

# Effects of Large Threshold Corrections in Supersymmetric Type-I Seesaw Model

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## Abstract

We investigate lepton flavor violating (LFV) radiative processes and the relic abundance of neutralino dark matter in supersymmetric type-I seesaw model. We carefully derive threshold corrections to the flavor off-diagonal elements of slepton mass matrix and up-type Higgs mass squared and find that they can be large in the case of large  $B_N^0$ . We examine how the branching ratios of LFV radiative decays and the relic abundance of neutralino dark matter can be significantly affected by the large threshold corrections. Soft scalar mass squared parameter of up-type Higgs scalar is also affected by the threshold corrections. Since the higgsino mass depends on the mass parameter for up-type Higgs, the LFV processes and the relic abundance of the neutralino dark matter are correlated with each other. We show that there are parameter regions where the predictions of the relic abundance of neutralino dark matter are consistent with WMAP observation and the branching ratios of LFV radiative decays are predicted to be testable in future experiments. We find that the masses of scalar supersymmetric particles are not necessarily small so that the branching ratios of LFV decays can be testable in future experiment, which is distinctive feature of this scenario.

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## I. INTRODUCTION

Seesaw mechanism has been invented to explain the smallness of observed neutrino masses relative to those of quarks and charged leptons. Among several varieties of seesaw models, the simplest version is type-I seesaw model which requires the standard model gauge singlet right-handed(RH) Majorana neutrinos and the existence of a huge mass scale which can be near or similar to the scale of grand unification. Supersymmetric(SUSY) version of the type-I seesaw model not only inherits this feature but also stabilizes the electroweak scale without fine-tuning, and provides a natural candidate for a dark matter. The neutrino Dirac type Yukawa couplings in SUSY type-I seesaw model are flavor off-diagonal, which give rise to neutrino mixing observed by neutrino oscillation experiments. Thanks to those Yukawa couplings flavor off-diagonal elements of the slepton mass matrix are induced by radiative corrections even though we take slepton mass matrix to be diagonal at the high energy scale. These flavor off-diagonal slepton masses can enhance branching ratios for lepton flavor violating(LFV) decays such as  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$  and  $\mu \rightarrow e\gamma$  compared to those in non SUSY seesaw models [1–7].

In our previous work, we have observed that threshold corrections to Higgs bilinear terms mediated by RH sneutrino can affect the minimization condition for the Higgs potential and thus the fine-tuning may be reduced when the mass splitting of RH sneutrinos becomes large[8]. We have shown that such a large mass splitting can be originated from large value of the B-term for RH sneutrino,<sup>1</sup>

$$\mathcal{L} = -\frac{1}{2}B_N M_R \tilde{N}^{*2} + h.c..$$

In addition, we have also shown that those threshold corrections can significantly affect the relic abundance of the neutralino dark matter in minimal supergravity scenario(mSUGRA). Thus some of the parameter space excluded by WMAP data in SUSY seesaw model without the threshold corrections can be consistent with WMAP data when we include the threshold

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<sup>1</sup> The large value of  $B_N$ , (i.e.  $B_N \gg m_{soft} \sim B(\sim \text{electroweak scale})$ ) can be obtained from the term of superpotential given by [9],

$$W = \int d^2\theta \lambda X N^2,$$

with  $M_R \equiv \lambda \langle X \rangle$ ,  $B_N \equiv \langle F_X \rangle / \langle X \rangle$ . In the case that  $\langle X \rangle \sim 10^{16}$  GeV and  $\langle F_X \rangle \sim 10^{21}$  GeV<sup>2</sup> which is the same order of SUSY breaking F-term,  $B_N \sim 100$  TeV is obtained.

corrections. However, we have not considered the flavor effects generated from the threshold corrections.

In this paper, we show that the threshold corrections can give rise to sizable contributions to lepton flavor violating phenomena, and extensively discuss how the branching ratios of LFV radiative decays are correlated with the relic abundance of neutralino dark matter. We believe that this observation is new although there are several literatures where the contributions of  $B$ -parameter and threshold corrections to lepton flavor violating phenomena have been studied [10, 11].

This paper is organized as follows. In section II, we derive the threshold corrections to Higgs and slepton mass squared parameters by using RGEs for corresponding parameters. We also calculate the finite terms which are not included in the approach using RGE method. In section III, we investigate how the branching ratios of LFV decays and the relic abundance of neutralino dark matter are affected by the threshold corrections and correlated with each other. In section IV, we devote to the numerical calculation and present our results. The concluding remarks will follow in section V. One-loop RGEs for slepton and Higgs masses including threshold corrections are presented in appendix.

## II. THE THRESHOLD CORRECTIONS

In SUSY type-I seesaw model, flavor off-diagonal elements of  $SU(2)$  slepton mass matrix can arise from radiative corrections mediated by RH neutrinos and sneutrinos even though the slepton mass matrix is taken to be flavor-diagonal at the high energy scale such as the GUT scale. These corrections are evaluated with the help of RGEs given in Ref.[2] under the assumption that soft scalar masses and scalar trilinear couplings are universal at the high energy scale. When the mass splittings of RH sneutrinos are large, threshold corrections arisen from integrating out heavy sectors should be taken into account. As we studied in [8], large mass splitting of RH sneutrinos is originated from large value of  $B$ -terms for RH sneutrinos. It turns out from our numerical estimation that those threshold corrections can dominate over the other radiative corrections to the flavor off-diagonal elements of  $SU(2)$  slepton mass matrix. It is also worthwhile to notice that both radiative corrections and threshold corrections to the flavor off-diagonal elements of slepton mass matrix can significantly contribute to Higgs mass squared parameters[8, 9].

Now, let us derive the threshold corrections to slepton and Higgs masses by integrating out one-loop RGEs in the case that the mass differences among three generations of RH sneutrinos are large. The superpotential in SUSY type-I seesaw model is given as

$$W = Y_{e,ij} H_1 \cdot L_i \bar{E}_j + Y_{\nu,ij} L_i \cdot H_2 \bar{N}_j + \frac{1}{2} M_{R,ij} \bar{N}_i \bar{N}_j - \mu H_1 \cdot H_2, \quad (1)$$

and soft SUSY breaking terms are written as

$$\begin{aligned} \mathcal{L}_{soft} = & -m_{\bar{E},ij}^2 \tilde{E}_i^\dagger \tilde{E}_j - m_{\tilde{L},ij}^2 \tilde{L}_i^\dagger \tilde{L}_j - m_{\tilde{N},ij}^2 \tilde{N}_i^\dagger \tilde{N}_j - m_{H_1}^2 H_1^\dagger H_1 - m_{H_2}^2 H_2^\dagger H_2 \\ & + (m_{H_1 H_2}^2 H_1 \cdot H_2 + h.c) \\ & - (A_{\nu,ij} \tilde{L}_i \cdot H_2 \tilde{N}_j^* + h.c) - (A_{e,ij} H_1 \cdot \tilde{L}_i \tilde{e}_{R,j}^* + h.c) - \frac{1}{2} (B_{ij}^2 \tilde{N}_i^* \tilde{N}_j^* + h.c) \quad ..(2) \end{aligned}$$

Redefining the chiral superfields,  $\bar{N}_i$ , we can take  $M_{R,ij}$  to be diagonal as

$$M_{R,ij} = M_{R,i} \delta_{ij}, \quad (3)$$

where  $M_{R,i}$  are real, positive and assumed to be hierarchical, i.e.  $M_{R,1} \ll M_{R,2} \ll M_{R,3}$ . Then, in the case of  $|B_{ij}^2| \ll |M_{R,i}^2 - M_{R,j}^2|$ , the mass eigenvalues of RH sneutrinos are approximately given as

$$\begin{aligned} M_{h,i}^2 &= m_{\tilde{N},ii}^2 + M_{R,i}^2 + |B_{ii}^2|, \\ M_{l,i}^2 &= m_{\tilde{N},ii}^2 + M_{R,i}^2 - |B_{ii}^2|, \end{aligned} \quad (4)$$

Here, the mass eigenstates,  $\tilde{N}_{h,i}$  and  $\tilde{N}_{l,i}$  are given by

$$\begin{aligned} \tilde{N}_i &= \frac{1}{\sqrt{2}} e^{i\Phi_i/2} (\tilde{N}_{h,i} + i\tilde{N}_{l,i}), \\ \tilde{N}_i^* &= \frac{1}{\sqrt{2}} e^{-i\Phi_i/2} (\tilde{N}_{h,i} - i\tilde{N}_{l,i}), \end{aligned} \quad (5)$$

where

$$\Phi_i = \arg(B_{ii}^2). \quad (6)$$

The hierarchical structure of RH neutrino and sneutrino masses is presented in Fig.1. The flavor off-diagonal part of the mass terms for RH sneutrinos is given as

$$\mathcal{L} = -\frac{1}{2} \sum_{i \neq j} \left( m_{h,ij}^2 \tilde{N}_{h,i} \tilde{N}_{h,j} + m_{l,ij}^2 \tilde{N}_{l,i} \tilde{N}_{l,j} + m_{hl,ij}^2 \tilde{N}_{h,i} \tilde{N}_{l,j} + m_{lh,ij}^2 \tilde{N}_{l,i} \tilde{N}_{h,j} \right), \quad (7)$$

where

$$\begin{aligned}
m_{h,ij}^2 &= \text{Re}(m_{\tilde{N},ij}^2 e^{-i(\Phi_i - \Phi_j)/2}) + \text{Re}(B_{ij}^2 e^{-i(\Phi_i + \Phi_j)/2}), \\
m_{l,ij}^2 &= \text{Re}(m_{\tilde{N},ij}^2 e^{-i(\Phi_i - \Phi_j)/2}) - \text{Re}(B_{ij}^2 e^{-i(\Phi_i + \Phi_j)/2}), \\
m_{hl,ij}^2 &= -\text{Im}(m_{\tilde{N},ij}^2 e^{-i(\Phi_i - \Phi_j)/2}) + \text{Im}(B_{ij}^2 e^{-i(\Phi_i + \Phi_j)/2}), \\
m_{lh,ij}^2 &= \text{Im}(m_{\tilde{N},ij}^2 e^{-i(\Phi_i - \Phi_j)/2}) + \text{Im}(B_{ij}^2 e^{-i(\Phi_i + \Phi_j)/2}), \\
&= m_{hl,ji}^2.
\end{aligned} \tag{8}$$

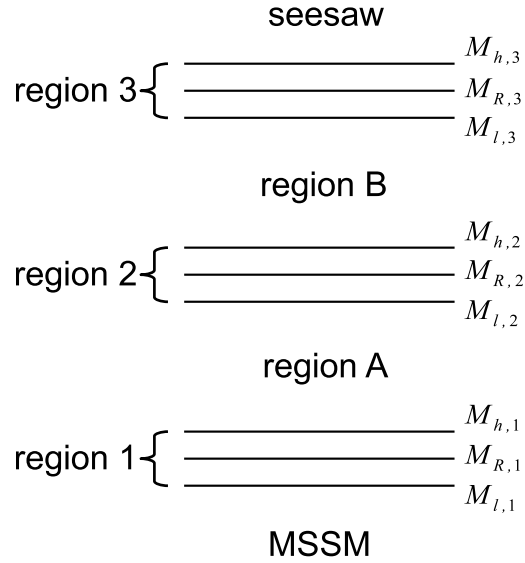


FIG. 1: Mass hierarchies of RH neutrino and sneutrino masses.  $M_{R,i}$  denote RH neutrino masses whereas  $M_{h,i}$  and  $M_{l,i}$  are heavier and lighter sneutrino masses, respectively

Let us calculate radiative corrections and threshold corrections to the slepton and Higgs mass squared parameters. We note that there exist two types of contributions, one is the logarithmic contributions which can be obtained from the RGEs and the other is finite contributions which are calculated in this paper by the diagrammatic approach.

### A. Leading log contributions

The leading log contributions to slepton and Higgs mass squared parameters can be obtained from RGEs. Let us first consider RGEs for slepton masses and up-type Higgs mass without taking into account the mass threshold contributions of RH sneutrinos. The explicit

expressions of the RGEs arisen from only the contributions due to the RH neutrinos and sneutrinos are given by for slepton masses [1, 2]

$$(16\pi^2)Q \frac{dm_{\tilde{L},ij}^2}{dQ} = 2(Y_\nu^* m_N^2 Y_\nu^T)_{ij} + 2(A_\nu^* A_\nu^T)_{ij} + (m_{\tilde{L}}^2 Y_\nu^* Y_\nu^T)_{ij} + (Y_\nu^* Y_\nu^T m_{\tilde{L}}^2)_{ij} + 2m_{H_2}^2 (Y_\nu^* Y_\nu^T)_{ij}, \quad (9)$$

and for up-type Higgs mass[3],

$$(16\pi^2)Q \frac{dm_{H_2}^2}{dQ} = 2\text{Tr}(Y_\nu^* m_N^2 Y_\nu^T) + 2\text{Tr}(Y_\nu^T m_{\tilde{L}}^2 Y_\nu^*) + 2m_{H_2}^2 \text{Tr}(Y_\nu^* Y_\nu^T) + 2\text{Tr}(A_\nu^* A_\nu^T). \quad (10)$$

We assume that soft SUSY breaking parameters are universal at the GUT scale, so that the following relations are valid,

$$m_{\tilde{L},ij}^2 = m_{\tilde{N},ij}^2 = m_0^2 \delta_{ij}, \quad m_{H_2}^2 = m_0^2, \quad A_{\nu,ij} = A_0 Y_{\nu,ij}, \quad B_{ij}^2 = B_N^0 M_{R,i} \delta_{ij}. \quad (11)$$

By integrating the above RGEs (9,10) between  $M_{R,k} \leq Q \leq M_{GUT}$ , we obtain flavor off-diagonal slepton masses and the radiative corrections to up-type Higgs masses, and their approximated expressions are given by

$$\delta m_{\tilde{L},ij}^2 \approx - \sum_k \frac{Y_{\nu,ik}^* Y_{\nu,kj}^T}{8\pi^2} (A_0^2 + 3m_0^2) \ln \frac{M_{GUT}}{M_{R,k}}, \quad (12)$$

and

$$\delta m_{H_2}^2 \approx - \sum_{i,k} \frac{Y_{\nu,ik}^* Y_{\nu,ki}^T}{8\pi^2} (A_0^2 + 3m_0^2) \ln \frac{M_{GUT}}{M_{R,k}}. \quad (13)$$

Here, we have used the fact that when  $Q^2 < M_{R,k}^2$  the chiral superfield  $\tilde{N}_k$  decouples, and thus  $Y_{\nu,ik}$  is set to zero.

Next, let us derive the corrections to the above relations (12,13) originated from RH sneutrino mass thresholds shown in Fig.1. We split a complex scalar  $\tilde{N}_i$  to two real scalars  $\tilde{N}_{h,i}$  and  $\tilde{N}_{l,i}$  as in Eq.(5) and then re-derive RGEs by using the method given in Ref.[12]. The complete results are presented in Appendix A. The terms of the RGEs which contribute

to the derivation of the threshold corrections are written as

$$\begin{aligned}
(16\pi^2)Q \frac{dm_{\tilde{L},ij}^2}{dQ} &\ni 2 \sum_k Y_{\nu,ik}^* Y_{\nu,kj}^T M_{R,k}^2 [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2) - 2\theta(Q^2 - M_{R,k}^2)] \\
&+ \sum_k [Y_{\nu,ik}^* A_{\nu,kj}^T e^{-i\Phi_k} + A_{\nu,ik}^* Y_{\nu,kj}^T e^{i\Phi_k}] M_{R,k} \\
&\times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)] \\
&+ \sum_k \left[ Y_{\nu,ik}^* |B_{kk}^2| Y_{\nu,kj}^T + \sum_{k'=1}^{k-1} (Y_{\nu,ik'}^* B_{k'k}^2 e^{-i\Phi_k} Y_{\nu,kj}^T + Y_{\nu,ik}^* B_{kk'}^{2*} e^{i\Phi_k} Y_{\nu,k'j}^T) \right] \\
&\times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)], \tag{14}
\end{aligned}$$

and

$$\begin{aligned}
(16\pi^2)Q \frac{dm_{H_2}^2}{dQ} &\ni 2 \sum_{i,k} Y_{\nu,ik}^* Y_{\nu,ki}^T M_{R,k}^2 [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2) - 2\theta(Q^2 - M_{R,k}^2)] \\
&+ 2 \sum_{i,k} \text{Re}(Y_{\nu,ik}^* A_{\nu,ki}^T e^{-i\Phi_k}) M_{R,k} [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)] \\
&+ \sum_{i,k} \left[ Y_{\nu,ik}^* |B_{kk}^2| Y_{\nu,ki}^T + 2 \sum_{j=1}^{k-1} \text{Re}(Y_{\nu,ij}^* B_{jk}^2 e^{-i\Phi_k} Y_{\nu,ki}^T) \right] \\
&\times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)]. \tag{15}
\end{aligned}$$

Note that the terms of RGEs given in Eq.(14) and (15) have been derived in the diagonal basis of Majorana mass matrix  $M_R$  by following the method appeared in [13]. Integrating the RGEs for the slepton masses given in Appendix A for the region of energy scale,  $M_{l,k}^2 \leq Q^2 \leq M_{h,k}^2$ , we obtain the threshold corrections as follows:

$$\begin{aligned}
\delta_{th}^k m_{\tilde{L},ij}^2 &\equiv m_{\tilde{L},ij}^2(M_{l,k}^2) - m_{\tilde{L},ij}^2(M_{h,k}^2) \\
&= \delta_{th1}^k m_{\tilde{L},ij}^2 + \delta_{th2}^k m_{\tilde{L},ij}^2, \tag{16}
\end{aligned}$$

where

$$\delta_{th1}^k m_{\tilde{L},ij}^2 = \frac{1}{16\pi^2} \left( 2Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{N},kk}^2 + Y_{\nu,ik}^* A_{\nu,kj}^T \frac{B_{kk}^{2*}}{M_{R,k}} + A_{\nu,ik}^* Y_{\nu,kj}^T \frac{B_{kk}^2}{M_{R,k}} \right), \tag{17}$$

$$\delta_{th2}^k m_{\tilde{L},ij}^2 = \frac{1}{16\pi^2} \sum_{m=1}^{k-1} (B_{mk}^2 B_{kk}^{2*} Y_{\nu,im}^* Y_{\nu,kj}^T + B_{mk}^{2*} B_{kk}^2 Y_{\nu,ik}^* Y_{\nu,mj}^T) \frac{1}{M_{R,k}^2}. \tag{18}$$

The terms which contain both  $A_\nu$  and  $B^2$  in Eq.(17) agree with Ref.[11] whereas disagree with Ref.[10]. The authors of Ref.[11] derived the threshold corrections by using the method

of analytic continuation into superspace[14–16] and the author of Ref.[10] derived the corrections by diagrammatic calculation. In the derivation of the above equations, we have used the following relations,

$$\ln \frac{M_{h,k}^2}{M_{R,k}^2} = \frac{|B_{kk}^2|}{M_{R,k}^2} + \frac{m_{\tilde{N},kk}^2}{M_{R,k}^2} - \frac{1}{2} \frac{|B_{kk}^2|^2}{M_{R,k}^4} + \mathcal{O}(M_{R,k}^{-3}), \quad \ln \frac{M_{h,k}^2}{M_{l,k}^2} = 2 \frac{|B_{kk}^2|}{M_{R,k}^2} + \mathcal{O}(M_{R,k}^{-3}). \quad (19)$$

Imposing universal condition for the soft SUSY breaking parameters at the GUT scale,  $\delta_{th}^k m_{\tilde{L},ij}^2$  is approximately written as

$$\delta_{th}^k m_{\tilde{L},ij}^2 \approx \frac{1}{8\pi^2} Y_{\nu,ik}^* Y_{\nu,kj}^T (m_0^2 + A_0 B_N^0). \quad (20)$$

We note that since  $B_{ij}^2 (i \neq j)$  are radiatively generated at one-loop level,  $\delta_{th2}^k m_{\tilde{L},ij}^2$  are two-loop contributions. To estimate how large it is, we solve RGE for  $B_{ij}^2$  given in Eq.(A5), and then obtain its value around at  $Q = M_{R,3}$ ,

$$B_{mk}^2(M_{R3}) = B_N^0 M_{R,k} \left[ \delta_{mk} - \frac{(Y_\nu^T Y_\nu^*)_{mk} (1 + \delta_{mk})}{8\pi^2} \ln \frac{M_{GUT}}{M_{R,3}} \right] - A_0 M_{R,k} \frac{(Y_\nu^T Y_\nu^*)_{mk} (1 + \delta_{mk})}{4\pi^2} \ln \frac{M_{GUT}}{M_{R,3}}, \quad (21)$$

where  $m \leq k$ . We notice that the term proportional to  $B_N^0$  in the RH side of Eq.(21) can be diagonalized by changing the basis of  $M_R$  into the diagonal basis at  $Q = M_{R,3}$ , but the term proportional to  $A_0$  can not be simultaneously diagonalized as proven in appendix C. Then, the contributions to  $\delta_{th2}^3 m_{\tilde{L},ij}^2$  are approximately given by

$$-\frac{B_N^0}{4\pi^2} \frac{A_0}{16\pi^2} \left[ \sum_{m=1}^2 (Y_{\nu,im}^* (Y_\nu^T Y_\nu^*)_{m3} Y_{\nu,3j}^T + Y_{\nu,i3}^* (Y_\nu^T Y_\nu^*)_{3m} Y_{\nu,mj}^T) \right] \ln \frac{M_{GUT}}{M_{R,3}}, \quad (22)$$

which turns out to be much smaller than  $\delta_{th1}^3 m_{\tilde{L},ij}^2$ , so that we can neglect these contributions.

Similarly, integrating Eq.(15) for the range of the scale,  $M_{l,k}^2 \leq Q^2 \leq M_{h,k}^2$ , we obtain the threshold corrections to the Higgs mass  $m_{H_2}^2$  given by,

$$\delta_{th}^k m_{H_2}^2 \approx \sum_i \frac{1}{8\pi^2} Y_{\nu,ik}^* Y_{\nu,ki}^T (m_0^2 + A_0 B_N^0). \quad (23)$$

## B. Scheme dependent finite terms

There are also finite terms which are renormalization scheme dependent. We obtain them by calculating corresponding Feynman diagrams and estimate how large they are. We used



$\overline{\text{MS}}$  ( $\overline{\text{DR}}$ ) scheme to subtract divergences. It turns out that these terms are small compared to the logarithmic contributions. To derive the scheme dependent terms, we first consider the divergent diagrams which arise from RH neutrino-sneutrino Yukawa interactions. The Feynman diagrams relevant to our purpose are shown in Figs.2,3. If we insert more flavor off-diagonal slepton masses into those diagrams, they can lead to only finite corrections proportional to the mass parameters such as  $B_{ij}^2$ ,  $m_{L,ij}^2$  and  $m_{N,ij}^2$  with  $i \neq j$ , which are at most at two-loop level because the mass parameters are radiatively generated at one-loop level under the assumption that soft masses are universal at the GUT scale.

The one-loop corrections to  $m_{L,ij}^2$  are obtained by calculating the diagrams (a)-(c)(g)(h) in Figs. 2,3. The corrections are composed of two contributions arisen from scalar loops and fermion loops,

$$\Sigma_{\tilde{L},ij} = \Sigma_{\tilde{L},ij}^s + \Sigma_{\tilde{L},ij}^f. \quad (24)$$

The scalar loop contributions are given by

$$\begin{aligned} -i\Sigma_{\tilde{L},ij}^s &= \frac{1}{2} (A_{\nu,ik}^* A_{\nu,kj}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T M_{R,k}^2) [I_2(m_{22}^2, M_{h,k}^2) + I_2(m_{22}^2, M_{l,k}^2)] \\ &+ \frac{1}{2} (Y_{\nu,ik}^* A_{\nu,kj}^T e^{-i\Phi_k} + A_{\nu,ik}^* Y_{\nu,kj}^T e^{i\Phi_k}) M_{R,k} [I_2(m_{22}^2, M_{h,k}^2) - I_2(m_{22}^2, M_{l,k}^2)] \\ &+ \frac{1}{2} Y_{\nu,ik}^* Y_{\nu,kj}^T [2I_1(m_{22}^2) + (1 + |\mu|^2/M_{h,k}^2)I_1(M_{h,k}^2) + (1 + |\mu|^2/M_{l,k}^2)I_1(M_{l,k}^2)] \\ &+ \frac{1}{2} Y_{\nu,ik}^* Y_{\nu,k'j}^T e^{\frac{i}{2}(\Phi_k - \Phi_{k'})} \left[ m_{h,kk'}^2 I_2(M_{h,k}^2, M_{h,k'}^2) + m_{l,kk'}^2 I_2(M_{l,k}^2, M_{l,k'}^2) \right. \\ &\quad \left. - i m_{hl,kk'}^2 I_2(M_{h,k}^2, M_{l,k'}^2) + i m_{hl,k'k}^2 I_2(M_{h,k'}^2, M_{l,k}^2) \right] + i \frac{p^2}{32\pi^2} Y_{\nu,ik}^* Y_{\nu,kj}^T, \end{aligned} \quad (25)$$

where  $m_{22}^2 = |\mu|^2 + m_{H_2}^2$ . The last momentum dependent term of Eq.(25) is obtained by expanding  $-i\Sigma_{\tilde{L},ij}^s$  with respect to external momenta  $\frac{p^2}{M_{R,k}^2}$  and by keeping the term which remains in the limit of large  $M_{R,k}$ . The loop functions  $I_1$  and  $I_2$  are given as,

$$\begin{aligned} I_1(m^2) &= Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2} \\ &= i \frac{m^2}{16\pi^2} \left[ \bar{\epsilon}^{-1} + 1 - \ln \frac{m^2}{Q^2} \right], \end{aligned} \quad (26)$$

$$\begin{aligned} I_2(m_1^2, m_2^2) &= Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_1^2} \frac{1}{k^2 - m_2^2} \\ &= \frac{i}{16\pi^2} \left[ \bar{\epsilon}^{-1} + 1 + \frac{m_1^2}{m_2^2 - m_1^2} \ln \frac{m_1^2}{Q^2} - \frac{m_2^2}{m_2^2 - m_1^2} \ln \frac{m_2^2}{Q^2} \right]. \end{aligned} \quad (27)$$

The fermion loop contributions are calculated with the fermion loop diagrams by keeping external momenta assumed to be small compared to  $M_{R,k}$  so as to derive the contributions

from wave function renormalization. The results are expressed as

$$-i\Sigma_{\tilde{L},ij}^f = -2 \sum_{k=1}^3 Y_{\nu,ik}^* Y_{\nu,kj}^T \left( 1 + \frac{|\mu|^2}{M_{R,k}^2} \right) I_1(M_{R,k}^2) - i\Sigma_{\tilde{L},ij}^p, \quad (28)$$

where  $\Sigma_{\tilde{L},ij}^p$  is momentum dependent part given as

$$-i\Sigma_{\tilde{L},ij}^p = \sum_{k=1}^3 \frac{i}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,kj}^T p^2 \left( \bar{\epsilon}^{-1} + \frac{1}{2} - \ln \frac{M_{R,k}^2}{Q^2} \right) + \mathcal{O}(M_{R,k}^{-1}). \quad (29)$$

The explicit form of  $-i\Sigma_{\tilde{L},ij}$  is shown in Appendix B. Then, the quadratic parts of effective Lagrangian for sleptons  $\tilde{L}_i$  can be written in  $\overline{\text{MS}}$  ( $\overline{\text{DR}}$ ) scheme as follows,

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{k_{\text{max}}} &= z_{\tilde{L},ij}^{(k_{\text{max}})}(Q^2) \partial_\mu \tilde{L}_i^* \partial^\mu \tilde{L}_j \\ &- \left( m_{\tilde{L},ij}^{2(k_{\text{max}})}(Q^2) + \delta m_{\tilde{L},ij}^{2(k_{\text{max}})}(Q^2) + \delta_{SD}^{(k_{\text{max}})} m_{\tilde{L},ij}^2 + \delta_{SI}^{(k_{\text{max}})} m_{\tilde{L},ij}^2 \right) \tilde{L}_i^* \tilde{L}_j, \end{aligned} \quad (30)$$

where the contributions of the loop diagrams mediated by heavy neutrino superfields,  $N_1 \sim N_{k_{\text{max}}}$  are included and the parameters are given by

$$z_{\tilde{L},ij}^{(k_{\text{max}})}(Q^2) = \delta_{ij} - \sum_{k=1}^{k_{\text{max}}} \frac{1}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,kj}^T \left( \ln \frac{M_{R,k}^2}{Q^2} - 1 \right), \quad (31)$$

$$\begin{aligned} \delta m_{\tilde{L},ij}^{2(k_{\text{max}})}(Q^2) &= \frac{1}{16\pi^2} \left[ \sum_{k=1}^{k_{\text{max}}} (A_{\nu,ik}^* A_{\nu,kj}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{H_2}^2 + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{N},kk}^2) \ln \frac{M_{R,k}^2}{Q^2} \right. \\ &\left. + \sum_{k \neq k',1}^{k_{\text{max}}} Y_{\nu,ik}^* Y_{\nu,k'j}^T m_{\tilde{N},kk'}^2 \ln \frac{\max(M_{R,k}^2, M_{R,k'}^2)}{Q^2} \right], \end{aligned} \quad (32)$$

$$\delta_{SD}^{(k_{\text{max}})} m_{\tilde{L},ij}^2 = -\frac{1}{16\pi^2} \sum_{k=1}^{k_{\text{max}}} (A_{\nu,ik}^* A_{\nu,kj}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{H_2}^2 + \sum_{k'=1}^{k_{\text{max}}} Y_{\nu,ik}^* Y_{\nu,k'j}^T m_{\tilde{N},kk'}^2), \quad (33)$$

$$\delta_{SI}^{(k_{\text{max}})} m_{\tilde{L},ij}^2 = \frac{1}{16\pi^2} \sum_{k=1}^{k_{\text{max}}} \left[ 2Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{N},kk}^2 + (Y_{\nu,ik}^* A_{\nu,kj}^T B_{kk}^{2*} + A_{\nu,ik}^* Y_{\nu,kj}^T B_{kk}^2) \frac{1}{M_{R,k}} \right] \quad (34)$$

Note that  $\delta_{SD} m_{\tilde{L},ij}^2$  and  $\delta_{SI} m_{\tilde{L},ij}^2$  are the scheme dependent and independent finite terms, respectively. One can normalize the kinetic term in canonical form by replacing  $\tilde{L}_i$  as,

$$\sqrt{z_{\tilde{L},ij}^{(k_{\text{max}})}} \tilde{L}_j \rightarrow \tilde{L}_n. \quad (35)$$

With the replacement, the effective Lagrangian is written as,

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{k_{\text{max}}} &= \partial_\mu \tilde{L}_i^* \partial^\mu \tilde{L}_i \\ &- \left( \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\text{max}})}}} m_{\tilde{L}}^{2(k_{\text{max}})} \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\text{max}})}}} + \delta m_{\tilde{L}}^{2(k_{\text{max}})} + \delta_{SD}^{(k_{\text{max}})} m_{\tilde{L}}^2 + \delta_{SI}^{(k_{\text{max}})} m_{\tilde{L}}^2 \right)_{ij} \tilde{L}_i^* \tilde{L}_j. \end{aligned} \quad (36)$$

Now let us apply the matching condition at the scale  $Q = M_{R,k_{\max}}$  to two effective theories, one of which contains the superfields,  $N_1 \sim N_{k_{\max}}$ , and the other contains only  $N_1 \sim N_{k_{\max}-1}$ . The effective Lagrangian with the  $k_{\max}$  active superfields should be the same as the one with the  $k_{\max} - 1$  active superfields at the matching scale;

$$\mathcal{L}_{eff}^{(k_{\max})}(Q = M_{R,k_{\max}}) = \mathcal{L}_{eff}^{(k_{\max}-1)}(Q = M_{R,k_{\max}}). \quad (37)$$

Then we can derive threshold corrections for the soft breaking mass for sleptons and for the Higgs particle. Applying the matching condition at  $Q^2 = M_{R,k_{\max}}^2$ , we obtain the relation,

$$\begin{aligned} m_{\tilde{L}}^{2(k_{\max}-1)}(M_{R,k_{\max}}) &= \sqrt{z_{\tilde{L}}^{(k_{\max}-1)}} \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\max})}}} m_{\tilde{L}}^{2(k_{\max})}(M_{R,k_{\max}}) \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\max})}}} \sqrt{z_{\tilde{L}}^{(k_{\max}-1)}} \\ &+ \delta_{SD}^{(k_{\max})} m_{\tilde{L}}^2 - \delta_{SD}^{(k_{\max}-1)} m_{\tilde{L}}^2 \\ &+ \delta_{SI}^{(k_{\max})} m_{\tilde{L}}^2 - \delta_{SI}^{(k_{\max}-1)} m_{\tilde{L}}^2. \end{aligned} \quad (38)$$

Here, we have used the relation,

$$\delta m_{\tilde{L}}^{2(k_{\max})}(M_{R,k_{\max}}) = \delta m_{\tilde{L}}^{2(k_{\max}-1)}(M_{R,k_{\max}}). \quad (39)$$

Inserting the wave function renormalizations given as,

$$\sqrt{z_{\tilde{L}nj}^{(k_{\max})}} = \delta_{nj} - \sum_{k=1}^{k_{\max}} \frac{1}{32\pi^2} Y_{\nu,nk}^* Y_{\nu,kj}^T (\ln \frac{M_{R,k}^2}{Q^2} - 1), \quad (40)$$

$$\left. \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\max})}}} \right|_{in} = \delta_{in} + \sum_{k=1}^{k_{\max}} \frac{1}{32\pi^2} Y_{\nu,ik}^* Y_{\nu,kn}^T (\ln \frac{M_{R,k}^2}{Q^2} - 1), \quad (41)$$

$$\left. \sqrt{z_{\tilde{L}}^{(k_{\max})}} \frac{1}{\sqrt{z_{\tilde{L}}^{(k_{\max}-1)}}} \right|_{ij} = \delta_{ij} + \frac{1}{32\pi^2} Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T (\log \frac{M_{R,k_{\max}}^2}{Q^2} - 1), \quad (42)$$

into Eq.(38), we finally obtain

$$\begin{aligned} m_{\tilde{L},ij}^{2(k_{\max}-1)}(M_{R,k_{\max}}) &= m_{\tilde{L},ij}^{2(k_{\max})}(M_{R,k_{\max}}) - \frac{1}{32\pi^2} \left[ Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}n}^T m_{\tilde{L},nj}^{2(k_{\max})}(M_{R,k_{\max}}) \right. \\ &+ \left. m_{\tilde{L},in}^{2(k_{\max})}(M_{R,k_{\max}}) Y_{\nu,nk_{\max}}^* Y_{\nu,k_{\max}j}^T \right] - \frac{1}{16\pi^2} \left[ Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T m_{\tilde{N},k_{\max}k_{\max}}^2 \right. \\ &+ \left. A_{\nu,ik_{\max}}^* A_{\nu,k_{\max}j}^T + Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T m_{H_2}^2 \right. \\ &+ \left. Y_{\nu,ik_{\max}}^* \left[ \sum_{k'=1}^{k_{\max}-1} Y_{\nu,k'j}^T m_{\tilde{N},k_{\max}k'}^2 \right] + \left[ \sum_{k=1}^{k_{\max}-1} Y_{\nu,ik}^* m_{\tilde{N},kk_{\max}}^2 \right] Y_{\nu,k_{\max}j}^T \right] \\ &+ \frac{1}{16\pi^2} \left[ 2Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T m_{\tilde{N},k_{\max}k_{\max}}^2 \right. \\ &+ \left. (Y_{\nu,ik_{\max}}^* A_{\nu,k_{\max}j}^T B_{k_{\max}k_{\max}}^{2*} + A_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T B_{k_{\max}k_{\max}}^2) \frac{1}{M_{R,k_{\max}}} \right], \end{aligned} \quad (43)$$

where we use  $B_{ij}^2 = B_0 M_{R,i} \delta_{ij}$  and both scheme dependent and independent threshold corrections are included. If  $B_0$  is larger than the other soft breaking parameters, i.e.,  $B_0 > A_\nu, m_{\tilde{N}}$ , the scheme dependent terms in second bracket in Eq.(43) are smaller than the scheme independent terms in third (final) bracket in Eq.(43). Therefore we can neglect the scheme dependent terms for the numerical analysis. We notice that the scheme independent terms are the same as those obtained from RG analysis given by Eq.(17).

Similarly, one can obtain the threshold corrections for bilinear part of the Higgs field  $H_2$ . Calculating the diagrams (d)(e)(f)(i) in Figs. 2,3, we get one-loop corrections to  $m_{H_2}^2$  which are divided by two contributions as follows. The scalar loop contributions are given by

$$\begin{aligned}
-i\Sigma_{H_2}^s &= \frac{1}{2} (A_{\nu,ik}^* A_{\nu,ki}^T + Y_{\nu,ik}^* Y_{\nu,ki}^T M_{R,k}^2) \left[ I_2(m_{\tilde{L},ii}^2, M_{h,k}^2) + I_2(m_{\tilde{L},ii}^2, M_{l,k}^2) \right] \\
&+ \frac{1}{2} (Y_{\nu,ik}^* A_{\nu,ki}^T e^{-i\Phi_k} + A_{\nu,ik}^* Y_{\nu,ki}^T e^{i\Phi_k}) M_{R,k} \left[ I_2(m_{\tilde{L},ii}^2, M_{h,k}^2) - I_2(m_{\tilde{L},ii}^2, M_{l,k}^2) \right] \\
&+ \frac{1}{2} Y_{\nu,ik}^* Y_{\nu,ki}^T [2I_1(m_{\tilde{L},ii}^2) + I_1(M_{h,k}^2) + I_1(M_{l,k}^2)] \\
&+ \sum_{i \neq j} Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{L},ji}^2 I_2(m_{\tilde{L},ii}^2, m_{\tilde{L},jj}^2) \\
&+ \frac{1}{2} Y_{\nu,ik}^* Y_{\nu,k'i}^T e^{\frac{i}{2}(\Phi_k - \Phi_{k'})} \left[ m_{h,kk'}^2 I_2(M_{h,k}^2, M_{h,k'}^2) + m_{l,kk'}^2 I_2(M_{l,k}^2, M_{l,k'}^2) \right. \\
&\left. + im_{hl,k'k}^2 I_2(M_{h,k'}^2, M_{l,k}^2) - im_{hl,kk'}^2 I_2(M_{h,k}^2, M_{l,k'}^2) \right] + i \frac{p^2}{32\pi^2} Y_{\nu,ik}^* Y_{\nu,ki}^T. \quad (44)
\end{aligned}$$

The fermion loop contributions are given by

$$-i\Sigma_{H_2}^f = -2 \sum_{k=1}^3 Y_{\nu,ik}^* Y_{\nu,ik} I_1(M_{R,k}^2) - i\Sigma_{H_2}^p, \quad (45)$$

where

$$-i\Sigma_{H_2}^p = \sum_{k=1}^3 \frac{i}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,ki}^T p^2 \left( \bar{\epsilon}^{-1} + \frac{1}{2} - \ln \frac{M_{R,k}^2}{Q^2} \right) + \mathcal{O}(M_{R,k}^{-1}). \quad (46)$$

The explicit expression of the total contributions to  $m_{H_2}^2$  is shown in Eq.(B2). Then the effective Lagrangian for Higgs ( $H_2$ ) field is given as,

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{k_{\text{max}}} &= z_{H_2} \partial_\mu H_2^* \partial^\mu H_2 \\
&- \left( m_{H_2}^{2(k_{\text{max}})}(Q^2) + \delta m_{H_2}^{2(k_{\text{max}})}(Q^2) + \delta_{SD}^{(k_{\text{max}})} m_{H_2}^2 + \delta_{SI}^{(k_{\text{max}})} m_{H_2}^2 \right) H_2^\dagger H_2. \quad (47)
\end{aligned}$$

where

$$z_{H_2}(Q^2) = 1 - \sum_{k=1}^{k_{\max}} \frac{1}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,ki}^T (\ln \frac{M_{R,k}^2}{Q^2} - 1), \quad (48)$$

$$\begin{aligned} \delta m_{H_2}^{2(k_{\max})}(Q^2) &= \frac{1}{16\pi^2} \left[ \sum_{k=1}^{k_{\max}} (A_{\nu,ik}^* A_{\nu,ki}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{L},ji}^2 + Y_{\nu,ik}^* Y_{\nu,ki}^T m_{\tilde{N},kk}^2) \ln \frac{M_{R,k}^2}{Q^2} \right. \\ &\quad \left. + \sum_{k \neq k', 1}^{k_{\max}} Y_{\nu,ik}^* Y_{\nu,k'i}^T m_{\tilde{N},kk'}^2 \ln \frac{\max(M_{R,k}^2, M_{R,k'}^2)}{Q^2} \right], \quad (49) \end{aligned}$$

$$\delta_{SD}^{(k_{\max})} m_{H_2}^2 = -\frac{1}{16\pi^2} \sum_{k=1}^{k_{\max}} (A_{\nu,ik}^* A_{\nu,ki}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{L},ji}^2) + \sum_{k'=1}^{k_{\max}} Y_{\nu,ik}^* Y_{\nu,ki}^T m_{\tilde{N},kk'}^2, \quad (50)$$

$$\begin{aligned} \delta_{SI}^{(k_{\max})} m_{H_2}^2 &= \frac{1}{16\pi^2} \sum_{k=1}^{k_{\max}} \left[ 2Y_{\nu,ik}^* Y_{\nu,ki}^T m_{\tilde{N},kk}^2 + (Y_{\nu,ik}^* A_{\nu,ki}^T B_{kk}^{2*} + A_{\nu,ik}^* Y_{\nu,ki}^T B_{kk}^2) \frac{1}{M_{R,k}} \right] \\ &\quad - \sum_{k=1}^{k_{\max}} \left[ \sum_{i \neq j} \frac{1}{16\pi^2} Y_{\nu,kj}^T m_{\tilde{L},ji}^2 Y_{\nu,ik}^* \frac{m_{\tilde{L},jj}^2 \log \frac{M_{R,k}^2}{m_{\tilde{L},jj}^2} - m_{\tilde{L},ii}^2 \log \frac{M_{R,k}^2}{m_{\tilde{L},ii}^2}}{m_{\tilde{L},jj}^2 - m_{\tilde{L},ii}^2} \right]. \quad (51) \end{aligned}$$

Using Eq.(47), one can obtain the soft mass of up-type Higgs including scheme dependent and independent threshold corrections as follows,

$$\begin{aligned} m_{H_2}^{2(k_{\max}-1)}(M_{R,k_{\max}}) &= m_{H_2}^{2(k_{\max})}(M_{R,k_{\max}}) \left[ 1 - \frac{1}{16\pi^2} Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}i}^T \right] \\ &\quad + \delta_{SD}^{(k_{\max})} m_{H_2}^2 - \delta_{SD}^{(k_{\max}-1)} m_{H_2}^2 + \delta_{SI}^{(k_{\max})} m_{H_2}^2 - \delta_{SI}^{(k_{\max}-1)} m_{H_2}^2 \\ &= m_{H_2}^{2(k_{\max})}(M_{R,k_{\max}}) \left[ 1 - \frac{1}{16\pi^2} Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}i}^T \right] \\ &\quad - \frac{1}{16\pi^2} \left[ A_{\nu,ik_{\max}}^* A_{\nu,k_{\max}i}^T + Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}j}^T m_{\tilde{L},ji}^2 \right. \\ &\quad \left. + Y_{\nu,ik_{\max}}^* \sum_{k'=1}^{k_{\max}-1} Y_{\nu,k'i}^T m_{\tilde{N},k_{\max}k'}^2 + \sum_{k=1}^{k_{\max}-1} Y_{\nu,ik}^* m_{\tilde{N},kk_{\max}}^2 Y_{\nu,k_{\max}i}^T \right. \\ &\quad \left. + Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}i}^T m_{\tilde{N},k_{\max}k_{\max}}^2 \right] + \frac{1}{16\pi^2} \left[ 2Y_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}i}^T m_{\tilde{N},k_{\max}k_{\max}}^2 \right. \\ &\quad \left. + (Y_{\nu,ik_{\max}}^* A_{\nu,k_{\max}i}^T B_{k_{\max}k_{\max}}^{2*} + A_{\nu,ik_{\max}}^* Y_{\nu,k_{\max}i}^T B_{k_{\max}k_{\max}}^2) \frac{1}{M_{R,k_{\max}}} \right] \\ &\quad - \sum_{i \neq j} \frac{1}{16\pi^2} Y_{\nu,k_{\max}j}^T m_{\tilde{L},ji}^2 Y_{\nu,ik_{\max}}^* \frac{m_{\tilde{L},jj}^2 \log \frac{M_{R,k_{\max}}^2}{m_{\tilde{L},jj}^2} - m_{\tilde{L},ii}^2 \log \frac{M_{R,k_{\max}}^2}{m_{\tilde{L},ii}^2}}{m_{\tilde{L},jj}^2 - m_{\tilde{L},ii}^2}. \quad (52) \end{aligned}$$

Here, similar to the case of slepton masses, the scheme dependent terms are smaller than the scheme independent terms.

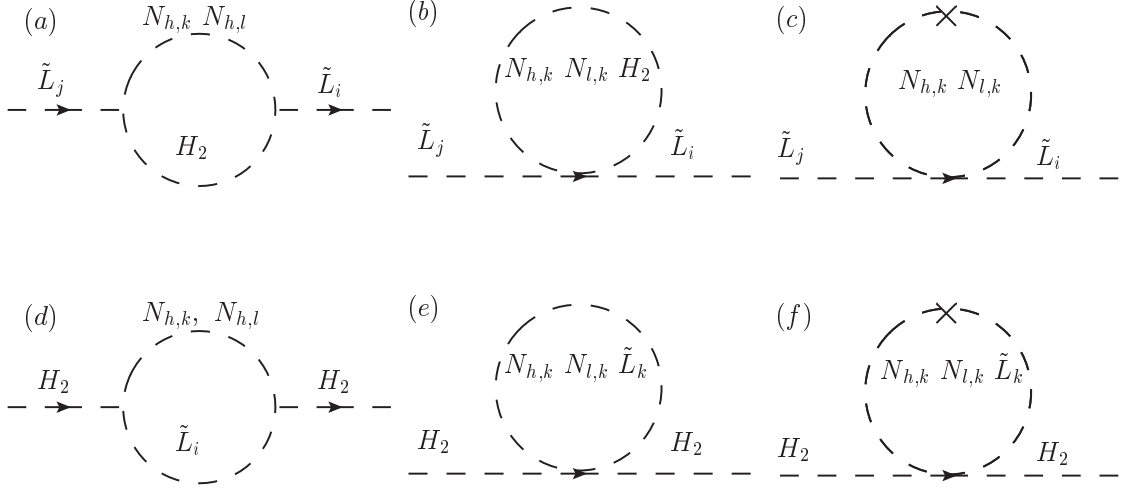


FIG. 2: Contributions from scalar degrees of freedom

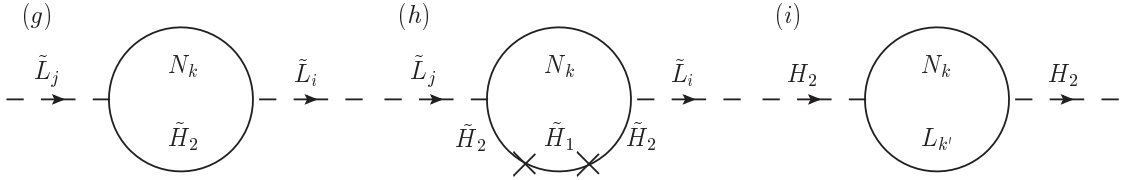


FIG. 3: Contributions from fermionic degrees of freedom

### III. LEPTON FLAVOR VIOLATING DECAYS AND THE RELIC ABUNDANCE OF NEUTRALINO DARK MATTER

As we have seen in the previous section, the threshold corrections to slepton and Higgs masses can be so large that they dominate over the RGE running effects which are approximately expressed in Eq.(12) and (13). In this paper, for our numerical calculation, we consider so called minimal supergravity scenario(mSUGRA) where gaugino masses, soft scalar masses and scalar trilinear couplings are universal at the GUT scale. In particular, we investigate the relic abundance of neutralino dark matter in the focus point region[17–19], which is one of the regions where the relic density is consistent with WMAP observation. In the region, the lightest supersymmetric particle (LSP) is a mixture state of bino and higgsino and the annihilation cross section for LSP is enhanced due to the appropriate portion of the higgsino component. In the seesaw model without threshold corrections, the focus point region is significantly affected by the neutrino Yukawa sector and shift to the energy

scale far from electroweak scale when the right-handed Majorana masses are sufficiently large [20–22]. In SUSY type-I seesaw model based on mSUGRA, the radiative corrections drive the up-type Higgs mass squared to be more negative than that in MSSM if we do not include the threshold corrections. This feature is due to the presence of RH neutrino and sneutrino sectors. In this case, the higgsino mass  $|\mu|$  is larger than that in MSSM, too. This in turn leads to larger relic abundance of neutralino dark matter compared to that in MSSM for fixed values of the soft scalar masses due to the small portion of the higgsino components in the LSP. However, if we include large threshold corrections mentioned above, the up-type Higgs mass squared is driven to be less negative which leads to smaller  $|\mu|$ . Therefore the portion of higgsino in the lightest neutralino state becomes large, and the right amount of relic abundance of neutralino dark matter consistent with WMAP observation can be obtained even in the parameter space of MSSM excluded by WMAP data [8].

Since the threshold corrections mentioned above produce flavor off-diagonal slepton masses, they are new additional source of lepton flavor violating phenomena. Since large threshold corrections lead to large flavor off-diagonal slepton masses, we anticipate that the amplitude of lepton flavor violating processes such as  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$  and  $\mu \rightarrow e\gamma$  can be enhanced due to the new source of lepton flavor violation. Therefore there exists a tension between the branching ratio of LFV decays and the relic abundance of neutralino dark matter.

Higgsino mass parameter  $\mu$  is determined by the minimization condition of the Higgs potential given as,

$$\frac{1}{2}m_Z^2 = -|\mu|^2 + \frac{m_{H_1}^2(m_Z^2) - m_{H_2}^2(m_Z^2) \tan^2 \beta}{\tan^2 \beta - 1}. \quad (53)$$

In the limit of large  $\tan \beta$ , this condition can be written as

$$\frac{1}{2}m_Z^2 \approx -|\mu|^2 - m_{H_2}^2(m_Z^2). \quad (54)$$

The radiative corrections to  $m_{H_2}^2$  are given as,

$$\delta m_{H_2}^2 \approx \sum_k \frac{Y_{\nu,ik}^* Y_{\nu,ki}^T}{8\pi^2} \left[ m_0^2 + A_0 B_N^0 - (A_0^2 + 3m_0^2) \ln \frac{M_{GUT}}{M_{R,k}} \right], \quad (55)$$

where the first two terms of RH side are the threshold corrections and the last term is the RG running effect. We see from Eq.(55) that in the case of the large threshold corrections to  $m_{H_2}^2$ , the first two contributions to  $\delta m_{H_2}^2$  dominate over the last one and thus Higgsino

mass parameter  $|\mu|^2$  becomes small. This lowers the abundance of neutralino dark matter compared to that of MSSM. From the constraint from WMAP observation, we can obtain the allowed parameter space.

Now, let us consider the radiative LFV decays and investigate how they are related with the relic abundance of neutralino dark matter. Branching ratios of the radiative LFV decays are approximately given by [3]

$$BR(l_i \rightarrow l_j \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|m_{\tilde{L},ij}^2|^2}{m_s^8} \tan^2 \beta, \quad (56)$$

where  $m_s$  is average SUSY scalar mass. For numerical calculations, we have used the complete formulas which are given by Eq.(51) in Ref.[2]. As we can see from Eq. (56), the branching ratios of the LFV decays are proportional to  $|m_{\tilde{L},ij}^2|^2$ . Including the threshold corrections,  $m_{\tilde{L},ij}^2$  can be approximately written as

$$m_{\tilde{L},ij}^2 \approx \sum_k \frac{Y_{\nu,ik}^* Y_{\nu,kj}^T}{8\pi^2} \left[ m_0^2 + A_0 B_N^0 - (A_0^2 + 3m_0^2) \ln \frac{M_{GUT}}{M_{R,k}} \right]. \quad (57)$$

Here, the first two terms of RH side are the threshold corrections given in Eq.(20). The last term comes from the RG running effects which is obtained by integrating out RGEs for  $m_{\tilde{L},ij}^2$ . Because the relative sign is opposite, both contributions cancel each other when their contributions are comparable. As the  $B_N^0$  becomes large, the first two terms gets dominant over the last term. It is worthwhile to notice that the size of  $B_N^0$  is limited by the constraints from the experiments.

#### IV. NUMERICAL CALCULATION

In this section, we present our numerical calculations of the relic abundance of the neutralino dark matter and the branching ratio of the LFV decays. For our numerical calculations, we use the micrOMEGAs package[23, 24] for the relic abundance and the SuSpect[25] for RGE running of soft SUSY breaking parameters with appropriate modification. Then we compare our numerical results with the experimental ones. We use the following upper bound for the branching ratio of the LFV decays :  $BR(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$  [26],  $BR(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$  [26] and  $BR(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$  [27].

The WMAP observation leads to the relic abundance of the cold dark matter,  $\Omega_{CDM} h^2$



given by[28],

$$\Omega_{CDM}h^2 = 0.111_{-0.015}^{+0.011} \quad (2\sigma).$$

For numerical calculations, we need to parameterize the neutrino Dirac Yukawa couplings in terms of light neutrino masses and mixing. From the superpotential, the Lagrangian for the neutrino sector is given as,

$$\mathcal{L} = -\frac{1}{2}Y_{\nu,ki}^T \overline{N_R^k} l_i \cdot H_2 - \frac{1}{2}Y_{\nu,ik} l_i^c \cdot H_2 N_R^{kc} - \frac{1}{2}\overline{N_R^k} M_{R,k} N_R^{kc} + h.c.. \quad (58)$$

After integrating out the heavy Majorana neutrinos, one obtains the dimension five operator.

$$\mathcal{L}_{\text{eff}} = +\frac{1}{2}(\overline{l_j^c} \cdot H_2) Y_{\nu,jk} \frac{1}{M_{R,k}} Y_{\nu,ki}^T (l_i \cdot H_2), \quad (59)$$

The mass terms for the left-handed light neutrinos are given by

$$\mathcal{L} = -\frac{1}{2}\overline{\nu_L^c} M_\nu \nu_L + h.c. , \quad (60)$$

where the left-handed light neutrinos  $\nu$  are presented in the flavor eigenstate,  $\nu^T = (\nu_e, \nu_\mu, \nu_\tau)$ , and  $M_\nu$  denotes the neutrino mass matrix written as

$$(M_\nu)_{ij} = -Y_{\nu,ik} M_{R,k}^{-1} Y_{\nu,kj}^T \langle H_2^0 \rangle^2. \quad (61)$$

The neutrino mass matrix  $M_\nu$  can be diagonalized by the unitary matrix  $U_\nu$  as follows:

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \equiv M_{\nu D}. \quad (62)$$

The unitary matrix  $U_\nu$  can be identified to  $U_{MNS}$ , defined by

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ \times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1). \quad (63)$$

where  $\delta, \alpha_1, \alpha_2$  are CP-violating phases,  $c_{ij} = \cos(\theta_{ij})$  and  $s_{ij} = \sin(\theta_{ij})$  with mixing angles  $\theta_{ij}$ . The current experimental values and bounds for the mixing angles are

$$\sin^2(2\theta_{12}) = 0.87 \pm 0.03, \quad \sin^2(2\theta_{23}) > 0.92, \quad \sin^2(2\theta_{13}) < 0.19. \quad (64)$$

Since we do not consider the CP-violation, the phases are set to zero. In our numerical calculations, we take  $\theta_{12} = 0.6$ ,  $\sin \theta_{23} = \cos \theta_{23} = 1/\sqrt{2}$  and  $\sin \theta_{13} = 0$ . It is well known that the neutrino Dirac Yukawa matrix can be written in terms of  $U_{MNS}$  and the diagonal forms of the mass matrices as follows [3];

$$Y_\nu^T = (i) \frac{1}{\langle H_2^0 \rangle} \sqrt{M_R} R \sqrt{M_\nu D} U_{MNS}^\dagger, \quad (65)$$

where  $R$  is a complex orthogonal matrix. We assume that  $R = 1$  for the sake of simplicity in our numerical calculations.

The combination  $(Y_\nu^* Y_\nu^T)_{ij}$  can be written as

$$(Y_\nu^* Y_\nu^T)_{ij} = \frac{1}{\langle H_2^0 \rangle^2} U_{\nu,ik} m_{\nu k} M_{R,k} U_{\nu,kj}^\dagger. \quad (66)$$

Assuming that neutrino masses are subject to normal hierarchy, we can take

$$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) = (0, \sqrt{\Delta m_{21}^2}, \sqrt{\Delta m_{21}^2 + \Delta m_{32}^2}), \quad (67)$$

where  $\Delta m_{21}^2$  and  $\Delta m_{32}^2$  are mass difference of neutrinos, and their experimental results are  $\Delta m_{21}^2 = (7.57 \pm 0.20) \times 10^{-5} \text{eV}^2$  and  $\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{eV}^2$ . We also assume that  $M_{R,1} \ll M_{R,2} \ll M_{R,3}$ . In this case,

$$(Y_\nu^* Y_\nu^T)_{ij} \sim \frac{1}{\langle H_2^0 \rangle^2} U_{\nu,i3} m_{\nu 3} M_{R,3} U_{\nu,3j}^\dagger. \quad (68)$$

Thanks to  $\sin \theta_{13} = 0$ ,  $(Y_\nu^* Y_\nu^T)_{13} \ll (Y_\nu^* Y_\nu^T)_{23}$ . As we can see from eq.(68), this implies that  $m_{L,13}^2 \ll m_{L,23}^2$ . Therefore, we anticipate that  $\text{BR}(\tau \rightarrow e\gamma) \ll \text{BR}(\tau \rightarrow \mu\gamma)$  despite the current experimental bounds are of same order. So, we do not present  $\text{BR}(\tau \rightarrow e\gamma)$  in this work.

Fig.4 shows how both the relic relic abundance and the branching ratios of the LFV decays simultaneously depend on the parameter  $B_N^0$  when light neutrino masses are hierarchical. We take universal soft scalar mass  $m_0$  to be 1TeV. Here, note that lightest SUSY particle is the lightest neutralino. We also take  $\tan \beta = 5(10)$  in the upper (lower) panels. The other input values of the parameters we take are presented in the caption of Fig.4. The left(right) two panels in Fig.4  $BR(\tau \rightarrow \mu\gamma)$  ( $BR(\mu \rightarrow e\gamma)$ ) *vs.* the relic abundance as a function of  $B_N^0$  is shown. In each panels, green solid and red dotted curves present the branching ratio of LFV and the relic abundance of the lightest neutralino, respectively. The gray dotted (upper) and blue solid (lower) horizontal lines show the current upper bound on the branching ratio from

experiments and the relic abundance of the dark matter obtained from WMAP, respectively. When  $B_N^0$  is so large that the prediction of  $\Omega_\chi h^2$  fit to the observed abundance of the dark matter,  $BR(\tau \rightarrow \mu\gamma)$  and  $BR(\mu \rightarrow e\gamma)$  are predicted to be quite large. In this case, the prediction of  $BR(\tau \rightarrow \mu\gamma)$  almost reaches to the experimental bound. Although there will be a chance to probe the LFV decays in the case of large value of  $B_N^0$  in future experiments, the size of  $B_N^0$  is limited by constraint coming from the relic abundance of neutralino dark matter. In Fig.5, we also show how the branching ratios of LFV depend on the universal soft scalar mass  $m_0$ . Here, there exist the valleys where the branching ratios are extremely suppressed, which are occurred due to cancellation between RG running effects and the threshold corrections. This is expected from Eq.(57), since the sign of the contribution from RG running effect to the soft SUSY breaking terms for slepton is opposite to the threshold correction. Therefore as  $m_0$  increases, the large threshold correction from  $B_N^0$  is cancelled and branching ratios for LFV become small. As can be seen from Fig. 5, the dotted (dashed) line starts from the point corresponding to  $m_0 = 700(1400)$  GeV below which electroweak symmetry is not radiatively broken due to too large corrections to  $m_{H_2}^2$ . In the case of large  $B_N^0$  such as  $400 \sim 600$  TeV, the branching ratio for  $\tau \rightarrow \mu\gamma$  and  $\mu \rightarrow e\gamma$  become so sizable that the LFV decay could be detected even when  $m_0$  is larger than 1TeV.

In the case of inverted hierarchy, three neutrino masses are given as

$$(m_{\nu 3}, m_{\nu 1}, m_{\nu 2}) = (m_{\nu 3}, \sqrt{m_{\nu 3}^2 + \Delta m_{32}^2 - \Delta m_{21}^2}, \sqrt{m_{\nu 3}^2 + \Delta m_{32}^2}), \quad (69)$$

and  $m_{\nu 3} < m_{\nu 1} < m_{\nu 2}$ . The threshold corrections to  $\delta m_{H_2}^2$  is proportional to  $\text{Tr}(Y_\nu^* Y_\nu^T)$ .  $\text{Tr}(Y_\nu^* Y_\nu^T)$  is approximately given as

$$\begin{aligned} \text{Tr}(Y_\nu^* Y_\nu^T) \approx & \left[ (|U_{\nu,12}|^2 + |U_{\nu,22}|^2 + |U_{\nu,32}|^2) m_{\nu 2} M_{R,2} \right. \\ & \left. + (|U_{\nu,23}|^2 + |U_{\nu,33}|^2) m_{\nu 3} M_{R,3} \right] / \langle H_2^0 \rangle^2. \end{aligned} \quad (70)$$

In the case of  $m_{\nu 3}/m_{\nu 2} \ll M_{R,2}/M_{R,3}$ , the first term in Eq.(70) is dominant. The ratio of  $\text{Tr}(Y_\nu^* Y_\nu^T)$  in the inverted hierarchy to that in the normal hierarchy is

$$\text{Tr}(Y_\nu^* Y_\nu^T) / \text{Tr}(Y_\nu^* Y_\nu^T)^{NOR} \sim M_{R,2} / M_{R,3}. \quad (71)$$

Therefore threshold corrections to  $m_{H_2}^2$  are much smaller than in the normal hierarchy case. This leads to rather larger abundance of the neutralino dark matter. On the other hand, the term  $(Y_\nu^* Y_\nu^T)_{21}$  is written as

$$(Y_\nu^* Y_\nu^T)_{21} \approx U_{\nu,22} U_{\nu,21}^\dagger m_{\nu,2} M_{R,2} / \langle H_2^0 \rangle^2. \quad (72)$$

This term is obviously larger than that in the normal hierarchy. In particular, for  $m_{\nu 3} \sim 0$ ,

$$(Y_\nu^* Y_\nu^T)_{21} / (Y_\nu^* Y_\nu^T)_{21}^{NOR} \sim 10. \quad (73)$$

Therefore we naively expect that  $BR(\mu \rightarrow e\gamma)$  becomes larger by 2 order of magnitude than that in the normal hierarchy for the same value of  $B_N^0$ . As a result, it becomes difficult to satisfy the constraint from the current bounds on  $BR(\mu \rightarrow e\gamma)$  and the relic abundance simultaneously.

When the neutrino masses are almost degenerate, i.e.  $m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3}$ ,  $(Y_\nu^* Y_\nu^T)_{21}$  is larger than that in the normal hierarchy, which makes the prediction of  $BR(\mu \rightarrow e\gamma)$  in this case substantially enhanced compared to the normal hierarchical case, and thus the constraint of  $BR(\mu \rightarrow e\gamma)$  becomes more severe when we consider the constraint of the relic abundance simultaneously. We present the branching ratios of the LFV decay  $\mu \rightarrow e\gamma$  and the relic abundance of the lightest neutralino as functions of  $B_N^0$  in Fig.6. The different figures correspond to different values of  $\tan\beta$ . As  $\tan\beta$  increases, larger value of  $B_N^0$  is preferred to accommodate both  $BR(\mu \rightarrow e\gamma)$  and the relic abundance of dark matter candidate.

## V. CONCLUSION

We have investigated the LFV radiative decays and the relic abundance of the neutralino dark matter in the SUSY seesaw model. We have carefully derived the threshold corrections to the flavor off-diagonal elements of slepton mass matrix and up-type Higgs mass squared and found that they can be so large in the case of large  $B_N^0$  that the branching ratios of the LFV decays and the relic abundance of neutralino dark matter can be significantly affected. Our numerical results show that there are parameter regions where the prediction of the relic abundance of neutralino dark matter is consistent with WMAP observation and the branching ratios of LFV radiative decays can be enhanced so as for them to be probed in future experiments [29, 30]. Although the origin of such large  $B_N^0$  is unclear, if such large B-term exists, the branching ratios of LFV decays are significantly enhanced even when  $m_0$  is not small. Therefore, the masses of scalar supersymmetric particles are not necessarily small so that the branching ratios of LFV decays can be testable in future experiment, which is distinctive feature of this scenario.

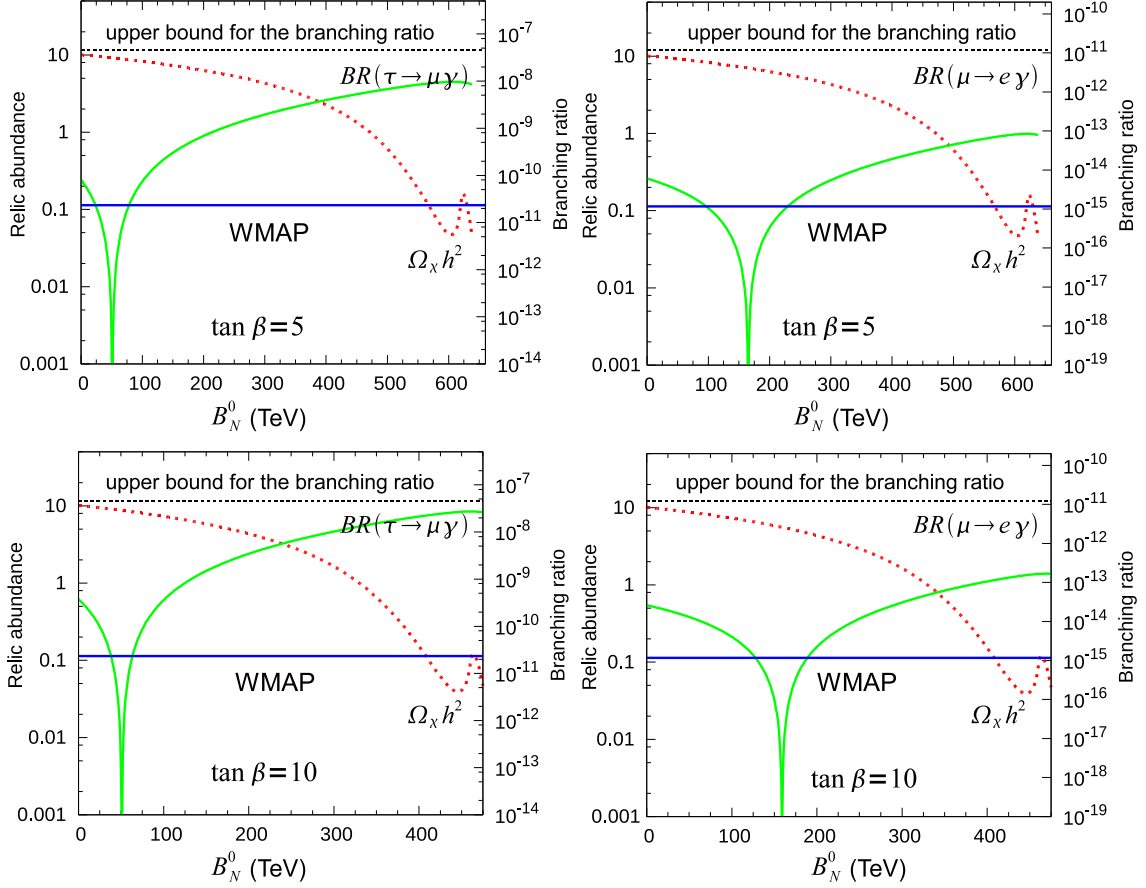


FIG. 4: The branching ratios of LFV decays and the relic abundance of the lightest neutralino are shown as functions of  $B_N^0$  with hierarchical neutrino mass case. Two left panels show  $BR(\tau \rightarrow \mu\gamma)$  *vs.* the relic abundance and two right panels show  $BR(\mu \rightarrow e\gamma)$  *vs.* the relic abundance. In each panel, green solid and red dotted curves represent the corresponding branching ratio and the relic abundance of the lightest neutralino, respectively. The gray dotted (upper) and blue solid (lower) straight lines represent the upper bound on the branching ratio given by experiments and the abundance of the dark matter obtained from WMAP, respectively. We take  $(M_{R,1}, M_{R,2}, M_{R,3}) = (10^{10}, 10^{12}, 10^{14})\text{GeV}$  and  $\tan\beta = 5$  for upper two panels and  $\tan\beta = 10$  for lower two panels. We also take  $m_0 = 1\text{TeV}$ ,  $A_0 = 300\text{GeV}$ ,  $m_{1/2} = 300\text{GeV}$  and  $\mu > 0$ .

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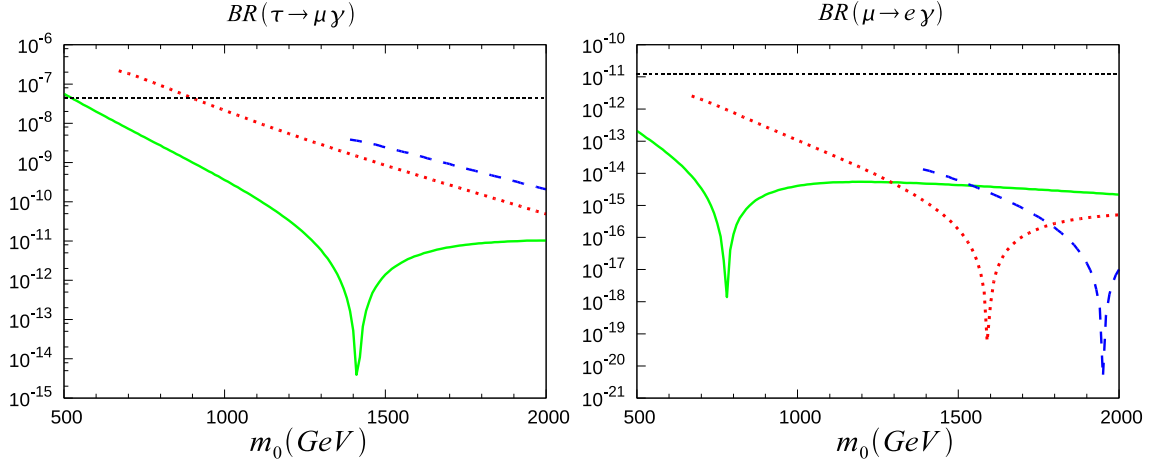


FIG. 5: The branching ratios of LFV decays are shown as functions of universal soft scalar mass  $m_0$  for hierarchical neutrino mass case. The left panel shows  $BR(\tau \rightarrow \mu\gamma)$  and the right panel shows  $BR(\mu \rightarrow e\gamma)$ . The gray dotted lines correspond to the experimental upper bound on the branching fractions. Green solid, red dotted and blue broken lines are calculated with  $B_N^0 = (100, 400, 600)\text{TeV}$ , respectively. We choose  $m_{1/2} = 300\text{GeV}$ ,  $A_0 = 300\text{GeV}$ ,  $\tan\beta = 10$ .

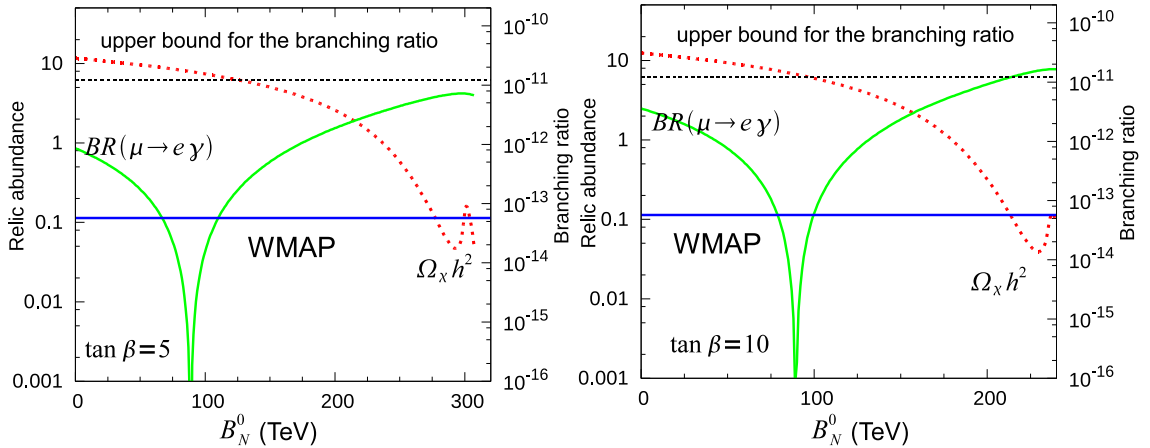


FIG. 6:  $Br(\mu \rightarrow e\gamma)$  vs. the relic abundance as function of  $B_N^0$  for degenerate neutrino case. The green solid and red dotted curves represent the branching ratio and the relic abundance of the lightest neutralino, respectively. The gray dotted (upper) and blue solid (lower) straight lines represent the upper bound on the branching ratio given by experiment. The parameters are the same as those in Fig.4.

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## APPENDIX A: RENORMALIZATION GROUP EQUATION

In this appendix, we present one-loop renormalization group equations for  $SU(2)$  slepton masses and Higgs mass squared parameters including the threshold effects. The one-loop RGE for the parameter  $B_{ij}^2$  is also presented. Here, we have omitted the same contributions as in MSSM. The RGEs for  $SU(2)$  slepton masses are given by

$$\begin{aligned}
(16\pi^2)Q \frac{dm_{\tilde{L},ij}^2}{dQ} &= \sum_k \left[ Y_{\nu,ik}^* m_{\tilde{N},kk}^2 Y_{\nu,kj}^T + \sum_{k'=1}^{k-1} (Y_{\nu,ik'}^* m_{\tilde{N},k'k}^2 Y_{\nu,kj}^T + Y_{\nu,ik}^* m_{\tilde{N},kk'}^2 Y_{\nu,k'j}^T) \right] \\
&\quad \times [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&+ \sum_k [A_{\nu,ik}^* A_{\nu,kj}^T + m_{22}^2 Y_{\nu,ik}^* Y_{\nu,kj}^T] [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&- 2|\mu|^2 \sum_k Y_{\nu,ik}^* Y_{\nu,kj}^T \theta(Q^2 - M_{R,k}^2) \\
&+ \sum_{k,k'} [Y_{\nu,ik}^* Y_{\nu,kk'}^T m_{\tilde{L},k'j}^2 + m_{\tilde{L},ik'}^2 Y_{\nu,k'k}^* Y_{\nu,kj}^T] \theta(Q^2 - M_{R,k}^2) \\
&+ 2 \sum_k Y_{\nu,ik}^* Y_{\nu,kj}^T M_{R,k}^2 [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2) - 2\theta(Q^2 - M_{R,k}^2)] \\
&+ \sum_k [Y_{\nu,ik}^* A_{\nu,kj}^T e^{-i\Phi_k} + A_{\nu,ik}^* Y_{\nu,kj}^T e^{i\Phi_k}] M_{R,k} \\
&\quad \times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)] \\
&+ \sum_k \left[ Y_{\nu,ik}^* |B_{N,k}| M_{R,k} Y_{\nu,kj}^T + \sum_{k'=1}^{k-1} (Y_{\nu,ik'}^* B_{k'k}^2 e^{-i\Phi_k} Y_{\nu,kj}^T + Y_{\nu,ik}^* B_{kk'}^{2*} e^{i\Phi_k} Y_{\nu,k'j}^T) \right] \\
&\quad \times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)], \tag{A1}
\end{aligned}$$

where  $m_{22}^2 = m_{H_2}^2 + |\mu|^2$ . The RGEs for Higgs mass squared parameters are given by

$$\begin{aligned}
(16\pi^2)Q \frac{dm_{H_2}^2}{dQ} &= \sum_{i,k} \left[ Y_{\nu,ik}^* m_{\tilde{N},kk}^2 Y_{\nu,ki}^T + \sum_{j=1}^{k-1} (Y_{\nu,ij}^* m_{\tilde{N},jk}^2 Y_{\nu,ki}^T + Y_{\nu,ik}^* m_{\tilde{N},kj}^2 Y_{\nu,ji}^T) \right] \\
&\quad \times [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&\quad + \sum_{i,j,k} (Y_{\nu,ki}^T m_{\tilde{L},ij}^2 Y_{\nu,jk}^*) [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&\quad + \sum_{i,k} (A_{\nu,ik}^* A_{\nu,ki}^T) [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&\quad + 2m_{H_2}^2 \sum_{i,k} Y_{\nu,ik}^* Y_{\nu,ki}^T \theta(Q^2 - M_{R,k}^2) \\
&\quad + 2 \sum_{i,k} Y_{\nu,ik}^* Y_{\nu,ki}^T M_{R,k}^2 [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2) - 2\theta(Q^2 - M_{R,k}^2)] \\
&\quad + 2 \sum_{i,k} \text{Re}(Y_{\nu,ik}^* A_{\nu,ki}^T e^{-i\Phi_k}) M_{R,k} [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)] \\
&\quad + \sum_{i,k} \left[ Y_{\nu,ik}^* |B_{N,k}| M_{R,k} Y_{\nu,ki}^T + 2 \sum_{j=1}^{k-1} \text{Re}(Y_{\nu,ij}^* B_{jk}^2 e^{-i\Phi_k} Y_{\nu,ki}^T) \right] \\
&\quad \times [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)], \tag{A2}
\end{aligned}$$

$$(16\pi^2)Q \frac{dm_{11}^2}{dQ} = |\mu|^2 \sum_{i,k} |Y_{\nu,ik}|^2 [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)], \tag{A3}$$

where  $m_{11}^2 \equiv |\mu|^2 + m_{H_1}^2$ . **We note that the RH-side of Eq.(A3) corresponds to the contribution from wave function renormalization.**

$$\begin{aligned}
(16\pi^2)Q \frac{dm_{H_1 H_2}^2}{dQ} &= \sum_{i,k} \mu |Y_{\nu,ik}|^2 e^{i\Phi_k} M_{R,k} [\theta(Q^2 - M_{h,k}^2) - \theta(Q^2 - M_{l,k}^2)] \\
&\quad + \sum_{i,k} \mu Y_{\nu,ik}^* A_{\nu,ik} [\theta(Q^2 - M_{h,k}^2) + \theta(Q^2 - M_{l,k}^2)] \\
&\quad + \sum_{i,k} |Y_{\nu,ik}|^2 m_{H_1 H_2}^2 \theta(Q^2 - M_{R,k}^2). \tag{A4}
\end{aligned}$$

The RGE for  $B_{ij}^2$  is given as,

$$\begin{aligned}
(16\pi^2)Q \frac{dB_{ij}^2}{dQ} &= 4 \left[ A_{\nu,ik}^T Y_{\nu,kj}^* M_{R,j} + M_{R,i} Y_{\nu,ik}^\dagger A_{\nu,kj} \right] \\
&\quad + 2 (Y_\nu^T Y_\nu^*)_{ik} (B^2)_{kj} + 2 (B^2)_{ik} (Y_\nu^T Y_\nu^*)_{jk}. \tag{A5}
\end{aligned}$$



The RG equation for  $M_R$  can be found, for instance, in [4],

$$(16\pi^2)Q \frac{dM_{R,ij}}{dQ} = 2 (Y_\nu^T Y_\nu^*)_{ik} M_{R,kj} + 2M_{R,ik} (Y_\nu^T Y_\nu^*)_{jk}. \quad (\text{A6})$$

## APPENDIX B: DIAGRAM CALCULATION

By summing the self-energies from both fermion and scalar loop diagrams, the one loop contribution to the slepton mass squared is given by,

$$\begin{aligned} -i\Sigma_{\tilde{L},ij} = & -\frac{i}{16\pi^2} \left[ -(\bar{\epsilon}^{-1} + 1)(A_{\nu,ik}^* A_{\nu,kj}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T (m_{H_2}^2 + m_{\tilde{N},kk}^2)) \right. \\ & + \sum_{k \neq k'} Y_{\nu,ik}^* Y_{\nu,k'j}^T m_{\tilde{N},kk'}^2 \\ & + (A_{\nu,ik}^* A_{\nu,kj}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{H_2}^2 + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{N},kk}^2) \ln \frac{M_{R,k}^2}{Q^2} \\ & + \sum_{k \neq k'} Y_{\nu,ik}^* Y_{\nu,k'j}^T m_{\tilde{N},kk'}^2 \ln \frac{\max(M_{R,k}^2, M_{R,k'}^2)}{Q^2} \\ & + 2Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{N},kk}^2 + (Y_{\nu,ik}^* A_{\nu,kj}^T B_{kk}^{2*} + A_{\nu,ik}^* Y_{\nu,kj}^T B_{kk}^2) \frac{1}{M_{R,k}} \\ & + \left. \sum_{k < k'} (Y_{\nu,ik}^* Y_{\nu,k'j}^T B_{kk'}^2 B_{k'k'}^{2*} + Y_{\nu,ik'}^* Y_{\nu,kj}^T B_{k'k}^{2*} B_{k'k'}^2) \frac{1}{M_{R,k'}} \right] \\ & + \frac{i}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,kj}^T p^2 [\bar{\epsilon}^{-1} + 1 - \ln \frac{M_{R,k}^2}{Q^2}]. \quad (\text{B1}) \end{aligned}$$

The contribution to the up type Higgs ( $H_2$ ) mass squared is given by,

$$\begin{aligned}
-i\Sigma_{H_2} = & -\frac{i}{16\pi^2} \left[ -(\bar{\epsilon}^{-1} + 1)(A_{\nu,ik}^* A_{\nu,ki}^T + Y_{\nu,ik}^* Y_{\nu,ki}^T)(m_{\tilde{L},ii}^2 + m_{\tilde{N},kk}^2) \right. \\
& + \sum_{k \neq k'} Y_{\nu,ik}^* Y_{\nu,k'i}^T m_{\tilde{N},kk'}^2 + \sum_{i \neq j} Y_{\nu,kj}^T m_{\tilde{L},ji}^2 Y_{\nu,ik}^* \\
& + (A_{\nu,ik}^* A_{\nu,ki}^T + Y_{\nu,ik}^* Y_{\nu,kj}^T m_{\tilde{L},ji}^2 + Y_{\nu,ik}^* Y_{\nu,ki}^T m_{\tilde{N},kk}^2) \ln \frac{M_{R,k}^2}{Q^2} \\
& + \sum_{k \neq k'} Y_{\nu,ik}^* Y_{\nu,k'i}^T m_{\tilde{N},kk'}^2 \ln \frac{\max(M_{R,k}^2, M_{R,k'}^2)}{Q^2} \\
& - \sum_{i \neq j} Y_{\nu,kj}^T m_{\tilde{L},ji}^2 Y_{\nu,ik}^* \frac{m_{\tilde{L},jj}^2 \log \frac{M_{R,k}^2}{m_{\tilde{L},jj}^2} - m_{\tilde{L},ii}^2 \log \frac{M_{R,k}^2}{m_{\tilde{L},ii}^2}}{m_{\tilde{L},jj}^2 - m_{\tilde{L},ii}^2} \\
& + 2Y_{\nu,ik}^* Y_{\nu,ki}^T m_{\tilde{N},kk}^2 + (Y_{\nu,ik}^* A_{\nu,ki}^T B_{kk}^{2*} + A_{\nu,ik}^* Y_{\nu,ki}^T B_{kk}^2) \frac{1}{M_{R,k}} \\
& + \sum_{k < k'} (Y_{\nu,ik}^* Y_{\nu,k'i}^T B_{kk'}^{2*} B_{k'k'}^{2*} + Y_{\nu,ik'}^* Y_{\nu,ki}^T B_{k'k}^{2*} B_{k'k'}^2) \frac{1}{M_{R,k'}} \left. \right] \\
& + \frac{i}{16\pi^2} Y_{\nu,ik}^* Y_{\nu,ki}^T p^2 [\bar{\epsilon}^{-1} + 1 - \ln \frac{M_{R,k}^2}{Q^2}]. \tag{B2}
\end{aligned}$$

## APPENDIX C: APPROXIMATE SOLUTIONS OF THE RENORMALIZATION GROUP EQUATIONS FOR $M_R$ AND $B^2$ .

As stated below Eq.(21), although the renormalization group running may induce the large flavor off-diagonal contribution to  $B_{ij}^2$  ( $i \neq j$ ) at a lower mass scale, by switching the basis of Majorana mass matrix  $M_R$  to the diagonal basis, we can keep  $B_{ij}^2$  in the basis almost diagonal because off-diagonal elements  $B_{ij}^2$  ( $i \neq j$ ) are doubly suppressed by a factor of  $\frac{A_0}{B_0}$  and one loop suppressed factor. The effect of the small off-diagonal elements in  $B_{ij}^2$  on slepton soft breaking term turns out to be smaller than the leading threshold corrections given in Eq.(20).

In this appendix, we first show that the large radiatively generated off-diagonal elements of  $B_{ij}^2$  corresponding to the second term of Eq.(21) are indeed rotated away in the diagonal basis for  $M_R$ . To show this, the renormalization group equations for  $B^2$  and  $M_R$  are solved in perturbative way, i.e., we use the approximation so that in the RH side of the renormalization group equations, all the couplings  $A_\nu$ ,  $Y_\nu$  and mass  $M_R$  are scale independent constants defined at GUT scale where the initial conditions for renormalization group equations are imposed. We also show that off-diagonal elements of the third term of Eq.(21) remain even after the rotation and are numerically small compared with the leading diagonal elements.

The solutions for Eq.(A5) and Eq.(A6) at  $Q = M_{R,3}$  with the boundary conditions in Eq.(11), are given as,

$$M_{Rij}(M_{R,3}) = M_{R,i}\delta_{ij} - 2(H_{ij}M_{R,j} + H_{ji}M_{R,i})t^{03}, \quad (C1)$$

$$\begin{aligned} B_{ij}^2(M_{R,3}) &= B_0[M_{R,i}\delta_{ij} - 2(H_{ij}M_{R,j} + H_{ji}M_{R,i})t^{03}] - 4A_0(H_{ij}M_{R,j} + H_{ji}M_{R,i})t^{03} \\ &= (B_0 + 2A_0)M_R(M_{R,3})_{ij} - 2A_0M_{R,i}\delta_{ij}, \end{aligned} \quad (C2)$$

where  $t^{03} = \frac{1}{16\pi^2} \log \frac{M_{GUT}}{M_{R,3}}$  and  $H = Y_\nu^T Y_\nu^*$ . Since the first term of  $B^2$  in Eq.(C2) is proportional to the running mass matrix of the heavy Majorana neutrinos, it is also diagonal in the diagonal basis for  $M_R$ . As for the second term proportional to  $A_0$ , it is changed into the non-diagonal one. To derive the unitary matrix  $O$  diagonalizing  $M_R(Q = M_{R,3})$

approximately, we first write the mass matrix at  $Q = M_{R,3}$  in the matrix form as,

$$M_R(Q = M_{R,3}) = \begin{pmatrix} M_{R1}(M_{R,3}) & -2H_{12}t^{03}(M_{R,1} + M_{R,2}) & -2H_{13}t^{03}(M_{R,1} + M_{R,3}) \\ -2H_{12}t^{03}(M_{R,1} + M_{R,2}) & M_{R2}(M_{R,3}) & -2H_{23}t^{03}(M_{R,2} + M_{R,3}) \\ -2H_{13}t^{03}(M_{R,1} + M_{R,3}) & -2H_{23}t^{03}(M_{R,2} + M_{R,3}) & M_{R3}(M_{R3}) \end{pmatrix}, \quad (\text{C3})$$

where we have used the property  $H_{ij} = H_{ji}$  since CP is assumed to be invariant. The diagonal elements at  $Q = M_{R,3}$  are given by

$$M_{Ri}(Q = M_{R,3}) = M_{R,i}(1 - 4H_{ii}t^{03}). \quad (\text{C4})$$

One can find the matrix  $O$  with which the mass matrix Eq.(C3) is diagonalized as,

$$O(M_{R,3})M_R(Q = M_{R,3})O^T(M_{R,3}) = D(Q = M_{R,3}), \quad (\text{C5})$$

where  $D$  is the diagonal matrix. The rotation given above corresponds to changing the basis  $N^c \rightarrow O^T N^c$ . In the new basis,  $B^2$  is given by,

$$B_{\text{new}ij}^2 \equiv (OB^2O^T)_{ij} = \{(B_0 + 2A_0)D_i(Q = M_{R,3}) - 2A_0M_{R,3}\}\delta_{ij} - 2A_0 \sum_{k=1}^2 (O_{ik}(M_{R,k} - M_{R,3})O_{jk}). \quad (\text{C6})$$

From Eq.(C6), the off diagonal elements of  $B_{\text{new}}^2$  are given as,

$$B_{\text{new}ij}^2 = -2A_0 \sum_{k=1}^2 O_{ik}(M_{R,k} - M_{R,3})O_{jk} \quad (i \neq j). \quad (\text{C7})$$

The diagonal elements are dominated by the term proportional to  $B_0M_{R,i}$ ,

$$B_{\text{new}ii}^2 = B_0D_i(M_{R,3}) + 2A_0(D_i - \sum_{k=1}^3 O_{ik}^2 M_{R,k}). \quad (\text{C8})$$

To write the off-diagonal elements of  $B_{\text{new}}^2$  explicitly, we introduce the parametrization for the orthogonal matrix  $O^T$  as,

$$O^T = \begin{pmatrix} c_{13}^N c_{12}^N & c_{13}^N s_{12}^N & s_{13}^N \\ -s_{23}^N s_{13}^N c_{12}^N - c_{23}^N s_{12}^N & -s_{23}^N s_{13}^N s_{12}^N + c_{23}^N c_{12}^N & c_{13}^N s_{23}^N \\ -c_{23}^N s_{13}^N c_{12}^N + s_{23}^N s_{12}^N & -c_{23}^N s_{13}^N s_{12}^N - s_{23}^N c_{12}^N & c_{13}^N c_{23}^N \end{pmatrix}, \quad (\text{C9})$$

where  $s_{ij}^N = \sin \theta_{ij}^N$  and  $c_{ij}^N = \cos \theta_{ij}^N$ . One can write  $B_{\text{new},ij}^2$  ( $i \neq j$ ) using the angles as

$$\begin{aligned} B_{\text{new},13}^2 &= -2A_0 c_{13}^N \{ (M_{R,1} - M_{R,2}) c_{12}^N s_{13}^N + (M_{R,2} - M_{R,3}) c_{23}^N (c_{23}^N c_{12}^N s_{13}^N - s_{23}^N s_{12}^N) \}, \\ B_{\text{new},23}^2 &= -2A_0 \{ c_{13}^N s_{12}^N s_{13}^N (M_{R,1} - M_{R,2}) + c_{13}^N c_{23}^N (s_{23}^N c_{12}^N + c_{23}^N s_{13}^N s_{12}^N) (M_{R,2} - M_{R,3}) \}, \\ B_{\text{new},12}^2 &= -2A_0 \{ c_{12}^N s_{12}^N ((M_{R,1} - M_{R,2}) - s_{13}^{N2} (M_{R,1} - M_{R,3})) \\ &\quad + s_{23}^N (s_{23}^N s_{12}^N c_{12}^N (1 + (s_{13}^N)^2) - c_{23}^N s_{13}^N \cos 2\theta_{12}^N) (M_{R,2} - M_{R,3}) \}. \end{aligned} \quad (\text{C10})$$

The angles  $\theta_{ij}^N$  can be determined by the diagonalization Eq.(C5). When  $M_{R,3} \gg M_{R,2}, M_{R,1}$ , one can determine  $s_{23}^N, s_{13}^N$  from the equation,

$$\begin{pmatrix} 0 & 0 & -2H_{13}t^{03}M_{R,3} \\ 0 & 0 & -2H_{23}t^{03}M_{R,3} \\ -2H_{13}t^{03}M_{R,3} & -2H_{23}t^{03}M_{R,3} & M_{R3}(M_{R3}) \end{pmatrix} \begin{pmatrix} s_{13}^N \\ c_{13}^N s_{23}^N \\ c_{13}^N c_{23}^N \end{pmatrix} = D_3 \begin{pmatrix} s_{13}^N \\ c_{13}^N s_{23}^N \\ c_{13}^N c_{23}^N \end{pmatrix}. \quad (\text{C11})$$

Then we find that  $D_3 \simeq M_{R,3}$ , and  $s_{13}^N$  and  $s_{23}^N$  are given as

$$\begin{aligned} s_{13}^N &\simeq -2H_{13}t^{03}, \\ s_{23}^N &\simeq -2H_{23}t^{03}, \end{aligned} \quad (\text{C12})$$

where we ignore the corrections of the order of  $O(\frac{M_{R,i}}{M_{R,3}})$  ( $i = 1, 2$ ). The determination of  $s_{12}^N$  is more involved. It is determined by diagonalizing the following  $2 \times 2$  matrix which can be obtained after the largest eigenvalue state is decoupled from the  $3 \times 3$  matrix in Eq.(C3).

$$\begin{aligned} &\begin{pmatrix} c_{12}^N & -s_{12}^N \\ s_{12}^N & c_{12}^N \end{pmatrix} \begin{pmatrix} M_{R,1} - M_{R,3}s_{13}^2 & -2H_{12}t^{03}M_{R,2} - M_{R,3}s_{13}s_{23} \\ -2H_{12}t^{03}M_{R,2} - M_{R,3}s_{13}s_{23} & M_{R,2} - M_{R,3}s_{23}^2 \end{pmatrix} \begin{pmatrix} c_{12}^N & s_{12}^N \\ -s_{12}^N & c_{12}^N \end{pmatrix} \\ &\simeq \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}. \end{aligned} \quad (\text{C13})$$

$s_{12}^N$  is approximately given by,

$$s_{12}^N \simeq -2H_{12}t^{03} - \frac{M_{R,3}}{M_{R,2}} s_{13}^N s_{23}^N, \quad (\text{C14})$$

where the following conditions are assumed to be satisfied,

$$M_{R,2} > (s_{13}^N)^2 M_{R,3}, (s_{23}^N)^2 M_{R,3}. \quad (\text{C15})$$

Using the formulae given in Eq.(C12), one can write the dominant terms for the first two equations in Eq.(C10),

$$\begin{aligned} B_{\text{new},13}^2 &= 2A_0 M_{R,3} (c_{12}^N s_{13}^N - s_{12}^N s_{23}^N), \\ B_{\text{new},23}^2 &= 2A_0 M_{R,3} (c_{12}^N s_{23}^N + s_{12}^N s_{13}^N). \end{aligned} \quad (\text{C16})$$

When  $s_{12}^N \ll 1$ , they are simplified as,

$$\begin{aligned} B_{\text{new},m3}^2 &= 2A_0 M_{R,3} s_{m3}^N \\ &= -4A_0 M_{R,3} H_{m3} t^{03}, \quad (m = 1, 2). \end{aligned} \quad (\text{C17})$$

Eq.(C17) shows that in the diagonal basis of  $M_R$ , the off diagonal elements of  $B_{m3}^2$  is given by the third term of Eq.(21) and it is small compared with the large diagonal element  $B_{\text{new}33}^2 \sim B_N^0 M_{R,3}$ .

Next we show that the variation of the threshold corrections Eq.(20) due to the change of the basis is two loop effect and thus negligibly small. When we change the basis as,

$$(N_R)^c \rightarrow O^T N_R^c, \quad (\text{C18})$$

$Y_\nu$  in Eq.(20) should be replaced by

$$Y_\nu \rightarrow Y_\nu O(M_{R,k})^T. \quad (\text{C19})$$

Then the threshold correction is replaced by the following equation,

$$\delta_{\text{th}}^k m_{Lij}^2 = \frac{1}{8\pi^2} (Y_\nu^* O^T)_{ik} (O Y_\nu^T)_{kj} (m_0^2 + A_0 B_N^0). \quad (\text{C20})$$

Now let us examine how large  $\delta_{\text{th}}^k m_{Lij}^2$  for the case  $k = 3$  could be after changing the basis as follows,

$$\delta_{\text{th}}^3 m_{Lij}^2 \simeq \frac{m_0^2 + A_0 B_N^0}{8\pi^2} \{Y_{\nu,i3}^* Y_{\nu,3j}^T + Y_{\nu,i3}^* (Y_{\nu,2j}^T s_{23}^N + Y_{\nu,1j}^T s_{13}^N) + (Y_{\nu,i2}^* s_{23}^N + Y_{\nu,i1}^* s_{13}^N) Y_{\nu,3j}^T\}. \quad (\text{C21})$$

Since  $s_{m3}^N = -(Y_\nu^T Y_\nu^*)_{m3} \frac{1}{8\pi^2} t^{03}$ , ( $m = 1, 2$ ), the variation  $\delta_{\text{th}}^3 m_{Lij}^2$  is the two loop order and thus small correction.

For the completeness, we check whether the conditions Eq.(C15) and  $s_{12}^N \ll 1$  which lead to Eq.(C17) are satisfied. For numerical estimation, We take  $M_{GUT} = 10^{16}(\text{GeV})$ ,  $M_{R,3} = 10^{14}(\text{GeV})$ ,  $M_{R,2} = 10^{12}(\text{GeV})$ , and  $M_{R,1} = 10^{10}(\text{GeV})$ . For  $H_{ij}$ , one may use the parametrization Eq.(65).

$$H_{ij} = (Y_\nu^T Y_\nu^*)_{ij} = \frac{1}{\langle H_2^0 \rangle^2} \sqrt{M_{R,i} M_{R,j}} (R m_\nu R^\dagger)_{ij}. \quad (\text{C22})$$

When R is a real orthogonal matrix,

$$|H_{ij}| < \frac{\sqrt{M_{R,i} M_{R,j}}}{v^2 \sin^2 \beta} \times (|m_{\nu 3} - m_{\nu 1}| + |m_{\nu 2} - m_{\nu 1}|). \quad (\text{C23})$$

Using  $\tan \beta = 5$  and  $v = 246(\text{GeV})$ , we obtain

$$\begin{aligned} |H_{12}| &\leq 1.0 \times 10^{-4}, \\ |H_{13}| &\leq 1.0 \times 10^{-3}, \\ |H_{23}| &\leq 1.0 \times 10^{-2}. \end{aligned} \tag{C24}$$

Since  $2t^{03} \simeq 0.06$ , we obtain

$$|s_{23}^N| \leq 6 \times 10^{-4}, \quad |s_{13}^N| \leq 6 \times 10^{-5}. \tag{C25}$$

Then Eq.(C15) is satisfied. We also note that  $s_{12}^N$  is as small as

$$|s_{12}^N| \leq 6 \times 10^{-6} + 3.6 \times 10^{-6} \sim 1.0 \times 10^{-5}. \tag{C26}$$

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