The Rational Origins of the State:
The "Necessary Accident" Theory of Early-States' Building Subsumed under the Kantian Categorical Frameworks

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Abstract

The early state came into existence as the necessary result of the innovation of metal tools under the condition that the selfish motives of preceding communities' chieftains for making the transaction of external trade more profitable by resorting to a military force, the net-benefit of whose use could be sure to be increased by innovating bronze weapons, later taken over by iron ones. Though the main propositions are based on the recent empirical studies of ancient history and evolution anthropology, they are subsumed under the categorical frameworks of the transcendental philosophy. To prove them, some expected hypotheses are inferred and deducted by the analysis of a two-stage game comprised of both the "network game with hierarchies" abstracting an irrigation economic community and the "two-stage bargaining game" modeling the process of bargaining in a foreign trade, both of which are modeled in accordance with the Kantian categorical frameworks

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"And yet, again, it will next to impossible to plant our city (polis) in a territory where it will need no imports." (The Republic of Plato, 1941, pp.56)

1．Introduction

Even band-communities had a military force aiming at aggressive war (Chagnon 1974). Likewise, the state is also a societal organization generating a power to enforce, called the "sovereignty," which plays crucial role in determining the actual level of the security of life and property. Furthermore, it is the well-known evidence that even the primitive band-communities, as well as clan communities just preceding early states, were networked vie inter-community trades (Ridley 1997). These empirical facts contradict the conventional concepts of the state based on "legitimatized monopoly in violence" and/or on the "opening-up of an external trade to kin-based communities".[1] Now is the time for asking fundamental questions on the state as follows: What is the difference between the state and the preceding communities? Under what conditions and by what motives the state is brought into existence? What on earth is the state? Why can a political system be stable only during a certain period of time? These questions are what I endeavor to answer in this paper. For this purpose, I categorize the main propositions under the cognitive frameworks of the transcendent philosophy, formulate the intuitive representations of early states in accordance with the categorized propositions and then construct a game theoretic model abstracting the process of building an early state in irrigation society, with a view to applying them to other types of states [2]

[1] Those concepts even now prevailing in social sciences originate in Weber (1911, 1924) and Engels (1878, 1884).

and to re-examining the traditional notions on the state. Finally, by the analysis of the base model, some hypotheses inferred from the main propositions are deductively proved. They are tested by reference to historical evidences.

Before explaining an outline of this paper, the philosophical criteria to judge the truth and objectivity of scientific propositions must be mentioned in spite of a digression. This is because as well as those criteria have yet to be established in social sciences, an immature state of the methodology of cognition is one of the reasons why arguments on the state are in a state of disorder. I note here that the cognitive frameworks of the transcendental philosophy pioneered by Kant (1787) should be considered as the criteria for judging the truth and objectivity. Coincidentally or not, the recent studies of evolution biology and neuroscience [3] have been providing empirical evidence to lend support to the main propositions of the transcendental philosophy, even if the former may not be explicitly conscious of such a relation with the latter. According to the main propositions of the transcendental philosophy, the cognition of *Homo sapiens*, classified into perception, intuition, category and generalized inference, is the processes of subsuming various types of representations under *a priori* cognitive frameworks classified into the time and space framework for sensibility, the categorical framework for understanding and the inference framework for reasoning based on categorical propositions. These cognitive frameworks may be considered as some of the "neural modules" biologically comprised of neural networks. [4]

First of all, the "relation" categories especially plays a key role in examining the main notions on the state, since they provide the processes of cognition with *a priori* frameworks for explicating what a state is, under what conditions and by what motives it is brought into existence, and finally how it is related to other states existing in the same period of time.

According to the first of the relation categories, called the "substance" (die Substanz) and the "accident" (die Akzidenz), the society is a substance in the sense that *Homo sapiens* have been organizing a type of society in order to adapt to the surroundings with which they were confronted, and that the purposes for organizing various types of the society in common are to provide for both necessities and peace as securely as possible, which are indispensable for the survival of both an individual organ and its descendents but cannot be securely provided by oneself. Therefore, it is necessary that the innate programs-for-survival tend to drive a group of individual organs to coordinate them for production including external trade, and to organize part of them into a military force. On the other hand, the state is an "accident," i.e., a type of the society, in the sense that it comes into being as a result of the transformation of other types of the society preceding to it by changing and/or replacing some elements characterizing those preceding types for the sake of adapting themselves to new surroundings. Therefore, "What are the new surroundings" is a key question to categorize the state in accordance with the "substance and accident" category. One of the main propositions of this paper is that those new surroundings are the innovation of metal tools, the application to a military force of which played a crucial role in causing the early state to come into existence.

The second of the relation categories, called the "causality" (die Kausalitaet) or "the cause and the effect" (die Ursache und die Wirkung), provides the cognitive frameworks according to which a happening (ein Geschehen), which means a change in the essential elements characterizing one type of the substance, is recognized as a combination between one representation and other ones prior to it, in such a way

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in which the preceding ones are recognized to have had an influence on the succeeding one. According to this cognitive framework, a state comes into existence as a "result" of some preceding causes but is not a "creature." The proposition of this paper is that the most crucial of the preceding influential causes is the innovation of metal tools, and that the entrepreneurial members of preceding societies who had an advantage in obtaining the metal goods were driven by the innate programs-for-survival to take the lead in transforming the existing social organizations into a new type of the society called the "early state." I note here that the innate programs-for-survival, called the "motives" in the fields of economics, keep on working as one of the essential elements of the society regardless of a difference in its types, and therefore that the motives, *selfish or not*, should be regarded as one of the "necessary conditions" for the early state to come into being as a result, but not the "cause."

The third of the relation categories, i.e., the "reciprocity" (die Wechselwirkung), provides the cognitive frameworks according to which plural representations existing at the same period of time are recognized as interrelated ones or as ones influencing one another. This category should be applied to explicating the relativity of the sovereignty of a state in inter-state surrounding without any common enforcer. Needless to say, the sovereignty means how much independent of any pressure from within and without the decision-making of a state can be.

Secondly, the "modality" category (die Modalität) provides the cognitive framework to evaluate how often a synthetic but not analytic proposition is likely to be realizable, classified into the "possibility modality," the "existence modality," and the "necessity modality." Taking it into consideration that

since some earliest-states came into being several thousand years ago, the states have been sure to be here and there, only the "necessity modality" should be taken up and be applied to the causality category.

Thirdly, the "quality" category (die Qualität) provides the intuitive framework according to which the degree of sovereignty is evaluated.

Finally, the "quantity" category (die Quantität) provides the intuitive framework according to which the size of a state represented by population, territory and economic power is evaluated in a single measure.

According to the cognitive frameworks mentioned above, the main synthetic propositions of this paper are formulated as follows: Firstly, an early state [5] came on the historical stage as a necessary result of the application to a military force of the innovation of metal tools, under the condition that the selfish motives of preceding communities' chieftains for making the transaction of external trade more profitable to them by resorting to the military force the net-benefit of whose use could be sure to be increased by innovating bronze weapons, taken over by iron ones later. Secondly, the early state is a type of the society and a transformation of some preceding type of the society. Thirdly, the sovereignty, i.e., the most essential element of the society, is relatively but not absolutely determined, and is dependent on other types of the quality category such as the degree of coordination and on the quantity category such as territory and population surrogating economic and military power.

The procedure of the proof of those synthetic propositions is as follows: Firstly, they are formalized under the categorical frameworks corresponding to them. Secondly, in accordance with the representations (images) of those propositions

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[5] The concept of the "early state" is not necessarily the same as that of its godparent, Classen and Skalnik (1978, 1981) which defined it as a society considered to have existed between chiefdom and full-grown states with regional territory. In this paper, according to the definition of Weber (1924) and the representation described by both Finley (1978) and Plutarch (1914-54), the early state is defined as a social organization ruled by *basileus* in the ancient Greek or *rex* in the ancient Rome who tried to have a monopoly in foreign trade by organizing immediate subordinates into a private military team living together inside a castle type of residence, surrounded by residential areas of small merchants and handicraftsmen, with farm lands spreading further away from them. In return for accepting the rule of the early king, those engaged in direct production were provided with protection from external threats, based on reciprocal contracts. In this paper such a concept of the early state is applied to the process of building an early state in irrigation society.

subsumed under the corresponding categories, the intuitive perceptions or phenomenon of observable early states are synthesized and unified in accordance with the above categorized propositions. Finally, the synthesized unification is combined into an abstract image (a representation of phenomenon), called "formal model" in economics. That is, an abstracted basic model is set up so that its basic assumptions and conditions are in accordance with the representations or images of the synthetic propositions subsumed under the categories. In this paper the basic model is formalized as a game-theoretic model. Finally, some expected hypotheses inferred from the basic model are deductively proved by the analysis of the basic model. According to the criteria for judging the truth and objectivity of the transcendent philosophy, the coincidence of the inferred hypotheses with those analytically-deducted results prove the objective truth of the synthetic propositions, called "the possibility of experiences" (die Möglichkeit of Erfarung). The truth is corroborated by reference to historical data (the hypotheses are tested).

What we should derive as the main propositions in the above procedure are as follows: The first is on the "causality" of an early state. It argues that if, under the new surrounding which the innovation of metal tools gives the opportunity to increase the bargaining power in external trade by resorting to a military force equipped with metal weapons, a self-interested chieftain can increase sufficiently his expected payoff by transforming the existing communities into a new social organization equipped with a standing military force, then he is motivated to build an early state. The second is on the" accidental" relation of an early state, arguing that the early state is a change in the situation of the society from the preceding clan-type of the society into a new type with a military force equipped with metal weapons in chaotic foreign-trade relations. That change in the situation is an adaption to a crucial change in some elements characterizing the preceding communities, and the crucial change is the innovation of metal tools taking over stone-made tools. The third is on the "reciprocity" of the early state, arguing that the sovereign power of an early state is reciprocally

interdependent on those of others existing in the same period time.

The expected result to be inferred and deducted from the analysis is that the bargaining power, i.e., an economic surrogate for the sovereignty in politics jargon, is a decreasing function of surrogate variables representing the influence of those other types of the society existing in the same period time. One corollary of the main propositions is deducted as follows: the membership size of the early state is larger than that of the preceding type of the society. It is because an increase in the size contributes to an increase in the bargaining power. This corollary explains the phenomenon that the size of a state tends to increase when it is faced with the external circumstance under which the sovereignty has to be intensified.

By the way, "inferences by reason" should be applied to the concept of the state in general. In this paper, the generalized topics on the state are just mentioned but not sufficiently addressed.

In what follows, this paper is organized as follows: In the second section, an outline of the main logic and the historical backgrounds are explained. In the third section, the traditional theories of the state are reviewed. In the fourth section, the base model of an irrigation society is set up on the basis of the discussions of the second and third section, and the stable nature of this society is examined not only in the analytical framework of a coalition game with hierarchies but also in the non-cooperative framework of a "link and claims" game. In the fifth section, an external trade is formalized in the analytical framework of a two-stage bargaining game. In the sixth section the main results are derived by inducting backwardly a two-stage game comprised of the base model and the two-stage bargaining game. In the seventh the propositions are proved by examining the results of the previous sections. In the eighth section, the bargaining-power theory is applied to the aristocracy and the ancient monarchy. In the ninth section it is shown that the process of forming the ancient monarchy of Japan can be explained according to the main propositions. The last section concludes this paper. Applicability and some policy implications

are mentioned. The main parts of mathematical proofs are left to the two appendixes to this paper.

2. An Outline of the Main Logic and the Historic Backgrounds

(2.1) An Outline of the Logic

The foreign trade in the Metal Age put the chieftains of kin-based communities — *clan* in English, *genos* in Greek, *gens* in Latin, *wuji* in Japanese — under a new external circumstance in the sense that the net-benefits of the use of a military force in the transaction of foreign trade were increased by the innovation of metal tools. If they could adapt to such a new circumstance, they could gain and secure bigger benefits from the foreign trade, the transaction in which, however, had to be carried out under anarchic conditions without any common enforcer. On the contrary, if they had failed to adapt, they might have been colonized as the worst. Such an opportunity and a peril drove those chieftains to strengthen the "bargaining power" in the transaction of the foreign trade. The bargaining power is an economic terminology to be used as surrogate for, or an equivalent to, the "sovereign power" in terms of politics. The pursuit of the bargaining power drove those chieftains to transform the traditional communities into an "enlarged" social organization armed with metal military tools. An increase in the size of society was necessary to integrate bargaining process and to bring about scale merits in the production of the means of payment, as well as to maintain such a strong military force as to be able to put the bargaining process at least on an equal footing. Depending on the relative strength of an early king's military forth over neighbor chieftains, an increase in the society size was achieved through an enlargement of early king's autarky or by way of forming a "star type of networked coalition" consisting of neighbor chieftains with an early king locating at the center of the network. The military forth was of a private nature, not only in the sense that the aim of maintaining it is to pursue the benefits of the chieftains but also in the sense that the cost was financed by their own economic and human resources. Therefore, when the net benefits to a chieftain of an increase in the bargaining power under an early state became sufficiently large, it was brought into existence as a result of the innovation of metal tools which motivated selfish chieftains to launch into a venturous enterprise, i.e., the transformation of the existing communities into an early state. Ueda (2010, 2009, 2008, 2007) called such a proposition the "bargaining power theory" of the state. Though some classical works [6] can inspire us to hypothetically infer this proposition, in this paper I induce it on the empirical bases of recent historical and anthropological studies on the external trade of ancient irrigation societies [7] under or in accordance with the categorical frameworks of the transcendent philosophy. The main synthetic proposition is proved deductively by the analysis of a basic model abstracting not only irrigation societies formalized by applying the hierarchical coalition game of Gemage (2004) but also the process of an external trade between a chieftain and a foreign counterpart [8] formalized by applying the two-stage bargaining model of Querido (2007). Furthermore, the stability nature of those preceding kin-based communities — the reason why they did not have to build a state to keep inner order — is demonstrated in another way by applying the "link and claim" game of Skipper and van den Noueland (2001) to the process of forming an irrigation society.

The categorical approach to the early state is applicable to other types of states, if we should make

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[6] For example, see Ortega (1921, 1930), Mommsen (1854), Engels (1888, 1878) and Plato (1941).

[7] As to the historical and archaeological study on external trades between ancient Japan and China, see Asai (2008), Matuki (2007) and Murakami (2007). Furthermore, see Okada (2008, 2004) regarding documental records written by some ancient Chinese dynasties. As to the historical studies on the irrigation systems in Japan, see Tude (2005,1989). As to the classical work on the ancient irrigation societies, see Wittforgel (1957) and Nakashima (1973).

[8] As a special case of the base model, it is applied to Basileus of the ancient Greek. See Wilson (1978) and Ridley (1997) as to reviews on anthropological studies on military actions organized by kin-based communities

an explicit distinction among four categories on the states-the accident category, the causality of the early state, the classification (the accident category and the causality one of other types of the state) and the stability of a state (the reciprocity). [9] Failure to distinguish these four concepts has brought serious confusions into arguments on the state. In order to overcome the failure, each of those should be argued under its corresponding cognitive framework of the transcendental philosophy. The main propositions on those four concepts are summarized in what follows.

Firstly, the early state is conceptualized as follows: it is such a new type of social organization as to be able to bring the "sovereign power" but not a mere violence into birth by combining economic and personnel resources with a military forth from the historical stage onward in which external trade entered into the Metal Age. The main purposes of the sovereign power are the same as those of military forth mobilized by the preceding kin-based communities in the sense that it is aimed at protecting against threat from within and without.[10] However, the early state ― and therefore, other types of states coming on the later historical stges ― is explicitly distinguished from those kin-based communities by the economic background of the former that though reciprocal exchanges characterizing the kin-based communities were left to personal contracts between chieftains and follower-members, the external trades of the early state were carried out under the historical conditions of the Metal Age. Since the conventional notions of the state, which originate in Weber and/or Marx = Engels, focused on a relative monopoly in violence as the origins of the power, they cannot distinguish the state from the preceding kin-based communities.

Secondly, the causality to explain what caused the early state come into existence is derived from the innovation of metal tools as a new impact influencing on the preceding communities, under the condition

that the members of those communities are always self-interestedly motivated or driven by the innate genes' programs-for-survival. Since the sovereign power of a state contributes to making property rights more effective, those subjects who can gain the largest net-benefits from bringing a power into birth are motivated to build a state even at their own cost. Though such a hero type of political entrepreneurs should be distinguished from a non-political mediocrity by differences in the emotional and instinctual neural systems, historical evidence shows that such a hero type of political entrepreneurs came on each epoch-making historical stage. It seems to be because of this selfish aspect of the motivation to bear the power that the "predation theory" of the state is apt to put the state in the perspective of violence.

Thirdly, according to the concept of the power defined by political philosophers represented by d'Entreves (1967), Arendt (1958) and Lutz (2006), the state is classified by who are the origins of the power, i.e., by who bears the cost of military force combined with both economic and personnel resources. It is because the power originates in the early king that the early state is called the" early kingship." This approach to classification of the state is applicable to other types of the states, *mutatis mutandis*.

Fourthly, the stability of a state is, strictly speaking, the stability of a political system. The "political system" determines who are entrusted to exercise the power by the original holder of the power. In the early state an early king and coalitional members not only bring the power of a state into birth but also exercise the power. In order to maintain the stability of the early kingship, therefore, the participants' constraints of both coalition members and other follower-members have to be satisfied. The ruler-ruled relations were formed by reciprocal contracts on a voluntary basis, and could be maintained to the extent that those contracts were fulfilled by both parties. It seems to be because of this stability aspect of the state that the

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[9] This topic was distinguished by Hegel (1824/1825), when he argued for the organic theory of the state.

[10] As a pioneering empirical study on such an organized war among primitive kin-based communities, see Chagnon (1974). Furthermore, see Wilson (1978) and Ridley (1997) as to the review of anthropological studies on military actions mobilized by primitive kin-based communities

"contractual theory" of the state is apt to consider a state to originate in contracts. The stability of a state is subsumed under the reciprocity category, and the actual degree of the sovereignty is an approximate measures. As long as the bargaining power can play the role of a surrogate for the sovereignty, the main determinants of the bargaining power may be recognized as the main factors influencing on the stability.

If the above inferences from the main propositions on the early are considered as the essential categorical frameworks, the categorical approach to the early state of this paper is applicable to other types of the state, and the traditional theories of the state should be reexamined from those four points of view.

(2.2) The Historical Backgrounds of an Early State

The chieftains of traditional irrigation communities, who had theretofore formed intra-community economic networks among those communities, were faced with the foreign trade which could bring new necessities, metal goods (heretofore, represented by iron resources[11]) into the Far East Asia of those days. These necessities were vital for increasing both economic productivity and military power. Although at first they might have passively joined in this new trading network, they could take advantage of the chieftainship to have exclusive benefits from this external trade. However, whilst the intra-community trades could be under a repeated-game setting, firstly because residential areas are stuck to those near irrigated lands and secondly because the relative military forth equipped with stone-made weapons was

not so distinguised as to be able to overwhelm others without too risky cost, they had to play with those new foreign counterparts in a finite-stage game setting under the condition that it is worth to resort to a military force equipped with iron weapons in terms of the net-benefits. Then, the validity of contracts concluded in each transaction in the foreign trade was doomed to reflect the relative strength of a power to enforce those contracts.[12] In this paper the power to enforce is called the "bargaining power" which is a surrogate for the sovereign power[13] in politics. It could be increased not only by integrating the process of transaction and the production of the means of payment but also by strengthening military forth. It was for the sake of increasing their payoffs that those chieftains pursued these two ways to increase the bargaining power. The pursuit after the bargaining power motivated them to coordinate the traditional communities into an enlarged social organ with a regular military force. As a result, those preceding communities were transformed into an early state, and the chieftains became an early king, called *basileus* in Greek, *rex* in Latin and *Ou* in Japanese. This causality to explain the origins of the early state is consistent with the "organic theory" of the state,[14] in the sense that both propositions imply that the more organically the members of a society are coordinated, the bigger common interest such as the sovereign power can be achieved.

On the other hand, the cost to a chieftain of forming an early state is for the most part comprised of the cost to maintain his private army, to produce a means of payment for imported necessities and to manage an irrigation system. They were financed by earnings

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[11] As to the empirical study on the origins of iron tools, see Muhly (1995).

[12] As to the pioneering work of such a difference between intra-community (domestic) trade and inter-community (external) trade, see Polanyi (1977, 1963).

[13] As to the original concept of the sovereignty, see Bodin (1576).

[14] By "organic model," I refer to the theories of the state argued by Plato, Aristotle, Cicero , Hegel (1824/1825), and Hardin (1995). They are common in arguing that a positive common interest can be given birth only by coordinating the members of a political organization into a networked division of labor functioning like an organic body. It is, in particular, Hegel that brought forth a consistent logic to reconcile the selfish motives to the achievement of a common interest. Strictly speaking, however, political philosophers considered as the contract theorist, such as Hobbes (1651), Spinoza (1677), Locke (1690) and Hume (1752), also argued for common interests such as a defense against foreign threat. In this respect, they can be subsumed under the organic theorist.

from the external trades and by farm rents. It might have been conventional to view the farm rents as an appropriation or benevolence without equivalent compensation, except for the benefits of protection from external threats. However, as to the farm rent of irrigation society, it should be recognized as farmers' payment not only for benefits obtained from joining in an irrigation system but also for loan of seeds, on the basis of a reciprocal-exchange contract. This point of view on the farm rent is justified on a rational basis, if we take it into consideration that the irrigation system is of a club nature and, furthermore, that the chieftain of an irrigation society had a technological monopoly in the safekeeping and species-improvement of seeds as well as in the construction and maintenance of a large-scaled irrigation system. On the farmers' side, they could flee to some traditional ways of life such as primitive field-farming or small-scaled irrigation farming. Owing to these options the farmers had, the chieftain was required to meet their participants' constraints. Accordingly, the process of forming an irrigation society is formalized on a voluntary reciprocity basis. That is why the irrigation society can be formalized by a "networked-coalition game with hierarchies" and why its stability in the sense of the core is derived.

Though the historical backgrounds explained in the above reflect the characteristics of irrigation societies in the Japanese Archipelago, they are also applicable to other irrigation societies such as ancient Egypt, Mesopotamia, and China, India and Sri Lanka,[15] *mutatis mutandis*. Furthermore, they are also applicable to the process of building the "early state" in ancient Greek,[16] if it is taken into allowance, firstly, that the economic bases of the community are comprised of both dry field farming and cattle-breeding,[17] and secondly that the early king is a "chief among the equals" whose election is much more influenced by *anassein iphi* (rule by forth)[18] than in those irrigation societies, since the chieftains of irrigation societies could also control follower-members by way of social functions such as the management of irrigation system.

3. The Reexamination of the Theories of the State

From the historical backgrounds explained in the previous section, the four propositions on the early state are derived, and summarized in what follows.

Firstly, the early state is defined as a social organization with the power of a state backed up by a military force, which was brought into birth under historically a new external circumstance, i.e., the innovation of metal tools by the application to military force of which the chieftains of the traditional kin-based communities could gain such a huge profit as to give them an incentive to take on the cost of transforming the existing communities into a new enlarged social organization equipped with a regular military force. The bargaining-power in the transaction of then-prevailing foreign trade could be increased by resorting to the military force which could contribute to enforcing the contracts of trade concluded under anarchic conditions.

Secondly, the driving engine of bringing it into birth is the selfish motives of those chieftains for gaining bigger profits from the foreign trade by resorting to a military force. The regular army of the early state was of a private nature in the sense that it was aimed at increasing the selfish benefits from a monopoly in foreign trade, as well as that at first it was financed by chieftain's private resources.

Thirdly, the power of a state originated in a chieftain-turned early king who could bring the power into birth by combining a military force with economic and personnel resources, subject to the participants' constraints of other members. It is

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[15] Gunawardana, R.A.L.H. (1981) as to the early state in Sri Lanka.

[16] See Weber (1924) and Finley (1978) as to the early kings of the ancient Greek.

[17] The production process is much less characterized with team production than that of irrigation society which is crucially dependent on an irrigation network.

[18] Plutarch (1914-54) gives us the image of the early king by way of the mythological stories on Theseus and Romulus.

because the power of an early state originated in the early king that the early state should be called the "early kingship."

Fourthly, the stability of an early kingship is dependent on how satisfactorily and extendedly the participants' constraints of both coalition members and follower-members can be met. In this sense, in order to maintain the stability of the political system of the early state, the exercise of the power of a state had to be justified by the implicit consents of those other members, although the power of a state originated in an early king.

The categorical approach to the early state is applicable to other types of the states appearing in the later historical stages,[19] if we follow the cognition framework of Kant (1787) and the "causal factors" defined in Spinoza's letter to Tirunhaus dated January in 1675 saying that the concept of a subject should include *causa efficiens*, i.e., the "causal factors" in which the subject has its origin.

Those causal factors are follows: *Firstly*, the "power" is a political concept defined as the ability to enforce one's will on others, and the early state was also invented to bring into birth an "equivalent" to the power in the sense that since the power of the early state was brought into birth by the selfish motives to have an advantageous position in foreign trade, the equivalent is called the "bargaining power" in economics terms. This term is such an inclusive concept as to include two extreme cases, i.e., conquest and surrender. It should be re-emphasized that since an organized military force for war is observed also in primitive kin-based communities, mere the existence of an organized military force is not sufficient to conceptualize. *Secondly*, the preceding communities were faced with the innovation of metal goods, the application to military force of which gave sufficient incentives for taking on the cost of transforming the preceding communities into a new social organization

aimed at increasing the bargaining power in the transaction of foreign trades under anarchy. *Thirdly*, the entrepreneurial agents who if they can adapt to the new external-trade circumstances, gain the big benefit of an increase in the bargaining power are motivated to take the risk of, or to participate in, building a new social organization. The power of a state originates in those subjects who contribute to the birth of it by bearing the cost on their own. *Fourthly*, as long as those entrepreneurial agents want to continue to gain the large benefits from adapting to the new external circumstance, they have to meet the participants' constraints of not only their coalition members but also follower-members such as economic agents engaged in production of the means of payment.

Though some political philosophers[20] could contribute to classifying the state by who are the origins of the power of a state, they did make clear neither how and why the power of a state originates in those who contributed to bringing it into birth, nor under what historical conditions the power was brought into birth.

Generally speaking, under anarchy modeled by the analytical framework of a finite-stage game, threat by force as *ultra ratio* plays a crucial role to conclude and keep contracts in advantageous condition. Therefore, the bargaining power in foreign trade carried out under such anarchy may be assumed to be determined by the strength of a power relative to foreign counterparts. However, the actual efficaciousness of the power, in particular, the sovereign power against external societies, is dependent on what a strong military forth the state has in reality. Such a role of military forth in determining the actual degree of the sovereignty might have led some schools of social sciences except the "contract theory" to focus only on violence in conceptualizing the state[21] or in justifying the exercise of power.[22]

However, if we take it into consideration that many

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[19] In this paper it is applied to the aristocracy and the ancient monarchy. Applicability to federalism can be confirmed by Hamilton's essays in the *Federalists*. As to the rational bases of federalism, see Riker (1962) and Alesina and Spolaore (2005). Problems with applicability to the modern mass-democracy are mentioned in the last section.

[20] See d'Entreves (1967), Arendt (1958) and Lutz (2006).

[21] The definition of the state based on violence is represented by Engels (1884), Weber (1911) and North (1981).

of primitive kin-based communities had also an organized force for wars, it is obvious that, in order to distinguish the state from those kin-based communities, we are required to find out other crucial factors than mere the existence of such an organized force. That crucial factor is the emergence of a new external circumstance with which the chieftains were faced in the historical stage at which exchange-trades had been prevailing under the condition of there being no common enforcer.

On the other hand, it should be noted that any state has its own preceding societies in which economic networks including external trades peculiar to those societies had been spontaneously grown, and that when faced with new external circumstances, some risk-taking entrepreneurial groups were driven to organize those existing societies into a new enlarged social organization with the aim of taking the opportunity of those new external circumstances by generating a power to enforce. Such a historical process leads us to view the state in a "dynamic perspective" in accordance with Ortega (1930), and to start formalizing the process of building a state from characterizing its preceding societies but not from an abstract society consisting of an ahistorical group of atomistic individuals. Such an ahistoric assumption, on which the traditional "contract theory" of the state[23] is based, makes it hard to solve the problem of collective action[24], but the categorical approach or the bargaining-power theory can solve it.

4. Irrigation Society with Canal system: The Base Model of the First Stage

For the purpose of proving the main propositions of the bargaining-power theory by the back-ward induction of a two-stage game, in this section the first stage is formalized as the process of networking an irrigation society and of producing and allocating its outputs. In the first subsection the base model of an irrigation society is set up in the analytical framework of a networked-coalition game. In the second subsection, the price of iron is defined. In the third subsection, the stability of the networked irrigation society is proved in the sense of the core. In the fourth, the stability is re-examined in the analytical framework of a non-cooperative "link and claim" game.

(4.1) Irrigation Society Networked with Canal System[25]

Irrigation system is an economic infrastructure indispensable for any irrigation society, but in order to set up and operate an irrigation system, the members of an irrigation society have to be coordinated into a networked coalition. On the other hand, various kinds of metal tools, represented by iron *en masse*, are vital for increasing both economic productivity and military power, but have to be procured by way of external trades with foreign counterparts.

In order to construct a formal model abstracting the essential characteristics of irrigation societies, suppose that a river is flowing down from mountain areas in its

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[22] Beginning with Augustinus, Machiavellri (1532), Engels (1884), Veblen (1889), Weber (1911), Oppenheimer (1927),[22] Caneiro (1970), Service (1971), North (1981), Olson(1993, 2000), McGuire and Olson (1996), Findly (1996), Kurrild-Klitgaard and Sevendsen (2003), and the conflict game models represented by Skerpardas (1992), Hirshleifer (2001), Moselle and Polak (2001) and Grossman (2003) are all subsumed under this category.

[23] By the contract theory, I refer to Hobbes (1651), Spinoza (1677), Locke (1690), Rousseau (1762) and the modern followers of some of them, represented by d'Entreves (1967), Rawls (1971, 2001), Nozick (1974), Buchanan (1975) and Lutz (2006).

[24] The classical works represented by Ostrom (1965), Wagner (1965) and Flohlich *et al*. assumed the existence of political entrepreneurship for collective action in order to solve the problem of collective action. Olson (1965) emphasized the lack of a motive for political entrepreneurship itself except for the case that the selective- incentives scheme can be applied to political organization. Olson (1993, 2000) solved the problem of collective action in politics by the" rational bandits" model.

[25] Refer to Nakashima (1973) and Wittfogel (1957) regarding the details of ancient irrigation systems. As to the irrigation society of Japan, see Tsude (1989).

riverhead region, and that a canal system for irrigation is set up by taking irrigation-water from one point of the river called *sluice gate*. To be concrete, this canal system is assumed as the following: A trunk canal[26] is constructed which can irrigate prospective n paddy fields, numbered ered 1,2,...,n in the order of distance from the sluice gate. I note here that "n" is a generic but not fixed number. This trunk canal can "technically" irrigate any number of paddy fields. Each paddy field is cultivated by one farmer. Paddy fields are developed and located one by one in line along the trunk canal.[27] In order to take irrigation-water to each paddy field from the trunk canal, each farmer has to construct one branch canal so as to be connected with it. A chieftain locating in the sluice gate coordinates those expected farmers to construct and maintain the trunk canal at the farmers' expense on an equality basis. The total cost of the trunk is denoted by K. The farmers bear not only the equal share in the cost of K as the entrance fee on a club good, but also the marginal cost of joining in the irrigation system. The latter cost is denoted by $C_i \equiv C(i)$, for i farmer, which is comprised of the cost to construct the i branch canal locating in the i ordered distance from the sluice gate called the *zero site* where the chieftain locates, and of the cost to communicate and transport between the i site and the *zero* site. In this section, the number i is treated as a natural number standing for i farmer, for $\forall i \in \{1, 2,..., n\}$. A set of a chieftain and n farmers is denoted by $N = \{0, 1, 2,..., n\}$, whose first element stands for the chieftain. The absolute number of the elements of N is defined by $|N| \equiv n$. If a new farmer joins in this irrigation system, his paddy field must locate in the next to the most-distanced site in the existing irrigation system. The cost function of the i branch canal, $C_i = C(i)$, is assumed to be an increasing function of the distance from the sluice gate, with an increasing rate. That is, the more distanced, the more rapidly it increases.

These assumptions are formulated by the relations (1) below.

(1) $C_i = C(i)$,
$\quad 0 = C(0) < C(1) < C(2) <, \cdots < C(n),$ *and*
$\quad C(i + 1) - C(i) < C(i + 2) - C(i + 1),$
$\quad \forall i \in \{0, 1, 2,..., n\}.$

The above assumptions on $C(i)$ are justified by assuming the technological characteristic that the further away from the sluice gate a farmer is, the costlier for him to communicate with the chieftain locating at zero site, to bring back loaned seeds and to transport a part of annual harvests for the payment of charges on the loaned seeds and on the consumption of irrigation-water.

The trunk canal is constructed by using the iron whose volume and technology are denoted by *M en masse*. They replace conventional tools, i.e., stoneware. Therefore, given a scale of irrigation system, K is assumed as a decreasing function of M,[28] defined by (2).

(2) $K = K(M); K(0) > 0, K'(M) < 0,$ *and* $K''(M) > 0.$

The above assumptions on K are justified, because, if the more of iron-tools replace the existing less-effective tools to construct one set of canal system, the less costly it can become, subject to the "as-usual" assumption on the second derivative.

If the trunk canal is constructed by the cooperative work of s farmers coordinated by the chieftain, the farmer i bears the cost amounting to $K(M)/s$, $\forall i \in \{1, 2,..., s\}$. Such a way of burden-sharing may appear to mean slave labor, but in a contractual term, it stands for *entrance fee* or *basic charge* for irrigation system.[29] Crops are harvested after each farmer is engaged in a farming work whose energy expenditure is denoted by e. It is assumed as a constant for all farmers. This

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[26] Whilst the trunk-canal system fits well to a multi-layered hierarchical society, a reservoir canal system to a star network. The base model is also applicable to the latter system, *mutatis mutandis*.

[27] Though paddy fields are assumed to be located in line, the model can be extended to the more complex irrigation systems in which each paddy field has its own sub- irrigation systems.

[28] Iron tools for construction of canals and for farming were usually lent to farmers by chieftains.

assumption is for simplicity but justified by the historical condition under which farmers on those days could not have so much option for leisure. The harvest on each cropland is assumed as an increasing function of the iron (iron tools), and defined by $f(M), f(0) > 0$, $f'(M) > 0$, and $f''(M) < 0$. This function is assumed to be the same for all croplands. Difference in the fertility of each crop land is reflected in the increasing marginal cost of the branch canal. A fixed percentage of $f(M)$, denoted by α, $0 \leq \alpha \leq 1$, is paid to the chieftain as *variable charges* for the consumption of irrigation-water and for the loaned seeds. It is considered as the contractual representation of so-called annual tributes.[30]

(4.2) Payment for Iron: The Terms of Trade

Iron, M, has to be procured by way of the foreign trade in which the chieftain (the *zero* player, hereafter) can take advantage of chieftainship to have a monopoly. According to the historical background, the foreign trade is classified into two cases. In the first case, the external trade is carried on under the existing community system, and the price of iron is a given P per unit of M because it is set by a foreign counterpart and he is a price-taker. In the second case, the foreign trade is carried on under an early state comprised of s members excluding the cieftain, and the price is a decreasing function of s, defined by $\phi(s)$, per unit of M. The size of a social organization, denoted by s, is taken as a surrogate for the consolidation effects on the bargaining process in the transaction of the foreign trade, the scale effects on producing the means of payment for M, and the military force.

The cost to govern the early state is denoted by $G(s)$ *en masse* that covers the cost of maintaining a regular military force and that of other administrative work.[31] In order to indicate the positive effects of forming a state on the bargaining power in the foreign trade, approximated by $\phi(s)$, it is assumed that $\phi_1 \equiv \partial \phi(s)/\partial s < 0$. On the other hand, in order to emphasize the costly nature of maintaining the state aiming at an increase in the bargaining power, it is assumed that $G' \equiv \partial G(s)/\partial s > 0$ and $G'' \equiv \partial^2 G(s)/\partial s^2 > 0$. Then, the total cost to the zero player of acquiring M, denoted by Ψ, is defined below.

$\Psi \equiv \Psi(M : P) = P \cdot M$, for the existing community with a given P, and

$\Psi \equiv \Psi(M, s) = \phi(s)M - G(s)$, for an early state with $|S|$ size of society.

In the fifth section, it is justified that $P > \phi(s)$.

(4.3) Hierarchical Network and Stability

If, in order for a group of members to produce a cooperative output, they have to be *ex ante* coordinated into a networked team, it is called the "networked coalition with hierarchies".[32] In this sub-section, the cooperative process of the irrigation society set up in the subsection (4.1) is formalized in the analytical framework of the networked coalition game with hierarchies.

The process of forming a hierarchical coalition begins with a two-player coalition and ends with a hierarchical coalition of $|N|$ size, for simplicity, under the condition that the superadditivity is satisfied until the coalition size gets to $n \equiv |N|$. The zero-player coordinates other members into a hierarchical network

[29] For example, according to the data documented by the centralized monarchy system in the 7th to 8th century, each farmer was liable to do "sixty day work" per year under the supervision of a local chief. This work is considered to be allocated to construction and maintenance of irrigation systems of the local community. On the other hand, each brunch canal is considered to have been maintained by the farmer engaged in farming along it.

[30] According to the above document, the tribute from annual harvest, called *So*, was about 3 to 5 per cent of the total harvest. Seeds were loaned at about fifty percent of interests. Payment in other products than paddy crops, called *Cyo* and *Yo*, can be also subsumed in $\alpha f(M)$ for simplicity.

[31] According to the ancient centralized dynasty system called the *Rituryo* system, the regular force was comprised of about 200 thousands military services and the cost of maintaining it was financed by the dynasty government. They were exempt from both payment in cloth called *Cho* and 60 days work for construction called *Zoyo*. These exemptions are considered as a payment to the military servicemen. The cost of constructing roads and metropolis was also financed by taxes.

[32] As to the details of the concept, see Demange (2004) and Bala and Goyal (2000).

and is at the top of any hierarchy if he is superior to others in managing the irrigation system. Suppose a generic stage of the process, denoted by a networked coalition, $S = \{0, 1, 2,..., s\}$, $1 \leq s \leq n$. Then, the zero-player offers those s members those contracts according to which if they join in an irrigation system, they are assured of payoffs satisfying the participants' constraints. The clauses of the contracts are classified into two types as follows: The first is comprised of the production technology or the means of production he offers and of the cost burden of each member. The second is of interest rates on loaned seeds and of charges on consumption of irrigation-water. The former is represented by $K(M)$, $f(M)$, $C(i)$ and $|S|$. The latter is by α. By abstracting the common factors, those contract clauses can be condensed into a three-element set[33], $\{\alpha, M, s\}$. For a given S, there exist various combinations of α with M. The combination is denoted by $a(S) = \{\alpha, M : \exists S\}$. Denoting a set of $a(S)$ by $A(S)$, $A(S) = \{a(S) : \forall S \subseteq N\}$. For mathematical simplicity, $A(S)$ is assumed to be compact. In order for those s farmers to accept a contract offer, $a(S)$, it has to meet the participants' constraints and must be feasible. The participants' constraints are defined by the opportunity cost, zero-normalized. The feasibility condition of $a(S)$ is satisfied, if the total payoffs are nonnegative.

If each of those s farmers accepts a contract offer $a(S)$ and it is feasible, the payoff of the zero-player, $\pi_0 = \pi_0(a(S))$ and that of i farmer, $\pi_i = \pi_i(a(S))$, $\forall i \in S \setminus \{0\}$, is defined by (3) and (4), respectively.

(3) $\quad \pi_0(a(S)) = |S| \alpha f(M) - \Psi$.
(4) $\quad \pi_i(a(S)) = (1 - \alpha) f(M) - K(M)/|S| - C(i) - e,$
$\quad\quad i \in S \setminus \{0\}$.

The above payoff functions are defined over the compact set $A(S)$ and are *continuous* over $A(S)$. If, furthermore, S is extended to the domain of positive real number, the payoff functions are also continuous over $A(S)$.

Since the opportunity cost of each player was zero-normalized, the participant's constraints of the zero-player and those of the farmer i are defined by (3)' and by (4)' in turn. Under the assumption of transferable utility, the feasibility of $a(S)$ is defined by (5).

(3)' $\quad \pi_0(a(S)) \geq 0$.
(4)' $\quad \pi_i(a(S)) \geq 0$.
(5) $\quad |S| \cdot f(M) - \Psi(M) - K(M) - \sum_{i=1}^{s} C(i) - |S| \cdot e$
$\quad\quad \equiv v(S) \geq 0$.

$v(s)$ of the right side of (5) means the value of the s-player cooperative game. In what follows, the constant parameter, e, is omitted without loss of generality.

Since the main assumptions of the above set-up, i.e., the superadditivity, the compactness and the continuous utility functions, meet the conditions of the hierarchical coalition game of Gemange (2004), we can prove the stability of the irrigation society along the same mathematical algorism.[34]

The stability nature of the irrigation society is summarized as *Proposition 1.* The general proof is left to *Appendix 1.*

Proposition 1: The irrigation society is stable in the sense that neither a player nor a coalition has an incentive to deviate from it, under the assumption that the superadditivity prevails and that utility functions are continuous over a compact set of variables. Furthermore, if chieftainship cannot be taken over by

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[33] Even if α is assumed as a given parameter, it does not influence the main conclusions of this paper, but in what follows, the general formalization is adopted.

[34] The mathematical algorithm begins with the process of the zero-player's maximizing his payoff in forming a two-player network subject to the participant's constraint of a farmer. Next, in the same way, he forms a three-player network subject to the participants' constraints of two farmers, and etc. The payoff allocations of $(n + 1)$ players' network satisfying such a procedure not only meet the participants' constraints of those networked members, but also do not give an incentive to make any coalition deviating from it. Furthermore, it is unique, if the zero-player is assumed as only one coordinator. The basic model can be extended to more complex types of irrigation system, as far as the main assumptions are maintained. For example, each branch canal can have its own hierarchical irrigation systems by extending smaller branch canals from it and connecting them. For another example, a canal system with reservoir is also formalized by a similar model, *mutatis mutandis*. The reservoir system is applicable to Sri Lanka.

any other member, then this stable system is unique. If the superadditivity stops at some size, then the society is divided into more than one irrigation systems called "heterarchy" each of which meets the superadditivity.

Proposition 1 not only proves that under the superadditive condition an irrigation society with hierarchies is stable in the sense that no member of the irrigation society has an incentive for deviating from it, but also implies that it is not right to trace the origins of the state back to the fissiparous tendencies of preceding societies, which some evolution archaeologists consider had caused those preceding societies to break up and to form a state on a contractual basis.[35]

(4.4) Stability Analysis in a Non-Cooperative Framework

Since any networked-coalition game with hierarchies is a cooperative game, it cannot describe the process of forming a network. In this subsection, the process itself is formulated by a "link and claims" game, and I prove that the payoff allocations defined in the previous subsection can be achieved not only as the Nash equilibrium but also the Strong Nash equilibrium.

Suppose that the "link and claims" game is played with $(n + 1)$ players, denoted by $N = \{0, 1, 2,..., n\}$. The strategy of i player is defined by (6).

(6) $r_i = \{d_0^i, d_1^i.., d_{i-1}^i, 0, d_{i+1}^i,..., d_n^i\},$
　　$\forall i \in N \equiv \{0, 1,..., n\}.$

The $(j + 1)$ th element of (6), denoted by d_j^i, signifies that if player j allows player i to secure $d_j^i > 0$ as a part of the payoff allocated to player i, the player i would like to form a link with the player j and that if not, the player i will not form a link with the player j, leading to the notation, $d_j^i = 0$. Only if both $d_j^i > 0$ and $d_i^j > 0$, the link between i and j is formed. Then, the payoff to the player i, denoted by $\Pi_i = \Pi_i(r_0, r_1,...,$

$r_n)$, is defined by (7).

(7) $\Pi_i(r_0, r_1,..., r_n) = \sum_{j \neq i, d_i^j > 0}^{n} d_j^i,$

$\forall i \in N, s.t., \Pi_0 + \Pi_1 +...+ \Pi_n = v(N).$

The strategy profile defined by (8) and (8)' below is the Nash equilibrium of the above game. (The proof is easy and omitted.)

(8) $r_0 = \{0, \Pi_0/n,..., \Pi_0/n\}, and$
(8)' $r_i = \{\Pi_i, 0,..., 0\}, i \in N\backslash\{0\}, i \in N\backslash\{0\}.$

Furthermore, if Π_0 and, $\Pi_i, i \in N\backslash\{0\}$, defined by (7) are replaced by π_0 and $\pi_i, i \in N\backslash\{0\}$ defined by (3) and (4), respectively, then the Nash equilibrium is the strong Nash equilibrium, too, in spite of $\pi_0 = \Pi_0 > 0.$[36] This is because the role of the center player, played by the chieftain in the above model, cannot be taken over by any other member. If it can be taken over by any member, the Nash equilibrium cannot meet the conditions of the strong Nash equilibrium.

5. Foreign Trade: the Second Stage of the Backward Induction

The intra-community economic network of an irrigation society is formed in a repeated-game setting. This is because any irrigation society is not free from adherence to farmland and because stone-tools have to be used, the power to enforce is not so different not only among a community's members but also among those communities. Therefore, economic networks among them are spontaneously grown on a voluntary basis and the chieftains of those communities are neither motivated nor capable to unite them into a state maintaining a regular military force equipped with metal tools.

On the other hand, foreign trades with foreign counterparts are carried out in the setting of a non-repeated game, in particular, if those foreigners have

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[35] For example, see Classen and Saknik (1978, 1981), and Carneiro (1970).

[36] In general, in order for the strategy profile of (8) and (8)' to be a strong Nash equilibrium, Π_0 must be zero. As to the proof, refer to the Theorem 4.1 and 4.2 of Slikker and Nouweland (2001).

other options for business connections and are free to shift their options backed up by having a recourse to a military forth. In this section an external trade of iron, carried on without any common enforcer, is formulated by a two-stage bargaining game between a chieftain called "buyer" and a foreign counterpart called "seller." It is only through this foreign trade that the buyer can obtain iron. It may be too costly to have so strong a regular army as to conquer the seller, but he wants to keep the external trade in as advantageous a condition as possible.

At the first phase[37] of the two-stage bargaining game, the seller offers a supply price, denoted by P, per one unit of the volume of iron, M. At the second phase, the buyer decides whether to accept or reject it. If he accepts, the contract is concluded and the buyer obtains M at the price of P. On the contrary, if he rejects the offer,[38] the bargaining process enters into conflict, and is settled so as to reflect the relative strength of a power to enforce. Both military power and economic power, and how much effectively the social organization is coordinated into an organic body are crucial factors determining the relative degree of the enforcement power. In each homeland, military forth such as a standing army may be or not may be waiting behind agents charged with bargaining on the spot. It is those agents including attendants and transporters that are involved with conflict on the spot. In any way, the extra cost to the buyer (seller) of exercising a military forth on the spot is assumed as a given parameter, denoted by $V_0(V)$. However, how the conflict is settled depends on whether the buyer is the chieftain of a preceding community or the king of an early state.

If the buyer is the chieftain of the preceding community which is categorized as a social organization with the military force being zero-normaligned, the probability of his winning in conflict is assumed to be a constant denoted by λ_0. On the other hand, if the buyer is the "king" of an early state, categorized as a social organization with a regular army, the probability of winning in conflict is considered to be more flexible and assumed to depend on the relative strength of the sovereign power he can exercise. The relation among the bargaining power (sovereign power), military power, economic power and their organic combination are formalized by the "Conflict Success Function"[39] (CSF, hereafter).

In the previous section, the number of the players, denoted by s, was defined as a natural number in order to make the explanation suitable to the analytical framework of a cooperative game. However, since differential calculus is required in this section, the space of s is extended to a real number,[40] if necessary. The superadditivity is not assumed in what follows, because the main topic is optimal decision on $\{\alpha, s, M\}$ by maximizing π_0, s. t., $\pi_i \geq 0$, $\forall i \in \{1, 2,..., s\}$, $s \geq n$.

(5.1) Foreign Trade without a Regular Military Force equipped with Iron Weapons

If the offered price P is accepted by the chieftain, the seller's payoff function, $\pi = \pi(P)$, and the chieftain's one, $\pi_0 = \pi_0(P)$ are defined by (9) and (9)' respectively, with α, s and M being given at the second stage of the backward induction process. To be simple, in what follows, it is assumed that the seller supplies iron at no cost.

(9) $\pi(P) = P$.

(9)' $\pi_0(P) = s\alpha f(M) - PM$.

All of α, s and M in (9)' are determined at the first stage in the backward induction process, and therefore, are recognized to be given at the second stage.

On the contrary, if the price is rejected by the buyer, the bargaining process proceeds to the second phase. The condition (10) is necessary for the buyer to choose "rejection" of the offered price P.

(10) $s\alpha f(M) - V_0 > s\alpha f(M) - PM$.

The above condition (10) means that M can be taken away by exercising a force at the cost of V_0 on the spot.

In the conflict, the chieftain and the seller expend V_0 and V, respectively. The chieftain wins the conflict with the probability of λ_0 assumed as a given parameter. If he wins the conflict, his payoff amounts to the value of $\{s\alpha f(M) - V_0\}$. If he loses, he has to pay $P \cdot M$ in return for M, and thus his payoff is reduced to the value of $\{s\alpha f(M) - PM - V_0\}$. Then, the expected payoff of the seller, $\pi(\alpha, s, M: \lambda_0)$, and that of the chieftain = buyer, $\pi_0(\alpha, s, M: \lambda_0)$, are defined by (11) and (11)' respectively.

(11) $\pi(\alpha, s, M: \lambda_0) = (1 - \lambda_0)(PM - V) + \lambda_0(-V)$.

(11)' $\pi_0(\alpha, s, M: \lambda_0) = \lambda_0\{s\alpha f(M) - V_0\}$
$\qquad\qquad + (1 - \lambda_0)\{s\alpha f(M) - PM - V_0\}$.

Denote by $P^* \equiv P^*(\alpha, s, M: \lambda_0)$ the maximum of P which the chieftain can accept at the first phase. Then, P^* is determined so as to solve the equation, $\pi_0(P) = \pi_0(\alpha, s, M: \lambda)$. By solving it and arranging the result, (12) is derived.

(12) $P^* \equiv P^*(\alpha, s, M: \lambda_0) = V_0 / (\lambda_0 M)$.

Substitute (12) into $\pi(P)$ and $\pi(\alpha, s, M: \lambda)$ and compare the results. Then the optimality of $P^*(\alpha, s, M:$

$\lambda_0)$ for the seller is confirmed by the inequality (*) below.

(*) $\pi(P^*) - \pi(\lambda_0) = V_0 / \lambda_0 > 0$.

It is obvious from (12) that the higher λ_0 is, the lower is P^*, *ceteris paribus*. This *ceteris paribus* causality between λ_0 and P^* motivates the chieftain to raise the probability of winning, anyhow. Then, if the probability of winning is considered to be increased by maintaining a regular army equipped with iron weapons, the chieftain is driven to transform the existing community into a new social organization with a regular army, which is usually an enlarged social organization to achieve an increase in the bargaining power. As a result, an early state comes into being.

(5.2) Foreign Trade with Regular Military Force equipped Iron Weapons

Suppose that the zero-player transforms the existing communities into an early state with regular army equipped with iron weapons. The cost of governing the early state is denoted by G. It is assumed as an increasing function of the state's members, denoted by s, with a slope becoming steeper in accordance with the "as usual" assumption on the cost function of economics. That is, $G = G(s)$, $G' > 0$ *and* $G'' > 0$.

On the other hand, it is assumed that the probability of winning in conflict is determined by CSF[41], defined by (13).

(13) $\lambda(s;\theta) = \dfrac{F(s)\theta_0}{F(s)\theta_o + \theta_1 V} = \dfrac{F(s)}{F(s) + \theta V}$.

It is defined as an increasing function of the "increasing function of s, denoted by $F(s)$," with θ_0 and θ_1 given. The function, $F(s)$, approximates how much effectively a set of the s members of the state are coordinated into a team type of organic body,[42] where $F'(s) > 0$ and $F''(s) < 0$. These assumptions on $F(s)$ are justified, if we take it into consideration, firstly that the personnel and logistic capacity of a

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[41] As to the original concept of the conflict success function, see Skaperdes (1992). The CSF is also taken as a proxy function measuring a sovereign power.

regular army must be backed up by both economic and human power, approximated by the size of society, secondly that the effects of those powers are subject to a gradually-increasing pattern, and finally that how effectively those physical factors can function depends on "how well-organized they are," which is represented by the functional form of $F(s)$.

$\theta \equiv \theta_1 / \theta_0$ on the right of (13), where θ_1 and θ_0 stands for the military technology of the seller and that of the buyer, respectively. It is easy to derive the signs of the first and second derivatives of $\lambda(s:\theta)$; $\partial \lambda / \partial s > 0$, $\partial^2 \lambda / \partial s^2 < 0$; $\partial \lambda / \partial \theta < 0$, and $\partial^2 \lambda / \partial \theta^2 > 0$.

Here, we can re-define λ_0 as $\lambda_0 = \inf_{s \in S} \lambda(s:\lambda)$. That is, λ_0 is achieved when s takes the minimum threshold value with θ being given.

By contrast, if there exists some s satisfying $\lambda^* \equiv Sup_{s \in S} \lambda(s:\theta) \doteqdot 1$, the buyer can acquire M only at the cost of V_0. Then, the buyer's payoff, denoted by $\pi_0(\lambda^*)$, approximates to $\{s \alpha f(M) - V_0 - G(s)\}$. Such an extreme case may fit well with the "predatory theory" of the state.

Then, if the offered price, P, is accepted by the buyer, the payoff of the seller, $\pi(P:\alpha, s, M)$, and that of the buyer, $\pi(P:\alpha, s, M)$, are defined by (14) and (14)' respectively.

(14) $\quad \pi(P:\alpha, s, M) = PM$.
(14)' $\quad \pi_0(P:\alpha, s, M) = s \alpha f(M) - PM - G(s)$.

On the contrary, if the buyer rejects the offer, the bargaining process falls into conflict and proceeds to the second phase of the bargaining process. It is noted here that in order to be rejected, the following inequality condition,[43] $s \alpha f(M) - V_0 - G(s) > s \alpha f(M) - PM - G(s)$ is required but it is essentially the same as (10). Then, the expected payoff of the seller, $\pi(\lambda(\alpha, s, M:\theta))$, and that of the zero player, $\pi_0(\lambda(\alpha, s,$

$M:\theta))$ are defined by (15) and (15)' respectively.

(15) $\quad \pi(\lambda(\alpha, s, M:\theta))$
$\quad = (1 - \lambda(\alpha, s, M:\theta))(PM - V)$
$\quad\quad + \lambda(\alpha, s, M:\theta)(-V)$.
(15)' $\quad \pi_0(\lambda(\alpha, s, M:\theta))$
$\quad = \lambda(\alpha, s, M:\theta)\{s \alpha f(M) - G(s) - V_0\}$
$\quad\quad + (1 - \lambda(\alpha, s, M:\theta))$
$\quad\quad\quad\quad \{s \alpha f(M) - G(s) - V_0 - PM\}$.

Denoting by $P^{**} \equiv P^{**}(\lambda(\alpha, s, M:\theta))$ the maximum of the offered price which the buyer can accept, it satisfies the equation, $\pi_0(P:\alpha, s, M) = \pi_0(\lambda(\alpha, s, M:\theta))$, or (14)'=(15)', and it is derived as (16) in the end.

(16) $P^{**} \equiv P^{**}(\lambda(\alpha, s, M:\theta)) = V_0 / \{\lambda(\alpha, s, M:\theta) \cdot M\}$

Whether P^{**} is optimal for the seller is examined by substituting (16) into (14) and (15) and then by comparing the results. The optimality is proved by deriving the following equality ; $\pi(\lambda(\alpha, s, M:\theta)) - \pi(P:\alpha, s, M, \lambda_0) = V + V_0 > 0$, for $P = P^{**}$.

When the buyer accepts P^{**}, then, his payoff function is defined by (17) which is derived from substituting (16) into (14)' or (15).'

(17) $\quad \pi_0(P^{**}:\alpha, s, M)$
$\quad = s \alpha f(M) - V_0 / \lambda(\alpha, s, M:\theta) - G(s)$
$\quad = \pi_0(\lambda(\alpha, s, M:\theta)), \text{for } P = P^{**}$.

6．Production and Distributions: the First Stage of the Backward Induction

The first stage of the backward induction is also classified into two cases: In the first case, the zero-player is the chieftain and determines the optimal-

[42] In other word, $F(s)$ stands for the" coordinated or organized degree of the members of a society argued by Hegel (1924/25), or a combination of the "exchange power" and the "coordination power" by Hardin (1995).

[43] In the case of the early state, it is realistic to assume that $V_0 = V_0(s)$ with $\partial V_0 / \partial s < 0$. Such an assumption is justified, if we take it consideration that the cost of guards can be reduced provided a regular army is standing behind. Since the assumption and the consideration can strengthen the causal logic below, it is for simplicity that the assumption of a constant V_0 is maintained in what follows..

value set of $\{\alpha, s, M\}$ which maximizes $\pi_0(P)$ defined by (9)' or $\pi_0(\alpha, s, M: \lambda_0)$ defined by (11)' subject to the constraint that $P = P^*(\alpha, s, M: \lambda_0) = V_0/(\lambda_0 \cdot M)$ with λ_0 given and that the participants' constraints of the farmers are satisfied. In what follows, the derived optimal-set is denoted by $\{\alpha(\lambda_0), s(\lambda_0), M(\lambda_0)\}$, and the value of $P^*(\alpha, s, M: \theta)$ substituted by those optimal variables is denoted by $P^*(\alpha(\lambda_0), s(\lambda_0), M(\lambda_0) : \lambda_0) \equiv P^*(\lambda_0)$.

In the second case, the zero-player derives the optimal-value set of $\{\alpha, s, M\}$ by maximizing $\pi_0(P: \alpha, s, M)$ for $P = P^{**}(\lambda(\alpha, s, M: \theta))$ defined by (14)' or $\pi_0(\lambda(\alpha, s, M: \theta)$ for $\phi(s) = P^{**}(\lambda(\alpha, s, M: \theta))$ defined by (15)', both of which result with (17) subjected to the constraint that $P^{**}(\alpha, s, M: \theta) = V_0/\{\lambda(s: \theta) M\}$, and that the participants' constraints of the farmers are met.

In what follows, the derived optimal-value set is denoted by $\{\alpha(\theta), s(\theta), M(\theta)\}$ and the value of $P^{**}(\lambda(\alpha, s, M: \theta))$ substituted by those optimal values is denoted by $P^{**}(\alpha(\theta), s(\theta), M(\theta): \theta) \equiv P^{**}(\theta)$.

It seems apparent from (16) that if M were set at a given value, then $\partial P^{**}/\partial s < 0$, in any case. This "ceteris paribus causality" between P^{**} and s may lead the zero-player to conjecture that he can make the better terms of trade by an increase in the bargaining power, which requires the transformation of the existing community into an enlarged political organization. His conjecture turns out right, as proved in *Appendix 2*. That is, even if not only the direct but the indirect effects of s on M are taken into consideration in making the decision at the first stage, the positive effects of an increase in the size of society on the better terms of trade are preserved. The main results of the backward induction are summarized in *Proposition 2*. (The mathematical proofs are given in *Appendix 2*.)

Proposition 2

Under the condition that $C'(s) \geq K(M)/s^2$, (i), (ii) and (iii) hold.

(i) $\partial M/\partial s > 0$, $\partial \alpha/\partial s < 0$.

(ii) $\partial s(\theta)/\partial \theta > 0$, $\partial \alpha(\theta)/\partial \theta < 0$, $\partial M(\theta)/\partial \theta \geq 0$.

(iii) $dP^{**}/ds < 0$, $P^*(\lambda_0)M(\lambda_0) > P^{**}(\theta)M(\theta)$.

The condition of *Proposition 2* means that an increase in the marginal cost of the irrigation system locating at the marginal site (s^{th} site), denoted by $C'(s)$, is larger than a decrease in the average cost $K(M)/s$, measured by $K(M)/s^2$, which is brought about by incorporating one more farmer into the irrigation system. This condition is justifiable as far as the cost of constructing a branch canal is in the increase at an increasing rate due to, for example, a drastic increase in the transportation cost.

The first inequality of (i) implies that the size of society is positively related to the demand for iron at the optimal. The second one means that the society size is negatively related to the zero-player's sharing in annual crops at the optimal.

The three inequalities of (ii) show the effects of a change in the relative military power of the foreign counterpart on the optimal values; $\alpha(\theta)$, s(θ), and $M(\theta)$. The implication of each sign are obvious.

The first part of (iii) implies that if the existing communities are transformed into an enlarged social organization with a regular army, the terms of trade are made more advantageous to the zero-player. The second part of (iii) means that in spite of more iron being imported, the total payment for them is smaller under the early state than under the preceding community. This means that the negative effects on the price of imported iron could offset the positive effects on the volume of imported iron.

7. The Rational Foundations of the Early State

According to *Proposition 1*, the chieftain of the preceding kin-based community is not motivated to have a military force aimed at keeping domestic economic networks in order. According to *Proposition 2*, however, even if it is maintained at private cost, he may be motivated to have a regular military force provided that he can get more profits from the transaction in the foreign trade by resorting to the military force the net-benefits of whose use could

increase by the innovation of metal goods. The increase in the net-benefits is accompanied with an increase in the size of society which represents the quantity aspect of not only territory but also economic and military power. Now, we have arrived at the final stage where we have to examine the truth and objectivity of the main synthetic propositions of this paper.

According to the criteria for judging the objective truth of a synthetic but not analytic proposition, the synthetic propositions, which are derived from unifying or combining the syntheses of various kinds of intuitions and/or categories so as to be subsumed under (or in accordance with) the cognitive frameworks of the pure categories, are "objectively true." The so-called "test of hypothesis" is already implied in these criteria for judging objectivity and truth, because phenomenon or empirical images are abstracted when they are subsumed under the categorical frameworks. (Such an objectivity and truth of synthetic proposition was called "the possibility of the experience" by Kant.) According to the empirical study of neuroscience, such Kantian criteria for judging the objective truth of a synthetic proposition are supported by the "neural modules" theory, which implies that although when all of those modules are set up has yet to be explicated, Homo sapiens have innately potential cognitive-frameworks in common. This is why regardless of nationality or gender, we think we are persuaded and convinced providing that an opinion or a view is explained along some logical framework. The causality is an example for such a common cognitive framework, let alone sensibility and emotional programs.

However, modern sciences call for revealing explicitly the test of hypothesis as qualification as truth. According to the procedure of the test, first of all some expected hypothesis should be deductively derived from the analysis of the main proposition. Next, the hypothesis has to be corroborated by experimentation or by reference to relevant empirical data serving as evidence, called the "test." In what follows in this section, the expected hypotheses are deducted by analyzing the base model. Historical evidence is referred to in the next two sections.

Before deducting the expected hypotheses from the analysis, the three main propositions of this paper are summarized below:

Firstly, the new circumstances in which metal tools were innovated and its application to military force could increase the expected net-benefits of resorting to the military force in the transaction of then-prevailing foreign trade drove the chieftains of the preceding communities to take this opportunity to have much more advantageous position in the foreign trade, i.e., to increase bargaining power in the transaction of the foreign trade by resorting to the military force strengthened by equipping with metal weapons, under the necessary condition that those chieftains had been steadily throughout motivated by self-interests. Thus, an early state came into existence as a result of the innovation of metal tools under the condition that the driving engine-factors had been the selfish motives of those chieftains. The self-interest motive is the economic version of the innate genes' programs-for-survival

Secondly, therefore, the early state is an "accidental situation" of the society as a substance and should be recognized not as a creature but as the transformation of the preceding kin-based communities into a new social organ which came into being as an effect of adaption to a change in the essential elements of the preceding type of the society. That change, called a new environment or surrounding, was the innovation of metal tools.

Thirdly, the bargaining power of an early state, the increase of which is the direct goal of those chieftains, is a surrogate for the sovereignty and is interdependent on one another's sovereignty. The quantity category such as the relative economic and military power represented by the relative size of society and the quality category such as the organic degree of a social organization and the relative military technology are the main determinants of the bargaining power.

The first expected hypothesis to be deducted from the analysis of the main propositions is as follows: that when faced with some new surrounding to be adapted to (i.e., faced with the innovation of metal goods), the

chieftain prefers an "early state type" of the society to the existing type of the society called the "preceding kin-based community just prior to it," under the condition that as the main driving-engine factor to bring about a change in the essential elements of the preceding community, he can take this opportunity to satisfy the innately-programmed self-interested motives by increasing his payoff.

The second expected hypothesis to be deducted from the analysis of the main propositions is as follows: the pursuit of the direct goal of his self-interested motives, i.e., the pursuit of an increase in the bargaining power in the foreign trade, led to the establishment of an early state as the end result, whether conscious of its historical implications or not. This second hypothesis is simultaneously proved in the process of the deductive proof of the first one.

The third expected hypothesis to be deducted from the analysis of the main propositions is as follows: that an increase in the bargaining power is consistent with an increase in the payoff of the chieftain, and therefore, that with given external factors such as opponents' military technology, the actual degree of the sovereignty of an early state is determined so as to meet the condition of maximizing his payoff. The deductive proof of this hypothesis is also addressed in the process of the proof of the first one. Thus, we can concentrate on the deductive proof of the first hypothesis in what follows.

For those purposes just mentioned in the above, it is enough to prove that the inequality (18) or its rewritten form (19) holds. Either way, it leads to the conclusion that the chieftain in the Metal Age prefers an early state to the existing community.

(18) $\pi_0(P^{**}(\theta))$

$= s(\theta)\alpha(\theta)f(M(\theta))$

$- \dfrac{V_0}{\lambda(\alpha(\theta), s(\theta), M(\theta):\theta)} - G(s(\theta))$

$> \pi_0(P^*(\lambda_0)) = s(\lambda_0)\alpha(\lambda_0)f(M(\lambda_0)) - \dfrac{V_0}{\lambda_0}.$

(19) $s(\theta)\alpha(\theta)f(M(\theta)) - s(\lambda_0)\alpha(\lambda_0)f(M(\lambda_0))$

$> G(s(\theta)) - \{\dfrac{V_0}{\lambda_0} - \dfrac{V_0}{\lambda(\alpha(\theta), s(\theta), M(\theta):\theta)}\}.$

The inequality (19) holds, if a combination of the following three conditions, (i), (ii) and (iii), are satisfied: the condition (i) that an increase in the share of annual harvests, i.e., the left side of (19), is large enough, the condition (ii) that $G(s(\theta))$ is small enough, and finally the condition (iii) that $\lambda(\alpha(\theta), s(\theta)M(\theta),:\theta)$ is large enough relatively to a given parameter, λ_0.

Whilst the left side of (19) means an increase in the "tributes" to the king obtained by transforming the existing community into an early state, the right side means an increase in its net cost. Therefore, both of (18) and (19) mean that in order for the chieftain to prefer an early state to the existing community, the net benefits to the chieftain of transforming into an early state must be positive. Let's examine the conditions for the positivity of the net-benefits in what follows.

The second term on the right side of (19) is positive because $\lambda_0 < \lambda(\alpha(\theta), s(\theta), M(\theta):\theta)$. If this positive second term is so large as to cause the inequality (19) to hold always, then the first hypothesis turns out to be proved.

However, when we take it into consideration that $G((\theta))$ is usually large, a change for the better in the terms of trade, defined by $\{V_0/\lambda_0 - V_0/\lambda(s(\theta):\theta)\}$ on the right side of (19), must be so sufficiently large as to satisfy the inequality (19). When the left side of (19) is large enough, the dependence of the required positivity of the chieftain's net-payoff on the better terms of trade is weakened. However, though both the inequality, $s(\theta) > s(\lambda_0)$, and the first part of (i) of Proposition 2 can contribute to the positivity of the left side of (19), the positivity itself of the left side cannot be proved, because how α changes is not obvious. On the other hand, as the left side becomes smaller, the right side must become smaller, too, in order for the net-payoff to continue to be positive.

The general conclusion is as follows: If there exist a range of the set whose elements consist of an increase

in the chieftain's share in the annual harvests, the cost of governance and an increase in the bargaining power and which can meet the inequality condition (19), the chieftain prefers the early state to the preceding community. However, if not, for example, if an increase in the chieftain's share in annual harvests is not so large and/or the cost of governance, $G(s)$, is not so small enough as to always assure the inequality (19), an increase in the bargaining power, approximated by $\{1/\lambda_0 - 1/\lambda\,(\alpha\,(\theta),s(\theta),M(\theta){:}\,\theta\,)\}$, must be large enough in order for the chieftain to prefer the early state to the preceding community and therefore, for him to be motivated to transform the latter into the former as the end result.

On the other hand, the actual degree of the bargaining power or that of the sovereignty is relatively determined in the sense that it is determined so as to meet the optimality condition of the maximization of his payoff. However, thanks to his direct motives for its increase, the bargaining power is sure to be higher in the early state than in the preceding community.

8. Application to the Aristocracy and the Ancient Monarchy

In this section, the main propositions above mentioned are applied to the aristocracy and to the ancient monarchy of an empire type. The aristocracy took over the early kingship in the ancient Athens or reconciled with it in the ancient Roma, and the ancient monarchy of an empire type was established vie federal system comprised of the early states with a hegemonic early king as the center of the federal coalition. Though the aristocracy is a political system in the Bronze Age and on the other hand, the ancient monarchy of an empire type is the one in the Iron Age, each type of the state is a transformation of the preceding type of the state caused by the appearance of much riskier surroundings or more profitable opportunity with which it was hard for one state only to deal.

(8.1) The Ancient Aristocracy: the "Rex＝Basileus" System

After the aristocracy took over the early kingship, the original meaning of *rex* in Lain and *basileus* in Greek were degraded to the status of a military commander entrusted with administrative work and management of religious institutions, appointed by the chieftain-turned aristocrats and approved by other "citizen" members. Since they were usually a lifetime officer and had some discretionary power due to the informational incompleteness of his military actions, they are often misunderstood as the same as the king in the Modern Age. However, they came on the historical stage when the aristocracy had been overwhelming. It came into being as the result of an endeavor to satisfy two requisites as follows: the first one that the early states had to organize themselves into an enlarged political union for the sake of attaining the larger economies of scale in the process of adapting to new external circumstances common to all of them such as Persian or Carthaginian threat, and the second one that the effective use of military force depends crucially on personal competence for military entrepreneurship. That is, some part of the functions taken by the kings of the early states were separated from them and entrusted to a new *rex* or a new *basileus* when the personal competence became crucial. The cost to the new rex or basileus of organizing a military force was financed by customs duty imposed on imported goods, though the core personnel of army corps were comprised of the aristocrat family. In this subsection, I show that the main propositions of this paper are applicable to this new *rex* system or the new *basileus* system under the aristocracy (hereafter, the *rex* system, for short).

I begin with the situation where h early states similar to each other as to the number of members and the level of productivity are separately engaged in an external trade with a new dominant foreign counterpart common to all of them. The early king i pays P for one unit of iron he acquires, denoted by M_i. He provides M_i for s_i farmers in exchange for sharing harvests the percentage of which is fixed at a given α. The probability of winning in the conflict which may

arise in the process of the transaction is denoted by λ_0, assumed as a given parameter. The optimal value of M_i and of s_i are determined by the same mathematical procedure as the previous sections, denoted by $M_i(\lambda) \equiv M_0$ and by $s_i(\lambda_0) \equiv s_0$, for $i = 1, 2,..., h$. Here, the new notations, $M(\lambda_0)$ and $s(\lambda_0)$, are defined as the following: $M(\lambda_0) = h \cdot M_0$ and $s(\lambda_0) = h \cdot s_0$. Then, the optimal payoff of the early king i is defined by (20).

$$(20) \quad \pi_i(s_0, M_0 : \lambda_0) = s_0 \, \alpha \, f(M_0) - \frac{V_0}{\lambda_0},$$
$$i = 1, 2..., h.$$

In the above, I note that $P^*(\lambda_0) \cdot M_0 = V_0 / \lambda_0$.

Next, suppose that a new rex is appointed by the early kings and entrusted with arranging the foreign trades and foreign affairs in which those early kings have been separately involved. The rex serves not only as negotiator in the foreign trade but also as military officer in charge of keeping the foreign trade in order. He bears the cost of doing these regular tasks, denoted by $G_0(M)$. It is assumed that $G_0' > 0$ and $G_0'' > 0$. He finances the cost to do those tasks by imposing customs duty on imported goods, represented by iron. The customs duty is denoted by β per unit of iron.

On the other hand, those h early kings make a political coalition comprised of $h \times s_0$ members excluding those h early kings under the assumption that each member state has the same member-size. When they join in the enlarged political union, each early king incurs the cost of governing this enlarged union in proportion to the member size of the early state he has been governing. It is denoted by $G_i(s_0) \equiv G(s_0)$, for $i = 1, 2,..., h$. He has to pay the customs duty in addition to the price of iron, in proportion to the share of iron they obtain, $1/h$.

Then, the payoff function of the new rex, denoted by $\Pi_0 \equiv \Pi_0(\beta, M: \theta, hs_0)$ and that of the early king i, denoted by $\Pi_i(\beta, M: \theta, sh_0)$, $i \in \{1, 2,..., h\}$, are defined by (21) and (22), in turn.

$$(21) \quad \Pi_0(\beta, M : \theta, hs_0) = \beta \frac{V_0}{\lambda(hs_0 : \theta)} - G_0(M).$$

$$(22) \quad \Pi_i(\beta, M : \theta, hs_0)$$

$$= s_0 \alpha f(M/h) - (1+\beta) \frac{V_0}{h\lambda(hs_0 : \theta)} - G(s_o),$$
$$i = 1, 2,..., h.$$

Note that the total value of imported iron is $PM = V_0 / \lambda(hs_0 : \theta)$ and that those h early kings share the imported iron, M, on an equal basis.

Under the prototypical new rex system, the early king-turned aristocrates maximize their payoffs subject to the participant constraint of the new rex. The process of the optimal decision made by those early kings is formalized by the Lagrangian (23) and its constraints (23)'.

$$(23) \quad \varphi(\beta, M : \theta, hs_0) = \sum_{i=1}^{h} [\alpha s_0 f(M/h) - G(s_0)$$
$$- \frac{1}{h}(1+\beta) \frac{V_0}{\lambda(hs_0 : \theta)} \ ]$$
$$+ \mu[\frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M)].$$

$$(23)' \quad \mu \geq 0, \Pi_0(\beta, M : \theta, hs_0) \geq 0,$$
$$0 < \beta < 1, \ and \ 0 < M.$$

Firstly, the necessary condition for β is shown by (24).

$$(24) \quad \frac{\partial \varphi}{\partial \beta} = -\sum_{i=1}^{h} [\frac{V_0}{h\lambda(hs_0 : \theta)}] + \mu \frac{1}{h} \frac{V_0}{\lambda(hs_0 : \theta)} = 0.$$

Taken it into consideration that $1/h$ is the share of iron allocating to the arly king i, it is obvious that $\mu = h > 0$.

Secondly, taking the positivity of μ into allowance, the necessary condition for μ is shown by (25).

$$(25) \quad \frac{\partial \varphi}{\partial \mu} = \frac{\beta V_0}{\lambda(hs_0 : \theta)} - G_0(M) = 0.$$

By arranging (25), the optimal value of β, denoted by $\beta(\theta)$, is derived from (26) below.

$$(26) \quad \beta(\theta) = G_0(M) \frac{\lambda(hs_0 : \theta)}{V_0}.$$

Taking it into consideration that $G_0(M)$ is assumed as an increasing function of M and its coefficient in (26) is given, the necessary condition (26) implies that β

has to move in the same direction as *M*. That is, if the early kings want to acquire more iron, they have to pay more to the new rex in the form of an increase in the customs duty.

Thirdly, from the necessary condition for *M* with the given relation, $\mu = h$, Eq.(27) is obtained.

$$(27)\quad G_0{}'(M(\theta)) = \frac{1}{h}\alpha\,s_0 f{}'(M(\theta)/h).$$

In the above, the optimal value of *M*, denoted by $M(\theta)$, is determined so as to equate the marginal cost to the new rex of acquiring $M(\theta)$ to a marginal increase in the annual tributes to each early king which is brought about by a marginal increase in the acquisition of iron. .

Each early king prefers joining in this enlarged political coalition, if the inequality condition (28) below is satisfied.

$$(28)\quad \Pi_i(\beta(\theta), M(\theta) : hs_0, \theta) > \Pi_i(s_0, M_0 : \lambda_0),$$
$$i \in \{1,2,...,h\}.$$

From the arrangement of the inequality (28), the inequality (28)' is derived.

$$(28)'\quad \alpha s_0[f(M(\theta)/h) - f(M_0)] - G(s_0) >$$
$$-[\frac{V_0}{\lambda_0} - \frac{1}{h}\{1+\beta(\theta)\}\frac{V_0}{\lambda(hs_0:\theta)}].$$

The left side of (28)' is a net increase in the tributes to the early king *i* of joining in the enlarged political coalition. On the other hand, the right side is a decrease in the amount of payment for the imported iron. If the winning probability, $\lambda(hs_0:\theta)$, can be made so large as to make the right side of (28)' negative and therefore to satisfy the inequality (28)' by forming the enlarged political union, then, each early king prefers joining in it.

On condition that $G(s_0)$ is not so small, however, $\lambda(hs_0:\theta)$ has to be large enough in order for the relation (28)' to hold, leading to the same conclusion as the first hypothesis to be inferred and deducted from the first of the main propositions of this paper.

As to the influence of the new *rex*, it is inferred

from (27) that the more competent the *rex* is, measured by $G_0{}'(M)$, the higher is the optimal volume of imported iron. This explains why the rex had to be entrusted with those special tasks common to all of the early kings. This conclusion is reinforced, if the technical effects of iron on the agricultural productivity, measured by $f{}'(M(\theta)/h)$, are more increased.

(8.2) The Ancient Monarchy of an Empire Type

If the political processes of building the ancient monarchy of an empire type are ignored, then, in spite of its opposite image the ancient monarchy of an empire type is subsumed under the constitutional monarchy in the sense that a lifelong ruler is entrusted with the exercise of power subjected to the participants' constraints of the state's members. Making use of the notations of the previous subsection, *mutatis mutandis*, the decision-making of the ancient monarch is formalized by the Lagrangian (29) and its constraints (29)'.

$$(29)\quad L(\beta, M : \theta, hs_0) = \beta\frac{V_0}{\lambda(hs_0:\theta)} - G_0(M)$$
$$+ \eta[\alpha s_0 f(M/h) - G(s_0)$$
$$-\frac{1}{h}(1+\beta)\frac{V_0}{\lambda(hs_0:\theta)} - A].$$

$$(29)'\quad \eta \geq 0;\ \pi_i(\beta, M : \theta, s_0) \geq A;\ 0 < \beta;\ 0 < M.$$

In the above formulation, the rex in the previous subsection was replaced with the ancient monarch. He is required to meet the participants' constraints of the kings of the early state, which are denoted by *A*. In return for resigning the kingship they had in the preceding early states, those kings secure themselves of the privileged status of an aristocrat in the ancient monarchy, even if their social status changed to the bureaucrats of the monarchy system.

First of all, from the necessary condition for β, we can derive the result that $\eta = h > 0$.

Next, by inserting that result to the necessary condition for *M*, (30) is obtained.

$$(30)\quad G_0{}'(M) = \alpha s_0 f{}'(M/h).$$

Finally, from the necessary condition for η, (31) and its rewritten version (31)' are derived.

(31) $\quad \alpha s_0 f(M/h) = G(s_0) + \dfrac{1}{h}(1+\beta)\dfrac{V_0}{\lambda(hs_0:\theta)} + A.$

(31)' $\quad \beta = \dfrac{h[\alpha s_0 f(M/h) - G(s_0) - A]}{V_0/\lambda(hs_0:\theta)} - 1.$

From comparing (30) with (27), it follows that the optimal value of M under the monarchy is larger than under the rex system. Therefore, economic welfare in terms of production level is better under the monarchy than under the rex system.

On the other hand, though (31)' and (26) are relevant to the customs duties, it is uncertain whether those duties are heavier in the monarchy or not. This is because the participants' constraints are changed from those of the rex to those of the former early kings.

Since A is the participants' constraints of those h early kings, it may be assumed that

$A \geq s_0 \alpha f(M_0) - V_0/\lambda_0 \equiv \pi_i(s_0, M_0: \lambda_0),$

$i \in \{1, 2,..., h\}.$

In the above, the identity part on the right side is defined by (20), *mutatis mutandis*. The above inequality at least assures the former early kings of the payoff obtained in the preceding early state.

Comparing the above inequality with (31), it is shown that the hypothesis inferred from the analysis of the above model of the ancient monarchy leads to the same conclusion as the first of the main propositions of this paper.

Furthermore, from the comparative analysis of (31) as to A, we can derive the results as follows: $\partial M/\partial A > 0$, and $\partial \beta/\partial A < 0$. The former means that the more restricted the participant's constraints of the former early kings denoted by A are (that is, the larger is A), then the higher the level of economic activity is in terms of M. The latter means that the larger is A, the lower the customs' duty is. By inserting these results

into the payoff function of the ancient monarch, it turns out that the larger A is, the smaller his payoff is. Under the constitutional monarchy subject to participant's constraint, therefore, the payoff of the ancient monarch may fall to the minimum level. The lower level of his payoff promotes political stability.[44]

A tyrant may take over the monarch, when the participants' constraints on the monarch are taken little care of by a ruler gripping the power of a state. Then, the tyrant's optimal decision is defined by (32).

(32) $\quad Max. \, \pi_0(\beta, M:\theta, s(\lambda)) = \beta \dfrac{V_0}{\lambda(s(\lambda):\theta)}$

$\quad - G_0(M), \, s.t., \, 0 < \beta, \, and \, 0 < M.$

From the first necessary condition of (32), it is obvious that $\partial \pi_0/\partial \beta > 0$, and $\partial \pi_0/\partial M < 0$. That is, the signs of those two derivatives demonstrate that the tyrant is intent both on maximizing tax revenues and on minimizing his duties.

9. Application to the Ancient Irrigation Society of Japan

In this section, I check the correspondence of the main propositions of this paper with the process of building the early states from around the second century BC to the second century AD in the Japanese Archipelago, and furthermore, apply them to the political process of forming the ancient monarchy of an empire type of Japan, beginning with the late third century AD and established in the late seventh century AD.

According to the official documents on the foreign relations between the early states in Japan and Chinese counterparts, we can infer that there were already around one hundred social organs of an early-state type in Japan before the second century BC in which Japan had been still in the Bronze Age.[45]

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[44] The mathematical procedure in subsection (2.4) is applicable to the proof of the political stability of the ancient monarchy.

[45] According to "Kansyo Chirishi" (the History of Han), there were already about one hundred "Kuni" each of which had been independently engaging in foreign trade with Chinese counterparts of those days, maybe, the branch offices of one of the seven countries in the War periods called Yan. The "Kuni" means a social organ which can meet the conditions for engaging in foreign trades, such as economic power, capability of maritime transportation, protected commerce center.

When Korean peninsula was put under the control of the Han dynasty and its local offices were set up in the 2nd century BC, the chiefdoms and the early states in the northern part of Kyushu region locating in the far west-south part of the Japanese Archipelagos began foreign trade with this new counterpart. Then, they were integrated into one "political union" the center player of which was called *Nakoku*.[46] This political union can be recognized explicitly as a coalition of early states according to the concept of this paper. Such a political union called the Kingship was observed in other regions called the Kingship of Izumo, the Kingship of Kibi, the Kingship of Yamato, and other two in the center part and eastern part of the Japanese Archipelago. Those political unions show some characteristics of an early state such as rulership and authority according to recent archaeological finding. The former one hundred "Kuni" and those political unions are recognized as the equivalent to the early state (early kingship) and the "polis" in the ancient Greek.

The aim of the unification was to make the better terms of trade in the foreign trade with a more powerful counterpart by integrating the processes of negotiation and by cutting the cost of producing the means of payment. Though paddy-planting had begun around 1000 BC in Japan,[47] iron tools began being utilized for agriculture in the later periods beginning with around 300 BC. The inference that the political union brought back iron resources vie the southern part of Korean peninsula is corroborated by the part of the Gisyo Benshinden in the ancient Chinese document called the "History of the Three Countries" in English. Given the military balance on those days, however, it was hard to increase λ (s: θ) due to the cost of $G(s)$ being too heavy.

After the Han dynasty broke down in the 2nd century AD, in particular, after the local agencies of long standing, called Rakurougun and Taihogun ruled by the pastoral Xianbei chieftains on those days, were withdrawn from Korean peninsula in 313 AD in the end, the network of external trade in the east north Asian region was put under anarchy. Under this new external circumstance, a new larger political coalition was formed by the hegemony of the preceding society in the central region of the Japanese Archipelagos, called the Kingship of Yamato. The new coalition is called the *Yamato* dynasty later.

The primary aim of this coalition was to keep the supply source of iron resources in stable order, or to carry out the diplomatic policy which can assure of procuring iron resources from the southern part of Chinese continent in advantageous terms. The Yamato coalition pursued the diplomatic policy of maintaining an external trade network with the hegemonic ruler, but not yet an emperor, of that southern part called East Jin or Sung. To maintain a large-scaled regular army was a necessary condition for that aim. Furthermore, the armed Yamato coalition had to pursue the diplomatic policy for Korean peninsula, the aim of which is to assist one of three hostile societies in the peninsula, called *Kudara*, in order to cope with other two ones and to protect the supply route of iron resources. It was enough for the Yamato to keep an advantageous external trade in order, because it was too costly to put the main part of the peninsula under direct control. That is, it was enough to increase λ (s: θ) relative to $G(s)$. On the other hand, irrigation technologies could be advanced drastically by making use of more iron resources, and iron tools became major agricultural tools in these periods. Both factors are considered to have contributed not only to an increase in the left hand of (19) but also to an increase in the second term of the right side.[48]

The above new external circumstances stimulated the kings of the early states and the chieftains of the

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[46] According to" Kansyo Chirishi," fifteen commerce centers were built in the south part of the peninsula called Hinban Gun in 108 BC. Since 82BC, those centers were abolished and integrated into the northern center called Rakuro Gun, as Han dynasty was waning. Then, those irrigation communities were required to re-organize themselves in to an enlarged political league, in order to adapt to the new external circumstance where political stability was lost.

[47] According to the archaeological study, rice planting in dry-field has much longer history.

[48] As to those processes, see Hirose (1997), pp.135-138, and pp.151-152, and Okada (2008).

preceding irrigation communities, which had been in geographically and historically an advantageous position, to form a larger political coalition with the aim of having a more favorable position in those foreign trades for iron resources. For this purpose, they transformed the existing societies into a more enlarged social organization with a strengthened regular army. It was called the Yamato dynasty and is considered to be subsumed in the category of the ancient monarchy. From the recent historical study, it is inferred that the ancient monarchy of Japan had been forming since the late 3rd to the early 4th century under the hegemony of the Yamato political coalition.

Since the late 6th century onward, however, the irrigation societies of Japan could gradually self-supply iron resources and were losing interests in the external trades for iron resources. It was for the sake of the defense against the military threat of the Tang dynasty that a centralized dynastic system under the ancient monarchy of an empire type was established in the 7th to 8th century. When the military threat waned away in the 8th century, the regular army comprised of 200-thousand soldiers was gradually disbanded and the centralized political system supporting it was actually being changed into a heterachic society in the late 8th century onward. In the end, the tribute which had continued to be presented by a state ruling the Korean peninsula on those days, called Silla, was rejected in the year 780. This meant the diplomatic declaration that Japan lost interests in diplomacy with the continent.[49]

10. Main Conclusions and Some Implications

In this paper I proved the three main propositions on the state in accordance with the criteria of the Kantian categorical framework for judging the truth and objectivity of the synthetic proposition. Those synthetic propositions of this paper are summarized below:

Firstly, the new circumstances, in which metal tools were innovated and its application to military force could increase the expected net-benefits of resorting to military force in the transaction of then-prevailing foreign trade, drove the chieftains of the preceding kin-based communities to take this opportunity to have much more advantageous position in the foreign trade, i.e., to increase a bargaining power in the transaction of the foreign trade by resorting to the military force strengthened by equipping with metal weapons, under the necessary condition that those chieftains had been steadily throughout motivated by self-interests. Thus, an early state came into existence as a result of the innovation of metal tools under the condition that the driving engine-factors had been the selfish motives of those chieftains. The self-interest motive is the economic version of the innate programs-for-survival of Homo sapiens.

Secondly, therefore, the early state is an "accidental situation" of the society as a substance and should be recognized not as a creature but as the transformation of the preceding kin-based communities into a new social organ which came into being as the effect of adaption to a change in the essential elements characterizing a preceding situation (i.e., the preceding kin-based community) of the society. The crucial change in the elements, so called a "new environment or surrounding," was the innovation of metal tools.

Thirdly, the bargaining power, the increase of which is the direct goal of those chieftains, is a surrogate for the sovereignty and is interdependent on one another's sovereignty. The quantity category such as the relative economic and military power represented by the relative size of society and the quality category such as the sovereignty is represented by the bargaining power.

All of the expected hypotheses to be deducted from the analysis of the main propositions can be drawn together to only one hypothesis as follows: when faced with some new surrounding to be adapted to (i.e., faced with the innovation of metal goods), the chieftain prefers an "early state type" of the society to the existing type of the society called the "preceding kin-based community just prior to it," under the necessary condition that as the driving-engine factor to bring about the transformation into a new type of the

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[49] See Matuki and Udagawa (1999), pp.81-121.

society, he can take this opportunity to satisfy the innately-programmed self-interested motives by increasing his payoff.

This hypothesis was proved deductively by analyzing the basic model subsumed under the categorical frameworks of the transcendental philosophy and then, corroborated by referring to the political processes of building the early states in the ancient periods of Japan. Furthermore, it was shown that the main propositions of this paper are applicable to the aristocracy (the rex ＝ basileus system) and to the ancient empire system.

Based on the main propositions, the conventional theories of the state were critically examined, leading to the conclusion as follows: it is because they are not explicitly based on the categorical frameworks of the transcendental philosophy that those discussions on the state have been in a state of disorder up till now.

In other paper (Ueda, 2011), I already showed the applicability of this paper to the bourgeois democracy, the modern constitutional monarchy and the feudal system. So, this second stage to the last of this paper is considered as a good opportunity to mention about how to address the problems with the modern mass-democracy from the view points of this paper. According to the category of the society as a "substance," the sovereignty is one of the most essential elements of the society as a substance and therefore, any type of the state cannot escape from an endeavor to keep the actual degree of the sovereignty at as a high level as possible. However, one of the key factors to determine the actual level of the sovereignty is the "organic degree" of a state." This is the Hegelian term to express how organically the state is coordinated into one team-like organization. In order to strengthen the sovereignty, the qualified members of the state are required to contribute to strengthening it in any way through some means such as personnel contributions and/or financial ones. According to the categories on the state, those members contributory to the sovereignty should be considered as the "qualified" ones. According to this criterion for the qualification, the modern mass-democracy with

universe suffrage may be a digression from the concept of the state in the sense that the qualified members have not been explicitly defined yet. From the viewpoint of the sovereignty, therefore, the modern mass-democracy may be considered as an abnormal situation of the society itself. That is why it is called the mass-tyrant. As Aristotle, Hegel, Nietzsche, Burke, and Tocqueville said, it is sure to be based on emotional bases such as envy, jealousy, or *ressentiment*. The French Revolution uncovered a Pandora's box having been confining them and Karl Marx gave those *ressentiment*-driven people the economic reasoning why they *should* be driven by those emotions.

Finally, this paper is concluded by suggesting how to apply the approach of this paper to the modern political integration. The process of the United States of America being formed can be interpreted as a venturous effort to keep external trades with Europe in as favorable condition as possible. If applied to the possibility of EU being transformed into a federal state, we should examine whether or not external threats common to the main member-countries becomes so serious that they prefer resigning at least some part of the sovereignty of a state to accepting any terms of trade for necessities vital for their survival (for example, energies). Such a worsening of the terms of trade is backed up by a difference in the military forth. The modern nation-state came into being as a result of the innovation of industrial technologies under the condition that the motives of industrial capitalists for securing markets in as favorable a condition as possible are the human driving-engine factor. It was a transformation into an enlarged social organ adaptable to the new surroundings. In order to promote the enlarged social organ to be stabilized, at first the concept of the nation was created as an "imagined community" (Anderson, 1991). As far as there are some economic backgrounds which drive the main member-countries to stick to their own special interests, for instance, in foreign affairs such as African problems, the traditional system of a nation state is more suitable to dealing with those special interests and on condition that the

surroundings are not drastically changed, it is impossible to integrate EU countries into a state.

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Appendix 1.
The Proof of Proposition 1

The proof follows the mathematical algorism of Demange (2004). As a preliminary arrangement, we begin with several definitions.[50]

Hierarchy: Hierarchy R is defined as $R^r(h) = 0$, for $h = 1, 2,..., n$. It means that the h player has r superiors in a hierarchy at the top of which the player zero is placed. He is in the r rank in the hierarchy.

Team: Given a hierarchy R, a coalition T is defined as a team, if there is a member i of T who is in a position superior to any other member j of T, and furthermore, the interval l $[i, j]$ is included in T. T^i is called the *full team* of i, which is composed of the i player and all his subordinates. D^i is called the *direct team* of i, which is composed of i player and all his direct subordinates.

Blocking Condition: Given a superadditive problem $(A(S); \pi_0, \pi_1,..., \pi_s, \forall S \subseteq N)$, a contract offer $a(N) \in A(N)$ is defined to be *blocked* by a coalitional team $S \subset N$, if and only if there exists $\exists b(S) \in A(S)$ such that $\pi_i(b(S)) > \pi_i(A(N))$, for $\forall i \in S$.

Ω-Stability: Let Ω denote a set of teams of N. Then, a contract offer $a(N) \in A(N)$ is *Ω-Stable*, if $a(N)$ is feasible and not blocked by any team coalition of Ω. .

Guarantee Levels: Given a feasible problem $(A(S); \pi_0, \pi_1,..., \pi_s, \forall S \subseteq N)$, where $A(S)$ is feasible for $\forall S \subseteq N$, the guarantee levels, denoted by g_n, g_{n-1}, g_1, g_0) $\equiv g$, are defined by the mathematical algorism as follows: *At the step 0,* the guarantee level of the player with the maximum rank n, denoted by g_n, it is determined by his reservation utility. That is, $g_n = 0$. *At the step r, $(r = 1, 2,..., n-1)$,* the guarantee level of the player with rank $n - r$, denoted by g_{n-r}, is determined by maximizing his payoff subject to the condition that the payoffs of the players with higher ranks are larger than or at least equal to their guarantee levels. That is, $g_{n-r} = Max_{a \in A(T^{n-r})}[\pi_{n-r}(a), s.t., \pi_k(a) \geq g_k$ for $\forall k \in T^{n-r} \setminus \{n-r\}]$. *At the step n,* the guarantee level of the player with the top superior status, denoted by, g_0 is determined by the following.

$$g_0 = Max_{a \in A(T^0)}[\pi_0(a), s.t., \pi_k(a) \geq g_k, \forall k \in T^0 \setminus \{0\}].$$

Hierarchical Outcome: The contract offer $a(N) \in A(N)$ which brings about the guarantee levels, $g = (g_n, g_{n-1},..., g_1, g_0)$ solved according to the above algorism, is defined as the *hierarchical outcome*.

Based on the above definitions, we can prove both existence and stability of the hierarchical outcome of more complex irrigation systems than the base model. *Theorem:* Given a hierarchy R, teams' set Ω, and a superadditive[51] problem $(A(S); \pi_0, \pi_1,..., \pi_s, \forall S \subseteq N)$, where $A(S)$ is feasible for $\forall S \subseteq N$, then, three propositions (i), (ii), and (iii) hold true as follows: (i) the finite guarantee levels g exist, (ii) the hierarchical outcome is not blocked by any team coalition of Ω, and (iii) Ω is the maximum stable set of teams which satisfies (i) and (ii).

The Proof of (i): By the assumptions on π_j for $\forall j \in N$, defined over the compact sets of $A(S)$ for $\forall S \subseteq N$, the superadditivity and the feasible contract offers, the existence of a finite g is obvious. For example, a finite set, $g = (g_n, g_{n-1},..., g_{n-(s-2)}, g_{n-(s-1)})$, for a team S, is obtained by setting π_i defined by (4) at g_i for $i \in S \setminus \{n-(s-1)\}$, if $\pi_{n-(s-1)}$ is maximized subject to π_i satisfying the participant's constraints for $\forall i \in S \setminus \{n-(s-1)\}$. Concretely speaking, g_n is set at zero, because the participant constraints are zero-normalized by the assumption of the base model. Next, π_{n-1} is set at g_{n-1} under the condition that the member $(n-1)$ is the one with $g_{n-1} = Max \pi_{n-1}$ subject to the constraint, $\pi_n \geq g_n$. To the extent that the super-additive condition is satisfied, the positivity of π_{n-1} is assured. This procedure can be continued until g_0 is set at the maximum of π_0 subject to the participants' constraints of other members.

If the chieftain's role cannot be taken over by any other player than the zero player, then the zero player is always at the top of any hierarchical coalition. It is,

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[50] As to the details of these definitions, see Demange (2004).

[51] The superadditive condition is defined as follows: For example, take $\{A(S); \pi_0^S, \pi_1^S,..., \pi_s^S, \forall S \subseteq N\}$ as a *problem*, where $A(S)$ is feasible. Then, the *problem* is *superadditive*, if and only if for $\forall S_1$ and $\forall S_2, S_2, S_1 \cap S_2 = empty$, there is $\exists a \in A(S_1 \cup S_2)$ such that $\pi_k(a) \geq \pi_k(a_1)$ for $\forall k \in S_1, \forall a_1 \in A(S_1)$, and $\pi_k(a) \geq \pi_k(a_2)$ for $\forall k \in S_2, \forall a_2 \in A(S_2)$.

for example, owing to his having some technological monopoly or authority. Then, the guarantee levels become unique.[52]

The Proof of (ii): If a team coalition $S \in \Omega$ could block the hierarchical outcome bringing forward a contract offer $a(N) \in A(N)$, then there is some other contract offer $b(S) \in A(S)$ such that $\pi_j(b(S)) > \pi_j (a(N))$ for $\forall j \in S$. But this contradicts with the definition of the guarantee levels of those players.

The Proof of (iii): Allow a non-team coalition to block the hierarchical outcome and take $\exists S_0 = \{k, k + 2,..., k + m\} \notin \Omega$. Then, we can constitute the Condorcet triple by selecting $S_1 = \{k, k + 1\}$ and $S_2 = \{k + 1, k + 2,..., k + m\}$ for $\forall k \geq 2$, so that $S_i \cap S_j = nonempty$ for $i, j = 0, 1,$ or $2, i \neq j$, and $S_0 \cap S_1 \cap S_2 = empty$. Thus, Ω is the maximal stable set of teams.   *Q.E.D.*

Appendix 2.
The Proof of Propoition 2

To begin with, the basic assumptions are summarized, *mutatis mutandis*, below.

$$\lambda(s) = \lambda_0, \ if \ s \leq s_0. \tag{A2-1}$$

$$\lambda(s) = \frac{F(s)}{F(s) + \theta} \equiv \lambda(s : \theta), \ if \ s > s_0. \tag{A2-2}$$

$$\lambda_0 \leq \lambda(s : \theta), \ if \ s \geq s_0. \tag{A2-3}$$

$$F(0) > 0, F' > 0, \ and \ F'' < 0. \tag{A2-4}$$

I note that in the dentition of (A2-2), V in (13) is normalized as unity without loss of generality. From the calculus we can derive the signs of the first and second derivatives as follows:

$$\partial\lambda / \partial s > 0, \partial^2\lambda / \partial s^2 < 0;$$
$$\partial\lambda / \partial\theta < 0, \partial\lambda^2 / \partial\theta^2 > 0. \tag{A2-5}$$

The two-stage game proceeds as follows: At the second (last) stage, subject to the condition that α, s and M are fixed at the first stage, the seller fixes the

optimal value of P at P^* with V_0 and λ_0 being given for the case of the existing community. On the other hand, he determines the optimal value of $\phi(s)$ so as to be equal to P^{**} with V_0 and θ given for the case of the early state. With a given set $\{\alpha, s, M\}$, those optimal values were derived as follows:

$$P^* \equiv P^*(\alpha, M, s : \lambda_0) = \frac{V_0}{\lambda_0 M}, \ with \ a \ given \ \lambda_0.$$

$$P^{**} \equiv P^{**}(\alpha, M, s : \theta) = \frac{V_0}{\lambda(s : \theta)M}, \ with \ a \ given \ \theta.$$

At the first stage, the chieftain derives the optimal values $\{\alpha(\lambda_0), s(\lambda_0), M(\lambda_0)\}$, subject to the constraint $P = P^*$ for the case of the preceding community. In the end, those values derive $P^*(\alpha(\lambda_0), s(\lambda_0), M(\lambda_0)) \equiv P^*(\lambda_0)$. On the other hand, he derives the optimal values $\{\alpha(\theta), s(\theta), M(\theta)\}$, subject to the constraint $\phi(s) = P^{**}$, for the case of the early state. Those derive $P^{**}(\alpha(\theta), s(\theta), M(\theta)) \equiv P^{**}(\theta)$. The main results between two sets of the optimal values are summarized by *Claim 2*. In what follows, we prove *Claim 2* by examining the optimality procedures at the first stage.

（Ⅰ）The Optimal Decision for the Early State
Under the condition that $\phi(s) = P^{**}$, the chieftain determines $\{\alpha, s, M\}$ so as to maximize his payoff, $\pi_0 (\alpha, s, M: \theta)$, subject to the participants' constraints of farmers defined by $\pi_s(\alpha, s, M: \theta) \geq 0$, where s means the farmer s with the highest rank in the irrigation system. If inner solutions are focused on, the payoff function of the chieftain, the participant condition of the farmer s, and the condition on $\phi(s)$ are rewritten as (A2-5) to (A2-8).

$$\pi_0(s, M, \alpha : \theta) = s\alpha f(M) - \phi(s)M - G(s). \tag{A2-5}$$

$$\pi_s(s, M, \alpha : \theta)$$
$$= (1 - \alpha)f(M) - \frac{K(M)}{s} - C(s) \geq 0. \tag{A2-6}$$

$$\phi(s) = \frac{V_0}{\lambda(s:\theta)M} = \frac{V_0\{F(s)+\theta\}}{F(s)M}. \qquad (A2\text{-}7)$$

$$s.t., \ 0 < s_0 < s, \ 0 < M, \ 0 < \alpha < 1.$$

The Lagrangian of the above , Γ, is defined by (A2-8).

$$\Gamma(s,M,\alpha:\theta)$$
$$= s\alpha f(M) - \frac{V_0\{F(s)+\theta\}}{F(s)} - G(s)$$
$$+ \mu\{(1-\alpha)f(M) - \frac{K(M)}{s} - C(s)\},$$

The Lagragian multiplier, $\mu \geq 0.$ $\qquad (A2\text{-}8)$

Taking the necessary condition for α into consideration, $(A2\text{-}9)$ is obtained.

$$\mu = s > 0. \qquad (A2\text{-}9)$$

The above result is derived from $\partial\Gamma/\partial\alpha = sf(M) - \mu f(M) = 0$, and from $f(M) > 0$ for $M \geq 0$.

Next, by taking the necessary condition for s subject to the condition that $0 < s$, $(A2\text{-}10)$ is obtained after μ is replaced with s.

$$\frac{\partial\Gamma}{\partial s} = \alpha f(M) + \frac{V_0\theta F'(s)}{F^2(s)} - G'(s)$$
$$+ \frac{K(M)}{s} - sC'(s) = 0 \qquad (A2\text{-}10)$$

By arranging the above equation, the optimal s is determined so as to satisfy $(A2\text{-}10)'$.

$$\alpha f(M) + \frac{V_0\theta F'(s)}{F^2(s)} - G'(s)$$
$$= s[C'(s) - \frac{K(M)}{s^2}] > 0. \qquad (A2\text{-}10)'$$

The left side is an increase in the net benefits to the chieftain of augmenting the member size by one, and the right means an increase in the net cost to the existing s farmers of one farmer being added under the assumption that all those s farmers have the same marginal cost as C'(s).

A necessary condition for μ is shown by $(A2\text{-}11)$, since $\mu > 0$.

$$\frac{\partial\Gamma}{\partial\mu} = (1-\alpha)f(M) - \frac{K(M)}{s} - C(s) = 0. \quad (A2\text{-}11)$$

Taking the total differential of $(A2\text{-}11)$, we obtain $(A2\text{-}12)$.

$$f(M)\frac{d\alpha}{ds} + [C'(s) - \frac{K(M)}{s^2}]$$
$$= [(1-\alpha)f'(M) - \frac{K'(M)}{s^2}]\frac{dM}{ds}. \qquad (A2\text{-}12)$$

Under the assumption of Claim 2, It is obvious from $(A2\text{-}12)$ that $\partial M/\partial s > 0$ and $\partial\alpha/\partial s < 0$. However, if α, s and M are allowed to move all at the same time, it is possible that the relation of $dM/ds > 0$ and that of $d\alpha/ds > 0$ can arise at the same time. This concludes the first part of (ⅰ) of Claim 2.

Incidentally, by taking the partial differentiation of $\partial M/\partial s$, i.e., $\partial M/\partial s = [C'(s) - K(M)/s^2]/[(1-\alpha)f'(M) - K'(M)/s]$ derived from $(A2\text{-}12)$, we can obtain that $\partial^2 M/\partial s^2 > 0$.

On the other hand, $M(\theta)$ is not determined as an inner solution, because under the condition that $K'(M) < 0, f'(M) > 0$ and $\mu > 0$, $(A2\text{-}13)$ has to hold.

$$\frac{\partial\Gamma}{\partial M} = s\alpha f'(M) + \mu\{(1-\alpha)f'(M) - \frac{K'(M)}{s}\} > 0.$$
$$(A2\text{-}13)$$

By the comparative statics of θ, we can derive (ⅱ) of Claim 2, as follows: Firstly, by taking the total differential of $(A2\text{-}10)$, the equation below is obtained.

$$0 = \frac{V_0 F'}{F^2}d\theta + f(M)d\alpha + \{\alpha f'(M)$$
$$+ \frac{K'(M)}{s}\}dM + \{V_0\theta\frac{\partial(F'/F^2)}{\partial s}$$
$$- G''(s) - \frac{K(M)}{s^2} - C'(s) - sC''(s)\}ds$$

It is obvious from the assumptions that the coefficient of $d\theta$ is positive, the one of $d\alpha$ is positive, and the one of ds is negative. Then, the desired results are obtained below.

$$\frac{\partial s(\theta)}{\partial \theta} > 0, \quad \frac{\partial \alpha(\theta)}{\partial \theta} < 0, \ and,$$

$$\frac{\partial M(\theta)}{\partial \theta} = \frac{V_0 F'/F^2}{-\alpha f'(M) - K'(M)/s} \geq (<) \, 0 \, ,$$

$$if \ \alpha f'(M) \leq (>) \frac{-K'(M)}{s} \quad (A2\text{-}14)$$

By taking the differential of ($A2$-11) of both M and s and by arranging the result, we obtain once again the first part of (i) of Claim 2, shown by ($A2$-15) in a different form.

$$\frac{\partial M(\theta)}{\partial s(\theta)} = \frac{C'(s) - K(M)/s^2}{(1-\alpha)f'(M) - K'(M)/s} \geq 0 \, (< 0),$$

$$if \ C'(S) - \frac{K(M)}{s^2} \geq 0 \, (< 0). \qquad (A2\text{-}15)$$

Since the assumption of Claim 2 is that $C'(s) \geq K(M)/s^2$, ($A2$-15) confirms that both $M(\theta)$ and $s(\theta)$ move in the same direction in response to a change in θ. Then, from ($A2$-14), it must hold that $\alpha f'(M) \leq - K'(M)/s$. Therefore, under the assumption of Claim 2, it must hold that $\partial M(\theta)/\partial \theta > 0$. This concludes the proof of (ii) of Claim 2.

Next, the negative effect of a change in s on P^{**} is easily derived as the following calculus shows:

$$\frac{dP^{**}}{ds} = \frac{-V_0[\partial \lambda/\partial s \cdot M + \lambda(s:\theta) \cdot \partial M/\partial s]}{\{\lambda(s:\theta)M\}^2} < 0.$$

Furthermore, if $s > s_0$, then $\lambda(s:\theta) > \lambda_0$. By consideration it into consideration, the following relations are derived.

$$P^*(\lambda_0)M(\lambda_0) = \frac{V_0}{\lambda_0} > \frac{V_0}{\lambda(s:\theta)}$$

$$= P^{**}(\theta)M(\theta), \quad for \ s > s_0.$$

This confirms (iii) of Claim 2.

(II) The Optimal Decision for Preceding Community

At the beginning of the first stage, the zero player (chieftain) replaces P with $P^*(\lambda_0) = V_0/(\lambda_0 M)$ and then, determines a set of the optimal values, $\{\alpha(\theta), s(\theta), M(\theta)\}$, by maximizing his payoff, subject to the participants' constraints of s farmers. The inner solution is derived by maximizing ($B2$-1) subject to both ($B2$-2) and the positive constraints on those variables.

$$\pi_0(s, M, \alpha : \lambda_0) = s\alpha f(M) - \frac{V_0}{\lambda_0}. \qquad (B2\text{-}1)$$

$$\pi_s(s, M, \alpha : \lambda_0) = (1-\alpha)f(M) - \frac{K(M)}{s} \quad (B2\text{-}2)$$
$$- C(s) \geq 0.$$
$$s.t., \ 0 < s < s_0, \ 0 < M, \ 0 < \alpha < 1.$$

The lagrangian, Φ, is defined by ($B2$-3).

$$\Phi(s, M, \alpha : \lambda_0)$$
$$= s\alpha f(M) - \frac{V_0}{\lambda_0} + \eta\{(1-\alpha)f(M)$$
$$- \frac{K(M)}{s} - C(s)\} \qquad (B2\text{-}3)$$

In the above, $\eta, \eta \geqq 0$, is the Lagrange multiplier.

The first necessary condition for α is derived by ($B2$-4) below.

$$\frac{\partial \Phi}{\partial \alpha} = sf(M) - \eta f(M) = 0, \quad f(M) > 0. \quad (B2\text{-}4)$$

From ($B2$-4), $s > 0$ and $f(M) > 0$, it is derived that $\eta = s > 0$.

Next, from the first necessary condition for s, ($B2$-5) is derived.

$$\frac{\partial \Phi}{\partial s} = \alpha f(M) + \eta \left[\frac{K(M)}{s^2} - C'(s)\right] = 0. \quad (B2\text{-}5)$$

Substituting s for η of ($B2$-5), we obtain ($B2$-5)' and, by arranging it, ($B2$-5)''.

$$\alpha f(M) + \frac{K(M)}{s} - sC'(s) = 0. \qquad (B2\text{-}5)'$$

$$C'(s) - \frac{K(M)}{s^2} = s\alpha f(M) > 0. \qquad (B2\text{-}5)''$$

Since the constraint ($B2$-2) is binding because $\eta = s > 0$, the necessary condition for η is shown by ($B2$-6).

$$\frac{\partial \Phi}{\partial \eta} = (1-\alpha)f(M) - \frac{K(M)}{s} - C(s) = 0. \quad (B2\text{-}6)$$

On the other hand, the sign of the first derivative of

M is always positive as shown below.

$$\frac{\partial \Phi}{\partial M} = s\alpha f'(M) + \eta[(1-\alpha)f'(M) - \frac{K'(M)}{s}] > 0.$$

From the total differential of (B2-6), we can derive both (B2-7) and (B2-8).

$$\frac{\partial M}{\partial s} = \frac{C'(s) - K(M)/s^2}{(1-\alpha)f'(M) - K'(M)/s} \geq 0. \qquad (B2\text{-}7)$$

$$\frac{\partial \alpha}{\partial s} = \frac{C'(s) - K(M)/s^2}{-f(M)} \leq 0. \qquad (B2\text{-}8)$$

The signs of the above derivatives are obvious, because from (B2-5)″ the sign of numerator of both (B2-7) and (B2-8) is positive, and because the sign of the denominator of (B2-7) is positive from the assumption $f'(M) > 0$ and $K'(M) < 0$. Since (B2-5)″ implies the assumption of Claim 2, that assumption can be dismissed for the case of the preceding community.

Thus, (i) of Claim 2 is also derived for the case of the preceding community.

Q.E.D.