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A STATISTICAL STUDY ON THE PREDICTION OF RANDOM NOISE FLUCTUATION AND
THE MUTUAL RELATIONSHIP BETWEEN VARIOUS NOISE EVALUATION INDICES IN AN
ACOUSTIC ENVIRONMENT

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ABSTRACT

Industrial growth carries with it an increasing environmental noise pollution problem. In order to evaluate quantitatively environmental random noise, many statistical noise descriptors (i.e., noise evaluation indices) are now being used. Percentile noise level L_x , equivalent noise level L_{eq} , traffic noise index TNI and perceived noise level PNL are some examples. Almost all these statistical indices are obtained in relationship with the whole probability distribution of environmental random noise. Noise surveys have indicated that road traffic noise is a major component of environmental random noise. In this doctoral thesis, a statistical method for prediction of road traffic noise generated by actual traffic flow of an arbitrary non-Poisson type is first discussed for practical application. Paying attention to the fact that sound insulation barriers are very often constructed to produce attenuation of environmental random noise, a method for predicting the stochastic insulation effect of a sound insulation barrier is then discussed for the case of arbitrary probability distribution of random noise incident on the barrier. In addition, the mutual relationships between various types of statistical noise evaluation indices in relation to L_x and L_{eq} , which play an important role in the field of noise evaluation and regulation problems, are discussed for the purpose of evaluating systematically real random noise. More specifically, a method for estimating an L_{eq} evaluation index is proposed using the moment statistics of random noise fluctua-

tion. Next, a method for estimation of L_{eq} is presented, using the known values of several specific L_x . A new trial estimating the original distribution form of random noise fluctuation is then presented using the known values of L_{eq} and several specific L_x , and a specific on-line measurement system based on the above-mentioned estimation theory for L_x and L_{eq} is constructed with the aid of a microcomputer by introducing an iterative process for extracting the moment statistics of random noise fluctuation. The theoretical approaches in this study are generally applicable to any kind of random phenomena. The theoretical results are experimentally confirmed by applying them to actually observed noise data.

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INTRODUCTION

Introduction

In recent years, problems resulting from environmental noise generated by various forms of transformation and industry have become more critical. In order to quantitatively assess real random noises, it is essential that a precise method for evaluation of these random noises taking human responses to these stochastic phenomena into consideration be found. When the problem of statistical evaluation of such stochastic phenomena is discussed, the following problems appear to be primary importance from the methodological viewpoint :

- (1) Many kinds of statistical noise evaluation indices, such as L_{eq} , TNI, L_{NP} and L_x ($x=5, 10, \dots$; i.e., (100-x) percentage point of noise level distribution) have been proposed, almost all of which can be obtained in close relation to the entire probability distribution.
- (2) Environmental random noises encountered in daily life give various kinds of probability distribution form other than the well-known Gaussian distribution, owing to the diversity of causes of fluctuation.
- (3) Two of the many noise evaluation indices, L_x and L_{eq} , play an important role in the field of noise evaluation and regulation.
- (4) In order to evaluate environmental random noise accurately, methods of prediction other than the well-known conventional methods derived from simplified models or standard probability expressions must be employed because of various factors. Not only physical

factors but also other more complicated factors related to environmental conditions, individual psychological response, etc., must be taken into account.

These practical points are dealt with in this doctoral thesis in a statistical method for prediction of noise level distribution and the mutual relationships between various kinds of evaluation indices in close relation to L_x and L_{eq} .

This thesis is divided into an Introduction, Parts I and II, consisting of Chapters 1-3 and Chapters 1-4 respectively the Conclusion and References. The outline of each part and chapter is given below.

In Part I, a general method for predicting the probability distribution form of environmental random noise is considered. In the problem of environmental noise control, a method for systematic prediction of the statistical properties of environmental noise is naturally of great importance. Furthermore, attention must be paid to the fact that environmental noise pollution is caused primarily by road traffic noise.

Chapter 1 discusses a statistical method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow. Until now, various approaches for predicting road traffic noise have been proposed from various points of view. These prediction methods can essentially be divided into the following two groups. One group comprises structural methods based on physical models. The other comprises functional methods based on mathematical models. The

former is constructed by accounting for various types of physical mechanisms such as traffic flow, propagation characteristics, etc. Thus, the more such physical mechanisms are taken into account in detail, the more complex the prediction method itself becomes. The latter is constructed by pre-establishing a mathematical model for the actual noise fluctuation itself. If only the latter method is used, quantitative inference of physical clues to practical noise control is not possible. Accordingly, a kind of hybrid method combining the latent advantages of the two prediction groups was proposed in a previous study. In order to establish a prediction method of this kind for road traffic noise generated from actual traffic flow of an arbitrary non-Poisson type, a new approach for practical use equivalent to the approach used for idealized Poisson type traffic flow is proposed in this chapter. The proposed method is applicable to treat any kind of road traffic noise fluctuation generated from actual non-Poisson type traffic flow, once information on the elementary response wave form of the level time pattern observed when one vehicle passes directly in front of an observation point has been obtained. The effectiveness of the proposed prediction method is experimentally confirmed by applying it to actually observed road traffic noise data.

Chapter 2 discusses a practical method for prediction of road traffic noise at a T-type road intersection based on the image method. In the previous chapter, a general method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow based on information on the elementary response wave form observed

experimentally is proposed. In order to establish a more effective method of prediction, a theoretical method for estimating the above-mentioned elementary response wave form, especially in a complex realistic acoustical environment must be found. In this chapter, a new approach for theoretical estimation of this elementary response wave form is proposed for a T-type road intersection with sound insulation barriers, using a well-known image method. The effectiveness of the proposed method is confirmed experimentally by applying it to road traffic noise data obtained at a T-type road intersection in a city area.

Chapter 3 describes a study on the probabilistic response of a sound insulation barrier. In the practical engineering field of noise control, sound insulation barriers are often constructed to produce attenuation of environmental random noise such as the road traffic noise discussed in the previous chapters. The acoustical design and evaluation problems of barriers have already been considered by many researchers. Almost all of these studies, however, were confined only to the effects on deterministic signals or the gross average evaluation of shielding effects. In this chapter, a method for prediction of the stochastic insulation effect of a sound insulation barrier in the case of arbitrary probability distribution of random noise incident on the barrier is proposed. The emphasis in this chapter is on how the output noise distribution form may be predicted using information on the statistical properties of random input noise and the frequency characteristics of the sound insulation barrier. The effectiveness of

the proposed prediction method is experimentally confirmed using actual noise data.

In Part II, the relationships between various statistical evaluation indices in relation to L_x and L_{eq} for precise evaluation of real environmental random noise are considered. As previously mentioned, two of these indices, L_x and L_{eq} , are very important in the field of noise evaluation and regulation. The establishment of a general theory of mutual relationship for these indices is of primary importance for systematic evaluation of environmental random noise. Studies for establishing mutual relationships of this kind have been carried out by many researchers. All most all these studies, however, have been limited to approximate methods derived from the assumption of an ordinary Gaussian distribution and/or practical methods derived by applying the conventional linear regression analysis method to actually observed data. Part II establishes a general theory for mutual relationship between various noise evaluation indices with emphasis on L_x and L_{eq} , taking the previously mentioned practical points into consideration.

Chapter 1 deals with a method for estimation of L_{eq} using the moment statistics of noise level fluctuation of an arbitrary non-Gaussian distribution type. To estimate the value of L_{eq} for an arbitrary non-Gaussian type random noise fluctuation, a new approach for practical application equivalent to that used for an ordinary Gaussian type is here proposed for the first time. The proposed estimation method is given in a generalized form universally applicable to any kind of random phenomena of a non-Gaussian property, including a

well-known simplified expression derived under assumption of ordinary Gaussian distribution. Finally, in order to confirm the effectiveness of this method, it is applied to actual road traffic noise data.

Chapter 2 describes the mutual relationships between several statistical indices connected with L_x and L_{eq} . A method for estimation of L_{eq} is presented using the known values of several specific L_x with the use of the mean and variance of the random noise fluctuation. The proposed estimation formula is also given in a generalized form and includes the well-known simplified expression mentioned above. Next, a new trial for estimation of the original distribution form of the random noise fluctuation is presented using the known values of L_{eq} and several specific L_x . The effectiveness of the proposed method is experimentally confirmed by applying it to actual road traffic noise data.

Chapter 3 also describes a method for estimation of the original distribution form of the random noise fluctuation, as discussed in Chapter 2. In this case, a more suitable expression of the probability distribution form is newly introduced in a form matched to the acoustic measurement on a decibel scale. The effectiveness of the proposed method is experimentally confirmed by applying it to actual road traffic noise data, and through comparisons of estimation accuracy between the proposed method and the method described in Chapter 2.

Chapter 4 discusses a precise method for calculation of L_x and L_{eq} noise evaluation indices using information on noise level fluctuation (moment statistics or cumulant statistics), based on the previous

discussion in Chapter 2. Furthermore, a specific on-line measurement system based on this method is constructed with the aid of a microcomputer by introducing an iterative process for extracting moment statistics of random noise fluctuation on a decibel scale. The effectiveness of the proposed method is experimentally confirmed by applying it to actual measurement of road traffic noise.

The conclusion summarizes the results obtained in this study, and future research works on statistical prediction and evaluation of this kind are described. References are given and a list of the author's publications is provided.

PART I

STATISTICAL METHOD FOR PREDICTION OF RANDOM NOISE FLUCTUATION

Chapter 1 A Method for Prediction of Road Traffic Noise Generated from Non-Poisson Type Traffic Flow

1.1 Introduction

Various methods for prediction of road traffic noise have been proposed.⁽¹⁾⁻⁽⁹⁾ From the methodological point of view, they can be divided into two basic groups, with one group comprising structural methods based on physical models and the other functional methods based on mathematical models. The former is designed to account for various types of physical mechanisms such as traffic flow, propagation characteristic, the geometrical structure of the road, etc. The more such physical mechanisms are taken into account, and the more detailed they are, the more complex the prediction method itself becomes. The latter, however, is constructed by pre-establishing a mathematical model for the actual noise fluctuation pattern itself. If only the latter method is used in the prediction of road traffic noise, it is not possible to infer quantitatively physical clues to practical noise control improvement. Accordingly, it would be effective to introduce some kind of hybrid method combining the latent advantages of both methods of prediction.⁽⁸⁾

In actual traffic flow, a general, arbitrary non-Poisson type flow distribution is more often seen than the idealized Poisson type, since traffic conditions are rarely ideal. Furthermore, actual road

traffic noises exhibit various types of probability distributions, due to the diversified causes of the noise fluctuations.

In this chapter, a practical method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow is proposed. The method is based on an approach equivalent to that used for an idealized Poisson type traffic flow. It allows for treatment of any kind of road traffic noise fluctuation, provided that information on the actual response wave form generated by the surrounding sound propagation environment of a level time pattern observed when one vehicle passes directly in front of an observation point can first be obtained.

The effectiveness of the proposed prediction method has been experimentally confirmed by applying it to observed road traffic noise data.

1.2 Theoretical consideration

1.2.1 Description of problem

Considering the wave form of noise level fluctuation at an observation point, as shown in Fig.1.1.1, let $w(\xi_i)(\xi_i \Delta t - \tau_i)$ be the normalized response wave form $w(0)=1$ observed when one vehicle passes directly in front of an observation point during a specific time

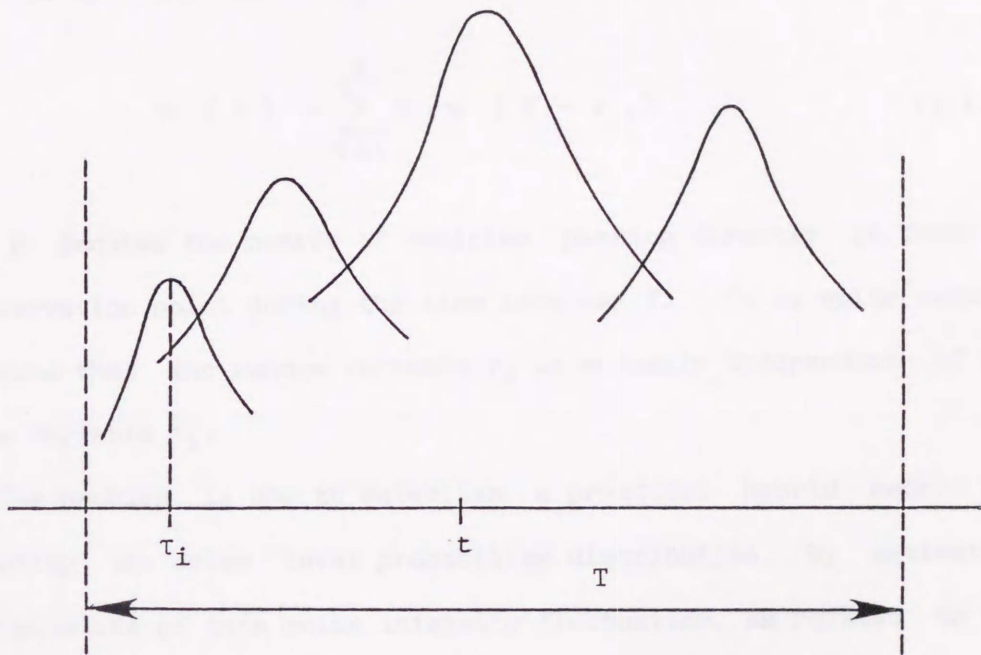


Fig.1.1.1 Time pattern of the noise level fluctuation generated by vehicles passing directly in front of an observation point.

interval, T . It is supposed that the i th vehicle generates the noise intensity Y_i at a time $t = \tau_i$. At this time, based on the additive property of energy quantities, the total noise intensity, $x(t)$, at a time t is directly expressed as

$$x(t) = \sum_{i=1}^N Y_i w(t - \tau_i), \quad (1.1.1)$$

where N denotes the number of vehicles passing directly in front of an observation point during the time interval T . It is quite natural to assume that the random variable Y_i is mutually independent of the random variable τ_i .

The problem is how to establish a practical hybrid method for predicting the noise level probability distribution by estimating the statistics of this noise intensity fluctuation, as related to the actual traffic flow and the elementary response wave form $w(\xi_i)$.

1.2.2 Arbitrary non-linear function type statistics for an arbitrary non-Poisson type traffic flow

The main purpose of this section is to present an explicit derivation of an expression for an arbitrary non-linear function type expectation value which is equivalent to that for a well-known Poisson type probability function. This will provide the advantage of being able to use the technical properties of Poisson's law. Let $z_i (\in [z_L, z_M])$ be the i th quantized amplitude with an amplitude difference in-

terval h . According to the previous work,⁽¹⁰⁾ the probability function $P(z_i)$ for a quantized random variable z_i can be generally expressed in a statistical series expansion form, as

$$P(z_i) = \sum_{n=0}^{\infty} b_n \left(\frac{d}{h} \right)^n \nabla^n P_x(z_i) \quad (1.1.2)$$

with

$$b_n \triangleq \frac{(-1)^n}{n!} \langle (z_i - x_i)^{(n)} \rangle_{z_i}, \quad (1.1.3)$$

where $P_x(x_i)$ is the basic probability function for a quantized random variable x_i with an amplitude difference interval h , which can be, artificially or arbitrarily, established in advance from the operational viewpoint. In Eq.(1.1.2), $\langle \cdot \rangle_{z_i}$ denotes an expectation operation with respect to z_i and $(\cdot)^{(n)}$ denotes the factorial function defined by

$$\xi_i^{(n)} \triangleq \xi_i (\xi_i - h) \cdot \cdot \cdot [\xi_i - (n-1)h] \quad (n \geq 1),$$

$$\xi_i^{(0)} \triangleq 1. \quad (1.1.4)$$

∇ denotes the backward difference operator :

$$\nabla f(z) \triangleq (1/d) [f(z) - f(z-h)] \quad (1.1.5)$$

with two arbitrary constants, d and h . In Eq.(1.1.5), the constant d is introduced for the purpose of relating the discrete amplitude type expression ($d=1$), and the continuous amplitude type expression ($d=h \rightarrow 0$). Thus, after introducing an arbitrary non-linear function $g(\cdot)$ with respect to z_i , its expectation value is directly given by using Eq.(1.1.2), as

$$\begin{aligned} \sum_{i=L}^M g(z_i) P(z_i) &= \sum_{n=0}^{\infty} b_n \left(\frac{d}{h}\right)^n \sum_{i=L}^M g(z_i) \nabla^n P_x(z_i) \\ &= \sum_{n=0}^{\infty} b_n \left(\frac{d}{h}\right)^n (-1)^n \sum_{i=L}^M \Delta^n g(z_i) P_x(z_i). \end{aligned} \quad (1.1.6)$$

Hereupon, the relationship :

$$\begin{aligned} G(z_i) \nabla F(z_i) &= \Delta \left\{ G(z_i) F(z_{i-1}) \right\} \\ &\quad - F(z_i) \Delta G(z_i) \end{aligned} \quad (1.1.7)$$

and a new type forward difference operation :

$$\Delta f(z) \triangleq (1/d) [f(z+h) - f(z)] \quad (1.1.8)$$

(Δ is the forward difference operator) have been used.

Upon employing, respectively, a quantized random variable z_i , the basic probability function $P_x(\cdot)$, with the number of vehicles being denoted by $N(\in[0, M])$ and a Poisson type probability function^{(11),(12)} :

$$P_x(N) = \frac{1}{N!} e^{-\lambda} \lambda^N \quad (1.1.9)$$

with

$$\lambda \triangleq \langle N \rangle_N, \quad (1.1.10)$$

Eq.(1.1.6) can easily be rewritten as

$$\begin{aligned} \langle g(N) \rangle_N & \triangleq \sum_{N=0}^M g(N) P(N) \\ & = \sum_{n=0}^{\infty} b_n (-1)^n \sum_{N=0}^M \Delta^n g(N) P_x(N) \\ & = \left\langle \sum_{n=0}^{\infty} b_n (-1)^n \Delta^n g(N) \right\rangle_x, \quad (1.1.11) \end{aligned}$$

for traffic flow with an arbitrary type probability function $P(N)$. Here, the two arbitrary constants are taken to be $h=d=1$ and $\langle \cdot \rangle_x$ denotes an expectation based on a Poisson type probability function $P_x(N)$. The expansion coefficient b_n is given by ⁽¹⁰⁾

$$b_n = \frac{(-1)^n}{n!} \left\{ \langle N^{(n)} \rangle_N - \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} b_k \lambda^{n-k} \right\},$$

$$b_0 = 1. \quad (1.1.12)$$

From Eq.(1.1.11), it should be noticed that the expectation value $\langle g(N) \rangle_N$ in terms of an arbitrary non-Poisson type probability function can be equivalently calculated based on an expectation operation in terms of an ordinary Poisson type probability function.

1.2.3 Prediction of road traffic noise by use of moment statistics for the noise intensity fluctuation

The conditional moment generating function $\langle e^{\theta x} | N \rangle_x$ with respect to the noise intensity fluctuation $x(t)$ can be regarded as an arbitrary non-linear function form $g(N, \theta)$. That is, all the characteristics of the non-Poisson distributions can be related to the conditional moment generating function $\langle e^{\theta x} | N \rangle_x$ thus producing the moment statistics $\langle x^m \rangle$:

$$g(N, \theta) \triangleq \langle e^{\theta x} | N \rangle_x. \quad (1.1.13)$$

The moment generating function $m(\theta) (\triangleq \langle \exp(\theta x) \rangle)$ can be directly derived by use of a relation equivalent to an ordinary Poisson type in Eq.(1.1.11), as follows :

$$m(\theta) = \langle \langle e^{\theta x} | N \rangle_x \rangle_N = \sum_{n=0}^{\infty} b_n (-1)^n \sum_{N=0}^{\infty} \Delta^n \langle e^{\theta x} | N \rangle_x \frac{1}{N!} e^{-\lambda} \lambda^N. \quad (1.1.14)$$

By substituting the difference calculation based on Eq.(1.1.1) :

$$\begin{aligned} \Delta^n \langle e^{\theta x} | N \rangle_x &= \Delta^n \langle e^{\theta Y_w} \rangle_{Y, w}^N \\ &= \langle e^{\theta Y_w} \rangle_{Y, w}^N (\langle e^{\theta Y_w} \rangle_{Y, w} - 1)^n \end{aligned} \quad (1.1.15)$$

into Eq.(1.1.14), one can subsequently obtain

$$m(\theta) = \sum_{n=0}^{\infty} b_n (-1)^n (\langle e^{\theta Y W} \rangle_{Y, W} - 1)^n \cdot e^{-\lambda} \sum_{N=0}^{\infty} \frac{1}{N!} \langle e^{\theta Y W} \rangle_{Y, W}^N \lambda^N. \quad (1.1.16)$$

Furthermore, by using two calculation procedures based on a Taylor series expansion :

$$\begin{aligned} (\langle e^{\theta Y W} \rangle_{Y, W} - 1)^n &= \left(\sum_{m=1}^{\infty} \frac{\theta^m}{m!} \langle Y^m \rangle \langle W^m \rangle \right)^n, \\ e^{-\lambda} \sum_{N=0}^{\infty} \frac{1}{N!} \langle e^{\theta Y W} \rangle_{Y, W}^N \lambda^N &= e \times p \left\{ \lambda \sum_{m=1}^{\infty} \frac{\theta^m}{m!} \langle Y^m \rangle \langle W^m \rangle \right\}, \end{aligned} \quad (1.1.17)$$

the moment generating function $m(\theta)$ can be consequently obtained as follows :

$$\begin{aligned} m(\theta) &= m_0(\theta) \sum_{n=0}^{\infty} b_n (-1)^n \left(\sum_{m=1}^{\infty} \frac{\theta^m}{m!} \langle Y^m \rangle \langle W^m \rangle \right)^n \\ &= \sum_{n=0}^{\infty} b_n (-1)^n \frac{\partial^n}{\partial \lambda^n} m_0(\theta) \end{aligned} \quad (1.1.18)$$

with

$$m_0(\theta) \triangleq e \times p \left\{ \lambda \sum_{m=1}^{\infty} \frac{\theta^m}{m!} \langle Y^m \rangle \langle W^m \rangle \right\}. \quad (1.1.19)$$

Needless to say, $m_0(\theta)$ denotes the moment generating function in the case of an idealized Poisson type traffic flow. Thus, the objective

moment generating function $m(\theta)$ can be given by simply differentiating $m_0(\theta)$ by the parameter λ . From the above equation, the m th order moment of $x(t)$ can easily be obtained as

$$\langle x^m \rangle \triangleq \left. \frac{\partial^m}{\partial \theta^m} m(\theta) \right|_{\theta=0} = \sum_{n=0}^{\infty} b_n (-1)^n \frac{\partial^n}{\partial \lambda^n} \langle x^m \rangle_0, \quad (1.1.20)$$

where $\langle x^m \rangle_0$ is the m th order moment of x for the case of an idealized Poisson type traffic flow. Furthermore, since the m th order cumulant of x for the case of an idealized Poisson type traffic flow is given by

$$\kappa_{m0} = \lambda \langle Y^m \rangle \langle W^m(\xi) \rangle \quad (1.1.21)$$

with

$$\langle W^m(\xi) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} W^m(\xi) d\xi,$$

(τ_i : uniform distribution)

(1.1.22)

the following two conditions are derived :

$$\left(\frac{\partial}{\partial \lambda} \right) \kappa_{m0} = \langle Y^m \rangle \langle W^m(\xi) \rangle,$$

$$\left(\frac{\partial^n}{\partial \lambda^n} \right) \kappa_{m0} = 0 \quad (n \geq 2).$$

(1.1.23)

By considering these two conditions and using the mathematical rela-

relationship between the moment statistics and the cumulant statistics, Eq.(1.1.20) can be expressed explicitly as

$$\langle x \rangle = \langle x \rangle_0 - b_1 \frac{\partial}{\partial \lambda} \kappa_{10},$$

$$\langle x^2 \rangle = \langle x^2 \rangle_0 - b_1 \left\{ \frac{\partial}{\partial \lambda} \kappa_{20} + 2 \langle x \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{10} \right\} + 2 b_2 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^2,$$

$$\langle x^3 \rangle = \langle x^3 \rangle_0 - b_1 \left\{ \frac{\partial}{\partial \lambda} \kappa_{30} + 3 \langle x \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{20} + 3 \langle x^2 \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{10} \right\}$$

$$+ 6 b_2 \left\{ \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right) \left(\frac{\partial}{\partial \lambda} \kappa_{20} \right) + \langle x \rangle_0 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^2 \right\}$$

$$- 6 b_3 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^3,$$

$$\langle x^4 \rangle = \langle x^4 \rangle_0 - b_1 \left\{ 4 \langle x^3 \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{10} + 6 \langle x^2 \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{20} \right.$$

$$\left. + 4 \langle x \rangle_0 \frac{\partial}{\partial \lambda} \kappa_{30} + \frac{\partial}{\partial \lambda} \kappa_{40} \right\}$$

$$+ b_2 \left\{ 12 \langle x^2 \rangle_0 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^2 + 24 \langle x \rangle_0 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right) \left(\frac{\partial}{\partial \lambda} \kappa_{20} \right) \right.$$

$$\left. + 6 \left(\frac{\partial}{\partial \lambda} \kappa_{20} \right)^2 + 8 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right) \left(\frac{\partial}{\partial \lambda} \kappa_{30} \right) \right\}$$

$$\begin{aligned}
& - 1 2 b_3 \left\{ 3 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^2 \left(\frac{\partial}{\partial \lambda} \kappa_{20} \right) - 4 \langle x \rangle_0 \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^3 \right\} \\
& + 2 4 b_4 \left\{ \left(\frac{\partial}{\partial \lambda} \kappa_{10} \right)^4 \right\}, \dots
\end{aligned} \tag{1.1.24}$$

In these equations, the expansion coefficient b_n ($n=1,2,\dots$) is explicitly expressed from its definition, as

$$\begin{aligned}
b_0 &= 1, & b_1 &= - \langle N \rangle_{N+\lambda}, \\
b_2 &= (1/2!) \left\{ \langle N^{(2)} \rangle_{N-2} \langle N \rangle_{N\lambda + \lambda^2} \right\}, \\
b_3 &= (1/3!) \left\{ - \langle N^{(3)} \rangle_{N+3} \langle N^{(2)} \rangle_{N\lambda} - 3 \langle N \rangle_{N\lambda^2 + \lambda^3} \right\}, \\
b_4 &= (1/4!) \left\{ \langle N^{(4)} \rangle_{N-4} \langle N^{(3)} \rangle_{N\lambda} + 6 \langle N^{(2)} \rangle_{N\lambda^2} \right. \\
& \quad \left. - 4 \langle N \rangle_{N\lambda^3} + \lambda^4 \right\}, \dots
\end{aligned} \tag{1.1.25}$$

By using the additive property of cumulant statistics with respect to the independent random variables, one can extend the above theory to cases in which several types of vehicle pass on a multi-laned road.

Therefore, by using the above estimated values for moment statistics $\langle x^m \rangle$ ($m=1,2,\dots$), the cumulative distribution function $Q(x)$ with respect to $x(t)$ can be expressed in the form of a statistical Laguerre series expansion universally applicable to arbitrary distribution types, as⁽¹³⁾

$$Q(x) = \int_0^{x/\lambda} P_r(u; m) du$$

$$+ P_{\Gamma} \left(\frac{x}{s}; m+1 \right) \sum_{n=3}^{\infty} A_n L_{n-1}^{(m)} \left(\frac{x}{s} \right) \quad (1.1.26)$$

with

$$m = \frac{\langle x \rangle^2}{\langle (x - \langle x \rangle)^2 \rangle}, \quad s = \frac{\langle (x - \langle x \rangle)^2 \rangle}{\langle x \rangle},$$

$$A_n \triangleq \frac{(n-1)! \Gamma(m+1)}{\Gamma(m+n)} \left\langle L_n^{(m-1)} \left(\frac{x}{s} \right) \right\rangle$$

$$= \sum_{k=0}^n \frac{(-1)^k}{k!} \frac{(n-1)! \Gamma(m+1)}{\Gamma(n-k+1) \Gamma(m+k)} \frac{\langle x^k \rangle}{s^k}, \quad (1.1.27)$$

where $P_{\Gamma}(u; m)$ is the well-known Gamma distribution defined by

$$P_{\Gamma}(u; m) \triangleq [1/\Gamma(m)] e^{-u} u^{m-1}. \quad (1.1.28)$$

Each order of the moment statistics $\langle x^m \rangle$ ($m=1,2,\dots,n$) is thus first obtained by Eq.(1.1.24) with the use of the expansion coefficients b_m ($m=1,2,\dots,n$) which characterize the non-Poisson properties. The estimated values based on each of these moment statistics $\langle x^m \rangle$ ($m=1,2,\dots,n$) thus influence each expansion coefficient A_m ($m=1,2,\dots,n$), as shown in Eq.(1.1.27). Therefore, each expansion coefficient A_m in the statistical Laguerre series expansion, Eq.(1.1.26) corre-

sponds to the non-Poisson distribution expansion coefficient, b_i ($1 \leq i \leq m$) in Eq.(1.1.25).

To sum up, the moment statistics $\langle x^m \rangle$ closely related to the non-Poisson properties influence each expansion coefficient A_m in the cumulative distribution function $Q(x)$, expressed in the form of a statistical Laguerre series expansion, Eq.(1.1.26).

1.3 Experimental consideration

1.3.1 Simplification of an elementary response wave form by use of triangular shape approximation⁽¹⁴⁾

The experimental procedures can be simplified by using a triangular shape to approximate an elementary response wave form due to the surrounding sound propagation environment when one vehicle passes directly in front of an observation point during the time interval T . For the purpose of determining $\langle x^m \rangle$, it is not necessary to accurately determine $w(\xi)$ itself. Only the resultant value of the definite integral $\int_{-T/2}^{T/2} w^m(\xi) d\xi$ need be obtained. The values of the cumulant statistics of the noise intensity fluctuation are not particularly sensitive to the exact wave form of $w(\xi)$, due to the effects of various types of smoothing operations.

The approximation method based on a triangular shape is as follows :

(1) The elementary response wave form of the actual time pattern is

first approximated by a triangular shape with the inclination α (dB/second) so that the total intensity of the approximated wave form is equal to that of the actual one, as shown in Fig.1.1.2.

- (2) After this approximation procedure, the approximated wave form on a decibel scale is transformed into a normalized wave form on an intensity scale, as shown in Fig.1.1.3, the approximated wave form $w(\xi)$ of the actual response wave thus given being

$$w(\xi) = e^{-\alpha|\xi|/M} \quad (M \triangleq 10 / \ln 10) \quad (1.1.29)$$

1.3.2 Predicted results based on the proposed method

The proposed prediction method has been applied to road traffic noise data observed near a national main road with two lanes (an up-lane and a down-lane). The arrangements for measuring road traffic noise at an observation point is shown in Fig.1.1.4. In this experiment, the passing vehicles are classified into two categories : heavy and light. The numbers of vehicles in each lane passing directly in front of an observation point within time intervals of 5 minutes were counted. The parameter λ was directly obtained by averaging the number of vehicles. The values of the moment statistics $\langle Y^m \rangle$ were obtained by averaging peak values of observed elementary response wave forms of noise intensities generated from light and heavy vehicles.

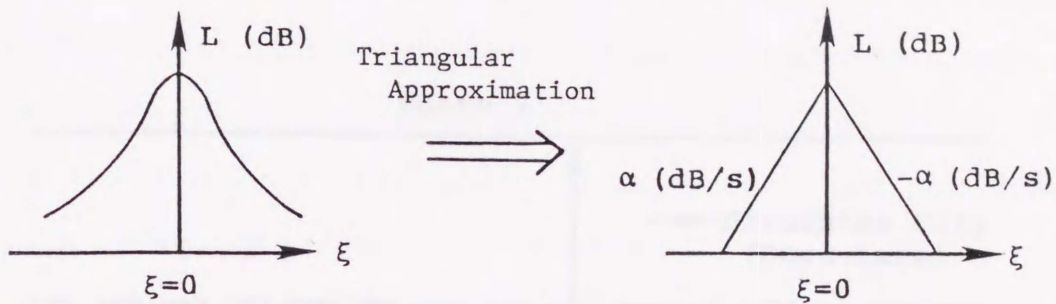


Fig.1.1.2 Simplified treatment of the actual wave form on a decibel scale by use of the triangular approximation.

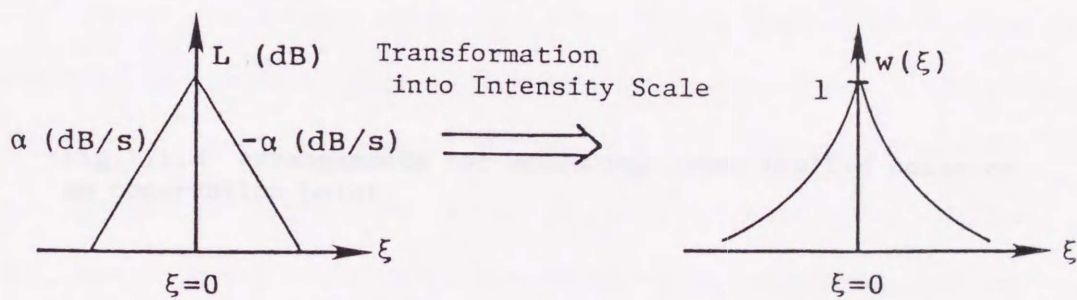


Fig.1.1.3 Transformation of an approximated response wave form on a decibel scale into a normalized response wave form on an intensity scale.

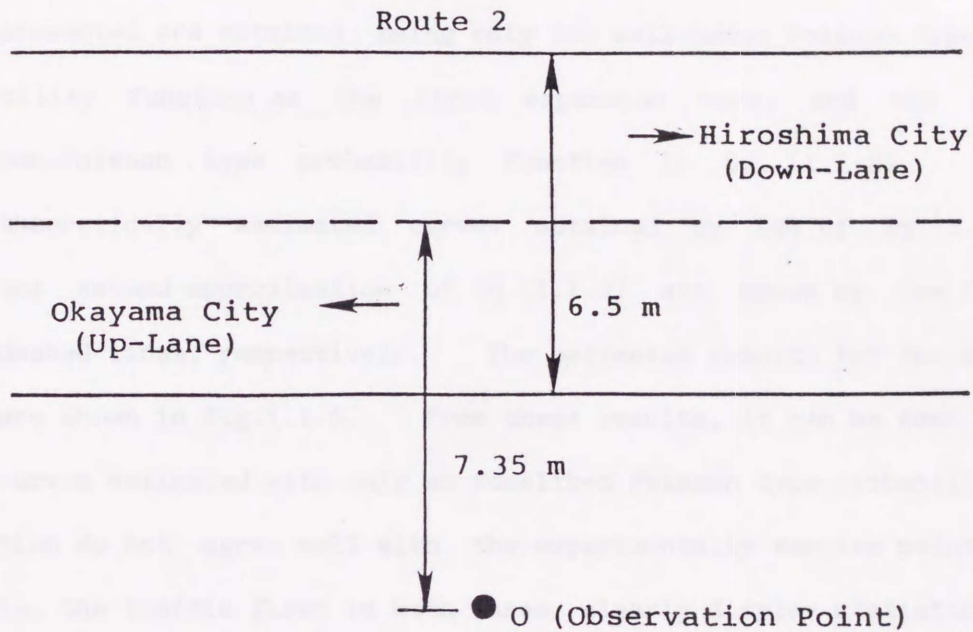


Fig.1.1.4 Arrangements for measuring road traffic noise at an observation point.

Figure 1.1.5, from up-lane traffic flow data, shows a comparison between the theoretically estimated curves and the experimentally sampled points for the cumulative distribution form of the number of passing vehicles. In this figure, the two theoretical curves presented are obtained using only the well-known Poisson type probability function as the first expansion term, and the arbitrary non-Poisson type probability function in Eq. (1.1.2). These theoretically estimated curves obtained by use of Eq.(1.1.9) and the second approximation of Eq.(1.1.2) are shown by the full and dashed lines, respectively. The estimated results for the down-lane are shown in Fig.1.1.6. From these results, it can be seen that the curves estimated with only an idealized Poisson type probability function do not agree well with the experimentally sampled points : that is, the traffic flows in both lanes clearly display statistical properties of non-Poisson type.

The observed noise data and two sets of traffic flow data (up- and down lanes) were used to obtain the results shown in Fig.1.1.7. This is a comparison between the theoretical curves predicted under the approximate assumption of an idealized Poisson type traffic flow and that obtained from the experimentally sampled points, for the cumulative distribution form of the A-weighted noise level fluctuation. A comparison between the theoretically predicted curves obtained by use of the proposed method based on the non-Poisson type traffic flow and that obtained from experimentally sampled points for the cumulative distribution form of the A-weighted noise level fluctu-

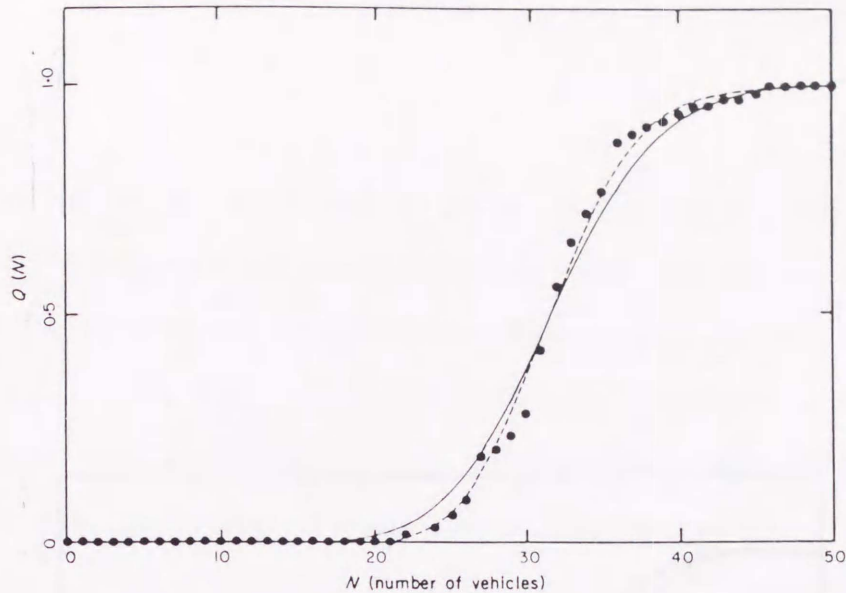


Fig.1.1.5 A comparison between experimentally sampled points and theoretically estimated curves obtained by using the unified series expansion expression(see Eq.(1.1.2)) for the cumulative distribution form of the number of passing vehicles on the up-lane of the road. Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as —, the first approximation corresponding to an idealized Poisson type probability function ; ----, the third approximation.

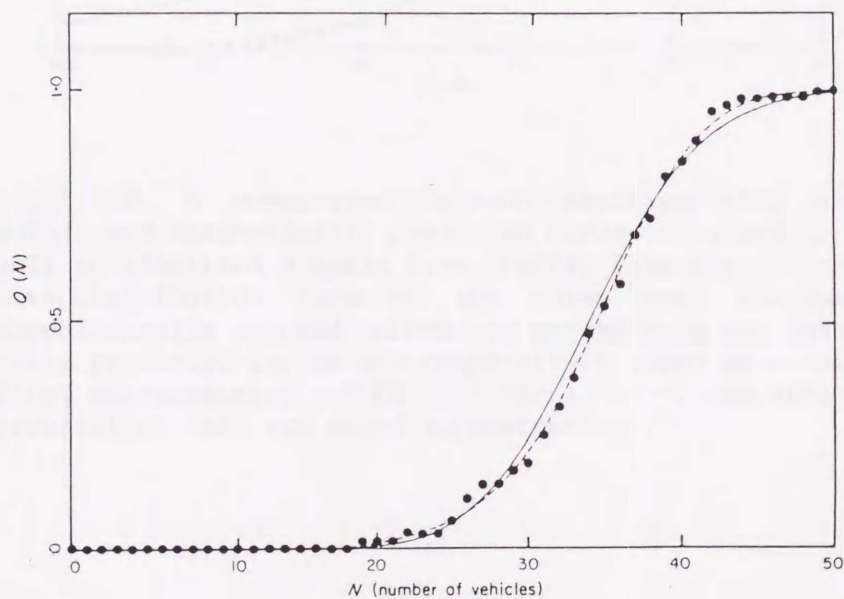


Fig.1.1.6 A comparison between experimentally sampled points and theoretically estimated curves obtained by using the unified series expansion expression(see Eq.(1.1.2)) for the cumulative distribution form of the number of passing vehicles on the down-lane of the road. Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as —, the first approximation corresponding to an idealized Poisson type probability function ; ----, the third approximation.

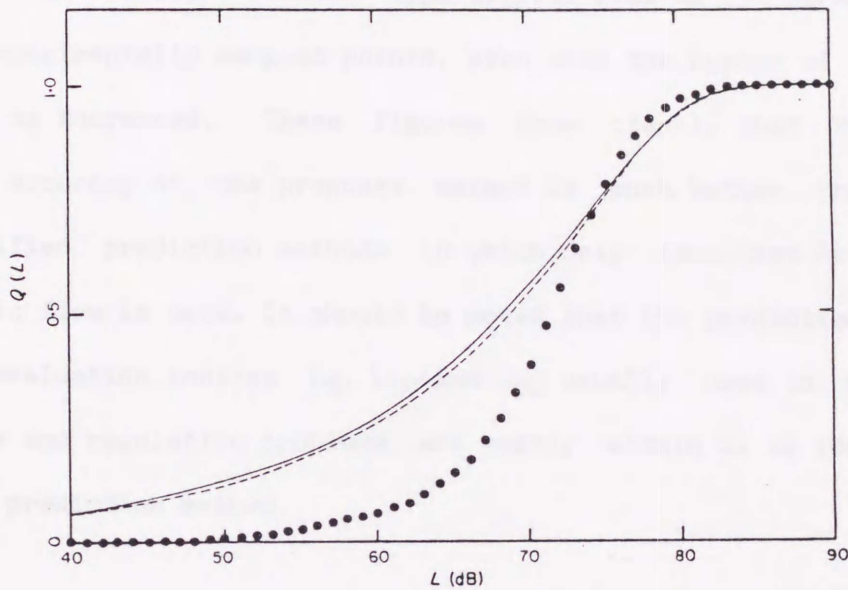


Fig.1.1.7 A comparison between experimentally sampled points and theoretically predicted curves obtained by using only an idealized Poisson type traffic flow for the cumulative distribution form of the noise level fluctuation. Experimentally sampled points are marked by ● and theoretically predicted curves are respectively shown as —, the first approximation of Eq.(1.1.26) ; ---, the second approximation (and the third approximation).

ation is shown in Fig.1.1.8. It is obvious that the successive addition of higher order expansion terms moves the theoretically predicted curves closer to the experimentally sampled points. However, as shown in Fig.1.1.7, the theoretically predicted curves based only on an idealized Poisson type traffic flow do not agree well with the experimentally sampled points, even when the number of correction terms is increased. These figures show clearly that the prediction accuracy of the proposed method is much better than that of simplified prediction methods in which only idealized Poisson type traffic flow is used. It should be noted that the prediction errors of the evaluation indices L_5 , L_{10} and L_{50} usually used in noise evaluation and regulation problems are mostly within ± 1 dB for the proposed prediction method.

1.4 Conclusion

This study has focussed on the establishment of a hybrid method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow, with emphasis on the methodological point of view.

Firstly, expressions for the statistical moments, of an arbitrary non-linear function type, for this arbitrary non-Poisson type traffic flow have been presented, as obtained by an approach equivalent to that used for an idealized Poisson type traffic flow. A unified explicit expression of series expansion type was then introduced

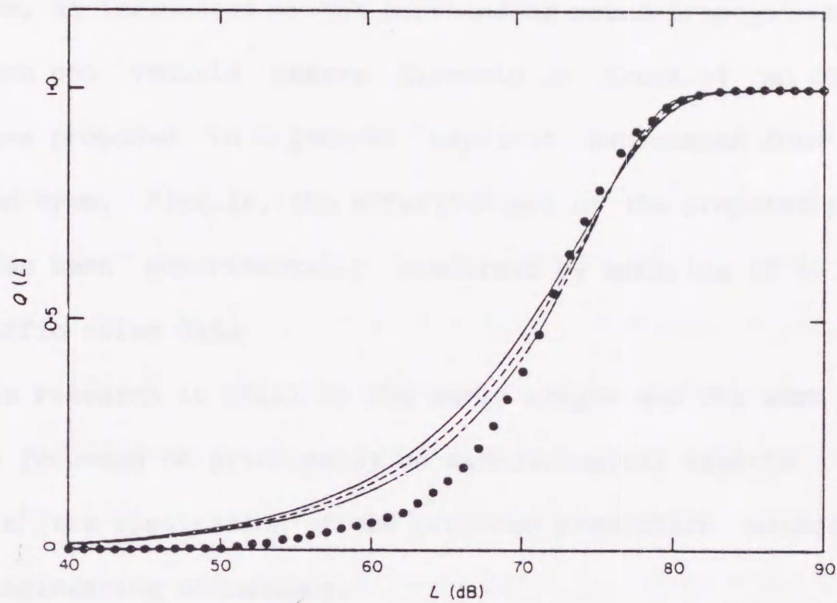


Fig.1.1.8 A comparison between experimentally sampled points and the theoretically predicted curves obtained by using the proposed method based on arbitrary non-Poisson type traffic flow for the cumulative distribution form of the noise level fluctuation. Experimentally sampled points are marked by ● and theoretically predicted curves are respectively shown as —, the first approximation of Eq. (1.1.26) ; ---, the second approximation ; - · - ·, the third approximation.

for vehicle number distribution in an arbitrary non-Poisson type traffic flow. Next, a unified statistical prediction method for road traffic noise, based on introduction of an elementary response wave form, as influenced by the surrounding sound propagation environment when one vehicle passes directly in front of an observation point, was proposed in a general explicit expression form of series expansion type. Finally, the effectiveness of the proposed prediction method has been experimentally confirmed by applying it to observed road traffic noise data.

This research is still in the early stages and the work reported here has focussed on principally on methodological aspects. Problems remain in the application of the proposed prediction method to other actual engineering situations.

Chapter 2 A Method for Prediction of Road Traffic Noise at a T-Type Road Intersection Based on the Image Method

2.1 Introduction

In Chapter 1, a general method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow was proposed, based on information on the elementary response wave form when one vehicle passes directly in front of an observation point. In order to establish a more effective prediction method, a theoretical method for estimating the above-mentioned elementary response wave form in complicated realistic acoustical environments must be found. In this chapter, a new approach for estimating theoretically this elementary response wave form at a T-type road intersection with sound insulation barriers is proposed, using a well-known image method.^{(15),(16)} The effectiveness of the proposed method has been experimentally confirmed by applying it to the road traffic noise data observed at a T-type road intersection in a city area.

2.2 Theoretical consideration

2.2.1 Outline of prediction theory

Let us consider the wave form of the noise intensity fluctuation $x(t)$ at an observation point, as stated in Chapter 1. By substituting

Eq.(1.1.25) into Eq.(1.1.24) in Chapter 1, the moment statistics of $x(t)$ can be resultantly expressed as

$$\begin{aligned}
 \langle x \rangle &= \langle N \rangle \langle Y \rangle \langle w(\xi) \rangle, \\
 \langle x^2 \rangle &= \langle N(N-1) \rangle \langle Y^2 \rangle \langle w(\xi) \rangle^2 + \langle N \rangle \langle Y^2 \rangle \langle w^2(\xi) \rangle, \\
 \langle x^3 \rangle &= \langle N(N-1)(N-2) \rangle \langle Y^3 \rangle \langle w(\xi) \rangle^3 + 3 \langle N(N-1) \rangle \langle Y^2 \rangle \langle Y \rangle \langle w^2(\xi) \rangle \langle w(\xi) \rangle + \langle N \rangle \langle Y^3 \rangle \cdot \langle w^3(\xi) \rangle, \\
 \langle x^4 \rangle &= \langle N(N-1)(N-2)(N-3) \rangle \langle Y^4 \rangle \langle w(\xi) \rangle^4 \\
 &+ 6 \langle N(N-1)(N-2) \rangle \langle Y^2 \rangle \langle Y \rangle^2 \langle w^2(\xi) \rangle \\
 &\cdot \langle w(\xi) \rangle^2 + 4 \langle N(N-1) \rangle \langle Y^3 \rangle \langle Y \rangle \langle w^3(\xi) \rangle \\
 &\cdot \langle w(\xi) \rangle + 3 \langle N(N-1) \rangle \langle Y^2 \rangle^2 \langle w^2(\xi) \rangle^2 + \langle N \rangle \cdot \langle Y^4 \rangle \langle w^4(\xi) \rangle, \dots \quad (1.2.1)
 \end{aligned}$$

Based on the additive property of cumulant statistics with respect to the independent random variables, one can extend this theory to cases in which several types of vehicle pass on a multi-laned road, as mentioned in Chapter 1. The above moment statistics $\langle x^m \rangle$ on an intensity scale can be transformed into the cumulant statistics κ_{Lm} on a decibel scale by using the following relationship⁽⁸⁾:

$$\frac{1}{x_0^m} \langle x^m \rangle = \exp \left\{ \sum_{i=1}^{\infty} \frac{1}{i!} \left(\frac{m}{M} \right) \kappa_{Li} \right\}. \quad (1.2.2)$$

$(x_0 \hat{=} 10^{-12} W / m^2, \quad M \hat{=} 10 / 1 n 10)$

Thus, after regarding the cumulant statistics κ_{Li} as unknown parameters and by solving the simultaneous equations derived from Eq.(1.2.2) ($i, m=1, 2, \dots, M$), the cumulant statistics κ_{Li} ($i=1, 2, \dots, M$) on a

decibel scale can be obtained. For example, the transformation formula for the cumulant statistics up to the fourth order is explicitly expressed by

$$\begin{aligned}
 \kappa_{L1} &= M \left\{ 4 \ln \frac{\langle X \rangle}{x_0} - 3 \ln \frac{\langle X^2 \rangle}{x_0^2} + \frac{4}{3} \ln \frac{\langle X^3 \rangle}{x_0^3} - \frac{1}{4} \ln \frac{\langle X^4 \rangle}{x_0^4} \right\}, \\
 \kappa_{L2} &= M^2 \left\{ -\frac{26}{3} \ln \frac{\langle X \rangle}{x_0} + \frac{19}{2} \ln \frac{\langle X^2 \rangle}{x_0^2} - \frac{14}{3} \ln \frac{\langle X^3 \rangle}{x_0^3} + \frac{11}{12} \ln \frac{\langle X^4 \rangle}{x_0^4} \right\}, \\
 \kappa_{L3} &= M^3 \left\{ 9 \ln \frac{\langle X \rangle}{x_0} - 12 \ln \frac{\langle X^2 \rangle}{x_0^2} + 7 \ln \frac{\langle X^3 \rangle}{x_0^3} - \frac{3}{2} \ln \frac{\langle X^4 \rangle}{x_0^4} \right\}, \\
 \kappa_{L4} &= M^4 \left\{ -4 \ln \frac{\langle X \rangle}{x_0} + 6 \ln \frac{\langle X^2 \rangle}{x_0^2} - 4 \ln \frac{\langle X^3 \rangle}{x_0^3} + \ln \frac{\langle X^4 \rangle}{x_0^4} \right\}.
 \end{aligned}
 \tag{1.2.3}$$

In order to evaluate the L_x noise evaluation indices (e.g., L_5, L_{10}, L_{50} and L_{90}), the cumulative distribution function $Q(L)$ of the noise level fluctuation L can be expressed in an orthonormal series expansion, as a statistical Hermite series expansion universally applicable to an arbitrary non-Gaussian distribution form, as follows⁽¹⁴⁾:

$$\begin{aligned}
 Q(L) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{L - \kappa_{L1}}{\sqrt{\kappa_{L2}}}} e^{-\frac{\xi^2}{2}} d\xi \\
 &= \sum_{n=3}^{\infty} B_n(\kappa_L) \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{(L - \kappa_{L1})^2}{2\kappa_{L2}} \right\} \\
 &\quad \cdot H_{n-1} \left(\frac{L - \kappa_{L1}}{\sqrt{\kappa_{L2}}} \right)
 \end{aligned}
 \tag{1.2.4}$$

with

$$B_n(\kappa_L) \triangleq \frac{1}{n!} \left\langle H_n \left(\frac{L - \kappa_{L1}}{\sqrt{\kappa_{L2}}} \right) \right\rangle, \quad (1.2.5)$$

where $H_n(\cdot)$ denotes the n th order Hermite polynomial.

2.2.2 Estimation of the elementary response wave form using the image method

Let us consider the sound propagation characteristic of a T-type road intersection as shown in Fig.1.2.1. There are homogeneous sound insulation barriers made of concrete material along both sides of the road. It is assumed that each vehicle drives in the middle of each lane with a proper constant speed, and the area across the river is regarded as a free field of sound. Figure 1.2.2 shows the geometrical construction of the T-type road intersection under consideration. First, an explicit expression for the sound propagation characteristic at each vehicle position can be derived by using a well-known image method.^{(15),(16)} The elementary response wave form in the time domain can then be evaluated by taking this sound propagation characteristic into consideration, together with the average speed of the passing vehicles. In this case, it was possible to assume that diffraction effects from high sound insulation barriers is negligibly small, through the well-known evaluation of sound insulation barriers using Maekawa's chart with a Fresnel number. Let

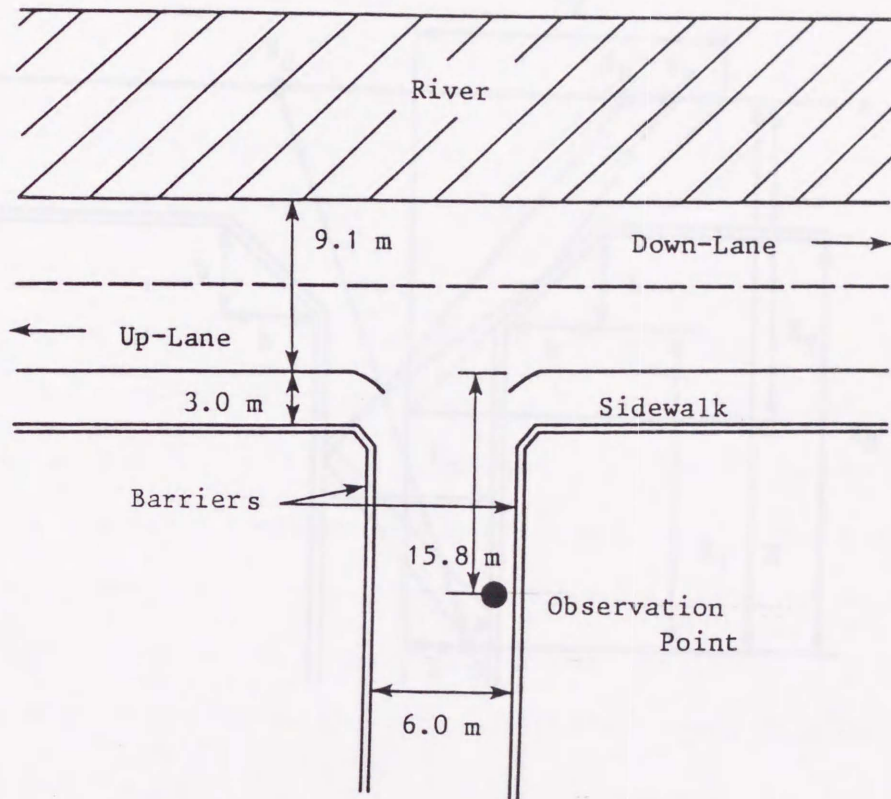


Fig.1.2.1 Arrangements for measuring road traffic noise at an observation point.

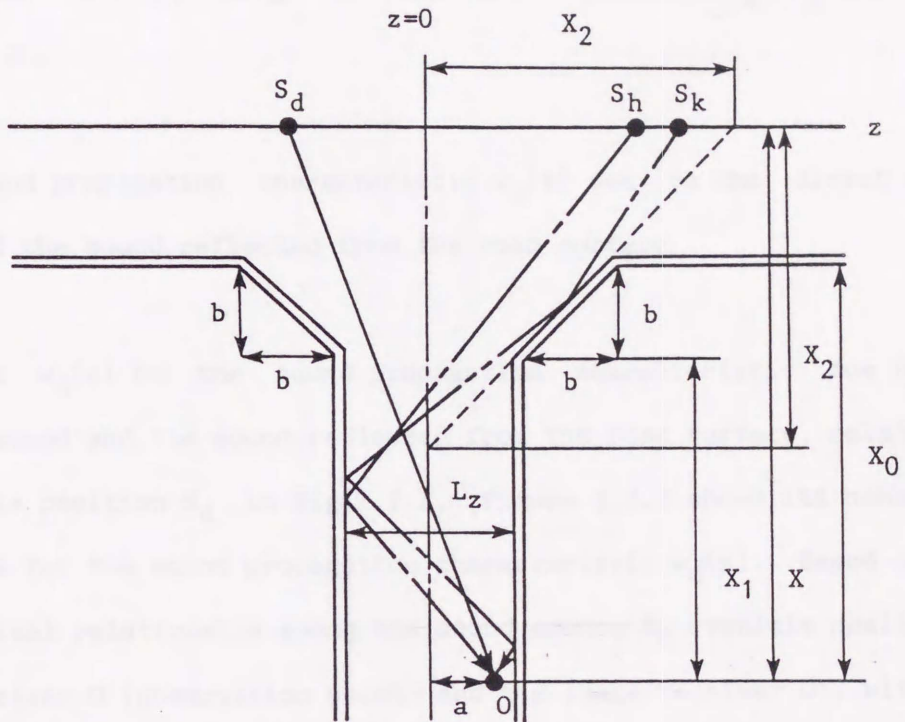


Fig.1.2.2 Geometrical construction of the T-type road intersection under consideration.

$w(z)$ be the sound propagation characteristic at an arbitrary vehicle position z . Here, $w(z)$ is classified into the following three categories (corresponding to each vehicle position S_d , S_h and S_k in Fig.1.2.2).

[A] Sound propagation characteristic $w_d(z)$ due to the direct sound and the sound reflected from the road surface

Let $w_d(z)$ be the sound propagation characteristic due to the direct sound and the sound reflected from the road surface, related to a vehicle position S_d in Fig.1.2.2. Figure 1.2.3 shows its schematic analysis for the sound propagation characteristic $w_d(z)$. Based on the geometrical relationship among the sound source S_d (vehicle position), the receiver O (observation point) and the image receiver O' , with the aid of the additive property of energy quantities, $w_d(z)$ is given by

$$w_d(z) = D(z_{d2} - z) D(z - z_{d1})$$

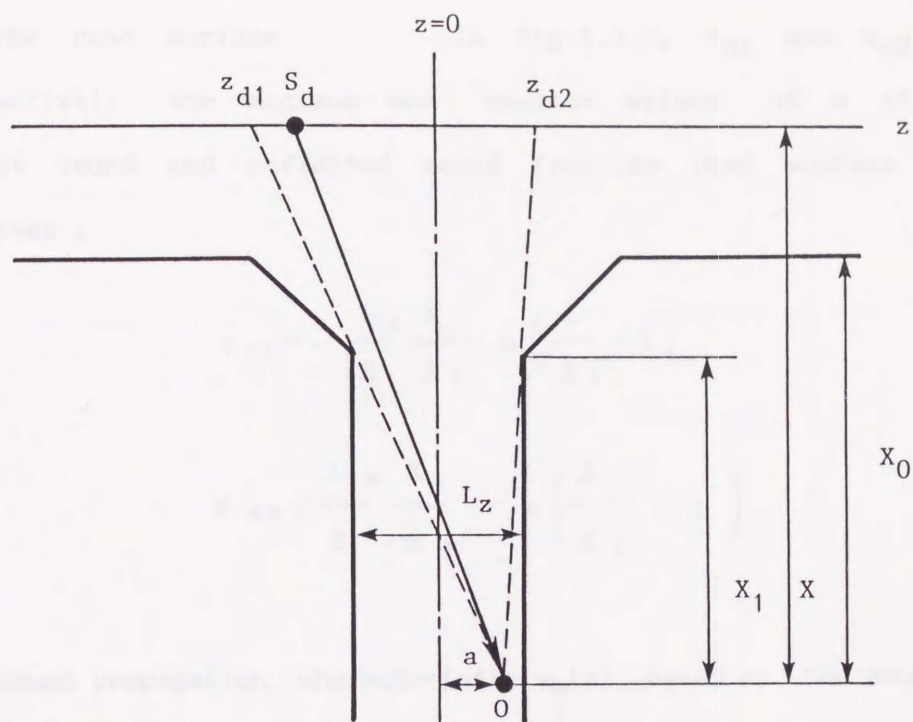
$$\cdot \frac{1}{4\pi} \left\{ \frac{1}{X^2 + (z - a)^2 + h^2} + \frac{R_1}{X^2 + (z - a)^2 + (2h_1 - h)^2} \right\},$$

(1.2.6)

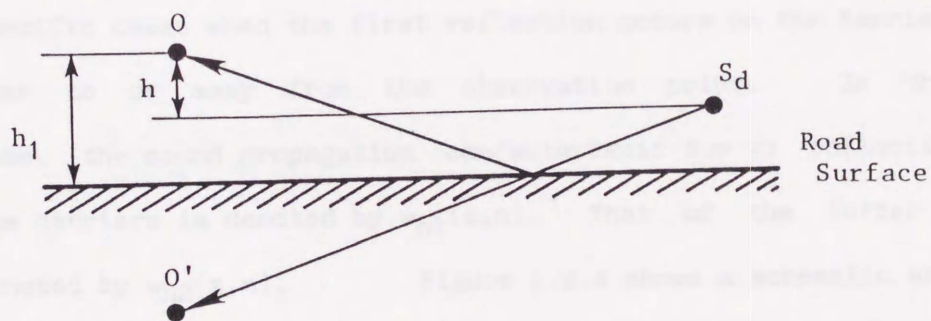
where $D(\cdot)$ is the truncation function defined as

$$D(\zeta) \triangleq \begin{cases} 1 & (\zeta > 0) , \\ 0 & (\zeta \leq 0) . \end{cases} \quad (1.2.7)$$

In Eq.(1.2.6), h denotes the difference in height between the sound source and the receiver O , and R_1 denotes the reflection coefficient



a) Schematic analysis of the direct sound.



b) Schematic analysis of sound reflection from the road surface.

Fig.1.2.3 Schematic analysis for the sound propagation characteristic $w_d(z)$.

of the road surface. In Fig.1.2.3, z_{d1} and z_{d2} denote respectively the minimum and maximum values of z at which direct sound and reflected sound from the road surface can be received :

$$z_{d1} = -\frac{L_z X}{2 X_1} - a \left(\frac{X}{X_1} - 1 \right),$$

$$z_{d2} = \frac{L_z X}{2 X_1} - a \left(\frac{X}{X_1} - 1 \right). \quad (1.2.8)$$

[B] Sound propagation characteristic $w_h(z)$ based on the sounds reflected from the barriers on both sides of the road

In this case, one can apply a well-known image method to two specific cases when the first reflection occurs on the barriers either near to or away from the observation point. In the former case, the sound propagation characteristic due to reflections from the barriers is denoted by $w_{h1}(z,n)$. That of the latter case is denoted by $w_{h2}(z,n)$. Figure 1.2.4 shows a schematic analysis of sound reflections from the barriers. Based on a geometrical analysis of n reflections from the barriers (denoted by O^n) through image receivers, one can easily obtain

$$w_h(z) = \sum_{n=1}^{\infty} [w_{h1}(z,n) + w_{h2}(z,n)]$$

$$= \sum_{n=1}^{\infty} \left\{ D(z_{h12} - z) D(z + z_{h11}) \right\}$$

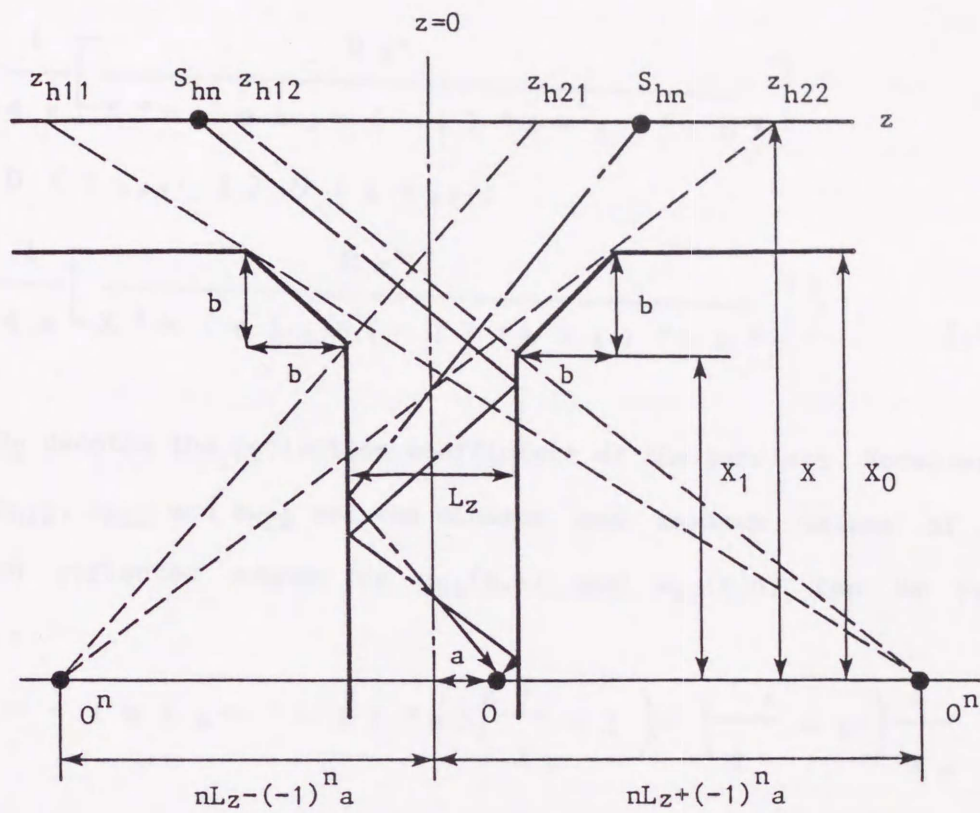


Fig.1.2.4 Schematic analysis of sound reflections from the sound insulation barriers.

$$\begin{aligned}
& \cdot \frac{1}{4\pi} \left[\frac{R_2 z^n}{X^2 + (nL_z + (-1)^n a + z)^2 + h^2} \right] \\
& + D(z_{h22} - z) D(z + z_{h21}) \\
& \cdot \frac{1}{4\pi} \left[\frac{R_2 z^n}{X^2 + (nL_z + (-1)^n a + z)^2 + h^2} \right] \Bigg\} , \quad (1.2.9)
\end{aligned}$$

where R_2 denotes the reflection coefficient of the barriers. Moreover, z_{h11} , z_{h12} , z_{h21} and z_{h22} are the minimum and maximum values of z at which reflected sounds for $w_{h1}(z,n)$ and $w_{h2}(z,n)$ can be received :

$$\begin{aligned}
z_{h11} &= - (nL_z + (-1)^n a) \left(\frac{X}{X_0} - 1 \right) - \left(\frac{L_z}{2} + b \right) \frac{X}{X_0} , \\
z_{h12} &= - (nL_z + (-1)^n a) \left(\frac{X}{X_0} - 1 \right) + \frac{L_z}{2} \frac{X}{X_1} , \\
z_{h21} &= (nL_z + (-1)^n a) \left(\frac{X}{X_0} - 1 \right) - \frac{L_z}{2} \frac{X}{X_1} , \\
z_{h22} &= (nL_z + (-1)^n a) \left(\frac{X}{X_0} - 1 \right) + \left(\frac{L_z}{2} + b \right) \frac{X}{X_0} .
\end{aligned} \quad (1.2.10)$$

[C] Sound propagation characteristic $w_k(z)$ due to sound reflection from the corner of the road intersection

Let $w_{k1}(z,n)$ be the sound propagation characteristic for a case in which the second reflection occurs at the barrier near the observation point after the first reflection at the corner, then, n reflections occur on the barriers. Alternatively, let $w_{k2}(z,n)$ be the sound propagation characteristic for a case in which the second reflection occurs at the barrier away from the same observation point. By consideration of both the image receivers and the image sources as shown in Fig.1.2.5 (S'_{k21} and S'_{k22}), one can obtain :

$$\begin{aligned}
 w_k(z) &= \sum_{n=1}^{\infty} [w_{k1}(z,n) + w_{k2}(z,n)] \\
 &= \sum_{n=1}^{\infty} \left\{ D(z_{k12} - z) d(z + z_{k11}) \right. \\
 &\quad \cdot \frac{1}{4\pi} \left[\frac{R z^{n+1}}{(X + z - X_2)^2 + (X_2 + nL_z + (-1)^n a)^2 + h^2} \right] \\
 &\quad + D(z_{k22} - z) D(z + z_{k21}) \\
 &\quad \left. \cdot \frac{1}{4\pi} \left[\frac{R z^{n+1}}{(X + z - X_2)^2 - (X_2 + nL_z + (-1)^n a)^2 + h^2} \right] \right\}, \tag{1.2.11}
 \end{aligned}$$

where z_{k11} , z_{k12} , z_{k21} and z_{k22} denote the minimum and maximum values of z at which reflected sounds for $w_{k1}(z,n)$ and $w_{k2}(z,n)$ can be received :

$$\begin{aligned}
 z_{k11} &= - \frac{nL_z + (-1)^n a + X_2}{nL_z + (-1)^n a + L_z/2 + b} X_0 + X - X_2, \\
 z_{k12} &= - \frac{nL_z + (-1)^n a + X_2}{nL_z + (-1)^n a + L_z/2} X_1 + X - X_2,
 \end{aligned}$$

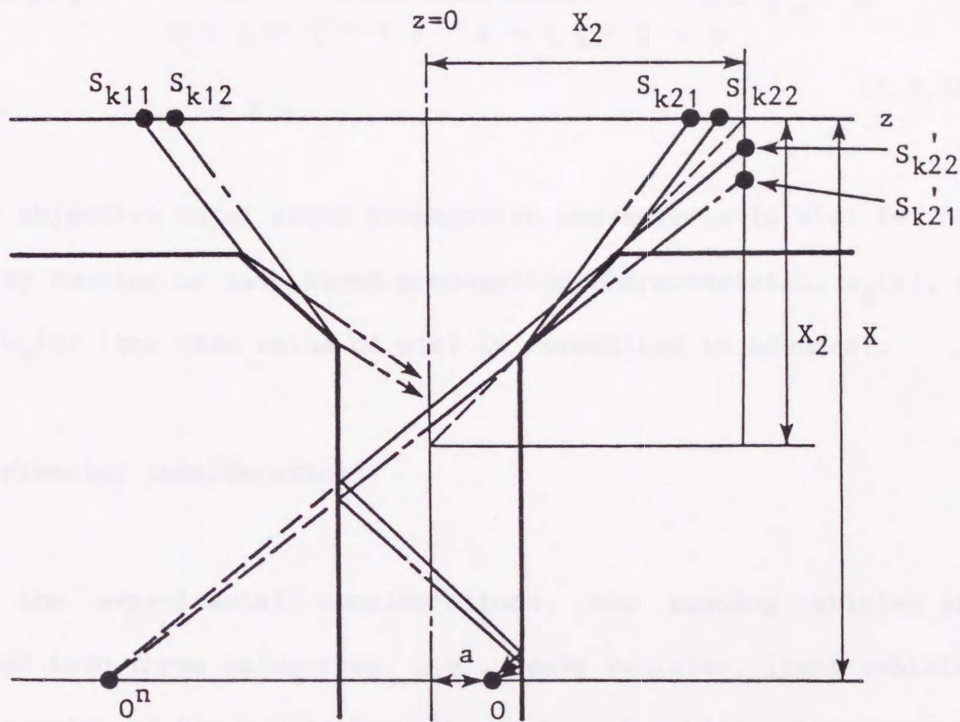


Fig.1.2.5 Schematic analysis of sound reflections from the corner of the road intersection.

$$\begin{aligned}
Z_{k21} &= \frac{n L_z - (-1)^n a + X_2}{n L_z + (-1)^n a + L_z / 2} X_1 - X + X_2, \\
Z_{k22} &= - \frac{n L_z - (-1)^n a + X_2}{n L_z + (-1)^n a + L_z / 2 + b} X_0 - X \\
&\quad + X_2.
\end{aligned}
\tag{1.2.12}$$

The objective total sound propagation characteristic $w(z)$ is obtained by summing up each sound propagation characteristic, $w_d(z)$, $w_h(z)$ and $w_k(z)$ (the peak value of $w(z)$ is normalized in advance).

2.3 Experimental consideration

In the experimental considerations, the passing vehicles are classified into three categories, i.e., heavy vehicles, light vehicles and motorcycles. Table 1.2.1 shows the number of vehicles in each lane passing directly in front of the observation point during the measurement time interval of 30 minutes. The values of moment statistics, $\langle Y^m \rangle$, were obtained by averaging peak values of the observed elementary response wave forms of the noise intensities generated from each type vehicle. Figure 1.2.6 shows an example of the estimated sound propagation characteristic $w(z)$ for the up-lane (the values of R_1 and R_2 are selected to 0.98). Based on the estimated sound propagation characteristics, Fig.1.2.7 shows a comparison between the theoretically predicted curves using the proposed method and the experimentally sampled points for the cumulative distribution form

Table 1.2.1 Number of vehicles passing directly in front of the observation point during the measurement time interval.

	Up-Lane	Down-Lane
Heavy vehicle	4	13
Light vehicle	48	145
Motorcycle	7	15

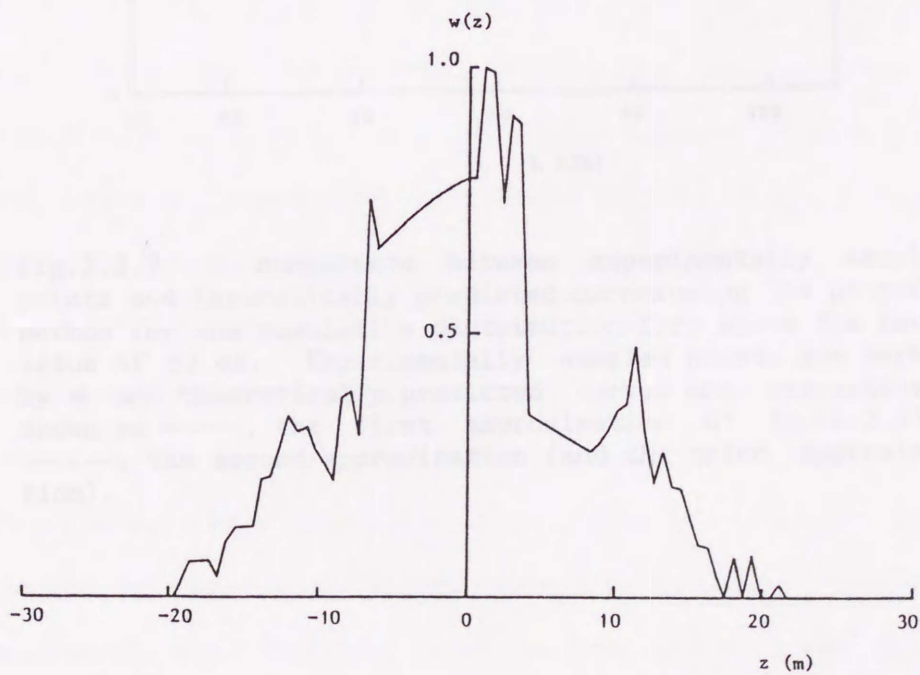


Fig.1.2.6 An example of the estimated sound propagation characteristic (Up-Lane).

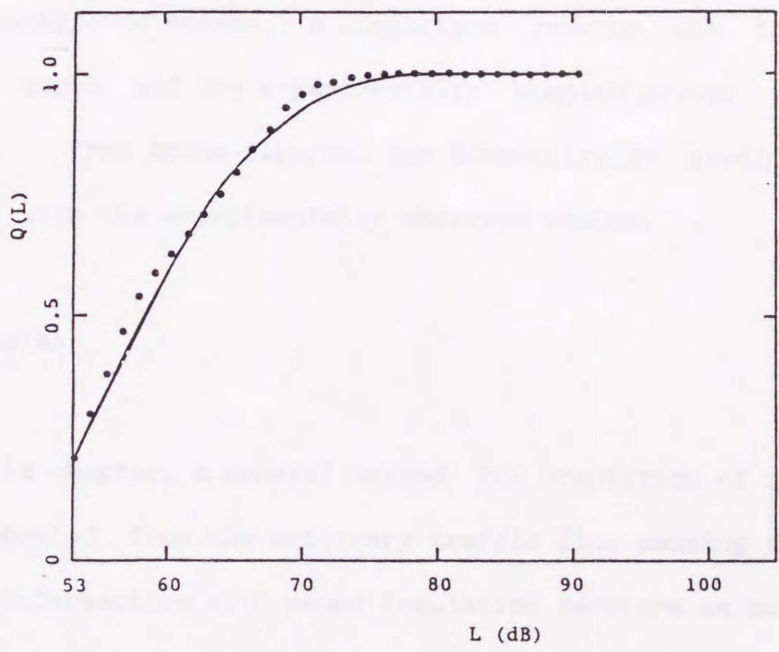


Fig.1.2.7 A comparison between experimentally sampled points and theoretically predicted curves using the proposed method for the cumulative distribution form above the level value of 53 dB. Experimentally sampled points are marked by ● and theoretically predicted curves are respectively shown as —, the first approximation of Eq.(1.2.4); ---, the second approximation (and the third approximation).

of the noise level fluctuation L . With consideration to the existence of background noises, a conditional cumulative distribution function with a restricted range above 53 dB has been predicted.⁽⁸⁾ Moreover, by introducing the convolution integral for two statistically independent noise intensities as another method of considering the existence of background noises, a comparison between the theoretically predicted curve and the experimentally sampled points is shown in Fig.1.2.8. From these figures, the theoretically predicted results agree well with the experimentally observed values.

2.4 Conclusion

In this chapter, a general method for prediction of road traffic noise generated from the arbitrary traffic flow passing through a T-type road intersection with sound insulation barriers on both sides of the road based on theoretical estimation of the sound propagation characteristic with the introduction of a well-known image method has been proposed. The effectiveness of the proposed prediction method has been experimentally confirmed by applying it to observed road traffic noise data.

This research is still in the early stages and the work reported here has focussed mainly on its methodological aspects. Accordingly, problems remain, e.g., application of the proposed method to the other actual engineering situations, such as other types of road intersections and/or the non-stationary traffic flow, etc.

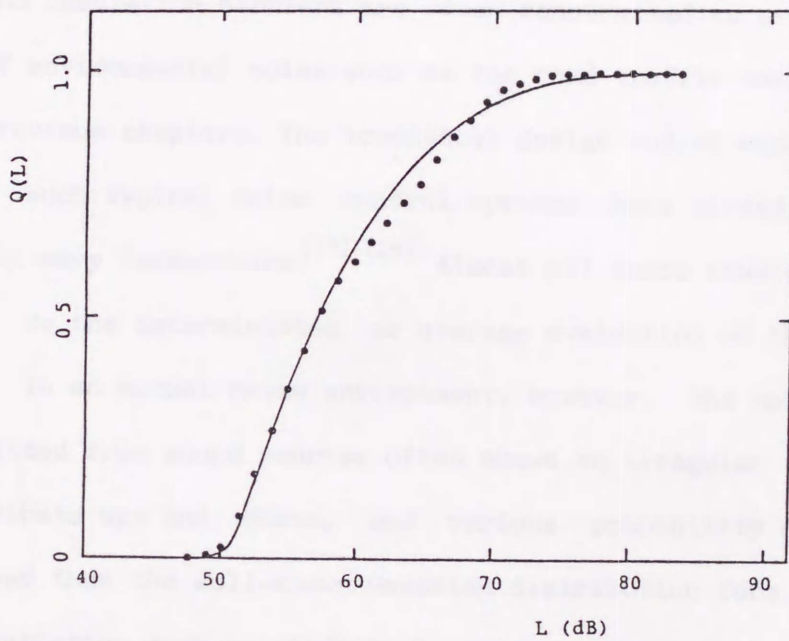


Fig.1.2.8 A comparison between experimentally sampled points and theoretically predicted curve using the convolution integral for the cumulative distribution form of the noise level distribution. Experimentally sampled points are marked by ● and theoretically predicted curve is shown as —.

Chapter 3 A Statistical Method for Prediction of the Sound Insulation Effect of Barriers

3.1 Introduction

Sound insulation barriers are often constructed to produce attenuation of environmental noise such as the road traffic noise discussed in the previous chapters. The acoustical design and/or evaluation problems of such typical noise control systems have already been considered by many researchers.⁽¹⁷⁾⁻⁽²⁵⁾ Almost all these studies have been confined to the deterministic or average evaluation of the shielding effects. In an actual noise environment, however, the noise fluctuation emitted from sound sources often shows an irregular time pattern with intricate ups and downs, and various probability distribution forms other than the well-known Gaussian distribution form. Furthermore, statistics such as median, L_x noise level and L_{eq} evaluation index are of considerable importance for actual noise evaluation and regulation problems. It is necessary therefore that an explicit expression of the output noise level or noise intensity distribution form be established in close relation to the frequency characteristic of the input random noise fluctuation and the sound insulation barrier.

From the methodological point of view, there are two approaches for prediction of the above probability distribution. One approach would be construction of a unified prediction method by reinforcement

of the existing sound insulation barrier, with due consideration of acoustic characteristic change. An alternative approach would be consideration of the resultant probability distribution due to a newly-constructed sound insulation barrier, rather than reinforcing or changing the existing system. In this chapter, a simplified and unified statistical evaluation method for predicting output noise level (or noise intensity) distribution based on the latter approach is theoretically proposed. The effectiveness of the proposed prediction method has been experimentally confirmed by applying it to actually observed noise data.

3.2 Theoretical consideration

3.2.1 General expression for noise intensity distribution

In order to predict the noise evaluation index, L_x , to find a general explicit expression of the probability density function with respect to the noise level or noise intensity fluctuations must first be found. As shown in Chapter 1, the generally applicable expression on the above-mentioned probability density function for noise intensity fluctuation is of a series expansion type taking a Gamma distribution as the first expansion term. More specifically, paying special attention to the fact that the noise intensity, x , always fluctuates in a non-negative region $[0, \infty)$, the probability density function $P(x)$ can be generally expressed in the form of a statistical Laguerre

series expansion⁽¹³⁾ :

$$P(x) = P_{\Gamma}(x; m, s) \left[1 + \sum_{n=3}^{\infty} b'_n L_n^{(m-1)}\left(\frac{x}{s}\right) \right] \quad (1.3.1)$$

with

$$m = \frac{\langle x \rangle^2}{\langle (x - \langle x \rangle)^2 \rangle},$$

$$s = \frac{\langle (x - \langle x \rangle)^2 \rangle}{\langle x \rangle}, \quad (1.3.2)$$

$$b'_n = \frac{\Gamma(m) n!}{\Gamma(m+n)} \left\langle L_n^{(m+1)}\left(\frac{x}{s}\right) \right\rangle$$

$$= \sum_{k=0}^n \frac{(-1)^k}{k!} \frac{\Gamma(m) n!}{\Gamma(n-k+1) \Gamma(m+k)} \frac{\langle x^k \rangle}{s^k} \quad (1.3.3)$$

and

$$P_{\Gamma}(x; m, s) = \frac{1}{\Gamma(m) s^m} e^{-x/s} x^{m-1}. \quad (1.3.4)$$

(Gamma distribution)

At this time, the cumulative distribution function, $Q(x) = \int_0^x P(\zeta) d\zeta$, for the noise intensity fluctuation, which is very important for the purpose of finding any L_x noise level, is expressed as

$$Q(x) = \int_0^{x/s} P_\Gamma(\xi; m) d\xi + P_\Gamma\left(\frac{x}{s}; m+1\right) \sum_{n=3}^{\infty} A_n L_{n-1}^{(m)}\left(\frac{x}{s}\right) \quad (1.3.5)$$

with

$$P_\Gamma(\xi; m) \triangleq \frac{1}{\Gamma(m)} e^{-\xi} \xi^{m-1} \quad (1.3.6)$$

(standard Gamma distribution)

and

$$A_n \triangleq \frac{(n-1)! \Gamma(m+1)}{\Gamma(m+n)} \left\langle L_n^{(m-1)}\left(\frac{x}{s}\right) \right\rangle = \sum_{k=0}^n \frac{(-1)^k}{k!} \frac{(n-1)! \Gamma(m+1)}{\Gamma(n-k+1) \Gamma(m+k)} \frac{\langle x^k \rangle}{s^k} \quad (1.3.7)$$

3.2.2 Relationship between frequency characteristics of a barrier and parameters contained in the cumulative distribution function

Let X_i ($i=1,2,\dots,N$) be the input noise intensity fluctuation existing in the i th frequency band-width of the noise fluctuation wave emitted from the sound source. And let the transfer coefficient a_i ($i=1,2,\dots,N$) reflect the intensity frequency characteristic of the sound insulation barrier at the center frequency, f_{ci} , of the i th octave band (or one-third-octave band). From the additive property of energy quantities, the over-all noise intensity, x , attenuated by the

sound insulation barrier at an observation point is given explicitly by

$$x = \sum_{i=1}^N a_i X_i. \quad (1.3.8)$$

Based on information on the statistical property of X_i and the intensity frequency characteristic a_i ($i=1,2,\dots,N$), the p th order moment, $\langle x^p \rangle$, for x can be easily estimated as

$$\langle x^p \rangle = \sum_{i+j+\dots+n=p} \frac{p!}{i! j! \dots n!} \cdot a_1^i a_2^j \dots a_N^n \langle X_1^i X_2^j \dots X_N^n \rangle. \quad (1.3.9)$$

Therefore, based on these lower and higher order moments, many values of parameters m , s and A_n can be calculated by use of Eqs.(1.3.2) and (1.3.7). The cumulative distribution function of the output noise intensity fluctuation can be evaluated by use of Eq.(1.3.5). The effect of the intensity frequency characteristic and the moment statistics of the incident random noise on the cumulative distribution function is explicitly reflected in each parameter m , s and A_n .

3.2.3 Estimation of the frequency characteristic of a sound insulation barrier

Let us consider the sound insulation barrier as shown in Fig.1.3.

1. Assuming that the fixed sound source can be regarded as a point source and that the barrier is sufficiently long in the horizontal

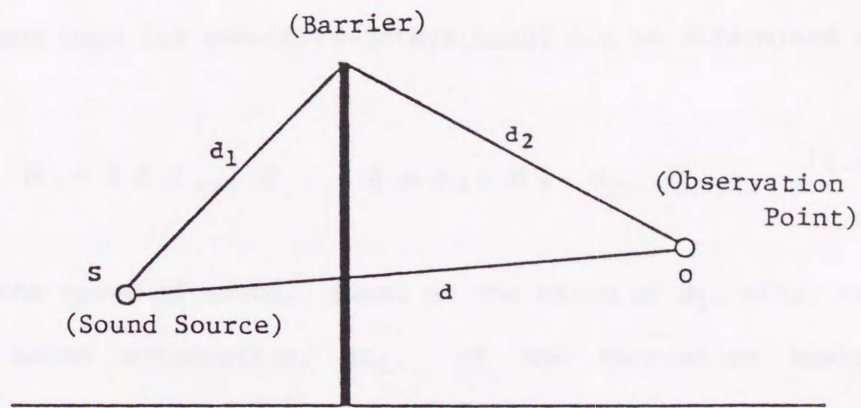


Fig.1.3.1 Location of sound insulation barrier.

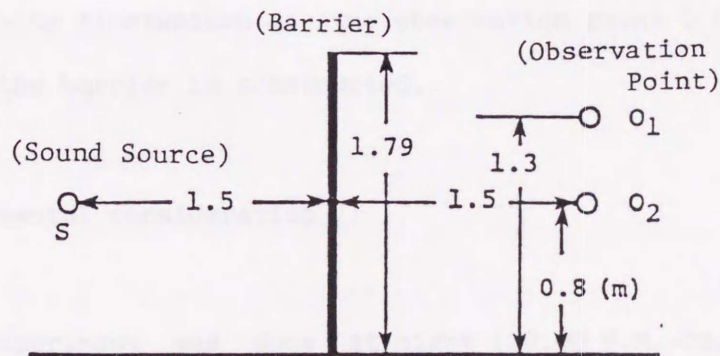


Fig.1.3.2 Layout of sound source, barrier and two observation points.

direction, the Fresnel number, N_i , at the center frequency, f_{ci} , of the i th octave band (or one-third-octave band) can be determined as

$$N_i = 2 \delta f_{ci} / C, \quad \delta = d_1 + d_2 - d, \quad (1.3.10)$$

where C is the speed of sound. Based on the value of N_i , after calculating the sound attenuation, ΔL_i , of the barrier on Maekawa's acoustical evaluation chart⁽¹⁷⁾ for barriers, the intensity frequency characteristic is explicitly evaluated by

$$a_i = 10^{-\Delta L_i / 10} \quad (i = 1, 2, \dots, N). \quad (1.3.11)$$

In this case, X_i ($i=1,2,\dots,N$) in Eq.(1.3.8) is the noise intensity fluctuation existing in the i th frequency band-width of the observed noise intensity fluctuation at the observation point 0 (see Fig.1.3.1), before the barrier is constructed.

3.3. Experimental consideration

The experiment was done at night (20.00 P.M.-03.00 A.M.) in a playground to avoid the effect of surrounding background noises. The layout of the sound source, the barrier and two observation points is shown in Fig.1.3.2. A barrier made of plywood panel (height : 1.79m, width : 6.32m, thickness : 13.5mm) was used.

A road traffic noise wave was used as one example of actual envi-

ronmental random noise. Using two sound level meters, two kinds of A-weighted over-all noise intensity fluctuation waves, x and x' , were recorded on a data recorder before and after construction of the barrier. For simplification of experimental procedures, the noise intensity fluctuation wave was observed by use of the octave band analysis.

The intensity frequency characteristic, a_i , of the barrier estimated using Maekawa's evaluation chart for a barrier is shown in Table 1.3.1. The effect of reflection from the ground has been taken into consideration.

Figure 1.3.3 shows a comparison between the theoretically predicted curves using the proposed method and the experimentally sampled points obtained from the observed data x , for the cumulative distribution form of the noise intensity fluctuation in a case in which the height of the observation point was 0.8 m. In another case in which the height of the observation point was 1.3 m, a comparison between theory and experiment is shown in Fig.1.3.4.

In each case, it can be seen that the theoretical curves come closer to the experimentally sampled points as the number of correction terms increases.

3.4 Conclusion

In this chapter, a statistical prediction method for output random noise fluctuation has been proposed in a unified type of



Table 1.3.1 The estimated values of intensity frequency characteristic, a_1 , for a sound insulation barrier.

Center frequency of octave band (Hz)	$h=0.8$ (m)	$h=1.3$ (m)
250	0.07771	0.19498
500	0.04023	0.05370
1000	0.01943	0.04897
2000	0.00996	0.02754
4000	0.00479	0.01479

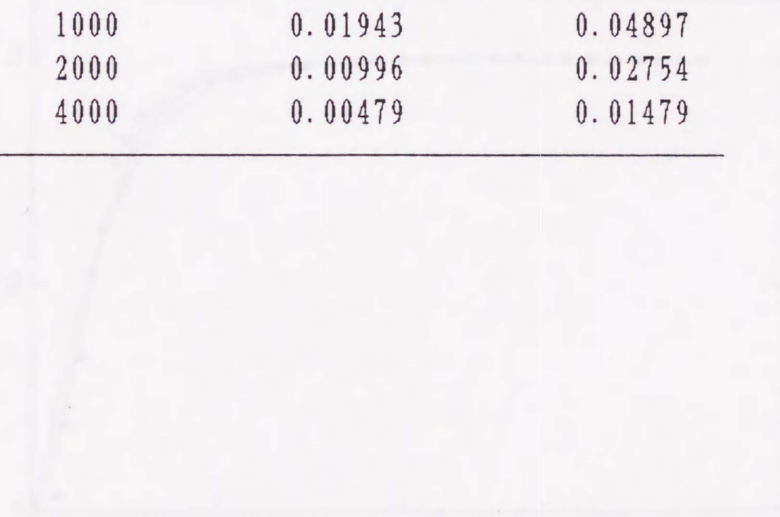


Fig. 1.3.2. Comparison between experimentally sampled points and theoretically predicted curves along the frequency axis for the same barrier with barrier form of the same geometry (insulating height of 1.3 m). Experimentally sampled points are marked by \square and theoretically predicted curves are respectively shown by —, the first approximation of Eq. (1.3.1), —, the second approximation —, and the third approximation.

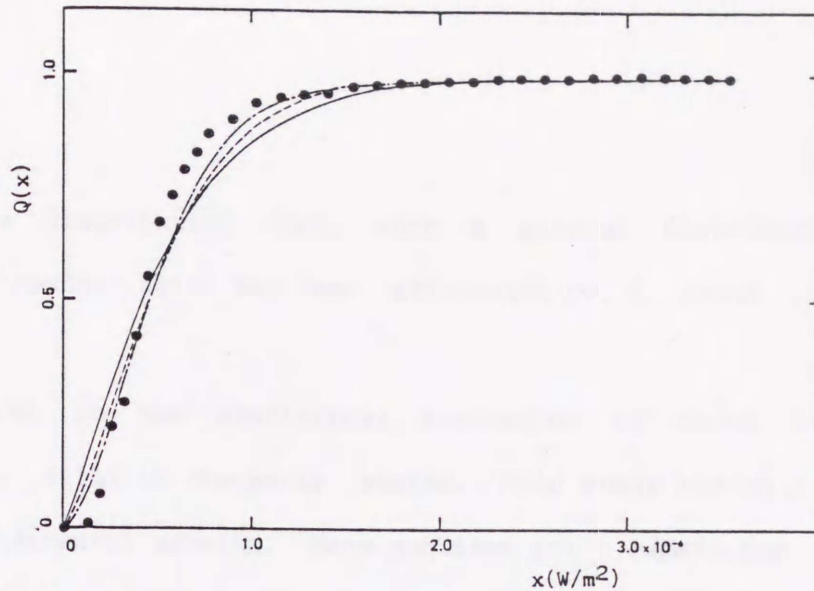


Fig.1.3.3 A comparison between experimentally sampled points and theoretically predicted curves using the proposed method for the cumulative distribution form of the noise intensity fluctuation (height of observation point : 0.8 m). Experimentally sampled points are marked by ● and theoretically predicted curves are respectively shown as —, the first approximation of Eq.(1.3.5) ; ---, the second approximation ; - · - ·, the third approximation.

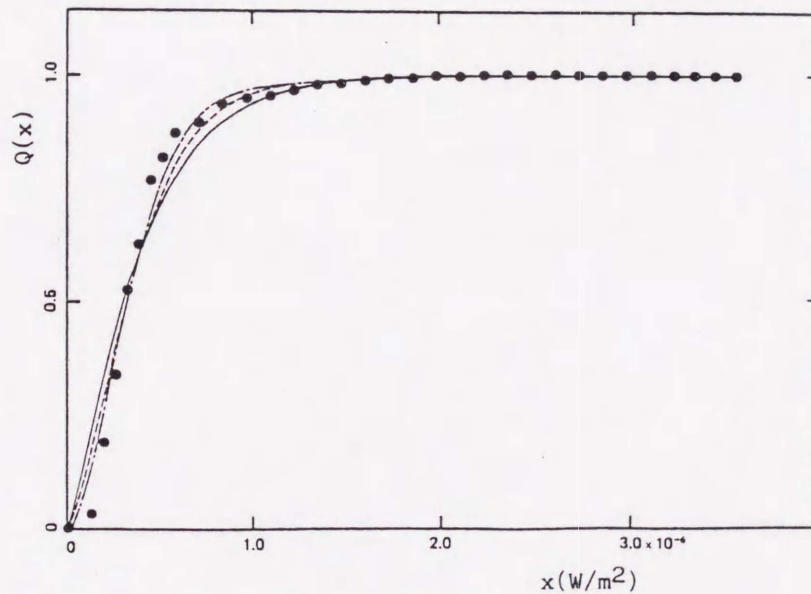


Fig.1.3.4 A comparison between experimentally sampled points and theoretically predicted curves using the proposed method for the cumulative distribution form of the noise intensity fluctuation (height of observation point : 1.3 m). Experimentally sampled points are marked by ● and theoretically predicted curves are respectively shown as —, the first approximation of Eq.(1.3.5) ; ---, the second approximation ; - · - ·, the third approximation.

probability distribution form, when a general distribution type stationary random noise has been attenuated by a sound insulation barrier.

Research on the statistical evaluation of sound insulation barriers is still in the early stages. This study has only focussed on some fundamental aspects. Many problems still remain for application to actual case which will be the subject of future studies.

PART II

MUTUAL RELATIONSHIP BETWEEN VARIOUS TYPES OF NOISE EVALUATION

INDICES CONNECTED WITH L_x AND L_{eq}

Chapter 1 A Method for Estimation of L_{eq} Noise Evaluation Index by
Use of Moment Statistics

1.1 Introduction

The following sampling method is often employed with the aid of a digital measurement technique to evaluate an L_{eq} noise evaluation index :

$$L_{eq} = 10 \log_{10} \frac{1}{N} \sum_{i=1}^N 10^{L_i/10}, \quad (2.1.1)$$

where N denotes the total number of the noise level L_i ($i=1,2,\dots,N$) sampled over a measurement time interval. Many instruments employ this evaluation method by using a usual digital sound level meter with quantized levels. From the viewpoint of signal processing, however, the following fundamental problems remain :

- (1) It is necessary to obtain many level data with a fairly fine sampling period, since the noise intensity fluctuation after the anti-logarithmic transformation of the sampled level datum :

$$x_i = x_0 \cdot 10^{L_i/10} \quad (x_0 : \text{reference noise intensity}) \quad (2.1.2)$$

fluctuates with large excursions, as compared with the original decibel-scaled level fluctuation.

- (2) In principle, the mean value of the noise intensity fluctuation

should be given by the sample mean operation based on the noise intensity data especially with an equally quantized intensity amplitude. If one uses the noise intensity fluctuation after transforming the measured noise level fluctuation with an equally quantized level amplitude into the intensity scale through the anti-logarithmic transformation, a calculation error of the intensity mean will occur because of the above level quantization.

When the noise evaluation index, L_{eq} , is calculated by the usual method (see Eq.(2.1.1)) based on its original definition with the use of actually measured data with digitalization, one can not avoid essentially the above two fundamental problems. Since this evaluation index is originally in the form of statistical information, a probabilistic relationship between this L_{eq} and the reliable and stable statistical information on noise level fluctuation should exist. If this relationship can be found theoretically and the objective L_{eq} from the statistical information on a decibel scale can be obtained by this theory, the above-mentioned fundamental problems can be avoided. That is, it is convenient to utilize explicitly the statistical information on the decibel-scaled level fluctuation itself with the aid of the theory for evaluating L_{eq} , since the above information is fairly stable and reliable, being based on the averaging operation supported by a large amount of data.⁽²⁶⁾ In this case, it would seem to be appropriate to derive first a general estimation method by using a unified expression on the noise level distribution. Furthermore, the actual environmental noise level fluctua-

tions encountered in daily life exhibit various types of probability distributions, due to the diversified types of noise fluctuation.

In consideration of these practical points, a general explicit expression for estimating L_{eq} using the moment statistics of the noise level fluctuation is proposed in this chapter after introduction of a general explicit expression on the arbitrary type noise level distribution form. More specifically, an expression for an arbitrary non-linear type expectation value is first derived based on an approach equivalent to that used for the well-known Gaussian distribution (this approach is quite similar to that discussed in Chapter 1, Part I). Next, an estimation method for the objective L_{eq} based on this expression is newly derived in a hierarchical form containing a well-known simplified estimation method derived under the assumption of ordinary Gaussian distribution as a special case.^{(27),(28)}

Finally, the effectiveness of the proposed estimation method has been experimentally confirmed by applying it to actually observed road traffic noise data.

1.2 Theoretical consideration

1.2.1 General theory for estimation of mean value of arbitrary non-linear function type random variables

To begin the analysis, let us introduce a statistical Hermite series expansion⁽¹³⁾ as a probability density function universally applicable to arbitrary non-Gaussian distribution forms of the noise level fluctuation L , as follows :

$$P(L) = N(L; \mu_L, \sigma_L^2) \left\{ 1 + \sum_{n=3}^{\infty} B_n H_n \left(\frac{L - \mu_L}{\sigma_L} \right) \right\} \quad (2.1.3)$$

with

$$\mu_L \triangleq \langle L \rangle, \quad \sigma_L^2 \triangleq \langle (L - \mu_L)^2 \rangle,$$

$$B_n \triangleq \frac{1}{n!} \left\langle H_n \left(\frac{L - \mu_L}{\sigma_L} \right) \right\rangle, \quad (2.1.4)$$

where $N(L; \mu_L, \sigma_L^2)$ denotes the well-known Gaussian distribution with the mean μ_L and the variance σ_L^2 :

$$N(L; \mu_L, \sigma_L^2) \triangleq \frac{1}{\sqrt{2\pi\sigma_L}} \exp \left[-\frac{(L - \mu_L)^2}{2\sigma_L^2} \right]. \quad (2.1.5)$$

By using the relationship between the Gaussian distribution and the Hermite polynomial⁽¹³⁾ :

$$N(L; \mu_L, \sigma_L^2) H_n \left(\frac{L - \mu_L}{\sigma_L} \right) = (-1)^n \sigma_L^n \cdot \frac{d^n}{dL^n} N(L; \mu_L, \sigma_L^2), \quad (2.1.6)$$

Eq.(2.1.3) can be rewritten as

$$P(L) = N(L; \mu_L, \sigma_L^2)$$

$$+ \sum_{n=3}^{\infty} B_n (-1)^n \sigma_L^n \frac{d^n}{dL^n} N(L; \mu_L, \sigma_L^2). \quad (2.1.7)$$

Based on the above relationship, after introducing an arbitrary non-linear function $g(\cdot)$ with respect to L , its expectation value is given by

$$\begin{aligned} \langle g(L) \rangle &\triangleq \int_{-\infty}^{\infty} g(L) P(L) dL \\ &= \int_{-\infty}^{\infty} g(L) N(L; \mu_L, \sigma_L^2) dL \\ &\quad + \sum_{n=3}^{\infty} B_n (-1)^n \sigma_L^n \int_{-\infty}^{\infty} g(L) \\ &\quad \cdot \frac{d^n}{dL^n} N(L; \mu_L, \sigma_L^2) dL. \end{aligned} \quad (2.1.8)$$

Furthermore, by introducing the following notation :

$$\langle g(L) \rangle_{\text{Gauss}} \triangleq \int_{-\infty}^{\infty} g(L) N(L; \mu_L, \sigma_L^2) dL \quad (2.1.9)$$

and carrying out the n -time integration by parts in the second term of Eq.(2.1.8), the following relationship can be obtained :

$$\langle g(L) \rangle = \langle g(L) \rangle_{\text{Gauss}} + \sum_{n=3}^{\infty} B_n \sigma_L^n \left\langle \frac{d^n}{dL^n} g(L) \right\rangle_{\text{Gauss}}. \quad (2.1.10)$$

The probabilistic boundary condition due to the reasonable limiting property given by

$$\frac{d^n}{dL^n} N(L; \mu_L, \sigma_L^2) \Big|_{L=-\infty \text{ or } L=\infty} = 0 \quad (2.1.11)$$

has been used for deriving the above relationship.

By using Eq.(2.1.10), the expectation value $\langle g(L) \rangle$ in terms of an arbitrary non-Gaussian distribution can be equivalently calculated based on an expectation operation in terms of an ordinary Gaussian distribution. The advantage of using Eq.(2.1.10) is that the well-known mathematical properties for Gaussian distribution can be utilized, since this probability distribution function plays an important role in the field of statistics and many related properties have already been investigated.

1.2.2 Estimation method of L_{eq} noise evaluation index

Consider an L_{eq} noise evaluation index defined as

$$L_{eq} \left(\triangleq 10 \log_{10} \frac{\langle x \rangle}{x_0} \right) = 10 \log_{10} \langle 10^{L/10} \rangle \quad (2.1.12)$$

After employing the following function form :

$$g(L) = 10^{L/10} \quad (2.1.13)$$

as a non-linear function $g(x)$ in Eq.(2.1.10), the following relationship can be derived by using the expectation operation in terms of a

well-known Gaussian distribution :

$$\begin{aligned} \langle g(L) \rangle_{\text{Gauss}} &\triangleq \int_{-\infty}^{\infty} 10^{L/10} \frac{1}{\sqrt{2\pi}\sigma_L} \exp\left[-\frac{(L-\mu_L)^2}{2\sigma_L^2}\right] dL \\ &= \exp(\mu_L/M + \sigma_L^2/2M^2) \quad (M \triangleq 10/1 \text{ n } 10), \end{aligned} \quad (2.1.14)$$

$$\left\langle \frac{d^n}{dL^n} g(L) \right\rangle_{\text{Gauss}} = \exp(\mu_L/M + \sigma_L^2/2M^2) / M^n. \quad (2.1.15)$$

Through substitution of Eqs.(2.1.14) and (2.1.15) into Eq.(2.1.10), the expectation value of the non-linear function, $g(x)$, based on the expectation operation in terms of an arbitrary non-Gaussian distribution can be expressed in a hierarchical form, as follows :

$$\begin{aligned} \langle g(L) \rangle &= \exp(\mu_L/M + \sigma_L^2/2M^2) \\ &\cdot \left[1 + \sum_{n=3}^{\infty} B_n \sigma_L^n / M^n \right]. \end{aligned} \quad (2.1.16)$$

Since the expansion coefficient B_n ($n=1,2,\dots$) is expressed in a specific form using its definition, as follows :

$$\begin{aligned} B_3 &= \frac{1}{3!} \left\langle H_3 \left(\frac{L-\mu_L}{\sigma_L} \right) \right\rangle = \frac{1}{6\sigma_L^3} \langle (L-\mu_L)^3 \rangle, \\ B_4 &= \frac{1}{4!} \left\langle H_4 \left(\frac{L-\mu_L}{\sigma_L} \right) \right\rangle = \frac{1}{24\sigma_L^4} [\langle (L-\mu_L)^4 \rangle - 3\sigma_L^4], \\ B_5 &= \frac{1}{5!} \left\langle H_5 \left(\frac{L-\mu_L}{\sigma_L} \right) \right\rangle = \frac{1}{120\sigma_L^5} \langle (L-\mu_L)^5 \rangle \\ &\quad - \frac{1}{12\sigma_L^3} \langle (L-\mu_L)^3 \rangle, \dots, \end{aligned} \quad (2.1.17)$$

Eq.(2.1.16) can consequently be expressed as

$$\langle g(L) \rangle = \exp(\mu_L/M + \sigma_L^2/2M^2) \left[1 + \frac{1}{6M^3} \langle (L - \mu_L)^3 \rangle + \frac{1}{24M^4} (\langle (L - \mu_L)^4 \rangle - 3\sigma_L^4) + \dots \right]. \quad (2.1.18)$$

After substitution of Eq.(2.1.18) into Eqs.(2.1.12) and (2.1.13), the objective explicit expression of hierarchically estimating L_{eq} can be obtained as

$$L_{eq} = \mu_L + \sigma_L^2/2M + M \ln \left[1 + \frac{1}{6M^3} \langle (L - \mu_L)^3 \rangle + \frac{1}{24M^4} (\langle (L - \mu_L)^4 \rangle - 3\sigma_L^4) + \dots \right]. \quad (2.1.19)$$

That is, from Eq.(2.1.19), L_{eq} can be given by the lower and higher order moment statistics for the noise level fluctuation of an arbitrary non-Gaussian distribution type. It should be noticed that the above estimation formula agrees completely with a well-known simplified estimation formula, derived under assumption of an ordinary Gaussian distribution as the first approximation^{(27),(28)}:

$$L_{eq} = \mu_L + 0.115 \sigma_L^2, \quad (2.1.20)$$

since the higher order expansion terms become zero for this case.

1.3 Experimental consideration

For the purpose of confirming the effectiveness of the proposed estimation method, two kinds of road traffic noise data measured using a digital sound level meter were employed. The first was measured near a national main road in a large city. At that time, the volume of traffic was 244 vehicles per 10 minutes. The second was measured near a country road in a rural district with a traffic volume of 19 vehicles per 10 minutes. The time interval for measurement and its sampling period were selected to be 10 minutes and 1 second, respectively. In addition, the values of L_{eq} in each case were simultaneously measured by a precision integrating sound level meter for L_{eq} . To simplify the notation of these cases, let us define the former case as " Case A " and the latter as " Case B ". Figure 2.1.1 shows the arrangements for measuring road traffic noise at an observation point for Case A. Figure 2.1.2 shows the arrangements for measuring road traffic noise at an observation point for Case B.

Figure 2.1.3 shows a comparison between the theoretically estimated curve using only the Gaussian distribution and the experimentally sampled points in the cumulative distribution form of the noise level fluctuation for Case A. From this figure, the theoretically estimated curve using only the Gaussian distribution agrees approximately with the experimentally sampled points. Thus, the data for Case A exhibits approximately the Gaussian distribution form. More-

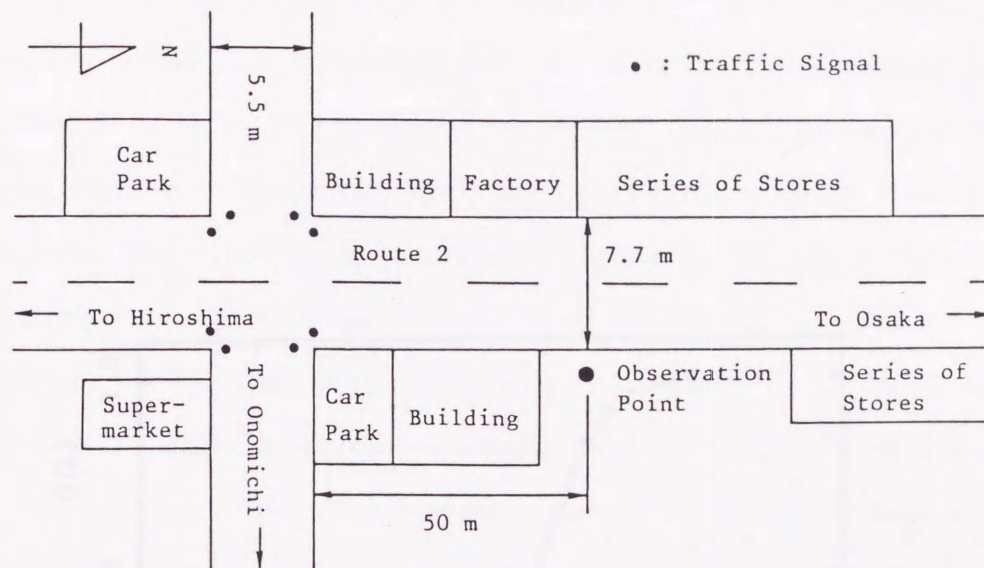


Fig.2.1.1 Arrangements for measuring road traffic noise at an observation point (Case A).

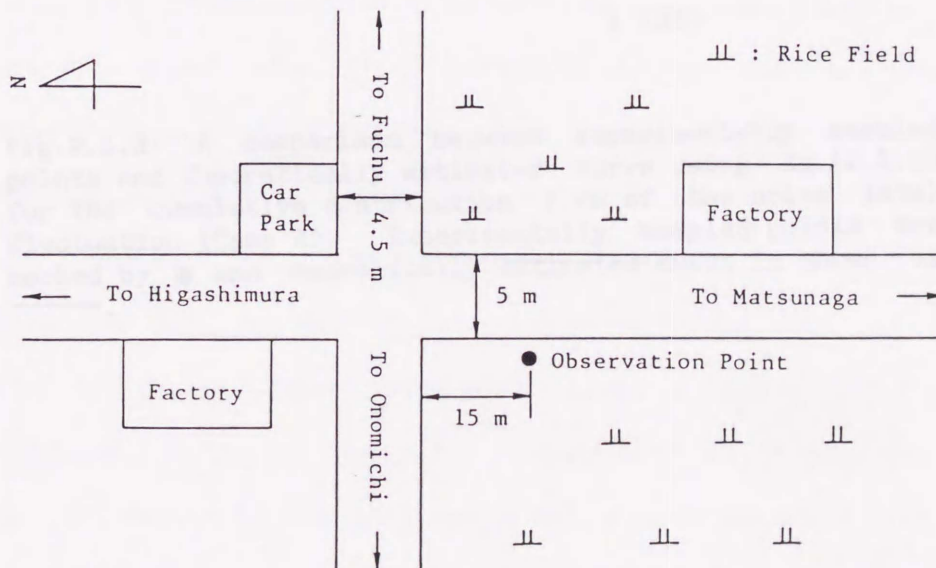


Fig.2.1.2 Arrangements for measuring road traffic noise at an observation point (Case B).

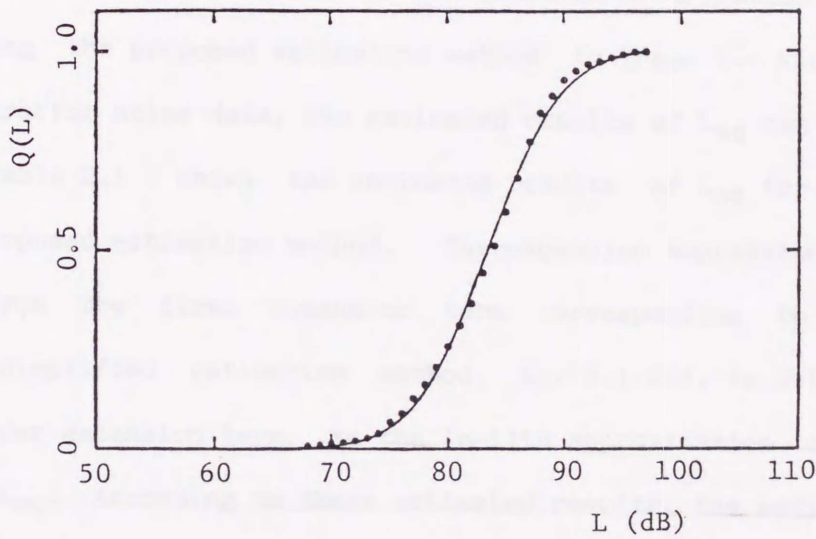


Fig.2.1.3 A comparison between experimentally sampled points and theoretically estimated curve using Eq.(2.1.5) for the cumulative distribution form of the noise level fluctuation (Case A). Experimentally sampled points are marked by ● and theoretically estimated curve is shown as —.

over, a comparison between the theoretically estimated curve using only the Gaussian distribution and the experimentally sampled points for Case B is shown in Fig.2.1.4. From this figure, the theoretically estimated curve using only the Gaussian distribution does not agree well with the experimentally sampled points. The data for Case B therefore strongly exhibits the non-Gaussian distribution form. After applying the proposed estimation method to these two kinds of actual road traffic noise data, the estimated results of L_{eq} can be obtained.

Table 2.1.1 shows the estimated results of L_{eq} for Case A using the proposed estimation method. The expansion expression of Eq.(2.1.19) from the first expansion term corresponding to the well-known simplified estimation method, Eq.(2.1.20), is defined to the n th order expansion term, as the $(n-1)$ th approximation of the estimated L_{eq} . According to these estimated results, the estimated value, using a well-known simplified estimation method derived under the assumption of an ordinary Gaussian distribution, agrees approximately with the measured value, since this data exhibits approximately the Gaussian distribution. More precisely, however, the estimated values using the proposed method tend agree well with the successive addition of higher order expansion terms. Table 2.1.2 shows the estimated results for Case B. According to these estimated results, the estimation accuracy using the well-known simplified estimation method is not sufficient for evaluation, since this data exhibits strongly the non-Gaussian distribution form. On the other hand, it is clear that the successive addition of higher order

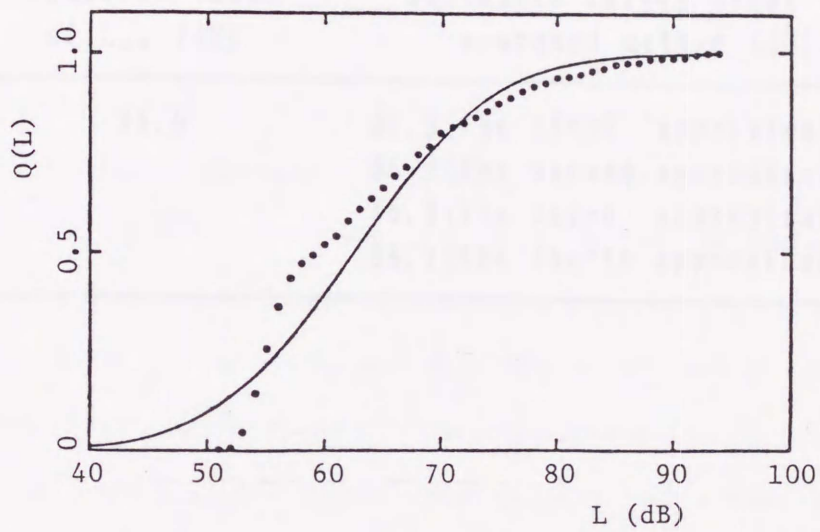


Fig.2.1.4 A comparison between experimentally sampled points and theoretically estimated curve using Eq.(2.1.5) for the cumulative distribution form of the noise level fluctuation (Case B). Experimentally sampled points are marked by ● and theoretically estimated curve is shown as —.

Table 2.1.1 The estimated results of L_{eq} using the proposed method (CaseA).

Measured value of L_{eq} (dB)	Estimated values using the proposed method (dB)
86.0	86.3 (the first approximation)
	86.2 (the second approximation)
	86.2 (the third approximation)
	86.1 (the fourth approximation)

Table 2.1.2 The estimated results of L_{eq} using the proposed method (CaseB).

Measured value of L_{eq} (dB)	Estimated values using the proposed method (dB)
76.4	72.9 (the first approximation)
	77.8 (the second approximation)
	78.6 (the third approximation)
	76.2 (the fourth approximation)

expansion terms moves the values estimated theoretically using the proposed method closer to the value measured experimentally. In each case, the estimated results agree well with the experimental results.

1.4 Conclusion

In this chapter, a new trial for estimating an L_{eq} noise evaluation index using the moment statistics of the noise level fluctuation has been proposed. More specifically, an explicit expression for an arbitrary non-linear function type expectation value which is equivalent to that for an ordinary Gaussian distribution has been first derived. Next, based on this convenient expression, a new trial of estimating L_{eq} using the moment statistics of the noise level fluctuation has been proposed, especially in a hierarchical form. Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise data.

This study is in the early stages and has therefore focussed only on the introduction of some methodological aspects. Accordingly, many future problems still remain. First, this method must be applied to many other actual cases to broaden and confirm its practical effectiveness. In addition, based on this method, a specific on-line measurement system for the sequential measurement of L_{eq} should be constructed for practical use.

Chapter 2 Mutual Relationship between Several Noise Evaluation Indices Connected with L_x and L_{eq}

2.1 Introduction

As is well-known, two noise evaluation indices, L_x and L_{eq} , play an important role in the field of noise evaluation and regulation problems. It is very important to find a general theory for the mutual relationships between several noise evaluation indices connected with L_x and L_{eq} , in order to grasp the systematical evaluation of environmental random noise. Up to now, various studies for the mutual relationship of this kind have already been investigated by many researchers. (27)-(34) Almost all these studies, however, were confined only to practical methods derived by applying directly the well-known linear regression analysis method to actually observed data or approximate methods derived under the assumption of a well-known Gaussian distribution. If a universally and objectively more precise relationship between these statistical evaluation indices is to be found, these conventional methods can not be used.

In this chapter, a general theory for the mutual relationship between several types of noise evaluation indices in relation to two typical indices, L_x and L_{eq} is proposed. More specifically, a general method for estimation of L_{eq} using several specific L_x is proposed in a general form including a simplified estimation method derived under the assumption of an ordinary Gaussian distribution as a spe-

cial case. Next, a new trial estimating the original noise level distribution using the measured values of L_{eq} and several specific L_x is proposed. The effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise data.

2.2 Theoretical consideration

2.2.1 Method for estimation of L_{eq} noise evaluation index by use of several specific L_x

Let us introduce the moment generating function $M_L(\theta)$ with respect to the noise level fluctuation L ($\Delta 10 \log_{10} x/x_0$), exhibiting an arbitrary non-Gaussian distribution form, as follows :

$$M_L(\theta) = \langle \exp(\theta \Delta 10 \log_{10} x/x_0) \rangle \quad (M \Delta 10 / \ln 10). \quad (2.2.1)$$

The mathematical relationship between the arbitrary order cumulant κ_{Ln} ($n=1,2,\dots$) with respect to L and the moment generating function $M_L(\theta)$ is given by

$$M_L(\theta) = \exp \left(\sum_{n=1}^{\infty} \frac{\kappa_{Ln}}{n!} \theta^n \right). \quad (2.2.2)$$

By replacing θ with $1/M$ in Eqs.(2.2.1) and (2.2.2), the following relationship can easily be obtained as

$$\langle x \rangle = x_0 \exp \left(\sum_{n=1}^{\infty} \frac{1}{n!} \cdot \frac{\kappa_{Ln}}{M^n} \right). \quad (2.2.3)$$

A substitution of Eq.(2.2.3) into the definition of L_{eq} thus yields a general expression of a series expansion type for estimating L_{eq} , as follows :

$$\begin{aligned} L_{eq} &\triangleq 10 \log_{10} \frac{\langle x \rangle}{x_0} \\ &= \kappa_{L1} + \frac{\kappa_{L2}}{2M} + \frac{\kappa_{L3}}{6M^2} + \frac{\kappa_{L4}}{24M^3} + \frac{\kappa_{L5}}{120M^4} + \dots \\ &= \mu_L + 0.115 \sigma_L^2 + 8.84 \times 10^{-3} \kappa_{L3} \\ &\quad + 5.09 \times 10^{-4} \kappa_{L4} + 2.34 \times 10^{-5} \kappa_{L5} + \dots \end{aligned} \quad (2.2.4)$$

When the noise level fluctuation exhibits an ordinary Gaussian distribution, Eq.(2.2.4) is reduced to a well-known simplified estimation method derived under the assumption of a well-known Gaussian distribution^{(27),(28)} :

$$L_{eq} = \mu_L + 0.115 \sigma_L^2 = L_{50} + 0.115 \sigma_L^2, \quad (2.2.5)$$

since the higher order cumulants κ_{Ln} ($n=3,4,\dots$) in this case are equal to zero.

A general explicit expression of the cumulative distribution function for the noise level fluctuation L is expressed in a statistical Hermite series expansion,⁽¹³⁾ as follows :

$$\begin{aligned}
Q(L) &= \int_{-\infty}^L N(\xi; \mu_L, \sigma_L^2) d\xi \\
&- \frac{\kappa_{L3}}{6\sigma_L^2} N(L; \mu_L, \sigma_L^2) H_2\left(\frac{L - \mu_L}{\sigma_L}\right) \\
&- \frac{\kappa_{L4}}{24\sigma_L^3} N(L; \mu_L, \sigma_L^2) H_3\left(\frac{L - \mu_L}{\sigma_L}\right) - \dots \quad (2.2.6)
\end{aligned}$$

According to the definition of L_X , the following relationship can easily be obtained :

$$\begin{aligned}
1 - \frac{X}{100} &= \int_{-\infty}^{L_X} N(\xi; \mu_L, \sigma_L^2) d\xi \\
&- \frac{\kappa_{L3}}{6\sigma_L^2} N(L_X; \mu_L, \sigma_L^2) H_2\left(\frac{L_X - \mu_L}{\sigma_L}\right) \\
&- \frac{\kappa_{L4}}{24\sigma_L^3} N(L_X; \mu_L, \sigma_L^2) H_3\left(\frac{L_X - \mu_L}{\sigma_L}\right) - \dots \quad (2.2.7)
\end{aligned}$$

The application of N kinds of specific L_X ($L_{X1}, L_{X2}, \dots, L_{XN}$) to Eq. (2.2.7) thus yields the N -dimensional simultaneous equations by regarding the cumulant statistics κ_{Ln} ($n=3,4,\dots$) as unknown parameters. From the practical point of view, the explicit expressions for estimating L_{eq} for cases with $N=1$ and $N=2$ are shown as

(1) $N=1$ (reference level L_{X1}) :

$$L_{eq} = \mu_L + 0.115\sigma_L^2 + 8.84 \times 10^{-3} \kappa_{L3} \quad (2.2.8)$$

with

$$\kappa_{L3} = \frac{6 \sigma_L^2 F(Z_1)}{N(Z_1; 0, 1) H_2(Z_1)}, \quad Z_1 \triangleq \frac{L_{x1} - \mu_L}{\sigma_L}, \quad (2.2.9)$$

where $F(z)$ is defined as

$$F(Z) \triangleq \int_{-\infty}^Z N(\xi; 0, 1) d\xi + \frac{X}{100} - 1. \quad (2.2.10)$$

(2) $N=2$ (reference levels : L_{x1} and L_{x2}) :

$$L_{eq} = \mu_L + 0.115 \sigma_L^2 + 8.84 \times 10^{-3} \kappa_{L3} + 5.09 \times 10^{-4} \kappa_{L4} \quad (2.2.11)$$

with

$$\kappa_{L3} = \frac{6 \sigma_L^2 [N(Z_2; 0, 1) H_3(Z_2) F(Z_1) - N(Z_1; 0, 1) H_3(Z_1) F(Z_2)]}{N(Z_1; 0, 1) N(Z_2; 0, 1) [H_2(Z_1) H_3(Z_2) - H_2(Z_2) H_3(Z_1)]},$$

$$\kappa_{L4} = \frac{24 \sigma_L^3 [N(Z_1; 0, 1) H_2(Z_1) F(Z_2) - N(Z_2; 0, 1) H_2(Z_2) F(Z_1)]}{N(Z_1; 0, 1) N(Z_2; 0, 1) [H_2(Z_1) H_3(Z_2) - H_2(Z_2) H_3(Z_1)]},$$

$$Z_1 \triangleq \frac{L_{x1} - \mu_L}{\sigma_L}, \quad Z_2 \triangleq \frac{L_{x2} - \mu_L}{\sigma_L}. \quad (2.2.12)$$

2.2.2 Method for estimation of noise probability distribution by use

of L_{eq} and several specific L_x

In this section, let us consider a general method for estimation of the original noise level distribution by using the measured values of L_{eq} and several specific L_x . To begin the analysis, a statistical Laguerre series expansion⁽¹³⁾ can be employed as a general expression of the noise intensity distribution. The cumulative distribution function of this expression is given after introducing a dimensionless variable, $\eta(=x/s)$, as follows :

$$\begin{aligned}
 Q(\eta) = & \int_0^\eta P_\Gamma(\xi; m) d\xi \\
 & + A'_3 \frac{\eta^m}{\Gamma(m)} \exp(-\eta) f_1(\eta) \\
 & + A'_4 \frac{\eta^m}{\Gamma(m)} \exp(-\eta) f_2(\eta) + \dots
 \end{aligned}
 \tag{2.2.13}$$

with

$$m = \mu_x^2 / \sigma_x^2, \quad s = \sigma_x^2 / \mu_x,$$

$$A'_n \triangleq \frac{n! \Gamma(m)}{\Gamma(m+n)} \left\langle L_n^{(m-1)} \left(\frac{x}{s} \right) \right\rangle, \tag{2.2.14}$$

where $f_1(\eta)$ and $f_2(\eta)$ are respectively defined as

$$\begin{aligned}
f_1(\eta) &\triangleq \eta^2 - 2(m+2)\eta + (m+2)(m+1), \\
f_2(\eta) &\triangleq -\eta^3 + 3(m+3)\eta^2 - 3(m+3)(m+2)\eta \\
&\quad + (m+3)(m+2)(m+1). \tag{2.2.15}
\end{aligned}$$

Moreover, μ_x and σ_x^2 are the mean value of x and its variance, respectively. The distribution parameters, m and s , can be rewritten by using L_{eq} , as follows :

$$\begin{aligned}
m &= 10 \frac{(0.2 L_{eq} + 2 \log_{10} x_0 - \log_{10} \sigma_x^2)}{\dots}, \\
s &= 10 \frac{(-0.1 L_{eq} - \log_{10} x_0 + \log_{10} \sigma_x^2)}{\dots} \tag{2.2.16}
\end{aligned}$$

From Eq.(2.2.13) and the definition of L_x , the following relationship can easily be obtained :

$$\begin{aligned}
1 - \frac{x}{100} &= \int_0^{\eta_x} P_\Gamma(\xi; m) d\xi \\
&\quad + A'_3 \frac{\eta_x^m}{\Gamma(m)} \exp(-\eta_x) f_1(\eta_x) \\
&\quad + A'_4 \frac{\eta_x^m}{\Gamma(m)} \exp(-\eta_x) f_2(\eta_x) + \dots, \\
\eta_x &\triangleq \frac{x_0}{s} 10^{L_x/10}. \tag{2.2.17}
\end{aligned}$$

Thus, the application of N kinds of specific L_x ($L_{x1}, L_{x2}, \dots, L_{xN}$) to Eq.(2.2.17) yields N -dimensional simultaneous equations, by regarding the expansion coefficients, A'_n ($n=3,4,\dots,N+2$), as unknown parameters. After estimating the distribution parameters, m , s and A'_n , using the known values of L_{eq} and several specific L_x , one can evaluate the original noise level distribution by substituting the estimated values of A'_n into Eq.(2.2.13). At this time, the cumulative distribution function $Q(L)$ with respect to L can be evaluated using the probability measure preserving transformation. From the practical point of view, the explicit expressions for estimating A'_n for cases with $N=1$ and $N=2$ are shown as

(1) $N=1$ (reference level : L_{x1}) :

$$A'_3 = \frac{6 \Gamma(m) G(x_1)}{\eta_1^m f_1(\eta_1) \exp(-\eta_1)} \quad (2.2.18)$$

with

$$\eta_1 \triangleq \frac{x_0}{s} 10^{L_{x1}/10}, \quad (2.2.19)$$

where $G(x)$ is defined as

$$G(x) \triangleq 1 - \frac{x}{100} - \int_0^x P_\Gamma(\xi; m) d\xi. \quad (2.2.20)$$

(2) N=2 (reference levels : L_{x1} and L_{x2})

$$A'_3 = \frac{6\Gamma(m) \{ \eta_2^m f_2(\eta_2) \exp(-\eta_2) G(x_1) - \eta_1^m f_2(\eta_1) \exp(-\eta_1) G(x_2) \}}{\eta_1^m \eta_2^m \exp\{-(\eta_1 + \eta_2)\} [f_1(\eta_1) f_2(\eta_2) - f_1(\eta_2) f_2(\eta_1)]},$$

$$A'_4 = \frac{24\Gamma(m) \{ \eta_1^m f_1(\eta_1) \exp(-\eta_1) G(x_2) - \eta_2^m f_2(\eta_2) \exp(-\eta_2) G(x_1) \}}{\eta_1^m \eta_2^m \exp\{-(\eta_1 + \eta_2)\} [f_1(\eta_1) f_2(\eta_2) - f_1(\eta_2) f_2(\eta_1)]},$$

(2.2.21)

with

$$\eta_1 \triangleq \frac{x_0}{s} 10^{L_{x1}/10}, \quad \eta_2 \triangleq \frac{x_0}{s} 10^{L_{x2}/10}.$$

(2.2.22)

2.3 Experimental consideration

For the purpose of confirming the effectiveness of the proposed estimation method, two kinds of road traffic noise data have been used. Road traffic noise was measured near a national main road in a large city, where the traffic volume was more than 3000 vehicles per hour, and near a national main road in a rural district with a traffic volume of about 1000 vehicles per hour. The measurement time interval and its sampling period were selected to be 1 hour and 1 second, respectively. To simplify the notation of these cases, let us define the former case as "Case A" and the latter as "Case B".

Figure 2.2.1 shows the arrangements for measuring road traffic noise at an observation point for Case A, and Fig. 2.2.2 shows the arrangements for measuring road traffic noise at an observation point for Case B.

Figure 2.2.3 shows a comparison between the theoretically estimated curves using the statistical Hermite series expansion, Eq.(2.2.6), and the experimentally sampled points in the cumulative distribution form of the noise level fluctuation for Case A. From this figure, the theoretically estimated curve using only the first expansion term (i.e., Gaussian distribution) agrees approximately with the experimentally sampled points. Thus, the data for Case A exhibits approximately the Gaussian distribution form. Figure 2.2.4 shows a comparison between the theoretically estimated curves and the experimentally sampled points for Case B. From this figure, the theoretically estimated curve using only the first expansion term (i.e., Gaussian distribution) does not agree well with the experimentally sampled points. Thus, the data for Case B exhibits the non-Gaussian distribution form. Figure 2.2.5 shows a comparison between the theoretically estimated curves using the statistical Laguerre series expansion and the experimentally sampled points in the cumulative distribution form for the noise intensity fluctuation for Case A. From this figure, the theoretically estimated curve using only the first expansion term (i.e., Gamma distribution) does not agree well with the experimentally sampled points. The data for Case A exhibits the non-Gamma distribution form. A comparison between the theoretically estimated curves and the experimen-

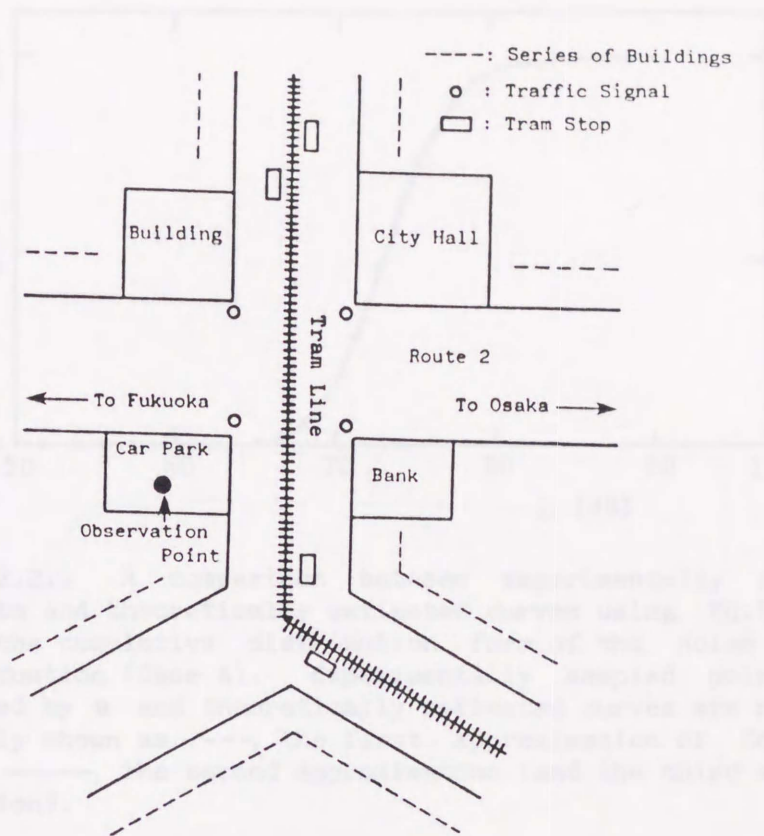


Fig.2.2.1 Arrangements for measuring road traffic noise at an observation point (Case A).

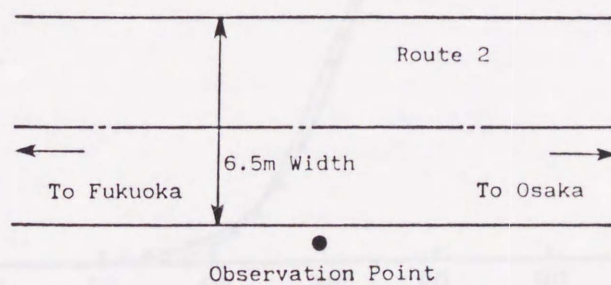


Fig.2.2.2 Arrangements for measuring road traffic noise at an observation point (Case B).

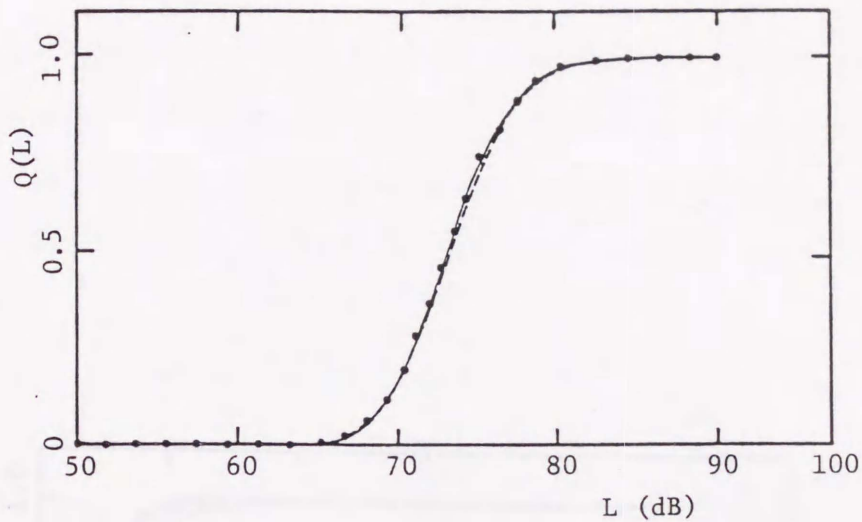


Fig.2.2.3 A comparison between experimentally sampled points and theoretically estimated curves using Eq.(2.2.6) for the cumulative distribution form of the noise level fluctuation (Case A). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.2.6) ; ———, the second approximation (and the third approximation).

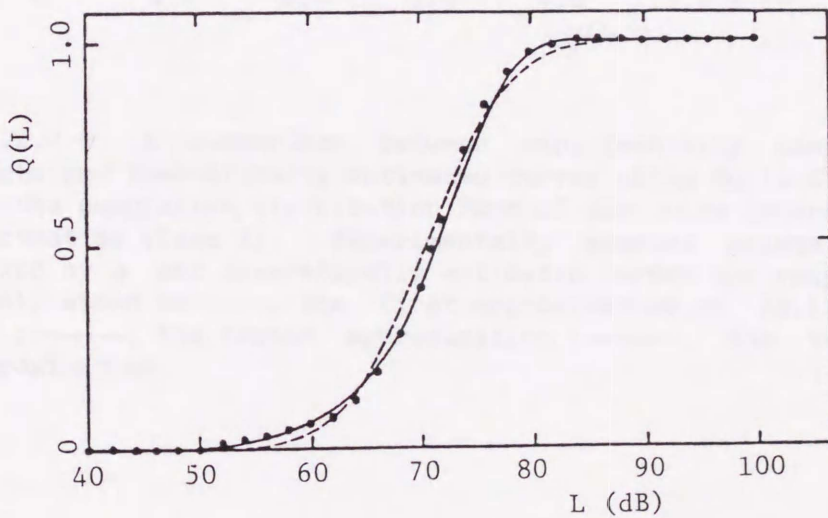


Fig.2.2.4 A comparison between experimentally sampled points and theoretically estimated curves using Eq.(2.2.6) for the cumulative distribution form of the noise level fluctuation (Case B). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.2.6) ; ———, the second approximation (and the third approximation).

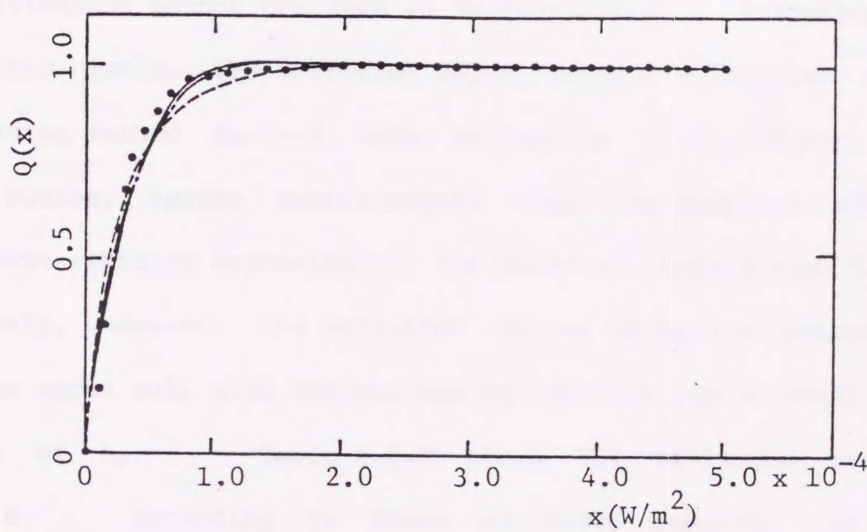


Fig.2.2.5 A comparison between experimentally sampled points and theoretically estimated curves using Eq.(2.2.13) for the cumulative distribution form of the noise intensity fluctuation (Case A). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.2.13); - · - · -, the second approximation; ———, the third approximation.

tally sampled points for Case B is shown in Fig.2.2.6. The theoretically estimated curve using only the first expansion term (i.e., Gamma distribution) agrees approximately with the experimentally sampled points. The data for Case B exhibits approximately the Gamma distribution.

Table 2.2.1 shows the estimated results of L_{eq} for Case A using the estimation method proposed in Section 2.2.1. According to these estimated results, the estimated value, using a well-known simplified estimation method derived under assumption of an ordinary Gaussian distribution, agrees approximately with the measured value, since this data exhibits approximately the Gaussian distribution form. More precisely, however, the estimated values using the proposed method tend to agree well with the successive addition due to other reference levels of L_x . Table 2.2.2 shows the estimated results for Case B. According to these estimated results, the accuracy of the well-known simplified estimation method is not sufficient for evaluation, since this data exhibits the non-Gaussian distribution form. The values estimated theoretically using the proposed method agree well with the measured value of L_{eq} , compared with the values estimated by the well-known simplified estimation method.

Figure 2.2.7 shows a comparison between the theoretically estimated curves using the method proposed in Section 2.2.2 and the experimentally sampled points in the cumulative distribution form of the noise level fluctuation for Case A. Moreover, Fig.2.2.8 shows the es-

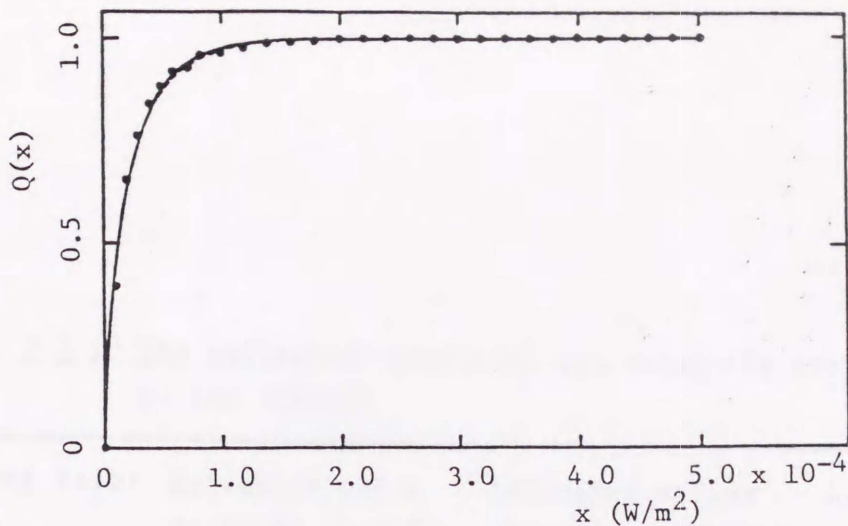


Fig.2.2.6 A comparison between experimentally sampled points and theoretically estimated curves using Eq.(2.2.13) for the cumulative distribution form of the noise intensity fluctuation (Case B). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.2.13); —, the second approximation (and the third approximation).

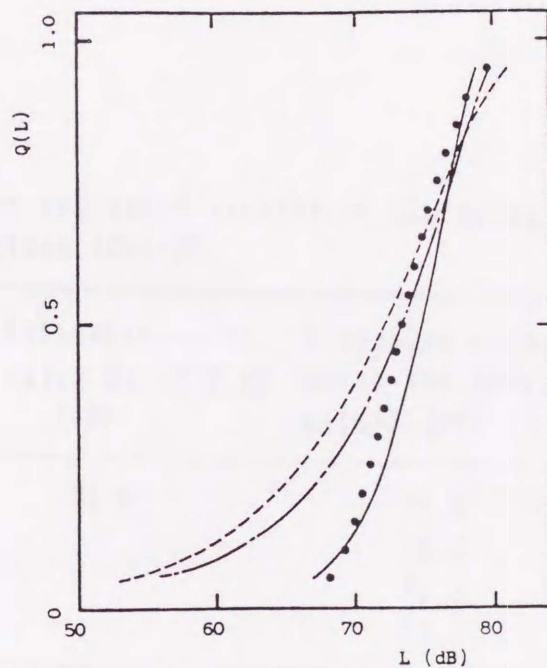


Fig.2.2.7 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case A). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation using L_{eq} and σ_x^2 ; - · - · -, the second approximation using L_{eq} , σ_x^2 and L_{50} ; —, the third approximation using L_{eq} , σ_x^2 , L_{10} and L_{90} .

Table 2.2.1 The estimated results of L_{eq} using the proposed method (CaseA).

Measured value of L_{eq} (dB)	Estimated value using Eq. (2.2.5) (dB)	Estimated values using the proposed method (dB)	L_x levels used in the proposed method
75.0	74.8	74.8	L_{10}
		74.9	L_{50}
		75.0	L_{10}, L_{50}
		74.9	L_{50}, L_{90}

Table 2.2.2 The estimated results of L_{eq} using the proposed method (CaseB).

Measured value of L_{eq} (dB)	Estimated value using Eq. (2.2.5) (dB)	Estimated values using the proposed method (dB)	L_x levels used in the proposed method
73.7	75.5	74.6	L_{10}
		74.7	L_{50}
		74.4	L_{10}, L_{50}
		74.3	L_5, L_{10}

estimated results for Case B. From these figures, it is obvious that the successive addition of higher expansion terms due to the reference levels of L_x moves the values estimated theoretically using the proposed method closer to the values measured experimentally.

To confirm the effectiveness of the proposed estimation method, road traffic noise data was also measured near an expressway in a rural district where the traffic volume was about 300 vehicles per hour. Table 2.2.3 shows the estimated results using the estimation method proposed in Section 2.2.1. Furthermore, in consideration of the effect of background noises, the estimated results for the original noise level distribution above L_{90} for Case B using the estimation method proposed in Section 2.2.2 are shown in Fig.2.2.9.

2.4 Conclusion

In this chapter, a general theory for the mutual relationship between several types of noise evaluation indices in relation to two typical indices, L_x and L_{eq} , has been proposed for the purpose of systematic evaluation of environmental noise. More specifically, a general method for estimation of L_{eq} using several specific L_x has been first proposed in a generalized form with inclusion of a well-known simplified estimation method derived under the assumption of an ordinary Gaussian distribution as a special case. Next, a new trial estimating the original noise level distribution using the measured values

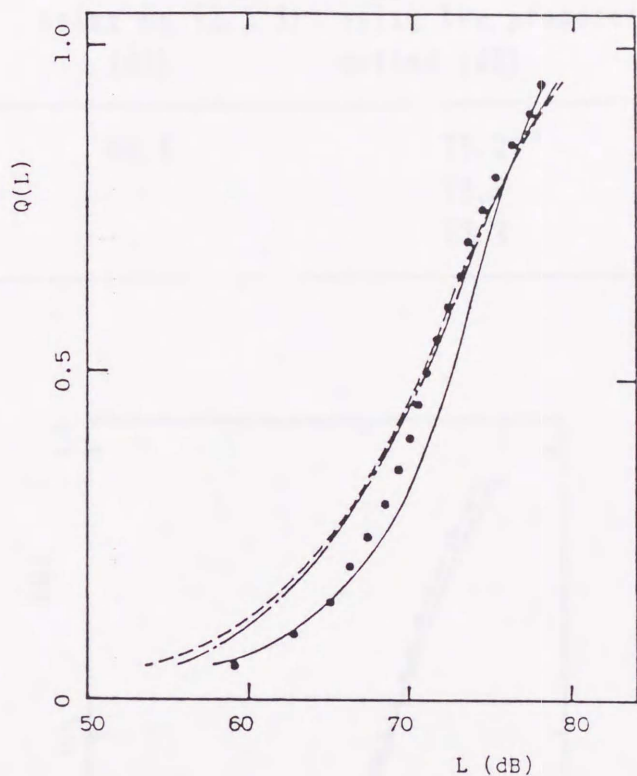


Fig.2.2.8 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case B). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation using L_{eq} and σ_x^2 ; - · - · -, the second approximation using L_{eq} , σ_x^2 and L_{50} ; —, the third approximation using L_{eq} , σ_x^2 , L_{10} and L_{90} .

Table 2.2.3 The estimated results of L_{eq} using the proposed method.

Measured value of L_{eq} (dB)	Estimated value using Eq. (2.2.5) (dB)	Estimated values using the proposed method (dB)	L_x levels used in the proposed method
72.5	68.4	73.2	L_{10}
		73.7	L_{50}
		73.3	L_5, L_{10}

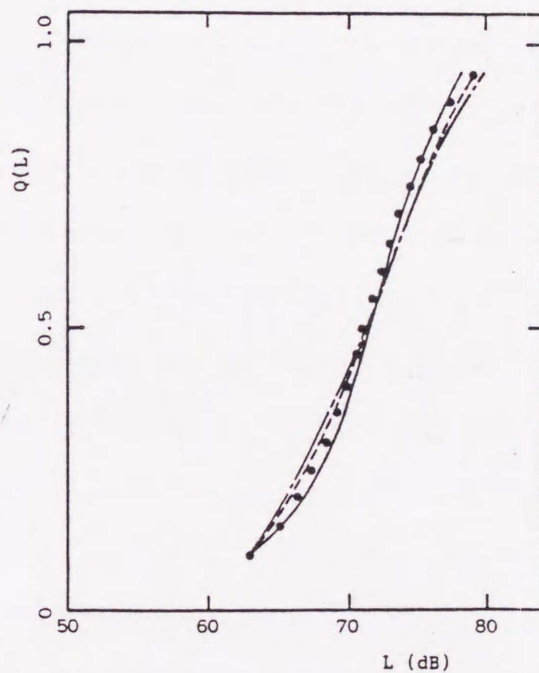


Fig.2.2.9 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form above the level value of L_{90} (Case B) by considering the effect of background noises. Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation using L_{eq} and σ_x^2 ; - · - · -, the second approximation using L_{eq} , σ_x^2 and L_{50} ; ———, the third approximation using L_{eq} , σ_x^2 , L_{10} and L_{50} .

of L_{eq} and several specific L_x has been proposed. The effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise data. The following points remain for future studies. First, this method must be applied to many other actual cases to broaden and confirm its practical effectiveness. It is necessary that similar mutual relationships be found with other noise evaluation indices such as L_{NP} , TNI , $WECPNL$, etc.

Chapter 3 Further Investigation of Method for Estimation of Noise Probability Distribution by Use of L_{eq} and Several Specific L_x

3.1 Introduction

In order to grasp quantitatively the mutual relationship between two noise evaluation indices, L_{eq} and L_x , somewhat systematical studies have already been considered in Chapter 2, and a method for estimation of L_{eq} was proposed using several specific L_x , with attention paid to the noise probability expression on a decibel scale. In addition, a new trial for estimation of the original noise level distribution using the measured values of L_{eq} and several specific L_x was also proposed with attention paid to the noise probability expression on an intensity scale (i.e., statistical Laguerre series expansion). This method would appear to be principally applicable to any kind of random phenomena. From the viewpoint of noise evaluation, however, such a probability expression on an intensity scale should be reconsidered at the first stage of study, especially in the form matched to the acoustic measurement on a decibel scale.

In this chapter, a further investigation in the statistical analysis for generally estimating the original noise level distribution on a decibel scale is proposed by first introducing a statistical orthonormal series expansion on an intensity scale taking the lognormal distribution as the first expansion term. The effectiveness of

the proposed method has been experimentally confirmed by applying it to actual road traffic noise data. The results estimated using the proposed method agree very well with the experimental values compared with the results estimated using the previous method.

3.2 Theoretical consideration

As touched on in the introduction, let us first introduce a statistical orthonormal series expansion, for the purpose of employing the noise probability expression on an intensity scale in the form matched to the acoustic measurement on a decibel scale, taking the lognormal distribution as the first expansion term. The above expression is shown as

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\} \cdot \left[1 + \sum_{n=3}^{\infty} C_n \phi_n(x)\right], \quad (2.3.1)$$

where two distribution parameters, μ and σ^2 , are given by the mean and variance of the noise intensity fluctuation x , as follows⁽³⁵⁾:

$$\mu \triangleq \ln \frac{\langle x \rangle^2}{\sqrt{\langle x^2 \rangle}}, \quad \sigma^2 \triangleq \ln \frac{\langle x \rangle^2}{\langle x^2 \rangle}, \quad (2.3.2)$$

and the expansion coefficient C_n is given by the orthonormal polynomial, $\phi_n(x)$, as follows:

$$C_n \triangleq \langle \phi_n(x) \rangle. \quad (2.3.3)$$

The orthonormal polynomial $\phi_n(x)$ is determined by Schmidt's orthogonalization procedure⁽³⁶⁾:

$$\phi_n(x) = \sum_{j=0}^n \lambda_{nj} x^j, \quad (2.3.4)$$

where the orthogonalization coefficient λ_{nj} is defined as

$$\lambda_{nj} = (d_n \cdot d_{n-1})^{-1/2} \cdot ((n+1, j+1) \text{ cofactor of } d_n(x)) \quad (2.3.5)$$

with

$$d_n \triangleq \begin{vmatrix} (x^0, x^0) & (x^0, x^1) & \cdots & (x^0, x^n) \\ (x^1, x^0) & (x^1, x^1) & \cdots & (x^1, x^n) \\ \vdots & \vdots & \ddots & \vdots \\ (x^n, x^0) & (x^n, x^1) & \cdots & (x^n, x^n) \end{vmatrix},$$

$$d_n(x) \triangleq \begin{vmatrix} (x^0, x^0) & (x^0, x^1) & \cdots & (x^0, x^j) & \cdots & (x^0, x^n) \\ (x^1, x^0) & (x^1, x^1) & \cdots & (x^1, x^j) & \cdots & (x^1, x^n) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (x^{n-1}, x^0) & (x^{n-1}, x^1) & \cdots & (x^{n-1}, x^j) & \cdots & (x^{n-1}, x^n) \\ x^0 & \cdots & x^1 & \cdots & x^j & \cdots & x^n \end{vmatrix}. \quad (2.3.6)$$

Here, each element of the above matrices is given by

$$\begin{aligned}
 (x^i, x^j) &\triangleq \int_0^{\infty} d(x) x^i x^j dx \\
 &= \int_0^{\infty} x^{i+j} \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\} dx \\
 &= \exp\left\{(i+j)\mu + \frac{1}{2}(i+j)^2\sigma^2\right\} \quad (2.3.7)
 \end{aligned}$$

with

$$d(x) \triangleq \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\}. \quad (2.3.8)$$

Since L_{eq} is originally evaluated from the averaged noise intensity, two distribution parameters, μ and σ^2 can be rewritten using L_{eq} and σ_x^2 (variance of x), as follows :

$$\begin{aligned}
 \mu &= 2 \ln x_0 + \frac{\ln 10}{5} L_{eq} - \frac{1}{2} \ln \left\{ \sigma_x^2 + x_0^2 \cdot 10^{\frac{L_{eq}}{5}} \right\}, \\
 \sigma^2 &= \ln \left\{ \sigma_x^2 + x_0^2 \cdot 10^{\frac{L_{eq}}{5}} \right\} - 2 \ln x_0 - \frac{\ln 10}{5} L_{eq}. \quad (2.3.9)
 \end{aligned}$$

The cumulative distribution function $Q(x)$ connected directly with an arbitrary L_x , is formulated from Eq.(2.3.1) and its definition, as follows :

$$\begin{aligned}
 Q(x) &\triangleq \int_0^x P(\xi) d\xi = \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} N(\xi; 0, 1) d\xi \\
 &+ \sum_{n=3}^{\infty} \sum_{j=0}^n C_n \lambda_{nj} \exp\{-2j\mu - j^2\sigma^2\} \\
 &\quad \cdot \int_{-\infty}^{\frac{\ln x - \mu - j\sigma^2}{\sigma}} N(\xi; 0, 1) d\xi \quad (2.3.10)
 \end{aligned}$$

with

$$N(\xi; 0, 1) \triangleq \frac{1}{\sqrt{2\pi}} \exp\{-\xi^2/2\}. \quad (2.3.11)$$

From the definition of L_X and Eq.(2.3.10), the following relationship can be easily obtained :

$$1 - \frac{X}{100} = \int_{-\infty}^{\frac{\ln X x - \mu}{\sigma}} N(\xi; 0, 1) d\xi + \sum_{n=3}^{\infty} \sum_{j=0}^n C_n \lambda_{nj} \exp\{-2j\mu - j^2\sigma^2\} \cdot \int_{-\infty}^{\frac{\ln X x - \mu - j\sigma^2}{\sigma}} N(\xi; 0, 1) d\xi, \quad (2.3.12)$$

where x_X denotes the intensity value corresponding to L_X , as follows :

$$x_X = x_0 \cdot 10^{L_X/10}. \quad (2.3.13)$$

Let x_{Xn} ($n=1,2,\dots,N$) be the intensity values corresponding to the specific L_{Xn} ($n=1,2,\dots,N$). After applying these x_{Xn} ($n=1,2,\dots,N$) values to Eq.(2.3.12), the N -dimensional simultaneous equations can be constructed by regarding the expansion coefficient C_n ($n=3,4,\dots,N+2$) as unknown parameters. From the estimated values of μ, σ^2 and C_n through the above procedures, one can estimate the original cumulative distribution function after substituting these values into Eq.(2.3.10). The objective cumulative distribution form $Q(L)$ on a decibel scale can of course be evaluated using the probability measure pre-

serving transformation :

$$Q(L) = Q(x) \left| \begin{array}{l} x = x_0 \cdot 10^{L/10} \end{array} \right. \quad (2.3.14)$$

From the practical point of view, the explicit expressions for estimating C_n for cases with $N=1$ and $N=2$ are shown as

(1) $N=1$ (reference level : L_{x1}) :

$$C_3 = \frac{G_1(x_{x1}; \mu, \sigma^2)}{F_1(x_{x1}; \mu, \sigma^2)} \quad (2.3.15)$$

(2) $N=2$ (reference levels : L_{x1} and L_{x2}) :

$$C_3 = \frac{G_1(x_{x1}; \mu, \sigma^2) F_2(x_{x2}; \mu, \sigma^2) - G_2(x_{x2}; \mu, \sigma^2) F_2(x_{x1}; \mu, \sigma^2)}{F_1(x_{x1}; \mu, \sigma^2) F_2(x_{x2}; \mu, \sigma^2) - F_1(x_{x2}; \mu, \sigma^2) F_2(x_{x1}; \mu, \sigma^2)},$$

$$C_4 = \frac{G_2(x_{x2}; \mu, \sigma^2) F_1(x_{x1}; \mu, \sigma^2) - G_1(x_{x1}; \mu, \sigma^2) F_1(x_{x2}; \mu, \sigma^2)}{F_1(x_{x1}; \mu, \sigma^2) F_2(x_{x2}; \mu, \sigma^2) - F_1(x_{x2}; \mu, \sigma^2) F_2(x_{x1}; \mu, \sigma^2)}, \quad (2.3.16)$$

where $F_1(\xi; \mu, \sigma^2)$, $F_2(\xi; \mu, \sigma^2)$, $G_1(\xi; \mu, \sigma^2)$ and $G_2(\xi; \mu, \sigma^2)$ are respectively defined as

$$F_1(\xi; \mu, \sigma^2) \triangleq \sum_{j=0}^3 \lambda_{3j} \exp(-2j\mu - j^2\sigma^2) \cdot \int_{-\infty}^{\frac{\ln \xi - \mu - j\sigma^2}{\sigma}} N(\xi; 0, 1) d\xi,$$

$$\begin{aligned}
F_2(\xi; \mu, \sigma^2) &\triangleq \sum_{j=0}^4 \lambda_{4j} \exp(-2j\mu - j^2\sigma^2) \\
&\quad \cdot \int_{-\infty}^{\frac{\ln \xi - \mu - j\sigma^2}{\sigma}} N(\xi; 0, 1) d\xi, \\
G_1(\xi; \mu, \sigma^2) &\triangleq 1 - \frac{X_1}{100} - \int_{-\infty}^{\frac{\ln \xi - \mu}{\sigma}} N(\xi; 0, 1) d\xi, \\
G_2(\xi; \mu, \sigma^2) &\triangleq 1 - \frac{X_2}{100} - \int_{-\infty}^{\frac{\ln \xi - \mu}{\sigma}} N(\xi; 0, 1) d\xi.
\end{aligned}
\tag{2.3.17}$$

3.3 Experimental consideration

In this section, the proposed method is applied to actual road traffic noise data to confirm its effectiveness. The arrangements for measuring road traffic noise at an observation point is shown in Fig.2.3.1. The road traffic noise data was recorded in advance using a data recorder in a measurement time interval of 2 hours. The data naturally exhibited non-stationary properties and has therefore been divided into two data groups with different fluctuation patterns of 1 hour, with sampling periods of 1 second. Let the first measurement interval be "Case A" and the second "Case B".

Figures 2.3.2 and 2.3.3 show the estimated results for the

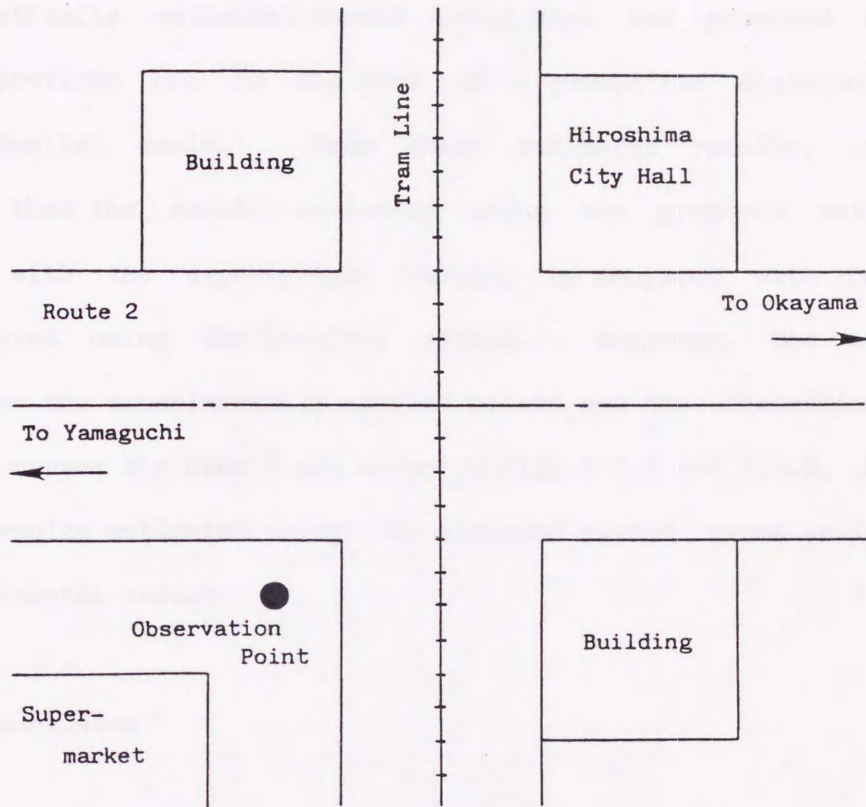


Fig.2.3.1 Arrangements for measuring road traffic noise at an observation point.

original noise level distribution for Case A. They show the comparisons between the experimentally sampled points and the theoretically estimated curves using both the proposed method and the previous one in the form of a cumulative distribution form on a decibel scale. From these estimated results, it can be seen that the results estimated using the proposed method agree well with the experimental values, as compared with the results estimated using the previous method. Moreover, the comparisons between the experimentally sampled points and the theoretically estimated curves for Case B are shown in Figs.2.3.4 and 2.3.5. Similarly, the results estimated using the proposed method agree well with the experimental values.

3.4 Conclusion

In this chapter, a new trial for estimation of noise level distribution has been proposed using the measured values of L_{eq} and several specific L_x , especially in close connection with the practical situation of actual acoustical measurement on a decibel scale. More specifically, a statistical orthonormal series expansion taking a lognormal distribution as the first expansion term has been first introduced for the purpose of employing the noise probability expression on an intensity scale especially in relation to the actual acoustic measurement on a decibel scale. The effect of the statistical properties of L_{eq} and several specific L_x on the above noise

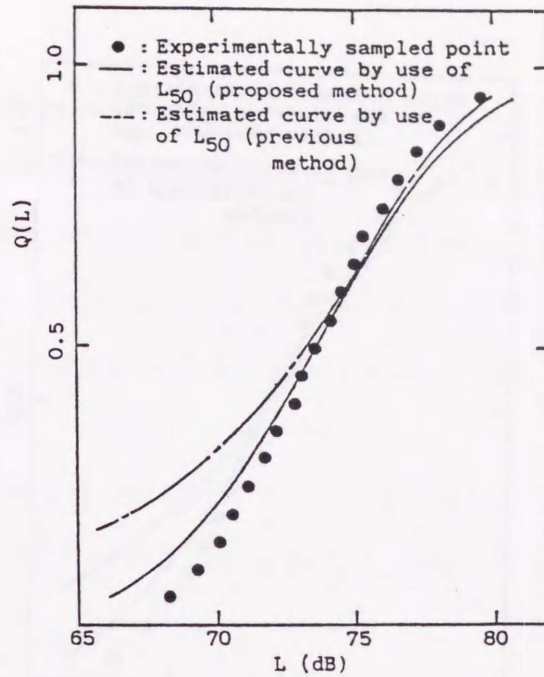


Fig.2.3.2 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case A).

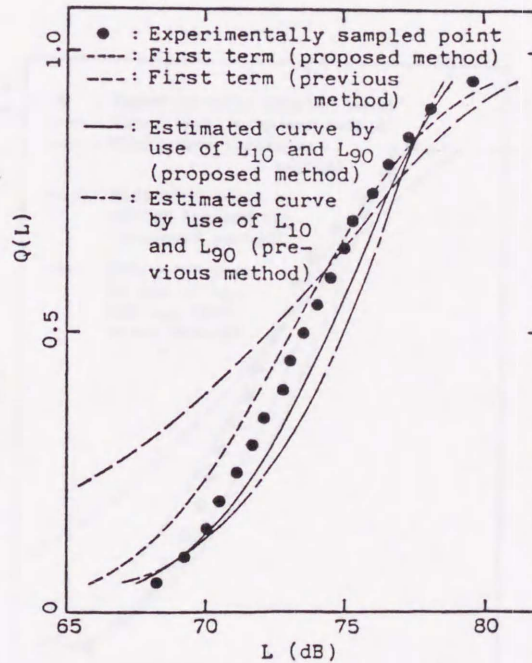


Fig.2.3.3 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case A).

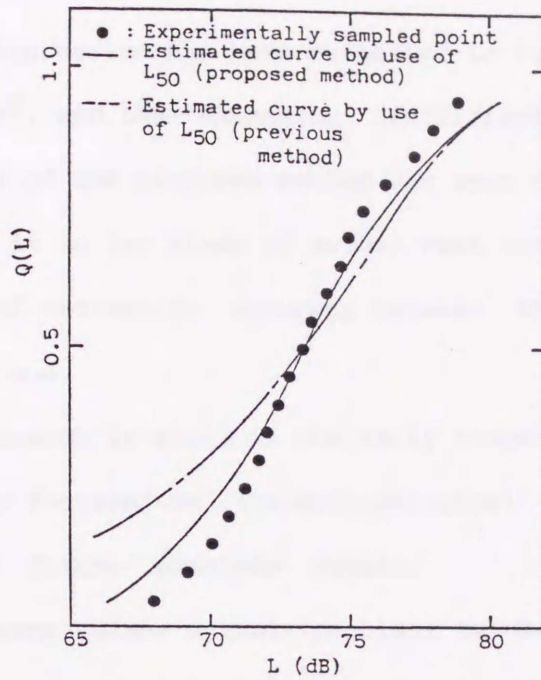


Fig.2.3.4 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case B).

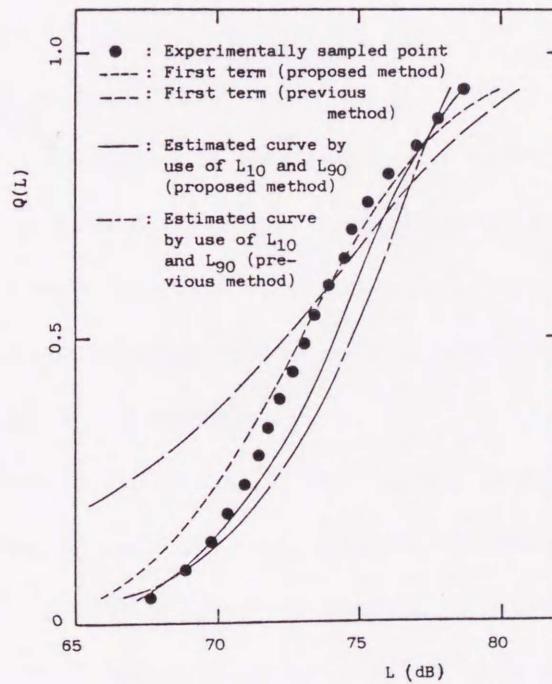


Fig.2.3.5 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case B).

probability expression has been reflected in two distribution parameters, μ and σ^2 , and the expansion coefficient C_n . Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to two kinds of actual road traffic noise data through comparisons of estimation accuracy between the proposed method and the previous one.

This research is still in the early stages so the work reported here has only focussed on its methodological aspects. Accordingly, several future problems remain. This method must be applied to many other actual problems to broaden and confirm its practical effectiveness. Furthermore, the variance on an intensity scale used here should be estimated from the other evaluation indices such as L_x and L_{eq} , as it is not usually employed in actual noise evaluation and regulation problems.

Chapter 4 Calculation of L_x and L_{eq} Noise Evaluation Indices by Use of Moment Statistics and Their Microcomputer-Aided On-Line Measurement

4.1 Introduction

As stated in the previous chapters, two noise evaluation indices, L_x and L_{eq} , play an important role in the field of noise evaluation and regulation problems. In order to evaluate these two indices, the usual measurement methods are given according to the original definition of each. That is, an L_x noise evaluation index is defined as the noise level exceeded by x percent throughout a total measurement time interval. In contrast, an L_{eq} noise evaluation index is defined as a constant noise level with a noise intensity value equal to the averaged intensity of the noise level fluctuation over a measurement time interval. The noise level fluctuation is generally measured these days in a quantized amplitude form at every discrete time period with a digital-type instrument. In cases in which this data is used to evaluate L_x and L_{eq} evaluation indices, the following fundamental problems still remain as seen from the viewpoint of signal processing :

- (1) In order to evaluate L_{eq} , it is necessary that many level data with a fairly fine sampling period be obtained, since the noise intensity fluctuation after anti-logarithmic transformation of the sampled level datum fluctuates with large excursions, as compared

with the original decibel-scaled level fluctuation.

- (2) In principle, the mean value of the noise intensity fluctuation should be given by the sample mean operation based on the noise intensity data, especially with an equally quantized intensity amplitude. If noise intensity fluctuation after transformation of the measured noise level fluctuation with an equally quantized level amplitude into the intensity scale through the anti-logarithmic transformation is used, a calculation error of the intensity mean will occur because of the above level quantization.
- (3) In order to evaluate L_x , a sampling period of an appropriate small value must be selected. The errors in L_x due to this sampling period have already been investigated by many researchers (e.g., see Refs.(37) and (38)). Furthermore, the value of L_x when x is small (e.g., L_1 and L_5) is statistically unstable in comparison with that of the median, since the amount of data exceeding x percent is small.^{(39),(40)}

In relation to the above problems, according to current measurement techniques, the sampling period may greatly influence the accuracy of the measurement result. In the case of the accuracy of L_{eq} especially, a fine sampling period related to the time constant of the integration giving the noise level will generally give a good approximation of the results obtained with true integration. If a measurement system is constructed therefore with the use of a microcomputer according to current practice, a huge memory capacity is necessary for long-term measurement with such a fine sampling period.

As mentioned in Chapter 2, explicit utilization of the statistical information on the decibel-scaled level fluctuation itself with the aid of the theory for estimating L_x and L_{eq} noise evaluation indices is very convenient (e.g., see Ref.(26)). The above statistical information is fairly stable and reliable, being based on the averaging operation supported by a large amount of data. In order to extract the statistical information, the sampling period of the noise level fluctuation can be greater than in current practice, referred to above. At the same time, each order moment statistics can be obtained successively by introducing an iterative calculation process. The problem of a huge memory capacity can therefore be solved by using this procedure.

In this chapter, a unified method for measuring two noise evaluation indices, L_x and L_{eq} , based on the theoretical study of Chapter 2 is proposed, using statistical information on the decibel-scaled level fluctuation under consideration. Based on this estimation method, a specific on-line measurement system for sequential measurement is constructed by use of a digital sound level meter and a microcomputer. Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise.

4.2 Theoretical consideration

4.2.1 General algorithm for estimating L_{eq}

Let us consider the noise level fluctuation L of an arbitrary non-Gaussian distribution type. According to the previous study in Chapter 2, a generalized expansion type expression for estimating L_{eq} is given by using the cumulant statistics of L , as follows :

$$L_{eq} (\triangleq 10 \log_{10} \langle x \rangle / x_0) = \mu_L + 0.115 \sigma_L^2 + 8.84 \times 10^{-3} \kappa_{L3} + 5.09 \times 10^{-4} \kappa_{L4} + \dots \quad (2.4.1)$$

From Eq.(2.4.1), it is possible to generally estimate an L_{eq} noise evaluation index by reflecting not only lower order cumulants but also higher order cumulants in a hierarchical form.

To establish an evaluation algorithm with the aid of a micro-computer, the cumulant κ_{Ln} ($n=1,2,\dots$) must be obtained by spending a small amount of its memory. To achieve this, the m th order moment of L can first be calculated by means of the following iterative process :

$$\langle L^m \rangle_N = \frac{N-1}{N} \langle L^m \rangle_{N-1} + \frac{1}{N} L_N^m. \quad (2.4.2)$$

From Eq.(2.4.2), the m th order moment $\langle L^m \rangle_N$ at the N th measurement time based on the memorized past value of the m th order moment, $\langle L^m \rangle_{N-1}$, at the $(N-1)$ th measurement time and the present datum, L_N , at the N th measurement time can be obtained in an iterative form.

After the m th order moment within the specific measurement time interval using the above procedure has been obtained, the resultant moment, $\langle L^n \rangle$ ($n=1,2,\dots,m$), can be transformed into the cumulant κ_{Ln}

(n=1,2,...,m), as follows⁽⁴¹⁾ :

$$\begin{aligned}
 \kappa_{L1} &= \langle L \rangle, & \kappa_{L2} &= \langle L^2 \rangle - \langle L \rangle \kappa_{L1}, \\
 \kappa_{L3} &= \langle L^3 \rangle - 2 \langle L \rangle \kappa_{L2} - \langle L^2 \rangle \kappa_{L1}, \\
 \kappa_{L4} &= \langle L^4 \rangle - 3 \langle L \rangle \kappa_{L3} - 3 \langle L^2 \rangle \kappa_{L2} - \langle L^3 \rangle \kappa_{L1}, \\
 \kappa_{L5} &= \langle L^5 \rangle - 4 \langle L \rangle \kappa_{L4} - 6 \langle L^2 \rangle \kappa_{L3} - 4 \langle L^3 \rangle \kappa_{L2} \\
 &\quad - \langle L^4 \rangle \kappa_{L1}, \dots
 \end{aligned} \tag{2.4.3}$$

The objective L_{eq} can thus be estimated by substituting the calculated value of cumulant statistics into Eq.(2.4.1).

4.2.2 General algorithm for estimating L_x

In order to estimate the arbitrary L_x noise evaluation indices (e.g., $L_5, L_{10}, L_{50}, L_{90}, \dots$), let us introduce a statistical Hermite series expansion as the cumulative distribution function universally applicable to the noise level fluctuation L of an arbitrary non-Gaussian distribution type, as follows⁽¹³⁾ :

$$\begin{aligned}
 Q(L) &= \int_{-\infty}^L N(\xi; \mu_L, \sigma_L^2) d\xi - \frac{\kappa_{L3}}{6\sigma_L^2} N(L; \mu_L, \sigma_L^2) \\
 &\quad \cdot H_2\left(\frac{L - \mu_L}{\sigma_L}\right) - \frac{\kappa_{L4}}{24\sigma_L^3} N(L; \mu_L, \sigma_L^2) H_3\left(\frac{L - \mu_L}{\sigma_L}\right) \\
 &\quad - \frac{\kappa_{L5}}{120\sigma_L^4} N(L; \mu_L, \sigma_L^2) H_4\left(\frac{L - \mu_L}{\sigma_L}\right) - \dots
 \end{aligned} \tag{2.4.4}$$

The value of the n th order Hermite polynomial, $H_n(\cdot)$, can be

calculated by the following recursive formula suitable for computer processing⁽⁴²⁾:

$$H_n(\xi) = \xi H_{n-1}(\xi) - (n-1) H_{n-2}(\xi). \quad (2.4.5)$$

In addition, the calculation of the first expansion term of Eq.(2.4.4) can easily be performed by using the well-known error function, as follows :

$$\int_{-\infty}^L N(\xi; \mu_L, \sigma_L^2) d\xi = 1 - \frac{1}{\sqrt{\pi}} \text{Erfc}\left(\frac{L - \mu_L}{\sqrt{2}\sigma_L}\right), \quad (2.4.6)$$

where Erfc(.) denotes the error function defined as

$$\text{Erfc}(\xi) \triangleq \int_{\xi}^{\infty} \exp(-t^2) dt. \quad (2.4.7)$$

At this time, the value of Erfc(.) can be accurately calculated by the following approximation⁽⁴³⁾:

$$\begin{aligned} \frac{2}{\sqrt{\pi}} \text{Erfc}(\xi) \approx & 1 / [1 + 0.0705230784\xi + 0.0422820123\xi^2 \\ & + 0.0092705272\xi^3 + 0.0001520143\xi^4 \\ & + 0.0002765672\xi^5 + 0.0000430638\xi^6]^{16}. \end{aligned} \quad (2.4.8)$$

Since the relationship between Q(L) and L_x is given by

$$1 - \frac{x}{100} = Q(L_x) \quad (2.4.9)$$

and $Q(L)$ is a monotone-increasing function, the objective L_x can be directly evaluated from the estimated curve of $Q(L)$ using the linear interpolation, even for measurement with a fairly large discrete level amplitude. This procedure is quite convenient for economizing calculation time, compared to the usual numerical integration. Consequently, after statistical information on the noise level fluctuation has been obtained in the manner described in the previous section, an arbitrary L_x can be calculated by this method. Furthermore, based on this estimation method, all the data of the noise level fluctuation need not be memorized and problem can be solved with a small amount of memory capacity when using a microcomputer for the measurement system.

4.3 Experimental consideration

4.3.1 Confirmation of effectiveness of the proposed estimation method

For the purpose of confirming the effectiveness of the proposed estimation method, a measurement system has been constructed using a digital sound level meter and a portable microcomputer. The block diagram of this measurement system is shown in Fig.2.4.1.

Two kinds of road traffic noise have been measured as typical examples of environmental noise. One was measured near a national main road in a large city with a traffic volume of 244 vehicles per 10 minutes, and the other near a country road in a rural district with a

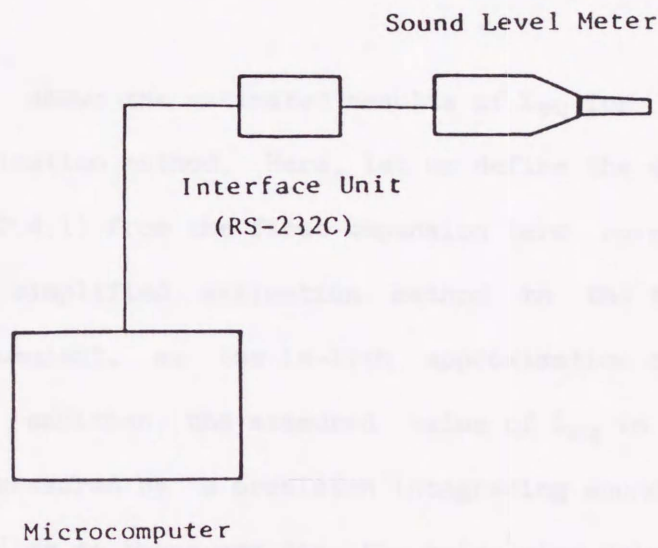


Fig.2.4.1 Block diagram of measurement system with the aid of a portable microcomputer.

traffic volume of 33 vehicles per 10 minutes. The measurement time interval and its sampling period were selected to be 10 minutes and 1 second, respectively. Here, let us define the former case as " Case A " and the latter as " Case B ". Figure 2.4.2 shows the arrangements for measuring road traffic noise at an observation point for Case A, and Fig.2.4.3 the arrangements for Case B.

Table 2.4.1 shows the estimated results of L_{eq} for Case A using the proposed estimation method. Here, let us define the expansion expression of Eq.(2.4.1) from the first expansion term corresponding to the well-known simplified estimation method to the term due to the n th order cumulant, as the $(n-1)$ th approximation of the estimated L_{eq} . In addition, the measured value of L_{eq} in this table shows the value measured by a precision integrating sound level meter for L_{eq} . According to these results, the calculated value, using a well-known simplified estimation method derived under assumption of a well-known Gaussian distribution, agrees approximately with the measured value. More specifically, however, the estimated values using the proposed method tend to agree well with the successive addition of higher order expansion terms. Table 2.4.2 shows the estimated results for Case B. According to these results, the estimation accuracy using the well-known simplified estimation method is not sufficient for evaluation. It is clear, though, that the successive addition of higher order expansion terms moves the values estimated theoretically using the proposed method closer

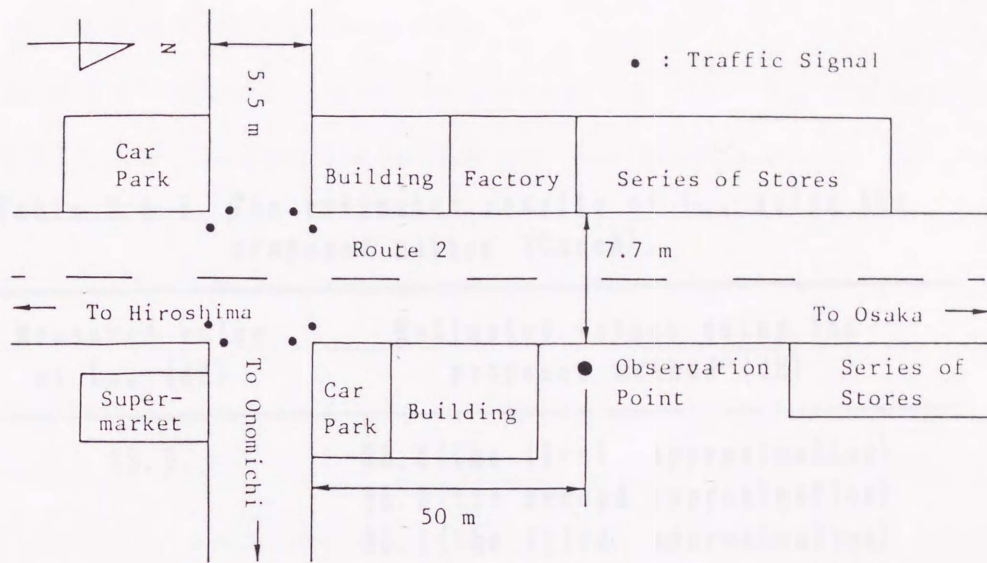


Fig.2.4.2 Arrangements for measuring road traffic noise at an observation point (Case A).

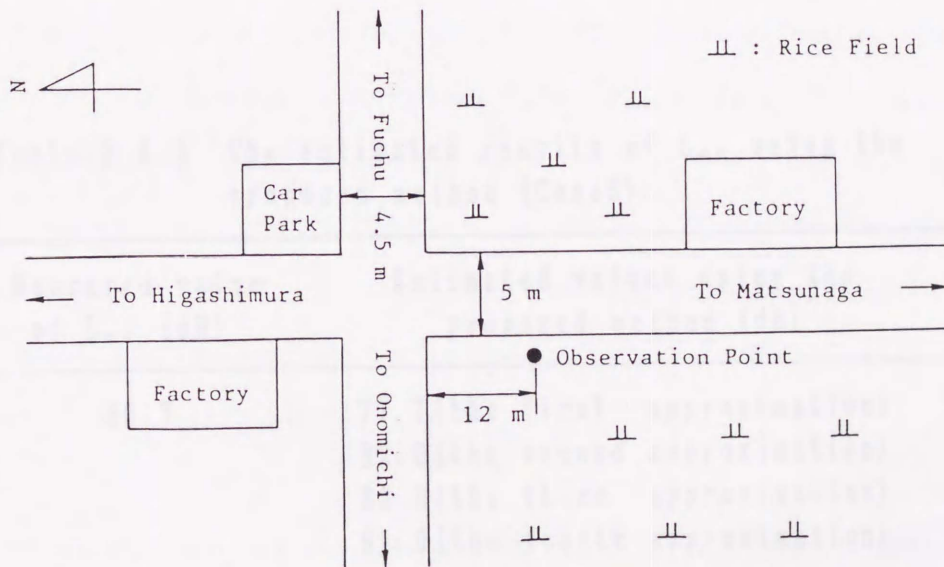


Fig.2.4.3 Arrangements for measuring road traffic noise at an observation point (Case B).

Table 2.4.1 The estimated results of L_{eq} using the proposed method (CaseA).

Measured value of L_{eq} (dB)	Estimated values using the proposed method (dB)
85.9	86.4 (the first approximation)
	86.0 (the second approximation)
	86.1 (the third approximation)
	86.1 (the fourth approximation)

Table 2.4.2 The estimated results of L_{eq} using the proposed method (CaseB).

Measured value of L_{eq} (dB)	Estimated values using the proposed method (dB)
80.7	77.7 (the first approximation)
	82.0 (the second approximation)
	82.8 (the third approximation)
	81.0 (the fourth approximation)

to the values measured experimentally.

Figure 2.4.4 shows a comparison between the theoretically estimated curves using the proposed method and the experimentally sampled points in the cumulative distribution form of the noise level fluctuation for Case A (arbitrary L_x noise levels are directly extracted by this distribution form). From this figure, the theoretically estimated curve due to the first approximation (i.e., Gaussian distribution) agrees approximately with the experimentally sampled points. More precisely, however, the estimated values using the proposed method tend to agree well with the measured values, with the successive addition of higher order expansion terms. Figure 2.4.5 shows the estimated results for Case B. In this figure, it can be seen that curve theoretically estimated by using only the first expansion term corresponding to the Gaussian distribution does not agree well with the experimentally sampled points. It is clear that the successive addition of higher order expansion terms moves the values estimated theoretically using the proposed method closer to the values measured experimentally. In each case, the estimation accuracy for both L_x and L_{eq} noise evaluation indices using the proposed estimation method is clearly sufficient.

4.3.2 Construction of an on-line measurement system for sequential measurement

In this section, an on-line measurement system for the sequential

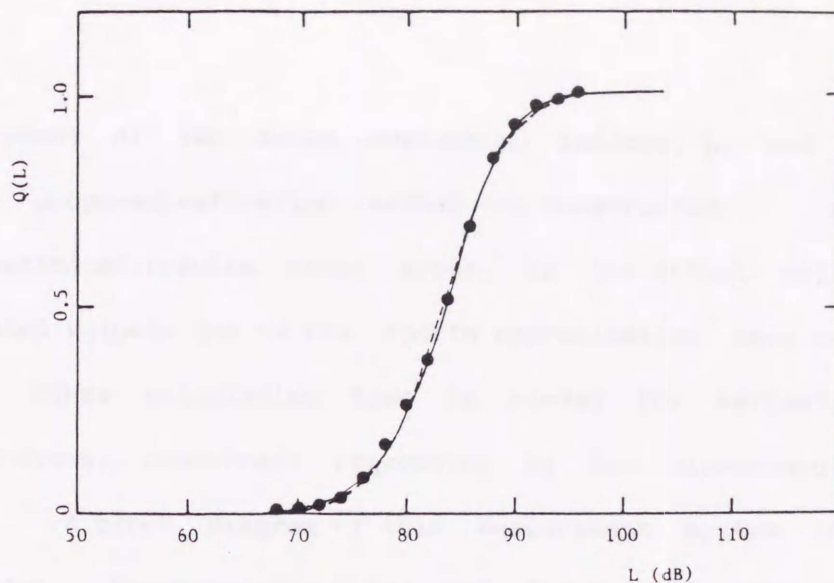


Fig.2.4.4 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case A). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.4.4) ; —, the second approximation (and the third and fourth approximations).

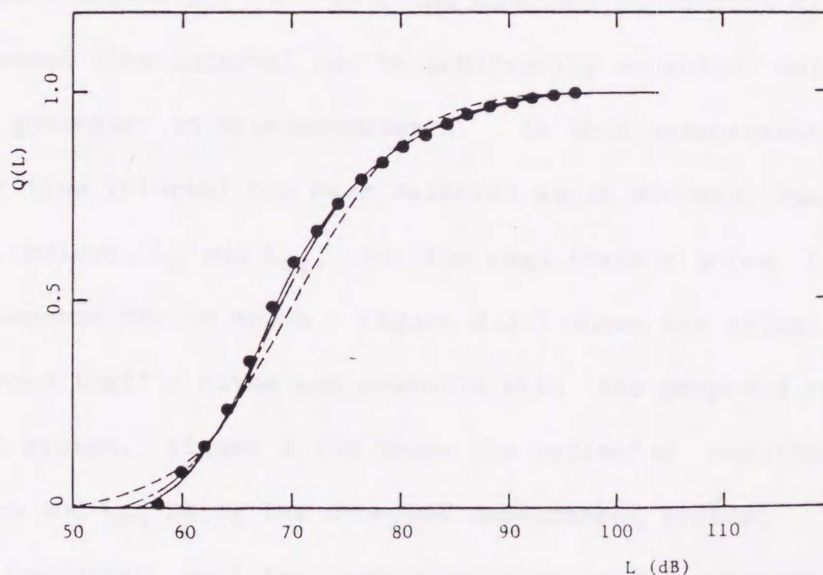
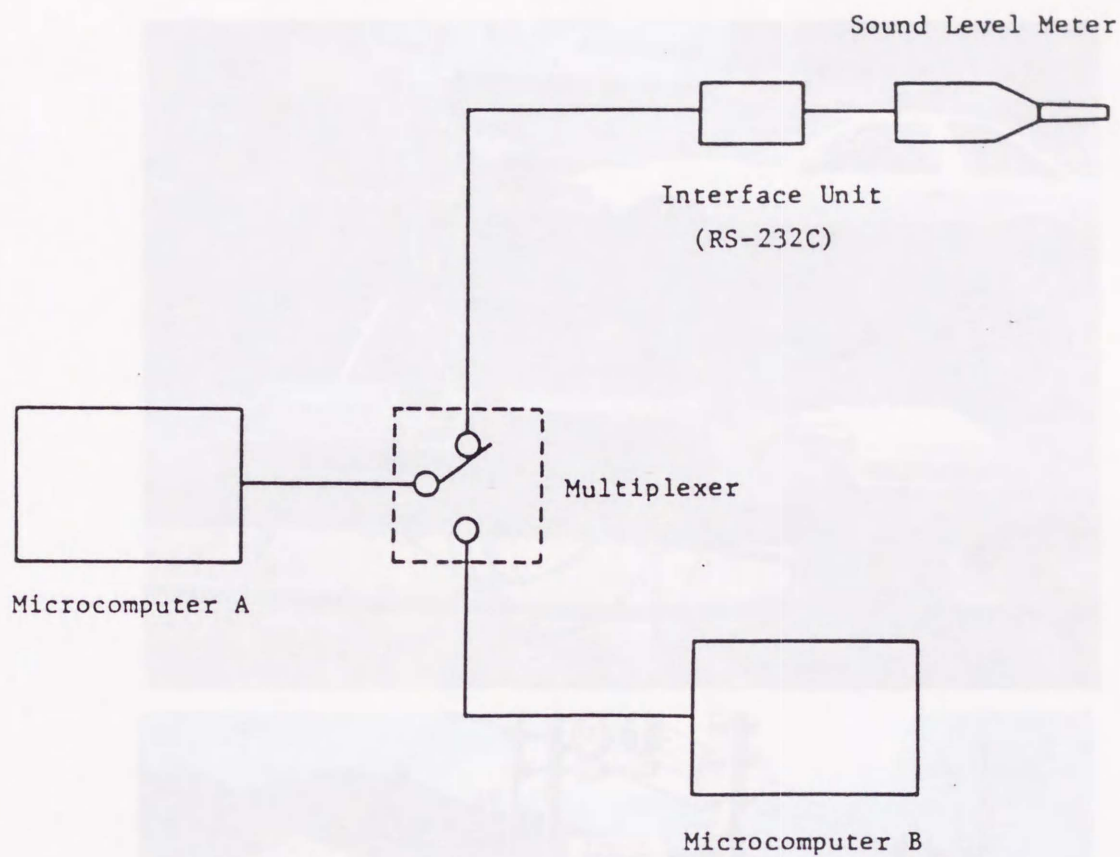


Fig.2.4.5 A comparison between experimentally sampled points and theoretically estimated curves using the proposed method for the cumulative distribution form of the noise level fluctuation (Case B). Experimentally sampled points are marked by ● and theoretically estimated curves are respectively shown as ----, the first approximation of Eq.(2.4.4) ; —, the second approximation (and the third approximation) ; —, the fourth approximation.

measurement of two noise evaluation indices, L_x and L_{eq} , based on the proposed estimation method is constructed. According to the estimated results shown above, in the actual estimation, the estimated values due to the fourth approximation have been employed. Since calculation time is needed for estimation of the two indices, concurrent processing by two microcomputers can be used. A block diagram of this measurement system is shown in Fig.2.4.6. Microcomputer A is used for the iterative process for extracting the moment statistics within each measurement time interval using Eq.(2.4.2) and the control system for the communication lines between the digital sound level meter and Microcomputer B with an RS-232C type interface. Microcomputer B is used for calculation of L_x and L_{eq} using the proposed estimation method after transforming the moment statistics $\langle L^n \rangle$ into the cumulant statistics κ_{Ln} . Each measurement time interval can be arbitrarily selected using the clock signal generator in Microcomputer A. In this measurement, each measurement time interval has been selected at 10 minutes. Two noise evaluation indices, L_x and L_{eq} , for the road traffic noise (Case A) have been measured for 24 hours. Figure 2.4.7 shows the actual location at which road traffic noise was measured with the proposed on-line measurement system. Figure 2.4.8 shows the estimated results of L_5 , L_{10} , L_{50} , L_{90} and L_{eq} using the proposed measurement system. The proposed system performed well for long-term measurement in each measurement time interval.



Microcomputer A : Iterative calculation of moment statistics and control system for communication lines

Microcomputer B : Calculation of L_x and L_{eq} using the proposed method

Fig.2.4.6 Block diagram of on-line measurement system for L_x and L_{eq} using the proposed method.



Fig.2.4.7 Actual scene of road traffic noise measurements using the proposed on-line measurement system.

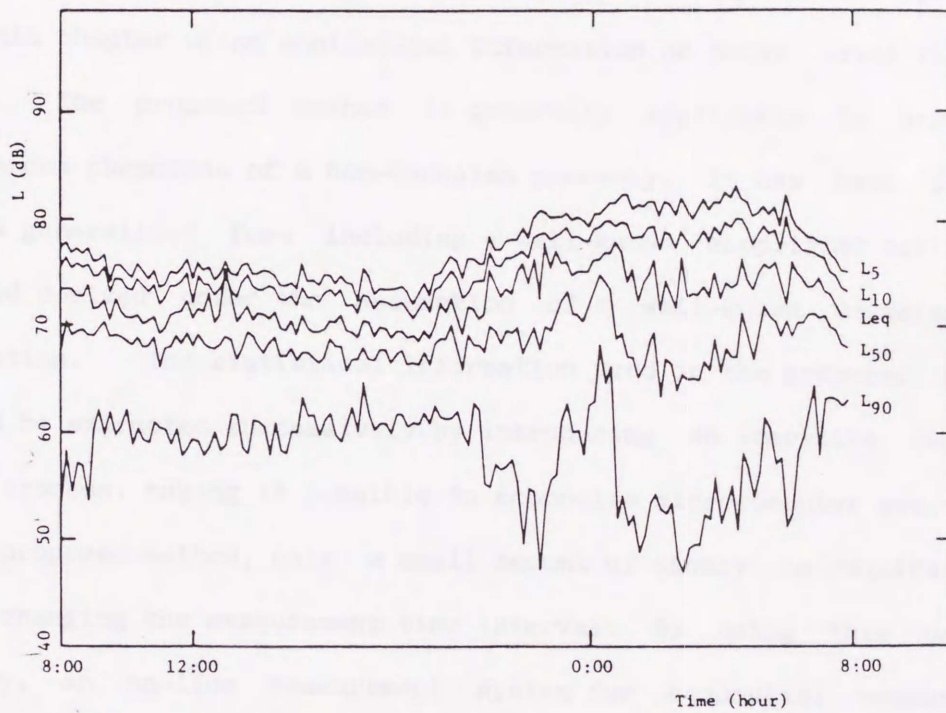


Fig.2.4.8 The estimated results of L_x and L_{eq} using the proposed on-line measurement system.

4.4 Conclusion

Based on the previous study in Chapter 2, a unified method for measuring two noise evaluation indices, L_x and L_{eq} has been proposed in this chapter using statistical information on noise level fluctuation. The proposed method is generally applicable to any kind of random phenomena of a non-Gaussian property. It has been derived in a generalized form including a well-known simplified estimation method derived under the assumption of a well-known Gaussian distribution. The statistical information used in the proposed method could be extracted successively by introducing an iterative calculation process, making it possible to economize microcomputer memory. In the proposed method, only a small amount of memory is required even when changing the measurement time interval. By using this unified theory, an on-line measurement system for sequential measurement through a long-term measurement time interval has been constructed using a digital sound level meter and a portable microcomputer.

Of course, this study is still in its early stage and has focussed only on its fundamental aspects. Problems remain in its application to other situations.

CONCLUSION

CONCLUSION

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CONCLUSION

The main results of this doctoral thesis are here summarized.

As titled in this study, a statistical prediction method for the noise level distribution and the mutual relationship between various kinds of noise evaluation indices in close relation to L_x and L_{eq} have been discussed.

In Part I, a general method for prediction of the probability distribution form of environmental random noises has been considered.

In Chapter 1, a statistical method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow has been proposed. Previously, various approaches for predicting road traffic noise have been proposed from various points of view. These prediction methods can essentially be divided into the following two groups. One group comprises structural methods based on physical models. The other comprises functional methods based on mathematical models. Accordingly, in the previous study, some kind of hybrid method combining the advantages latent in both prediction groups was proposed. In order to establish this kind of prediction method, a new approach equivalent to that used for an idealized Poisson type traffic flow has been proposed in this chapter for practical use. The proposed prediction method can treat any kind of road traffic noise fluctuation generated from actual non-Poisson type traffic flow, once information has been obtained on the elementary response wave form of the level time pattern observed when one vehicle passes directly in

front of an observation point. The effectiveness of the proposed prediction method has been experimentally confirmed by applying it to actually observed road traffic noise data.

In Chapter 2, a practical method for prediction of road traffic noise at a T-type road intersection has been proposed based on the image method. In Chapter 1, a general method for prediction of road traffic noise generated from an arbitrary non-Poisson type traffic flow was proposed based on the information on the elementary response wave form observed experimentally. In order to establish a more effective prediction method, a theoretical method for estimating the above elementary response wave form must be found, especially in a complicated realistic acoustical environment. In this chapter, a new approach for estimating theoretically this elementary response wave form has been proposed for the case of a T-type road intersection with sound insulation barriers, using a well-known image method. The effectiveness of the proposed method has been experimentally confirmed by applying it to the road traffic noise data observed at a T-type road intersection in a city area.

In Chapter 3, a study on the stochastic response of a sound insulation barrier has been discussed. In the practical engineering field of noise control, a sound insulation barrier is very often constructed to produce attenuation of environmental random noises such as the road traffic noise discussed in the previous chapters. The acoustical design and/or the evaluation problem of barriers have already been considered by many researchers. Almost all these studies, however,

were confined only to the effects on deterministic signals or the gross average evaluation of shielding effects. In this chapter, a prediction method of the stochastic insulation effect of a sound insulation barrier has been proposed for the case of an arbitrary probability distribution for the random noise incident on the barrier. The emphasis in this chapter has been focussed on how the output noise distribution form using information on the statistical properties of the random input noise and the frequency characteristics of the sound insulation barrier can be predicted. The effectiveness of the proposed prediction method has been experimentally confirmed using actual noise data.

In Part II, in order to evaluate precisely complicated real environmental random noise, the mutual relationship between various kinds of statistical evaluation indices in relation to L_x and L_{eq} has been considered. In the field of noise evaluation and regulation problems, two noise evaluation indices, L_x and L_{eq} , play an important role. Accordingly, the establishment of a general theory for the above mutual relationship is of basic importance to obtain systematical evaluation of actual environmental random noises. Previously, various studies for finding mutual relationships of this kind have already been considered by many researchers. Almost all of these studies, however, were confined to approximate methods derived under the assumption of an ordinary Gaussian distribution and/or practical methods derived by applying the conventional linear regression analysis method to actually observed data. From the above

practical points of view, a general theory for mutual relationships between various noise evaluation indices in close relation to L_x and L_{eq} has been presented in Part II.

In Chapter 1, a method for estimation of an L_{eq} noise evaluation index has been proposed by use of moment statistics of the noise level fluctuation of arbitrary non-Gaussian distribution type. To estimate the value of L_{eq} for an arbitrary non-Gaussian type random noise fluctuation, a new approach equivalent to that used for the Gaussian type has been first proposed for practical application. The proposed estimation method was given in a generalized form universally applicable to any kind of random phenomena of a non-Gaussian property, including a well-known simplified expression derived under assumption of an ordinary Gaussian distribution. Finally, it has been applied to actual noise data to confirm its effectiveness.

In Chapter 2, the mutual relationship between several statistical indices connected with L_x and L_{eq} has been described. An estimation method of L_{eq} is presented using the known values of several specific L_x with the use of the mean and variance of the random noise fluctuation. The proposed estimation formula was also given in a generalized form including the well-known simplified expression mentioned above. Next, a new trial estimating the original distribution form of the random noise fluctuation has been presented using the known values of L_{eq} and several specific L_x . The effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise data.

In Chapter 3, a method for estimation of the original distribution form of the random noise fluctuation has been also proposed. In this case, a more suitable expression of noise probability distribution on an intensity scale has been newly introduced in a form matched to acoustic measurement on a decibel scale. The effectiveness of the proposed method has been experimentally confirmed by applying it to actual road traffic noise data, and through comparisons of estimation accuracy between the proposed method and the previous one derived in Chapter 2.

In Chapter 4, based on the previous discussion in Chapter 2, a precise calculation method of L_x and L_{eq} noise evaluation indices has been proposed by use of information on the noise level fluctuation (i.e., moment statistics or cumulant statistics). Based on this estimation method, a specific on-line measurement system has been constructed with the aid of a microcomputer by introducing an iterative process for extracting the moment statistics of random noise fluctuation on a decibel scale. The effectiveness of the proposed method has been experimentally confirmed by applying it to actual measurement of road traffic noise.

This study is still in its early stages and the work presented here has focussed primarily on fundamental aspects. Many problems remain for future research. The following studies, for example, are left for the future :

- (1) A reasonable evaluation method for complex situations in which environmental noise consists of various sound sources such as the

road traffic noise, the aircraft noise, the construction noise, etc. is required.

- (2) A systematical evaluation method for various types of noise reduction systems in a room, such as sound absorbing materials, sound insulation walls, etc. must be considered.
- (3) It is also necessary that a reasonable method for evaluating more complicated real environmental noise attenuated by various reduction systems acoustically coupled each other be established.
- (4) More general probability expressions must be introduced by considering more severe actual phenomena in a form matched to acoustic measurements : i.e., non-stationary random processes, restricted fluctuation range, etc.
- (5) Based on the discussion described in this study, practical methods must be established for concrete countermeasures to noise pollution using actual noise reduction systems (sound absorbing materials, sound insulation walls, signal control systems for traffic flow, etc.).

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A New Method of Estimating Parameters of a Nonlinear Stochastic System by the Least Squares

Abstract: A new method for estimating parameters of a nonlinear stochastic system by the least squares is proposed. The method is based on the least squares method and the Kalman filter algorithm. The method is applied to the estimation of parameters of a nonlinear stochastic system. The results show that the method is effective and accurate.

1. Introduction

In the past few years, the estimation of parameters of a nonlinear stochastic system has attracted much attention. In the early 1970s, the least squares method was used for the estimation of parameters of a nonlinear stochastic system. However, this method is only applicable to the estimation of parameters of a nonlinear stochastic system when the system is linear. In the early 1980s, the Kalman filter algorithm was used for the estimation of parameters of a nonlinear stochastic system. However, this method is only applicable to the estimation of parameters of a nonlinear stochastic system when the system is linear. In the early 1990s, the least squares method was used for the estimation of parameters of a nonlinear stochastic system. However, this method is only applicable to the estimation of parameters of a nonlinear stochastic system when the system is linear.

In this paper, a new method for estimating parameters of a nonlinear stochastic system by the least squares is proposed. The method is based on the least squares method and the Kalman filter algorithm. The method is applied to the estimation of parameters of a nonlinear stochastic system. The results show that the method is effective and accurate.

2. Theoretical Derivation

Consider a nonlinear stochastic system described by the following equation:

$$\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) + w(t)$$

where $x(t)$ is the state vector, $u(t)$ is the control input, $w(t)$ is the process noise, and $A(x(t))$ and $B(x(t))$ are nonlinear functions of the state vector $x(t)$.

$$y(t) = C(x(t))x(t) + v(t)$$

where $y(t)$ is the output, $v(t)$ is the measurement noise, and $C(x(t))$ is a nonlinear function of the state vector $x(t)$.

The least squares method is used for the estimation of parameters of the nonlinear stochastic system. The results show that the method is effective and accurate.

The Kalman filter algorithm is used for the estimation of parameters of the nonlinear stochastic system. The results show that the method is effective and accurate.

3. Simulation Results

The simulation results show that the proposed method is effective and accurate. The results are compared with the least squares method and the Kalman filter algorithm. The results show that the proposed method is more accurate than the other two methods.

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4. Conclusion

The proposed method for estimating parameters of a nonlinear stochastic system by the least squares is effective and accurate. The method is based on the least squares method and the Kalman filter algorithm.

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