The Oblique and Non-Oblique Corrections in Technicolor Model without Exact Custodial Symmetry

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Abstract

We discuss whether there is a realistic Technicolor model under the constraints of Oblique and Non-oblique corrections from the precision measurements.

To satisfy the constraint of oblique correction, a one-family Technicolor model without exact custodial symmetry was proposed by Appelquist and Terning. We construct effective Lagrangian including technimesons for the one-family Technicolor model without exact custodial symmetry. Tree level contributions to oblique correction parameters S and U due to spin 1 technimesons are computed with the effective Lagrangian. An isospin breaking term which is associated with technilepton vector mesons gives a negative contribution to the electroweak radiative correction parameter S due to mixing between $I = 0$ and $I = 1$ vector mesons.

To satisfy the constraint of non-oblique correction, $Z b\bar{b}$ vertex correction, the effects of diagonal extended technicolor interaction was studied by Wu. By means of the effective lagrangian approach, we discuss the effects of extended technicolor(ETC) gauge interaction to the oblique and non-oblique corrections. It is shown that the T parameter is unacceptably large when the $Z b\bar{b}$ vertex correction and S parameter are consistent with the experiments in the ETC model. Hence, some difficulty is still remained in the ETC mechanism.

1 Introduction

The oblique corrections, S and T parameters, severely put constraint on QCD-like technicolor models and the present data shows that the models can not satisfy the constraints[1]. In the QCD-like model, S is about $0.1 \times N_{doublet} \times N_{TC} > 0$ and T vanishes because of exact custodial symmetry (chiral symmetry $SU(N_d)_L \otimes SU(N_d)_R$). However, S parameter that has been obtained from many experimental data is negative. From the recently experimental data, 1-family technicolor model($N_d = 8$) is completely ruled out and 1-doublet model may be also. Accordingly, we need to modify the technicolor model to satisfy the constraints of the oblique parameters. It is shown that if these models have the splitting of the masses between up-type and down-type particles (isospin breaking), they may be able to satisfy the constraints from oblique correction S by considering one-loop approximation. In ref.[2] Appelquist and Terning show a one-family technicolor model with isospin breaking in technilepton sector. This model has the following features. (1)isospin breaking of technilepton doublet, (2)the existence of two scales which are one of techniquark sector and one of technilepton sector. Because of the feature (1) S is reduced and because of (2) T is also reduced. In our work[3], we construct the effective lagrangian on this technicolor model without exact custodial symmetry and calculate the oblique corrections by considering the effects of the (axial)vectormeson that are composed by the technifermions.

On the other hand, for the vertex corrections of Zbb, which is called non-oblique correction, there is a discrepancy between the prediction of the standard model (SM) and the experimental data at LEP. The experimental value $R_b = 0.2202 \pm 0.0020$ was different from the value $R_b^{SM} = 0.2157$ predicted by SM with top quark mass $m_t = 175 GeV$. In the extended technicolor model(ETC), the con-

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tribution from sideways ETC interaction was first studied by the authors of Ref.[4]. However, this discrepancy could not be explained by the effects of the sideways ETC interaction. The discrepancy becomes larger by this effect[4, 5, 6]. In the recent works [7], it is shown that the diagonal extended technicolor(ETC) interaction may solve the Zbb problem, i.e., the discrepancy between the experiment and the prediction of the SM in Zbb vertex. If the contribution of the diagonal interaction to Zbb vertex is large enough to cancel the other corrections for the Zbb vertex, the discrepancy will be explained.

In order to build realistic technicolor models, the constraints of oblique correction and non-oblique correction must be satisfied at once. However, such large effect from diagonal ETC interaction that can explain the discrepancy of R_b also contributes to the oblique corrections because the effect comes from the breaking of the isospin symmetry in the right handed ETC interaction.[8] It is necessary to break the isospin symmetry to generate the mass difference between top and bottom quarks. Hence, the T parameter must receive large contribution from the ETC interactions. The diagrams such as Fig.16 [8]($\mathbf{A}$[7] and $\mathbf{B}$) must contribute to the oblique correction T [1]. We study the effect of the diagonal ETC interaction for the oblique corrections in the case that the non-oblique correction of the Zbb vertex is consistent with the experimental data in a realistic one-family model with small S parameter[2](the model without exact custodial symmetry[3]).

This thesis is developed as follows. In chapter 2 and 3, we briefly review about Oblique and non-oblique corrections and the Technicolor Model. In chapter 4, we construct the effective lagrangian including technimesons for a technicolor model without exact custodial symmetry. By using this method, the oblique corrections are computed. It is shown that S parameter receives negative contribution from

the $\rho - \omega$ mixing. In chapter 5, the contribution from the isospin breaking to vertex corrections in technilepton sector is described in the case that N_{TC} is large. The effects from only the sideways ETC interaction is considered. In the case that there is the isospin breaking in the technilepton sector, the difference of the vertex corrections between $Z\tau\tau$ and $W\tau\nu$ is shown. In chapter 6, we shown that contribution from diagonal ETC interaction to T parameter is unacceptably large when the Zbb vertex correction is consistent with experiments in the case that N_{TC} is small. Chapter 7 is devoted to the conclusion.

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2 Oblique and Non-Oblique Corrections

The standard model is precisely tested on the pole of Z boson at LEP. The experiments probe its predictions with sufficient accuracy. If a few new physics exists, the effects from the new particles must appear in the precision measurements for the low energy phenomena. The effects appear through some corrections as shifting of the standard model parameters. One is the radiative correction of the weak gauge boson . This is called "Oblique" correction[1]. Other one is the vertex correction called "Non-Oblique" correction[4]. The oblique correction does not depend on the process we consider but the non-oblique correction depend. We can obtain some constraints of their corrections on the new physics from the precision electroweak experiment.

2-1 Oblique Correction

The Oblique correction is the radiative correction on the self energy of the electro weak gauge bosons. The radiative correction is

$$\Pi_{ab}^{\mu\nu}(q) = \Pi_{ab}(q^2)g^{\mu\nu} + (q^\mu q^\nu terms), \tag{2.1}$$

with $a, b = A, W^\pm, Z$. If new physics exists, the effect must appear in the radiative corrections.

$$\Pi_{ab}(q^2) = \Pi_{ab}^{SM}(q^2) + \delta\Pi_{ab}(q^2) \tag{2.2}$$

The first term on the right-hand side represents the SM contribution, while all new physics effects are contained in the second term. If the scale of the new physics is sufficiently large compared with the mass scale of the weak gauge bosons, the $\Pi_{ab}(q^2)$ can be described by a Taylor expansion.

$$\delta\Pi_{ab}(q^2) = \delta\Pi_{ab}(0) + q^2\frac{d}{dq^2}\delta\Pi_{ab}(q^2)\mid_{q^2=0} + O(\frac{q^2}{M_{new}^2}) \tag{2.3}$$

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Therefore, there are eight parameter,

$$\delta\Pi_{AA}(0) \qquad \delta\Pi_{ZA}(0) \qquad \delta\Pi_{ZZ}(0) \qquad \delta\Pi_{WW}(0)$$

$$\frac{d}{dq^2}\delta\Pi_{AA}(q^2)\,|_{q^2=0} \quad \frac{d}{dq^2}\delta\Pi_{ZA}(q^2)\,|_{q^2=0} \quad \frac{d}{dq^2}\delta\Pi_{ZZ}(q^2)\,|_{q^2=0} \quad \frac{d}{dq^2}\delta\Pi_{WW}(q^2)\,|_{q^2=0}$$

$$(2.\ 4)$$

Two of these, by gauge invariance,

$$\delta\Pi_{AA}(0) = 0, \quad \delta\Pi_{ZA}(0) = 0.$$

When the tree input parameters, α, M_Z and G_F renormalized, three linear combinations of remaining six quantities can be eliminated. The effects from new physics can be described by three combinations which called "Oblique correction", S, T and U. The oblique parameters defined by Peskin and Takeuchi [1] is

$$S = -16\pi\frac{d}{dp^2}\delta\Pi_{3Y}(p^2)\,|_{p^2=0}, \qquad\qquad (2.\ 5)$$

$$\alpha T = \frac{g^2 + g'^2}{M_Z^2}[\delta\Pi_{11}(0) - \delta\Pi_{33}(0)], \qquad\qquad (2.\ 6)$$

$$U = 16\pi\frac{d}{dp^2}[\delta\Pi_{11}(p^2) - \delta\Pi_{33}(p^2)]\,|_{p^2=0}. \qquad\qquad (2.\ 7)$$

By using this parameters, one can compare the effect from new physics(techniparticle ...) with experimental data. But in this definition, the assumption that the new particle is very heavy compared with the mass of weak gauge boson is used. If there are a few new light particles, this notation may not use in terms of the Taylor expansion of self-energy.

Toy Model (QED with a massive gauge boson)

As a simple example to explain the oblique correction, we consider a toy model[9] in which a massive $U(1)$ gauge boson field A_μ couples to a fermion current J_μ.

The Lagrangian for the toy model is

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + eJ_\mu A^\mu. \qquad\qquad (2.\ 8)$$

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There are the self-energy from the interaction $eJ_\mu A^\mu$. For this lagrangian a self-energy is

$$
\begin{aligned}
A_\mu \Pi(q^2) A^\mu &= A_\mu M^2 (a_0 + a_1 \frac{q^2}{M^2} + a_2 \frac{q^4}{M^4} + \cdots) A^\mu \\
&= A_\mu M^2 (a_0 + a_1 \frac{-\Box}{M^2} + a_2 \frac{(-\Box)^2}{M^4} + \cdots) A^\mu.
\end{aligned}
\qquad (\,2.\,9\,)
$$

where $a_n \propto (\frac{M^2}{M^2_{New}})^n$. M is a pole mass of a propagator of the gauge boson. After the renormarization for the self energy, the effective lagrangian is

$$
\begin{aligned}
\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} &+ \frac{1}{2}\tilde{m}^2 A_\mu A^\mu \\
&+ \tilde{e} J_\mu A^\mu + A_\mu \Pi(q^2) A^\mu.
\end{aligned}
\qquad (\,2.\,10\,)
$$

The equation of motion for the gauge field A_μ in the lagrangian eq.(\,2.\,8\,)is

$$
\Box A_\mu = -\tilde{e} J_\mu - \tilde{m}^2 A_\mu
\qquad (\,2.\,11\,)
$$

Using this equation of motion,

$$
A_\mu \Box^n A^\mu \sim (-1)^n (M^{2n} A_\mu A^\mu + \sum_{m=0}^{n-1} \tilde{e} J_\mu (-\Box)^m A^\mu),
\qquad (\,2.\,12\,)
$$

and this formula substitute in eq.(\,2.\,9\,).

$$
A_\mu \Pi(-\Box) A^\mu = A_\mu \Pi(M^2) A^\mu + \tilde{e} J_\mu \sum_{n=1} (\frac{-\Box}{M^2})^{n-1} \sum_{m=n} a_m A^\mu
\qquad (\,2.\,13\,)
$$

the total effective lagrangian is

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\tilde{m}^2 + \Pi(M^2)) A_\mu A^\mu \\
&+ \tilde{e} J_\mu (1 + \frac{1}{2}\sum_{n=1} (\frac{-\Box}{M^2})^{n-1} \sum_{m=n} a_m) A^\mu
\end{aligned}
\qquad (\,2.\,14\,)
$$

In this lagrangian, the vertex is

$$
i\Lambda^\mu(q^2) = i\tilde{e}\gamma^\mu (1 + \frac{1}{2}\sum_{n=1} (\frac{q^2}{M^2})^{n-1} \sum_{m=n} a_m)
\qquad (\,2.\,15\,)
$$

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Convensionary,

$$i\Lambda^\mu(q^2) = i\tilde{e}\gamma^\mu(1 + \frac{1}{2}\frac{\Pi(q^2) - \Pi(M^2)}{q^2 - M^2})$$ (2. 16)

while the mass of A_μ is

$$M^2 = \tilde{m}^2 + \Pi(M^2)$$ (2. 17)

Therefore we can understand that the dependence from the fermions in new physics appear in obserbables of basic theory. It is same for the electro-weak model (Standard model).

Electroweak Model

We discuss the parametrization of oblique correction to the standard model[9]. We suppose the existence of new physics and the selfenergy from the new physics. The total effective lagrangian of SM is

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \mathcal{L}_{new}$$ (2. 18)

where

$$\begin{aligned}
\mathcal{L}_{new} &= \frac{1}{2}A_\mu\Pi_\gamma(-\Box)A^\mu + \frac{1}{2}Z_\mu\Pi_Z(-\Box)Z^\mu \\
&+ W_\mu^\dagger\Pi_W(-\Box)W^\mu + A_\mu\Pi_{Z\gamma}(-\Box)Z^\mu
\end{aligned}$$ (2. 19)

the equations of motion are

$$\begin{aligned}
\Box A^\mu &= -\tilde{e}J_{em}^\mu + \cdots \\
\Box Z^\mu &= -\frac{\tilde{e}}{\tilde{s}\tilde{c}}J_{nc}^\mu - \tilde{m}_Z^2 Z^\mu + \cdots \\
\Box W^\mu &= -\frac{\tilde{e}}{\sqrt{2}\tilde{s}}J_{cc}^\mu - \tilde{m}_W^2 W^\mu + \cdots
\end{aligned}$$ (2. 20)

By using the same method with the example of toy model, the effective lagrangian is

$$\mathcal{L}_{eff} = kineticterms$$

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$$+ \quad (\tilde{m}_Z^2 \tilde{c}^2 + \Pi(M_W^2))W_\mu^\dagger W^\mu + \frac{1}{2}(\tilde{m}_Z^2 + \Pi(M_Z^2))Z_\mu Z^\mu$$

$$+ \quad \tilde{e}J_{em}^\mu(1 + \frac{1}{2}\frac{\Pi_\gamma(q^2)}{q^2})A_\mu$$

$$+ \quad \tilde{e}J_{em}^\mu \frac{1}{2}\frac{\Pi_{Z\gamma}(q^2)}{q^2}Z_\mu$$

$$+ \quad \frac{\tilde{e}}{\tilde{s}\tilde{c}}J_{nc}^\mu(1 + \frac{1}{2}\frac{\Pi_Z(q^2) - \Pi_Z(M_Z^2)}{q^2 - M_Z^2})Z_\mu \qquad (2.21)$$

$$+ \quad \frac{\tilde{e}}{\sqrt{2}\tilde{s}}J_{cc}^\mu(1 + \frac{1}{2}\frac{\Pi_W(q^2) - \Pi_W(M_W^2)}{q^2 - M_W^2})W_\mu^\dagger + h.c.$$

Because we replace the renormalized quantities $\tilde{e}, \tilde{s}$ and $\tilde{m}_Z$ with the experimental values, we choose the three measured electroweak observables α, G_F and M_Z. These observables is measured very well.

$$4\pi\alpha = e^2 = \tilde{e}(1 + \frac{\Pi_\gamma(q^2)}{q^2}|_{q^2=0}) + O(\tilde{e}^4)$$

$$M_Z^2 = m_Z^2 = \tilde{m}_z^2 + \Pi_Z(M_Z^2) \qquad (2.22)$$

$$G_F = \frac{\sqrt{2}e^2}{8s^2c^2m_z^2} = \frac{\sqrt{2}\tilde{e}^2}{8\tilde{s}^2\tilde{c}^2\tilde{m}_z^2}$$

Therefore, the tilded quantities are

$$\tilde{e} = e(1 - \frac{1}{2}\frac{\Pi_\gamma(q^2)}{q^2}|_{q^2=0})$$

$$\tilde{m}_Z^2 = m_Z^2 - \Pi_Z(M_Z^2)$$

$$\tilde{s}^2 = s^2[1 + \frac{c^2}{c^2 - s^2}(-\frac{\Pi_\gamma(q^2)}{q^2}|_{q^2=0} - \frac{\Pi_W(0)}{M_W^2} + \frac{\Pi_Z(M_Z^2)}{M_Z^2}] \qquad (2.23)$$

And substituting the tilded quantities, the vertex is

$$i\Lambda_{em}^\mu(q^2) = -ieQ\gamma^\mu(1 + \frac{1}{2}\frac{\Pi_\gamma(q^2)}{q^2} - \frac{\Pi_\gamma(q^2)}{q^2}|_{q^2=0}) \qquad (2.24)$$

$$i\Lambda_{nc}^\mu(q^2) = -i\frac{e}{sc}\gamma^\mu(1 + \frac{1}{2}\frac{\Pi_Z(q^2) - \Pi_Z(M_Z^2)}{q^2 - M_Z^2} + \frac{1}{2}\frac{\Pi_W(0)}{M_W^2} + \frac{1}{2}\frac{\Pi_Z(M_Z^2)}{M_Z^2})$$

$$\times \quad [T_3L - Q(s^2 + \frac{s^2c^2}{c^2 - s^2}(-\frac{\Pi_\gamma(q^2)}{q^2}|_{q^2=0}$$

$$-\frac{\Pi_W(0)}{M_W^2} + \frac{\Pi_Z(M_Z^2)}{M_Z^2}) - sc\frac{\Pi_{z\gamma}(q^2)}{q^2})] \qquad (2.25)$$

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$$i\Lambda_{cc}^{\mu}(q^2) = -i\frac{e}{s\sqrt{2}}\gamma^{\mu}L(1 + \frac{1}{2}\frac{\Pi_W(q^2) - \Pi_W(M_W^2)}{q^2 - M_W^2}$$
$$+ \frac{1}{2(c^2 - s^2)}(s^2\frac{\Pi_\gamma(0)}{q^2}\big|_{q^2=0} + c^2\frac{1}{2}\frac{\Pi_W(0)}{M_W^2} - c^2\frac{1}{2}\frac{\Pi_Z(M_Z^2)}{M_Z^2})) \qquad (2.26)$$

$$m_W^2 = c^2 m_Z^2 - c^2\Pi_Z(m_Z^2)$$
$$- \frac{c^2 s^2 m_Z^2}{(c^2 - s^2)}(-\frac{\Pi_\gamma(q^2)}{q^2}\big|_{q^2=0} - \frac{\Pi_W(0)}{M_W^2} + \frac{\Pi_Z(M_Z^2)}{M_Z^2})$$
$$+ \Pi_W(M_W^2) \qquad (2.27)$$

The oblique parameters defined in ref.[9] on the mass of the weak gauge bosons is

$$\alpha S = 4s_W^2 c_W^2[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}]$$
$$- 4s_W c_W(c_W^2 - s_W^2)\delta\Pi'_{ZA}(0) - 4s_W^2 c_W^2\delta\Pi'_{AA}(0), \qquad (2.28)$$

$$\alpha T = \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2}, \qquad (2.29)$$

$$\alpha U = 4s_W^2[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}]$$
$$- 4s_W^2 c_W^2[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}]$$
$$- 4s_W^4\delta\Pi'_{AA}(0) - 8s_W^3 c_W\delta\Pi'_{ZA}(0), \qquad (2.30)$$

$$\alpha V = \delta\Pi'_{ZZ}(M_Z^2) - \frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}, \qquad (2.31)$$

$$\alpha W = \delta\Pi'_{WW}(M_W^2) - \frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}, \qquad (2.32)$$

$$\alpha X = -s_W c_W[\frac{\delta\Pi_{ZA}(M_Z^2)}{M_Z^2} - \delta\Pi'_{ZA}(0)], \qquad (2.33)$$

where $\delta\Pi_{XX}$ is a gauge bosons' self-energy for the beyond standard model and the prime show differentiation with respect to q^2. Using the notation of oblique correction, the vertex in electroweak model and W mass is shown as following:

$$i\Lambda_{em}^{\mu}(0) = -ieQ\gamma^{\mu} \qquad (2.34)$$

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$$i\Lambda_{nc}^{\mu}(M_Z^2) = -i\frac{e}{sc}\gamma^{\mu}(1 + \frac{1}{2}\alpha T + \frac{1}{2}\alpha V)$$

$$\times \quad [T_3L - Q(s^2 + \frac{\alpha S}{4(c^2 - s^2)} - \frac{c^2s^2\alpha T}{c^2 - s^2} + \alpha X)] \qquad (\ 2.\ 35\ )$$

$$i\Lambda_{cc}^{\mu}(M_W^2) = -i\frac{e}{s\sqrt{2}}\gamma^{\mu}L(1 - \frac{\alpha S}{4(c^2 - s^2)}$$

$$+\frac{c^2\alpha T}{2(c^2 - s^2)} + \frac{\alpha U}{8s^2} + \frac{1}{2}\alpha W) \qquad (\ 2.\ 36\ )$$

$$M_W^2 = M_W^{SM\,2}(1 - \frac{\alpha S}{2(c^2 - s^2)} + \frac{c^2\alpha T}{c^2 - s^2} + \frac{\alpha U}{4s^2} \qquad (\ 2.\ 37\ )$$

It is possible to express a wide variety of precision electroweak observables in terms of only the six parameters. If there are a few new light particles, we must use this notation. When the new particles have very heavy mass compared with the mass of weak gauge bosons, this notation becomes same as the notation defined by Peskin and Takeuchi. Then, in approximation that $M_W, M_Z \sim 0$, V, W and X vanishes. In other words, by only the three parameters S, T and U, we can examine the existence of the new heavy particles in the precision electroweak measurement.

2-2 Non-Oblique Correction

The contribution from new physics to vertex correction is usually very small, because the interaction among the ordinary fermion and the new particles is very weak. However, Such as Extended Technicolor Model that the interaction produce the large mass, for example top mass, there is the large vertex correction.(See the captor 5 and 6.)

Especially, recent experimental data at LEP shows that there are the deviations from the prediction of standard model in the Z boson partial width ratio. The experimental value

$$R_b = 0.2202 \pm 0.0020, \qquad (\ 2.\ 38\ )$$

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where

$$R_b = \frac{\Gamma(Z \to bb)}{\Gamma(Z \to hadrons)}, \qquad (\ 2.\ 39\ )$$

has already been different from the value $R_b^{SM} = 0.2157$ predicted by standard model with top mass $m_t = 175 GeV$. This standard model prediction is that added the leading standard model corrections such as fig.2 to the value $R_b^0 = 0.2197$ at the tree level(see fig.1)[10]. Hence, this discrepancy may be evidence of beyond
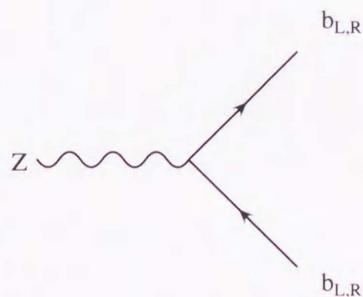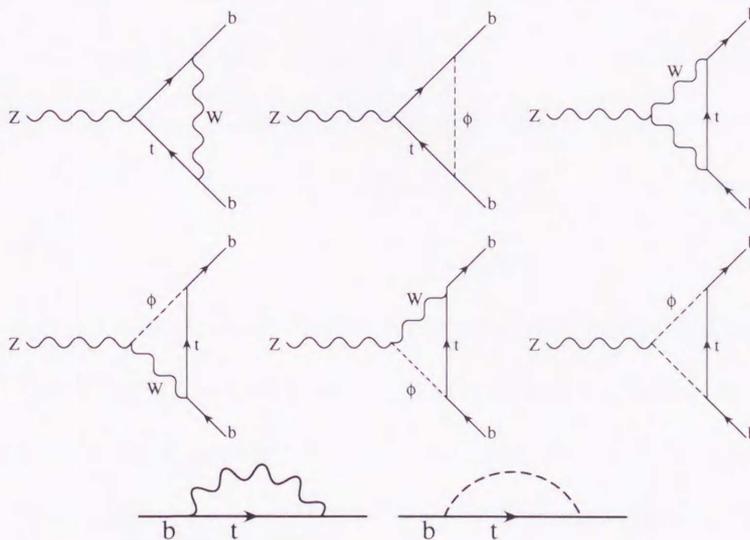

Figure 1:

Figure 2: The leading standard-model corrections to R_b, in t'Hooft-Feynman gauge.

the standard model. We must examine the non-oblique correction for the vertex of $Z - bb$ in the all models of new physics. At present, I do not know the model that is satisfying the constraint of the correction without extended technicolor model.[11]

3 Review of Technicolor Model

The standard model is consistent with almost all the experiments. The predictions of SM show the agreement with the recent precision experiments. However, SM has a few problems. One of them is the problem which is called "fine tuning problem" or "naturalness problem". If the Higgs potential in SM remain as a fundamental terms,

$$-m^2\varphi^\dagger\varphi + \frac{\lambda}{4}(\varphi^\dagger\varphi)^2 \qquad (3.1)$$

until certain very high energy scale Λ which is GUT scale($\Lambda \sim 10^{15} GeV$), the parameters in the terms must be fine tuned. The parameter receive the radiative corrections. The correction is proportion to Λ^2. Hence,

$$m^2 \rightarrow m^2 - \lambda\frac{\Lambda^2}{16\pi^2}. \qquad (3.2)$$

Because the potential must produce the symmetry breaking on electroweak scale ($v \sim 250$ GeV) to give the realistic weak gauge boson mass, we must take m^2 a very large mass to cancel the large value $\lambda\frac{\Lambda^2}{16\pi^2}$. We must be fine tuned like $1.00000000000000000\cdots001 - 1 = 0.00000000000000000\cdots001$. We think that this is an unnatural situation. To avoid this unnaturality, some beyond the standard model should exist.

One of the candidates as the beyond the standard model is Technicolor model. In this model, the Higgs is not elementary particle but a composite particle of the new fermion called technifermions with the additional strong interaction (technicolor interaction). The condensate $< \bar{T}T >\neq 0$ occur the electroweak $SU(2)_L \otimes U(1)_Y$ symmetry breaking. The mass correction for the fermion is

$$\delta m \sim mg^2(\ln\frac{\Lambda}{m})^n, \qquad (3.3)$$

and proportion to the power of $ln\Lambda$. When the coupling, g, is enough small, the correction is small compared with the mass of the fermion. Hence, the model that included just fermions and gauge bosons must not need the fine tuning such as the scalar potential of SM.

3-1 QCD

To break the electroweak symmetry, there is no need for elementary Higgs bosons. For example, we consider QCD. Ignore the small electroweak couplings of quarks. Then, their interactions respect a large global chiral flavor symmetry, $SU(2)_L \otimes SU(2)_R$ for (u, d). The strong QCD interactions of quarks cause this chiral symmetry breaking by the condensates

$$< \bar{u}u > = < \bar{d}d > = -\Delta_q \sim -4\pi f_\pi^3 \qquad (3.4)$$

The chiral symmetry is broken to $SU(2)_V$. Hence, there are $2^2 - 1 = 3$ massless Goldstone bosons, the decay constant of the bosons is $f_\pi = 93 MeV$.

Here, to examine the Higgs mechanism, we restore the ignored electroweak interactions. Then, the self energy of the weak gauge bosons are

$$\Pi_{\mu\nu}^{ab}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu})\frac{g_a g_b f_\pi^2}{4q^2} + \cdots \qquad (3.5)$$

where $(a, b) = Z, W$. This term show the mass of the weak gauge boson. In other words, the strong QCD interaction acts the role of Higgs mechanism. The electro weak symmetry $SU(2)_L \otimes U(1)_Y$ has broken to $U(1)_{em}$ and the weak bosons, W and Z, gain the mass. The mass are

$$M_W = \frac{1}{2}g f_\pi, M_Z = \frac{1}{2}\sqrt{g^2 + g'^2} f_\pi. \qquad (3.6)$$

While photon remains massless. The three degree of Goldstone boson is eaten by the longitudinal mode of the weak gauge boson.

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However, in the case that the QCD interaction induce the electro weak symmetry breaking, the mass is too small to predict the realistic mass. $M_W \sim 53 MeV$ and $M_Z \sim 60 MeV$. The measured values are about 1500 times larger.

Here, we consider whether the QCD scale, $\Lambda_{QCD} \sim 300 MeV$, is natural one or not. The scale is defined as that the QCD coupling $\alpha(\Lambda_{QCD}^2) \sim 1$. In one loop level renormarization equation for the coupling is

$$\frac{d}{d ln Q^2} \alpha_{QCD} = -\frac{11 - N_f}{4\pi} \alpha_{QCD}, \qquad (3.\ 7)$$

where N_f is the number of flavor. Put the boundary condition on the GUT scale as following,

$$\alpha_{QCD}(\Lambda_{QCD}^2) = \alpha_{GUT}. \qquad (3.\ 8)$$

Then,

$$\frac{1}{\alpha_{QCD}(Q^2)} - \frac{1}{\alpha_{GUT}} = \frac{11 - N_f}{4\pi} ln \frac{Q^2}{\Lambda_{GUT}^2}. \qquad (3.\ 9)$$

The ratio of the scales between QCD and GUT is

$$\frac{\Lambda_{QCD}^2}{\Lambda_{GUT}^2} = \exp[-\frac{4\pi}{11 - N_f}(\frac{1}{\alpha_{GUT}} - 1)] \qquad (3.\ 10)$$

Because $\alpha_{GUT} \sim O(10^{-2})$, the ratio becomes very small value of order 10^{-31}. Hence, such theory can naturally produce the very small scale such as Λ_{QCD}. We can understand that there is not the naturalness problem in the theory included only fermions and gauge bosons.

3-2 Technicolor Model

By QCD interaction, the breaking scale is too small. Assume that there is a new asymptotically free gauge interaction which called "Technicolor". We consider the

one doublet (U, D) of technifermions. They are massless and has the chiral flavor symmetry (custodial symmetry). Then, such as the case of QCD, the strong techni-color interaction cause the condensation of technifermions and the condensate break the chiral symmetry $SU(2)_L \otimes SU(2)_R$ to $SU(2)_V$.

$$< \bar{U}U >=< \bar{D}D >= -\Delta_T \sim -4\pi F_\pi^3 \qquad (3. 11)$$

where F_π is a decay constant of the massless Goldstone bosons (techni pions). By the same way of the case of QCD, the weak gauge bosons gain the mass.

$$M_W = \frac{1}{2}gF_\pi, M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}F_\pi \qquad (3. 12)$$

The scale Λ_{TC} at which technicolor interactions become strong is determined by the weak scale, $F_\pi = 246 GeV$.

One-family Technicolor Model

As realistic model, there is a one-family model. Introduced fermions are 4 doublets, three doublet with color charge (techniquarks) and one doublet without color charge (technileptons).

$$\begin{pmatrix} U \\ D \end{pmatrix}^1_L, \quad \begin{pmatrix} U \\ D \end{pmatrix}^2_L, \quad \begin{pmatrix} U \\ D \end{pmatrix}^3_L, \quad \begin{pmatrix} N \\ E \end{pmatrix}_L$$
$$U^1_R, \qquad U^2_R, \qquad U^3_R, \qquad N_R \qquad (3. 13)$$
$$D^1_R, \qquad D^2_R, \qquad D^3_R, \qquad E_R$$

where the index, 1,2,3, of U and D show the $SU(3)$ color. The condensations of these technifermions cause the chiral symmetry $SU(8)_L \otimes SU(8)_R$ breaking to $SU(8)_V$ and give the mass to weak gauge boson as Higgs mechanism. In this model, the value of decay constant of techni pion is $246/\sqrt{N_d}GeV = 123GeV$.

3-3 Extended Technicolor Model

This model must explain the mass of ordinary fermions. To generate the mass, the

technicolor interaction is extended. The extended gauge group is $SU(N_{TC}+3)$. First, at the scale M_1 $SU(N_{TC}+3)$ is broken to $SU(N_{TC}+2)$, then, the first generation (u, d, ν_e, e) gain the mass. The second, at the scaleM_1, $SU(N_{TC}+2)$ is broken to $SU(N_{TC}+1)$, the second generation (c, s, ν_μ, μ) gain the mass. Finery, at the scale M_3, $SU(N_{TC}+1)$ is broken to $SU(N_{TC})$, the third generation $(t, b, \nu_\tau. \tau)$ gain mass. The remained $SU(N_{TC})$ interaction become strong and cause the condensate of technifermions at scale Λ_{TC}. The difference of the scales generate the differences of mass between generations.

$$
\begin{aligned}
m_1 &\sim \frac{g_{ETC}^2(M_1)}{M_1^2} < \bar{T}T > \sim \frac{g_{ETC}^2(M_1)}{M_1^2} 4\pi F_\pi^3 \\
m_2 &\sim \frac{g_{ETC}^2(M_2)}{M_2^2} < \bar{T}T > \sim \frac{g_{ETC}^2(M_2)}{M_2^2} 4\pi F_\pi^3 \\
m_3 &\sim \frac{g_{ETC}^2(M_3)}{M_3^2} < \bar{T}T > \sim \frac{g_{ETC}^2(M_3)}{M_3^2} 4\pi F_\pi^3
\end{aligned}
\tag{3.14}
$$

Hence, because the ETC gauge interaction produces the differences between generations, the scales must satisfy the following relation,

$$M_1 >> M_2 >> M_3$$

The mass differences in same generation is induced from the difference of coupling between light-handed ETC interactions and right-handed's. Consider for the quarks of the third generation. There are $SU(N_{TC}+1)_L$, $SU(N_{TC}+1)_{UR}$ and

$SU(N_{TC} + 1)_{DR}$ ETC gauge interactions,

$$ETC \quad : \quad SU(N_{TC}+1)_L \quad SU(N_{TC}+1)_{UR} \quad SU(N_{TC}+1)_{DR}$$

$$coupling \quad : \quad \xi_L^t g_{ETC} \qquad \xi_R^t g_{ETC} \qquad \xi_R^b g_{ETC}$$

$$multiplet \quad : \quad \begin{pmatrix} T^1 \\ T^2 \\ . \\ . \\ T^{N_{TC}} \\ q \end{pmatrix}_L \quad \begin{pmatrix} U^1 \\ U^2 \\ . \\ . \\ U^{N_{TC}} \\ t \end{pmatrix}_R \quad \begin{pmatrix} D^1 \\ D^2 \\ . \\ . \\ D^{N_{TC}} \\ b \end{pmatrix}_R$$

Then, the masses of top quark and bottom quark are

$$m_t \quad \sim \quad \xi_L^t \xi_R^t \frac{g_{ETC}^2}{M_3^2} <\bar{U}U> \sim \xi_L^t \xi_R^t \frac{g_{ETC}^2}{M_3^2} 4\pi F_\pi^3$$

$$m_b \quad \sim \quad \xi_L^t \xi_R^b \frac{g_{ETC}^2}{M_3^2} <\bar{D}D> \sim \xi_L^t \xi_R^b \frac{g_{ETC}^2}{M_3^2} 4\pi F_\pi^3 \qquad (\ 3.\ 15\ )$$

The isospin breaking of top and bottom comes from the difference of the couplings. This mechanism is same for the leptons and other generations. We must consider that there are eigen coupling for flavors.
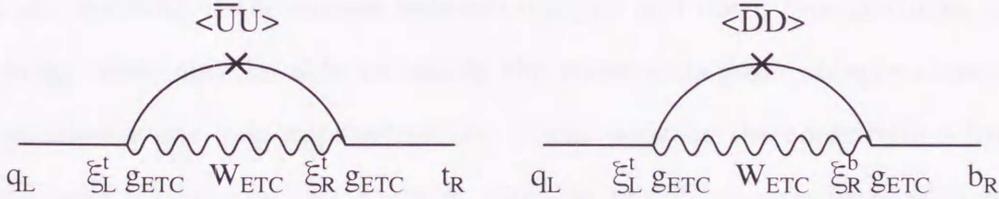

Figure 3: The masses of top and bottom quarks produced by ETC interaction

3-4Oblique corrections in Technicolor model

Recent precision measurement show that the ordinary technicolor model may be ruled out[16]. Because the model can not satisfy the constraints of Oblique correction and non-Oblique correction. Especially, QCD-like technicolor model completely was ruled out by the constraint of S parameter.

In one-loop approximation, the S and T parameters are

$$S = \frac{N_{TC}}{6\pi} \sum_{i=1}^{doublet} (1 - Y_L^i \ln \frac{m_U^i}{m_D^i}), \qquad (\,3.\ 16\,)$$

$$T = \frac{N_{TC}}{16\pi s^2 c^2 M_Z^2} \sum_{i=1}^{doublet} (m_U^2 + M_D^2 - \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2}). \qquad (\,3.\ 17\,)$$

Because of the custodial symmetry in the QCD-like model, the isospin is conserved:

$$m_U = m_D.$$

$$(\,3.\ 18\,)$$

Hence,

$$S = \frac{N_{TC} N_{doublet}}{6\pi} > 0 \qquad (\,3.\ 19\,)$$

$$T = 0 \qquad (\,3.\ 20\,)$$

The value of the S parameter is positive. However, from experiment the constraint favor the negative value. Accordingly, we need to modify the technicolor model to satisfy the constraints of the oblique parameters. It is shown that if these models have the splitting of the masses between up-type and down-type particles (isospin breaking), they may be able to satisfy the constraints from oblique correction S by considering one-loop approximation. There must be the contribution from the second term to the S in eq.(3. 16). However the T becomes large with positive sign.

To avoid the difficulty for the oblique correction, the model without exact custodial symmetry was proposed by appelquist and Terning[2]. The global symmetry breaking pattern is that $SU(6)_L \otimes SU(6)_R \otimes SU(2)_L \otimes U(1)_{2R} \otimes U(1)_{8L} \otimes U(1)_{8R} \otimes U(1)_V$ is broken into $SU(6)_V \otimes U(1)_{2V} \otimes U(1)_{8V} \otimes U(1)_V$. The feature of the model is that the custodial symmetry was broken in only the technilepton sector.

4 Effective Lagrangian for a Technicolor Model without Exact Custodial Symmetry

It has been shown that Technicolor models are strongly constrained by precision measurements of electorweak parameters. In particular, scaling-up QCD one-family Technicolor model with exact custodial symmetry seem to be already excluded by studying an oblique correction parameter S [1].

$$S_{theory} = 0.28 \times 4 = 1.1$$

$$S_{exp}^{New} = -0.42 \pm 0.36^{-0.08}_{+0.17}$$

where S_{theory} is an estimation with vector and axial vector meson dominance assumption for scaling-up QCD one-family Technicolor model. (See appendix E.) A factor of 4 comes from the fact that the model contains four $SU(2)$ doublets of technifermions. S_{exp} is quoted from [17] and the reference point of the standard model is taken at $m_t = 150\ GeV$ and $m_H = 1\ TeV$. However, according to ref.[2], this is not the case for Technicolor models without exact custodial symmetry. A realistic model is proposed for a one-family Technicolor model. In their model, isospin breaking is introduced for a light technilepton doublet. The doublet contributes to the radiative correction parameter S in negative sign while keeping ρ parameter nearly equal to 1. In their analysis, free technifermion model is used to compute S parameter.

In this paper, we compute the oblique corrections (S, T and U) in a more realistic way. In ref.[2], the oblique corrections are computed within one-loop approximation of technifermions. (See Fig.4(a).) However, if Technicolor theory is an ultraviolet asymptotically free and an infrared confining theory like QCD, this includes only the part of the non-perturbative effects which come from the use

of constituent technifermion masses. In fact, the correction like Fig.4(b) must be important at $q^2 = 0$ (q : external momentum of gauge boson) where the oblique corrections are defined. If the infinite series of the perturbative expansions are summed up, the contribution must be well described by the one pole exchange diagram of technimeson in Fig.4(c) . Therefore, in order to incorporate non-perturbative effects
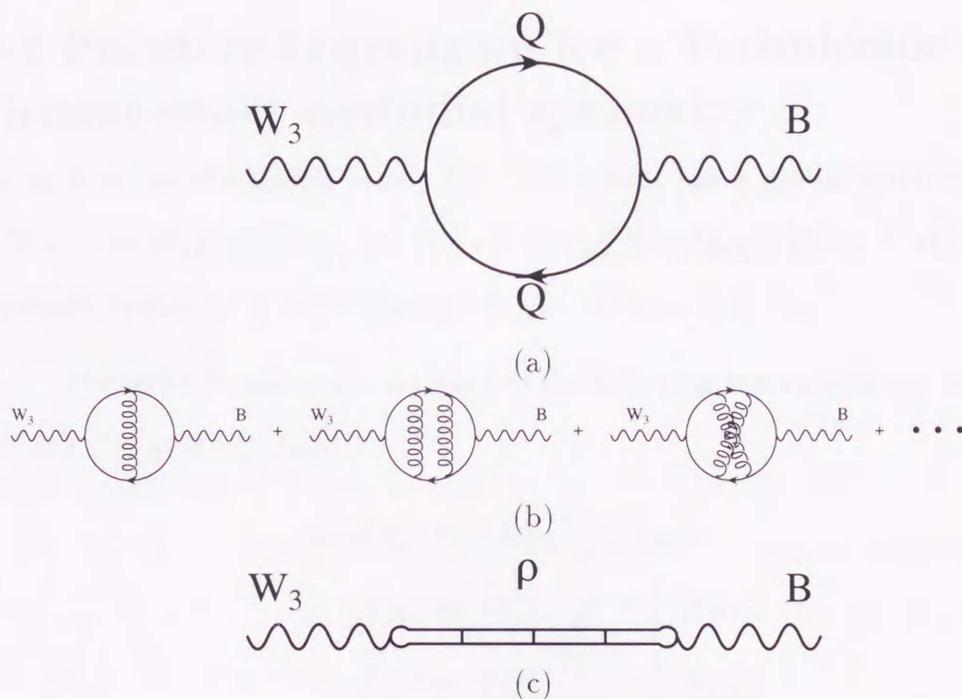
Figure 4: (a):The Feynman diagram for the contribution to S in a free technifermion model. (b): The perturbative corrections to Figure (a). (c): A typical nonperturbative contribution to S due to one pole exchange of techni ρ meson.

of technicolor, we need to describe Technicolor theory in terms of technimesons. We construct an effective Lagrangian with low lying technimesons for the one-family Technicolor model without exact custodial symmetry. By using the effective Lagrangian, we can compute non-perturbative effect of bound states of technifermions on the radiative correction parameters S, T and U. Our paper is organized in the

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following way. In section 4-1, we review feature of the model. With some assumptions on the low lying technimesons' spectrum, we construct a low energy effective Lagrangian. In section 4-2, S, T, and U parameters are computed with the Lagrangian. It is shown that S parameter receives negative contribution due to the mixing between isosinglet and isotriplet techni-vector mesons. Section 4-3 is devoted to finding the range of the parameters for negative S.

4-1 Effective Lagrangian for a Technicolor model without exact custodial symmetry

Let us describe the model briefly [2]. The model has a global symmetry : $G = SU(6)_L \otimes SU(6)_R \otimes SU(2)_L \otimes U(1)_{2R} \otimes U(1)_{8L} \otimes U(1)_{8R} \otimes U(1)_V$ [1] which is spontaneously broken to $H = SU(6)_V \otimes U(1)_{2V} \otimes U(1)_{8V} \otimes U(1)_V$.

The technifermions are assigned to the following representations of $SU(3)_c \otimes SU(2) \otimes U(1)_Y \otimes SU(N_{TC})$:

$$
\begin{aligned}
(U, D)_L &= (3, 2, \frac{Y_{Lq}}{2}, N_{TC}), \\
U_R &= (3, 1, \frac{Y_{Lq}}{2} + \frac{1}{2}, N_{TC}), \\
D_R &= (3, 1, \frac{Y_{Lq}}{2} - \frac{1}{2}, N_{TC}), \\
(N, E)_L &= (1, 2, \frac{Y_{Ll}}{2} + \frac{1}{2}, N_{TC}), \\
E_R &= (1, 1, \frac{Y_{Ll}}{2} - \frac{1}{2}, N_{TC}),
\end{aligned}
$$

where $Y_{Lq}(Y_{Ll})$ is hypercharge of lefthanded techniquark(technilepton) ($Y_{Lq} = 1/3$, $Y_{Ll} = -1$). The following mass spectrum is assumed for technifermions:

- $M_U = M_D$,

- $M_N < M_E$.

---

[1]Some part of G must be broken explicitly in order that unnecessary massless physical Nambu-Goldstone bosons disappear. See appendix **C** for the details.

$SU(6)_V$ symmetry is preserved because techniquarks are degenerate, while $SU(2)_V$ symmetry is explicitly broken due to isospin breaking of technileptons. To proceed further, we need to know the technimesons' spectrum of the model. Since the model does not have the same global symmetry as that of QCD ($SU(2)_L \otimes SU(2)_R$ in chiral limit) and underlying dynamics may be also different from that, we can not simply scale up the mesons' spectrum in QCD. Here we make use of the global symmetry as a guide to construct an effective Lagrangian. Global symmetry strongly constraints the structure of effective Lagrangian as well as properties of bound states included in effective Lagrangian. Concerned with technimesons, which are bound states of technifermions, we need to make a few assumptions. In this paper, we include only Nambu Goldstone Bosons (NGB), Pseudo Nambu Goldstone Bosons (PNGB) and spin 1 mesons. For the purpose of studying tree level contribution to the oblique correction parameters, S,T and U, other mesons with higher spins (spin≥ 2) can be ignored because they do not contribute to self energy corrections of gauge bosons. Further we only keep $O(p^2)$ terms of NGBs and PNGBs and ignore their loop effects and $O(p^4)$ counter terms. About spin 1 mesons, we employ the approach of including vector mesons into chiral Lagrangian [18][19] and extend it to our case. In table(1), technimesons and their technifermion contents as well as their J, P, C and I are listed. (For NGB and PNGB sector, we quote them from ref.[2].) Note that charged technilepton NGBs ($\Pi^\pm$) do not have definite parity because $\Pi^\pm$ are NGBs associated with $SU(2)_L$ not $SU(2)_A$. (Note that we do not have full $SU(2)_R$ symmetry.) In the same way, exotic left-handed charged vector mesons(A_L) are introduced so that they interact with $\Pi^\pm$ and $SU(2) \otimes U(1)$ gauge bosons etc. without loss of the invariance. Corresponding to spin 0, vector, and axial and left-handed vector mesons, the effective Lagrangian consists of three parts: $\mathcal{L}_S$, $\mathcal{L}_V$, and $\mathcal{L}_A$: The explicit form for them will be presented below.

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	J^{pc}	$Number$		$Isospin$	$charge$
P^α	0^-	35	$\bar{Q}\gamma_5 T^\alpha Q$		
Π^3	0^{-+}	1	$\bar{L}\gamma_5\tau^3 L$	1	0
θ_8	0^{-+}	1	$\bar{Q}\gamma_5 Q - 3\bar{L}\gamma_5 L$	0	0
$\Pi^\pm$	0	2	$\bar{L}\frac{1}{2}(1-\gamma_5)\tau^\pm L$	1	± 1
$V_{6\mu}^\alpha$	1^-	35	$\bar{Q}\gamma_\mu\lambda^\alpha Q$		
$\omega_{6\mu}$	1^{--}	1	$\bar{Q}\gamma_\mu Q$	0	0
$\rho_{2\mu}$	1^{--}	1	$\bar{L}\gamma_\mu\tau^3 L$	1	0
$\omega_{2\mu}$	1^{--}	1	$\bar{L}\gamma_\mu L$	0	0
$A_{6\mu}^\alpha$	1^+	35	$\bar{Q}\gamma_\mu\gamma_5 T^\alpha Q$		
$A_{8\mu}$	1^{++}	1	$\bar{Q}\gamma_\mu\gamma_5 Q - 3\bar{L}\gamma_\mu\gamma_5 L$	0	0
$A_{2\mu}$	1^{++}	1	$\bar{L}\gamma_\mu\gamma_5\tau^3 L$	1	0
$A_{L\mu}^\pm$	1	2	$\bar{L}\gamma_\mu\frac{1}{2}(1-\gamma_5)\tau^\pm L$	1	± 1

Table 1: J,P,C,I and electric charge of the technimesons incorporated in the effective Lagrangian. In the third column, the numbers of corresponding technimesons are shown. In the fourth column, techniquark contents with the same J,P,C,I and electric charge are shown.

NGB and PNGB sector

In order to construct the Lagrangian of NGBs and PNGBs, we employ the non-linear realization approach with exponentialized fields for NGBs and PNGBs [20]:

$$\xi_6 = \exp(i\sum_{\alpha=1}^{35}\frac{T^\alpha P^\alpha}{F_6} + i\frac{1}{4\sqrt{3}}\frac{\theta_8}{F_8}),$$
(4. 1)

$$\xi_2 = \exp(i\sum_{a=1}^{2}\frac{\tau^a \Pi^a}{2F_L} + i\frac{\tau^3\Pi^3}{2F_2} - i\frac{3}{4\sqrt{3}}\frac{\theta_8}{F_8}),$$
(4. 2)

$$\xi_1 = \exp(-i\frac{\tau^3\Pi^3}{2F_2} + i\frac{3}{4\sqrt{3}}\frac{\theta_8}{F_8}),$$
(4. 3)

where, T^α are generator of $SU(6)$ and τ^a, τ^3 are Pauli matrices. P^α are 35 NGBs for broken $SU(6)_A$ symmetry, which are techniquark bound states. $\Pi^a(a=1,2)$ and Π^3 are NGBs for broken $SU(2)_L(\tau^a : a=1,2)$ and $U(1)_{2A}$ symmetry respectively, which are technilepton bound states. θ_8 is a NGB for broken $U(1)_{8A}$ symmetry. F_6, F_L, F_2 and F_8 are decay constants for these NGBs.

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This part of the Lagrangian consists of $O(p^2)$ terms of NGBs and PNGBs.

$$\mathcal{L}_S = F_6^2 tr(\alpha_{6\perp})^2 + F_L^2 tr(\alpha_{L\perp})^2 + F_2^2 tr(\alpha_{2\perp})^2 + F_8^2 tr(\alpha_{8\perp})^2$$
$$+ \beta_\perp tr(\alpha_{8\perp}\hat{\tau}^3\alpha_{2\perp}) + \mathcal{L}'_{br}, \tag{4.4}$$

where,

$$\hat{\tau}^3 = \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix}, \tag{4.5}$$

$$\alpha_{6\perp\mu} = 2\sum_{\alpha=1}^{35} \begin{pmatrix} T^\alpha & \\ & 0 \end{pmatrix} tr[T^\alpha \frac{\xi_6\nabla_{R\mu}\xi_6^\dagger - \xi_6^\dagger\nabla_{L\mu}\xi_6}{2i}], \tag{4.6}$$

$$\alpha_{L\perp\mu} = -2\sum_{a=1}^{2} \frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^a \end{pmatrix} tr[\frac{\tau^a}{2} \frac{\xi_2^\dagger\hat{\nabla}_{L\mu}\xi_2}{2i}], \tag{4.7}$$

$$\alpha_{2\perp\mu} = 2\frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix} tr[\frac{\tau^3}{2} \frac{\xi_1^\dagger\hat{\nabla}_{R\mu}\xi_1 - \xi_2^\dagger\hat{\nabla}_{L\mu}\xi_2}{2i}], \tag{4.8}$$

$$\alpha_{8\perp\mu} = 2\frac{1}{4\sqrt{3}} \begin{pmatrix} I_6 & \\ & -3I_2 \end{pmatrix} [tr\{\frac{1}{4\sqrt{3}} \frac{\xi_6\nabla_{R\mu}\xi_6^\dagger - \xi_6^\dagger\nabla_{L\mu}\xi_6}{2i}\}$$
$$+ tr\{\frac{-3}{4\sqrt{3}} \frac{\xi_1^\dagger\hat{\nabla}_{R\mu}\xi_1 - \xi_2^\dagger\hat{\nabla}_{L\mu}\xi_2}{2i}\}], \tag{4.9}$$

In this paper, $\begin{pmatrix} A & \\ & B \end{pmatrix}$ always means a 8×8 matrix in which A (B) is a 6×6 (2×2) submatrix. I_n is a $n \times n$ unit matrix. $\nabla_{R(L)}$ are covariant derivatives for $SU(2)_L \otimes U(1)_Y \otimes SU(3)_c$ gauge interactions and are given by,

$$\nabla_{R\mu} = \partial_\mu + ig'(\frac{\tau^3}{2} \otimes I_3)B_\mu + ig'(\frac{Y_{LQ}}{2} \otimes I_3)B_\mu$$
$$+ ig_c \sum_{m=1}^{8} (I_2 \otimes \lambda^m)G_\mu^m, \tag{4.10}$$

$$\nabla_{L\mu} = \partial_\mu + ig \sum_{A=1}^{3} (\frac{\tau^A}{2} \otimes I_3)W_\mu^A + ig'(\frac{Y_{LQ}}{2} \otimes I_3)B_\mu$$
$$+ ig_c \sum_{m=1}^{8} (I_2 \otimes \lambda^m)G_\mu^m, \tag{4.11}$$

$$\hat{\nabla}_{R\mu} = \partial_\mu + ig'\frac{\tau^3}{2}B_\mu + ig'\frac{Y_{Ll}}{2}B_\mu, \tag{4.12}$$

$$\hat{\nabla}_{L\mu} = \partial_\mu + ig \sum_{A=1}^{3} \frac{\tau^A}{2}W_\mu^A + ig'\frac{Y_{Ll}}{2}B_\mu. \tag{4.13}$$

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where, W_μ^A, B_μ and G_μ^m are $SU(2) \otimes U(1)_Y \otimes SU(3)_c$ gauge bosons and λ^m are generator of $SU(3)_c$. Let us return to the Lagrangian (4. 4). Note that F_L is not degenerate with F_2 because custodial symmetry is broken in the technilepton sector and Π_L form an irreducible representation under $U(1)_{2V}$ which Π^3 does not belong to. The difference between F_L and F_2 gives rise to small deviation of ρ parameter from 1 (see (4. 74)). $\mathcal{L}'_{br}$ consists of explicit breaking terms which make physical NGB massive. Without $\mathcal{L}'_{br}$ and $SU(2)_L \otimes U(1)_Y$ gauge interaction, we have 3 color singlet physical massless NGBs which are linear combinations of $\theta_8, \Pi^3, \Pi^\pm, P^3$ and $P^\pm$. Here $P^\pm$ and P^3 are $SU(2)$ triplet and color singlet bound states of techniquarks. By adding $\mathcal{L}'_{br}$, we can make them massive. (See appendix C for the details.)

Now let us show that the Lagrangian (4. 4) is invariant under G. To prove this, we must know the transformations of $\alpha_\perp$s. First of all, the transformation of ξs are given by [20];

$$\xi'_6 = g_{L6}\xi_6 h_6^\dagger = h_6\xi_6 g_{R6}^\dagger, \qquad (4.\ 14)$$

$$\xi'_2 = g_{L2}\xi_2 h_2^\dagger, \qquad (4.\ 15)$$

$$\xi'_1 = g_{R2}\xi_1 h_2^\dagger, \qquad (4.\ 16)$$

$$\frac{\theta'_8}{F_8} = \frac{\theta_8}{F_8} + \phi_{R8} - \phi_{L8}. \qquad (4.\ 17)$$

In (4. 14)-(4. 17), g_Ls and g_Rs are chiral transformations which correspond to G while hs are vectorial transformations which correspond to the unbroken symmetry H. $g_{L6}(g_{L2})$ and $g_{R6}(g_{R2})$ are the chiral transformations in the techniquark (technilepton) sector. They are parametrized by the following equations.

$$g_{L6} = \exp(iT_6^\alpha \phi_{L6}^\alpha + i\frac{1}{4\sqrt{3}}\phi_{L8} + i\frac{1}{4}\phi_V), \qquad (4.\ 18)$$

$$g_{R6} = \exp(iT_6^\alpha \phi_{R6}^\alpha + i\frac{1}{4\sqrt{3}}\phi_{R8} + i\frac{1}{4}\phi_V), \qquad (4.\ 19)$$

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$$g_{L2} = \exp\left(i\sum_{a=1}^{2}\tau^a\phi_{L2}^a + i\tau^3\phi_{L2}^3 - i\frac{3}{4\sqrt{3}}\phi_{L8} + i\frac{1}{4}\phi_V\right), \qquad (4.\,20)$$

$$g_{R2} = \exp\left(i\tau^3\phi_{R2}^3 - i\frac{3}{4\sqrt{3}}\phi_{R8} + i\frac{1}{4}\phi_V\right), \qquad (4.\,21)$$

where $\phi_{L6}(\phi_{R6})$, $\phi_{L2}^a(\phi_{R2}^3)$, $\phi_{L8}(\phi_{R8})$ and ϕ_V are the parameters of $SU(6)_L$ $(SU(6)_R)$, $U(1)_{8L}$ $(U(1)_{8R})$, $SU(2)_L$ $(U(1)_{2R})$ and $U(1)_V$ respectively. h_6 (h_2) is a vectorial transformation in the techniquark (technilepton) sector. They are defined by,

$$h_6 = \hat{h}_6 h_{Q1}, \qquad (4.\,22)$$

$$\hat{h}_6 = \exp(iT_6^\alpha \phi_{V6}^\alpha) \quad \in SU(6)_V, \qquad (4.\,23)$$

$$h_{Q1} = \exp\left(i\frac{1}{2\sqrt{3}}\phi_{Q1}\right) \quad \in U(1)_V, \qquad (4.\,24)$$

$$h_2 = \hat{h}_2 h_{L1}, \qquad (4.\,25)$$

$$\hat{h}_2 = \exp\left(i\frac{\tau^3}{2}\phi_{V2}\right) \quad \in U(1)_{2V}, \qquad (4.\,26)$$

$$h_{L1} = \exp\left(i\frac{1}{2}\phi_{L1}\right) \quad \in U(1)_V, \qquad (4.\,27)$$

where ϕ_{V6} and ϕ_{Q1} are parameters for $SU(6)_V$ and $U(1)_V$ transformations in the techniquark sector while ϕ_{V2} and ϕ_{L1} are parameters for $U(1)_{2V}$ and $U(1)_V$ transformations in the technilepton sector. These parameters in the vectorial transformations depend on g_L, g_R and ξ. By using (4.\,14)-(4.\,17), it is not difficult to see that $\alpha_{6\perp\mu}$, $\alpha_{L\perp\mu}$ and $\alpha_{2\perp\mu}$ transform as;

$$\alpha'_{6\perp\mu} = \hat{h}_6 \alpha_{6\perp\mu} \hat{h}_6^\dagger, \qquad (4.\,28)$$

$$\alpha'_{2L\mu} = \hat{h}_2 \alpha_{2L\mu} \hat{h}_2^\dagger, \qquad (4.\,29)$$

$$\alpha'_{2\perp\mu} = \alpha_{2\perp\mu}, \qquad (4.\,30)$$

$$\alpha'_{8\perp\mu} = \alpha_{8\perp\mu}. \qquad (4.\,31)$$

With (4.\,28)-(4.\,31), the Lagrangian (4.\,4) is invariant under G.

Vector meson(1^{--}) sector

In addition to NGBs and PNGBs, we incorporate vector mesons into the effective Lagrangian. Corresponding to unbroken symmetry: $SU(6)_V \otimes U(1)_{2V} \otimes U(1)_{8V} \otimes U(1)_V$, we introduce 38 vector mesons, 35 of which are techniquark bound states belonging to the adjoint representation of $SU(6)_V$. Corresponding to $U(1)_{qV}$ ($U(1)_V$ for techniquark sector), $U(1)_{lV}$ ($U(1)_V$ for technilepton sector) and $U(1)_{2V}$, three neutral vector mesons, *techni* ω_q ($\omega_{6\mu}$), *techni* ω_l ($\omega_{2\mu}$), and *techni* ρ_l ($\rho_{2\mu}$) are introduced. For the techniquark sector of the effective Lagrangian, we can just extend the approach of [18] [19] into the larger symmetry, i.e, chiral $SU(6)_L \otimes SU(6)_R \otimes U(1)_{6V}$. On the other hand, for technilepton sector, non-trivial isospin breaking terms are introduced. Let us record vector meson part of the effective Lagrangian first,

$$
\begin{aligned}
\mathcal{L}_V &= \frac{1}{2} tr F_{V_6}^{\mu\nu} F_{V_6\,\mu\nu} + \frac{1}{2} tr F_{V_2}^{\mu\nu} F_{V_2\,\mu\nu} + \frac{1}{2} tr F_{V_{\omega 6}}^{\mu\nu} F_{V_{\omega 6}\,\mu\nu} + \frac{1}{2} tr F_{V_{\omega 2}}^{\mu\nu} F_{V_{\omega 2}\,\mu\nu} \\
&\quad - M_{V_6}^2 tr(V_{6\mu} - \frac{i}{G_6}\alpha_{6\|\mu})^2 - M_{V_{\omega 6}}^2 tr(V_{\omega 6\mu} - \frac{i}{G_{\omega 6}}\alpha_{\omega 6\|\mu})^2 \\
&\quad - M_{V_2}^2 tr(V_{2\mu} - \frac{i}{G_2}\alpha_{2\|\mu})^2 - M_{V_{\omega 2}}^2 tr(V_{\omega 2\mu} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu})^2 \\
&\quad + \alpha_V tr F_{V_2}^{\mu\nu} \hat{\tau}^3 F_{V_{\omega 2}\,\mu\nu} \\
&\quad - \beta_V tr(V_{2\mu} - \frac{i}{G_2}\alpha_{2\|\mu}) \hat{\tau}^3 (V_{\omega 2\mu} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu}),
\end{aligned}
\tag{4.32}
$$

where

$$
\alpha_{6\|\mu} = 2 \sum_{\alpha=1}^{35} \begin{pmatrix} T^\alpha & \\ & 0 \end{pmatrix} tr[T^\alpha \frac{\xi_6 \nabla_{R\mu} \xi_6^\dagger + \xi_6^\dagger \nabla_{L\mu} \xi_6}{2i}],
\tag{4.33}
$$

$$
\alpha_{\omega 6\|\mu} = 2 \frac{1}{2\sqrt{3}} \begin{pmatrix} I_6 & \\ & 0 \end{pmatrix} tr[\frac{1}{2\sqrt{3}} \frac{\xi_6 \nabla_{R\mu} \xi_6^\dagger + \xi_6^\dagger \nabla_{L\mu} \xi_6}{2i}],
\tag{4.34}
$$

$$
\alpha_{2\|\mu} = 2 \frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix} tr[\frac{\tau^3}{2} \frac{\xi_1^\dagger \hat{\nabla}_{R\mu} \xi_1 + \xi_2^\dagger \hat{\nabla}_{L\mu} \xi_2}{2i}],
\tag{4.35}
$$

$$
\alpha_{\omega 2\|\mu} = 2 \frac{1}{2} \begin{pmatrix} 0 & \\ & I_2 \end{pmatrix} tr[\frac{1}{2} \frac{\xi_1^\dagger \hat{\nabla}_{R\mu} \xi_1 + \xi_2^\dagger \hat{\nabla}_{L\mu} \xi_2}{2i}].
\tag{4.36}
$$

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Vector mesons are decomposed into their component fields,

$$V_{6\mu} = i \sum_{\alpha=1}^{35} \begin{pmatrix} T^\alpha & \\ & 0 \end{pmatrix} \rho_{6\mu}^\alpha,$$ (4. 37)

$$V_{\omega 6\mu} = i\frac{1}{2\sqrt{3}} \begin{pmatrix} I_6 & \\ & 0 \end{pmatrix} \omega_{6\mu},$$ (4. 38)

$$V_{2\mu} = i\frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix} \rho_{2\mu},$$ (4. 39)

$$V_{\omega 2\mu} = i\frac{1}{2} \begin{pmatrix} 0 & \\ & I_2 \end{pmatrix} \omega_{2\mu}.$$ (4. 40)

The quantities defined above transform under G in the following way,

$$\begin{cases} \alpha'_{6\|\mu} &= \hat{h}_6 \alpha_{6\|\mu} \hat{h}_6^\dagger - i\hat{h}_6 \partial_\mu \hat{h}_6^\dagger \\ V'_{6\mu} &= \hat{h}_6 V_{6\mu} \hat{h}_6^\dagger + \frac{1}{G_6} \hat{h}_6 \partial_\mu \hat{h}_6^\dagger, \end{cases}$$ (4. 41)

$$\begin{cases} \alpha'_{\omega 6\|\mu} &= \alpha_{\omega 6\|\mu} - ih_{Q1} \partial_\mu h_{Q1}^\dagger \\ V'_{\omega 6\mu} &= V_{\omega 6\mu} + \frac{1}{G_{\omega 6}} h_{Q1} \partial_\mu h_{Q1}^\dagger, \end{cases}$$ (4. 42)

$$\begin{cases} \alpha'_{2\|\mu} &= \alpha_{2\|\mu} - i\hat{h}_2 \partial_\mu \hat{h}_2^\dagger \\ V'_{2\mu} &= V_{2\mu} + \frac{1}{G_{\omega 2}} \hat{h}_2 \partial_\mu \hat{h}_2^\dagger, \end{cases}$$ (4. 43)

$$\begin{cases} \alpha'_{\omega 2\|\mu} &= \alpha_{\omega 2\|\mu} - ih_{L1} \partial_\mu h_{L1}^\dagger \\ V'_{\omega 2\mu} &= V_{\omega 2\mu} + \frac{1}{G_{\omega 2}} h_{L1} \partial_\mu h_{L1}^\dagger. \end{cases}$$ (4. 44)

In (4. 32), the terms proportional to α_V and β_V break isospin symmetry. The term with the coefficient α_V generates mixing between *techni* ω_l ($\omega_{2\mu}$ ($I = 0$)) and *techni* ρ_l ($\rho_{2\mu}$ ($I = 1$)) through the kinetic term while the term whose coefficient is β_V generates the mixing through the mass term.

Axial vector meson (A) and left-handed vector meson (A_L) sector

This part of the Lagrangian are given by,

$$\mathcal{L}_A = \frac{1}{2} tr F_{A_6}^{\mu\nu} F_{A_6\,\mu\nu} + \frac{1}{2} tr F_{A_L}^{\mu\nu} F_{A_L\,\mu\nu} + \frac{1}{2} tr F_{A_2}^{\mu\nu} F_{A_2\,\mu\nu} + \frac{1}{2} tr F_{A_8}^{\mu\nu} F_{A_8\,\mu\nu}$$

$$- M_{A_6}^2 tr(A_{6\mu} - \frac{i}{\lambda_6} \alpha_{6\perp\mu})^2 - M_{A_8}^2 tr(A_{8\mu} - \frac{i}{\lambda_8} \alpha_{8\perp\mu})^2$$

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$$- M_{A_L}^2 tr(A_{L\mu} - \frac{i}{\lambda_L}\alpha_{L\perp\mu})^2 - M_{A_2}^2 tr(A_{2\mu} - \frac{i}{\lambda_2}\alpha_{2\perp\mu})^2$$

$$- \alpha_A \frac{2}{\sqrt{3}} tr F_{A_2}^{\mu\nu} \hat{\tau}^3 F_{A_8\,\mu\nu}$$

$$+ \beta_A \frac{2}{\sqrt{3}} tr(A_{2\mu} - \frac{i(1+\delta)}{\lambda_2}\alpha_{2\perp\mu})\hat{\tau}^3(A_{8\mu} - \frac{i(1+\delta')}{\lambda_8}\alpha_{8\perp\mu})$$

$$+ \frac{1}{4} \frac{\beta_A^2 M_{A2}^2 \delta^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2} \frac{1}{\lambda_2^2} tr(\alpha_{2\perp\mu})^2$$

$$+ \frac{1}{4} \frac{\beta_A^2 M_{A8}^2 \delta'^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2} \frac{1}{\lambda_8^2} tr(\alpha_{8\perp\mu})^2$$

$$+ \frac{2}{\sqrt{3}} \frac{\delta\delta'\beta_A M_{A2}^2 M_{A8}^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2} \frac{1}{\lambda_2\lambda_8} tr(\alpha_{2\perp\mu}\hat{\tau}^3\alpha_{8\perp\mu}). \qquad (\,4.\,45\,)$$

The decomposition into component fields is given by,

$$A_{6\mu} = i\sum_{\alpha=1}^{35} \begin{pmatrix} T^\alpha & \\ & 0 \end{pmatrix} a_{6\mu}^\alpha, \qquad\qquad (\,4.\,46\,)$$

$$A_{8\mu} = i\frac{1}{4\sqrt{3}} \begin{pmatrix} I_6 & \\ & -3I_2 \end{pmatrix} a_{8\mu}, \qquad\qquad (\,4.\,47\,)$$

$$A_{L\mu} = i\sum_{a=1}^{2} \frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^a \end{pmatrix} a_{L\mu}^a, \qquad\qquad (\,4.\,48\,)$$

$$A_{2\mu} = i\frac{1}{2} \begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix} a_{2\mu}. \qquad\qquad (\,4.\,49\,)$$

They transform under G in the following way,

$$A'_{6\mu} = \hat{h}_6 A_{6\mu} \hat{h}_6^\dagger, \qquad\qquad (\,4.\,50\,)$$

$$A'_{8\mu} = A_{8\mu}, \qquad\qquad (\,4.\,51\,)$$

$$A'_{L\mu} = \hat{h}_2 A_{L\mu} \hat{h}_2^\dagger, \qquad\qquad (\,4.\,52\,)$$

$$A'_{2\mu} = A_{2\mu}. \qquad\qquad (\,4.\,53\,)$$

The terms proportional to α_A and β_A break isospin symmetry. We have already seen the counterparts of these terms in the vector meson sector (4. 32). Moreover, two additional parameters δ and δ' come in (4. 45). In particular, the presence of δ gives rise to a new isospin breaking effect on S and U as we show in section

4-2. We also note that there are no inhomogeneous terms for the transformation of axial (A) and left-handed (A_L) vector mesons under G. Therefore the mixing terms between these vector mesons and PNGB appear. This effect results in redefinition of the coefficients of $O(p^2)$ terms of NGB and PNGB sector (4. 4). In order to avoid the redefinition of the coefficients, we just need to add the appropriate $O(p^2)$ terms in axial and left-handed vector meson sector. The $O(p^2)$ terms of NGBs and PNGBs in (4. 45) are chosen so that the $O(p^2)$ terms in (4. 4) should not be altered after eliminating A and A_L with their equation of motion. (See appendix **D** for the details of the procedure.)

4-2 S,T and U parameter

The electroweak radiative correction parameter S, T and U are defined in terms of the gauge boson self-energy:

In order to compute technimesons contribution to S, T and U, we need to expand the effective Lagrangian in terms of their component fields explicitly. Because we only consider their tree level contribution here, it is suffice to keep technicolor singlet and color singlet technimesons (vector, axial and left-handed vector meson) in the expansion. It is spin 1 technimesons that contribute to S and U within the tree level approximation. In the following, we compute S, T and U in techniquark sector and in technilepton sector, respectively. The latter computation will tell us how differently custodial symmetry breaking terms contribute to S compared with techniquark sector where custodial symmetry is exact.

The techniquark sector

In this sector, color singlet and $SU(2)$ triplet part of techni-vector mesons $V_6(1^-)$

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and $A_6(1^+)$ contribute to S. Since $SU(2)$ singlet vector mesons such as $V_{\omega 6}$ and A_8 do not couple to W_3, they do not contribute to S. The mixing terms between the vector mesons and gauge bosons are given by :

$$\mathcal{L}_{6int} = -M_{V_6}^2 tr(V_{6\mu} - \frac{i}{G_6}\alpha_{6\|\mu})^2 - M_{A_6}^2 tr(A_{6\mu} - \frac{i}{\lambda_6}\alpha_{6\perp\mu})^2 \qquad (4.54)$$

$$= 2M_{V_6}^2[\frac{\rho_6}{2} - \frac{\sqrt{3}}{4G_6}(gW_3 + g'B)]^2 + 2M_{A_6}^2[\frac{a_6}{2} + \frac{\sqrt{3}}{4\lambda_6}(gW_3 - g'B)]^2 .$$

$\rho_6(a_6)$ are color singlet and isotriplet vector(axial vector) mesons. Therefore, these terms induce $W_3 - (\rho_6, a_6)$ and $B - (\rho_6, a_6)$ mixings, which give rise to a contribution to S through the Feynman diagram (Fig.5). We obtain S in techniquark sector.
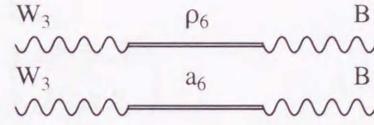
Figure 5: The Feynman diagrams for the contribution to S in the techniquark sector.

$$igg'S_q = -16\pi\frac{d}{dq^2}[(i\sqrt{3}\frac{M_{V6}^2}{2G_6}g)\frac{-i}{q^2 - M_{V6}^2}(ig'\sqrt{3}\frac{M_{V6}^2}{2G_6})]\mid_{q^2=0}$$

$$-16\pi\frac{d}{dq^2}[(i\sqrt{3}\frac{M_{A6}^2}{2\lambda_6}g)\frac{-i}{q^2 - M_{A6}^2}(-ig'\sqrt{3}\frac{M_{A6}^2}{2\lambda_6})]\mid_{q^2=0} . \qquad (4.55)$$

$$S_q = 4\pi[\frac{3}{G_6^2} - \frac{3}{\lambda_6^2}]. \qquad (4.56)$$

The technilepton sector

In this sector, there are isospin breaking terms,

$$\mathcal{L}' = \alpha_V tr F_{V_2}^{\mu\nu} \tau^3 F_{V_\omega 2\,\mu\nu}$$

$$- \beta_V tr(V_2 - \frac{i}{G_2}\alpha_{2\|\mu})\hat{\tau}^3(V_{\omega 2} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu})$$

$$- \alpha_A \frac{2}{\sqrt{3}} tr F_{A_2}^{\mu\nu}\hat{\tau}^3 F_{A_8\,\mu\nu}$$

$$+ \beta_A \frac{2}{\sqrt{3}} tr(A_{2\mu} - \frac{i(1+\delta)}{\lambda_2}\alpha_{2\perp\mu})\hat{\tau}^3(A_{8\mu} - \frac{i(1+\delta')}{\lambda_8}\alpha_{8\perp\mu}). \qquad (4.57)$$

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Because of the presence of these terms, isospin of vector mesons is not a conserved quantity and mixing between I=0 and I=1 vector mesons can occur. Then, $SU(2)$ singlet vector mesons such as *techni* ω_l (ω_2) can contribute to S through the mixing terms. Therefore , we need to diagonalize mass terms and kinetic terms of vector mesons. By expanding the effective Lagrangian, we obtain:

$$
\begin{aligned}
\mathcal{L}_V &= -\frac{1}{4} \begin{pmatrix} \partial_{[\mu,\rho_{2\nu]}} & \partial_{[\mu,\omega_{2\nu]}} \end{pmatrix} \begin{pmatrix} 1 & \alpha_V \\ \alpha_V & 1 \end{pmatrix} \begin{pmatrix} \partial_{[\mu,\rho_{2\nu]}} \\ \partial_{[\mu,\omega_{2\nu]}} \end{pmatrix} \\
&+ \frac{1}{2} \begin{pmatrix} \rho_2 & \omega_2 \end{pmatrix} \begin{pmatrix} M_{V2}^2 & \frac{\beta_V}{2} \\ \frac{\beta_V}{2} & M_{V\omega2}^2 \end{pmatrix} \begin{pmatrix} \rho_2 \\ \omega_2 \end{pmatrix} \\
&- \frac{1}{2} \begin{pmatrix} \rho_2 & \omega_2 \end{pmatrix} \begin{pmatrix} M_{V2}^2 & \frac{\beta_V}{2} \\ \frac{\beta_V}{2} & M_{V\omega2}^2 \end{pmatrix} \begin{pmatrix} \frac{gW_3 + g'B}{G_2} \\ \frac{2Y_{Ll}g'B}{G_{\omega2}} \end{pmatrix},
\end{aligned}
\tag{4.58}
$$

$$
\begin{aligned}
\mathcal{L}_A &= -\frac{1}{4} \begin{pmatrix} \partial_{[\mu,a_{2\nu]}} & \partial_{[\mu,a_{8\nu]}} \end{pmatrix} \begin{pmatrix} 1 & \alpha_A \\ \alpha_A & 1 \end{pmatrix} \begin{pmatrix} \partial_{[\mu,a_{2\nu]}} \\ \partial_{[\mu,a_{8\nu]}} \end{pmatrix} \\
&+ \frac{1}{2} \begin{pmatrix} a_2 & a_8 \end{pmatrix} \begin{pmatrix} M_{A2}^2 & \frac{\beta_A}{2} \\ \frac{\beta_A}{2} & M_{A8}^2 \end{pmatrix} \begin{pmatrix} a_2 \\ a_8 \end{pmatrix} \\
&+ \frac{1}{2} \begin{pmatrix} a_2 & a_8 \end{pmatrix} \begin{pmatrix} M_{A2}^2 & \frac{\beta_A}{2}(1 + \delta') \\ \frac{\beta_A}{2}(1 + \delta) & M_{A8}^2 \end{pmatrix} \begin{pmatrix} \frac{gW_3 - g'B}{\lambda_2} \\ 0 \end{pmatrix},
\end{aligned}
\tag{4.59}
$$

where

$$
\partial_{[\mu,V_{\nu]}} = \partial_\mu V_\nu - \partial_\nu V_\mu.
$$

The kinetic and mass terms can be diagonalized by doing the following transformations successively.

$$
\begin{pmatrix} \rho_2^m \\ \omega_2^m \end{pmatrix} = U_{mV} Z_V U_{DV} \begin{pmatrix} \rho_2 \\ \omega_2 \end{pmatrix},
\tag{4.60}
$$

$$
\begin{pmatrix} a_2^m \\ a_8^m \end{pmatrix} = U_{mA} Z_A U_{DA} \begin{pmatrix} a_2 \\ a_8 \end{pmatrix},
\tag{4.61}
$$

where U_D, Z and U_m are defined by,

$$
U_{DV(A)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},
\tag{4.62}
$$

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$$Z_{V(A)} = \begin{pmatrix} (1 - \alpha_{V(A)})^{\frac{1}{2}} & 0 \\ 0 & (1 + \alpha_{V(A)})^{\frac{1}{2}} \end{pmatrix}, \qquad (4.63)$$

$$U_{mV(A)} = \begin{pmatrix} c_{V(A)} & -s_{V(A)} \\ s_{V(A)} & c_{V(A)} \end{pmatrix}. \qquad (4.64)$$

U_D is a 45 degree rotation matrix to diagonalize the kinetic terms. Z is a scale transformation to keep correct normalization for the kinetic terms. Finally U_m is a rotation matrix which diagonalizes the mass terms. U_m relates the mass matrices to their diagonal forms in the following way.

$$\begin{pmatrix} M_\rho^2 & 0 \\ 0 & M_\omega^2 \end{pmatrix} = U_{mV} Z_V^{-1} U_{DV} \begin{pmatrix} M_{V2}^2 & \frac{\beta_V}{2} \\ \frac{\beta_V}{2} & M_{V\omega 2}^2 \end{pmatrix} U_{DV}^{-1} Z_V^{-1} U_{mV}^{-1} \qquad (4.65)$$

$$= U_{mV} \begin{pmatrix} \frac{1}{2(1-\alpha_V)}(M_{V2}^2 + M_{V\omega 2}^2 - \beta_V) & \frac{1}{2\sqrt{(1-\alpha_V^2)}}(M_{V2}^2 - M_{V\omega 2}^2) \\ \frac{1}{2\sqrt{(1-\alpha_V^2)}}(M_{V2}^2 - M_{V\omega 2}^2) & \frac{1}{2(1+\alpha_V)}(M_{V2}^2 + M_{V\omega 2}^2 + \beta_V) \end{pmatrix} U_{mV}^{-1},$$

$$\begin{pmatrix} M_{a2}^2 & 0 \\ 0 & M_{a8}^2 \end{pmatrix} = U_{mA} Z_A^{-1} U_{DA} \begin{pmatrix} M_{A2}^2 & \frac{\beta_A}{2} \\ \frac{\beta_A}{2} & M_{A8}^2 \end{pmatrix} U_{DA}^{-1} Z_A^{-1} U_{mA}^{-1} \qquad (4.66)$$

$$= U_{mA} \begin{pmatrix} \frac{1}{2(1-\alpha_A)}(M_{A2}^2 + M_{A8}^2 - \beta_A) & \frac{1}{2\sqrt{(1-\alpha_A^2)}}(M_{A2}^2 - M_{A8}^2) \\ \frac{1}{2\sqrt{(1-\alpha_A^2)}}(M_{A2}^2 - M_{A8}^2) & \frac{1}{2(1+\alpha_A)}(M_{A2}^2 + M_{A8}^2 + \beta_A) \end{pmatrix} U_{mA}^{-1}.$$

In (4.65) and (4.66), M_ρ and M_ω (M_{a2} and M_{a8}) are eigenvalues of vector (axial vector) mesons mass matrix. With these transformations, the interaction term between gauge bosons and vector mesons is given by:

$$\begin{aligned}
\mathcal{L}_{int} = &-\frac{1}{2} \begin{pmatrix} \rho_2^m & \omega_2^m \end{pmatrix} \begin{pmatrix} M_\rho^2 & 0 \\ 0 & M_\omega^2 \end{pmatrix} U_{mV} Z_V U_{DV} \begin{pmatrix} \frac{gW_3 + g'B}{G_2} \\ \frac{2Y_{L l}g'B}{G_{\omega 2}} \end{pmatrix} \\
&+\frac{1}{2} \begin{pmatrix} a_2^m & a_8^m \end{pmatrix} \begin{pmatrix} M_{a2}^2 & 0 \\ 0 & M_{a8}^2 \end{pmatrix} U_{mA} Z_A U_{DA} \begin{pmatrix} \frac{gW_3 - g'B}{\lambda_2} \\ 0 \end{pmatrix} \\
&+\frac{1}{4} \begin{pmatrix} a_2^m & a_8^m \end{pmatrix} U_{mA} Z_A^{-1} U_{DA} \begin{pmatrix} 0 & \beta_A \delta' \\ \beta_A \delta & 0 \end{pmatrix} \begin{pmatrix} \frac{gW_3 - g'B}{\lambda_2} \\ 0 \end{pmatrix}, \qquad (4.67)
\end{aligned}$$

where,

$$\begin{pmatrix} M_\rho^2 & 0 \\ 0 & M_\omega^2 \end{pmatrix} U_{mV} Z_V U_{DV} \qquad (4.68)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} M_\rho^2[c_V(1-\alpha_V)^{\frac{1}{2}} - s_V(1+\alpha_V)^{\frac{1}{2}}] & M_\rho^2[-c_V(1-\alpha_V)^{\frac{1}{2}} - s_V(1+\alpha_V)^{\frac{1}{2}}] \\ M_\omega^2[c_V(1+\alpha_V)^{\frac{1}{2}} + s_V(1-\alpha_V)^{\frac{1}{2}}] & M_\omega^2[c_V(1+\alpha_V)^{\frac{1}{2}} - s_V(1-\alpha_V)^{\frac{1}{2}}] \end{pmatrix},$$

$$\begin{pmatrix} M_{a2}^2 & 0 \\ 0 & M_{a8}^2 \end{pmatrix} U_{mA} Z_A U_{DA} \qquad (4.\ 69)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} M_{a2}^2[c_A(1-\alpha_A)^{\frac{1}{2}} - s_A(1+\alpha_A)^{\frac{1}{2}}] & M_{a2}^2[-c_A(1-\alpha_A)^{\frac{1}{2}} - s_A(1+\alpha_A)^{\frac{1}{2}}] \\ M_{a8}^2[c_A(1+\alpha_A)^{\frac{1}{2}} + s_A(1-\alpha_A)^{\frac{1}{2}}] & M_{a8}^2[c_A(1+\alpha_A)^{\frac{1}{2}} - s_A(1-\alpha_A)^{\frac{1}{2}}] \end{pmatrix},$$

$$U_{mA} Z_A^{-1} U_{DA} \begin{pmatrix} 0 & \beta_A \delta' \\ \beta_A \delta & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2(1-\alpha_A^2)}} \times \qquad (4.\ 70)$$

$$\begin{pmatrix} -\beta_A \delta[c_A(1+\alpha_A)^{\frac{1}{2}} + s_A(1-\alpha_A)^{\frac{1}{2}}] & \beta_A \delta'[c_A(1+\alpha_A)^{\frac{1}{2}} - s_A(1-\alpha_A)^{\frac{1}{2}}] \\ \beta_A \delta[s_A(1+\alpha_A)^{\frac{1}{2}} - c_A(1-\alpha_A)^{\frac{1}{2}}] & \beta_A \delta'[s_A(1+\alpha_A)^{\frac{1}{2}} + c_A(1-\alpha_A)^{\frac{1}{2}}] \end{pmatrix}$$

By computing the Feynman diagram (Fig.6), the sum of the contribution to S from technilepton sector and techniquark sector (4. 56) is,

$$\begin{aligned} S &= 4\pi\Big[\frac{3}{G_6^2} - \frac{3}{\lambda_6^2} + \frac{1}{G_2^2} - \frac{1}{\lambda_2^2} + Y_{Ll}\frac{2\alpha_V}{G_2 G_{\omega 2}}\Big] \\ &\quad - \frac{\pi}{2}\frac{\beta_A^2 \delta^2}{\lambda_2^2(1-\alpha_A^2)}\Big[\{c_A(1+\alpha_A)^{\frac{1}{2}} + s_A(1-\alpha_A)^{\frac{1}{2}}\}^2 \frac{1}{M_{a2}^4} \\ &\quad + \{-s_A(1+\alpha_A)^{\frac{1}{2}} + c_A(1-\alpha_A)^{\frac{1}{2}}\}^2 \frac{1}{M_{a8}^4}\Big]. \qquad (4.\ 71) \end{aligned}$$

The term proportional to $1/G_6^2$ $(1/\lambda_6^2)$ comes from vector (axial vector) mesons
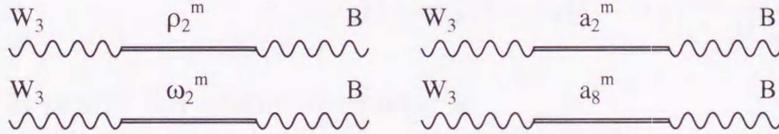

Figure 6: The Feynman diagrams for the contribution to S in the technilepton sector.

in the techniquark sector while the term proportional to $1/G_2^2$ $(1/\lambda_2^2)$ comes from vector (axial vector) mesons in the technilepton sector. As isospin breaking effects, we obtain two kinds of new contributions to S: One is the term proportional to Y_{Ll}. The other is the term proportional to δ. The former comes from the mixing between $I = 0$ and $I = 1$ vector mesons in the techniquark sector. Note that Y_{Ll} is -1 in

one family technicolor model. Therefore, when we change only α_V without changing the other parameters, the minimum of S can be obtained for $\alpha_V = 1^2$. The latter comes from the exchange of two neutral axial vector mesons whose masses are given by M_{a2} and M_{a8}. For T parameter, we obtain the same expression as that is given in ref.[2]. For completeness, we give the explicit form here.

$$M_W^2 = \frac{1}{4}g^2\{3F_6^2 + F_L^2\}, \qquad (4.72)$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)\{3F_6^2 + F_2^2\}, \qquad (4.73)$$

$$\alpha T = \rho - 1 = \frac{F_L^2 - F_2^2}{3F_6^2 + F_2^2}. \qquad (4.74)$$

With the assumption:$F_L, F_2 \ll F_6$, T parameter can be very small even if there is a splitting between F_L and F_2 as stated by the authors of [2]. Finally we compute U parameter. The result is given by,

$$
\begin{aligned}
U = 4\pi[&\frac{1}{G_2^2} + \frac{1}{\lambda_2^2} - \frac{1}{\lambda_L^2} \\
&+ \frac{1}{8}\frac{\beta_A^2\delta^2}{\lambda_2^2(1-\alpha_A^2)}\{[c_A(1+\alpha_A)^{\frac{1}{2}} + s_A(1-\alpha_A)^{\frac{1}{2}}]^2\frac{1}{M_{a2}^4} \\
&+ [s_A(1+\alpha_A)^{\frac{1}{2}} - c_A(1-\alpha_A)^{\frac{1}{2}}]^2\frac{1}{M_{a8}^4}\}].
\end{aligned}
\qquad (4.75)
$$

U parameter is zero if the isospin symmetry is exact. Since the isospin symmetry is broken in the technilepton sector of the present model, U parameter is not necessarily zero. The left-handed vector meson a_L contributes to $\delta\Pi_{11}$
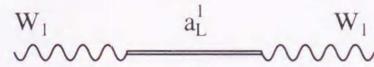

Figure 7: The Feynman diagram for the contribution to U due to the left-handed vector meson.

part of U (Fig.7), which is given by the term proportional to $1/\lambda_L^2$. On the other hand, $I = 1$ vector (axial vector) meson contributes to $\delta\Pi_{33}$ part of U, which is

---

[2]Only ω_2^m which consists of $N\bar{N}$ component contributes to S in that case, because $E\bar{E}$ component of vector meson decouples and ideal mixing is realized. This confirms the conjecture of the importance of ω given in ref.[2].

given by the term proportional to $1/G_2^2$ ($1/\lambda_2^2$). If the isospin were conserved, $\delta\Pi_{11}$ and $\delta\Pi_{33}$ should have been cancelled each other. This is not the case in the present model.

4-3 The range of parameters for negative S

In this section, we explore the parameter region where S parameter is negative since negative S is favored under the present experimental fits. If the future experimental constraint on S is improved, we can do more complete analysis. Since we do not know the underlying dynamics of the present model, we do not have any guiding principle to determine the parameters of our model without experimental information. Therefore instead of trying to predict S in our model, we determine the allowed region of the parameters of the effective Lagrangian by imposing the present experimentally allowed region for S. Since we have many parameters, we further need to limit ourselves into the small parameter space to draw some definite conclusions. Here we simply assume that S is dominated by only vector mesons and the contribution of axial vector mesons can be ignored[3] . Under the assumption of vector (1^{--}) dominance, S is given by,

$$
\begin{aligned}
S &= 4\pi\left[\frac{3}{G_6^2} + \frac{1}{G_2^2} - \alpha_V \frac{2}{G_2 G_{\omega 2}}\right] \\
&\geq 4\pi\left[\frac{3}{G_6^2} + \frac{1}{G_2^2} - \frac{2}{G_2 G_{\omega 2}}\right].
\end{aligned}
\tag{4.76}
$$

Here we have substituted $Y_{Ll} = -1$ in (4. 71) and neglect axial vector contribution. In the second line of (4. 76), the inequality holds because α_V can take its value between -1 and 1 and $\alpha_V = 1$ is the condition to have minimum value of S. Note that the bound for α_V comes from the condition for the positive semi-definiteness

---

[3]This is a good approximation in the case of the scaling-up QCD technicolor model as we have shown in appendix **E** (E.5)

of the kinetic terms of vector mesons (4. 63). In the following we assume that $\alpha_V = 1$ and impose the condition of negative S. The condition for $S \leq 0$ now leads a relation,

$$\frac{G_6}{G_2}(\frac{2G_6}{G_{\omega 2}} - \frac{G_6}{G_2}) \geq 3.$$
(4. 77)

This region is shown in Fig.5 in the parameter space (G_6/G_2 vs $G_6/G_{\omega 2}$). We can get the lower bound for one of the parameter, $G_6/G_{\omega 2}$.

$$\frac{G_6}{G_{\omega 2}} \geq \sqrt{3}$$
(4. 78)

Hence $G_{\omega 2}$ must be smaller than G_6 in order to make S negative. This means that the coupling strength between *techni* ω_l and gauge bosons are stronger than that between *techni* ρ_q and gauge bosons. Note that the coupling strength between gauge bosons and vector mesons is proportional to $1/G$. Since the negative contribution to S is proportional to Y_{Ll} and Y_{Ll} part of hypercharge interaction couples to *techni* ω_l, strong coupling between *techni* ω_l and hypercharge gauge boson B is preferred to get negative S.
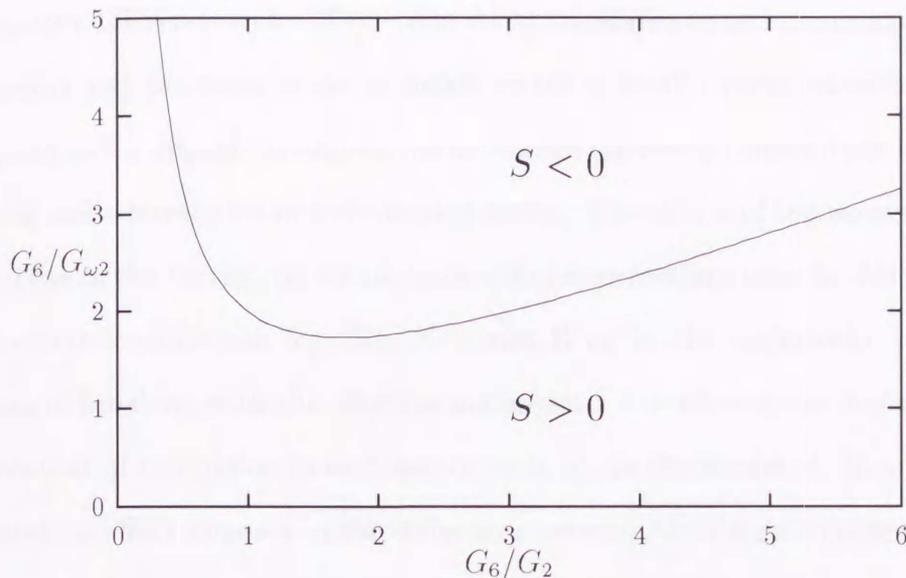
40

Figure 8: $S < 0$ $(S > 0)$ region with vector(1^{--}) dominance assumption for $\alpha_V = 1$. G_6, G_2 and $G_{\omega 2}$ are coupling constants associated with vector mesons.

5 The Vertex Correction in The Model without Exact Custodial Symmetry

In the last chapter, we constructed the effective Lagrangian for a one-family technicolor model without exact custodial symmetry and discussed the constraints for the oblique corrections. The most distinctive feature from the traditional technicolor theory is the isospin breaking in technilepton sector[2]. The symmetry that is to be satisfied in the technilepton sector is $SU(2)_L \otimes U(1)_{2R} \otimes U(1)_V$ global symmetry and this is broken to $U(1)_{2V} \otimes U(1)_V$ global symmetry when technileptons condense. This sector does not have QCD-like symmetry (chiral $SU(2)_L \otimes SU(2)_R$ symmetry). In the model [2, 3], we find that the constraints for oblique corrections can be satisfied because of the feature of isospin breaking in the technilepton sector. The S parameter is reduced by $\rho - \omega$ mixing[3] that is produced by the difference features from QCD in technilepton sector. While, the model can also reduce the T parameter if it

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has two difference scales of the pion decay constants in techniquark and technilepton sectors and the scale in technilepton sector is small compared with that in techniquark sector enough to neglect the difference the decay constants of between charged pion and neutral pion in technilepton sector. The effects of the isospin breaking must appear in the vertex correction (non-oblique correction) too. In this letter, we study the vertex correction for Zbb, $Z\tau\tau$ and $W\tau\nu$ in the technicolor model including isospin breaking with the effective lagrangian. The corrections depend on the decay constant of technipion in each sector [4, 5, 6]. In the model [2, 3], one of the isospin breaking effect appears in the difference between the decay constant of the charged technipion and that of neutral technipion in the technilepton sector. The differences directly appear in the differences of the vertex corrections between $Z\tau\tau$ and $W\tau\nu$. However, from the constraint of the oblique correction T, the difference between the decay constants of the technipion in technilepton sector must be small enough compared with decay constant in the techniquark sector. The other larger effect of the isospin breaking in the technilepton sector comes from the technivectormesons that is composed by technileptons. We can consider that there are a few lighter technivectormesons compared with the others in the techniquark sector in technilepton sector, because the decay constant of technipion in technilepton sector is lighter than one in techniquark sector. Hence, the dependence for the vertex corrections for $Z\tau\tau$ and $W\tau\nu$ from the light technivectormeson may be larger than the dependence from heavy mesons in techniquark sector. If there is a difference between dependence of the vertex corrections, it should be measured in the future precision measurements. In QCD-like model, the chiral $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ global symmetry is broken to $SU(2)_V \otimes U(1)_V$. There are three degenerated thechnivectormesons(ρ^0 and $\rho^\pm$) for the $SU(2)$ symmetry, three degenerated techniaxialvectormesons($a^0, a^\pm$) for $SU(2)$ symmetry and techni ω meson for $U(1)_V$ symmetry. However, this model

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is different from QCD. The model predicts the existence of a neutral ρ meson for $U(1)_{2V}$ and a techni ω meson for $U(1)_V$ symmetry. There must not be charged ρ mesons that degenerate with neutral techni ρ meson. There is also a neutral techniaxialvectormeson and there is not charged one. For the left degree of freedom in broken symmetry, there are exotic charged left-handed meson[4] that is composed by technileptons. While, because we expect that the breaking of the $SU(2)_L \otimes SU(2)_R$ to $SU(2)_L \otimes U(1)_{2R}$ in technilepton sector to be happened on higher energy scale, the masses of right-handed meson for the breaked symmetry must be heavier than other mesons. Accordingly, there are the neutral technivectormeson and techniaxialvectormeson which contribute to the vertex correction for the $Z\tau\tau$, while the charged technivectormesons are absent and there is only exotic charged left-handed meson which should contribute to the correction for the $W\tau\nu$. Hence, we may gain some hint about the evidence of isospin breaking in technilepton sector through the difference of the vertex corrections between $Z\tau\tau$ and $W\tau\nu$ in the precision measurements.

5-1 The Vertex Corrections

The vertex corrections depend on the Extended Technicolor Model (ETC). The Lagrangian [5] which describes the ETC gauge interaction[6] of one family technicolor model between the third family and Technifermion is,

$$
\begin{aligned}
\mathcal{L}_{ETC(3-TC)} &= g_{ETC}\xi_L^t \bar{Q}_L^i W_{ETC}^\mu \gamma_\mu q_L^i \\
&+ g_{ETC}\xi_R^t \bar{t}_R^i W_{ETC}^\mu \gamma_\mu U_R^i + g_{ETC}\xi_R^b \bar{b}_R^i W_{ETC}^\mu \gamma_\mu D_R^i + h.c.
\end{aligned}
$$

---

[4]There is the dependence of the left-handed meson to U parameter and the U has finite values. The meson will be constricted by the U parameter.

[5]Similarly some diagonal ETC gauge interactions between the same family also exist [6]. The vertex corrections for this interaction as depicted in Fig.9(b) is exist. The vertex is effectively same with Fig.10, except for the order of technicolor's number N_{tc} and the sign. By mean of $1/N$ expansion we can ignore this dependence, but if N_{TC} is small, we need to consider the effects.

[6]Here, we assume the simple ETC model.

$$+ \quad g_{ETC}\xi_L^\tau \bar{L}_L W_{ETC}^\mu \gamma_\mu l_L$$

$$+ \quad g_{ETC}\xi_R^\tau \bar{\tau}_R W_{ETC}^\mu \gamma_\mu E_R + h.c., \qquad (5.1)$$

where $Q_L^i = (U^i, D^i)_L$, U_R^i and D_R^i represent techniquarks, $q_L^i = (t^i, b^i)_L$, t_R^i and b_R^i represent the third family of quarks and " i " is the color index of QCD. $L_L = (N, E)_L$, E_R represent the technilepton, $l_L = (\nu, \tau)_L$ and τ_R represent the third family of leptons. g_{ETC} is a coupling of ETC interaction. W_{ETC} is an ETC gauge boson which mediates between the third family of ordinary fermions and techni fermions. $\xi_L^{t(\tau)}$ is a coefficient of left handed coupling and $\xi_R^{t(b,\tau)}$ is one of right handed coupling. Since the left handed fermion which belongs to $SU(2)$ doublet, the couplings of up-side and down-side in the doublet are the same as each other.

From eq.(5.1) the masses of ordinary fermions are given as,

$$m_t \sim \xi_L^t \xi_R^t \frac{g_{ETC}^2}{M_{ETC}^2} <\bar{U}U> \sim \xi_L^t \xi_R^t \frac{g_{ETC}^2}{M_{ETC}^2} 4\pi F_6^3, \qquad (5.2)$$

$$m_b \sim \xi_L^t \xi_R^b \frac{g_{ETC}^2}{M_{ETC}^2} <\bar{D}D> \sim \xi_L^t \xi_R^b \frac{g_{ETC}^2}{M_{ETC}^2} 4\pi F_6^3, \qquad (5.3)$$

$$m_\tau \sim \xi_L^\tau \xi_R^\tau \frac{g_{ETC}^2}{M_{ETC}^2} <\bar{E}E> \sim \xi_L^\tau \xi_R^\tau \frac{g_{ETC}^2}{M_{ETC}^2} 4\pi F_2^3, \qquad (5.4)$$

where M_{ETC} is the mass of the ETC gauge boson and $<\bar{Q}Q>$ is the condensation of technifermions. F_6 is the decay constants of technipion in techniquark sector and F_2 is that in technilepton sector. Here we used the relation of naive dimensional analysis $<\bar{Q}Q> \sim 4\pi F_Q^3$ [23].

Now, the vertex correction under consideration is shown in Fig.9(a). Because we assume that the ETC gauge boson is much heavier than the weak gauge boson, we can shrink the gauge propagator as shown in Fig.10. The ETC interaction in eq.(5.1) becomes the following effective four-fermi interaction after Fierz transformation,

$$\mathcal{L}_{int} = \quad - \quad \frac{1}{2}\xi_L^{t\,2} \frac{g_{ETC}^2}{M_{ETC}^2}(\bar{q}_L\gamma^\mu\tau^A q_L)(\bar{Q}_L\gamma_\mu\tau^A Q_L)$$

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$$- \frac{1}{2}\xi_L^{\tau\,2}\frac{g_{ETC}^2}{M_{ETC}^2}(\bar{l}_L\gamma^\mu\tau^A l_L)(\bar{L}_L\gamma_\mu\tau^A L_L). \qquad (5.5)$$

Then we replace the left handed technifermion current by chiral current [23, 4, 5, 6] that is the Noether current for $SU(2)_L$ symmetry in our effective Lagrangian [3]. We assume that technivectormesons in techniquark sector can be ignored when their masses are very heavy, $M_{V6}, M_{\omega 6} \sim 1$ TeV, compared with weak gauge boson masses. On the other hand, it is expected that the masses of the technivectormesons in the

(a)   (b)
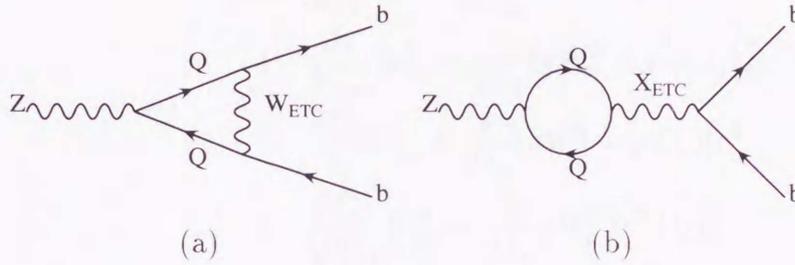
Figure 9: The Feynman diagram for the contribution to the vertex correction according to (a)sideways ETC gauge interaction and (b) diagonal ETC gauge interaction.
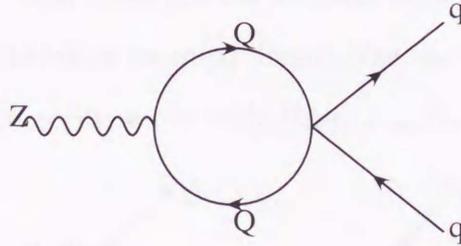
Figure 10: The Feynman diagram in which the ETC gauge boson propagators are shrunk in the upper figures.

technilepton sector are lighter than those in the techniquark sector in this model [2], because the pion decay constants in the technilepton sector are much smaller than those in the techniquark sector. Then the contribution of these light technivector

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mesons may be large and can not be ignored.

$$
\begin{aligned}
\mathcal{L}_{int} = & -\frac{1}{2}\xi_L^{t\,2}\frac{g_{ETC}^2}{M_{ETC}^2}[\bar{q}_L\gamma^\mu\{ \quad - \quad F_6^2\frac{\sqrt{3}}{2}(gW_\mu^3 - g'B_\mu)\frac{\tau^3}{\sqrt{3}} - F_6^2\frac{\sqrt{3}}{\sqrt{2}}(gW_\mu^\pm)\frac{\tau^\mp}{\sqrt{3}}\}q_L] \\
& -\frac{1}{2}\xi_L^{\tau\,2}\frac{g_{ETC}^2}{M_{ETC}^2}[\bar{l}_L\gamma^\mu\{ \quad - \quad F_2^2\frac{1}{2}(gW_\mu^3 - g'B_\mu)\tau^3 - F_L^2\frac{1}{\sqrt{2}}(gW_\mu^\pm)\tau^\mp \\
& + \frac{M_{V2}^2}{G_2}[\rho_{2\mu}^3 - \frac{1}{2G_2}(gW_\mu^3 + g'B_\mu)]\tau^3 \\
& + \frac{M_{\omega2}^2}{G_{2\omega}}[\omega_{2\mu} - \frac{1}{2G_{6\omega}}2Y_{Ll}g'B_\mu]Y_{Ll} \\
& + \frac{\beta_V}{2G_2}[\omega_{2\mu} - \frac{1}{2G_{2\omega}}2Y_{Ll}g'B_\mu]\tau^3 \\
& + \frac{\beta_V}{2G_{2\omega}}[\rho_{2\mu}^3 - \frac{1}{2G_2}(gW_\mu^3 + g'B_\mu)]Y_{Ll} \\
& + \frac{M_{A2}^2}{\lambda_2}[a_{2\mu}^3 + \frac{1}{2\lambda_2}(gW_\mu^3 - g'B_\mu)]\tau^3 \\
& + \frac{M_{AL}^2}{\sqrt{2}\lambda_L}[a_{L\mu}^\pm + \frac{1}{2\lambda_L}gW_\mu^\pm]\tau^\mp\}l_L], \qquad (5.6)
\end{aligned}
$$

where, we followed the same notation as ref.[3]. ρ_2 and ω_2 are technivectormesons, a_2 is a techniaxialvectormeson and $a_L^\pm$ is a exotic charged left-handedmeson. M_V, M_ω, M_{A2} and M_{AL} are their masses. In the technilepton sector, because of the presence of the isospin breaking terms, there are the mixings between neutral vectormesons, techni-ρ and techni-ω. Therefore we must diagonalize the mixing when we compute the effects of the technivectormeson in technilepton sector.
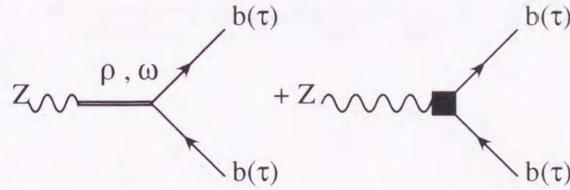

Figure 11: The Feynman diagram to compute the vertex correction by effective lagrangian approach. The first shows the effects of thechnivectormesons. The second shows the effects of the thecnipion.

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With eq.(5. 6), the vertex corrections are the following,

$$\delta g_L^{Zb\bar{b}} = \frac{1}{4}\xi_L^{t\,2}\frac{g_{ETC}^2}{M_{ETC}^2}F_6^2\sqrt{g^2+g'^2},\qquad(5.\ 7)$$

$$\delta g_L^{Z\tau^+\tau^-} = \frac{1}{4}\xi_L^{\tau\,2}\frac{g_{ETC}^2}{M_{ETC}^2}F_2^2\sqrt{g^2+g'^2}+\delta\bar{g}_{LV}^{Z\tau^+\tau^-}+\delta\bar{g}_{LA}^{Z\tau^+\tau^-},\qquad(5.\ 8)$$

$$\delta g_L^{W\tau\nu} = -\frac{1}{2\sqrt{2}}\xi_L^{\tau\,2}\frac{g_{ETC}^2}{M_{ETC}^2}F_L^2 g+\delta\bar{g}_{LL}^{W\tau\nu},\qquad(5.\ 9)$$

where $\delta\bar{g}_L^{Z\tau\tau}$ is the correction from the effect from the technivectormesons as shown in Fig.11. Substituting g_{ETC}^2/M_{ETC}^2 from eq.(5. 2) into eqs.(5. 7)-(5. 9), we find

$$\delta g_L^{Zb\bar{b}} = \frac{1}{4}\frac{\xi_L^t}{\xi_R^t}\frac{m_t}{4\pi F_6}\sqrt{g^2+g'^2},\qquad(5.\ 10)$$

$$\delta g_L^{Z\tau^+\tau^-} = \frac{1}{4}\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}m_t\frac{F_2^2}{4\pi F_6^3}\sqrt{g^2+g'^2}+\delta\bar{g}_{LV}^{Z\tau\tau}+\delta\bar{g}_{LA}^{Z\tau\tau},\qquad(5.\ 11)$$

$$\delta g_L^{W\tau\nu} = -\frac{1}{2\sqrt{2}}\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}m_t\frac{F_L^2}{4\pi F_6^3}g+\delta\bar{g}_{LL}^{W\tau\nu}.\qquad(5.\ 12)$$

The correction from the vectormesons is,

$$\delta\bar{g}_{LV}^{Z\tau\tau} = \frac{1}{4}\xi_L^{\tau\,2}\frac{g_{ETC}^2}{M_{ETC}^2}\ \Big\{\ \frac{1}{G_2^2}\Big[\frac{M_\rho^2}{2}A_\rho^2\frac{p^2}{p^2-M_\rho^2}+\frac{M_\omega^2}{2}A_\omega^2\frac{p^2}{p^2-M_\omega^2}\Big]\frac{g^2-g'^2}{\sqrt{g^2+g'^2}}$$
$$+\ \frac{1}{G_{2\omega}^2}\Big[\frac{M_\rho^2}{2}B_\rho^2\frac{p^2}{p^2-M_\rho^2}+\frac{M_\omega^2}{2}B_\omega^2\frac{p^2}{p^2-M_\omega^2}\Big]\frac{2g'^2}{\sqrt{g^2+g'^2}}\qquad(5.\ 13)$$
$$+\ \frac{1}{G_2 G_{2\omega}}\Big[\frac{M_\rho^2}{2}A_\rho B_\rho\frac{p^2}{p^2-M_\rho^2}+\frac{M_\omega^2}{2}A_\omega B_\omega\frac{p^2}{p^2-M_\omega^2}\Big]\sqrt{g^2+g'^2}\Big\},$$

with

$$A_\rho = c_V(1-\alpha_V)^{\frac{1}{2}}-s_V(1+\alpha_V)^{\frac{1}{2}},\qquad(5.\ 14)$$

$$B_\rho = -c_V(1-\alpha_V)^{\frac{1}{2}}-s_V(1+\alpha_V)^{\frac{1}{2}},\qquad(5.\ 15)$$

$$A_\omega = c_V(1+\alpha_V)^{\frac{1}{2}}+s_V(1-\alpha_V)^{\frac{1}{2}},\qquad(5.\ 16)$$

$$B_\omega = c_V(1+\alpha_V)^{\frac{1}{2}}-s_V(1-\alpha_V)^{\frac{1}{2}},\qquad(5.\ 17)$$

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where we followed the notation of ref.[3]. α_V is a parameter which indicates the isospin breaking (the mixing between techni-ρ and techni-ω in their kinetic terms of them), and c_V and s_V represent $cos\theta_V$, $sin\theta_V$, where θ_V is the mixing angle to diagonalize the $\rho - \omega$ mixing terms. The corrections from the techniaxialvectormeson and the left-handedmeson are

$$\delta \bar{g}_{LA}^{Z\tau\tau} = -\frac{1}{2}\xi_L^{\tau 2} \frac{g_{ETC}^2}{M_{ETC}^2}[\frac{M_{A2}^2}{2\lambda_2^2}\frac{p^2}{p^2 - M_{A2}^2}]\sqrt{g^2 + g'^2}, \qquad (5.18)$$

$$\delta \bar{g}_{LL}^{W\tau\nu} = \frac{1}{2\sqrt{2}}\xi_L^{\tau 2} \frac{g_{ETC}^2}{M_{ETC}^2}[\frac{M_{AL}^2}{2\lambda_L^2}\frac{p^2}{p^2 - M_{AL}^2}]g. \qquad (5.19)$$

The condition for decay constant of techni-pion

Now, we impose the constraints for the decay constants of technipion and consider some conditions which satisfy them. In the present model, in order to satisfy the constraint of the oblique correction, the pion decay constant in the technilepton sector must be much smaller than the decay constant in techniquark sector. We search the values of decay constant which satisfy the conditions, and compute the vertex corrections for Zbb, $Z\tau\tau$ and $W\tau\nu$. First, we can obtain the constraints from the relation between weak-gauge boson masses and the decay constants. In a one-family technicolor model with custodial symmetry the constraint is $4F_\pi^2 \simeq (250)^2(GeV)^2$. On the other hand, because in the present model the decay constant in the technilepton sector are different from that in the techniquark sector, the constraint is

$$3F_6^2 + F_2^2 \simeq (250)^2(GeV)^2. \qquad (5.20)$$

The second constraint is obtained from T parameter [1] which indicates the breaking of custodial symmetry. The condition is obtained from the constraint of T parameter [1]. The upper bound of T parameter is

$$T < 0.5. \qquad (5.21)$$

T parameter is given by [3],

$$\alpha T = \frac{F_L^2 - F_2^2}{3F_6^2 + F_2^2}.$$

Combining eq.(5. 20) with eq.(5. 21), we obtain the constraint between F_L and F_2,

$$F_L^2 - F_2^2 < 300 \ (GeV)^2. \tag{5. 22}$$

The last constraint is obtained from the ratios of masses of ordinary fermions m_τ : $m_b : m_t \sim 1 : 3 : 100$. From mass formulae in eqs.(5. 2) - (5. 4), we obtain

$$\xi_L^\tau \xi_R^\tau F_2^3 : \xi_L^t \xi_R^b F_6^3 : \xi_L^t \xi_R^t F_6^3 \sim 1 : 3 : 100. \tag{5. 23}$$

To determine the decay constants, we need to make some assumptions on the coupling constants ξs. Here we assume that the difference between the masses of the ordinary quark and the lepton comes from the differences of the decay constants of technipion in each sector. There are two cases roughly. One of them is that the difference of the decay constants is due to the difference between the masses of the up-type quark (t) and the lepton (τ). The other is that the difference is due to the difference between the down-type quark (b) and the lepton. Correspondingly, we assume the relations among the couplings ξs, i.e., (A) $\xi_L^\tau \xi_R^\tau = \xi_L^t \xi_R^t$ and (B) $\xi_L^\tau \xi_R^\tau = \xi_L^t \xi_R^b$. For both cases, we can determine the values of the pion decay constants with the constraints on eq.(5. 20), eq.(5. 22) and eq.(5. 23) .

(A) $\xi_L^\tau \xi_R^\tau = \xi_L^t \xi_R^t$: $F_6 = 143 GeV$, $F_2 = 31 GeV$, $F_L = 35 GeV$

(B) $\xi_L^\tau \xi_R^\tau = \xi_L^t \xi_R^b$: $F_6 = 135 GeV$, $F_2 = 90 GeV$, $F_L = 92 GeV$

In both cases, we compute the vertex correction for $Z b\bar{b}$, $Z\tau\tau$ and $W\tau\nu$ without including the correction due to the technivectormesons ($\delta\bar{g}$) as shown Table 2, Table 3 and Table 4 respectively. For comparison, as case(C), we show the vertex correction for the case when the decay constants in technilepton and techniquark sector

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are degenerate. Here, we find that the contributions of technipion for their vertex corrections ($\delta g - \delta \bar{g}$) become large, as the difference between the decay constants in the techniquark sector and the technilepton sector is becoming smaller.

	(F_6, F_2)	$\delta g_L^{Zb\bar{b}}$	$\frac{\delta\Gamma}{\Gamma}$
(A)	$(143GeV, 31GeV)$	$0.0181(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$	$-11\%(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$
(B)	$(135GeV, 90GeV)$	$0.0192(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$	$-11\%(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$
(C)	$(125GeV, 125GeV)$	$0.0207(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$	$-13\%(\frac{m_t}{175})\frac{\xi_L^t}{\xi_R^t}$

Table 2: The value of the vertex correction of $Zb\bar{b}$ and an amount of shifting the $Zb\bar{b}$ width from the standard model in a one-family technicolor model without exact custodial symmetry for each cases.

	(F_6, F_2)	$\delta g_L^{Z\tau^+\tau^-} - \delta \bar{g}_L^{Z\tau\tau}$	$\frac{\delta\Gamma}{\Gamma}$
(A)	$(143GeV, 31GeV)$	$0.0009(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$	$-0.5\%(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$
(B)	$(135GeV, 90GeV)$	$0.0085(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$	$-4.9\%(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$
(C)	$(125GeV, 125GeV)$	$0.0207(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$	$-12\%(\frac{m_t}{175})\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}$

Table 3: The value of the vertex correction of $Z\tau\tau$ and an amount of shifting the $Z\tau\tau$ width from the standard model except for the contribution from the technivectormesons in a one-family technicolor model without exact custodial symmetry for each cases.

5-3 Other effects for the vertex corrections

Next, we consider the correction, $\delta \bar{g}_L^{Z\tau\tau}$, which comes from the technivectormesons in the technilepton sector. For simplicity, we put $\alpha_V \sim 1$, $c_V \sim 1$ and $s_V \sim 0$ in the factors in eqs.(5. 14)-(5. 17), and substitute for $\frac{g_{ETC}^2}{M_{ETC}^2}$ from eq.(5. 2) in eq.(5. 14). Then eq.(5. 14) becomes,

$$\delta \bar{g}_L^{Z\tau\tau} = \frac{1}{4}\frac{\xi_L^{\tau 2}}{\xi_L^t \xi_R^t}\frac{m_t}{4\pi F_6^3} \quad \{ \quad \frac{1}{G_2^2}[M_\omega^2 \frac{p^2}{p^2 - M_\omega^2}]\frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}}$$

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	(F_6, F_L)	$\delta g_L^{W\tau\nu_\tau} - \delta\bar{g}_{LL}^{W\tau\nu}$	$\frac{\delta\Gamma}{\Gamma}$
(A)	$(143GeV, 35GeV)$	$-0.0015(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$	$-0.5\%(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$
(B)	$(135GeV, 92GeV)$	$-0.0126(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$	$-3.8\%(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$
(C)	$(125GeV, 125GeV)$	$-0.0292(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$	$-9\%(\frac{m_t}{175})\frac{\xi_L^{\tau\,2}}{\xi_L^t\xi_R^t}$

Table 4: The value of the vertex correction of $W\tau\nu$ and an amount of shifting the $W\tau\nu$ width from the standard model in a one-family technicolor model without exact custodial symmetry for each cases.

$$+ \quad \frac{1}{G_{2\omega}^2}[M_\omega^2\frac{p^2}{p^2 - M_\omega^2}]\frac{2g'^2}{\sqrt{g^2 + g'^2}}$$

$$+ \quad \frac{1}{G_2 G_{2\omega}}[M_\omega^2\frac{p^2}{p^2 - M_\omega^2}]\sqrt{g^2 + g'^2}\} \qquad (5.24)$$

Here at the scale of $p^2 \simeq M_Z^2$, we find that the contribution becomes large in the following cases. (1) The technivectormeson's mass is close to the gauge-boson's mass. (2) The couplings G_2 and $G_{2\omega}$ are becoming smaller. Because the smaller values for G_2 and $G_{2\omega}$ are favored to satisfy the constraint of the oblique correction S [3], the contribution from the technivectormesons will also be large. In Fig., we present the behavior of $\delta g_L^{Z\tau\tau}$ including the contribution of the technivectormesons $\delta\bar{g}_{LV}^{Z\tau\tau}$ and the techniaxialvectormesons $\delta\bar{g}_{LA}^{Z\tau\tau}$ and the behavior of $\delta g_L^{W\tau\nu}$ including the contribution of the left-handedmesons $\delta\bar{g}_{LL}^{W\tau\nu}$ as a function of M_ω or M_{AL} for several sets of values of G_2 and $G_{2\omega}$. In QCD, the mass of a ρ meson is 770 MeV and a_1 meson is 1260 MeV. Hence, we assume that the mass of techniaxialvectormeson M_{A2} is about $\frac{1260}{770}M_\omega$ in the calculation of the vertex correction $\delta\bar{g}_{LA}^{Z\tau\tau}$. Moreover, we assume that the mass of the left-handed meson M_{AL} also be $\frac{1260}{770}M_\omega$. In the previous work [3], we find that the value of G_6^2 is 31.5 and λ_6^2 is 106 when the custodial symmetry for a doublet is exact. Here, we assume that

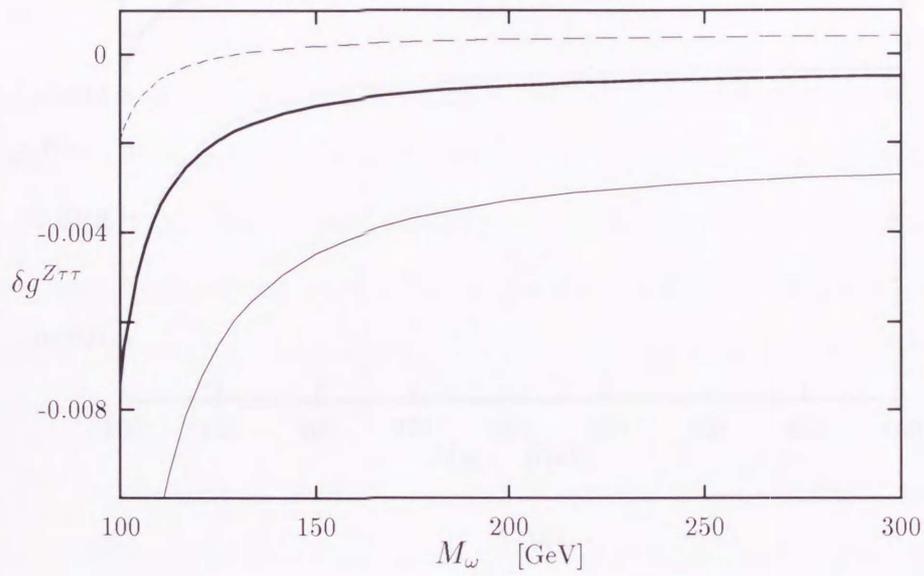$$G_6^2 : \lambda_6^2 = 31.5 : 106 = G_2^2, G_\omega^2 : \lambda_2^2, \lambda_L^2.$$

The upper bound of $G_{2\omega}$ [3] which makes S^V (the correction from the technivectormesons) to be negative is,

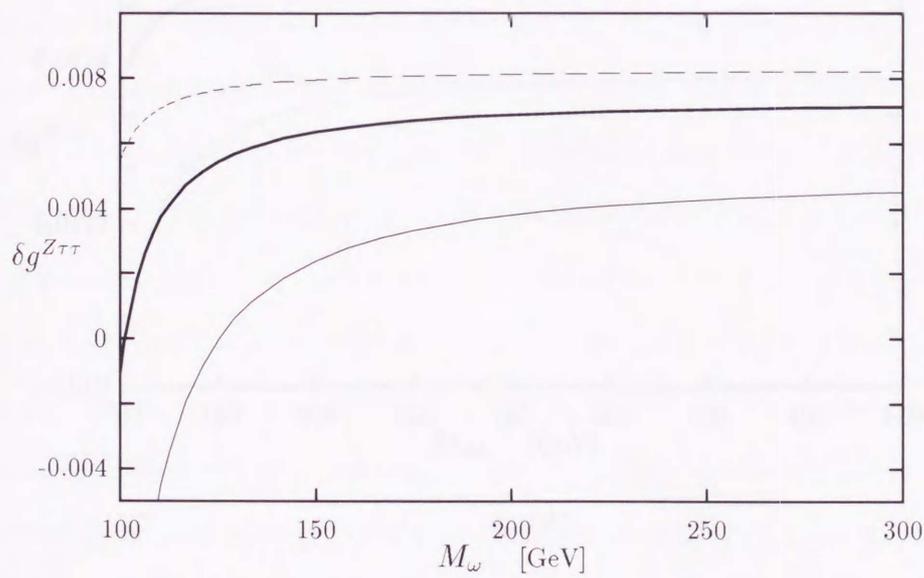$$G_{2\omega} < \frac{G_6}{\sqrt{3}} \sim \frac{5.61}{\sqrt{3}} \sim 3.24. \qquad (5.\ 25)$$

Therefore we plot the graph in the following three cases in Fig..

(1) $G_2, G_{2\omega} = 5.61$

(2) $G_2, G_{2\omega} = 3.24$ ($S^V = 0$)
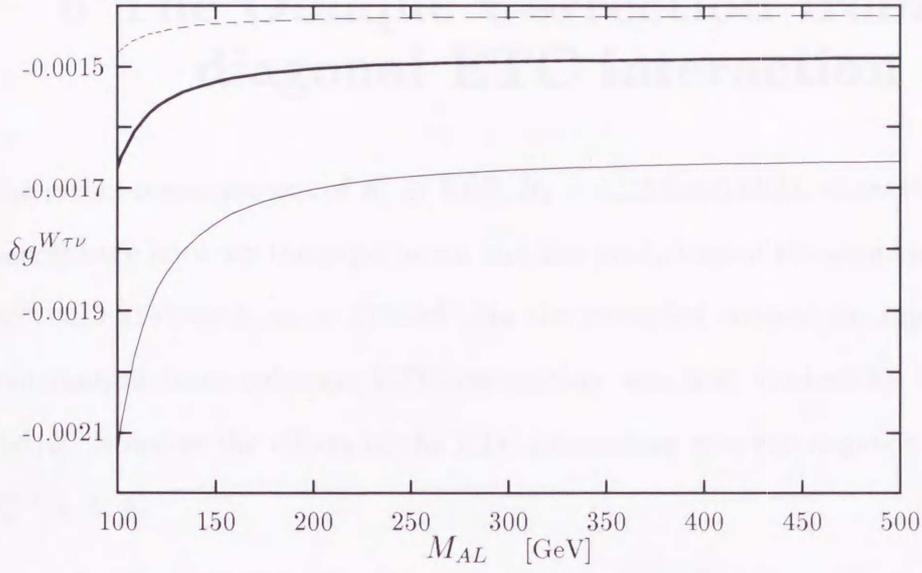
(3) $G_2, G_{2\omega} = 2$ ($S^V = -2$)

The case(1) is one with positive S like the traditional technicolor model with custodial symmetry. The case(2) is one with $S^V = 0$, and the case(3) is an extreme case with $S^V = -2$. Here we obtain the suppression on the vertex correction for $Z\tau\tau$ due to the vector meson when S is negative (Fig.). While, the correction for $W\tau\nu$ is small and enhanced by the effects of the technivectormesons, because there is only left-handed technivectormesons in the present model. We find that the difference between the vertex corrections of $Z\tau\tau$ and $W\tau\nu$ in terms of the contribution from the technivectormesons appear. In other words, the difference will be the evidence of the isospin breaking in technilepton sector.
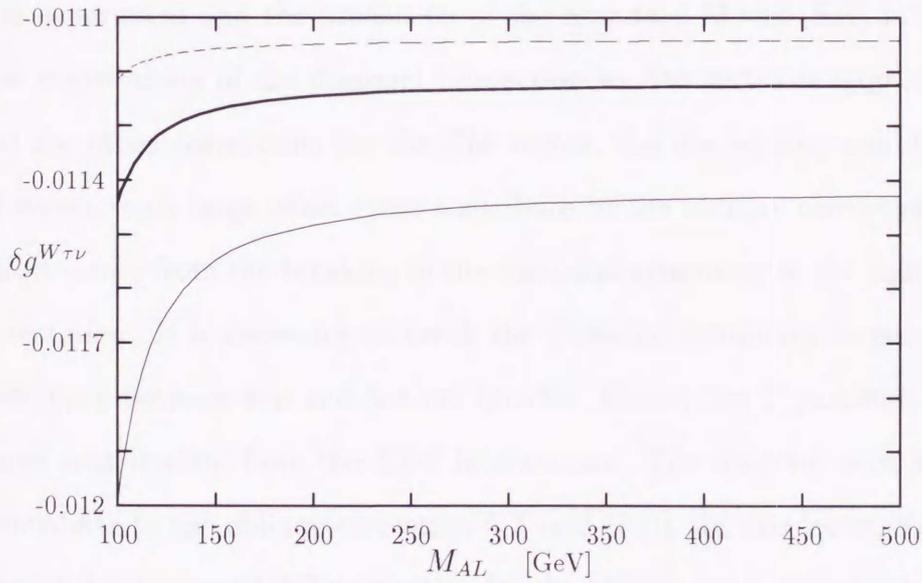
0

-0.004

$\delta g^{Z\tau\tau}$

-0.008

100 150 M_ω 200 [GeV] 250 300

(a)

0.008

0.004

$\delta g^{Z\tau\tau}$

0

-0.004

100 150 200 M_ω [GeV] 250 300

(b)

(c)

(d)

Figure 12: In (a) and (b) (F_6, F_2, F_L) = (143GeV,31GeV,35GeV) and (c) and (d) (F_6, F_2, F_L) = (135GeV,90GeV,92GeV), plotting the $\delta g_L^{Z\tau\tau}$ as a function of M_ω and $\delta g_L^{W\tau\nu}$ as a function of M_{AL} for the each casees, (1) $G_{2\omega} = 5.61$ with a dashline, (2) $G_{2\omega} = 3.24$ ($S^V = 0$) with a thickline and (3) $G_{2\omega} = 2$ ($S^V = -2$) with a thinline.

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6 The Oblique Correction from the diagonal ETC interaction

The recent measurement of R_b at LEP, $R_b = 0.2202 \pm 0.0020$, show that there is the discrepancy between the experiment and the prediction of the standard model(SM), $R_b^{SM} = 0.2157$ with $m_t = 175 GeV$. In the extended technicolor model(ETC), the contribution from sideways ETC interaction was first studied by the authors of Ref.[4]. However the effects of the ETC interaction give the negative corrections to R_b^{SM}[4, 5, 6].

In the recent works [7], it is shown that the diagonal extended technicolor(ETC) interaction may solve the Zbb problem, i.e., the discrepancy between the experiment and the prediction of the Standard Model (SM) in Zbb vertex. If the contribution of the diagonal interaction to Zbb vertex is large enough to cancel the other corrections for the Zbb vertex, the discrepancy could be explained. However, such large effect must contribute to the oblique corrections because the effect comes from the breaking of the custodial symmetry in the right handed ETC interaction. It is necessary to break the custodial symmetry to generate the mass difference between top and bottom quarks. Hence, the T parameter must receive large contribution from the ETC interactions. The diagram such as Fig.16[7]must contribute to the oblique correction S,T and U [1]. In this letter, we study the effect of the diagonal ETC interaction for the oblique corrections in the case that the non-oblique correction of the Zbb vertex is consistent with the experimental data in a realistic one-family model with the small S parameter[2](the model without exact custodial symmetry[3]).

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[7]In Ref.[7], the contribution from Fig.16(A) has been calculated but the contribution from Fig.16(B) is not considered.

6-1 Non-Oblique correction

We consider the non-oblique correction in the model with small S parameter, i.e., the technicolor model without exact custodial symmetry. The ETC model used in this talk is that the model that the horizontal symmetry $SU(N_{TC} + 1)$ is broken into $SU(N_{TC})$. In the multiplet of $SU(N_{TC} + 1)$, the third generation of ordinary fermions and the techni-fermions are contained. The lagrangian for the diagonal ETC interaction in the one-family technicolor model is

$$
\begin{aligned}
\mathcal{L}_{ETC(3-TC)}^{D} = g_{ETC} X_{ETC}^{\mu} \frac{1}{\sqrt{2N_{TC}(N_{TC}+1)}} \big[\quad & \xi_L^t(\bar{Q}_L^i \gamma_\mu Q_L^i - N_{TC}\bar{q}_L^i \gamma_\mu q_L^i) \\
+ \quad & \xi_R^t(\bar{U}_R^i \gamma_\mu U_R^i - N_{TC}\bar{t}_R^i \gamma_\mu t_R^i) \\
+ \quad & \xi_R^b(\bar{D}_R^i \gamma_\mu D_R^i - N_{TC}\bar{b}_R^i \gamma_\mu b_R^i) \\
+ \quad & \xi_L^\tau(\bar{L}_L \gamma_\mu L_L - N_{TC}\bar{l}_L \gamma_\mu l_L) \\
+ \quad & \xi_R^\nu(\bar{N}_R \gamma_\mu N_R - N_{TC}\bar{\nu}_R \gamma_\mu \nu_R) \\
+ \quad & \xi_R^\tau(\bar{E}_R \gamma_\mu E_R - N_{TC}\bar{\tau}_R \gamma_\mu \tau_R)\big]
\end{aligned}
$$

$$(6.\ 1)$$

where $Q_L^i = (U^i, D^i)_L$, U_R^i and D_R^i represent techniquarks, $q_L^i = (t^i, b^i)_L$, t_R^i and b_R^i represent the third family of quarks and " i " is the color index of QCD. $L_L = (N, E)_L$, E_R represent the technilepton, $l_L = (\nu, \tau)_L$ and τ_R represent the third family of leptons. g_{ETC} is a coupling of ETC interaction. X_{ETC} is diagonal ETC gauge boson which mediates between the third family of ordinary fermions and techni fermions. N_{TC} is the number of the technicolor. $\frac{1}{\sqrt{2N_{TC}(N_{TC}+1)}}$ is the normalization factor of the generator of horizontal symmetry $SU(N_{TC} + 1)$. $\xi_L^{t(\tau)}$ is a coefficient of left handed coupling and $\xi_R^{t(b,\tau)}$ is one of right handed coupling. Since the left handed fermion belongs to $SU(2)$ doublet, the couplings of up-type quark and down-type quark in the doublet are the same as each other.

The effective lagrangian is

$$\mathcal{L}_{int} = \frac{1}{2}\frac{g_{ETC}^2}{q^2 - M_X^2} \quad \frac{1}{2N_{TC}(N_{TC}+1)}$$

$$\times \quad [\quad \xi_L^t \bar{Q}_L^i \gamma^\mu Q_L^i + \xi_R^t \bar{U}_R^i \gamma^\mu U_R^i + \xi_R^b \bar{D}_R^i \gamma^\mu D_R^i$$

$$- \quad N_{TC}\xi_L^t \bar{q}_L^i \gamma_\mu q_L^i - N_{TC}\xi_R^t \bar{t}_R^i \gamma_\mu t_R^i - N_{TC}\xi_R^b \bar{b}_R^i \gamma_\mu b_R^i$$

$$+ \quad \xi_L^\tau \bar{L}_L \gamma^\mu L_L + \xi_R^\nu \bar{N}_R \gamma^\mu N_R + \xi_R^\tau \bar{E}_R \gamma^\mu E_R \qquad (6.2)$$

$$- \quad N_{TC}\xi_L^\tau \bar{l}_L \gamma_\mu l_L - N_{TC}\xi_R^\nu \bar{\nu}_R \gamma_\mu \nu_R - N_{TC}\xi_R^\tau \bar{\tau}_R \gamma_\mu \tau_R]^2,$$

where M_X is the mass of ETC gauge boson. Below the TC chiral symmetry breaking scale, the current of techniquarks are replaced by the Noether current[23, 4, 5, 6, 25] in the effective chiral lagrangian with $SU(2N_c)_L \otimes SU(2N_c)_R \otimes U(1)_V$ in techniquark sector [3, 25]. Here, we separate the right-handed current into τ^3 and singlet components of $SU(2)$,

$$\xi_R^t \bar{U}_R^i \gamma^\mu U_R^i + \xi_R^b \bar{D}_R^i \gamma^\mu D_R^i = \frac{\xi_R^t + \xi_R^b}{2}\bar{Q}_R^i \gamma^\mu Q_R^i + \frac{\xi_R^t - \xi_R^b}{2}\bar{Q}_R^i \tau^3 \gamma^\mu Q_R^i \quad (6.3)$$

Explicitly, the right-handed currents of techniquark are replaced by the following Noether current of the effective lagrangian.

$$\sum_{i=1}^3 \bar{Q}_R^i \gamma^\mu Q_R^i \sim -3\frac{M_{6\omega}}{G_{6\omega}}[\omega_6^\mu - \frac{\sqrt{3}}{2G_{6\omega}}2Y_{Lq}g'B^\mu]\frac{Y_{Lq}}{\sqrt{3}}, \qquad (6.4)$$

$$\sum_{i=1}^3 \bar{Q}_R^i \tau^3 \gamma^\mu Q_R^i \sim 3F_6^2\frac{1}{2}(gW^\mu - g'B^\mu)$$

$$-3\frac{M_{V6}}{G_6}[\rho_6^\mu - \frac{\sqrt{3}}{2G_6}(gW^\mu + g'B^\mu)]\frac{1}{\sqrt{3}}$$

$$-3\frac{M_{A6}}{\lambda_6}[a_6^\mu + \frac{\sqrt{3}}{2\lambda_6}(gW^\mu - g'B^\mu)]\frac{1}{\sqrt{3}}, \qquad (6.5)$$

where, ω_μ and ρ_μ are techni-omega meson and techni-rho meson that is composed by techniquarks and $M_{\omega 6}$ and M_{V6} are their masses. a_μ is a techni-axialvectormeson and M_{A6} is its mass. G and λ are the couplings which are related to the techni-vectormesons. The F_6 is the decay constant of technipion in techniquark sector.

We can neglect the technilepton contribution to the oblique corrections because the coefficients of ETC coupling or decay constant F_2 in technilepon sector is much smaller than that of techniquark in order to generate the mass difference between techniquark and technilepton. Besides this reason, in the model with small S parameter[2][3], the decay constant F_2 must be much smaller than that in the

Figure 13: The contribution to the non-oblique correction according to sideway ETC gauge interaction.

techniquark sector to satisfy the experimental bound of T parameter,.

While, the non-oblique corrections for Zbb vertex [7] are given by

$$\delta g_L^{ETC} = \delta g_{LS}^{ETC} + \delta g_{LD}^{ETC}, \qquad (6.6)$$

where, the contribution from the side-way ETC gauge interaction of Fig.13[4, 5]is

$$\delta g_{LS}^{ETC} = \frac{1}{8}\xi_L^{t\,2}\frac{g_{ETC}^2}{M_{ETC}^2}F_\pi^2\sqrt{g^2 + g'^2}, \qquad (6.7)$$

and the contribution from the diagonal ETC interaction of Fig.14 is

$$\delta g_{LD}^{ETC} = -\frac{3}{8}\xi_L^t(\xi_R^t - \xi_R^b)\frac{g_{ETC}^2}{M_{ETC}^2}\frac{1}{N_{TC}+1}F_\pi^2\sqrt{g^2 + g'^2}. \qquad (6.8)$$

If the effect of the ETC gauge interaction, i.e., eq.(6. 6) explain the difference between the experimental data of R_b and the prediction of SM, the parameter $\xi_R^t - \xi_R^b$ must be larger than $\xi_L(N_{TC}+1)/3$ and small M_X/g_{ETC} is favored. Since S parameter is proportional to N_{TC}, the small N_{TC} is favored to be consistent with the experimental constraint for S. Therefore we choose $N_{TC} = 2$. The parameter
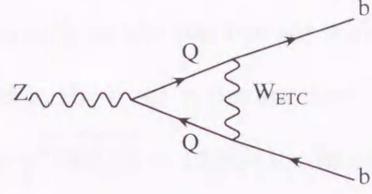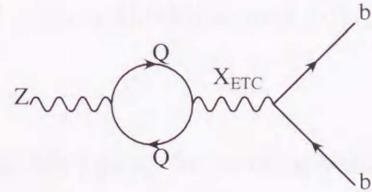

Figure 14: The contribution to the vertex correction according to diagonal ETC gauge interaction.

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ξ_L^t is taken to be unity for simplicity. Comparing the mass of between top quark and bottom quark, ξ_R^t is much larger than ξ_R^b. Hence, we treat ξ_R^t as the parameter which show the breaking of custodial symmetry. In the model with small S parameter[2] (the model without exact custodial symmetry[3]), $F_\pi \sim \sqrt{250^2/3} \sim 144 GeV$. In eq.(6. 11), we put $\lambda^2 = 106$ (See ref.[3]). Here, we define a ratio of the ETC correction to R_b

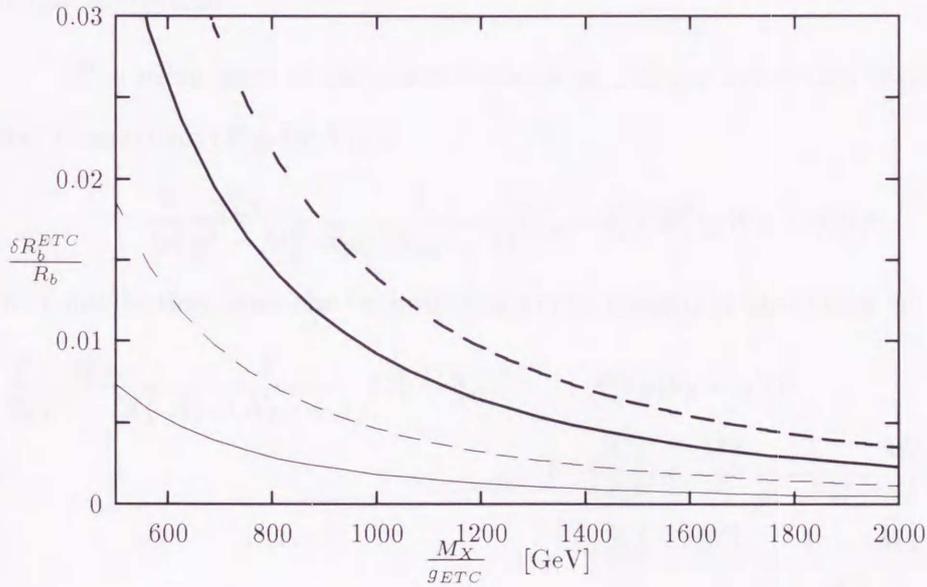$$\frac{\delta R_b^{ETC}}{R_b} \sim (1 - R_b)\frac{2g_L\delta g_L^{ETC}}{g_L^2 + g_R^2}.$$

Figure 15: $\frac{\delta R_b^{ETC}}{R_b}$ as a function of M_X/g_{ETC} for following values for $\xi_R^t - \xi_R^b$. (a) 1.2 with a thinline, (b) 1.5 with a dashed thinline, (c) 2 with a thickline and (d) 2.5 with a dashed thickline.

In Fig.15, the ratio presented as the functions of M_X/g_{ETC} for several values of ξ_R^t. Because the ξ_R^t must be larger than $\xi_L(N_{TC}+1)/3 = 1$, we choose the following values for $\xi_R^t - \xi_R^b \sim \xi_R^t$. (a)1.2, (b)1.5, (c)2 and (d)2.5. If the contribution to R_b from the ETC model explains the experimental data in 1 σ level, the $\delta R_b^{ETC}/R_b$ must larger than about 0.012. Then, in Fig.15, it is shown that the mass of ETC

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gauge boson M_X/g_{ETC} must be smaller than about 700 GeV in case (b), 900 GeV in (c), 1100 GeV in (d).

6-2 The Oblique correction from diagonal ETC interaction

In the same ETC model that we calculated the contributions to Zbb vertex correction, we study the contributions from the diagonal ETC interaction to oblique corrections. In the case $N_{TC} = 2$, there are two diagrams that contribute to the oblique correction.

The main part of the contributions to oblique correction from the diagonal ETC interaction (Fig.16(A)) is

$$\frac{9}{64} \frac{g_{ETC}^2}{p^2 - M_X^2} \frac{1}{N_{TC}(N_{TC}+1)} (\xi_R^t - \xi_R^b)^2 F_\pi^4 (gW_3 - g'B)^2. \qquad (6.9)$$

The contribution from the techni(axial)vectormesons is also given by,

$$\frac{9}{32} \frac{g_{ETC}^2}{p^2 - M_X^2} \frac{1}{N_{TC}(N_{TC}+1)} (\xi_R^t - \xi_R^b)^2 \qquad F_\pi^2 (gW_3 - g'B) \qquad (6.10)$$

$$\times \quad \{ \quad [\frac{M_V^2}{G^2} + \frac{M_V^2}{G} \frac{1}{p^2 - M_V^2} \frac{M_V^2}{G}](gW_3 + g'B)$$

$$- \quad [\frac{M_A^2}{\lambda^2} + \frac{M_A^2}{\lambda} \frac{1}{p^2 - M_A^2} \frac{M_A^2}{\lambda}](gW_3 - g'B)\}$$

$$+ \frac{9}{64} \frac{g_{ETC}^2}{p^2 - M_X^2} \frac{1}{N_{TC}(N_{TC}+1)} (\xi_R^t - \xi_R^b)^2 \quad \{ \quad [\frac{M_V^2}{G^2} + \frac{M_V^2}{G} \frac{1}{p^2 - M_V^2} \frac{M_V^2}{G}](gW_3 + g'B)$$

$$- \quad [\frac{M_A^2}{\lambda^2} + \frac{M_A^2}{\lambda} \frac{1}{p^2 - M_A^2} \frac{M_A^2}{\lambda}](gW_3 - g'B)\}^2.$$

Using eq.(6. 4) and eq.(6. 5), we obtain the contributions to the oblique parameters[1] from Fig.16(A)

$$S^{dETC} = -\frac{9}{2}\pi \frac{(\xi_R^t - \xi_R^b)^2}{N_{TC}(N_{TC}+1)} [\frac{g_{ETC}^2}{M_X^4} F_\pi^4 + 2\frac{g_{ETC}^2}{M_X^2} \frac{1}{\lambda^2} F_\pi^2], \qquad (6.11)$$

$$\alpha T_A^{ETC} = \frac{9}{32} \frac{(\xi_R^t - \xi_R^b)^2}{N_{TC}(N_{TC}+1)} \frac{g_{ETC}^2}{M_X^2} F_\pi^4 \frac{g^2 + g'^2}{M_Z^2}. \qquad (6.12)$$

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From this analysis, the contribution to the S parameter for the diagonal ETC gauge interaction is negative[8].

There is also another two loop contribution to the oblique correction T from the diagonal ETC interaction (Fig.16(B)). Below the ETC scale, the contribution is obtained from the following four-fermi lagrangian:

$$-\frac{1}{4}\frac{g_{ETC}^2}{M_X^2}\frac{1}{N_{TC}(N_{TC}+1)}\quad [\quad \xi_L^{t\,2}(\bar{Q}_L^i\gamma_\mu Q_L^i)^2 \qquad\qquad (6.\ 13)$$

$$+\ \frac{(\xi_R^t+\xi_R^b)^2}{4}(\bar{Q}_R^i\gamma^\mu Q_R^i)^2 + \frac{(\xi_R^t-\xi_R^b)^2}{4}(\bar{Q}_R^i\tau^3\gamma^\mu Q_R^i)^2].$$

After Fierz transformation, the lagrangian becomes to

$$-\frac{1}{4}\frac{g_{ETC}^2}{M_X^2}\frac{1}{N_{TC}(N_{TC}+1)}\quad [\quad \frac{\xi_L^{t\,2}}{2}\sum_{A=0}^{3}(\bar{Q}_L^i\gamma_\mu\tau^A Q_L^i)^2$$

$$+\ \frac{(\xi_R^t+\xi_R^b)^2}{8}\sum_{A=0}^{3}(\bar{Q}_R^i\gamma^\mu\tau^A Q_R^i)^2 \qquad (6.\ 14)$$

$$+\ \frac{(\xi_R^t-\xi_R^b)^2}{8}\{(\bar{Q}_R^i\gamma^\mu Q_R^i)^2 + (\bar{Q}_R^i\gamma^\mu\tau^3 Q_R^i)^2$$

$$-\sum_{a=1}^{2}(\bar{Q}_R^i\gamma_\mu\tau^a Q_R^i)^2\}],$$

where, $\tau^a(a=1,2,3)$ is the Pauli matrix and τ^0 is a unit matrix. Note that the sign in the third term different with the other terms. We replace the currents of technifermion by the Noether current. Then, the contribution to T from Fig.16(B) is given by

$$-\ \frac{3}{32}\frac{g_{ETC}^2}{M_X^2}\frac{F_\pi^4}{N_{TC}^2(N_{TC}+1)}[\xi_L^2+\frac{(\xi_R^t+\xi_R^b)^2}{4}+\frac{(\xi_R^t-\xi_R^b)^2}{4}](gW_3-g'B)^2$$

$$-\ \frac{3}{32}\frac{g_{ETC}^2}{M_X^2}\frac{F_\pi^4}{N_{TC}^2(N_{TC}+1)}[\xi_L^2+\frac{(\xi_R^t+\xi_R^b)^2}{4}-\frac{(\xi_R^t-\xi_R^b)^2}{4}]\sum_{a=1}^{2}(gW^a)^2\ (6.\ 15)$$

---

[8]We only consider the contribution from techniquarks. The S^{ETC} of eq.(6. 11) has negative sign[26] but the contribution is small compared with that to the T parameter. However there may be the large contribution to S from the other fermions.

Hence, only the terms of a factor of $(\xi_R^t - \xi_R^b)^2$ only contribute to T from Fig.16(B).

The contribution to T parameter is

$$\alpha T_B^{ETC} = \frac{3}{32} \frac{(\xi_R^t - \xi_R^b)^2}{N_{TC}^2(N_{TC}+1)} \frac{g_{ETC}^2}{M_X^2} F_\pi^4 \frac{g^2 + g'^2}{M_Z^2}. \qquad (6.\ 16)$$

The total contribution to T from the diagonal ETC interaction is

$$T^{ETC} = T_A^{ETC} + T_B^{ETC}. \qquad (6.\ 17)$$

In Fig.17, we plot the behavior of the contribution to oblique correction T from diagonal ETC interaction (eq.(6. 17)), by choosing the same values for ξ_R^t as those in Fig.15. For the values of M_x/g_{ETC} which satisfy the experimental constraint of R_b, the contribution to S from ETC negligible compared with that from TC (The typical TC contribution to S is $0.1 N_{TC}$ from a one doublet technifermion.). T receives large value. In Fig.17, it is shown that the value of T must be larger than about 0.9 in the cases (b),(c) and (d) for $1\ \sigma$ level of experiment of R_b. This value contradict with the experimental bound of T ($T_{exp} < 0.5$). In the model with small S[2], the situation is worse because T parameter already receives the contribution from the custodial symmetry breaking in technilepton sector. Hence, it is not favored that the T receives the additional contribution from ETC interaction. It is difficult that the discrepancy between the SM and the experiment for the R_b is explained by the contribution of the diagonal ETC gauge interaction, because the contribution to T parameter contradicts with the experimental bound.
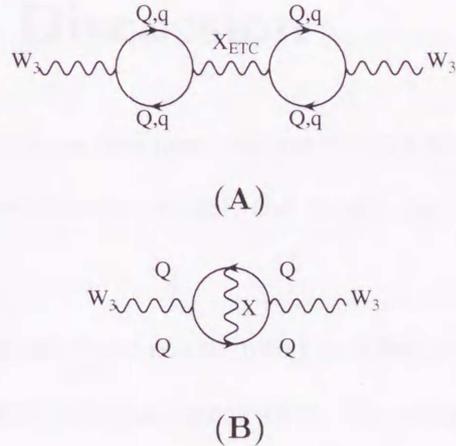

(A)

(B)

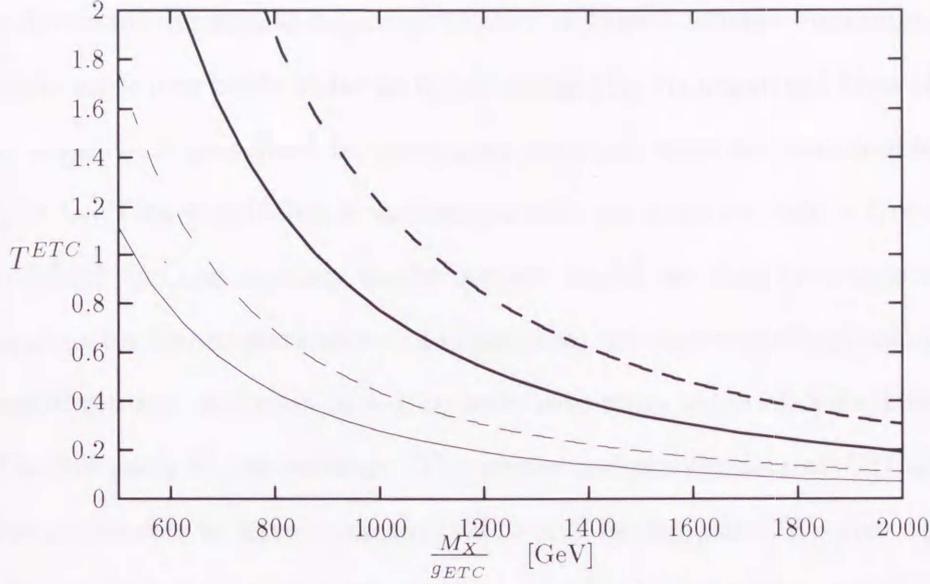Figure 16: The Feynman diagrams for the contribution to the oblique correction from diagonal ETC interaction.

Figure 17: T^{ETC} as a function of M_X/g_{ETC} for following values for $\xi_R^t - \xi_R^b$. (a) 1.2 with a thinline, (b) 1.5 with a dashed thinline, (c) 2 with a thickline and (d) 2.5 with a dashed thickline.

7 Conclusions and Discussion

In this thesis, we discussed the constraints of the oblique and non-oblique corrections for technicolor model. To construct a realistic technicolor model, the model must satisfy the constraints at once.

For the constraints from oblique corrections, we have constructed an effective Lagrangian for a technicolor model without exact custodial symmetry. By using the Lagrangian, we have computed tree level contribution to S and U from spin 1 technimesons. We have shown that in a realistic one-family model, the *techni* ρ_l and the *techni* ω_l mixing can contribute to S parameter with negative sign. The most important term in our effective Lagrangian is the mixing in the kinetic term, $tr F_\rho \hat{\tau}^3 F_\omega$. S is independent of the coefficient of the mass mixing term β_V. We have also studied the condition to have minimum value of S under the vector meson (1^{--}) dominance. We find that the vector meson consists of $N\bar{N}$ component must

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be dominant dynamical degree of freedom to take S to have minimum value ($\alpha_V = 1$) This argument holds as far as hypercharge (Y_{Ll}) is negative. Thus the mechanism for negative S presented in this paper does not work for one doublet model with $Y_L = 0$. This conclusion is consistent with an analysis with a free technifermion model [2]. On the contrary to the present model, we may introduce a small isospin breaking for techniquark sector. In that case, the corresponding parameter of isospin breaking term, α_V must be -1 to have minimum value of S because hypercharge of techniquark Y_{Lq} is positive. The vector mesons consists of $D\bar{D}$ component will play major role to have minimum value of S in that case. We also note that exotic lefthanded charged vector mesons are naturally introduced in our framework. They contribute to U due to the mixing with $W^{\pm}$. U can be both negative and positive depending on the parameters.

There is an important distinction between our computation and that with a free technifermion model even if the parts of the expression of S, T and U look similar to each other. The distinction is that our computation certainly incorporates the nonperturbative diagrams which are not taken into account of in the free technifermion one loop diagrams. Once the parameters of our Lagrangian are determined either experimentally or theoretically, the results presented here would be more reliable. There are many things to be done in this direction. The origin of the isospin breaking must be studied. Also we need to relate the parameters of the effective Lagrangian to more fundamental interaction for example, by modeling technicolor by Nambu Jona-Lasinio model [22]. The difference between our computation and that with a free technifermion model will also be clarified by this kind of study.

We have also described the vertex correction of $Z b\bar{b}$, $Z\tau\tau$ and $W\nu\tau$ in the one family extended technicolor model without exact custodial symmetry in the case that N_{TC} is large. The values of the corrections can not be determined precisely, since the corrections include a few unknown parameters ξs. The corrections which are obtained in this work is much larger than those in one doublet model. If we suitably choose each unknown parameter ξ, we will be able to obtain the vertex corrections which satisfy the constraint from the experiment. When $\xi = 1$, the vertex corrections are so large that this model is ruled out. Then, in order to reduce the values in this case, we may have to consider the other ETC model or walking technicolor. However, we find that if the the difference between the vertex corrections for $Z\tau\tau$ and $W\tau\nu$ is measured in experiment, it is the evidence of the isospin breaking in the technilepton sector. It comes from not only the difference between the decay constants but a large contribution to $Z\tau\tau$ vertex due to the technivectormesons. The contributions of the vectormesons for $Z\tau\tau$ reduce the value which takes account of only technipion contribution (the first term of the eq.(5. 8)). The vertex correction for the $Z\tau\tau$ can be negative due to this effect. While, the contribution for $W\tau\nu$ is not changed so large. Hence, the difference between the corrections for the $Z\tau\tau$ and the $W\tau\nu$ appear. We expect that in the near future the precession measurements (in LEP200, JLC etc.) of the vertex corrections of $W\tau\nu$ will determine whether the isospin of technilepton sector breaks or not.

The contribution to the vertex correction of $Z b b$ from the diagonal ETC gauge interaction become large with positive sign when the $\xi_R^t - \xi_R^b$ is larger than $\xi_L^t (N_{TC} + 1)/3$. However, because the such large $\xi_R^t - \xi_R^b$ breaks the custodial symmetry significantly, T must receive the contribution from the diagonal ETC interaction. In this letter, we consider the case that N_{TC} is small value[7] because of keeping S

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small value[2]. Let us mention possible corrections to our calculation. There are two types of corrections. One is perturbative technicolor correction to four-fermi operators (eq.(6. 3) and eq.(6. 14)), and the other is matrix element of four-fermi operators. We neglected the perturbative technicolor corrections and assumed that the coefficient of four-fermi operators will not be changed much from the lowest tree level ETC gauge boson exchanged diagrams. The validity of these assumption must be checked in the further investigate. By taking account of the uncertainties of our calculation, we point out that the proposal of Ref.[7] is on very dangerous ground rather than conclude that our calculation in this letter completely rule out the proposal. If our calculation is not altered after incorporate the corrections, we may concluded that It is difficult that the $\xi_R^t g_{ETC}/M_X$ becomes large enough to explain the discrepancy for R_b , unless the other mechanism suppress the T parameter in this model.

Acknowledgement

Appendix

In this appendix, we provide some useful formulas needed to derive the results given in the text.

A Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_S &= F_6^2 tr(\alpha_{6\perp})^2 + F_L^2 tr(\alpha_{L\perp})^2 + F_2^2 tr(\alpha_{2\perp})^2 + F_8^2 tr(\alpha_{8\perp})^2 \\
&+ \beta_\perp tr(\alpha_{8\perp}\hat{\tau}^3\alpha_{2\perp}),
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
\mathcal{L}_V &= \frac{1}{2}tr F_{V_6}^{\mu\nu} F_{V_6\,\mu\nu} + \frac{1}{2}tr F_{V_2}^{\mu\nu} F_{V_2\,\mu\nu} + \frac{1}{2}tr F_{V_{\omega 6}}^{\mu\nu} F_{V_{\omega 6}\,\mu\nu} + \frac{1}{2}tr F_{V_{\omega 2}}^{\mu\nu} F_{V_{\omega 2}\,\mu\nu} \\
&- M_{V_6}^2 tr(V_{6\mu} - \frac{i}{G_6}\alpha_{6\|\mu})^2 - M_{V_{\omega 6}}^2 tr(V_{\omega 6\mu} - \frac{i}{G_{\omega 6}}\alpha_{\omega 6\|\mu})^2 \\
&- M_{V_2}^2 tr(V_{2\mu} - \frac{i}{G_2}\alpha_{2\|\mu})^2 - M_{V_{\omega 2}}^2 tr(V_{\omega 2\mu} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu})^2 \\
&+ \alpha_V tr F_{V_2}^{\mu\nu}\hat{\tau}^3 F_{V_{\omega 2}\,\mu\nu} \\
&- \beta_V tr(V_{2\mu} - \frac{i}{G_2}\alpha_{2\|\mu})\hat{\tau}^3(V_{\omega 2\mu} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu}),
\end{aligned} \tag{A.2}
$$

$$
\begin{aligned}
\mathcal{L}_A &= \frac{1}{2}tr F_{A_6}^{\mu\nu} F_{A_6\,\mu\nu} + \frac{1}{2}tr F_{A_L}^{\mu\nu} F_{A_L\,\mu\nu} + \frac{1}{2}tr F_{A_2}^{\mu\nu} F_{A_2\,\mu\nu} + \frac{1}{2}tr F_{A_8}^{\mu\nu} F_{A_8\,\mu\nu} \\
&- M_{A_6}^2 tr(A_{6\mu} - \frac{i}{\lambda_6}\alpha_{6\perp\mu})^2 - M_{A_8}^2 tr(A_{8\mu} - \frac{i}{\lambda_8}\alpha_{8\perp\mu})^2 \\
&- M_{A_L}^2 tr(A_{L\mu} - \frac{i}{\lambda_L}\alpha_{L\perp\mu})^2 - M_{A_2}^2 tr(A_{2\mu} - \frac{i}{\lambda_2}\alpha_{2\perp\mu})^2 \\
&- \alpha_A \frac{2}{\sqrt{3}} tr F_{A_2}^{\mu\nu}\hat{\tau}^3 F_{A_8\,\mu\nu} \\
&+ \beta_A \frac{2}{\sqrt{3}} tr(A_{2\mu} - \frac{i(1+\delta)}{\lambda_2}\alpha_{2\perp\mu})\hat{\tau}^3(A_{8\mu} - \frac{i(1+\delta')}{\lambda_8}\alpha_{8\perp\mu}) \\
&+ \frac{1}{4}\frac{\beta_A^2 M_{A2}^2\delta^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2^2} tr(\alpha_{2\perp\mu})^2 \\
&+ \frac{1}{4}\frac{\beta_A^2 M_{A8}^2\delta'^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_8^2} tr(\alpha_{8\perp\mu})^2
\end{aligned}
$$

$$+\ \frac{2}{\sqrt{3}}\frac{\delta\delta'\beta_A M_{A2}^2 M_{A8}^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2\lambda_8}tr(\alpha_{2\perp\mu}\hat{\tau}^3\alpha_{8\perp\mu}).\qquad(\text{A.3})$$

B   Decomposition into fields' components

In terms of fields' components, $\alpha_\perp$s and $\alpha_\parallel$s are given by:

$$\alpha_{6\perp\mu} = -\frac{1}{2\sqrt{3}F_6}\{\sum_{a=1}^{2}\tau^a(\partial_\mu P^a + \frac{\sqrt{3}F_6}{2}gW_\mu^a)$$

$$+\tau^3(\partial_\mu P^3 + \frac{\sqrt{3}F_6}{2}(gW_\mu^3 - g'B_\mu))\}\otimes I_3,\qquad(\text{B.1})$$

$$\alpha_{L\perp\mu} = -\frac{1}{2F_L}\sum_{a=1}^{2}\tau^a\{\partial_\mu\hat{\Pi}^a + \frac{F_L}{2}gW_\mu^a\},\quad \hat{\Pi}^a \equiv \frac{1}{2}\Pi^a,\qquad(\text{B.2})$$

$$\alpha_{2\perp\mu} = -\frac{1}{2F_2}\tau^3\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\},\qquad(\text{B.3})$$

$$\alpha_{8\perp\mu} = -\frac{1}{4\sqrt{3}F_8}\begin{pmatrix} I_6 & \\ & -3I_2\end{pmatrix}\partial_\mu\theta_8,\qquad(\text{B.4})$$

$$\qquad(\text{B.5})$$

$$A_{6\mu} - \frac{i}{\lambda_6}\alpha_{6\perp\mu} = \frac{i}{2\sqrt{3}}\tau^3\otimes I_3[a_{6\mu}^3 + \frac{1}{\lambda_6 F_6}\{\partial_\mu P^3 + \frac{\sqrt{3}F_6}{2}(gW_\mu^3 - g'B_\mu)\}$$

$$+\ \frac{i}{2\sqrt{3}}\sum_{a=1}^{2}\tau^a\otimes I_3[a_{6\mu}^a + \frac{1}{\lambda_6 F_6}\{\partial_\mu P^a + \frac{\sqrt{3}F_6}{2}gW_\mu^a\}],\ (\text{B.6})$$

$$A_{L\mu} - \frac{i}{\lambda_L}\alpha_{L\perp\mu} = \frac{i}{2}\sum_{a=1}^{2}\tau^a[a_{L\mu} + \frac{1}{\lambda_L F_L}\{\partial_\mu\hat{\Pi}^a + \frac{F_L}{2}gW_\mu^a\}],\qquad(\text{B.7})$$

$$A_{2\mu} - \frac{i}{\lambda_2}\alpha_{2\perp\mu} = \frac{i}{2}\tau^3[a_{2\mu} + \frac{1}{\lambda_2 F_6}\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_{3\mu} - g'B_\mu)\}],\qquad(\text{B.8})$$

$$A_{8\mu} - \frac{i}{\lambda_8}\alpha_{8\perp\mu} = \frac{i}{4\sqrt{3}}\begin{pmatrix} I_6 & \\ & -3I_2\end{pmatrix}[a_{8\mu} + \frac{1}{\lambda_8 F_8}\partial_\mu\theta_8],\qquad(\text{B.9})$$

$$V_{6\mu} - \frac{i}{G_6}\alpha_{6\parallel\mu} = \frac{i}{2\sqrt{3}}\tau^3\otimes I_3[\rho_{6\mu}^3 - \frac{\sqrt{3}}{2G_6}(gW_{3\mu} + g'B_\mu)]$$

$$-\ \frac{i}{2\sqrt{3}}\sum_{a=1}^{2}\tau^a\otimes I_3[\rho_{6\mu}^a - \frac{\sqrt{3}}{2G_6}(gW_\mu^a)],\qquad(\text{B.10})$$

$$V_{\omega6\mu} - \frac{i}{G_{\omega6}}\alpha_{\omega6\parallel\mu} = \frac{i}{2\sqrt{3}}I_6[\omega_{6\mu} - \frac{\sqrt{3}}{2G_{\omega6}}(2Y_{Ll}g'B_\mu)],\qquad(\text{B.11})$$

$$V_{2\mu} - \frac{i}{G_2}\alpha_{2\|\mu} = \frac{i}{2}\tau^3[\rho_{2\mu} - \frac{1}{2G_2}(gW_{3\mu} + g'B_\mu)], \qquad (\text{B.12})$$

$$V_{\omega 2\mu} - \frac{i}{G_{\omega 2}}\alpha_{\omega 2\|\mu} = \frac{i}{2}I_2[\omega_{2\mu} - \frac{1}{2G_{\omega 2}}(2Y_{Ll}g'B_\mu)]. \qquad (\text{B.13})$$

By substituting these expressions, we obtain :

$$
\begin{aligned}
\mathcal{L}_S = \quad & \frac{1}{2}\{(\partial_\mu P^3)^2 + (\partial_\mu \Pi^3)^2 + (\partial_\mu \theta_8)^2\} + \frac{1}{2}\sum_{a=1}^{2}\{(\partial_\mu P^a)^2 + (\partial_\mu \hat{\Pi}^a)^2\} \\
+ \quad & \frac{1}{2}\sum_{a=1}^{2}(F_L\partial_\mu\hat{\Pi}^a + \sqrt{3}F_6\partial_\mu P^a)gW_\mu^a + \frac{1}{2}(F_2\partial_\mu\Pi^3 + \sqrt{3}F_6\partial_\mu P^3)(gW_\mu^3 - g'B_\mu) \\
+ \quad & \frac{1}{8}(3F_6^2 + F_2^2)(gW_\mu^3 - g'B_\mu)^2 + \frac{1}{8}(3F_6^2 + F_L^2)(gW_\mu^a)^2 \\
- \quad & \frac{\sqrt{3}\beta_\perp}{4F_8F_2}\partial_\mu\theta_8\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\} + \mathcal{L}'_{br}, \qquad (\text{B.14})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_V = \quad - & \frac{1}{4}(\partial_{[\mu}\rho_{6\nu]})^2 - \frac{1}{4}(\partial_{[\mu}\omega_{6\nu]})^2 \\
- \quad & \frac{1}{4}(\partial_{[\mu}\rho_{2\nu]})^2 - \frac{1}{4}(\partial_{[\mu}\omega_{2\nu]})^2 - \frac{\alpha_V}{2}(\partial_{[\mu}\rho_{2\nu]})(\partial_{[\mu}\omega_{2\nu]}) \\
+ \quad & \frac{1}{2}M_{V6}^2\{\rho_{6\mu}^3 - \frac{\sqrt{3}}{2G_6}(gW_\mu^3 + g'B_\mu)\}^2 + \frac{1}{2}M_{V6}^2\{\rho_{6\mu}^a - \frac{\sqrt{3}}{2G_6}gW_\mu^a\}^2 \\
+ \quad & \frac{1}{2}M_{\omega 6}^2\{\omega_{6\mu} - \frac{\sqrt{3}}{2G_{\omega 6}}2g'Y_{Lq}B_\mu\}^2 \\
+ \quad & \frac{1}{2}M_{V2}^2\{\rho_{2\mu} - \frac{1}{2G_2}(gW_\mu^3 + g'B_\mu)\}^2 + \frac{1}{2}M_{\omega 2}^2\{\omega_{2\mu} - \frac{1}{2G_{\omega 2}}2g'Y_{Ll}B_\mu\}^2 \\
+ \quad & \frac{\beta_V}{2}\{\rho_{2\mu} - \frac{1}{2G_2}(gW_\mu^3 + g'B_\mu)\}\{\omega_{2\mu} - \frac{1}{2G_{\omega 2}}2g'Y_{Ll}B_\mu\}, \qquad (\text{B.15})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_A = \quad - & \frac{1}{4}(\partial_{[\mu}a_{6\nu]}^a)^2 - \frac{1}{4}(\partial_{[\mu}a_{8\nu]})^2 \\
- \quad & \frac{1}{4}(\partial_{[\mu}a_{2\nu]})^2 - \frac{\alpha_A}{2}(\partial_{[\mu}a_{8\nu]})(\partial_{[\mu}a_{2\nu]}) \\
- \quad & \frac{1}{4}\sum_{a=1}^{2}(\partial_{[\mu}a_{L\nu]}^a)^2 \\
+ \quad & \frac{1}{2}M_{A6}^2\sum_{a=1}^{2}[a_{6\mu}^a + \frac{1}{\lambda_6 F_6}\{\partial_\mu P^a + \frac{\sqrt{3}F_6}{2}gW_\mu^a\}]^2 \\
+ \quad & \frac{1}{2}M_{A6}^2[a_{6\mu}^3 + \frac{1}{\lambda_6 F_6}\{\partial_\mu P^3 + \frac{\sqrt{3}F_6}{2}(gW_\mu^3 - g'B_\mu)\}]^2
\end{aligned}
$$

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$$+ \quad \frac{1}{2}M_{A8}^2[a_{8\mu} + \frac{1}{\lambda_8 F_8}\partial_\mu\theta_8]^2$$

$$+ \quad \frac{1}{2}M_{A2}^2[a_{2\mu}^3 + \frac{1}{\lambda_2 F_2}\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}]^2$$

$$+ \quad \frac{\beta_A}{2}[a_{2\mu}^3 + \frac{1+\delta}{\lambda_2 F_2}\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}][a_{8\mu} + \frac{1+\delta'}{\lambda_8 F_8}\partial_\mu\theta_8]$$

$$+ \quad \frac{1}{2}M_{AL}^2\sum_{a=1}^{2}[a_{L\mu}^a + \frac{1}{\lambda_L F_L}\{\partial_\mu\hat{\Pi}^a + \frac{F_L}{2}gW_\mu^a\}]^2$$

$$+ \quad \frac{1}{8}\frac{\beta_A^2 M_{A2}^2\delta^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2^2 F_2^2}\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}^2$$

$$+ \quad \frac{1}{8}\frac{\beta_A^2 M_{A8}^2\delta'^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_8^2 F_8^2}\partial_\mu\theta_8^2$$

$$- \quad \frac{1}{2}\frac{\delta\delta'\beta_A M_{A2}^2 M_{A8}^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{F_2 F_8\lambda_2\lambda_8}\partial_\mu\theta_8\{\partial_\mu\Pi^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}, \qquad (\text{ B.16 })$$

We fix the gauge into unitary gauge. This corresponds to the following replacement.

$$(gW_\mu^3 - g'B_\mu) \quad \rightarrow \quad (gW_\mu^3 - g'B_\mu) - \frac{2}{3F_6^2 + F_2^2}(\sqrt{3}F_6\partial_\mu P^3 + F_2\partial_\mu\Pi^3), \qquad (\text{ B.17 })$$

$$gW_\mu^a \quad \rightarrow \quad gW_\mu^a - \frac{2}{3F_6^2 + F_L^2}(\sqrt{3}F_6\partial_\mu P^a + F_2\partial_\mu\hat{\Pi}^a), \qquad (\text{ B.18 })$$

We also redefine the axial vector and left-handed vector mesons.

$$a_{6\mu}^a \quad \rightarrow \quad a_{6\mu}^a - \frac{1}{\lambda_6 F_6}\partial_\mu P^a, \qquad (\text{ B.19 })$$

$$a_{6\mu}^3 \quad \rightarrow \quad a_{6\mu}^3 - \frac{1}{\lambda_6 F_6}\partial_\mu P^3, \qquad (\text{ B.20 })$$

$$a_{8\mu} \quad \rightarrow \quad a_{8\mu} - \frac{1}{\lambda_8 F_8}\partial_\mu\theta_8, \qquad (\text{ B.21 })$$

$$a_{2\mu}^3 \quad \rightarrow \quad a_{2\mu}^3 - \frac{1}{\lambda_2 F_2}\partial_\mu\Pi^3, \qquad (\text{ B.22 })$$

$$a_{L\mu}^a \quad \rightarrow \quad a_{L\mu}^a - \frac{1}{\lambda_L F_L}\partial_\mu\hat{\Pi}^a. \qquad (\text{ B.23 })$$

Then, $\mathcal{L}_S$ and $\mathcal{L}_A$ are written in terms of physical degrees of freedom.

$$\mathcal{L}_S = \frac{1}{2}\{(\partial_\mu\tilde{\Pi}^3)^2 + (\partial_\mu\tilde{\Pi}^a)^2 + (\partial_\mu\theta_8)^2\}$$

$$- \frac{\sqrt{3}\beta_\perp}{4F_8 F_2}\partial\theta_8\{\cos\chi_3\partial_\mu\tilde{\Pi}^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}$$

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$$
+ \frac{1}{8}(3F_6^2 + F_2^2)(gW_\mu^3 - g'B_\mu)^2 + \frac{1}{8}(3F_6^2 + F_L^2)(gW_\mu^a)^2
$$
$$
+ \mathcal{L}'_{br}, \tag{B.24}
$$

$$
\begin{aligned}
\mathcal{L}_A = & -\frac{1}{4}(\partial_{[\mu,}a_{6\nu]}^a)^2 - \frac{1}{4}(\partial_{[\mu,}a_{8\nu]})^2 \\
& -\frac{1}{4}(\partial_{[\mu,}a_{2\nu]})^2 - \frac{\alpha_A}{2}(\partial_{[\mu,}a_{8\nu]})(\partial_{[\mu,}a_{2\nu]}) \\
& -\frac{1}{4}\sum_{a=1}^{2}(\partial_{[\mu,}a_{L\nu]}^a)^2 \\
& +\frac{1}{2}M_{A6}^2\sum_{a=1}^{2}[a_{6\mu}^a + \frac{\sqrt{3}}{2\lambda_6}gW_\mu^a]^2 \\
& +\frac{1}{2}M_{A6}^2[a_{6\mu}^3 + \frac{\sqrt{3}}{2\lambda_6}(gW_\mu^3 - g'B_\mu)]^2 \\
& +\frac{1}{2}M_{A8}^2[a_{8\mu}]^2 \\
& +\frac{1}{2}M_{A2}^2[a_{2\mu} + \frac{1}{2\lambda_2}(gW_\mu^3 - g'B_\mu)]^2 \\
& +\frac{\beta_A}{2}[a_{2\mu} + \frac{1}{2\lambda_2}(gW_\mu^3 - g'B_\mu)][a_{8\mu}] \\
& +\frac{1}{2}M_{AL}^2\sum_{a=1}^{2}[a_{L\mu}^a + \frac{1}{2\lambda_L}gW_\mu^a]^2 \\
& +\frac{\beta_A\delta'}{2\lambda_8 F_8}a_2^3\partial_\mu\theta_8 + \frac{\beta_A\delta}{2\lambda_2 F_2}a_8\{cos\chi_3\partial_\mu\tilde{\Pi}^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\} \\
& +\frac{1}{8}\frac{\beta_A^2 M_{A2}^2\delta^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2^2 F_2^2}\{cos\chi_3\partial_\mu\tilde{\Pi}^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\}^2 \\
& +\frac{1}{8}\frac{\beta_A^2 M_{A8}^2\delta'^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_8^2 F_8^2}(\partial_\mu\theta_8)^2 \tag{B.25} \\
& -\frac{1}{8}\frac{\delta\delta'\beta_A^3}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{F_2 F_8\lambda_2\lambda_8}\partial_\mu\theta_8\{cos\chi_3\partial_\mu\tilde{\Pi}^3 + \frac{F_2}{2}(gW_\mu^3 - g'B_\mu)\},
\end{aligned}
$$

where,

$$
\begin{pmatrix} \tilde{\Pi}^a \\ \tilde{P}^a \end{pmatrix} = \begin{pmatrix} cos\chi_L & -sin\chi_L \\ sin\chi_L & cos\chi_L \end{pmatrix} \begin{pmatrix} \hat{\Pi}^a \\ \hat{P}^a \end{pmatrix}, \tag{B.26}
$$

$$
\begin{pmatrix} \tilde{\Pi}^3 \\ \tilde{P}^3 \end{pmatrix} = \begin{pmatrix} cos\chi_3 & -sin\chi_3 \\ sin\chi_3 & cos\chi_3 \end{pmatrix} \begin{pmatrix} \Pi^3 \\ P^3 \end{pmatrix}, \tag{B.27}
$$

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$$cos\chi_L = \frac{\sqrt{3}F_6}{\sqrt{3F_6^2 + F_L^2}}, \quad sin\chi_L = \frac{F_L}{\sqrt{3F_6^2 + F_L^2}}, \qquad (\text{ B.28 })$$

$$cos\chi_3 = \frac{\sqrt{3}F_6}{\sqrt{3F_6^2 + F_2^2}}, \quad sin\chi_3 = \frac{F_2}{\sqrt{3F_6^2 + F_2^2}}. \qquad (\text{ B.29 })$$

C Pseudo Nambu Goldstone boson sector

$$
\begin{aligned}
\mathcal{L}_S &= \frac{1}{2}\{(\partial_\mu \tilde{\Pi}^3)^2 + (\partial_\mu \tilde{\Pi}^a)^2 + (\partial_\mu \theta_8)^2\} \\
&+ \frac{1}{8}(3F_6^2 + F_2^2)(gW_\mu^3 - g'B_\mu)^2 + \frac{1}{8}(3F_6^2 + F_L^2)(gW_\mu^a)^2 \\
&- \frac{\sqrt{3}\beta_\perp}{4F_8 F_2}\partial\theta_8\{\cos\chi_3 \partial_\mu \tilde{\Pi}^3 + \frac{1}{2}(gW_\mu^3 - g'B_\mu)\} + \mathcal{L}'_{br}. \qquad (\text{ C.1 })
\end{aligned}
$$

Now we are ready for giving the explicit form for $\mathcal{L}'_{br}$. Because $\tilde{\Pi}^3$, $\tilde{\Pi}^a$ and θ_8 are NGBs associated with broken global symmetry, we can introduce the following mass terms as $\mathcal{L}'_{br}$.

$$\mathcal{L}'_{br} = -\frac{1}{2}\begin{pmatrix} \tilde{\Pi}_3 & \theta_8 \end{pmatrix}\begin{pmatrix} m_3^2 & \frac{\beta_S}{2} \\ \frac{\beta_S}{2} & m_8^2 \end{pmatrix}\begin{pmatrix} \tilde{\Pi}_3 \\ \theta_8 \end{pmatrix} - \frac{1}{2}m_a^2 \tilde{\Pi}_a^2 \qquad (\text{ C.2 })$$

The mass terms break the global symmetry without loss of $SU(2) \otimes U(1)$ gauge invariance and these NGBs become PNGBs.

D $O(p^2)$ terms in axial vector and left-handed vector sector

In this appendix, we show how to determine the $O(p^2)$ terms which consist of NGBs and PNGBs in this sector. As explained in the text, we add the $O(p^2)$ terms so that

the incorporation of axial and left-handed vector mesons does not change the decay constants of NGBs and PNGBs. By doing so, $\rho\,(T)$ parameter depends only on the parameters in $\mathcal{L}_s$. Though it is just the matter of the definition of the parameters of $O(p^2)$ terms in $\mathcal{L}_s$, our choice is convenient because the the parameters in $\mathcal{L}_s$ are directly related to physical quantities such as decay constants. Further $\rho\,(T)$ parameter is independent of the parameters in $\mathcal{L}_A$ with the procedure adopted here. Let us discuss $0(p^2)$ terms which consist of $\alpha_{6\perp}$ and $\alpha_{L\perp}$ first. These terms are related to A_6 and A_L. The equations of motion of A_6 and A_L up to $O(p^2)$ are,

$$
\begin{aligned}
A_6 &= \frac{i}{\lambda_6}\alpha_{6\perp}, \\
A_L &= \frac{i}{\lambda_L}\alpha_{L\perp}.
\end{aligned}
$$

(D.1)

By substituting these into (4. 45), we do not have $O(p^2)$ terms of NGBs and PNGBs. Therefore the $O(p^2)$ terms which are already present in (4. 45) is enough. $O(p^2)$ terms which consist of $\alpha_{2\perp}$ and $\alpha_{8\perp}$ have more complicated coefficients as shown in (44). We have determined them in the following way. Let us focus on a part of $\mathcal{L}_A$

$$
\begin{aligned}
\mathcal{L}_0 = &\quad \frac{1}{2}M_{A_8}^2(a_{8\mu} - \frac{1}{\lambda_8}\hat{\alpha}_{8\perp\mu})^2 + \frac{1}{2}M_{A_2}^2(a_{2\mu} - \frac{1}{\lambda_2}\hat{\alpha}_{2\perp\mu})^2 \\
&+ \beta_A\frac{1}{2}(a_{2\mu} - \frac{(1+\delta)}{\lambda_2}\hat{\alpha}_{2\perp\mu})(a_{8\mu} - \frac{(1+\delta')}{\lambda_8}\hat{\alpha}_{8\perp\mu}) \\[2mm]
= &\quad \frac{1}{2}\left((a_2 - \frac{1}{\lambda_2}\hat{\alpha}_{2\perp}) \quad (a_8 - \frac{1}{\lambda_8}\hat{\alpha}_{8\perp})\right)\begin{pmatrix} M_{A2}^2 & \frac{\beta_A}{2} \\ \frac{\beta_A}{2} & M_{A8}^2 \end{pmatrix}\begin{pmatrix} (a_2 - \frac{1}{\lambda_2}\hat{\alpha}_{2\perp}) \\ (a_8 - \frac{1}{\lambda_8}\hat{\alpha}_{8\perp}) \end{pmatrix} \\
&+ \frac{1}{2}\left((a_2 - \frac{1}{\lambda_2}\hat{\alpha}_{2\perp}) \quad (a_8 - \frac{1}{\lambda_8}\hat{\alpha}_{8\perp})\right)\begin{pmatrix} 0 & \beta_A\delta' \\ \beta_A\delta & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{\lambda_2}\hat{\alpha}_{2\perp} \\ -\frac{1}{\lambda_8}\hat{\alpha}_{8\perp} \end{pmatrix} \\
&+ \frac{\beta_A}{2}\frac{\delta\delta'}{\lambda_2\lambda_8}\hat{\alpha}_{2\perp}\hat{\alpha}_{8\perp},
\end{aligned}
$$

(D.2)

where we have used the following notation;

$$
A_{2\mu} = i\frac{1}{2}\begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix}a_{2\mu},
$$

(D.3)

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$$A_{8\mu} = i\frac{1}{4\sqrt{3}}\begin{pmatrix} I_6 & \\ & -3I_2 \end{pmatrix} a_{8\mu}, \qquad (\text{D.4})$$

$$\alpha_{2\perp\mu} = \frac{1}{2}\begin{pmatrix} 0 & \\ & \tau^3 \end{pmatrix}\hat{\alpha}_{2\perp\mu} \qquad (\text{D.5})$$

$$\alpha_{8\perp\mu} = \frac{1}{4\sqrt{3}}\begin{pmatrix} I_6 & \\ & -3I_2 \end{pmatrix}\hat{\alpha}_{8\perp\mu} \qquad (\text{D.6})$$

The equations of motion for a_2 and a_8 up to $O(p^2)$ are:

$$\begin{pmatrix} a_2 - \frac{1}{\lambda_2}\hat{\alpha}_{2\perp} \\ a_8 - \frac{1}{\lambda_8}\hat{\alpha}_{8\perp} \end{pmatrix} = \frac{1}{2}\frac{1}{M_{A2}^2 M_{A8}^2 - \frac{\beta_A^2}{4}}\begin{pmatrix} M_{A8}^2 & -\frac{\beta_A}{2} \\ -\frac{\beta_A}{2} & M_{A2}^2 \end{pmatrix}\begin{pmatrix} 0 & \beta_A\delta' \\ \beta_A\delta & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\lambda_2}\hat{\alpha}_{2\perp} \\ \frac{1}{\lambda_8}\hat{\alpha}_{8\perp} \end{pmatrix}(\text{D.7})$$

By substituting this into $\mathcal{L}_0$, we obtain;

$$\begin{aligned}
\mathcal{L}_c = \quad & -\frac{1}{4}\frac{\beta_A^2 M_{A2}^2 \delta^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2^2}tr(\alpha_{2\perp\mu})^2 \\
+ \quad & -\frac{1}{4}\frac{\beta_A^2 M_{A8}^2 \delta'^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_8^2}tr(\alpha_{8\perp\mu})^2 \\
+ \quad & -\frac{2}{\sqrt{3}}\frac{\delta\delta'\beta_A M_{A2}^2 M_{A8}^2}{M_{A2}^2 M_{A8}^2 - \frac{1}{4}\beta_A^2}\frac{1}{\lambda_2\lambda_8}tr(\alpha_{2\perp\mu}\tau^3\alpha_{8\perp\mu}) \qquad (\text{D.8})
\end{aligned}$$

By subtracting $\mathcal{L}_c$ from $\mathcal{L}_0$, we obtain $O(p^2)$terms which consist of $\alpha_{2\perp}$ and $\alpha_{8\perp}$ in (4. 45). These are the desired counter terms which kill the effect of axial vector mesons and left-handed vector mesons on ρ (T) parameter.

E   S in scaling-up QCD technicolor model

For the completeness, we compute S_{theory} of scaling-up QCD technicolor model which is quoted in the introduction. The scaling-up QCD technicolor model has $SU(2)_L \otimes SU(2)_R$ global symmetry and $N_{TC} = N_c = 3$. Therefore we only need to study the SU(2) subsector of one-family model. S in this model is given by;

$$S = 4\pi[\frac{1}{G_V^2} - \frac{1}{\lambda_A^2}]. \qquad (\text{E.1})$$

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The G_V and λ_A are defined in the same way as G_6 and λ_6. G_V and λ_A are determined by $\rho \to \pi\pi$ and $a_1 \to \gamma\pi$ decays .

$$\Gamma_{\rho\pi\pi} = \frac{1}{48\pi}m_\rho(\frac{m_\rho^2}{2G_V f_{QCD}^2})^2(1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}}, \qquad (\text{ E.2 })$$

$$\Gamma_{a\gamma\pi} = \frac{\alpha}{24 f_{QCD}^2 G_A^2}\frac{(m_a^2 - m_\pi^2)^3}{m_a^3}, \qquad (\text{ E.3 })$$

where f_{QCD} is the pion decay constant. By using the following values,

$$f_{QCD} = 93 MeV, \quad m_\pi = 140 MeV, \quad m_\rho = 768 MeV, \quad m_a = 1260 MeV,$$

$$\Gamma_{\rho\pi\pi} = 152 MeV, \quad \Gamma_{a\gamma\pi} = 0.64 MeV,$$

we obtain;

$$G_V^2 = 31.5, \quad G_A^2 = 106. \qquad (\text{ E.4 })$$

This leads to the following estimation of S_{theory} for the scaling-up QCD technicolor model which is quoted in the text.

$$S_{theory} = (0.40 - 0.12) = 0.28 \qquad (\text{ E.5 })$$

0.40 comes from the contribution of $I = 1$ vector meson while 0.12 comes from $I = 1$ axial vector meson. Because they are dimensionless quantities, the scaling relations between the parameters in QCD and that of the technicolor are given by,

$$G_6 = G_V, \quad \lambda_6 = \lambda_A. \qquad (\text{ E.6 })$$

This equalities hold if the underlying dynamics of the techniquark sector is the same as that of QCD.

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