Stability of the Friedmann Universe in the Poincaré Gauge Theory1)

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Abstract

In General Relativistic cosmology, it is well known that quantum effects due to vacuum polarization make the Friedmann universe unstable4).

Here, in the Poincaré Gauge Theory, solutions of the Friedmann universes stable against these quantum effects are obtained under linear approximation for the cases k=0,±1 of the radiation dominant universe(RDU), the matter dominant universe(MDU) and the de Sitter universe(dSU). These solutions are small oscillations around the standard Big Bang solution of General Relativity and exist for each era of RDU, MDU and dSU, respectively, if we choose the parameters of the Poincaré Gauge Theory, the total entropy of the universe and others properly.

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§ 1. Introduction

It is well known that quantum effects of matter fields play important roles in General Relativistic(GR) cosmologies of the early universe such as the inflationary universe scenarios[5)] (especially, the origin of fluctuation[6)] and the reheating problem[7)]), the damping of anisotropies[8)], the matter generation by particle productions[9)] and so on.

Among them, there is the problem of instability of the universe[4)]. That is, if we add quantum effects due to vacuum polarization of quantized matter fields[10)] to the right hand side (RHS) of the Einstein equation (semi-classical picture of GR)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left(T^{cl}_{\mu\nu} + \langle 0|T_{\mu\nu}|0\rangle\right), \qquad (1.1)$$

then the Friedmann universe[4)] and the Minkowski spacetime[11)] become unstable. Here, G is the gravitational constant and the first and second terms in RHS are the classical and quantum parts of the energy-momentum tensor, respectively. This instability causes serious difficulties in cosmology as will be reviewed in §3.

The exact theory of quantum gravity[12)] has possibility to avoid this instability. But at present such a theory has not yet been completed;moreover, it is not clear whether such a theory indeed solves the stability problem. In this paper an alternative way is investigated;that is, we treat the stability problem within the semi-classical picture of the Poincaré Gauge

Theorty (PGT).

In this picture, it will be shown that under linear approximation there exist solutions of the Friedmann universe stable against quantum effects for the cases k=0,±1 of RDU, MDU and dSU, if we choose the parameters of PGT, the total entropy of the universe and others properly. (The radiation dominant universe occupies almost all the era of the early universe[13] and the de Sitter universe is essential to the inflationary universe scenarios[5]. See §2.)

The Poincaré Gauge Theory[14] is a gauge theory for extended gravity. Its gauge group is $T \otimes L_{\mathrm{internal}}$, where T is the translational gauge group and L_{internal} is the internal Lorentz gauge group. It contains GR and the New General Relativity (NGR)[15] as its special cases. The underlying spacetime manifold of this gravitational theory is the Riemann-Cartan spacetime characterized by curvature and torsion. The torsion couples with the intrinsic spin of matter. Because spinor fields are representations of the Lorentz group, they can easily be introduced into this theory.

Quantum effects due to vacuum polarization in PGT are investigated in Ref.16):It is shown therein under the assumption of asymptotic freedom and multiplicative renormalizability, that at high temperature the theory is asymptotically comformally invariant and that particles become massless. These results are then used to obtain the expression for $\langle 0|T_{\mu\nu}|0\rangle$.

This paper is organized as follows. In §2 we survey the classical Friedmann universe solutions of GR with k=0,±1 for later convienience. In §3 we explain briefly how quantum effects due to vacuum polarization make the Friedmann universe unstable in GR and give some observational consequences caused by it. In §4 a survey of classical PGT equations for the scale parameter A(t) and the torsion S(t) of the Friedmann universe with k=0,±1 is given. In §5 it is shown under linear approximation for the cases k=0,-1 and for the case k=1 excluding the era at which $\dot{A} \simeq 0$, that there is a classical, stable solution of RDU for the era of the universe where $T \lesssim m_p$ (m_p is the Planck mass and $\simeq 10^{19}$ GeV) if we choose the parameters of PGT and the total entropy of the universe properly. This solution describes a small oscillation around the standard Big Bang solution (SBBS) of GR with frequency being $\lesssim 10^{-2} m_p$. In §6 the conditions for the parameters of PGT and the total entropy of the universe under which quantum effects of matter fields due to vacuum polarization dose not break this classical stability are given. The cases of MDU and dSU are considered in the same manner in §7 and §8, respectively. The last section is dedicated to summary and discussions.

§ 2. A survey of the standard Big Bang solution of GR

We consider a homogeneous and isotropic space(i.e. the Friedmann universe) with the metric

$$ds^2 = -dt^2 + \frac{A^2(t)}{\left(1+ \frac{k}{4}r^2\right)}(dx^2 + dy^2 + dz^2) \qquad (2.1)$$

where A(t) is the scale parameter, $r^2=x^2+y^2+z^2$ and k=0,1,-1 corresponding to the flat, closed and open universe, respectivly[13]. (We use the unit with c=ħ=1.) The Einstein equation for $A_0(t)$ is

$$k + \dot{A}_0^2 = \frac{8\pi G}{3}\rho_{cl}A_0^2 = \frac{8\pi}{3m_p^2}\rho_{cl}A_0^2 \equiv \frac{1}{6a}\rho_{cl}A_0^2 \qquad (2.2)$$

where we define

$$a = \frac{m_p^2}{16\pi}. \qquad (2.3)$$

Here the suffix 0 means the classical solution of GR.

In the following we investigate three important types of the Friedmann universe;RDU, MDU and dSU. The radiation dominant universe occupies almost all the era of the early universe at which significant phenomena such as the nucleosynthesis[13] had happened. After RDU the matter dominant universe occupies the very long era until the present time[13]. Though there are many scenarios of the inflationary universe, all of them contain the era of the de Sitter expansion[5].

The classical energy density of these universe ρ_{cl} is expressed by

$$\rho_r = \frac{D_r}{A_0^4} \quad (\mathrm{RDU}), \tag{2.4}$$

$$\rho_d = \frac{D_m}{A_0^3} \quad (\mathrm{MDU}) \tag{2.5}$$

and

$$\rho_v = \mathrm{const.} \quad (\mathrm{dSU}), \tag{2.6}$$

where the values of constants D_r, D_m and ρ_v are estimated as follows. The value of D_r after the phase transition(PT)[5] of the Grand Unified Theory(GUT)[17] can be estimated from the present value of S_t (S_t is the total entropy of the universe in a volume specified by the radius A_0), that is, $D_r = S_t^{4/3} \gtrsim 10^{112}$[5]. It is considered that the value of D_r before PT is smaller than this value by a factor $\gtrsim 10^{110}$ according to the inflationary universe scenarios[5]. The present value of D_m is estimated from observations of the total energy density of present universe, the result is $10^{58} \lesssim D_m/m_p \lesssim 10^{59}$[13]. (According to this uncertainty, we can not determine the present value of k[13].) The value of ρ_v depends on the energy scale at which PT happened. In the case of SU(5)-GUT phase transition, for example, $\rho_v \simeq 10^{60}(\mathrm{GeV}^4)$[17]. In order to make the following discussions as general as possible, we treat these constants as free parameters and give them special values when they are needed.

The equation (2.2) can then be expressed as

$$A_0^2(\ k + \dot{A}_0^2\ ) = U_0 \quad (\mathrm{RDU}), \tag{2.7}$$

$$A_0(\ k + \dot{A}_0^2\ ) = v_0 \quad (\mathrm{MDU}) \tag{2.8}$$

and

$$A_0^{-2}(\ k + \dot{A}_0^2) = w_0 \qquad (\mathrm{dSU}), \qquad (2.9)$$

where we define

$$U_0 = \frac{D_r}{6a}\ , \qquad (2.10)$$

$$v_0 = \frac{D_m}{6a} \qquad (2.11)$$

and

$$w_0 = \frac{\rho_v}{6a}\ . \qquad (2.12)$$

The solutions of these equations are given in Ref.13). For the case k=1, the solution has a maximum $A^R_{max}=U_0^{1/2}$(RDU), a maximum $A^M_{max}=v_0$(MDU) and a minimum $A^d_{min}=w_0^{-1/2}$(dSU), respectively. We can show that in the region

$$A_0 \ll U_0^{1/2} \qquad (k=\pm1,\mathrm{RDU}), \qquad (2.13)$$

$$A_0 \ll v_0 \qquad (k=\pm1,\mathrm{MDU}) \qquad (2.14)$$

and

$$A_0 \gg w_0^{-1/2} \qquad (k=\pm1,\mathrm{dSU}), \qquad (2.15)$$

the effects of the curvature expressed by the term of k in (2.7)~(2.9) become negligible, so that the equations (2.7)~(2.9) result in those of the flat universe (k=0).

In §6, 7 and 8 we will find stable solutions of RDU, MDU and dSU for the cases k=0,±1 with quantum effects due to vacuum polarization, respectively. We can show that in the regions (2.13) and (2.14) these solutions result in those of the flat universe obtained in Ref.2).

§ 3. The instability of the Friedmann universe in GR

In this section, we show the instability of the Friedmann universe with quantum effects in GR[4)] for the case k=0 of RDU and serious difficulties of cosmology which arise from it.

In GR, quantum effects due to vacuum polarization of matter field at one loop level in a background gravitational field are given[10)] for massless and conformal invariant theory by

$$\rho_q = \frac{6\lambda\tilde{\alpha}}{A^4}\left(A^2\dot{A}A^{(3)} + A\dot{A}^2\ddot{A} - \frac{1}{2}A^2\ddot{A}^2 - \frac{3}{2}\dot{A}^4 - k\dot{A}^2\right)$$

$$+ \frac{6\lambda\tilde{\beta}}{A^4}\left(\frac{1}{2}\dot{A}^4 + k\dot{A}^2\right) , \qquad (3.1)$$

$$\mathrm{Tr} \equiv \langle 0|T_\mu{}^\mu|0\rangle = -\rho_q + 3p_q$$

$$= -\frac{6\lambda\tilde{\alpha}}{A^3}\left(A^2A^{(4)} + 3A\dot{A}A^{(3)} - 5\dot{A}^2\ddot{A} + A\ddot{A}^2 - 2k\ddot{A}\right)$$

$$- \frac{12\lambda\tilde{\beta}}{A^3}(k + \dot{A}^2)\ddot{A} \qquad (3.2)$$

where $A^{(i)}$ $(i=3,4,..)$ are i-th derivatives of A and

$$\lambda = \frac{1}{2880\pi^2} \quad \text{(natural unit)} , \qquad (3.3)$$

$$\tilde{\alpha} = N_\phi + 6N_\psi + 12N_A , \qquad (3.4)$$

$$\tilde{\beta} = N_\phi + 11N_\psi + 62N_A \qquad (3.5)$$

and N_ϕ,N_ψ and N_A are the number of species of scalar, spinor and vector fields, respectively. In the following we set

$\tilde{\alpha} = \tilde{\beta} = O(10^2)$.

Adding (3.1) to RHS of (2.7) and setting k=0, we obtain the GR equation of the scale parameter A(t) which includes quantum effects as

$$A^2\dot{A}^2 = U_0 + \frac{\tilde{\alpha}\lambda}{2a}\left(2A^2\dot{A}A^{(3)} - A^2\ddot{A}^2 + 2A\dot{A}^2\ddot{A} - 3\dot{A}^4\right) + \frac{\tilde{\beta}\lambda}{2a}\dot{A}^4 . \qquad (3.6)$$

With $r = A\dot{A}$, (3.6) is rewritten as

$$\ddot{r} = \frac{a(r^2 - U_0^{1/2})}{\tilde{\alpha}\lambda r} + \left(\frac{\tilde{\beta}}{2\tilde{\alpha}} - 1\right)\frac{r^3}{A^4} + \frac{r\dot{r}}{A^2} + \frac{\dot{r}^2}{2r} . \qquad (3.7)$$

In the following, we owe to the paper of T.V.Ruzumaikina and A.A.Ruzumaikin in Ref.4) for some mathematics. Using "particle position":R and "time":τ defined as

$$R \equiv (A\dot{A})^{3/2} = r^{3/2} , \qquad (3.8)$$

$$\tau \equiv 12^{-3/4}A^3 , \qquad (3.9)$$

respectively, (3.7) is rewritten in a form without velocity term as follows

$$\frac{d^2R}{d\tau^2} = \frac{2a}{3\tilde{\alpha}\lambda}\tau^{-2/3}\left(R^{-1/3} - U_0R^{-5/3}\right) - \frac{\tilde{\beta}}{12\tilde{\alpha}}\tau^{-2}R . \qquad (3.10)$$

This is the equation of a particle moving in a potential. When $m_p t \gg 1$ and $R = O(U_0^{3/4})$, we can show that the second term is much smaller than the first term in (3.10) and that the potential has a *maximum* at $R = U_0^{3/4}$ (This is due to the positive sign of $A^{(3)}$-term in RHS of (3.6)). The outline of the potential is drawn in

Fig.1. The resting solution at this maximum corresponds to the classical GR solution of RDU($A_0\dot{A}_0=\sqrt{U_0}$). However the particle is unstable and must roll down the hill of the potential sooner or later, so that evolution of the universe become much different from SBBS.

When $R \ll U_0^{3/4}$, we can show that $\dot{A}(\tau) \ll \dot{A}_0(\tau)$, so that the Hubble constant become much smaller than the value of SBBS which is in agreement with the observation[13]. Further, the helium mass fraction which is very sensitive to the expansion rate when the universe is a few minutes old[13] becomes much smaller than the value of SBBS. When $R \gg U_0^{3/4}$, we can get opposite results based on the relation $\dot{A}(\tau) \gg \dot{A}_0(\tau)$. The other cosmological observables which are sensitive to $\dot{A}$ such as the fluctuations[6] etc. will be much different from the observations too. These are serious difficulties in cosmology of GR with quantum effects.

§ 4. The classical PGT equations for the Friedmann universe

It is shown[18)] that under the condition of homogeneity, isotropy and parity conservation only the following components of the torsion tensor remain nonvanishing;

$$\begin{cases} T^{1}{}_{\cdot 10} = T^{2}{}_{\cdot 20} = T^{3}{}_{\cdot 30} \equiv S(t) \neq 0 \\ \text{other components} = 0 \end{cases} \qquad (4.1)$$

Here, 0 and i(=1,2,3) are indices for time and space components, respectively. The classical equation for A(t) of the Friedmann universe in PGT[19)] is

$$k + \left(\dot{A} + \frac{f\dot{F}A}{3B} \right)^2 = \frac{\rho_{cl} + \frac{1}{3}fF^2 - 9\beta A^{-2}(\dot{A}^2 + k)}{6B} , \qquad (4.2)$$

where F is the scalar curvature in the presense of torsion;

$$F = \frac{1}{2b}\Big(\rho_{cl} - 3p_{cl} - 18\beta A^{-2}(k + \dot{A}^2 + A\ddot{A})\Big) . \qquad (4.3)$$

Here ρ_{cl} and p_{cl} are the classical energy density and pressure, respectively, and B is given by

$$B = b + \frac{2}{3}fF. \qquad (4.4)$$

Because F contains $\ddot{A}$, the classical equation (4.2) is a third order differential equation for A. The constants f and b are given by

$$f = \frac{1}{4}(a_5 + 12a_6), \qquad (4.5)$$

$$b = a - \frac{3}{2}\beta , \qquad (4.6)$$

where a_5 , a_6 and β are three of nine parameters of PGT[14]. (See also Appendix B of Ref.2).)

The condition of the propagating torsion with positive-definite energy and positive mass restricts the parameters[20] as

$$f > 0 \qquad (4.7)$$

and

$$\beta \neq 0 . \qquad (4.8)$$

The torsion field is then given by

$$S = - \frac{f\dot{F}}{3B} . \qquad (4.9)$$

§ 5. The classical stable PGT solution for RDU

In this section we show that under linear approximation the classical stable PGT solution of the Friedmann universe for RDU with $k=0,\pm 1$ exists for the era $T \lesssim m_p$ under proper conditions for f, β and D_r(in the following, we use D_r instead of the total entropy S_t).

We introduce new functions

$$x \equiv A^2(k + \dot{A}^2) \tag{5.1}$$

and

$$X \equiv x - kA^2 = A^2\dot{A}^2, \tag{5.2}$$

where x becomes U_0 when A is A_0. The equation (4.2) then becomes

$$\ddot{x} = \frac{2b^2}{3f\beta}X + \frac{\dot{x}^2}{2X} - \frac{k\dot{x}}{\sqrt{X}}$$

$$\pm \frac{2b^{\frac{3}{2}}\sqrt{X}}{3\sqrt{6}f\beta}\left(D_r - 9\beta x - 6kbA^2 + \frac{18kf\beta}{b\sqrt{X}}\dot{x} + \frac{27f\beta^2}{4b^2X}\dot{x}^2\right)^{\frac{1}{2}}\left(1 - \frac{3f\beta\dot{x}}{b^2A^2\sqrt{X}}\right)^{\frac{1}{2}}, \tag{5.3}$$

where we have used (2.4) as ρ_{cl}. We restrict ourselves to the case in which the $\dot{x}$-terms are much smaller than the remaining terms, because it is difficult to solve (5.3) exactly: Consistency of this assumption is justified later(see below (5.29)). Then we get

$$\ddot{x} = \frac{2b^2}{3f\beta}X + \frac{\dot{x}^2}{2X} - \frac{k\dot{x}}{\sqrt{X}}$$

$$\pm \frac{2b^{\frac{3}{2}}\sqrt{X}M_r}{3\sqrt{6}f\beta}\left(1 + \frac{9kf\beta}{b\sqrt{X}M_r^2}\dot{x} + \frac{27f\beta^2\dot{x}^2}{8b^2XM_r^2} - \frac{3f\beta\dot{x}}{2b^2A^2\sqrt{X}}\right) , \qquad (5.4)$$

where $$M_r \equiv \left(D_r - 9\beta x - 6kbA^2 \right)^{1/2} . \qquad (5.5)$$

It will be confirmed later that the argument of the square root in (5.5) is positive for our solution(see below (5.12)). This equation is interpreted as that for a particle moving in a potential V(x) with additional $\dot{x}$-dependent forces, where

$$-\frac{dV(x)}{dx} = \frac{2b^2}{3f\beta}\left(X \pm \frac{X^{\frac{1}{2}}M_r}{\sqrt{6b}}\right). \qquad (5.6)$$

(Strictly speaking, this picture is justified only under linear approximation(see (5.11))). We choose the negative sign in RHS of (5.6) so that an equilibrium point at which $dV/dx=0$ is $x=U_0$ which corresponds to SBBS of GR. Then whether this point is a minimum or maximum point of the potential depends on the sign of β(note that $f>0$(see (4.7))).

To stabilize the universe this point must be a *minimum* point of the potential. We can show this is realized when

$$\beta < 0 . \qquad (5.7)$$

Let us seek a solution of x in the form of a small oscillation around U_0. Putting

$$x = A^2(k + \dot{A}^2) = U_0 + \varepsilon , \tag{5.8}$$

and assuming that $\varepsilon/U_0 << 1$, or for definiteness of our argument, that

$$O(\varepsilon/U_0) \lesssim 10^{-4} , \tag{5.9}$$

we get a linearized equation for $\delta A = A - A_0$, the deviation from A_0 satisfiying (2.7);

$$\delta\dot{A} + \frac{U_0}{A_0^3\dot{A}_0}\delta A = \frac{1}{2A_0^2\dot{A}_0}\varepsilon . \tag{5.10}$$

Applying linear approximation to (5.4), we have the following equation for ε;

$$\ddot{\varepsilon} = \frac{ab}{3f\beta}\varepsilon + \frac{\dot{A}_0}{A_0}\dot{\varepsilon} - \frac{2k}{A_0\dot{A}_0}\dot{\varepsilon} + O\left(\varepsilon^2, \dot{\varepsilon}^2, \varepsilon\dot{\varepsilon}, \varepsilon\delta A, \delta A^2, ..\right). \tag{5.11}$$

For the cases k=0 and k=-1 Eq.(5.11) can be considered to be valid for all eras of the universe, since $\dot{A}_0 \neq 0$.

For the case k=1, however, $\dot{A}_0$ vanishes at the maximum point $A^R_{max} = U_0^{1/2}$, so in the following we shall restrict ourselves to the era satisfying

$$0 < A_0 \leqq (1-\theta)U_0^{1/2} \ , \qquad (5.12)$$

where θ is a small but finite, positive constant. The smaller θ becomes, the smaller value of ε/U_0 is needed to show the stability of RDU in the following discussions. For the case (5.9), we safely can take $\theta \simeq 0.1$. So, we fix θ to this value in the following. When (5.7) and (5.12) are satisfied, we can show that the argument in the square root in (5.5) is positive.

It is proper to solve the equation (5.11) by the WKB method. However, the WKB solution is too complicated to estimate quantum effects such as (3.1) or (3.2). So, we adopt harmonic oscillater approximation which is justified under the conditions

$$\left|\frac{ab}{3f\beta}\varepsilon\right| \gg \left|\frac{\dot{A}_0}{A_0}\dot{\varepsilon}\right| \qquad (5.13a)$$

and

$$\left|\frac{ab}{3f\beta}\varepsilon\right| \gg \left|\frac{2k}{A_0\dot{A}_0}\dot{\varepsilon}\right| \ . \qquad (5.13b)$$

Under these conditions we obtain approximate solution

$$\varepsilon = \xi_0 \cos\kappa t \qquad (5.14)$$

with

$$O(\xi_0/U_0) \lesssim 10^{-4} \qquad (5.15)$$

and

$$\kappa^2 = \frac{ab}{3f(-\beta)} \cdot \tag{5.16}$$

The two conditions (5.13) are equivalent to

$$\kappa A_0 \gg 1 \tag{5.17a}$$

and

$$D_r \gg 72f , \tag{5.17b}$$

respectively. The condition (5.17b) is needed only for the case $k=\pm1$. Because the value of κ is found to be $\sim\sqrt{a/2f}$(see (6.15)) and $A_0 T = D_r^{1/4}$ for adiabatic expansion of the universe(T is the temperature of the universe), the condition (5.17a) becomes

$$T \ll \frac{1}{(32\pi f)^{1/2}} D_r^{1/4} m_p . \tag{5.18}$$

If RHS of (5.18) $\gg m_p$, we may expect that the era (5.18) includes the era $T \lesssim m_p$ where the semi-classical picture is available[10),12)]; so we demand

$$D_r \gg (32\pi f)^2 . \tag{5.19}$$

Under this condition, the following discussions are justified for $T \lesssim m_p$.

Putting (5.14) into (5.10), we obtain

$$\delta A(t) = \tfrac{1}{2}\dot{A}_0(t)\int_{t_0}^{t} \frac{\xi_0 \cos\kappa t'}{A_0^2(t')\dot{A}_0^2(t')}\,dt', \qquad (5.20)$$

where we have chosen the integration constant so that $\delta A(t_0)=0$. The numerator in the integrand has time scale $\sim f^{1/2}t_p$, while the denominator has cosmic time scale, so we expect that the latter can be regarded as constant in comparison with the former, then we obtain

$$A_0 + \delta A = A_0\left(1 + \frac{1}{2}\cdot\frac{U_0}{A_0^2\dot{A}_0}\cdot\frac{\xi_0}{U_0}\cdot\frac{1}{\kappa\dot{A}_0}\sin\kappa t \right). \qquad (5.21)$$

We can show that the second term in the parenthesis of (5.21) is much smaller than the first term using (5.12),(5.15) and (5.17a).

For the cases k=0 and k=−1, this is a stable solution of the universe throughout its history;it is called the *trembling universe*" (Ref.2)), because it describes a small oscillation around SBBS of GR. In particular, for the case k=0 ($A_0(t)=\sqrt{2}U_0^{1/4}t^{1/2}$), (5.21) coincides with the result of Ref.2). (Note that ξ_0/U_0 is equal to $2\eta_0/u_0$ of Ref.2).) For the case k=1, however, this solution can be justified only for the era satisfying (5.12).

In the rest of this section we make some preparations for §6. It is difficult to estimate complicated quantum effects such as (3.1) and (3.2) throughout all the era of the universe, so we need restrict the era of the universe to

$$\kappa A_0 \gtrsim \frac{U_0}{\xi_0} \quad . \tag{5.22}$$

Then we can show the following relations which will be needed in §6.

$$A^2\dot{A}\ddot{A} \simeq -\frac{\kappa\xi_0}{2}\sin\kappa t = \frac{\dot{\varepsilon}}{2} - A\dot{A}(\dot{A}^2+k), \tag{5.23}$$

$$A^2\dot{A}A^{(3)} \simeq -\frac{\kappa^2\xi_0}{2}\cos\kappa t = \frac{\ddot{\varepsilon}}{2}, \tag{5.24}$$

$$A^2\dot{A}A^{(4)} \simeq \frac{\kappa^3\xi_0}{2}\sin\kappa t \tag{5.25}$$

and

$$A^2\dot{A}A^{(5)} \simeq \frac{\kappa^4\xi_0}{2}\cos\kappa t \quad . \tag{5.26}$$

In RHS of (5.23), the inequality $\dot{\varepsilon}/2 \gtrsim A\dot{A}(\dot{A}^2+k)$ holds when (5.22) is satisfied[21]. Then F and $\dot{F}$ can be expressed as

$$F = -\frac{9\beta}{b}\left(\frac{k+\dot{A}^2+A\ddot{A}}{A^2}\right) \simeq -\frac{9\beta}{2b}\cdot\frac{\dot{\varepsilon}}{A^3\dot{A}} \tag{5.27}$$

and

$$\dot{F} = -\frac{9\beta}{b}\left(\frac{k+\dot{A}^2+A\ddot{A}}{A^2}\right)^{\cdot} \simeq -\frac{9\beta}{b}\cdot\frac{A^{(3)}}{A} \simeq -\frac{9\beta}{2b}\cdot\frac{\ddot{\varepsilon}}{A^3\dot{A}} \quad . \tag{5.28}$$

The use of (5.28) in (4.9) gives the following expression for the torsion:

$$S(t) \simeq -\frac{a}{3\beta}\cdot\frac{\xi_0}{U_0}\cdot\frac{\cos\kappa t}{t}. \qquad (5.29)$$

Using (5.15),(5.22) and (5.23) (in the case k=1 we need (5.12) in addition), we can confirm that the $\dot{\varepsilon}$-terms (namely, the $\dot{x}$-term) in (5.3) are much smaller than the remaining terms.

We can show as before (see (5.18)) that the era (5.22) is equivalent to

$$T \lesssim \frac{1}{(32\pi f)^{1/2}}\cdot\frac{\xi_0}{U_0} D_r^{1/4} m_p. \qquad (5.30)$$

Further we demand

$$D_r \gtrsim (32\pi f)^2 \left(\frac{U_0}{\xi_0}\right)^4 \qquad (5.31)$$

in order that the era (5.30) includes $T \lesssim m_p$.

§ 6. The PGT solution of RDU stable against quantum effects

Quantum effects due to vacuum polarization of matter fields at one loop level in a background gravitational field in PGT are investigated by Buchbinder, Odintsov and Shapiro[16]. They have shown under the assumption of multiplicative renormalizability and asymptotic freedom that the theory is asymptotically comformally invariant and that matter fields become massless at high temperature. Then, they obtain the expression for $<0|T_{\mu\nu}|0>$ at one loop approximation. It consists of two parts;the one is the same as GR and the other is made of the axial-vector part of the torsion tensor $a_i=\frac{1}{6}\varepsilon_{ijmn}T^{jmn}$. However, in homogeneous and isotropic space a_i vanishes[18] (see (4.1)), hence quantum effects in PGT have the same form in GR given by (3.1)~(3.5).

In the following, we treat the cases where masses of matter fields can be ignored;$m_i=0$, $T >> m_i$ and $T << m_i$, where m_i are the masses of particles concerned. Before PT, all masses are exactly zero[5]. In the case $T << m_i$, we may expect that the masses dose not contribute to quantum effects because of the decoupling theorem[22] and we shall discuss this case in §7 as the case MDU. If $T \simeq m_i$, the masses of particles cannot be ignored and different proper treatments are needed.

Due to these quantum effects, ρ_{cl} and p_{cl} are modified like

$$\rho_{cl} \rightarrow \rho_{cl} + \rho_q \quad \left(\text{i.e., } U_0 \rightarrow U_0\left(1 + \frac{\rho_q}{\rho_{cl}}\right)\right), \tag{6.1}$$

$$p_{cl} \rightarrow p_{cl} + p_q \, . \tag{6.2}$$

Accordingly, the scalar curvature F and its time derivative $\dot{F}$ are changed like

$$F \rightarrow F + \Delta F \equiv - \frac{9\beta}{b}\left(\frac{k + \dot{A}^2 + A\ddot{A}}{A^2}\right) - \frac{1}{2b}Tr \tag{6.3}$$

and

$$\dot{F} \rightarrow \dot{F} + \Delta\dot{F} \equiv - \frac{9\beta}{b}\left(\frac{k + \dot{A}^2 + A\ddot{A}}{A^2}\right)^{\cdot} - \frac{1}{2b}\dot{T}r \ , \tag{6.4}$$

respectively. The equation for A with quantum effects are then obtained from (4.2) and (4.3) by making the above replacement of ρ_{cl}, p_{cl}, F and $\dot{F}$. Since $\dot{T}r$ contains $A^{(5)}$, the equation for A with quantum effects is a 5th-order differential equation. From this equation, we can obtain an equation for ε of (5.8) with quantum corrections. It has a form of (5.11) with the following replacement of ε, $\dot{\varepsilon}$ and ε,

$$\varepsilon \quad \rightarrow \varepsilon\left(1 - \frac{U_0}{\varepsilon}\cdot\frac{\rho_q}{\rho_{cl}}\right), \tag{6.5}$$

$$\frac{\dot{A}_0}{A_0}\varepsilon \rightarrow \frac{\dot{A}_0}{A_0}\varepsilon\left(1 - \frac{\Delta F}{F}\right), \tag{6.6}$$

$$\frac{2k}{A_0\dot{A}_0}\dot{\varepsilon} \rightarrow \frac{2k}{A_0\dot{A}_0}\dot{\varepsilon}\left(1 + \frac{\Delta F}{F}\right) \tag{6.7}$$

and

$$\ddot{\varepsilon} \rightarrow \ddot{\varepsilon}\left(1 + \frac{\Delta\dot{F}}{\dot{F}}\right) . \tag{6.8}$$

In the following we show that the second terms on the RHS of (6.5)~(6.8) which represent quantum corrections contribute only to the negligible terms in RHS of (5.11), so that the classical stable solution expressed by (5.14) and (5.20) is still valid.

First we demand

$$\frac{\rho_q}{\rho_{cl}} \lesssim \frac{\xi_0^2}{U_0^2} \tag{6.9}$$

so that the effect of ρ_q in (6.5) is absorbed into the negligible term $O(\varepsilon^2)$ in RHS of (5.11). Secondly we demand

$$\frac{\Delta F}{F} \lesssim 1 \tag{6.10}$$

so that the effects of ΔF in (6.6) and (6.7) are the same order with the second and third term in RHS of (5.11), respectively. (They are negligible because of (5.17).) Lastly we demand

$$\frac{\Delta \dot{F}}{\dot{F}} \lesssim \frac{\xi_0}{U_0} \tag{6.11}$$

so that the effect of $\Delta \dot{F}$ in (6.8) is $O(\varepsilon^2)$.

Using the expressions for ρ_q and Tr given by (3.1) and (3.2) respectively, and employing the relations (5.23)~(5.26), it can be shown that the inequalities (6.9)~(6.11) are satisfied, if the following conditions are satisfied in addition to (5.22);

$$f \gtrsim 2\lambda\tilde{\alpha}\cdot\frac{U_0}{\xi_0}\ , \qquad (6.12)$$

$$-\beta \gtrsim a \qquad (6.13)$$

and

$$D_r \gtrsim 6\lambda\tilde{\alpha}\left(\frac{U_0}{\xi_0}\right)^2 . \qquad (6.14)$$

Here, we have used

$$\kappa^2 \sim \frac{m_p^2}{32\pi f} \qquad (6.15)$$

which is obtained when (6.13) is satisfied.

As an example, let us briefly outline the arguments leading to (6.12). Taking the first term of RHS of (3.1) as ρ_q, the inequality (6.9) becomes

$$\frac{\rho_q}{\rho_{cl}} \sim \frac{\frac{6\lambda\tilde{\alpha}}{A^4}\cdot A^2\dot{A}A^{(3)}}{D_r/A^4} \sim 2\lambda\tilde{\alpha}\frac{\xi_0}{U_0}\cdot\frac{1}{f} \lesssim \frac{\xi_0^2}{U_0^2}, \qquad (6.16)$$

where we have used (3.1) and (5.24). From this relation we obtain (6.12). Repeating similar analyses for each term in ρ_q, we obtain (6.12),(6.14) and (5.22) as the sufficient conditions for (6.9).

To summarize, the conditions under which linearized stable solution of RDU with quantum correction exists for $T \lesssim m_p$ are (5.9),(5.17b),(5.19),(5.31),(6.12),(6.13) and (6.14), in addition to these we need (5.12) for the case k=1.

The parameter region restricted by (5,9),(5.17b),(5.19),(5.31),(6.12) and (6.14) is shown in Fig.2, where f, ξ_0/U_0 and D_r are parametrized as

$$f = 10^m , \tag{6.17}$$

$$\frac{\xi_0}{U_0} = 10^n \tag{6.18}$$

and

$$D_r = 10^p , \tag{6.19}$$

and set p to, for example, $p_0=112$. From Fig.2 we notice that the smaller the value of D_r becomes, the narrower the stable region becomes. In conclusion, for RDU with k=0,±1 we need

$$f \gtrsim 10^{1.6} , \tag{6.20}$$

$$D_r \gtrsim 10^{23.2} \tag{6.21}$$

and (6.13) to stabilize the universe for $T \lesssim m_p$.

In RDU $\rho_r >> \rho_d$, so we have set $\rho_d=0$ up to this point. However, stricrly speaking, ρ_d can not been perfectly neglected; in RHS of (4.3) we obtain $\rho_{cl}-3p_{cl}=\rho_r-3p_r+\rho_d-3p_d=\rho_d$ because of $\rho_r=3p_r$ and $p_d=0$. In this case, we have additonal D_m-terms to RHS of (5.3),(5.4),(5.6) and (5.11). For the value $D_m/m_p=10^{58}$ and $D_r= 10^{112}$, we can show that these terms can be neglected and give no influences to the above discussions if we demand

$$f \lesssim 34 . \tag{6.22}$$

We need not this condition for the case of pure radiation, for example, for RDU before PT where all particles have no mass.

§ 7. The case of the matter dominant universe(MDU)

The temperature of MDU($T \lesssim 10^{-12.6}$ GeV$^{13)}$) is sufficiently low in comparison with masses of leptons, quarks, Higgs, massive gauge bosons and so on. So, it is plausible to assume that the effects of these massive particles disappear from vacuum polarization because of the decoupling theorem$^{22)}$. Therefore, the case of MDU can be treated by repeating almost same analysis of RDU. So in the following we only point out main differences between MDU and RDU.

We introduce a function

$$y \equiv A(k + \dot{A}^2) \qquad (7.1)$$

which becomes v_0 when A is A_0. We obtain the classical equation for y from (2.5) and (4.2) as

$$\ddot{y} = \frac{2b^2}{3f\beta}Y - \frac{5Y^{1/2}}{2A^{3/2}}\dot{y} - \frac{\dot{y}^2}{2Y}$$

$$\pm \frac{2b^{\frac{3}{2}}\sqrt{Y}M_m}{3\sqrt{6}f\beta}\left[1- \frac{9f\beta}{b}\left(\frac{D_m}{6bA} -2k\right)\frac{\dot{y}}{\sqrt{AY}M_m^2} + \frac{9f\beta^2\dot{y}^2}{4b^2YM_m^2}\right]^{\frac{1}{2}}\left(1- \frac{3f\beta\dot{y}}{b^2A^3}\right)^{\frac{1}{2}}, \qquad (7.2)$$

where

$$Y \equiv y - kA \qquad (7.3)$$

and

$$M_m \equiv \left(D_m - 9\beta y - 6kbA\right)^{\frac{1}{2}} . \qquad (7.4)$$

Then, as in RDU we choose the negative sign in RHS of (7.2) and we demand $\beta<0$, so that the potential has a minimum at $y=v_0$ and the universe is stable. In this argument, we have assumed the inequality

$$\frac{f}{(m_pA)^3}\cdot\frac{D_m}{m_p} \ll 1 \qquad (7.5)$$

so that the potential has only one equilibrium point.

Applying linear approximation, we have the equations for small deviations ε and δA as

$$\ddot{\varepsilon} = \frac{ab}{3f\beta}\varepsilon - \left(\frac{5\sqrt{Y}}{2A_0^{3/2}} + \frac{Y}{A_0^2} + \frac{k}{\sqrt{YA_0}}\cdot - \frac{D_m}{12b\sqrt{YA}_0^{3/2}} \right)\varepsilon + O(\varepsilon^2) \qquad (7.6)$$

and

$$\delta\dot{A} + \frac{v_0}{2A_0^2\dot{A}_0}\delta A = \frac{1}{2A_0\dot{A}_0}\varepsilon\ . \qquad (7.7)$$

We obtain a solution of harmonic oscillater approximation as

$$y = v_0 + \xi_0\cos\kappa t \quad (v_0 \gg \xi_0), \qquad (7.8)$$

where κ is given by (5.16), if the following conditions are satisfied:

$$\begin{cases} \kappa A_0 \gg 1 & (k=0,-1) \qquad (7.9a) \\ \kappa A_0 \gg 1 \ \text{ and } \ A_0 < 0.9v_0 & (k=1) \qquad (7.9b) \end{cases}$$

and

$$\left\{\begin{array}{lll} \dfrac{f^{1/2}}{3^{1/2}}\cdot\dfrac{(16\pi)^{3/2}}{(m_pA)^{5/2}}\left(\dfrac{D_m}{m_p}\right)^{3/2} << 1 & (k=0) & (7.10a) \\ f^{1/2}\dfrac{(16\pi)^{3/2}}{(m_pA)^2}\cdot\dfrac{D_m}{m_p} << 1 & (k=\pm 1). & (7.10b) \end{array}\right.$$

Using $AT=D_r^{1/4}$, the condition (7.9a) becomes

$$T << \frac{1}{(32\pi f)^{1/2}}D_r^{1/4}m_p . \qquad (7.11)$$

Here we define T_1 at which $\rho_d=\rho_r$, and below which $\rho_d>\rho_r$, that is, the universe is MDU. We demand that the RHS of (7.11) >> T_1, then we may expect that the era (7.11) includes the matter dominant era $T \lesssim T_1$; so we obtain

$$D_r >> (32\pi f)^2\left(\frac{T_1}{m_p}\right)^4 . \qquad (7.12)$$

For our universe, T_1 is estimated to be $O(10^{4.5}$ x $2.7^oK)=$ $O(10^{-12.6}\ \mathrm{GeV}^{13)})$.

Finally, δA is given from (7.7) and $\varepsilon=\xi_0\cos\kappa t$ for MDU as

$$\delta A(t) = \frac{1}{2}\dot{A}_0(t)\int_{t_0}^{t}\frac{\xi_0\cos\kappa t'}{A_0(t')\dot{A}_0^2(t')}dt' . \qquad (7.13)$$

Using the same approximation as that for the RDU (see below (5.20)), we obtain

$$A_0 + \delta A = A_0\left(1 + \frac{v_0}{2\dot{A}_0 A_0}\cdot\frac{\xi_0}{v_0}\cdot\frac{1}{\kappa A_0}\sin\kappa t \right) . \qquad (7.14)$$

For the case $k=0(A_0=\left(\frac{9}{4}v_0\right)^{1/3}t^{2/3})$, (7.14) coincides with the result of Ref.2). When the condition (7.9) is satisfied, we can show that the second term in the parenthesis of (7.14) is much smaller than the first term.

In order to estimate quantum effects, we need to restrict the era of the universe to

$$\kappa A_0 \gtrsim \frac{v_0}{\xi_0} , \qquad (7.15)$$

equivalently,

$$T \lesssim \frac{1}{(32\pi f)^{1/2}}\cdot\frac{\xi_0}{v_0}D_r^{1/4}m_p . \qquad (7.16)$$

Here we demand

$$D_r \gtrsim (32\pi f)^2\left(\frac{v_0}{\xi_0}\right)^4\left(\frac{T_1}{m_p}\right)^4 \qquad (7.17)$$

so that the era (7.16) includes $T \lesssim T_1$.

It can be shown that the conditions under which quantum effects do not break the classical stability of the universe for $T \lesssim T_1$, are (6.13),

$$f \gtrsim 2\lambda\tilde{\alpha}\cdot\frac{v_0}{\xi_0} \qquad (7.18)$$

and

$$\frac{D_m}{m_p} \gtrsim 1. \qquad (7.19)$$

To summarize, the conditions under which linearized stable solution of MDU with quantum correction exists are (7.5),(7.10), (7.12),(7.17),(7.18),(7.19),(6.13) and

$$\xi_0/v_0 \lesssim 10^{-4} \quad . \tag{7.20}$$

In addition to these we need the condition $A_0 < 0.9v_0$ for the case k=1.

In Fig.3, we show the region restricted by (7.5),(7.10), (7.12),(7.17),(7.18) and (7.20). There we parametrize as (6.17), (6.19), $\xi_0/v_0=10^n$, $D_m/m_p=10^q$ and $m_pA=10^r$, and set, for example, $p=p_1=20$, q=58 and r=58. (From observation m_pA is estimated to be $\gtrsim 10^{58}$ for MDU[13].) From Fig.3, we conclude that to stabilize MDU with k=0,±1 the conditions (6.13),(6.20) and

$$D_r \gtrsim 10^{-103.2} \tag{7.21}$$

are needed.

The scalar curvature and the torsion are given by

$$F \simeq \frac{9\beta\kappa\xi_0}{2b\dot{A}_0A_0^2}\sin\kappa t \ , \tag{7.22}$$

$$\dot{F} \simeq \frac{9\beta\kappa^2\xi_0}{2b\dot{A}_0A_0^2}\cos\kappa t \tag{7.23}$$

and

$$S(t) \simeq -\frac{2a}{3\beta}\cdot\frac{\xi_0}{A_0^2}\cdot\frac{\cos\kappa t}{t} \ . \tag{7.24}$$

§ 8 The case of the de Sitter universe(dSU)

Let us treat the de Sitter universe with k=0,±1, introducing a function

$$z \equiv A^{-2}(\ k + \dot{A}^2\ ) \qquad (8.1)$$

which becomes w_0 when A is A_0. We obtain the classical equation for z from (4.2) and $\rho_{cl}=\rho_v$ as

$$\ddot{z} = \frac{2b^2}{3f\beta A^2}Z - \left(z - \frac{\rho_v}{9\beta}\right)\frac{8Z}{A^2} - \frac{7\sqrt{Z}\dot{z}}{A} + \frac{Az\dot{z}}{\sqrt{Z}} + \frac{A^2\dot{z}^2}{2Z}$$

$$\pm \frac{2b^{\frac{3}{2}}\sqrt{Z}M_{d1}}{3\sqrt{6}f\beta A}\left[1 + \frac{4fM_{d2}}{3b^2}\left(1 + \frac{9\beta A\dot{z}}{4M_{d1}^2\sqrt{Z}}\right)\right]^{\frac{1}{2}}\left(1+ \frac{4fM_{d2}}{3b^2}\right)^{\frac{1}{2}}, \qquad (8.2)$$

where

$$Z \equiv zA^2 - k\ , \qquad (8.3)$$

$$M_{d1} \equiv \left(\rho_v - 9\beta z - \frac{6kb}{A^2}\right)^{\frac{1}{2}} \qquad (8.4)$$

and

$$M_{d2} \equiv \rho_v - 9\beta z + \frac{9\beta A\dot{z}}{4\sqrt{Z}}\ . \qquad (8.5)$$

Then, as in RDU we choose the negative sign in RHS of (8.2) and we demand $\beta<0$, so that the potential has a minimum at $z=w_0$ and the universe is stable.

Applying linear approximation, we have the equations for small deviations ε and δA as

$$\ddot{\varepsilon} = \left(\frac{ab}{3f\beta} - 12w_0 + \frac{8k}{A_0^2}\right)\varepsilon + \left(\frac{7k}{A_0\dot{A}_0} - \frac{7w_0A_0}{\dot{A}_0}\right)\dot{\varepsilon} + O(\varepsilon^2) \qquad (8.6)$$

and

$$\delta\dot{A} - \frac{w_0A_0}{\dot{A}_0}\delta A = \frac{A_0^2}{2\dot{A}_0}\varepsilon\ . \qquad (8.7)$$

We obtain a solution of harmonic oscillater approximation as

$$z \ = \ w_0 + \ \delta_0\cos\kappa' t \quad (\ w_0 >> \delta_0\ ) \qquad (8.8)$$

with

$$\kappa' = \left(\frac{ab}{3f(-\beta)} + 12w_0\right)^{1/2}, \qquad (8.9)$$

if the following conditions are satisfied,

$$\frac{\rho_v}{m_p^4} << \frac{3}{12544\pi^2}\cdot\frac{1}{f} \qquad (8.10)$$

and

$$\begin{cases} \kappa A_0 >> 1 & (k=0,-1) \qquad (8.11a) \\ \kappa A_0 >> 1 \ \text{ and } \ A_0 \gtrsim \frac{\sqrt{2}}{\sqrt{w_0}} = \sqrt{2}\cdot A^d_{min} & (k=1)\ \ , \qquad (8.11b) \end{cases}$$

where $1/\sqrt{w_0}$ is the minimum scale parameter of the de Sitter universe($=A^d_{min}$) with $k=1(A_0=\frac{1}{\sqrt{w_0}}\cosh\sqrt{w_0}t)$. The first term in the parenthesis of RHS of (8.9) is much larger than the second term, so $\kappa' \simeq \kappa$. The second condition of (8.11b) is needed, because the equations for ε and δA become singular at $A_0=A^d_{min}$. Using

$AT=D_r^{1/4}$, the condition (8.11a) becomes

$$T \ll \frac{1}{(32\pi f)^{1/2}} D_r^{1/4} m_p \ . \qquad (8.12)$$

We demand here that the RHS of (8.12) $\gg \rho_v^{1/4}$, then we may expect that the era (8.12) includes the de Sitter universe era with $T \lesssim \rho_v^{1/4}$;so we obtain

$$D_r \gg (32\pi f)^2 \cdot \frac{\rho_v}{m_p^4} \ . \qquad (8.13)$$

In order that the second condition of (8.11b)($A_0 \gtrsim \sqrt{2} A^d_{min}$) is satisfied for the era $T \lesssim \rho_v^{1/4}$, we demand

$$D_r \gtrsim \left(\frac{3}{16\pi}\right)^2 \cdot \frac{m_p^4}{\rho_v} \ , \qquad (8.14)$$

which becomes

$$\begin{cases} D_r \gtrsim 10^{13.6} \quad (\rho_v^{1/4} = 10^{15} \text{ GeV}) & (8.15a) \\ D_r \gtrsim 10^{33.6} \quad (\rho_v^{1/4} = 10^{10} \text{ GeV}). & (8.15b) \end{cases}$$

We obtain the solution of δA from (8.7) and $\varepsilon = \xi_0 \cos\kappa t$ as

$$\delta A(t) = \frac{1}{2}\dot{A}_0(t) \int_{t_0}^{t} \frac{\xi_0 \cos\kappa t'}{A_0(t')\dot{A}_0^2(t')} dt' . \qquad (8.16)$$

With the same approximation as used for the RDU (see below (5.20)), we have

$$A_0 + \delta A = A_0\left(1 + \frac{A_0^2 w_0}{2\dot{A}_0}\cdot\frac{\delta_0}{w_0}\cdot\frac{1}{\kappa A_0}\sin\kappa t \right) . \qquad (8.17)$$

For the case $k=0(A_0=\frac{1}{\sqrt{w_0}}\exp(\sqrt{w_0}t))$, (8.17) becomes

$$A_0 + \delta A = A_0\left(1 + \frac{1}{2}\cdot\frac{\delta_0}{w_0}\cdot\frac{\sqrt{w_0}}{\kappa}\sin\kappa t\right) . \qquad (8.18)$$

When (8.11) is satisfied, we can show that the second term in the parenthesis of (8.17) is much smaller than the first term.

In order to estimate quantum effects, we needs two conditions

$$\kappa A_0 \gtrsim \frac{w_0}{\delta_0} \qquad (8.19)$$

and

$$\frac{w_0^{1/2}}{\kappa} \lesssim \frac{\delta_0}{w_0} . \qquad (8.20)$$

The condition (8.19) is equivalent to

$$T \lesssim \frac{1}{(32\pi f)^{1/2}}\cdot\frac{\delta_0}{w_0}D_r^{1/4}m_p . \qquad (8.21)$$

Further we demand

$$D_r \gtrsim (32\pi f)^2\left(\frac{w_0}{\delta_0}\right)^4\cdot\frac{\rho_v}{m_p^4} \qquad (8.22)$$

so that the era (8.21) incldes $T \lesssim \rho_v^{1/4}$.

When the inequality (8.19) is satisfied, we can show that the condition under which quantum effects do not break the classical stability of the universe is (6.13),

$$f \gtrsim 2\lambda\tilde{\alpha}\cdot\frac{w_0}{\delta_0} \tag{8.23}$$

and

$$\frac{\rho_v}{m_p^4} \gtrsim \frac{1}{128\pi^2 f}\left(\frac{\delta_0}{w_0}\right)^3 . \tag{8.24}$$

To summarize, the conditions under which linearized stable solution of dSU exist are (8.10),(8.13),(8.14),(8.20),(8.22), (8.23),(8.24),(6.13) and

$$\delta_0/w_0 \lesssim 10^{-4}. \tag{8.25}$$

In Fig.4, we show the region restricted by (8.10),(8.13),(8.20), (8.22),(8.23),(8.24) and (8.25). Here we parametrize as (6.17), (6.19), $\delta_0/w_0=10^n$ and $\rho_v/m_p^4=10^{-4s}$, and set, for example, $p=p_2=60$ and s=4(region I corresponding to vacuum energies of PT of SU(5)-GUT;$\rho_v^{1/4}=10^{15}$ GeV) and s=9(region II corresponding to $\rho_v^{1/4}$ $=10^{10}$ GeV). We notice that the lower the vacuum energy becomes, the wider the stable region becomes. For SU(5)-GUT case, it is concluded from Fig.4 and (8.15a) that to stabilize dSU with k=0,±1 we need (6.13),

$$f \gtrsim 10^{1.8} \tag{8.26}$$

and

$$D_r \gtrsim 10^{8.4} \quad (\gtrsim 10^{13.6} \quad \text{for } k=1) \tag{8.27}$$

For the case $\rho_v^{1/4}=10^{10}$ GeV, on the other hand, we obtain

$$f > 10^{6.8} \tag{8.28}$$

and

$$D_r \gtrsim 10^{18.4} \quad (\gtrsim 10^{33.6} \quad \text{for } k=1) \tag{8.29}$$

Lastly, we obtain

$$F \simeq \frac{9\beta\kappa\delta_0 A_0}{2b\dot{A}_0} \sin\kappa t \,, \tag{8.30}$$

$$\dot{F} \simeq \frac{9\beta\kappa^2\delta_0 A_0}{2b\dot{A}_0} \cos\kappa t \tag{8.31}$$

and

$$S(t) \simeq -\frac{3a\beta\delta_0 A_0}{2b^2\dot{A}_0} \cos\kappa t \,. \tag{8.32}$$

§ 9. Summary and discussions

We have shown that at classical level of PGT the three types of the Friedmann universe(RDU, MDU and dSU) with $k=0,\pm 1$ are stable under linear and harmonic oscillater approximation if the parameters of PGT are chosen properly. Then, we have shown that quantum effects due to vacuum polarization at one loop level do not break this classical stability of the universe if we choose the parameters of PGT, the the total entropy of the universe and others properly.

In this section, we summarize the conditions which are needed for stable RDU, MDU and dSU in common, using simple notation $\Delta=\xi_0/U_0=\zeta_0/v_0=\delta_0/w_0$, where $\Delta << 1$ under linear approximation (for definiteness we require $O(\Delta) \lesssim 10^{-4}$). The conditions individual to each three universes are listed in Table 1.

At the classical level of PGT, the three types of the Friedmann universe with $k=0,\pm 1$ are stable under the conditions $f>0$ and $\beta<0$;then there exist the small oscillative solutions around SBBS of GR with the common frequency $\kappa=\sqrt{\frac{ab}{3f(-\beta)}}$(for dSU the frequency is $\kappa' \simeq \kappa$).

In the presence of quantum effects due to vacuum polarization we can show that the three types of the Friedmann universe are still stable for each era under the conditions $f \gtrsim 2\lambda\tilde{\alpha}\Delta^{-1}$, $-\beta \gtrsim a$ and $\kappa A \gtrsim \Delta^{-1}$. The last condition result in individual conditions for D_r (see Table 1).

From these and the individual conditions listed in Table 1, we obtain the stable parameter regions of RDU, MDU and dSU, respectively(see Figs.2, 3 and 4).

For RDU and MDU the stable regions are basically triangle regions surrounded by three lines, as for RDU(Fig.2),(1),(3) and (4). These lines (1),(3) and (4) express the inequalities (5.15),(5.31) and (6.12), respectively. The condition (5.15) justifies linear approximation. Under the condition (5.31) quantum effects can be estimated and suppressed for $T \lesssim m_p$. We notice that the larger the value of D_r becomes, the upper the line (3) is located and the wider the stable region becomes. This means that the more radiation(entropy) becomes, the more stable the universe has a tendency to be. The condition (6.12) also suppresses quantum effects. We conclude the necessary condition for f and D_r in Table.2. Because the coupling between torsion and fermion fields is given by $\sim 1/\sqrt{f}$ in PGT[14)], the conditions $f \gtrsim 10^{1.6}$ mean that this coupling can be treated by perturbation method. After PT we know $D_r \gtrsim 10^{112}$ from observation[13)], so conditions for D_r in Table 2 are satisfied. Before PT, we may consider these conditions of Table.2 restrict the value of D_r.

For dSU we notice that the lower ρ_V becomes, the wider the stable region becomes, and that f is restricted by $10^{1.8} \lesssim f \lesssim 10^6$ and $10^{6.8} \lesssim f \lesssim 10^{26}$ for the region I and II, respectively. The necessary conditions for f and D_r are listed in Table.2 and above discussions for f and D_r are also applicable for dSU case.

In Table 3, we show comparison between GR and PGT in the classical and the semi-classical theory. Why the universe can be stable in PGT? To see this, let us compare the equation of GR which contains quantum effects(see (3.6) and (3.10)) with classical equation of PGT(see (5.4)). In GR with quantum effects, the potential has a maximum and the universe becomes unstable(see Fig.1) because of the positive sign of $A^{(3)}$-term in RHS of (3.6). However, in PGT, we can choose the parameters f and β freely under the restriction of (4.7) and (4.8) so that the potential has a minimum point which corresponds to SBBS of GR.

There still remain unsolved problems. First, for the case k=1 we have shown the stability for the era except neighbourhood of the singular point $\dot{A}=0$(see (5.12)). We can say that the smaller θ becomes, the narrower the stable region becomes. Secondly, for the case $T \simeq m_i$ we do not know the form of quantum effects due to vacuum polarization in PGT, so that we have not shown stability of the universe.

So far in this paper we investigate the stability problem with the semi-classical picture, that is, at the level of one-loop quantum correction. With the full quantum theory in which the gravitational field is quantized as well as the matter field, there is a possibility that we can treat the stability problem more completely and solve above remaining problem. Now this is being investigated.

Table 1

The various conditions for classical and semi-classical stability of RDU, MDU and dSU with $k=0,\pm1$.

	RDU*	MDU	dSU
(1) classical stability	$f > 0 \qquad \beta < 0$		
		$\frac{fD_m}{m_p^4 A^3} << 1$	
(2) classical small oscillative solution	$x=U_0+\xi_0\cos\kappa t$ $(\xi_0/U_0 << 1)$	$y=v_0+\xi_0\cos\kappa t$ $(\xi_0/v_0 << 1)$	$z=w_0+\delta_0\cos\kappa' t$ $(\delta_0/w_0 << 1)$
	$\kappa^2=\frac{ab}{3f(-\beta)}$		$\kappa'^2=\kappa^2+12w_0\simeq\kappa^2$
(3) avoidance of singularity at $\dot{A}\simeq0$ (only k=1)	$0<A_0<0.9U_0^2$	$0<A_0<0.9v_0$	$A_0>\sqrt{2}A_{min}$
(4) harmonic osci. approximation is available	$\kappa A_0 >> 1$		
(4′) conditions: (4) for individual era	$D_r>>(32\pi f)^2$ $(T \lesssim m_p)$	$D_r>>(32\pi f)^2\ \frac{T_1}{m_p}$ $(T \lesssim T_1)$	$D_r>>(32\pi f)^2\ \frac{\rho_v}{m_p^4}$ $(T \lesssim \rho_v^{1/4})$
(5) harmonic osci. approximation is available	$D_r >> 72f$ (only k=±1)	$\begin{cases} (16\pi)^{\frac{3}{2}}\frac{f^{1/2}D_m}{m_p^3A^2} << 1 & (k=\pm1) \\ \frac{(16\pi)^{\frac{3}{2}}}{3^{1/2}}\frac{f^{1/2}D_m^{3/2}}{m_p^4A^{5/2}} << 1 & (k=0) \end{cases}$	$\frac{\rho_v}{m_p^4} << \frac{3}{12544\pi^2}\cdot\frac{1}{f}$
(6) the conditions under which quantum effects are estimated	$\kappa A_0 \gtrsim \frac{U_0}{\xi_0}$	$\kappa A_0 \gtrsim \frac{v_0}{\xi_0}$	$\kappa A_0 \gtrsim \frac{w_0}{\delta_0}$ $\frac{w_0^{1/2}}{\kappa'} \lesssim \frac{\delta_0}{w_0}$
(6′) conditions: (7) for individual era	$D_r\gtrsim(32\pi f)^2\left(\frac{U_0}{\xi_0}\right)^4$ $(T \lesssim m_p)$	$D_r\gtrsim(32\pi f)^2\left(\frac{v_0}{\xi_0}\right)^4\left(\frac{T_1}{m_p}\right)^4$ $(T \lesssim T_1)$	$D_r\gtrsim(32\pi f)^2\left(\frac{w_0}{\delta_0}\right)^4\frac{\rho_v}{m_p^4}$ $(T \lesssim \rho_v^{1/4})$
(7) stability with quantum corrections	$f \gtrsim 2\lambda\tilde{\alpha}\ \frac{U_0}{\xi_0}$	$f \gtrsim 2\lambda\tilde{\alpha}\ \frac{v_0}{\xi_0}$	$f \gtrsim 2\lambda\tilde{\alpha}\ \frac{w_0}{\delta_0}$
		$-\beta \gtrsim a$	
	$D_r \gtrsim 6\lambda\tilde{\alpha}\left(\frac{U_0}{\xi_0}\right)^2$ (only k=±1)	$\frac{D_m}{m_p} \gtrsim 1$ (only k=±1)	$\frac{\rho_v}{m_p^4} \lesssim \frac{1}{128\pi^2 f}\left(\frac{\delta_0}{w_0}\right)^3$

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* If ρ_d is not neglected in RDU, we need the condition $f < 34$ in addition to above conditions.

Table 2

The necessary conditions for f and D_r under which RDU, MDU and dSU are stable at individual temperature regions.

	f	D_r
RDU $(T \lesssim m_p)$	$\gtrsim 10^{1.6}$	$\gtrsim 10^{23.2}$
MDU $(T \lesssim T_1, D_m/m_p = 10^{58} = m_p A)$	$\gtrsim 10^{1.6}$	$\gtrsim 10^{-103.2}$
dSU $(T \lesssim \rho_v^{1/4} = 10^{15}$GeV;SU5-GUT)	$10^{1.8} \lesssim f \lesssim 10^{6}$	$\gtrsim 10^{8.4}$ ($\gtrsim 10^{13.6}$ for k=1)
dSU $(T \lesssim \rho_v^{1/4} = 10^{10}$GeV)	$10^{6.8} \lesssim f \lesssim 10^{26}$	$\gtrsim 10^{18.4}$ ($\gtrsim 10^{33.6}$ for k=1)

Table 3

The comparison between GR and PGT about the order of the differential equation and stability.

	classical theory	semi-classical theory
GR	1st-order	3rd-order unstable
PGT	3rd-order stable	5th-order (effectively 3rd-order) stable at least in certain parameter region

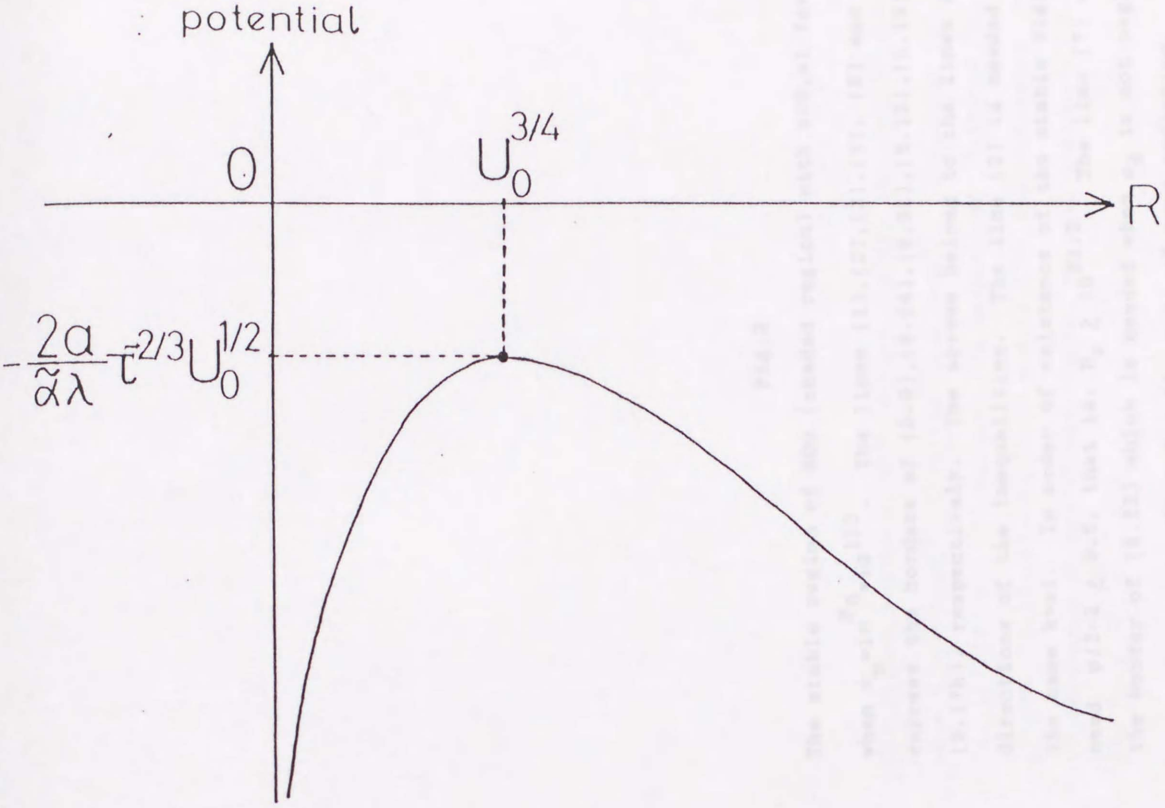

Fig.1

The potential of the motion equation (3.10) for the case $m_p t \gg 1$ and $R=O(U_0^{3/4})$. It takes the form

$$\text{pot.} = \frac{-a}{\tilde{\alpha}\lambda}\tau^{-2/3}\left(R^{2/3} + U_0 R^{-2/3}\right) \qquad \left(\begin{array}{l}\text{integration constant}\\ \text{is set to zero}\end{array}\right)$$

and has a maximum at $R=U_0^{3/4}$. The resting solution at this point corresponds to the classical RDU solition of GR.

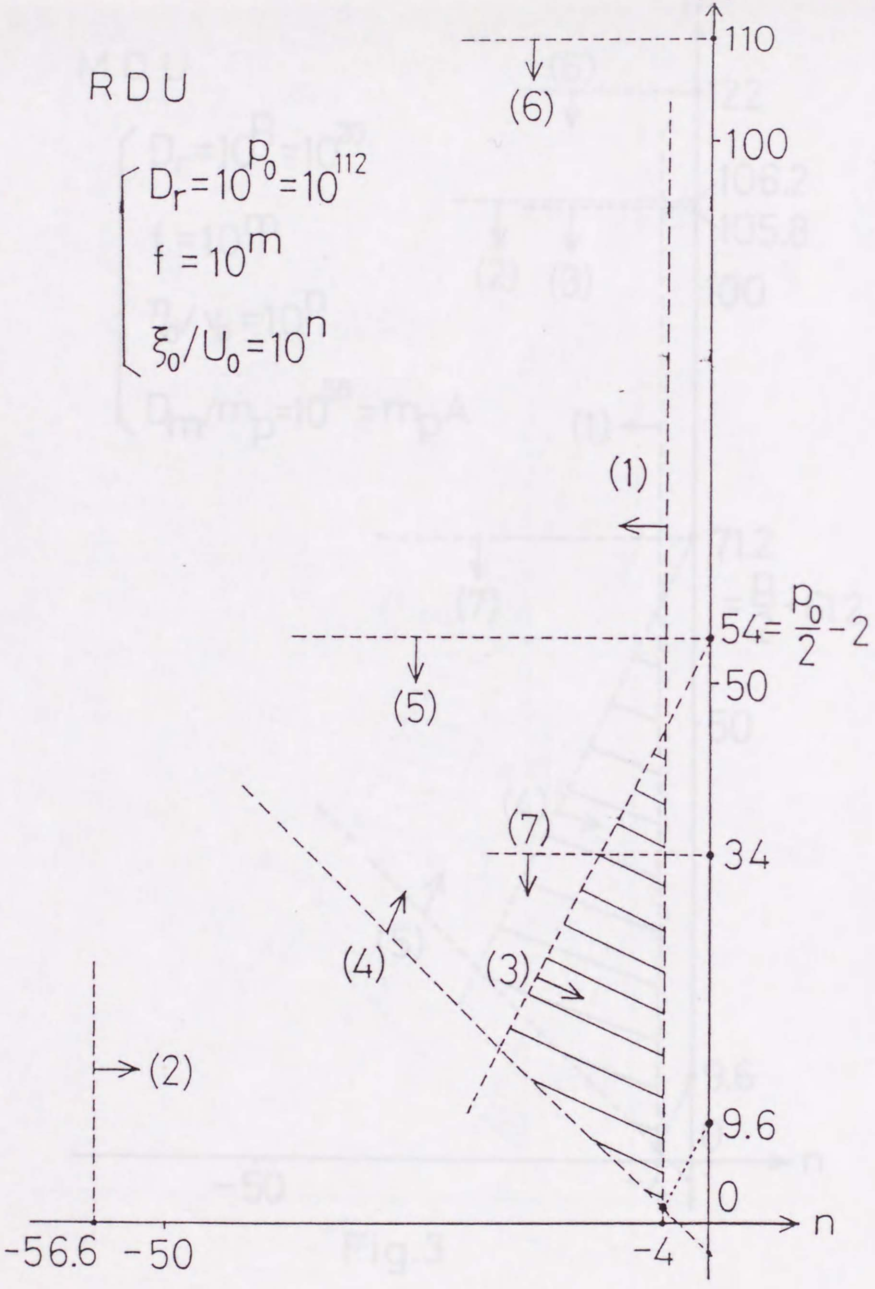

Fig.2

Fig.2

The stable region of RDU (shaded region) with k=0,±1 for $T \lesssim m_p$, when $D_r=10^{p_0}=10^{112}$. The lines (1),(2),(3),(4), (5) and (6) express the borders of (5.9),(6.14),(5.31),(6.12),(5.19) and (5.17b), respectively. The arrows belong to the lines show directions of the inequalities. The line (2) is needed only for the case k=±1. In order of existance of the stable region, we need $p/2-2 \gtrsim 9.6$, that is, $D_r \gtrsim 10^{23.2}$. The line (7) expresses the border of (6.22) which is needed when ρ_d is not neglected. We need not this line for the case of pure radiation.

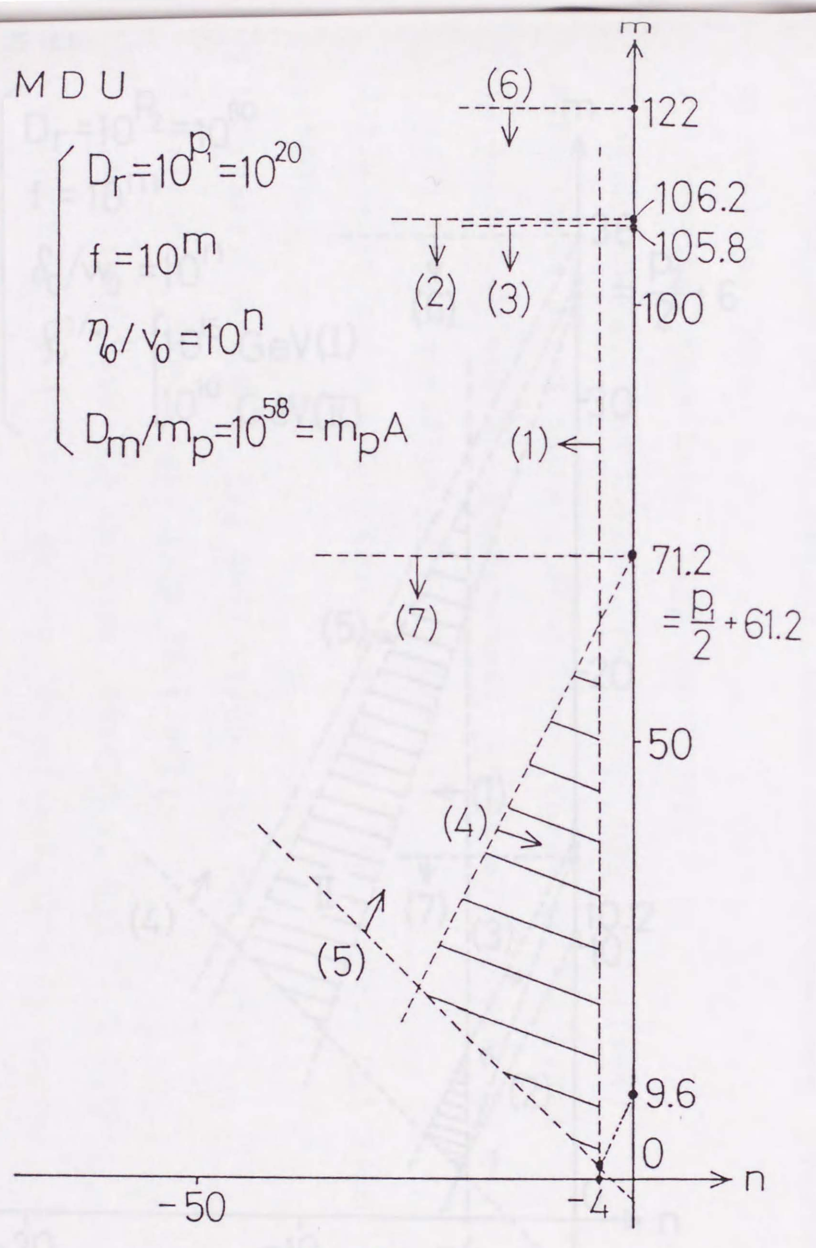

Fig.3

Fig.3

The stable region of MDU (shaded region) with k=0,±1 for $T \lesssim T_1$, when $D_r=10^{p_1}=10^{20}$, $D_m/m_p=10^{58}$, $m_pA=10^{58}$. The lines (1),(2),(3),(4),(5),(6) and (7) express the borders of (7.20),(7.10a),(7.10b),(7.17),(7.18),(7.5) and (7.12), respectively. In order of existance of the stable region, we need $p/2+61.2 \gtrsim 9.6$, that is, $D_r \gtrsim 10^{-103.2}$.

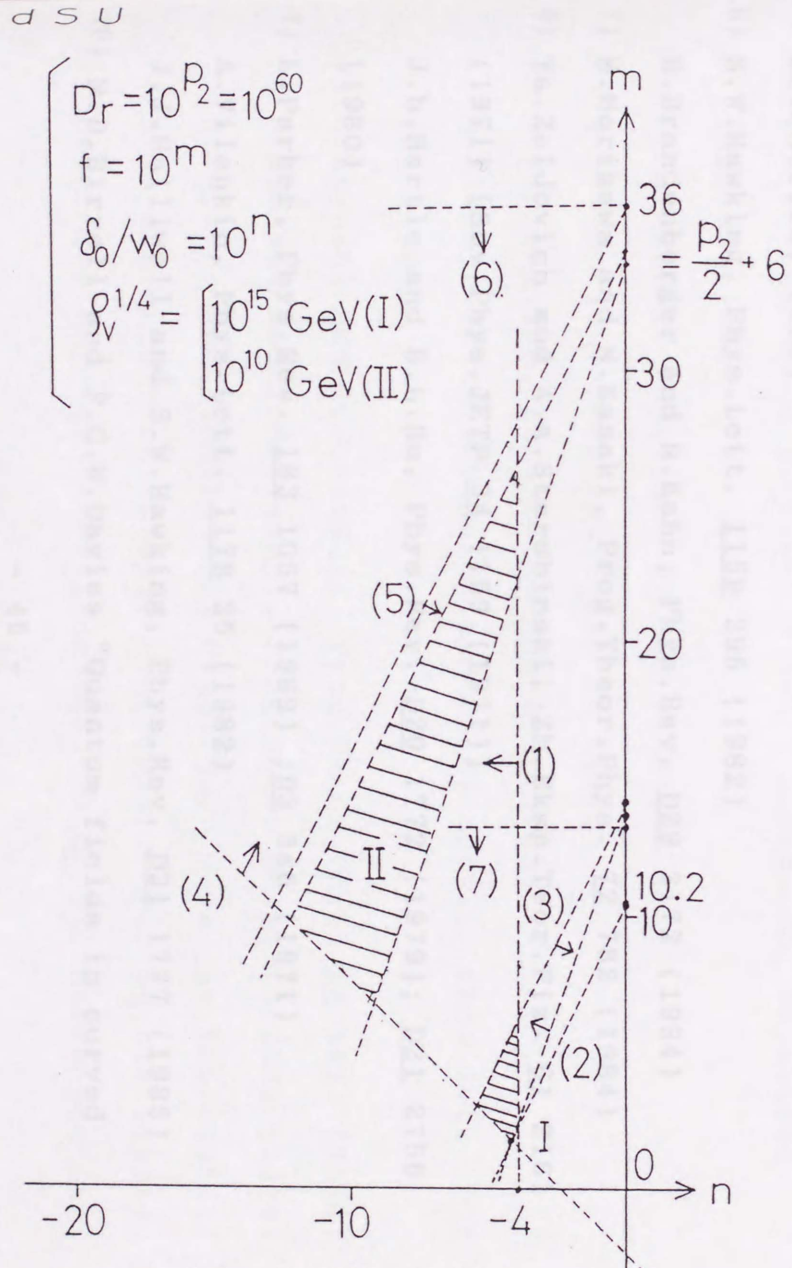

Fig.4

Fig.4

The stable region of dSU (shaded region) with k=0,±1 for $T \lesssim \rho_v^{1/4}$, when $D_r = 10^{p_2} = 10^{60}$, $\rho_v^{1/4} \simeq 10^{15}$ GeV (region I) and $\rho_v^{1/4} \simeq 10^{10}$ GeV (region II). The lines (1),(2),(3),(4),(5),(6) and (7) express the borders of (8.25),(8.24),(8.20),(8.23),(8.22),(8.13) and (8.10) for the case $\rho_v^{1/4} \simeq 10^{15}$GeV, respectively. In order of existance of the region I, we need $p/2+6 \gtrsim 10.2$ that is $D_r \gtrsim 10^{8.4}$. Similarly, for region II we need $D_r \gtrsim 10^{18.4}$.

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$$A^2\dot{A}\ddot{A} \simeq - \frac{\kappa\xi_0}{2}\cdot\sin\kappa t = \frac{\dot{\varepsilon}}{2} .$$

(The relations (5.24)~(5.26) remain same.) However, (5.23)~(5.26) are sufficient for the estimation of quantum effects. So in the following we shall restrict ourselves to the era which satisfies (5.22).

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