PHYSICAL REVIEW D **83,** 034005 (2011)

Nonet meson properties in the Nambu–Jona-Lasinio model with dimensional versus cutoff regularization

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(Received 2 October 2010; published 7 February 2011; corrected 9 February 2011)

The Nambu–Jona-Lasinio model with a Kobayashi-Maskawa-'t Hooft term is one low energy effective theory of QCD which includes the $U_A(1)$ anomaly. We investigate nonet meson properties in this model with three flavors of quarks. We employ two types of regularizations, the dimensional and sharp cutoff ones. The model parameters are fixed phenomenologically for each regularization. Evaluating the kaon decay constant, the η meson mass and the topological susceptibility, we show the regularization dependence of the results and discuss the applicability of the Nambu–Jona-Lasinio model.

DOI: 10.1103/PhysRevD.83.034005 PACS numbers: 12.39.−x, 11.10.Kk, 11.30.Rd

I. INTRODUCTION

The unique QCD Lagrangian is found by imposing the local $SU_c(3)$ gauge symmetry, Lorentz invariance, locality, and renormalizability in four space-time dimensions. It is believed to be a fundamental theory of strong interaction between quarks and gluons. One of the features of QCD dynamics is ''asymptotic freedom'' which legitimates perturbation theory at short distances. Unfortunately nonperturbative effects cannot be avoided in the confinement phase which takes place at a low energy scale, i.e., where the QCD coupling is not small.

The Nambu–Jona-Lasinio (NJL) model [1] is a well-known and often used low energy effective theory of QCD [2–4]. Nambu and Jona-Lasinio have introduced a four-fermion interaction to describe the attractive force between fermions. In this model, chiral symmetry is spontaneously broken by a nonvanishing expectation value for a composite operator constructed by the fermion and antifermion fields and the fermion mass is dynamically generated. The NJL model and its generalizations are extremely useful in the study of the light meson properties at low energy.

The four-fermion interaction is a dimension six operator in four space-time dimensions, therefore, the model is nonrenormalizable and depends on the regularization procedure. In order to regularize fermion loop integrals one usually introduces a momentum scale Λ to cut off integration momenta higher than Λ. Another regularization, the dimensional one, is an analytic regularization; one calculates loop integrals as analytic functions of the space-time dimensions and uses them for the value of the space-time dimensions less than four [5–9]. Other methods are also studied in NJL type models, e.g., the smooth cutoff [10,11], the Pauli-Villars [12], and the Fock-Schwinger proper-time regularizations [13,14].

In the present paper we study the NJL model in the dimensional regularization and compare the results with ones obtained in the sharp cutoff regularization. The NJL model in both regularizations describes well the dynamical breaking of chiral symmetry and the π and σ meson properties in vacuum [8]. Yet at finite density (especially when the cutoff scale is close to the Fermi momentum) important contributions to the quark loop integrals are dropped in the cutoff regularization scheme [15], while dimensional regularization leads to results consistent with QCD even in the region of asymptotic freedom, therefore, we find a strong regularization dependence in the extended NJL model with an attractive force in the color antitriplet channel for a large chemical potential [9]. Such a high density state may be realized in astrophysical objects. It is expected that the model can be tested by observing the structure of dense stars.

Recently, issues related to the regularization of NJL type models for the system at finite temperature, T, and chemical potential, μ, have been attracting a lot of attention of several groups. The dependence on μ of the cutoff was proposed in combination with a running coupling constant in Ref. [16]. This procedure has been used in Ref. [17] to obtain a quark matter equation of state in the NJL model which is relevant for the stability analysis of neutron stars. An extension of the model to high temperatures and densities proposed in Ref. [18] is used in Ref. [16] in connection with an implicit regularization scheme. The effects of the cutoff regularization procedure in several thermodynamic quantities were also analyzed in the Polyakov–Nambu–Jona-Lasinio model [19].

Non-negligible regularization dependence is observed even when considering the system at $T = \mu = 0$. The cutoff scale is usually taken to be lower than η' meson mass ($m_{\eta'} \simeq 958$ MeV) to reproduce light meson properties [4,20,21]. Then the $m_{\eta'}$ in the low energy effective theory is not a well-defined quantity. Λ is phenomenologically fixed and it is considered as a scale above which the effective model may lose its validity. Furthermore, the cutoff regularization may break some symmetry of the Lagrangian. The cutoff regularization may cause some unexpected effects in the system where the strange quark and $U_A(1)$ anomaly play an important role. Therefore we launch a plan to study the three-flavor system with a $U_A(1)$ anomaly by using the dimensional regularization.

In the present paper the extended NJL model including the Kobayashi-Maskawa-'t Hooft term [22,23] is regarded as a low energy effective theory of QCD with a $U_A(1)$ anomaly. We consider three-flavor light quarks, u, d, and s, and investigate the nonet meson properties. In Sec. II. we introduce the model Lagrangian and briefly review regularization schemes. In Sec. III we calculate the meson masses, decay constants, and topological susceptibility in the leading order of $1/N_c$ expansion. The results depend on the regularization parameters. In Sec. IV we evaluate the kaon decay constant, the η meson mass, m_η, and the topological susceptibility χ as a function of the space-time dimension D associated with loop integrals. In Sec. V the dependence of physical quantities on the current up-quark mass, m_u, is studied. After phenomenologically fixing the model parameters, we discuss the two regularizations dependence of the results. This paper shows that the model Lagrangian is not satisfactory in either of regularizations. Some concluding remarks are given in Sec. VI.

II. NJL MODEL WITH $U_A(1)$ ANOMALY

The NJL model is one of the simplest models to describe the dynamical symmetry breaking. It is often used to study the light meson properties at a low energy scale. Since the $U_A(1)$ anomaly induces the mass difference between η and η' mesons, the NJL model should be extended to include the contribution from the $U_A(1)$ anomaly in evaluating the nonet meson system constructed out of the three-flavor light quarks.

A. Model set up

Kobayashi and Maskawa have introduced an interaction written in the determinant of the composite operator constructed by quark and antiquark fields in the nonet meson system [22]. In QCD the θ-term can be transformed to the determinant term by a $U_A(1)$ transformation. Thus, we can include the contribution of the $U_A(1)$ anomaly through the determinant term in the NJL model. It should be noted that there is an alternative way to introduce the $U_A(1)$ anomaly in the NJL model even in the two-flavor case [24]. In the present paper we consider the four- and six-fermion interaction invariant under $SU_L(3) \otimes SU_R(3)$ global flavor symmetry and start with the Lagrangian,

$$\mathcal{L}_{\rm NJL} = \sum_{i,j=1}^{3} \bar{q}_i(i\not{\partial} - \hat{m})_{ij}q_j + \mathcal{L}_4 + \mathcal{L}_6, \tag{1}$$

where

$$\mathcal{L}_4 = G\sum_{a=0}^{8}\bigg[\bigg(\sum_{i,j=1}^{3}\bar{q}_i\lambda_a q_j\bigg)^2 + \bigg(\sum_{i,j=1}^{3}\bar{q}_i i\gamma_5\lambda_a q_j\bigg)^2\bigg], \tag{2}$$

$$\mathcal{L}_6 = -K[\det\bar{q}_i(1-\gamma_5)q_j + \text{H.c.}], \tag{3}$$

the subscripts (i, j) are the flavor indices, $\hat{m}$ expresses the current quark mass matrix, λ_a are the Gell-Mann matrices in the flavor space, and G and K represent the effective coupling constants for four- and six-fermion interaction, respectively. The determinant in $\mathcal{L}_6$ concerns the matrix elements labeled by the flavor indices, ij. The order of the coupling constants are supposed to be $GN_c \simeq O(1)$, $KN_c^2 \simeq O(1)$, therefore, we work in the framework of the $1/N_c$ expansion. In this paper we do not care about the flavor mixing and set the mass matrix to have a diagonal form, $\hat{m} = \text{diag}(m_u, m_d, m_s)$. The current quark mass explicitly breaks the global $SU(3)$ flavor symmetry. Below we consider the $SU(2)$ isospin symmetric case and take $m_u = m_d$ for simplicity.

The chiral condensates $\langle\bar{u}u\rangle$, $\langle\bar{d}d\rangle$, and $\langle\bar{s}s\rangle$ generate the constituent quark masses, m_u^*, m_d^*, and m_s^* inside mesons. To evaluate the constituent quark mass we solve the gap equations, which are derived by differentiating the thermodynamic potential with respect to m_u^*, m_d^*, and m_s^*. The gap equations are obtained in the leading order of the $1/N_c$ expansion [2–4],

$$m_u^* = m_d^* = m_u + 4G(i\mathop{\rm tr}_{s,c}S^u) + 2K(i\mathop{\rm tr}_{s,c}S^d)(i\mathop{\rm tr}_{s,c}S^s), \tag{4}$$

$$m_s^* = m_s + 4G(i\mathop{\rm tr}_{s,c}S^s) + 2K(i\mathop{\rm tr}_{s,c}S^u)(i\mathop{\rm tr}_{s,c}S^d), \tag{5}$$

where the symbol $\mathop{\rm tr}_{s,c}$ stands for the trace in spinor and color indices. $\mathop{\rm tr}_{s,c}S^i$ represent the chiral condensates, $-i\mathop{\rm tr}_{s,c}S^u \equiv \langle\bar{u}u\rangle$ and $-i\mathop{\rm tr}_{s,c}S^s \equiv \langle\bar{s}s\rangle$, which are given by the trace of the quark propagator inside mesons,

$$-i\operatorname*{tr}_{s,c} S^i = N_c \operatorname*{tr}_{s} \int \frac{d^D p}{i(2\pi)^D} S^i(p),$$
$$S^i(p) \equiv \frac{1}{\not p - m_i^* + i\epsilon}. \tag{6}$$

Here we indicate the space-time dimension for internal quark fields by D. Below we omit the subscripts in $\operatorname*{tr}_{s,c}$ for notational simplicity.

B. Regularization schemes

The fermion loop integral in Eq. (6) is divergent in four space-time dimensions. To obtain a finite result we have to regularize it. The four- and six-fermion interactions are written in terms of the dimension six and nine operators, respectively. Thus, the operators in $\mathcal{L}_4$ and $\mathcal{L}_6$ are irrelevant in four dimensions. It means that the results depend on regularization procedures. Here we use two different procedures; one is the three-momentum sharp cutoff regularization and the other is the dimensional regularization.

In the three-momentum sharp cutoff method, we cut off the space momentum component integral above the scale, Λ,

$$\int \frac{d^D p}{(2\pi)^D} \to \int \frac{dp_0}{2\pi} \int^\Lambda \frac{d^3 p}{(2\pi)^3}. \tag{7}$$

In the dimensional regularization scheme, we regularize the divergent integral with the help of analytic continuation of the integral as a function of the space-time dimension D to a noninteger value less than four,

$$\int \frac{d^D p}{(2\pi)^D} \to \frac{2(4\pi)^{-D/2}}{\Gamma(D/2)} \int_0^\infty dp\, p^{D-1}. \tag{8}$$

This is a kind of an analytic regularization. We regard the space-time dimensions D in the fermion loop integral as one of the parameters of the effective model of QCD. The dimensional regularization is applied to momentum integrals only for internal fermion lines.

The common parameters of the models considered here are the coupling constants G and K and the current quark masses $m_u(= m_d)$, m_s. In the cutoff scheme, the cutoff scale Λ is one more parameter. On the other hand, in the dimensional regularization, we consider the space-time dimension D as one of the model parameters. In this case we have to introduce one more parameter, the renormalization scale M_0, to obtain results with the correct mass dimension. Thus, the parameters in these two regularization methods are aligned as follows:

$$\text{Cutoff}: G, K, m_u(= m_d), m_s, \Lambda,$$
$$\text{Dimensional}: G, K, m_u(= m_d), m_s, D, M_0.$$

All the parameters should be fixed phenomenologically.

III. MESON MASS AND DECAY CONSTANT

In this section we shall evaluate the properties of the nonet meson system. Here we calculate the meson mass, meson decay constant and topological susceptibility in the two regularizations.

A. π and K masses

First, we consider pion and kaon. The masses of these mesons are obtained by observing the pole structure in their propagators. Employing the random-phase approximation and the $1/N_c$ expansion, the meson propagators are given by [3,4]

$$\Delta_P(k^2) = \frac{2K_\alpha}{1 - 2K_\alpha \Pi_P(k^2)} + O(N_c^{-1}), \tag{9}$$

where the index α denotes the isospin channel and P stands for the meson species, π and K. The flavor-dependent effective couplings K_α are defined by

$$K_3 \equiv G + \tfrac{1}{2} K i\, \mathrm{tr} S^s, \quad \text{for } \pi^0, \tag{10}$$

$$K_6 \equiv G + \tfrac{1}{2} K i\, \mathrm{tr} S^u, \quad \text{for } K^0, \bar{K}^0. \tag{11}$$

In the leading order of the $1/N_c$ expansion the self-energy for each meson, P, is given by

$$\Pi_P(k^2)\delta_{\alpha\beta} = \int \frac{d^D p}{i(2\pi)^D} \mathrm{tr}[\gamma_5 T_\alpha S^i(p + k/2) \times \gamma_5 T_\beta^\dagger S^j(p - k/2)], \tag{12}$$

where the trace runs over flavor, spinor, and color indices. The $SU(3)$ matrices, T_α, corresponding to different channels are, $T_3 = \lambda_3$ for π^0, $T_6 = (\lambda_6 + i\lambda_7)/\sqrt{2}$ for K^0, and $T_6^\dagger$ for $\bar{K}^0$. Thus, the self-energy for π^0, K^0, and $\bar{K}^0$ read

$$\Pi_\pi(k^2 = m_\pi^2) = 2\Pi_5^{uu}(k^2 = m_\pi^2), \tag{13}$$

$$\Pi_K(k^2 = m_K^2) = 2\Pi_5^{sd}(k^2 = m_K^2), \tag{14}$$

where $\Pi_5^{ij}(k^2)$ is the loop integral

$$\Pi_5^{ij}(k^2) = \int \frac{d^D p}{i(2\pi)^D} \mathrm{tr}[\gamma_5 S^i(p + k/2)\gamma_5 S^j(p - k/2)] = \frac{1}{2}\left(\frac{i\,\mathrm{tr} S^i}{m_i^*} + \frac{i\,\mathrm{tr} S^j}{m_j^*}\right) + \frac{1}{2}[k^2 - (m_i^* - m_j^*)^2] I_{ij}(k^2), \tag{15}$$

with

$$I_{ij}(k^2) = \int \frac{d^D p}{i(2\pi)^D} \frac{N_c \operatorname*{tr}_{s} 1}{(p^2 - m_i^{*2})[(p - k)^2 - m_j^{*2}]}. \tag{16}$$

Here we set $\operatorname*{tr}_{s} 1$ to be 4 in the cutoff and $2^{D/2}$ in the dimensional regularization schemes, respectively.

Substituting the solution of the gap equations (4) and (5) in the self-energy expression for π^0, K^0 and $\bar{K}^0$ and evaluating the pole structure of the denominator in Eq. (9), we obtain the on-shell conditions for pion and kaon masses,

$$0 = \frac{m_u}{m_u^*} - 2K_3 k^2 I_{uu}(k^2)|_{k^2=m_\pi^2}, \tag{17}$$

$$\begin{aligned} 0 = 1 &- \frac{m_s^* - m_s}{2m_u^*} - \frac{m_u^* - m_u}{2m_s^*} \\ &- 2G\left(\frac{i\,\mathrm{tr}S^u - i\,\mathrm{tr}S^s}{m_u^*} + \frac{i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u}{m_s^*}\right) \\ &- 2K_6[k^2 - (m_s^* - m_u^*)^2] I_{us}(k^2)|_{k^2=m_K^2}. \end{aligned} \tag{18}$$

These equations are used to determine the values of the constituent quark masses m_u^* and m_s^*. Because of the $SU(2)$ isospin symmetry the equations for the charged pion and kaon cases are the same as for the neutral ones. The QED effect is also important for the mass differences between the neutral and charged mesons. It should be noted that Eq. (18) becomes Eq. (17) in the limit $m_s \to m_u$ because the $SU(3)$ flavor symmetry is restored in this limit.

B. π and K decay constants

Next, we consider the pion and kaon decay constants. The decay constant f_P is defined by the matrix element of the axial current between meson and vacuum states,

$$\begin{aligned} ik_\mu f_P \delta_{\alpha\beta} = -M_0^{4-D} \int \frac{d^D p}{(2\pi)^D} \\ \times \mathrm{tr}\left[\gamma_\mu \gamma_5 \frac{T_\alpha}{2} S^i g_{Pqq}(0) \gamma_5 T_\beta^\dagger S^j\right], \end{aligned} \tag{19}$$

where we introduce a renormalization scale M_0 to define the decay constant with a correct mass dimension, dim $(f_P) = 1$ for dimensional scheme. The trace runs over flavor, spinor, and color indices. The meson-to-quark-quark coupling g_{Pqq} is given by

$$g_{Pqq}(k^2)^{-2} = M_0^{4-D} \frac{\partial \Pi_P(k^2)}{\partial k^2}. \tag{20}$$

Inserting Eqs. (13) and (14) into Eq. (20), we obtain the pion and kaon decay constants,

$$f_\pi^2 M_0^{D-4} = m_u^{*2} I_{uu}(0), \tag{21}$$

$$\begin{aligned} f_K^2 M_0^{D-4} = \frac{1}{J_{us}(0)} \Bigg[m_u^* I_{us}(0) + \mathrm{tr}_s 1 \cdot N_c (m_s^* - m_u^*) \\ \times \int_0^1 dx \int \frac{d^D p}{i(2\pi)^D} \frac{x}{(p^2 - L_{us}(0) + i\epsilon)^2} \Bigg]^2, \end{aligned} \tag{22}$$

where J_{us} is defined by

$$\begin{aligned} J_{us}(k^2) = I_{ij}(k^2) + 2\mathrm{tr}_s 1 \cdot N_c (m_s^* - m_u^*)^2 \\ \times \int_0^1 dx \int \frac{d^D p}{i(2\pi)^D} \frac{x(1-x)}{(p^2 - L_{us}(k^2) + i\epsilon)^3}. \end{aligned} \tag{23}$$

L_{ij} is defined in the Appendix [see Eq. (A1)]. As confirmed in the case with m_π and m_K, Eq. (22) corresponds to Eq. (21) in the $m_s \to m_u$ limit due to the restoration of the $SU(3)$ flavor symmetry.

C. η and η' mesons

As is well known, the octet state, η_8, and the singlet state, η_0, are mixed in the real world. Thus, the eigenstates, η and η', with diagonal mass matrix are described as mixed states of η_8 and η_0. In the random-phase approximation the propagator of the $\eta - \eta'$ system is given by [3,4]

$$\boldsymbol{\Delta}^+(k^2) = 2\boldsymbol{K}^+[1 - 2\boldsymbol{K}^+ \boldsymbol{\Pi}(k^2)]^{-1}, \tag{24}$$

where $\boldsymbol{K}^+$ and $\boldsymbol{\Pi}$ are the 2×2 matrices

$$\boldsymbol{K}^+ = \begin{pmatrix} K_{00} & K_{08} \\ K_{80} & K_{88} \end{pmatrix}, \tag{25}$$

$$\boldsymbol{\Pi} = \begin{pmatrix} \Pi_{00} & \Pi_{08} \\ \Pi_{80} & \Pi_{88} \end{pmatrix}, \tag{26}$$

with

$$\begin{aligned} K_{00} &= G - \frac{1}{3} K(i\,\mathrm{tr}S^s + 2i\,\mathrm{tr}S^u), \\ K_{88} &= G - \frac{1}{6} K(i\,\mathrm{tr}S^s - 4i\,\mathrm{tr}S^u), \\ K_{08} &= K_{80} = -\frac{\sqrt{2}}{6} K(i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u), \end{aligned}$$

and

$$\begin{aligned} \Pi_{00}(k^2) &= \frac{2}{3}[2\Pi_5^{uu}(k^2) + \Pi_5^{ss}(k^2)], \\ \Pi_{88}(k^2) &= \frac{2}{3}[\Pi_5^{uu}(k^2) + 2\Pi_5^{ss}(k^2)], \\ \Pi_{08}(k^2) &= \Pi_{80}(k^2) = \frac{2\sqrt{2}}{3}[\Pi_5^{uu}(k^2) - \Pi_5^{ss}(k^2)]. \end{aligned}$$

To obtain the η and η' meson masses we diagonalize the inverse propagator via an orthogonal transformation [4,21]. Thus, the η and η' meson masses are given by the solution to the following equations:

$$\begin{aligned} A(k^2) + C(k^2) - \sqrt{\{A(k^2) - C(k^2)\}^2 + 4\{B(k^2)\}^2} = 0 \\ \text{for } k^2 = m_\eta^2, \end{aligned} \tag{27}$$

$$A(k^2) + C(k^2) + \sqrt{\{A(k^2) - C(k^2)\}^2 + 4\{B(k^2)\}^2} = 0 \quad \text{for } k^2 = m_{\eta'}^2, \tag{28}$$

where

$$A(k^2) = K_{88} - 2\Pi_{00}(k^2)\det\boldsymbol{K}^+,$$
$$B(k^2) = -K_{08} - 2\Pi_{08}(k^2)\det\boldsymbol{K}^+,$$
$$C(k^2) = K_{00} - 2\Pi_{88}(k^2)\det\boldsymbol{K}^+.$$

The mixing angle θ_η is found to be

$$\tan(2\theta_\eta) = \frac{2B(k^2)}{C(k^2) - A(k^2)}. \tag{29}$$

The off-diagonal matrix elements in Eq. (24) disappear and the mixing angle θ_η vanishes in the limit $m_s \to m_u$. In this limit Eq. (27) coincides with Eqs. (17) and (18) and the mass spectrum for the octet mesons degenerates. Since the $U_A(1)$ anomaly breaks the degeneracy between the octet and the singlet mesons, a different on-shell condition is obtained from Eq. (28).

D. Topological susceptibility

To compare the QCD axial current with the NJL axial current we introduce the topological charge density $Q(x)$ [4],

$$Q(x) \equiv \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} = 2K\mathrm{Im}[\det\bar{q}(1-\gamma_5)q], \tag{30}$$

where g is the strong coupling constant of QCD and $F^a_{\mu\nu}$ is the field strength for gluons. The topological susceptibility χ is defined by the correlation function between the topological charge densities at different points,

$$\chi = \int d^4x \langle 0|TQ(x)Q(0)|0\rangle_{\text{connected}}. \tag{31}$$

It describes some global feature of QCD dynamics. In the leading order of the $1/N_c$ expansion it is given by [25]

$$\begin{aligned}\chi = \frac{4K^2}{M_0^{D-4}}(i\,\mathrm{tr}S^u)^2\Big[&(i\,\mathrm{tr}S^u)(i\,\mathrm{tr}S^s)\Big(\frac{2i\,\mathrm{tr}S^s}{m_u^*} + \frac{i\,\mathrm{tr}S^u}{m_s^*}\Big)\\ &+ \Big\{\frac{1}{\sqrt{6}}(2i\,\mathrm{tr}S^s + i\,\mathrm{tr}S^u)(\Pi_{00}(0), \Pi_{08}(0))\\ &+ \frac{1}{\sqrt{3}}(i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u)(\Pi_{08}(0), \Pi_{88}(0))\Big\}\boldsymbol{\Delta}^+(0)\\ &\times \Big\{\frac{1}{\sqrt{6}}(2i\,\mathrm{tr}S^s + i\,\mathrm{tr}S^u)\begin{pmatrix}\Pi_{00}(0)\\ \Pi_{08}(0)\end{pmatrix}\\ &+ \frac{1}{\sqrt{3}}(i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u)\begin{pmatrix}\Pi_{08}(0)\\ \Pi_{88}(0)\end{pmatrix}\Big\}\Big].\end{aligned} \tag{32}$$

We compare this result with the one obtained by the lattice QCD.

IV. PARAMETER SETTING IN THE DIMENSIONAL REGULARIZATION

In this section we discuss the physical scale and the parameter setting for the model with the dimensional regularization. In the study with the help of the NJL model, parameters are usually determined by fitting the physical quantities $\{m_\pi, f_\pi, m_K, X\}$, where there are several choices for X. Here we fit the parameters by choosing $X = m_{\eta'}$. We reproduce these four observables in the dimensional regularization scheme without fixing two of the model parameters, m_u and D. As for the up-quark mass, m_u, we fix it by hand and test several values. The dimension, D, is kept to be a free parameter. Then we calculate some meson characteristics as functions of D. Here we evaluate the D dependence of some observed physical quantities: the kaon decay constant f_K, η meson mass m_η, and topological susceptibility χ.

A. Parameter setting

The model has 6 parameters m_u, m_s, G, K, M_0, and D, as is already mentioned in Sec. II. Here we set $m_u(= m_d)$ at 3, 4, 5 and 6 MeV, and study the region, $2 < D < 4$, in which an UV stable fixed point appears for G. Following the previous study [4], we fit the other 4 parameters $\{M_0, G, K, m_s\}$ by using the measured physical observables [26],

$$\begin{aligned} m_\pi &= 138\ \text{MeV}, & m_K &= 495\ \text{MeV},\\ f_\pi &= 92\ \text{MeV}, & m_{\eta'} &= 958\ \text{MeV}.\end{aligned} \tag{33}$$

These quantities are calculated by analyzing Eqs. (17), (18), (21), and (28), respectively. We solve these equations under the constraints imposed by the gap equations for m_u^* and m_s^*. Eliminating the term $Ki\,\mathrm{tr}S^s$ from Eqs. (4) and (17), we obtain m_u^* as a function of m_u and D,

$$0 = \frac{m_u}{m_u^*} - \frac{1}{2}\frac{m_u^* - m_u}{i\,\mathrm{tr}S^u}k^2 I_{uu}(k^2)|_{k^2=m_\pi^2}. \tag{34}$$

We numerically evaluate this expression to plot m_u^* as a function of D in Fig. 1. Inserting this m_u^*, function of D, into Eq. (21), we also describe the renormalization scale M_0 as a function of m_u and D. The D dependence of M_0 is shown in Fig. 2. From Figs. 1 and 2, we confirm that m_u^* and M_0 in the 3-flavor NJL model have a behavior similar to that in the 2-flavor model [8].

Next we would like to fix the remaining 3 parameters $\{G, K, m_s\}$. From Eqs. (4), (5), and (18), the coupling constants G and K are obtained as functions of m_u, m_u^*, and m_s^*,

$$G(m_s^*) = \frac{m_u^* - m_u}{4i\,\mathrm{tr}S^u} - \frac{1}{2}K(m_s^*)i\,\mathrm{tr}S^s, \tag{35}$$

$$K(m_s^*) = \frac{g(m_s^*)}{h(m_s^*)}, \tag{36}$$

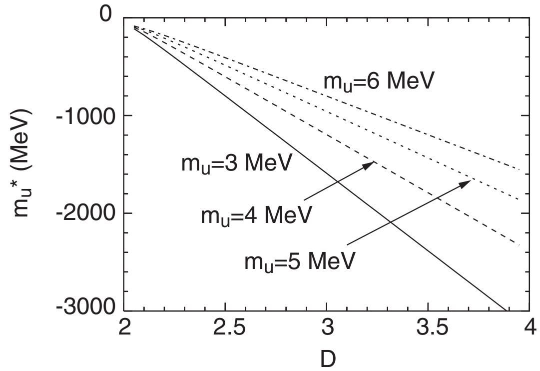

FIG. 1. The solution of the gap equation for m_u^* as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

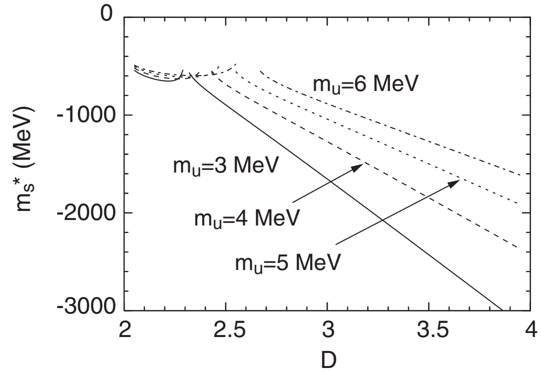

FIG. 3. The solution of the gap equation m_s^* as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

where $g(m_s^*)$ and $h(m_s^*)$ are given by

$$g(m_s^*) = -1 + \frac{m_u^* - m_u}{2m_s^*} + \frac{m_u^* - m_u}{2i\,\mathrm{tr}S^u}\left[\frac{i\,\mathrm{tr}S^u}{m_u^*} + \frac{i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u}{m_s^*} + \{m_K^2 - (m_u^* - m_s^*)^2\}I_{us}(m_K^2)\right],$$

$$h(m_s^*) = (i\,\mathrm{tr}S^s - i\,\mathrm{tr}S^u)\left[\frac{i\,\mathrm{tr}S^u}{m_u^*} + \frac{i\,\mathrm{tr}S^s}{m_s^*} + \{m_K^2 - (m_u^* - m_s^*)^2\}I_{us}(m_K^2)\right].$$

Substituting Eqs. (35) and (36) into Eq. (18), we describe the current strange quark mass, m_s, as a function of m_u, m_u^*, and m_s^*. Then the η' meson mass (28) is also expressed by a function of m_u, m_u^*, and m_s^*. From the expression for $m_{\eta'}$ and Eq. (34) we evaluate m_s^* numerically.

The D dependence of m_s^* is shown in Fig. 3. We should note that qualitatively the solution for m_s^* has a similar behavior to m_u^*. Decreasing the dimension from four, we observe that the absolute value of m_s^* goes down almost linearly. A different tendency takes place near the dimension two, a discontinuity is observed at $D \simeq 2.5$. A physical solution for m_s^* does not appear, neither it is not seen for m_u^* in Fig. 1.

Once m_s^* is obtained, $G(m_s^*)$, $K(m_s^*)$, $m_s(m_s^*)$ are calculated by inserting the solution for m_s^*. The numerical results for G, K, and m_s are shown in Figs. 4–6, respectively. In Fig. 4 we observe a similar behavior of the four-fermion coupling, G, to one in the 2-flavor NJL model [8], while we do not have a consistent solution for the dimension where no solution is found for m_s^*. In Fig. 5 we see that K gets drastically large near the dimension where a consistent solution is lost. Finally in Fig. 6 it is found that the value of the current strange quark mass m_s is roughly constant for the dimension larger than that corresponding to the discontinuity point, but it falls down as D decreases below the discontinuity point.

In Figs. 3–6, we observe a discontinuity around $D \simeq 2.5$ where the behavior drastically changed. It comes from the fact that the values of the couplings G and K become divergent when m_s^* approaches m_u^*. The self-energy $\Pi_5^{ii}(k^2 = m_{\eta'}^2)$ is also divergent at $m_{\eta'}^2 = 4m_i^{*2}$. Therefore, m_s^* has no physical solution around $m_s^* \simeq m_u^* \simeq m_{\eta'}/2$, as is numerically confirmed in Fig. 3. This is the reason why the behavior of m_s^* is different from that of m_u^* near the dimension two. The constituent strange quark mass, m_s^*, cannot develop a value smaller than $m_{\eta'}/2$.

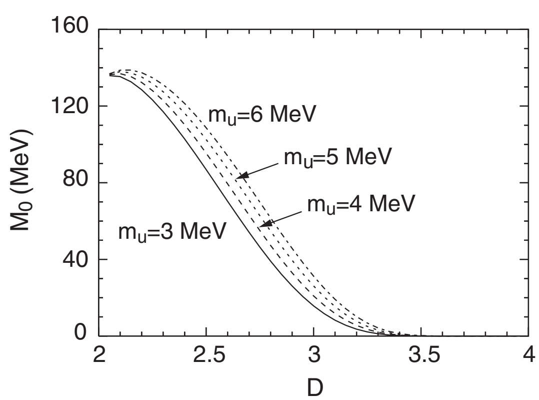

FIG. 2. The renormalization scale M_0 as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

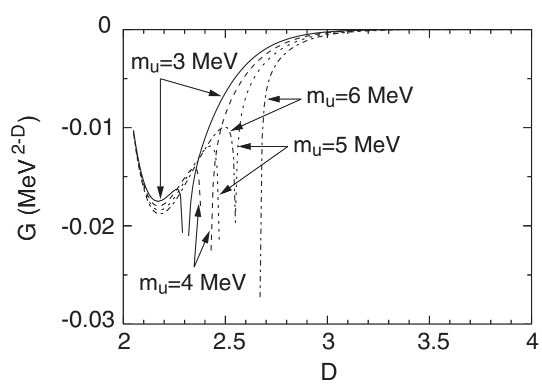

FIG. 4. The 4-fermion coupling G as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

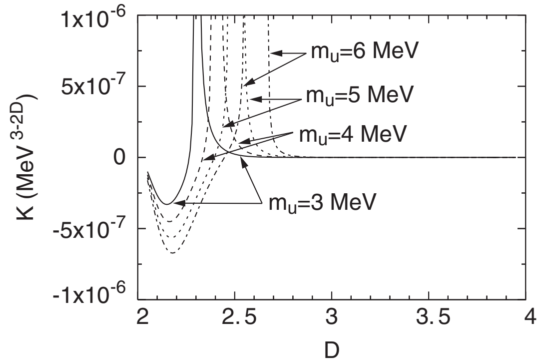

FIG. 5. The 6-fermion coupling K as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

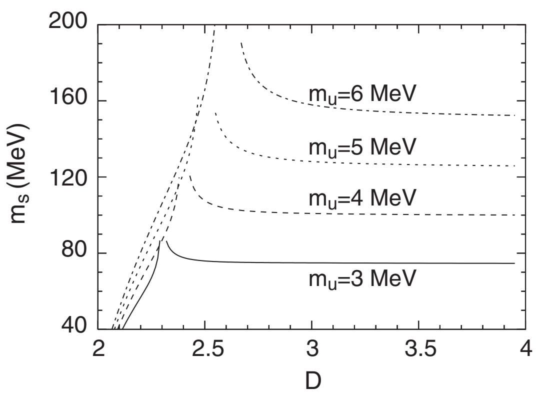

FIG. 6. The strange quark mass m_s as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

Below the dimension for the discontinuity the η' meson propagator contains a imaginary part which corresponds to the decay width.

B. Physical quantities

We have fixed the model parameters in the previous subsection except for the up-quark mass m_u and the dimension, D. We are now ready for evaluating various meson properties as a function of D for a fixed m_u. Employing the obtained parameters and the chiral condensates, m_u^* and m_s^*, one can numerically calculate the kaon decay constant, f_K, the η meson mass, m_η, and the topological susceptibility, χ. The behavior of f_K, m_η, and χ are displayed in Figs. 7–9, respectively. These values are roughly constant near four dimensions. They rapidly fall down as D decreases near two dimensions. We again observe a discontinuity around $m_s^* \simeq m_u^* \simeq m_{\eta'}/2$. The topological susceptibility blows up around the discontinuity.

As is seen in Fig. 7, the decay constant f_K is smaller than the observed one, 110 MeV, in the region, $2 < D < 4$. The model has to be improved to describe the kaon decay. We can fit the η meson mass to $m_\eta = 548$ MeV by tuning the dimension of the fermion loop integrals, D. The topological susceptibility is calculated to be $\chi^{1/4} = 170 \pm 7$,

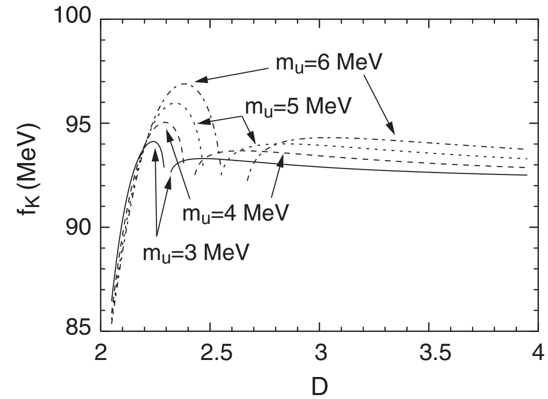

FIG. 7. The kaon decay constant f_K as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

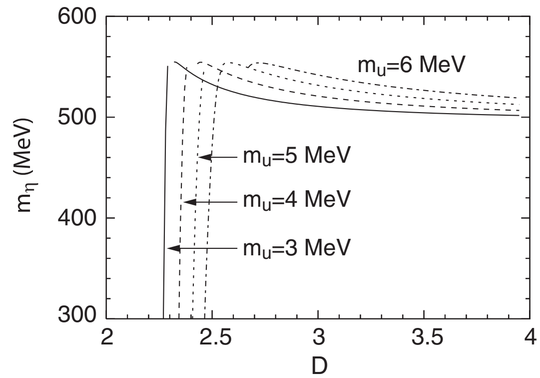

FIG. 8. The η mass m_η as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

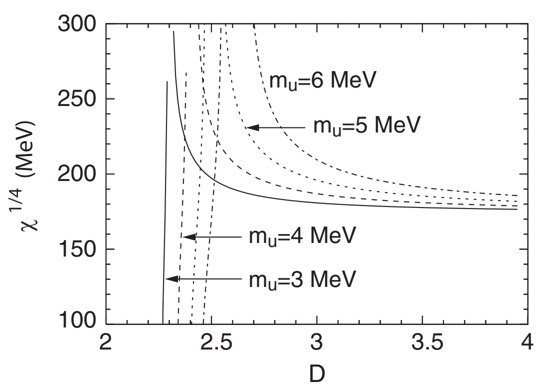

FIG. 9. The topological susceptibility χ as a function of D for $m_u = 3$, 4, 5, and 6 MeV.

174 ± 7 MeV in lattice simulations [27] and $\chi^{1/4} = 179$ MeV in the Witten-Veneziano mass formula [28,29]. We can also fit the topological susceptibility in the interval, $2 < D < 4$. Consequently the dimension for the fermion loop integrals, D, can be fixed to reproduce the η meson mass or the topological susceptibility. We will discuss this matter in the next section where we need to fit an additional parameter D.

V. QUARK MASS DEPENDENCE

In the cutoff regularization all the model parameters can be fixed by fitting the physical quantities $\{m_\pi, f_\pi, m_K, m_{\eta'}\}$ for a given m_u. In the dimensional regularization an additional observable, X', needs to be fitted due to one more parameter involved in this case. Therefore, there are five physical quantities, i.e., the set of them, $\{m_\pi, f_\pi, m_K, m_{\eta'}, X'\}$, which are needed to fit all the model parameters. In this section we consider two different observables for X'; one is the η meson mass, $X' = m_\eta$, another is the topological susceptibility, $X' = \chi$. By using the experimental/empirical values of m_η and χ we can define two different parameter sets and evaluate the meson properties. Then some physical quantities are evaluated for $m_u = 3$, 4, 5, 5.5, and 6 MeV. We calculate the physical quantities in the both regularizations to compare them with each other for each given m_u.

A. Physical quantities in the dimensional regularization

To show the validity of the model as a low energy effective theory of QCD, we evaluate parameters of the model to reproduce the input sets $\{m_\pi, f_\pi, m_K, m_{\eta'}, m_\eta\}$ or $\{m_\pi, f_\pi, m_K, m_{\eta'}, \chi\}$ in the dimensional regularization and discuss the other (output) physical quantities. There exist two solutions to reproduce $m_\eta = 548$ MeV for $m_u \lesssim 5$ MeV. In Table I we show m_s, f_K, χ, $\langle\bar{u}u\rangle_r$ under the condition that $m_\eta = 548$ MeV. The subscript r in $\langle\bar{u}u\rangle_r$ indicates that the quantity is renormalized, $\langle\bar{u}u\rangle_r = \langle\bar{u}u\rangle M_0^{4-D}$. The last lines in Tables I and II show the experimental/empirical values. The light quark masses are evaluated at 1 GeV [26]. The empirical value of $\langle\bar{u}u\rangle \simeq \langle\bar{s}s\rangle$ is evaluated by using the Gell-Mann-Oakes-Renner relation [30]. We observe that the topological susceptibility χ is always larger than the one in the lattice simulation and the Witten-Veneziano mass formula.

In Tables II and III we fix the parameters of the model in order to reproduce $\chi^{1/4} = 170$ or 179 MeV and then we calculate m_s, f_K, m_η, $\langle\bar{u}u\rangle_r$ for $m_u = 3$, 4, 5, 5.5, 6 MeV.

TABLE I. The physical quantities in the dimensional regularization (in units of MeV, except for D), m_η are fixed at 548 MeV. The last line shows the experimental/empirical values.

m_u	m_s	f_K	$\chi^{1/4}$	$-\langle\bar{u}u\rangle_r^{1/3}$	D
3.0	84.9	93.0	244	301	2.289
3.0	79.0	93.2	225	301	2.372
4.0	118	93.2	254	274	2.378
4.0	106	93.5	225	273	2.522
5.0	156	93.2	275	254	2.465
5.0	134	93.9	224	254	2.687
5.5	148	94.0	224	246	2.775
6.0	162	94.2	224	239	2.867
3.4–6.8	94.5–176	110	170–179	228–287	

TABLE II. The physical quantities in the dimensional regularization (in units of MeV, except for D), $\chi^{1/4}$ are fixed at 170 MeV. The last line shows the experimental/empirical values.

m_u	m_s	f_K	m_η	$-\langle\bar{u}u\rangle_r^{1/3}$	D
3.0	77.1	93.6	481	302	2.280
4.0	105	94.2	478	274	2.360
5.0	134	94.8	475	255	2.434
5.5	150	95.1	473	247	2.467
6.0	166	95.4	471	240	2.500
3.4–6.8	94.5–176	110	548	228–287	

TABLE III. The physical quantities for the dimensional regularization (in units of MeV, except for D), $\chi^{1/4}$ are fixed at 179 MeV.

m_u	m_s	f_K	m_η	$-\langle\bar{u}u\rangle_r^{1/3}$	D
3.0	78.0	93.5	498	301	2.281
3.0	74.8	92.8	507	300	3.222
4.0	106	94.1	495	274	2.363
4.0	100	92.9	507	272	3.895
5.0	136	94.7	491	255	2.438
5.5	152	95.0	489	247	2.472
6.0	168	95.3	487	240	2.505

As is shown in Fig. 9, we find two solutions in both sides of the discontinuity to satisfy $\chi^{1/4} = 179$ MeV for $m_u \lesssim 4$ MeV. In any case, we obtain the value for the η meson mass which is smaller than the observed one, $m_\eta = 548$ MeV.

B. Physical quantities in the cutoff regularization

In this section we fix the model parameters to reproduce the measured values of Eq. (33) in the cutoff regularization for a given m_u. The quantities, $\{m_s, G, K, \Lambda\}$, are found following the same procedure as in the previous section. In Table IV we show the parameters, G, K and Λ for $m_u = 3$, 4, 5, 5.5, and 5.87 MeV. It is observed that the cutoff scale Λ decreases as m_u increases. No solution is found to simultaneously satisfy Eqs. (17) and (21) for $m_u \gtrsim 5.87$ MeV. In the last lines of Tables IV and V we consider another set of parameters from Ref. [21]. These values [21,31] are derived by using $m_u = 5.5$ MeV, $m_\pi = 135$ MeV, $f_\pi = 92.4$ MeV, $m_K = 497.7$ MeV, and $m_{\eta'} = 957.8$ MeV as input parameters. These input parameters are about the same as ours. However, we obtain different results, especially for $K\Lambda^5$ and m_η in Tables IV and V, respectively. These differences come from the definition of the η' meson mass. We define $m_{\eta'}$ from the real part of Eq. (28) [4]. On the other hand, Ref. [21] treats $m_{\eta'}$ properly, as the pole of the propagator $\mathbf{\Delta}^+$ in the complex plane of k^2 [32]. In Table V the quantities, m_s, f_K, m_η, χ, $\langle\bar{u}u\rangle$, are shown. In the cutoff regularization the kaon

TABLE IV. The parameter list for cutoff scale Λ, 4-fermion coupling G, and 6-fermion coupling K.

m_u (MeV)	Λ (MeV)	$G\Lambda^2$	$K\Lambda^5$
3.0	960	1.55	8.34
4.0	797	1.60	8.38
5.0	682	1.71	8.77
5.5	631	1.81	9.17
5.87	580	2.09	10.1
5.5	602.3	1.835	12.36

TABLE V. The physical quantities in the cutoff regularization (in units of MeV).

m_u	m_s	f_K	m_η	$\chi^{1/4}$	$-\langle \bar{u}u\rangle^{1/3}$
3.0	89.5	113	451	160	301
4.0	110	107	457	158	273
5.0	128	101	473	160	253
5.5	136	97.3	482	163	245
5.87	139	93.3	501	172	240
5.5	140.7	95.39	514.8	179.0	241.9

decay constant, f_K, is consistent with the observed value for a smaller m_u, while the η meson mass and the topological susceptibility are smaller than the experimental/empirical values. We obtain only smaller η meson mass than the experimental one, similarly to the situation in the dimensional regularization with fixed topological susceptibility χ.

Thus none of the regularizations can describe all of the nonet meson properties. We can fit the η meson mass or the topological susceptibility in the dimensional regularization but we obtain only a smaller kaon decay constant in either case. In the Pauli-Villars regularization it has been found that $m_u = m_d = 2.7$ MeV, $m_s = 92$ MeV, $f_K = 131$ MeV, and $m_\eta = 526$ MeV where the input parameters m_π, m_K, f_π and $m_{\eta'}$ have been used [12].

VI. CONCLUSION

We studied nonet meson properties in the three-flavor NJL model with the dimensional and sharp cutoff regularizations in the leading order of the $1/N_c$ expansion. We employed m_u, m_π, m_K, f_π, $m_{\eta'}$ as input parameters and fix the model parameters, m_s, G, K, M_0, and m_s, G, K, Λ in the dimensional and cutoff regularizations, respectively. In the case of the dimensional regularization the dimension, D, is still a free parameter. Thus, we evaluate the kaon decay constant, the η meson mass and the topological susceptibility as a function of D.

The constituent up-quark mass, m_u^*, and the renormalization scale M_0 behave in a similar way as the two-flavor case. No consistent solution for m_s^* is found and the effective coupling, G and K, is divergent around $m_s^* \simeq m_u^* \simeq m_{\eta'}/2$. Below this region η' meson propagator develops an imaginary part. The kaon decay constant, f_K, is smaller than the observed value, 110 MeV, for $2 < D < 4$.

We found that m_s, f_K, m_η, χ are almost constant near four dimensions. Fitting the parameters at the lower dimension and then taking the four dimensional limit, obtained values for m_s, f_K, m_η, χ are not divergent. The results do not depend on the regularization parameter. It bring us an interesting idea of making a regularization independent prediction in the NJL model.

We have evaluated the physical quantities, m_s, f_K, χ, m_η, and $\langle \bar{u}u\rangle$, in both the dimensional and the cutoff regularizations for a fixed m_u. In the dimensional regularization the topological susceptibility, χ, develops a larger value than the one obtained in the lattice simulation and Witten-Veneziano mass formula for the fixed m_η. On the other hand, the η meson acquires a smaller mass than the observed one for the fixed χ.

In the cutoff regularization the cutoff scale significantly depends on m_u; the cutoff Λ increases with decreasing m_u. It is interesting to note that the coupling constants become smaller when one takes the larger cutoff, which is consistent with the renormalization group argument where the coupling strength becomes smaller with increasing the energy scale. This cutoff effect is also confirmed numerically in the NJL model through changing the cutoff in the temporal direction [33].

In Tables. II, III, and IV some difficulty is seen in tuning f_K, m_η, and χ simultaneously. Thus, this paper shows that the model based on the Lagrangian (1) is not satisfactory in either of the regularizations. It teaches us that new terms should be added to the Lagrangian to improve the model at least in one of the regularizations. We are especially interested in including vector-type and multifermion interactions. It is also interesting to apply our result to a system at finite temperature and chemical potential and we hope to report on the problems in future.

ACKNOWLEDGMENTS

The authors would like to thank Y. Hoshino and Y. Kitadono for fruitful discussions. H. K. is supported by Grant No. NSC-99-2811-M-033-017 from the National Science Council (NSC) of Taiwan. Discussions during the 2009 ‘‘International Workshop on Strong Coupling Gauge Theories in LHC Era’’(SCGT 09) and the YIPQS international workshop on ‘‘New Frontiers in QCD 2010’’ were useful to complete this work.

APPENDIX A: INTEGRALS

In this section, we demonstrate several integrals both in the cutoff and dimensional regularization schemes which are required in Secs. II and III.

1. The cutoff regularization

The chiral condensates $i\,\mathrm{tr}S^i$ in Eq. (6) are important quantities because they concern gap equations. They take the following form after the integration is performed:

$$\underset{s,c}{i\,\mathrm{tr}}\,S^i = \frac{N_c}{2\pi^2} m_i^* \left\{ \Lambda\sqrt{\Lambda^2 + m_i^{*2}} - m_i^{*2} \ln\frac{\Lambda + \sqrt{\Lambda^2 + m_i^{*2}}}{\sqrt{m_i^{*2}}} \right\}. \tag{A1}$$

The next integral to carry out is $I_{ij}(k^2)$ in Eq. (16) which determines the self-energies of mesons, Π_P, Eq. (12). It takes the forms

$$I_{ij}(k^2) = \frac{N_c}{2\pi^2}\int_0^1 dx \left\{ -\frac{\Lambda}{\sqrt{\Lambda^2 + L_{ij}(k^2)}} + \ln\frac{\Lambda + \sqrt{\Lambda^2 + L_{ij}(k^2)}}{\sqrt{L_{ij}(k^2)}} \right\}, \tag{A2}$$

for $(m_i^* - m_j^*)^2 < k^2 < (m_i^* + m_j^*)^2$ and $k^2 = 0$,

$$\begin{aligned} I_{ij}(k^2) &= \frac{N_c}{2\pi^2}\int_0^1 dx \left[-\frac{\Lambda}{\sqrt{\Lambda^2 + L_{ij}(k^2)}} + \ln\left\{\Lambda + \sqrt{\Lambda^2 + L_{ij}(k^2)}\right\}\right] \\ &\quad + \frac{N_c}{4\pi^2}\left[2 - \frac{1}{2}\ln(m_i^{*2} m_j^{*2}) - \frac{\Delta_{ij}}{2k^2}\ln\frac{m_i^{*2}}{m_j^{*2}} + \frac{\nu_{ij}}{2k^2}\ln\frac{(k^2-\nu_{ij})^2 - \Delta_{ij}}{(k^2+\nu_{ij})^2 - \Delta_{ij}} + i\pi\frac{\nu_{ij}}{2k^2}\right], \end{aligned} \tag{A3}$$

for $(m_i^* + m_j^*)^2 < k^2$. In Eqs. (A2) and (A3) we use the following notations:

$$L_{ij}(k^2) = m_i^{*2} - \Delta_{ij}x - k^2 x(1-x), \tag{A4}$$

$$\nu_{ij}(k^2) = \sqrt{k^4 - 2k^2(m_i^{*2} + m_j^{*2}) + \Delta_{ij}^2}, \tag{A5}$$

$$\Delta_{ij} = m_i^{*2} - m_j^{*2}. \tag{A6}$$

Finally, Eqs. (22) and (23), involved in the derivation of the kaon decay constant, are calculated as

$$f_K^2 = \frac{1}{J_{us}(0)}\left[m_u^* I_{us}(0) - \frac{N_c}{2\pi^2}(m_s^* - m_u^*) \times \int_0^1 dx x \left\{ \frac{\Lambda}{\sqrt{\Lambda^2 + L_{us}(0)}} - \ln\frac{\Lambda + \sqrt{\Lambda^2 + L_{us}(0)}}{\sqrt{L_{us}(0)}} \right\}\right]^2, \tag{A7}$$

$$J_{us}(k^2) = I_{us}(k^2) - \frac{N_c}{4\pi^2}\Lambda^3 (m_s^* - m_u^*)^2 \times \int_0^1 dx \frac{x(1-x)}{L_{us}(k^2)[\Lambda^2 + L_{us}(k^2)]^{3/2}}. \tag{A8}$$

2. The dimensional regularization

The corresponding integrals in the dimensional regularization are performed as

$$\underset{s,c}{i\,\mathrm{tr}}\,S^i = \frac{N_c}{(2\pi)^{D/2}}\Gamma\left(1 - \frac{D}{2}\right) m_i^* (m_i^{*2})^{((D/2)-1)}, \tag{A9}$$

$$f_K^2 = \frac{1}{M_0^{D-4} J_{us}(0)}\left[m_u^* I_{us}(0) + \frac{N_c}{(2\pi)^{D/2}}\Gamma\left(2 - \frac{D}{2}\right) \times (m_s^* - m_u^*)\int_0^1 dx x L_{us}(0)^{((D/2)-2)}\right]^2, \tag{A10}$$

$$J_{us} = I_{us}(k^2) - \frac{N_c}{(2\pi)^{D/2}}\Gamma\left(3 - \frac{D}{2}\right)(m_s^* - m_u^*)^2 \times \int_0^1 dx x(1-x) L_{us}(k^2)^{((D/2)-3)}. \tag{A11}$$

Regarding I_{ij}, we employ different expressions for numerical calculations depending on the value of k^2;

$$I_{ij} = \frac{N_c}{(2\pi)^{D/2}}\Gamma\left(2 - \frac{D}{2}\right)\int_0^1 dx L_{ij}^{((D/2)-2)}(k^2), \tag{A12}$$

for $(m_i^* - m_j^*)^2 < k^2 < (m_i^* + m_j^*)^2$ and $k^2 = 0$,

$$\begin{aligned} I_{ij}(k^2) &= \frac{N_c}{(2\pi)^{D/2}}\Gamma\left(2 - \frac{D}{2}\right)\left[\frac{2}{D-2}\nu_{ij}^{((D/2)-2)} \times \left\{ a_-^{((D/2)-1)} F\left(2 - \frac{D}{2}, \frac{D}{2} - 1, \frac{D}{2}; -\frac{a_-}{a_+ - a_-}\right)\right.\right. \\ &\quad \left. + (1 - a_+)^{((D/2)-1)} F\left(2 - \frac{D}{2}, \frac{D}{2} - 1, \frac{D}{2}; -\frac{1 - a_+}{a_+ - a_-}\right)\right\} \\ &\quad \left. + e^{i\pi(2-(D/2))}(k^2)^{(1-(D/2))}\nu_{ij}^{D-3} B\left(\frac{D}{2} - 1, \frac{D}{2} - 1\right)\right], \end{aligned} \tag{A13}$$

for $(m_i^* + m_j^*)^2 < k^2$, respectively. Here, F denotes the hypergeometric function and $a_\pm$ is defined by

$$a_\pm(k^2) = \frac{k^2 + \Delta_{ij} \pm \nu_{ij}(k^2)}{2k^2}. \tag{A14}$$

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