(\rho-\alpha A_2^e)} A_2 - \frac{\lambda}{\beta(1-\lambda)} = 0,$$

and

$$(12)\quad A_1 = [\lambda\alpha^2 + (1-\lambda)\{1-\beta(\rho-\alpha A_2^e)(1+\alpha A_2)\}]^{-1} [\lambda\alpha(\alpha A_1^e + \bar{y}) + (1-\lambda)\pi^* - \beta(1-\lambda)(\rho - \alpha A_2^e)(\alpha A_1^e A_2 + \pi^*)].$$

Section A of Matsukawa Okamura and Taki [2009] provides the details of the solution.

We can use Eqs. (11) and (12) to define the central bank's best-response function $(A_1, A_2) = B(A_1^e, A_2^e)$. In fact, these equations can be solved recursively to obtain a unique solution to the problem. First, find the negative root of the quadratic equation (11), in which A_1 is not included. As shown in Section A of Matsukawa, Okamura and Taki [2009], the quadratic equation (11) always has two real roots, and the positive one is not relevant here. Therefore, we focus our attention on the negative root and denote it as $A_2 = f(A_2^e)$. Second, substituting A_2, obtained in this manner, into (12) yields the solution for A_1.

Constructing Discretionary Equilibria

We now turn to the characterization of a discretionary equilibrium. We show that there are two discretionary equilibria: one with deflation bias and the other with high inflation bias. A necessary and sufficient condition for the central bank's *feedback coefficients*, (A_1^D, A_2^D), to be chosen in a discretionary equilibrium is that $(A_1^D, A_2^D) = B(A_1^D, A_2^D)$. Setting $A_2^e = A_2 = A_2^D$ in (11), we obtain

$$(A_2^D)^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho - \alpha A_2^D)^2}{\alpha\beta(1-\lambda)(\rho-\alpha A_2^D)} A_2^D - \frac{\lambda}{\beta(1-\lambda)} = 0.$$

Rearranging terms yields

$$(13)\quad (1-\lambda)(A_2^D)^2 + \frac{(1-\lambda)(1-\beta\rho^2)}{\alpha\beta\rho} A_2^D + \frac{\lambda}{\beta} = 0.$$

Setting $A_2^e = A_2 = A_2^D$ and $A_1^e = A_1 = A_1^D$ in (12), and after rearranging, we have

$$(14)\quad A_1^D = \frac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}} + \pi^*.$$

The discretionary equilibria of our model can be characterized by the central bank's *feedback coefficients* (A_1^D, A_2^D) satisfying Eqs. (13) and (14). It is important to note that the inflation rate in a discretionary equilibrium, A_1^D, can be lower than the optimum rate π^* when the absolute value of A_2^D is large enough to make $1-\beta(\rho-\alpha A_2^D)$ negative. In addition, if $\bar{y} > 0$ is sufficiently large, or the central bank targets an output level sufficiently higher than the natural level, an activist policy brings about deflation, rather than high inflation in a discretionary equilibrium. We now have

Theorem 1: Under discretion, the activist monetary policy, which is very sensitive to changes in the output gap, brings about a lower inflation rate than the optimum. Furthermore, if the central bank targets a sufficiently higher level of output than the natural level, it can bring about deflation in a discretionary equilibrium.

Two Types of Discretionary Equilibria

It is easy to show that the sign of the discriminant of (13) is the same as the sign of $\phi(\lambda, \rho, \alpha, \beta) = (1-\lambda)\{1-2\beta\rho^2+\beta^2\rho^4\} - 4\lambda\alpha^2\rho^2\beta$. It is also easy to show that $\frac{\partial\phi}{\partial\lambda} < 0$, $\frac{\partial\phi}{\partial\rho} < 0$, $\frac{\partial\phi}{\partial\alpha} < 0$, and $\frac{\partial\phi}{\partial\beta} < 0$, for $0 < \lambda < 1$. For example, the first inequality implies that combinations of (ρ, λ) are more likely to make the discriminant of the quadratic Eq. (13) negative as the weight put on output gap stabilization (λ) increases, given ρ and α. In other words, as long as the central bank does not place too much weight on output stability, there exist two discretionary equilibria. Figure I illustrates the regions of ρ and λ, where the roots of the quadratic equation (13) consist of real roots, and of complex pairs for $\alpha = 0.5$, 0.9 and $\beta = .99$. The discriminant of (13) is negative for the set of (ρ, λ) located at the northeast of the curves shown in the figure.

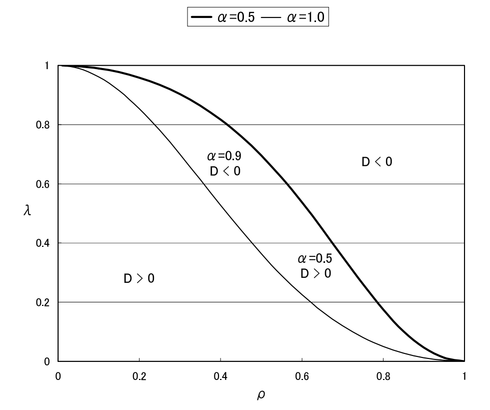


Figure Ⅰ The Discriminant of Equation (13)
$\beta = .99$

When the quadratic equation (13) has a positive discriminant, the two distinct real roots are negative, by Viète's formulas. Let A_2^{**} be the one with greater absolute value and A_2^{*} be the one with smaller absolute value. Denote the intercept associated with A_2^{**} and A_2^{*} as A_1^{**} and A_1^{*}. In the discussion that follows, the discretionary monetary policy represented by (A_1^{**}, A_2^{**}) is called G-active monetary policy and the resulting equilibrium is called the G-active equilibrium. Meanwhile, the discretionary monetary policy represented by (A_1^{*}, A_2^{*}) and the resulting equilibrium are called conventionally active or C-active monetary policy, and the C-active equilibrium.

In a G-active equilibrium, A_2^{**} can be large enough (in absolute value) to make the denominator of (14) negative. Then, for a positive target level of output gap, $\bar{y}$, A_1^{**} is lower than π^*. In particular, if the central bank tries to target an output level sufficiently higher than the natural level, i.e., if $\bar{y}$ is sufficiently large, the equilibrium inflation rate under G-active monetary policy, A_1^{**} can be negative. That is, G-active monetary policy can bring about deflation, rather than high inflation in a discretionary equilibrium. In contrast, the resulting inflation rate is higher than the social optimum in the C-active equilibrium.

Theorem 2: As long as the central bank does not place too much weight on output stability, there exist two discretionary equilibria. If the central bank tries to target a sufficiently high output, relative to the natural-rate level, the resulting inflation rate is lower (higher) than the target level, or social optimum, in the G-active (C-active) equilibrium.

Examples

Given the parameter values, one can see that Eq. (11) determines A_2 as a function of A_2^e alone. In Figure Ⅱ, the vertical axis represents A_2, the feedback coefficient on $y_{\tau-1}$ chosen by the central bank, and the horizontal axis represents A_2^e, the value of A_2 expected by the public. As mentioned above, the quadratic equation (11) always has two real roots and the positive one (dashed line) is not relevant. Therefore, we focus on the negative root (solid line) denoted by $A_2 = f(A_2^e)$.

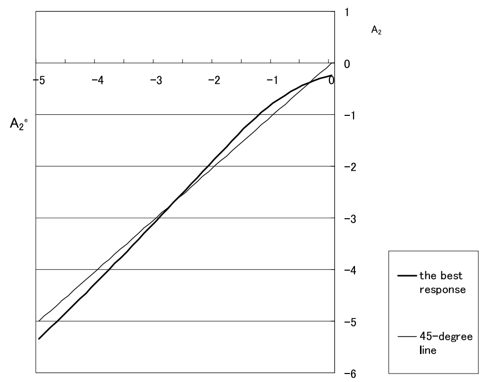


Figure Ⅱ Discretionary Equilibria
$A_2^{**} = -2.6609$, $A_2^{*} = -0.37963$
$\alpha = 0.5$, $\beta = .99$, $\rho = 0.5$, $\lambda = 0.8$

Figure Ⅱ shows this function for our benchmark case ($\alpha = .5, \beta = .99, \rho = .5, \lambda = .5$). The quadratic equation (13) has two real roots for this parameter configuration, and the curve $A_2 = f(A_2^e)$ intersects the 45-degree line at $A_2^{**} = -2.6609$ (the G-active equilibrium) and $A_2^{*} = -0.37963$ (the C-active equilibrium).

It is easy to obtain the feedback coefficient A_1^{**}, because (14) means that A_1^{**} is linear in $\bar{y}$ and π^*. For example, suppose that the optimum inflation rate is one percent ($\pi^* = 0.01$), and that the central bank desires to make the output level four percent higher than its natural level $\bar{y}$ in our benchmark case (α =.5, β =.99, ρ =.5, λ =.5). Subsequently, in the G-active equilibrium, the rate of inflation chosen by the central bank observing $y_{t-1} = 0$, is $A_1^{**} = 0.01 - 0.0062 \times 4 = -0.0148$. That is, despite the one percent target, the central bank chooses a 1.48 percent rate of deflation in the G-active equilibrium. Interestingly, deflation instead of inflation results in a discretionary equilibrium, in contrast to the argument of BG. Deflation arises because $\beta(\rho - \alpha A_2^{**}) > 1$. Therefore, the denominator of the expression $A_1^{**} = \frac{\alpha \lambda \bar{y}}{(1-\lambda)\{1-\beta(\rho - \alpha A_2^{**})\}} + \pi^*$ is negative. It follows immediately that the higher the level of output gap desired by the central bank, the higher the possibility that a deflationary equilibrium will emerge.

We now examine the C-active equilibrium. Consider our benchmark case, where α =.5, ρ = 0.5, λ = 0.5, β =.99. Since $A_1^* - \pi^* = 0.01577$ for $\bar{y} = 0.01$, if the optimum levels of output gap and inflation are $\bar{y} = 0.04$ and $\pi^* = 0.01$, respectively, the inflation rate in the C-active equilibrium will be $\pi = 0.01 + 0.01577 \times 4 = 0.07308$, i.e., 7.308 percent inflation. As in the BG model, the resulting inflation rate is higher than the social optimum.

Comparison to Previous Literature

The BG static model used unemployment as the state variable. We replace unemployment by output gap and reformulate it as follows. The central bank minimizes a simple quadratic form: $E_\tau\{\lambda(y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2\}$, subject to $y_\tau = \alpha(\pi_\tau - \pi_\tau^e) + \eta_\tau$, where we use the notation of Section Ⅱ. It is straightforward to show that the inflation rate in the BG discretionary equilibrium is $A_1^{BG} = \frac{\alpha \lambda \bar{y}}{1-\lambda}$, which is higher than the optimum level π^*. In a discretionary equilibrium, *the expected rate of inflation is sufficiently high so that the marginal cost of inflation just balances the marginal gain from reducing unemployment* (BG, p.599).

Consider an economy described by the key elements, (1), (2), and (6). In this economy, the rational expectations hypothesis implies that the public forms inflation expectations rationally, in accordance with (6): $\pi_\tau^e = E_\tau \pi_\tau$. Therefore,

(15) $\pi_\tau^e = A_1 + A_2 y_{\tau-1}$.

This condition implies that the process of expectations formation can be described using the same feedback rule as that chosen by the central bank. In contrast, this paper assumes expectations formation of the form (3). In our model, (15) holds in discretion equilibria, but $A_1 = A_1^e$ and $A_1 = A_2^e$ are equilibrium conditions and not assumptions.

In the discretionary regime, the central bank moves after the public expects the inflation rate rule. In other words, given the anticipated inflation rate rule (3), the central bank chooses the optimum feedback rule (6). In a rational expectations equilibrium, however, the public and the central bank move simultaneously, therefore, the public cannot be surprised by the central bank.

Using the same methods as in the DGK, the rational expectations equilibrium of this economy can be obtained:

(16) $A_1^{\#} = \dfrac{\alpha\lambda\bar{y}}{(1-\lambda)(1-\rho\beta)} + \pi^*,\ A_2^{\#} = \dfrac{-\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}$.

Setting $\bar{y} = 0$ (and fixing the relative weight on inflation variability in the objective function to unity, $1 - \lambda = 1$), (16) reduces to the rational expectations equilibrium obtained by DGK.

Now, let us compare the rational expectations equilibrium of DGK with the discretionary equilibria obtained in this paper, $A_1^D = \frac{\alpha \lambda \bar{y}}{(1-\lambda)\{1-\beta(\rho - \alpha A_2^D)\}} + \pi^*$. The difference between these two rates is the term $\alpha\beta A_2^D$ in the denominator. Since the central bank minimizes the expected discounted losses, given the feedback coefficients anticipated by the public in a discretionary equilibrium, the degree of output persistence faced by the central bank is $\rho - \alpha A_2^e$, instead of ρ. Therefore, $A_1^{\#}$ and A_1^D differ by $\alpha\beta A_2^D$ in the denominator.

Finally, consider the optimal reaction function under commitment. It is a reasonable conjecture that the feedback coefficients associated with the minimum of the central bank's loss function are $A_1 = \pi^*$ and $A_2 = 0$[4]. Section B of Matsukawa, Okamura and Taki [2007] presents the details of the proof. It is also straightforward to show that $A_2^* < A_2^{\#} < 0$[5] (See Section B of Matsukawa, Okamura and Taki [2011]).

Intuition for G-active Equilibrium

If the central bank were to choose a higher inflation rate in a G-active equilibrium, where the rate of inflation is lower than the optimum rate, π^*, it could decrease both the cost of inflation and output cost. It is natural, therefore, to ask, “Why doesn't the central bank raise the inflation rate in a G-active equilibrium?” To answer this question, we study the effects of a one-time deviation of inflation from an arbitrary feedback rule, $\pi_\tau = \tilde{A}_1 + \tilde{A}_2 y_{\tau-1}$. We suppose that in the initial period, $\tau = 0$, the central bank chooses this feedback rule, and that it is anticipated by the public: $A_1^e = \tilde{A}_1$ and $A_2^e = \tilde{A}_2$. In this discussion, we also assume that $y_0 = 0$, for simplicity. In the first period, $\tau = 1$, the central bank chooses another inflation rate, $\pi_1 > \pi_0$, although the previous rule $\pi_\tau = \tilde{A}_1 + \tilde{A}_2 y_{\tau-1}$ continues to be in place, and is anticipated by the public in subsequent periods, $\tau \geq 2$ (Figure Ⅲ-1, upper panel).

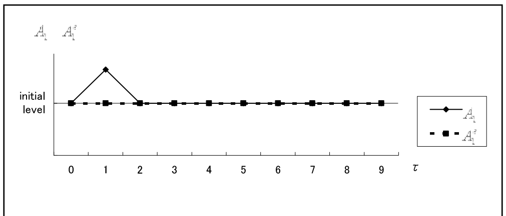
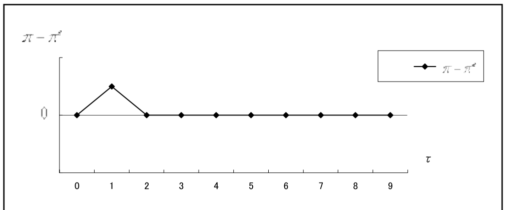
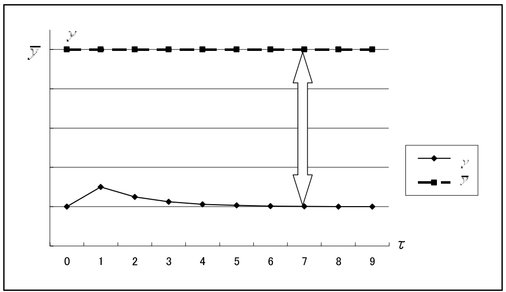


Figure Ⅲ-1: The effects of one-time deviation of inflation





The paths of inflation and output gap are depicted in the middle and lower panels of Figure Ⅲ-1. In the first period, the central bank conducts a surprise inflation policy: $\pi_1 > \pi_1^e = \pi_0$ (Figure Ⅲ-1, middle panel). Under the maintained assumption that $y_0 = 0$, this policy produces a positive output gap, $y_1 = \alpha(\pi_1 - \pi_1^e) > 0$ in the same period. Since $\pi_\tau = \pi_\tau^e$ holds for $\tau \geq 2$, it follows that $y_\tau = \rho y_{\tau-1} + \alpha(\pi_\tau - \pi_\tau^e) = \rho y_{\tau-1} = \rho^{\tau-1} y_1 > 0$, for $\tau \geq 2$. As the lower panel of Figure Ⅲ-1 shows, the output gap approaches zero from above, which keeps both the actual and expected inflation rates below their original paths, because $\pi_{\tau+1} = \pi_{\tau+1}^e = \tilde{A}_1 + \tilde{A}_2 y_\tau < \tilde{A}_1 = \pi_0$ holds for $\tau \geq 2$.

Let h denote one-time deviation of inflation from the original path: $h = \pi_1 - \pi_0$. Differentiating the authority's loss function L, with respect to h, we can decompose the resulting marginal effects of a one-time deviation of inflation into three parts: $\frac{\partial L}{\partial h} = \frac{\partial}{\partial h} \sum_{\tau=1}^{\infty} \beta^\tau E_1\{\lambda(y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2\} = (1-\lambda)\{\Delta^\pi(\tau = 1) + \Delta^\pi(\tau \geq 2)\} + \lambda\Delta^y$, where, $\Delta^\pi(\tau = 1) = \frac{\partial \beta(\pi_1 - \pi^*)^2}{\partial h}$, $\Delta^\pi(\tau \geq 2) = \frac{\partial \sum_{\tau=2}^{\infty} \beta^\tau E_1(\pi_\tau - \pi^*)^2}{\partial h}$ and $\Delta^y = \frac{\partial \sum_{\tau=1}^{\infty} \beta^\tau E_1(y_\tau - \bar{y})^2}{\partial h}$. In addition, we define $\Delta^\pi = \Delta^\pi(\tau = 1) + \Delta^\pi(\tau \geq 2)$. Since $|y_\tau - \bar{y}| < |y_0 - \bar{y}| = \bar{y}$ for all periods, τ, the marginal output cost of a one-time deviation of inflation is negative (marginal gain): $\Delta^y < 0$. Note that since the costs are evaluated by quadratic functions, the marginal output cost (the marginal gain from positive output gaps, Δ^y) and the marginal costs of inflation ($\Delta^\pi(\tau = 1)$, $\Delta^\pi(\tau \geq 2)$, Δ^π) are proportional to the vertical distances $|y_\tau - \bar{y}|$ and $|\pi_\tau - \pi^*|$ indicated in Figures Ⅲ-1 and Ⅲ-2.

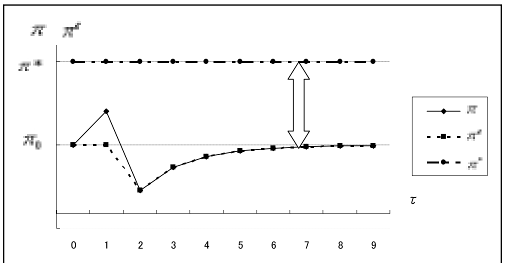
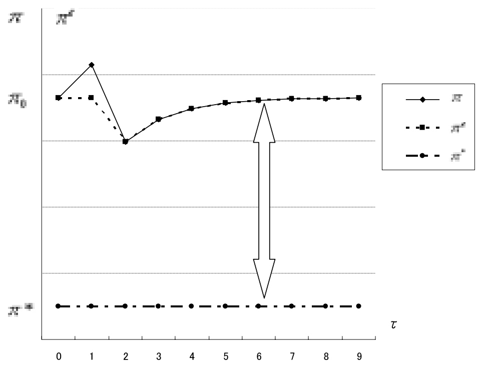


Figure Ⅲ-2: The Paths of Inflation and Expected Inflation (A Deflationary Environment)



(An Inflationary Environment)

If $|\tilde{A}_2|$ is large enough, $|\pi_\tau - \pi_0| = |\tilde{A}_2 y_{\tau-1}|$ is significantly larger for $\tau \geq 2$ than $|\pi_1 - \pi_0|$, and the effect in the first period ($\Delta^\pi(\tau = 1)$) is dominated by the effects in later periods ($\Delta^\pi(\tau \geq 2)$) (Figure Ⅲ-2). We now distinguish between two cases: an inflationary environment, where the actual rate of inflation is higher than the optimum rate ($\pi > \pi^*$), and a deflationary environment, where the actual rate of inflation is lower than the optimum rate ($\pi < \pi^*$).

First, consider a deflationary environment. Since $|\pi_1 - \pi^*| < |\pi_0 - \pi^*|$ for $\tau = 1$ and $|\pi_\tau - \pi^*| > |\pi_0 - \pi^*|$ for $\tau \geq 2$, it follows that $\Delta^\pi(\tau = 1) < 0$ and $\Delta^\pi(\tau \geq 2) > 0$ (Figure Ⅲ-2, the first panel). Noting that $\Delta^\pi = \Delta^\pi(\tau = 1) + \Delta^\pi(\tau \geq 2)$, we conclude that $\Delta^\pi > 0$ because $\Delta^\pi(\tau \geq 2) > 0$ is the dominating effect. If the initial rate of inflation, π_0, is far below π^*, $|\pi_\tau - \pi^*|$ is large relative to $|y_\tau - \bar{y}|$, for all $\tau \geq 0$. Consequently, the weighted sum of $\Delta^\pi(> 0)$ and $\Delta^y(< 0)$ becomes positive: $\lambda\Delta^\pi + (1 - \lambda)\Delta^y > 0$ [6]. If the initial rate of inflation, π_0, is below but not too far below π^*, $|\pi_\tau - \pi^*|$ is small relative to $|y_\tau - \bar{y}|$, for all $\tau \geq 0$, and the weighted sum, $\lambda\Delta^\pi + (1 - \lambda)\Delta^y$, becomes negative. The value of π_0, for which this weighted sum becomes zero, corresponds to the G-active equilibrium (A_1^{**}, A_2^{**}). These results are summarized in the upper panel of Table.

Second, consider an inflationary environment. Since $|\pi_1 - \pi^*| > |\pi_0 - \pi^*|$ for $\tau = 1$ and $|\pi_\tau - \pi^*| < |\pi_0 - \pi^*|$ for $\tau \geq 2$, it follows that $\Delta^\pi(\tau = 1) > 0$ and $\Delta^\pi(\tau \geq 2) < 0$, implying that $\Delta^\pi < 0$ because $\Delta^\pi(\tau \geq 2)$

Table: The effects of one-time deviation of inflation from a fixed path

Case 1: $|\tilde{A}_2|$ is large as in the G-active equilibrium

		$\Delta^\pi(\tau = 1)$	$\Delta^\pi(\tau \geq 2)$	Δ^π	Δ^y	$\lambda\Delta^y + (1 - \lambda)\Delta^\pi$
I		+	−	−	−	−
D	π_0 close to π^*	−	+	+	−	−
	Equilibrium	−	+	+	−	0
	π_0 far below π^*	−	+	+	−	+

Case 2: $|\tilde{A}_2|$ is small as in the C-active equilibrium

		$\Delta^\pi(\tau = 1)$	$\Delta^\pi(\tau \geq 2)$	Δ^π	Δ^y	$\lambda\Delta^y + (1 - \lambda)\Delta^\pi$
I	π_0 close to π^*	+	−	+	−	−
	Equilibrium	+	−	+	−	0
	π_0 far below π^*	+	−	+	−	+
D		−	+	−	−	−

Note I: inflationary environment ($\pi > \pi^*$), D: deflationary environment ($\pi < \pi^*$).

$\Delta^\pi(\tau = 1) = \dfrac{\partial\beta(\pi_1 - \pi^*)^2}{\partial h}$, $\Delta^\pi(\tau \geq 2) = \dfrac{\partial\sum_{\tau=2}^{\infty}\beta^\tau E_1(\pi_\tau - \pi^*)^2}{\partial h}$, $\Delta^\pi = \Delta^\pi(\tau = 1) + \Delta^\pi(\tau \geq 2)$

and $\Delta^y = \dfrac{\partial\sum_{\tau=2}^{\infty}\beta^\tau E_1(y_\tau - \bar{y})^2}{\partial h}$, where $h = A_1 - \tilde{A}_1$.

is the dominating effect (Figure Ⅲ-2, the second panel). Then, the weighted sum $\lambda\Delta^{\pi}+(1-\lambda)\Delta^{y}$ is also negative, and we conclude that an active equilibrium cannot exist in an inflationary environment. This result is also presented in the upper panel of Table.

If $|\tilde{A}_2|$ is small enough, $|\pi_1 - \pi_0|$ is significantly larger than $|\pi_\tau - \pi_0| = |\tilde{A}_2 y_{\tau-1}|$ for $\tau \geq 2$, and the effect in the first period ($\Delta^{\pi}(\tau = 1)$) dominates those in the later periods ($\Delta^{\pi}(\tau \geq 2)$)[7]. Proceeding in the same manner, we can show that a C-active equilibrium exists only in an inflationary environment. Table, lower panel, summarizes these results.

The Stability of Equilibria

Equations (11) and (12) implicitly define two functions of the form $A_2(A_2^e)$ and $A_1(A_2^e, A_1^e)$. Differentiating these functions with respect to A_2^e and A_1^e, and evaluating them in the discretionary equlibria, we have: $\left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^{**}} > 1$, and $0 < \left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^{*}} < 1$. (See Section C of Matsukawa, Okamura and Taki [2011].) These results imply that when the public anticipates a more (less) active policy than that chosen in a G-active equilibrium, the best response for the central bank is to choose an even more (even less) active policy than that anticipated by the public. In contrast, when the public anticipates more (less) active policy than that chosen in a C-active equilibrium, the best response for the central bank is to choose a less (more) active policy than that anticipated by the public.

To illustrate, given a set of parameters, $\alpha = .5$, $\rho = 0.5$, $\lambda = 0.5$, $\beta = .99$, the graph of the central bank's best response (11) depicted in Figure Ⅱ cuts the diagonal from below (from above), as A_2^e approaches $A_2^{**}(A_2^{*})$ from below, implying that $A_2^{**}(A_2^{*})$ is unstable (stable).

It is also straightforward to show that $\left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{**},\, A_2=A_2^{**}} > 1$, in a G-active equilibrium, and $0 < \left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{*},\, A_2=A_2^{*}}$, in a C-active equilibrium. The sequential nature of the solution allows us to conclude that a G-active (C-active) equilibrium is unstable (stable). Note that the instability of the G-active equilibrium does not mean the instability of the dynamic system (5). In fact, the degree of persistence at the G-active equilibrium is $|\rho - \alpha A_2 - \alpha A_2^e| = |\rho| < 1$; thus, the system is stable around this equilibrium.

Ⅵ. Conclusion and Discussion

In this paper, we developed a dynamic version of the BG model, in which the dynamics of the model are driven by the persistency in output gap. The model reveals an interesting implication for activist monetary policy under the BG discretionary regime. Our main conclusions are as follows.

Except for the case of the double root, there are two (if any) discretionary equilibria, which are characterized as deflationary and high-inflationary equilibria. We call the former the G-active equilibrium and refer to the associated monetary policy as G-active. The central bank's feedback rule chosen in the G-active equilibrium has a larger (in absolute value) coefficient on the lagged output gap, and corresponds to monetary policy during the Greenspan era. The latter is called the C-active equilibrium and the associated monetary policy is referred to as C-active, which corresponds to monetary policy before 1979. We showed that a G-active equilibrium might result in a deflationary equilibrium. In addition, we showed that a G-active equilibrium is unstable and that a C-active equilibrium is stable.

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Footnotes

1 Orphanides (2003) shows that Federal Reserve policies over many periods, virtually since the founding of the institution, can be broadly interpreted in terms of the Taylor-rule framework with surprising consistency.

2 The model is obviously an extremely simple one. In particular, the absence of a nominal interest rate reaction function is not entirely realistic.

3 DGK assumes that current realizations of supply shocks are in the information set of the central bank at the time it makes policy decisions, but not in that of the private sector (see Okamura, Matsukawa and Taki [2006]). In the present model, however, the central bank's information set does not include current realizations of supply shocks.

4 If current realizations of supply shocks are in the information set of the central bank but not in that of the private sector, monetary policy is effective to the extent that the feedback coefficient on supply shocks has non-zero feedback coefficient in the central bank's reaction function (see Okamura, Matsukawa and Taki [2006]).

5 Section D of Matsukawa Okamura and Taki [2007] presents the proof of this result.

6 Of course, extreme cases in which the central bank puts little weight on inflation stability or output stability (the value of λ near the ends of interval [0, 1]), should be neglected.

7 For example, $|A_2^*| = 0.37963$ in the C-active equilibrium for our benchmark case ($\alpha = .5$, $\beta = .99$, $\rho = .5$, $\lambda = .5$).