Self-Reproduction in Two- and Three-Dimensional Reversible Cellular Automata (With Movies)

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Abstract

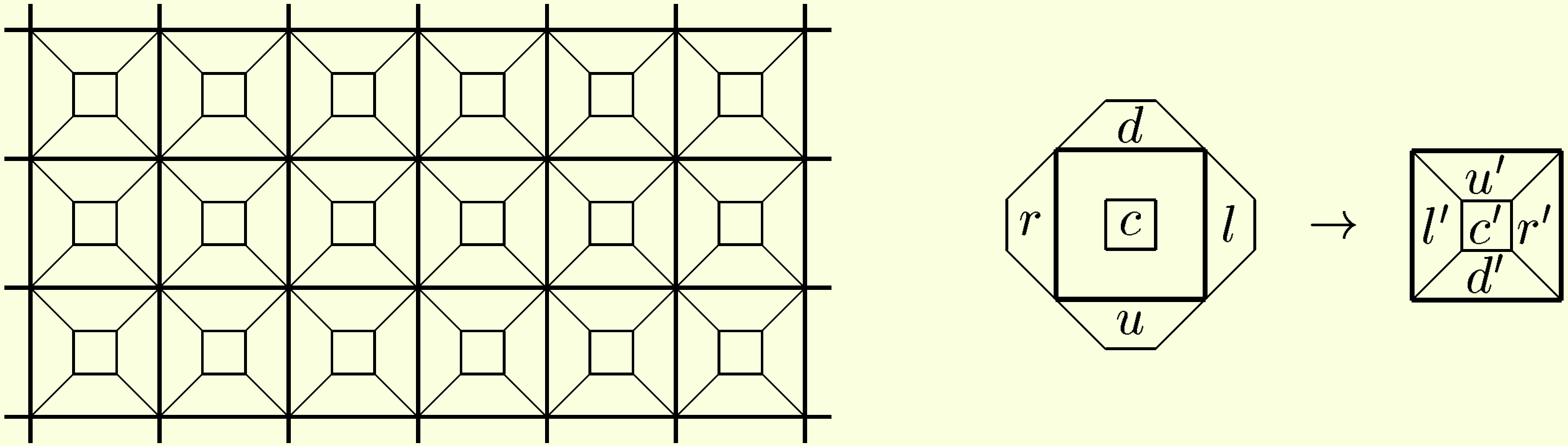
Self-reproduction in cellular automaton (CA) was first studied by von Neumann [Neumann, 1966]. Later, Langton proposed a CA in which simple objects called Langton's loop can self-reproduce [Langton, 1984]. Here, we show that Langton's type self-reproduction is possible even in reversible CAs (RCAs). First, a two-dimensional (2-D) RCA SR_{2D} is given. There, varieties of objects called Worms and Loops of almost any shape can self-reproduce using the shape-encoding method. It is then extended to a 3-D RCA SR_{3D}. In the 3-D case, there are much more varieties of self-reproducing objects, though the basic method is essentially the same. Examples of movies showing self-reproducing processes in SR_{2D} and SR_{3D} are given in this slide.

Reversible Cellular Automaton (RCA)

- It is a CA whose global transition function is one-to-one.
- It is considered as an abstract model for describing spatiotemporal phenomena having physical reversibility.
- It is also studied as a model of reversible computing (see e.g., [Morita, 2008], [Morita, 2011]).
- We use the framework of a partitioned cellular automaton (PCA) to design self-reproducing RCAs.
- A PCA is a subclass of a usual CA.
- It is known that the global transition function of a PCA is one-to-one iff its local transition function is one-to-one.
- Hence, a PCA makes it easy to design an RCA.

Partitioned Cellular Automaton (PCA)

- In a 2-dimensional 5-neighbor PCA, the cellular space, and the form of a local transition function is as below:



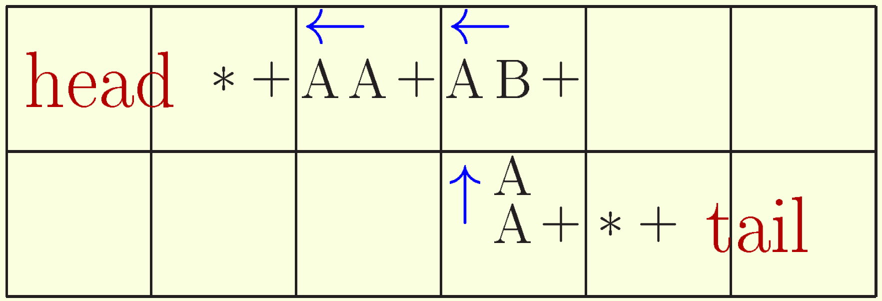
- Each square cell is divided into 5 parts, where each part has its own set of states.
- The local transition function determines the next state (c', u', r', d', l') of each cell by the tuple (c, u, r, d, l) of the present states of the nearest parts of the 5 neighbor cells.
- The local function is applied to all the cells synchronously.

The 2-D Reversible PCA (RPCA) SR_{2D}

- SR_{2D} is a 5 neighbor RPCA, in which various kinds of objects can self-reproduce [Morita, Imai, 1996].
- Each part of a cell has the state set $\{\#, *, +, -, A, B, C, D\}$, and hence the cell has 8^5 states in total.
- The states A, B, C, and D are used for describing a shape of an object, i.e., symbols for a code word. The states $*, +$, and $-$ are for transferring and controlling symbols A, B, C, and D. The state $\#$ is a quiescent (blank) state.
- Since the local transition function is given in [Morita, Imai, 1996], we do not describe it here.
- SR_{2D} employs the shape encoding method, where an object can generate its "genetic code" dynamically.

A Worm in SR_{2D}

- A Worm is a simple object acting as a wire on which signals travel from the tail to the head.



- Signals are interpreted as commands, and they are executed at the head of a Worm to create a shape.
- The shape of a Worm is “encoded” into commands at the tail, and retracts the encoded part. It is called the “shape-encoding” method.

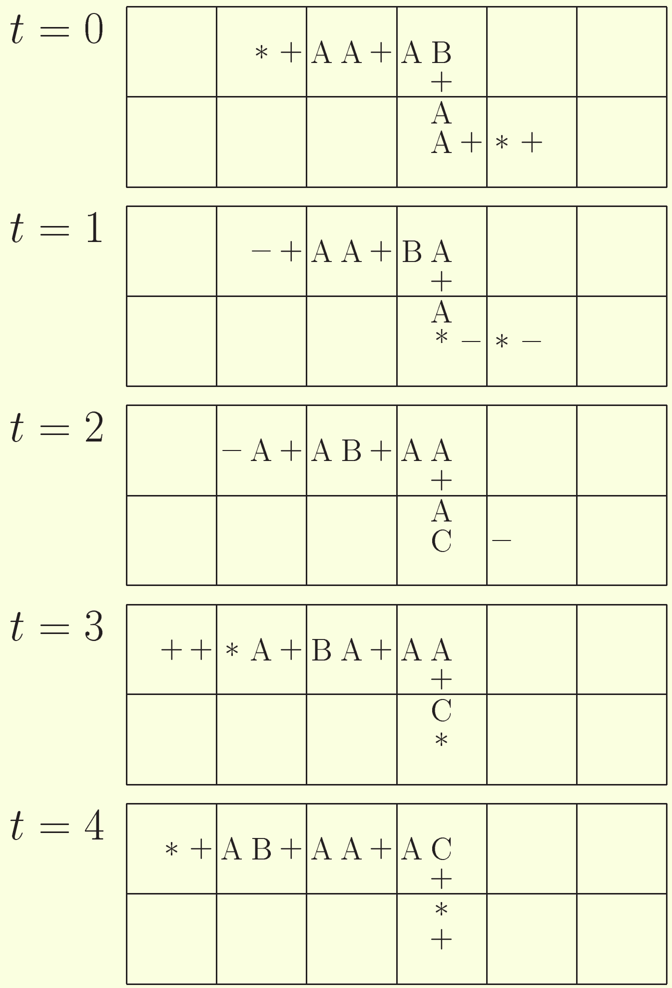
Commands in SR_{2D}

- A command consists of two signals chosen from $\{A, B, C\}$.
- Each command has the meaning as described below.

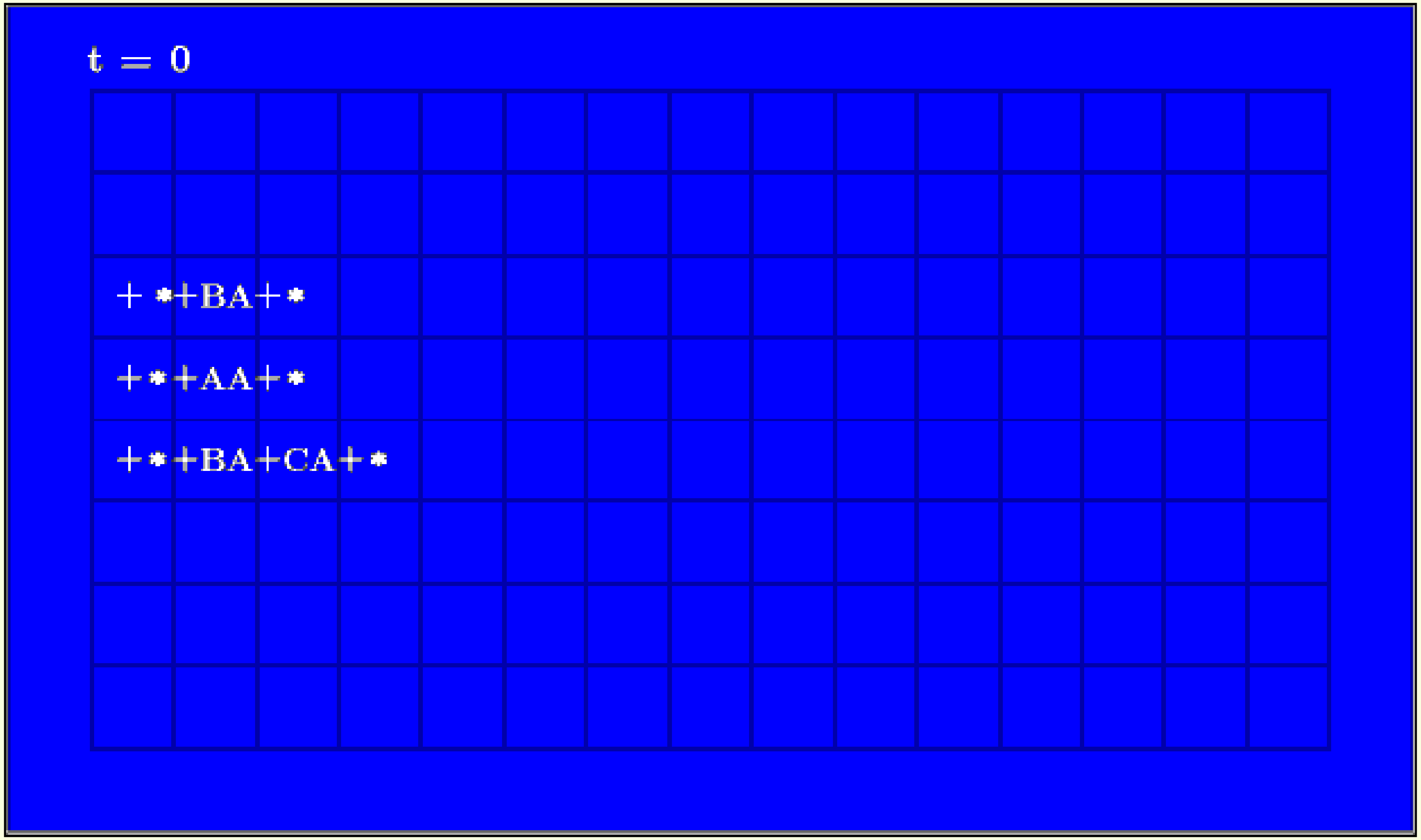
Command		Operation
First signal	Second signal	
A	A	Advance the head straight ahead
A	B	Advance the head turning leftward
A	C	Advance the head turning rightward
B	A	Branch the wire in three ways
B	B	Branch the wire in two ways (making leftward branch)
B	C	Branch the wire in two ways (making rightward branch)

Movement of a 2-D Worm

- A worm crawls in the cellular space of SR_{2D} as below.

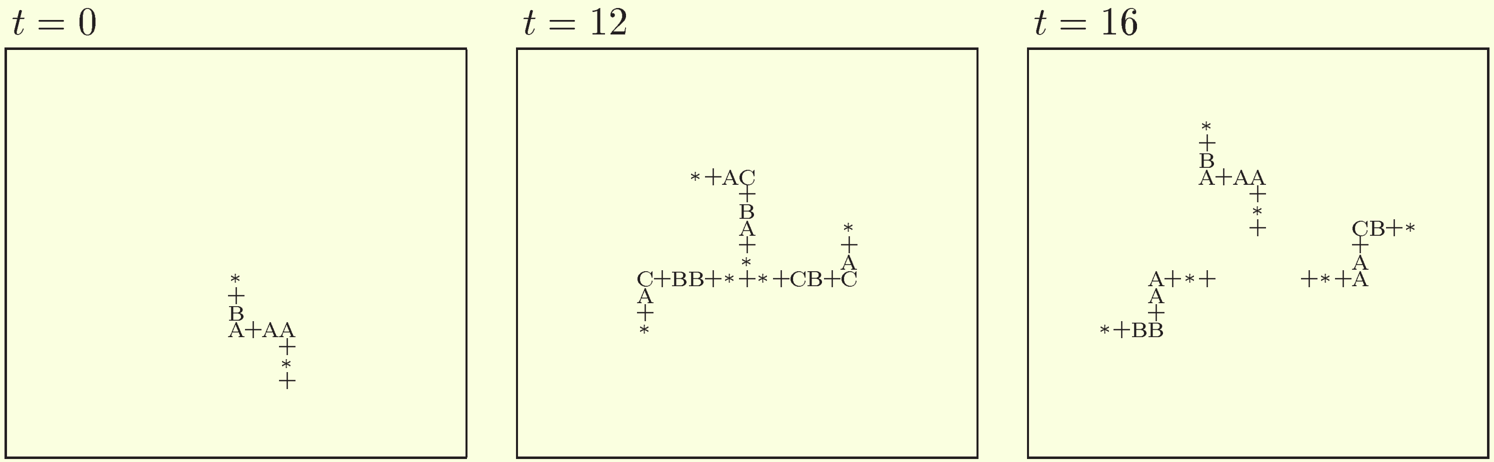


Behaviors of 2-D Worms (Movie)



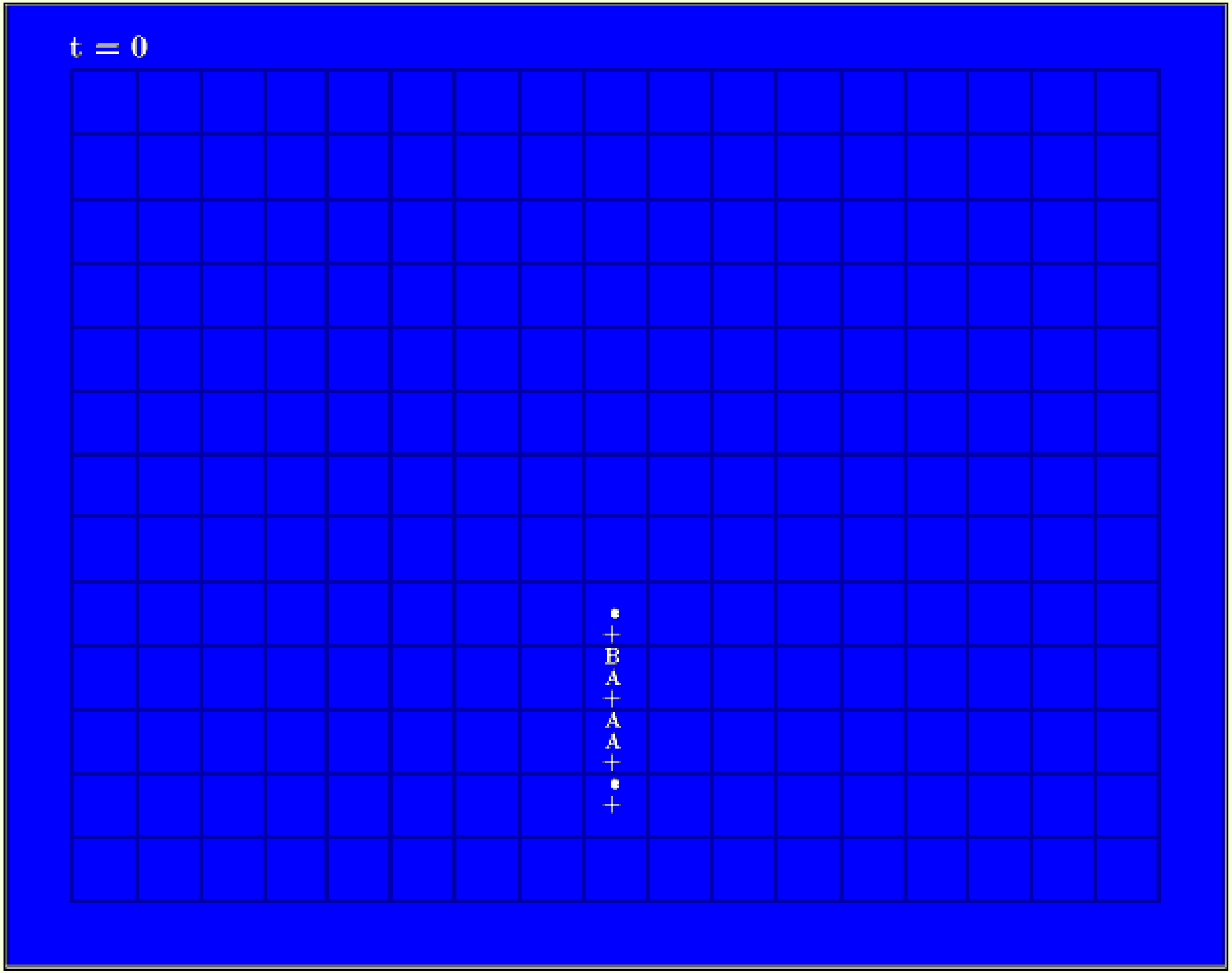
Self-Reproduction Process of a 2-D Worm

- By giving a branch command BA to a Worm, its head branches in three directions.
- Signals are copied at the branching point.
- When the tail reaches the branching point, three branches are split. Thus, three worms of the same shape appear.

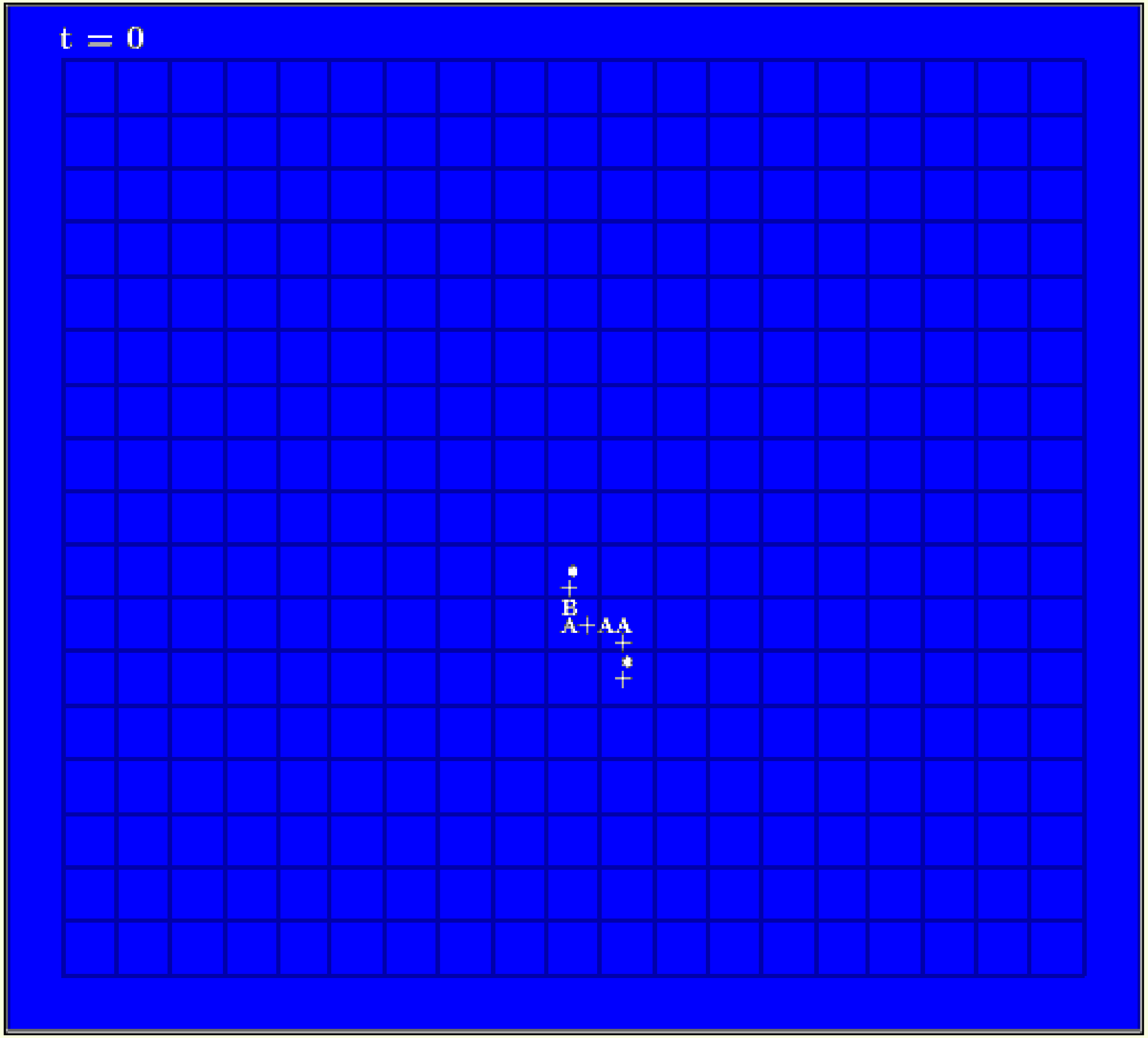


- Since the shape-encoding method is used, a Worm of almost any shape can reproduce itself provided that it does not touch itself in this process.

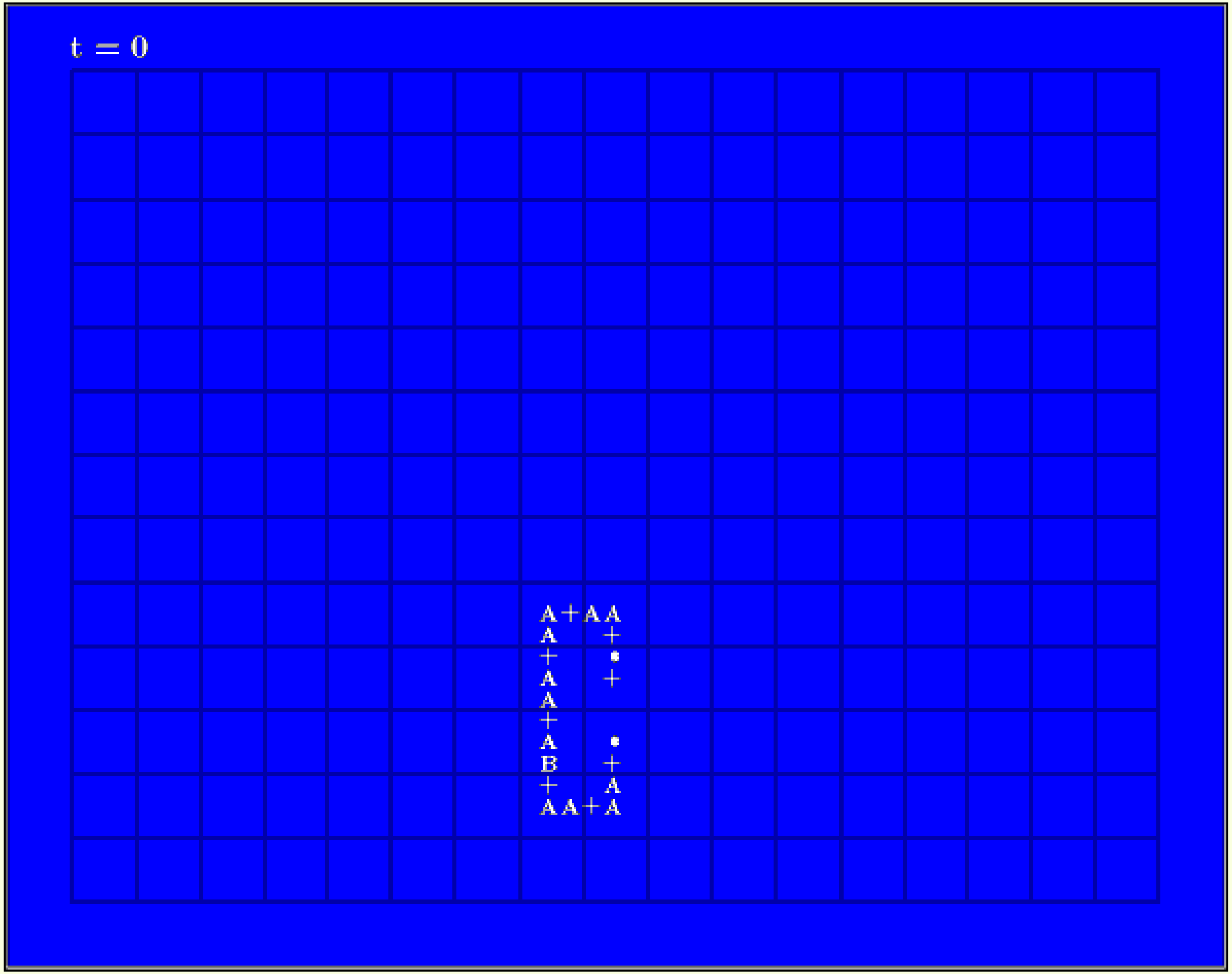
Self-Reproduction Movie of a 2-D Worm (1)



Self-Reproduction Movie of a 2-D Worm (2)

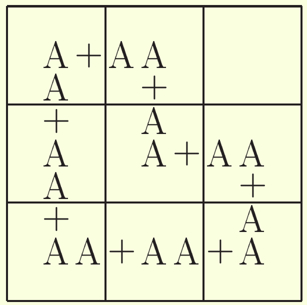


Self-Reproduction Movie of a 2-D Worm (3)



Loop in SR_{2D}

- A Loop is a simple closed wire.



- By putting a special command DB into a Loop, it creates an "arm" to reproduce itself as shown in the next page.
- The arm works like a Worm. Hence, the basic mechanism of self-reproduction is similar to a Worm.

Self-Reproduction Process of a 2-D Loop

$t = 0$

```
A+AA+AA+AA
A        +
+        A
A  AA+A  A
A  +  A  +
+  A  +  A
A  A  AA+A
A  +
+  A
A  A+AA+AA
A        +
+        A
BD+AA+AA+A
```

$t = 50$

```
A+AB+A++AA
A        +
+        A
B  AA+A  A
A  +  A  +
+  A  +  A
B  A  AA+A
A  +
+  A
C  A+AA+AA
A        +
+        A
CA+AA+CA+*A+BA+BA+AA+AA+AA+AA+AA+AA+AA+AA+*
```

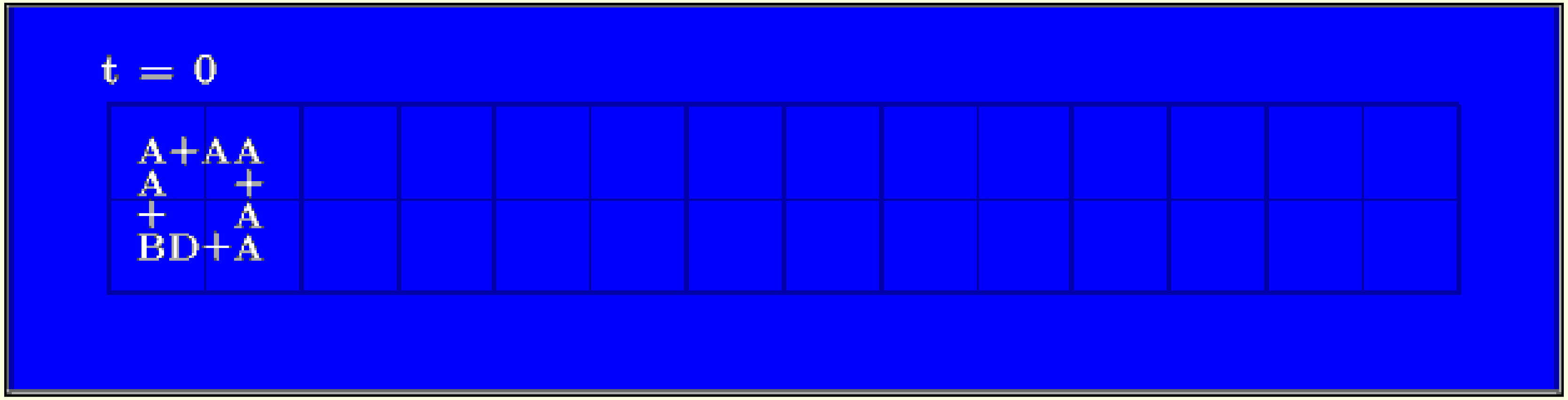
$t = 100$

```
A+AA+AA+AA
A        +
+        A
A  AA+A  A
A  +  A  +
+  A  +  A                                                  –
A  A  AA+A                                                  A
A  +                                                        +
+  A                                                        A
A  A+AA+AA                                                  C+AC+AB
A        +                                                        +
+        A                                                        A
AA+AA+AA+A         – AA+AA+AA+AA+AA+AA+BA+AA+AA+AA+BA+AA+AA+BA+AA+B
```

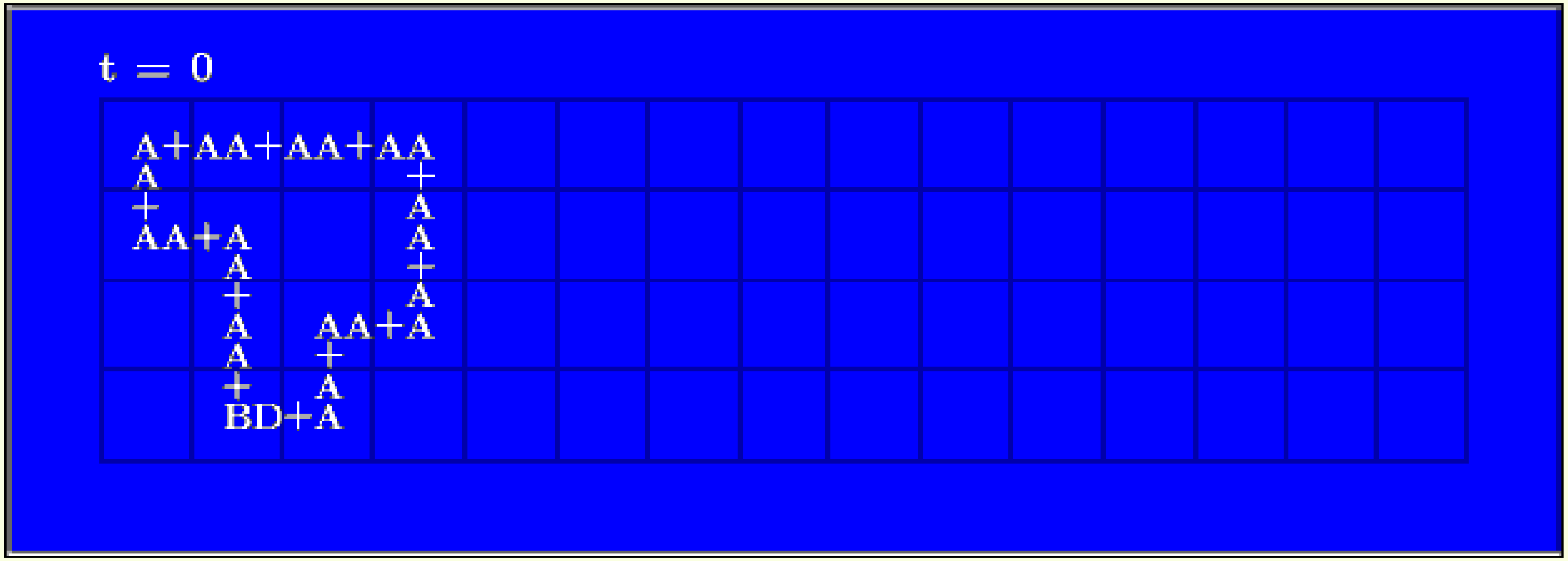
$t = 152$

```
A+AA+AA+AA                                               A+AA+AA+AA
A        +                                               A        +
+        A                                               +        A
A  AA+A  A                                               A  AA+A  A
A  +  A  +                                               A  +  A  +
+  A  +  A                                               +  A  +  A
A  A  AA+A                                               A  A  AA+A
A  +                                                     A  +
+  A                                                     +  A
A  A+AA+AA                                               A  A+AA+AA
A        +                                               A        +
+        A                                               +        A
AA+AA+AA+A                                               BD+AA+AA+A
```

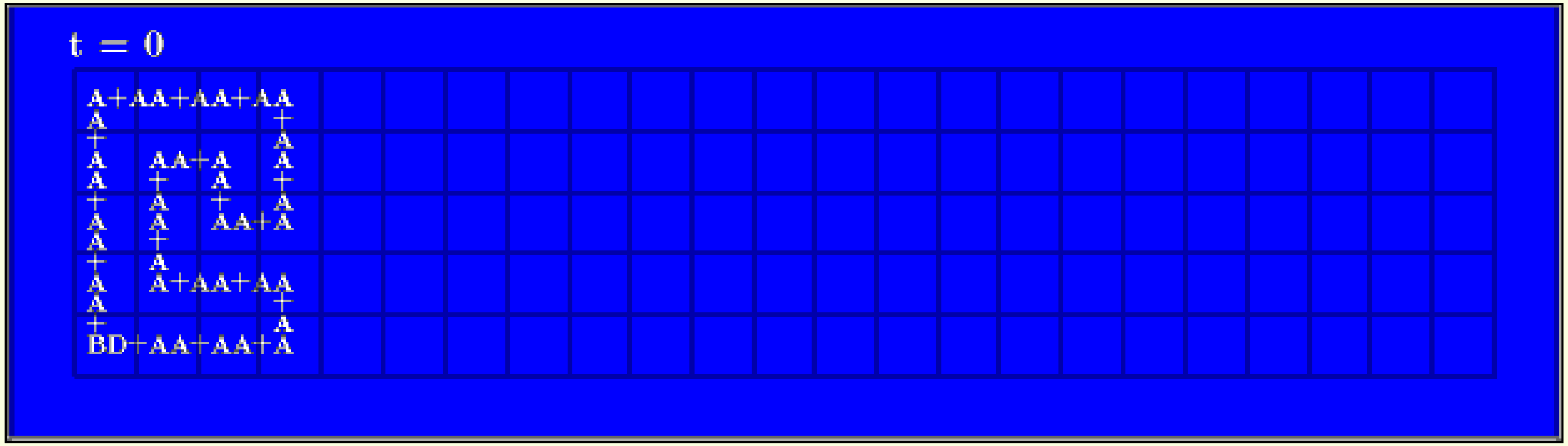
Self-Reproduction Movie of a 2-D Loop (1)



Self-Reproduction Movie of a 2-D Loop (2)



Self-Reproduction Movie of a 2-D Loop (3)

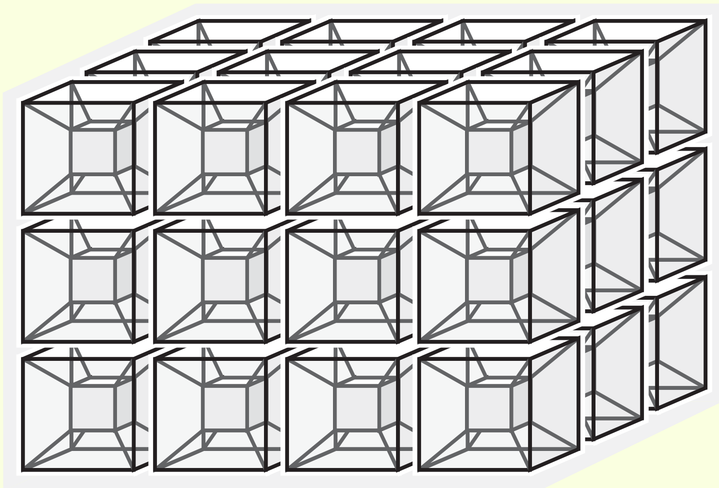


Why Is Self-Reproduction Possible in a "Reversible" CA?

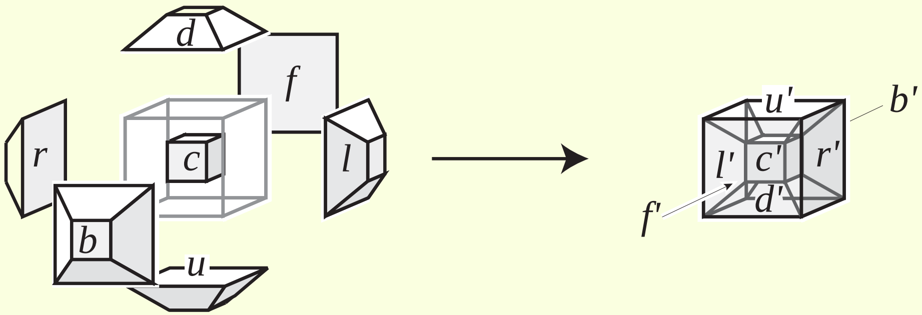
- The following operations are used in SR_{2D} for the self-reproduction of Worms and Loops.
  - Decoding a command to create a shape.
  - Encoding a shape into a command.
  - Copying a command sequence.
- The above operations can be performed in a reversible way, since no information is erased in these processes.

Three-Dimensional (3-D) PCA

- A 3-D PCA is a natural extension of a 2-D PCA.
- The cellular space of a 3-D 7-neighbor PCA is as follows, where each cell has seven parts.

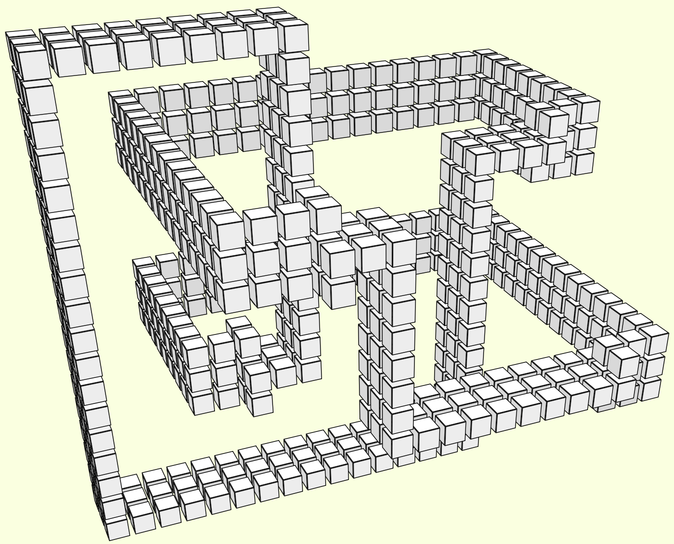


- The form of its local transition function is as below.

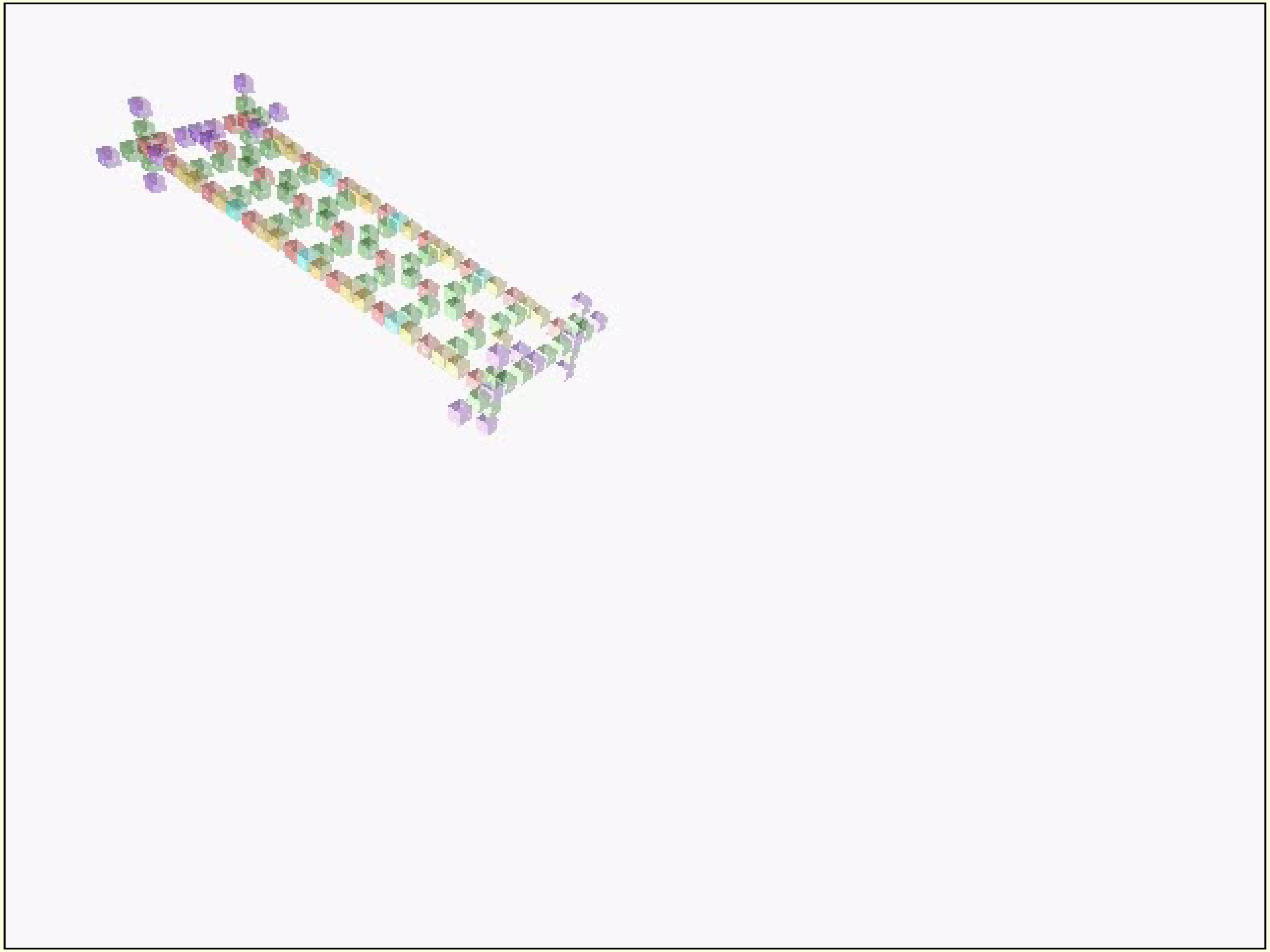


The 3-D RPCA SR_{3D}

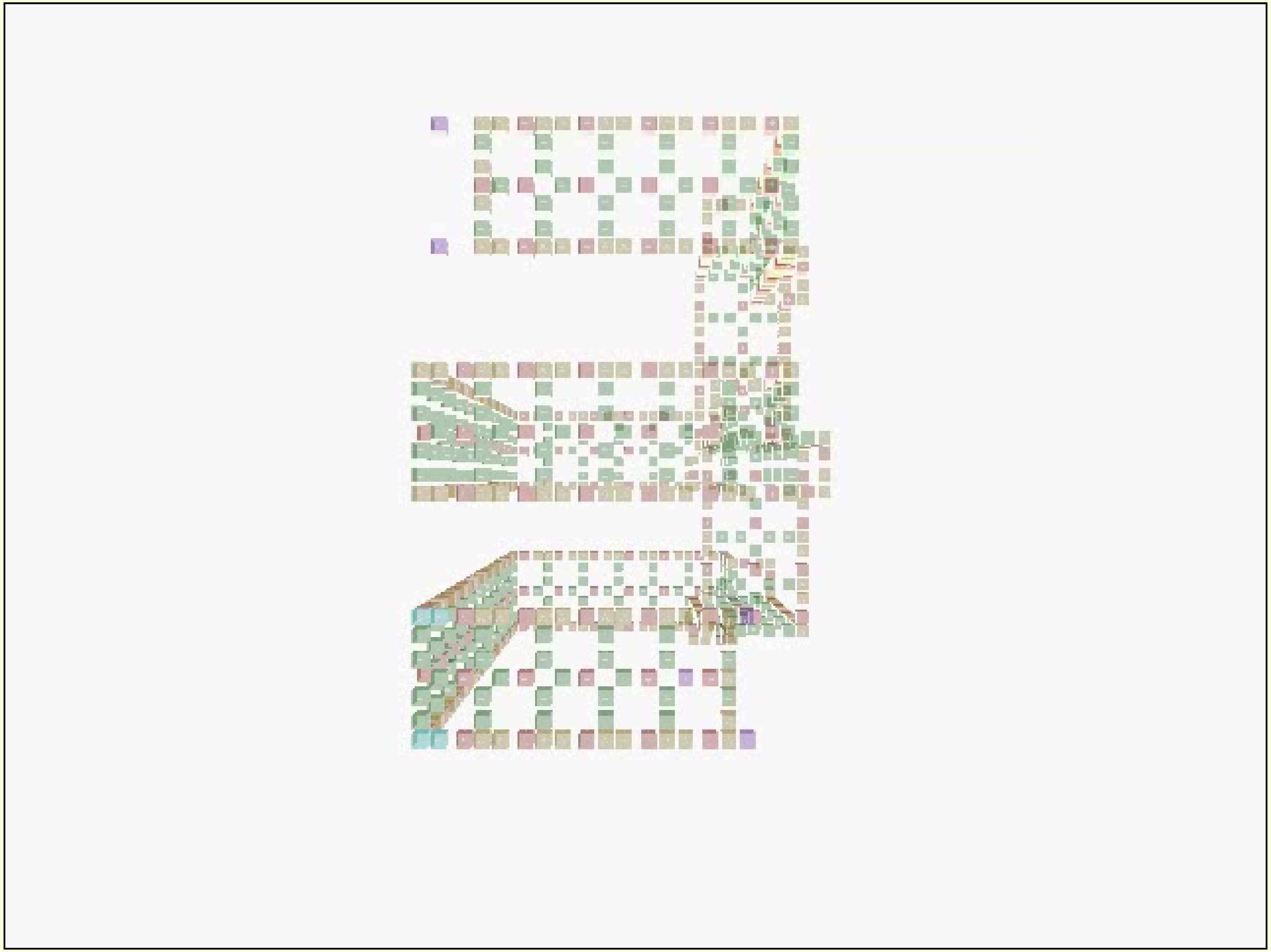
- SR_{3D} is a 7-neigbor 3-D RPCA [Imai, Hori, Morita, 2001].
- Each part of a cell has 9 states. Thus, the cell has 9^7 states in total.
- Basic mechanism is similar to the 2-D RPCA SR_{2D}.
- However, varieties of possible shapes and modes of self-reproduction are much larger.
- Even a Loop having a knot can self-reproduce.



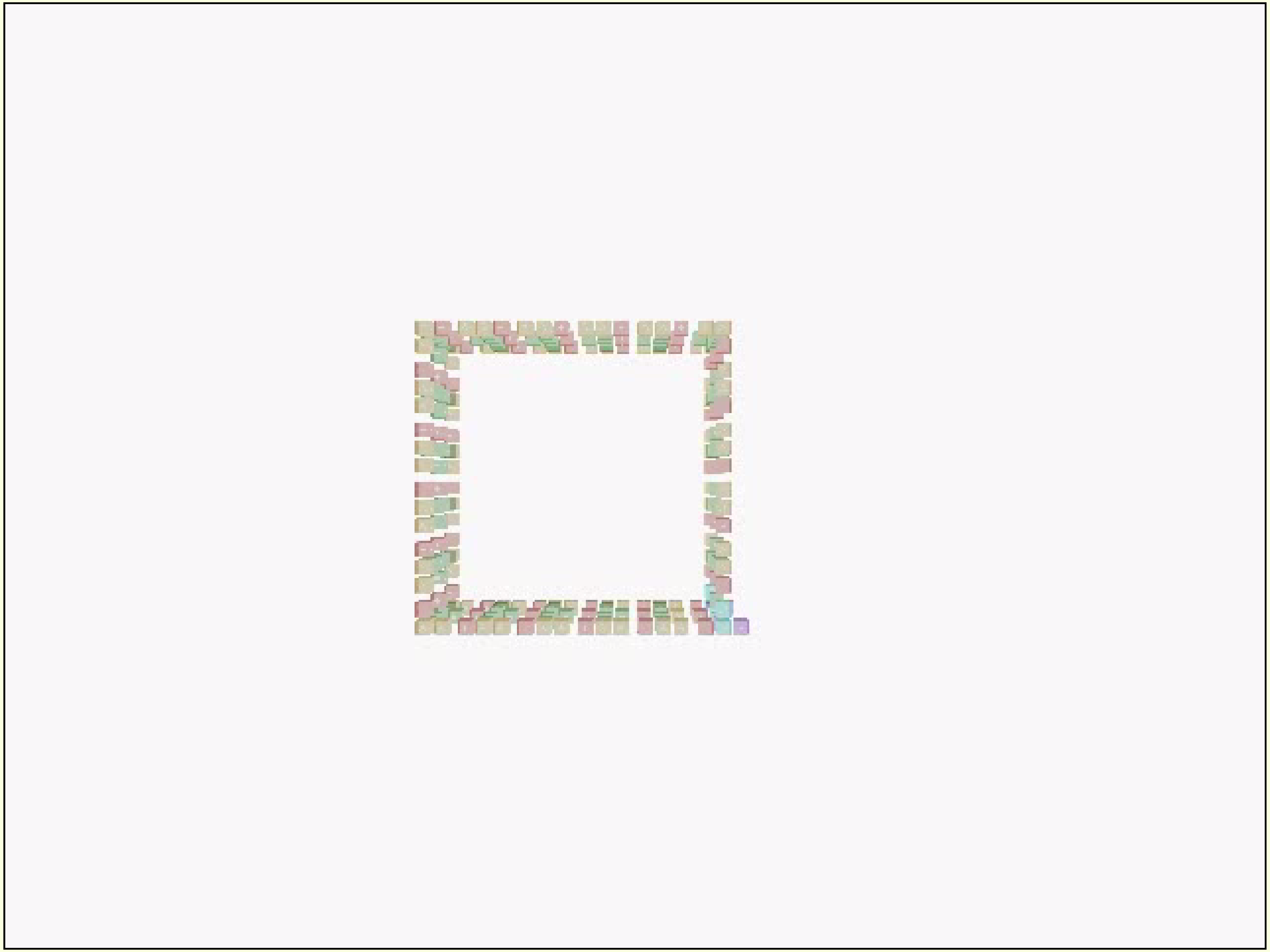
Self-Reproduction Movie of a 3-D Worm (1)



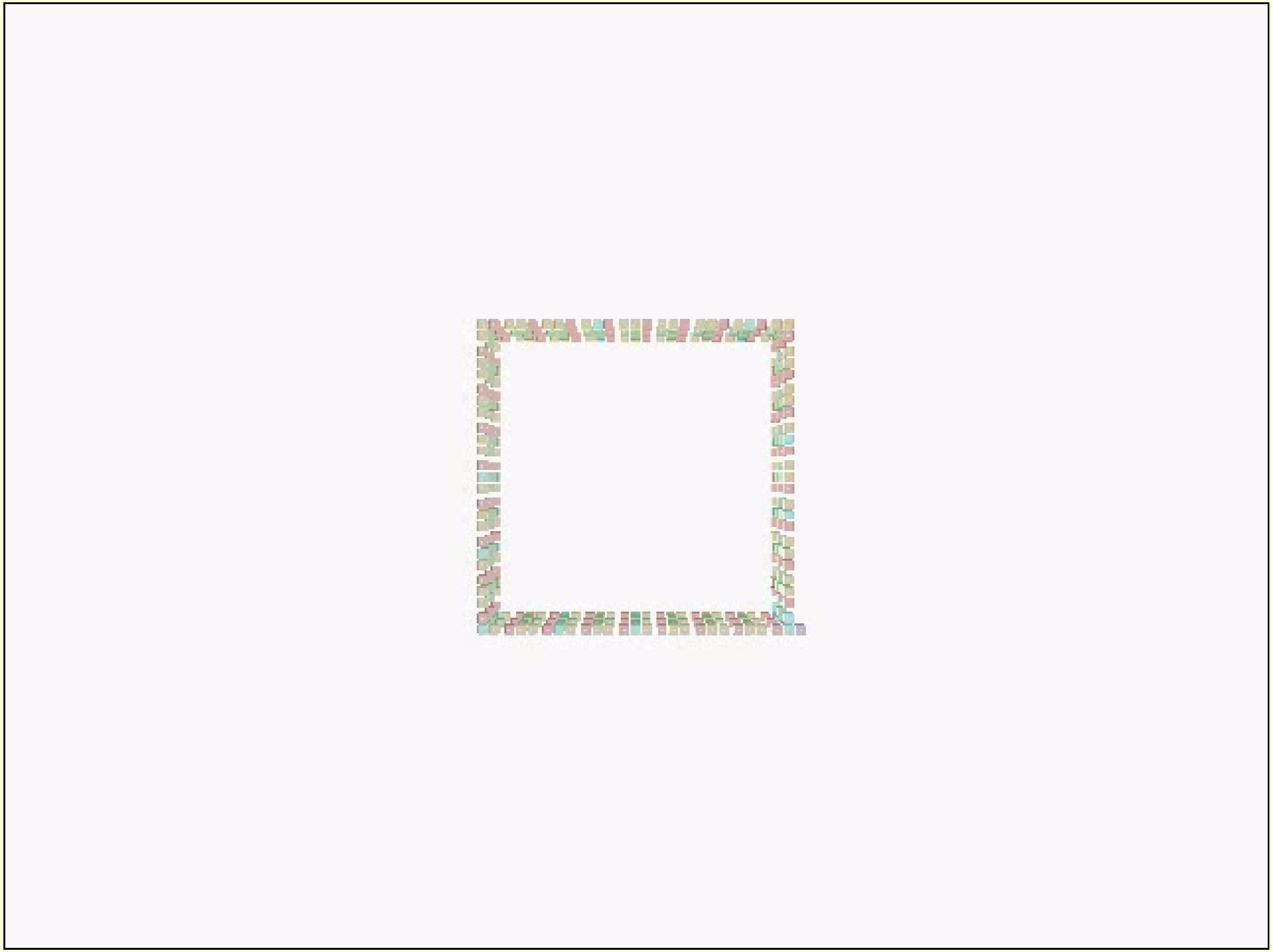
Self-Reproduction Movie of a 3-D Worm (2)



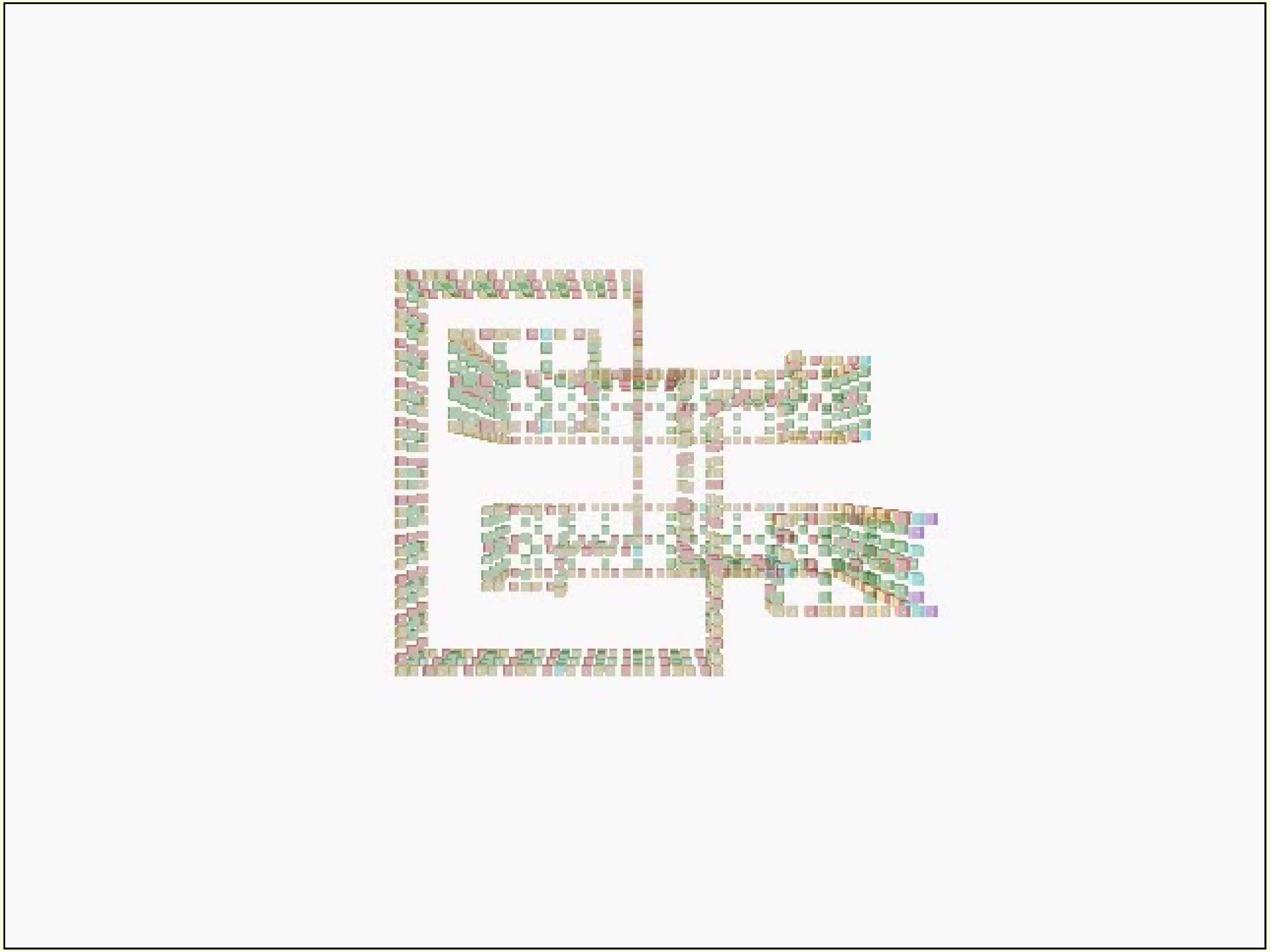
Self-Reproduction Movie of a 3-D Loop (1)



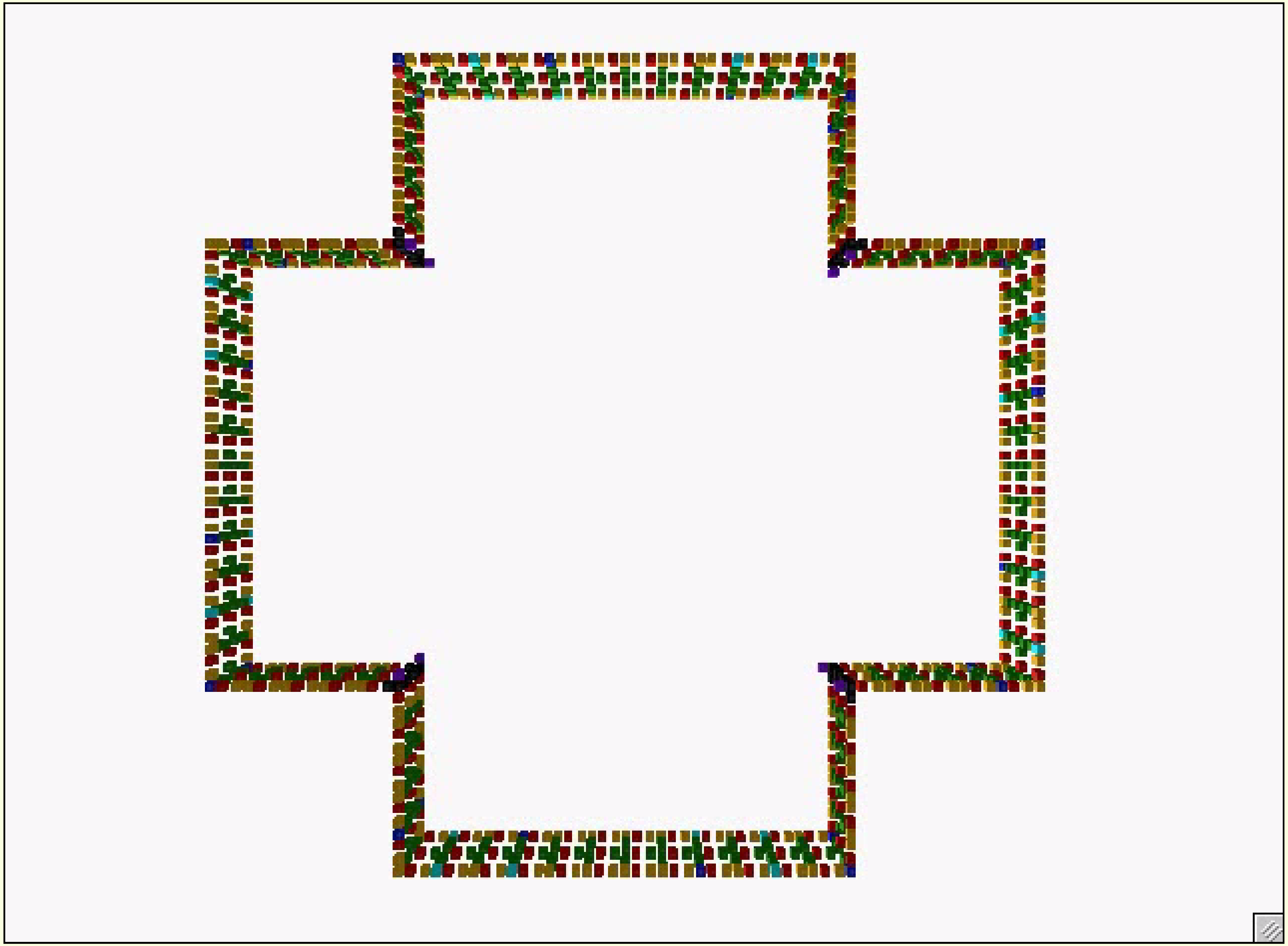
Self-Reproduction Movie of a 3-D Loop (2)



Self-Reproduction Movie of a 3-D Loop (3)



Self-Reproduction Movie of a 3-D Loop (4)



Concluding Remarks

- A 2-D reversible cellular automaton (RCA) SR_{2D}, and a 3-D RCA SR_{3D} are given.
- In these RCAs, Worms and Loops of almost any shape can self-reproduce.
- The reason self-reproduction is possible in reversible CAs is that transformations between shapes and their codes, and copying code sequences are all performed reversibly.

References (1)

[Imai, Hori, Morita, 2001] Imai, K., Hori, T., and Morita, K., Self-reproduction in three-dimensional reversible cellular space, *Artif. Life*, **8**, 155–174 (2002).

[Langton, 1984] Langton, C.G., Self-Reproduction in Cellular Automata, *Physica D*, **10**, 135–144 (1984).

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[Neumann, 1966] von Neumann, J., *Theory of Self-Reproducing Automata* (ed. A.W. Burks), The University of Illinois Press, Urbana (1966).