The Effects of Dynamic Feedbacks on LS and MM Estimator Accuracy in Panel Data Models: Some Additional Results

Kazuhiko Hayakawa*
Department of Economics, Hiroshima University

June 10, 2010

Abstract

In this paper, we show that the order of magnitude of the finite sample bias of the GMMld$^{(2)}$ estimator of Bun and Kiviet (2006) reduces from $O(T/N)$ to $O(1/N)$ if the original level model is transformed by the upper triangular Cholesky factorization of the inverse of the pseudo variance matrix of error component u_i wherein true values of the variances of individual effects and disturbances may not be used. Some variants of the system GMM estimator that are associated with the Cholesky-transformed model are also discussed.

*The author is deeply grateful to the referee for the helpful comments provided. This research is supported by a Grant-in-Aid for Scientific Research (KAKENHI 20830056, 22730178) of the JSPS.

1 Introduction

In the context of a panel AR(1) model, Alvarez and Arellano (2003) demonstrate that the first-difference GMM estimator with an optimal weighting matrix, which is identical to the GMMfl$^{(2)}$ estimator in Bun and Kiviet (2006, hereafter BK), is consistent when both N and T are large, while that with a non-optimal weighting matrix is inconsistent. This seems to imply that the choice of a weighting matrix is important for the bias when we allow N and T to be large.

Although BK recognize this feature, for a model in levels, they consider GMMld estimators in 2SLS form, which becomes optimal only when the variance of individual effects σ_η^2 is zero, and conjecture in footnote 4 (p.422) that the results may change if the optimal weighting matrix is used. The purpose of this paper is to show that whether an optimal weighting matrix is used or not is inconsequential; the important ingredient for a reduction in the order of magnitude of the finite sample bias is whether the model is transformed in such a way that an endogeneity bias reduces from $O(1)$ to $O(1/T)$ (see Remark 4 below).

To demonstrate this, we first introduce a model transformed by the upper triangular Cholesky factorization of the inverse of the pseudo variance matrix of the error components where true values of variances of individual effects and disturbances may not be used, and then consider GMM estimators for this transformed model, which we call the GMMcd estimator. Consequently, we show that the GMMcd estimators have the same order of magnitude of finite sample bias as GMMfl estimators. For instance, if $Z_{di}^{(2)}$ is used in the GMMcd estimator, the order of magnitude of the finite sample bias is $O(1/N)$, which is identical to that of the GMMfl$^{(2)}$ estimator.

We also show that the GMMcd is identical to the GMM estimator for original level model using a particular weighting matrix if all available instruments are used, which is parallel to the relationship between the first-difference GMM estimator using an optimal weighting matrix and the GMMfl$^{(2)}$ estimator.

We further consider other various GMM estimators that are associated with the Cholesky-transformed model. For instance, we consider a GMM estimator for the Cholesky-transformed model using $Z_{di}^{(1),(0)}$ or a system GMM estimator that uses the Cholesky-transformed model instead of the original level model.

The structure of this paper is as follows. In the next section, we review the setup of BK, introduce the Cholesky-transformed model, and define several GMM estimators. In Section 3, we derive the finite sample biases of the estimators, and in Section 4, we provide the simulation results. Finally, Section 5 concludes.

2 Review of the setup and transformed model

BK consider the following model:

$$\begin{aligned} y_{it} &= \gamma y_{i,t-1} + \beta x_{it} + \eta_i + \varepsilon_{it} \\ &= W_{it}'\delta + u_{it}, \end{aligned}$$

where $|\gamma| < 1$, $\eta_i \sim iid(0, \sigma_\eta^2)$, and $\varepsilon_{it} \sim iid(0, \sigma_\varepsilon^2)$ for $i = 1, ..., N$ and $t = 0, ..., T$. See BK for the properties of y_{it} and x_{it}, and the assumption on the initial conditions. Stacking over time, we have

$$y_i = \gamma y_{i(-1)} + \beta x_i + \eta_i \iota_T + \varepsilon_i$$

$$= W_i\delta + u_i,$$

where $y_i = (y_{i1}, ..., y_{iT})'$, $y_{i(-1)} = (y_{i0}, ..., y_{i,T-1})'$, $x_i = (x_{i1}, ..., x_{iT})'$, $\iota_T = (1, ..., 1)'$, $\varepsilon_i = (\varepsilon_{i1}, ..., \varepsilon_{iT})'$, $W_i = (y_{i(-1)}, x_i)$, and $u_i = \eta_i\iota_T + \varepsilon_i$.

Let us consider a GMM estimator for the equation in levels:

$$\dot{y}_i = \dot{W}_i\delta + \dot{u}_i \qquad (i = 1, ..., N), \tag{1}$$

where $\dot{y}_i = (y_{i2}, ..., y_{iT})'$, $\dot{W}_i = (W_{i2}, ..., W_{iT})'$, and $\dot{u}_i = (u_{i2}, ..., u_{iT})'$. Since the moment conditions for this model are given by $E(Z_{di}^{(h)\prime}\dot{u}_i) = 0$ $(h = 2, 1, 0)$, we consider the following GMM estimator:

$$\begin{aligned}\widehat{\delta}_{\mathrm{GMMld}(r)^{(h)}} &= \left[\left(\sum_{i=1}^N \dot{W}_i' Z_{di}^{(h)}\right)\left(\sum_{i=1}^N Z_{di}^{(h)\prime} V_{T_1(r)} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^N Z_{di}^{(h)}\dot{W}_i\right)\right]^{-1} \\ &\quad \times\left(\sum_{i=1}^N \dot{W}_i' Z_{di}^{(h)}\right)\left(\sum_{i=1}^N Z_{di}^{(h)\prime} V_{T_1(r)} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^N Z_{di}^{(h)}\dot{y}_i\right),\end{aligned} \tag{2}$$

where $V_{T_1(r)} = I_{T_1} + r\iota_{T_1}\iota_{T_1}'$, $0 \leq r < \infty$ and $T_1 = T - 1$. Note that this estimator is optimal when $r = r_o \equiv \sigma_\eta^2/\sigma_\varepsilon^2$ and not optimal when $r \neq r_o$. The GMMld$^{(h)}$ estimator BK consider is a special case of GMMld$(r)^{(h)}$ with $r = 0$, i.e.,

$$\begin{aligned}\widehat{\delta}_{\mathrm{GMMld}^{(h)}} &= \widehat{\delta}_{\mathrm{GMMld}(0)^{(h)}} \\ &= \left[\left(\sum_{i=1}^N \dot{W}_i' Z_{di}^{(h)}\right)\left(\sum_{i=1}^N Z_{di}^{(h)\prime} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^N Z_{di}^{(h)}\dot{W}_i\right)\right]^{-1} \\ &\quad \times\left(\sum_{i=1}^N \dot{W}_i' Z_{di}^{(h)}\right)\left(\sum_{i=1}^N Z_{di}^{(h)\prime} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^N Z_{di}^{(h)}\dot{y}_i\right)\end{aligned}$$

Note that BK conjecture in footnote 4 (p.422) that the difference in the order of magnitude of GMMfl$^{(h)}$ and GMMld$^{(h)}$ may be the result from the fact that GMMld$^{(h)}$ does not use the optimal weight. We show that what is important for a reduction in bias is whether an original level model is transformed by a particular GLS type transformation that reduces the endogeneity bias from $O(1)$ to $O(1/T)$, and whether or not an optimal weighting matrix is used is not important. To demonstrate this, let us consider an alternative model.

As an alternative model to (1), let us consider a model transformed by the upper triangular Cholesky factorization of $V_{T_1(r)}^{-1}$:

$$V_{T_1(r)}^{-1/2}\,\dot{y}_i = V_{T_1(r)}^{-1/2}\dot{W}_i + V_{T_1(r)}^{-1/2}\dot{u}_i \qquad (i = 1, ..., N), \tag{3}$$

which we rewrite as follows:

$$y_i^+ = W_i^+\delta + u_i^+ \qquad (i = 1, ..., N). \tag{4}$$

Note that this transformation is a *pseudo* GLS transformation since r may not be equal to r_o. If $r = r_o$, this transformation is a genuine GLS transformation.

In (3), $V_{T_1(r)}^{-1/2}$ is the upper triangular Cholesky factorization of $V_{T_1(r)}^{-1}$ as follows:[1]

$$\begin{aligned} V_{T_1(r)}^{-1} &= I_{T_1} - \left(1 - \frac{1}{1+T_1 r}\right)\frac{1}{T_1}\iota_{T_1}\iota_{T_1}' \quad = \quad A_{T_1} + B_{T_1(r)}, \qquad (5)\\ A_{T_1} &= I_{T_1} - \frac{1}{T_1}\iota_{T_1}\iota_{T_1}' \quad B_{T_1(r)} = \left(\frac{1}{1+T_1 r}\right)\frac{1}{T_1}\iota_{T_1}\iota_{T_1}', \text{and} \\ V_{T_1(r)}^{-1/2} &= diag\left(\sqrt{\frac{T_1-1+\frac{1}{r}}{T_1+\frac{1}{r}}}, \sqrt{\frac{T_1-2+\frac{1}{r}}{T_1-1+\frac{1}{r}}}, ..., \sqrt{\frac{1+\frac{1}{r}}{2+\frac{1}{r}}}, \sqrt{\frac{\frac{1}{r}}{1+\frac{1}{r}}}\right) \\ &\times \begin{bmatrix} 1 & \frac{-1}{T_1-1+\frac{1}{r}} & \frac{-1}{T_1-1+\frac{1}{r}} & & \cdots & \frac{-1}{T_1-1+\frac{1}{r}} \\ 0 & 1 & \frac{-1}{T_1-2+\frac{1}{r}} & & \cdots & \frac{-1}{T_1-2+\frac{1}{r}} \\ 0 & 0 & 1 & & & \\ \vdots & \vdots & & \ddots & \ddots & \vdots \\ & & & & 1 & \frac{-1}{1+\frac{1}{r}} \\ 0 & 0 & & \cdots & 0 & 1 \end{bmatrix}. \qquad (6) \end{aligned}$$

Note that the relationship between (1) and (4) is very similar to that between the model in first differences and the model in the forward orthogonal deviations. This can be seen by observing that the model in forward orthogonal deviations is obtained by multiplying the upper triangular Cholesky factorization of the inverse of the covariance matrices of the error $\Delta\varepsilon_i$ to the original first difference model, while (4) is obtained by multiplying the upper triangular Cholesky factorization of the inverse of the pseudo covariance matrix $V_{T_1(r)}$ to the original level model[2].

Let us denote the $(t-1)$th row of (4) as

$$y_{it}^{+} = W_{it}^{+'}\delta + u_{it}^{+} \qquad (t = 2, ..., T;\ i = 1, ..., N). \qquad (7)$$

Using (6), (7) can be written as[3]

$$(b_t y_{it}^* + k_t y_{it}) = (b_t W_{it}^* + k_t W_{it})'\delta + (b_t v_{it}^* + k_t u_{it}) \qquad (8)$$
$$(t = 2, .., T-1;\ i = 1, ..., N),$$

where variables with $*$ are in forward orthogonal deviations and

$$\begin{aligned} b_t^2 &= \frac{(T-t+1)(T-t)}{(T-t+1+\frac{1}{r})(T-t+\frac{1}{r})} = 1 - \frac{2(T-t)+1+\frac{1}{r}}{r(T-t+1+\frac{1}{r})(T-t+\frac{1}{r})}, \text{and} \qquad (9)\\ k_t &= \frac{1}{r\sqrt{T-t+1+\frac{1}{r}}\sqrt{T-t+\frac{1}{r}}} = O\left(\frac{1}{T-t}\right). \qquad (10) \end{aligned}$$

From (9) and (10), we find that, as $T-t$ and/or r gets larger, $k_t \to 0$ and $b_t \to 1$. This implies that the transformed model (7) can be approximated by the model in forward orthogonal deviations if $T-t$ and/or r is large.

We now consider a GMM estimation of (4). Since u_{it}^{+} still contains individual effects, the first-differenced instruments $Z_{di}^{(h)}$ $(h = 0, 1, 2)$ must be used. Specifically,

[1] This form is obtained by Hayakawa (2008).

[2] The pseudo covariance matrix corresponds to $V_{T_1(r)}$ with arbitrary r which can be different from r_o.

[3] For derivation, see Hayakawa (2008)

the GMM estimator of model (4), which uses $Z_{di}^{(h)}$ $(h = 2, 1, 0)$, has the following form:

$$\widehat{\delta}_{\mathrm{GMMcd}(r)^{(h)}} = \left[\left(\sum_{i=1}^{N} W_i^{+\prime} Z_{di}^{(h)}\right)\left(\sum_{i=1}^{N} Z_{di}^{(h)\prime} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{di}^{(h)} W_i^{+}\right)\right]^{-1} \times \left(\sum_{i=1}^{N} W_i^{+\prime} Z_{di}^{(h)}\right)\left(\sum_{i=1}^{N} Z_{di}^{(h)\prime} Z_{di}^{(h)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{di}^{(h)} y_i^{+}\right).$$

We use the acronym "cd" to indicate that the model is transformed by Cholesky factorization and that differenced instruments are used. GMMcd$(r)^{(h)}$ is optimal when $r = r_o$ and not optimal when $r \neq r_o$ since u_{it}^{+} is serially correlated and heteroskedastic unless $r = r_o$. This can be observed by

$$E(u_{it}^{+2}) = \sigma_\varepsilon^2\left[1 + \frac{1}{r\left(T - t + 1 + \frac{1}{r}\right)\left(T - t + \frac{1}{r}\right)}\left(\frac{r_o}{r} - 1\right)\right],$$

$$E(u_{it}^{+} u_{is}^{+}) = \frac{\sigma_\varepsilon^2 d_t d_s}{\left(T - t + \frac{1}{r}\right)\left(T - s + \frac{1}{r}\right)}\frac{1}{r}\left(\frac{r_o}{r} - 1\right), \qquad (t > s)$$

where $d_t^2 = (T - t + \frac{1}{r})/(T - t + 1 + \frac{1}{r})$.

Further, for $h = 2, 1$, $\widehat{\delta}_{\mathrm{GMMcd}(r)^{(h)}}$ can be written as

$$\widehat{\delta}_{\mathrm{GMMcd}(r)^{(h)}} = \left[\sum_{t=2}^{T} W_t^{+\prime} M_{Z_{dt}}^{(h)} W_t^{+}\right]^{-1}\left[\sum_{t=2}^{T} W_t^{+\prime} M_{Z_{dt}}^{(h)} y_t^{+}\right] \qquad (h = 2, 1),$$

where $W_t^{+} = (W_{1t}^{+}, ..., W_{Nt}^{+})'$, $y_t^{+} = (y_{1t}^{+}, ..., y_{Nt}^{+})'$, and $M_{Z_{dt}}^{(h)}$ is as defined in BK.

Lastly, we consider a system of the model transformed by forward orthogonal deviations and the Cholesky-transformed model (4). For $k = 21, 11, 00$, we define the GMMs$^{(k)}$ and GMMs$(r)^{(k)}$ estimators as follows[4]:

$$\widehat{\delta}_{\mathrm{GMMs}^{(k)}} = \left[\left(\sum_{i=1}^{N} W_i^{\dagger\prime} Z_{si}^{(k)}\right)\left(\sum_{i=1}^{N} Z_{si}^{(k)\prime} Z_{si}^{(k)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{si}^{(k)} W_i^{\dagger}\right)\right]^{-1} \times \left(\sum_{i=1}^{N} W_i^{\dagger\prime} Z_{si}^{(k)}\right)\left(\sum_{i=1}^{N} Z_{si}^{(k)\prime} Z_{si}^{(k)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{si}^{(k)} y_i^{\dagger}\right) \quad (k = 21, 11, 00),$$

$$\widehat{\delta}_{\mathrm{GMMs}(r)^{(k)}} = \left[\left(\sum_{i=1}^{N} W_i^{\ddagger\prime} Z_{si}^{(k)}\right)\left(\sum_{i=1}^{N} Z_{si}^{(k)\prime} Z_{si}^{(k)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{si}^{(k)} W_i^{\ddagger}\right)\right]^{-1} \times \left(\sum_{i=1}^{N} W_i^{\ddagger\prime} Z_{si}^{(k)}\right)\left(\sum_{i=1}^{N} Z_{si}^{(k)\prime} Z_{si}^{(k)}\right)^{-1}\left(\sum_{i=1}^{N} Z_{si}^{(k)} y_i^{\ddagger}\right) \quad (k = 21, 11, 00),$$

where $W_i^{\dagger} = \left(W_i^{*\prime}, W_i^{\prime}\right)'$, $y_i^{\dagger} = \left(y_i^{*\prime}, y_i^{\prime}\right)'$, $W_i^{\ddagger} = \left(W_i^{*\prime}, W_i^{+\prime}\right)'$, $y_i^{\ddagger} = \left(y_i^{*\prime}, y_i^{+\prime}\right)'$, $Z_{si}^{(21)} = diag\left(Z_{li}^{(2)}, Z_{di}^{(1)}\right)$, $Z_{si}^{(11)} = diag\left(Z_{li}^{(1)}, Z_{di}^{(1)}\right)$, and $Z_{si}^{(00)} = diag\left(Z_{li}^{(0)}, Z_{di}^{(0)}\right)$.

[4] Chigira and Yamamoto (2006) consider a GMM estimator similar to GMMs$^{(00)}$ where the number of instruments is $O(1)$.

Note that, for $k = 21, 11$, $\widehat{\delta}_{\text{GMMs}^{(k)}}$ and $\widehat{\delta}_{\text{GMMs}(r)^{(k)}}$ can be written as

$$\widehat{\delta}_{\text{GMMs}^{(h1)}} = \left[\sum_{t=1}^{T-1} W_t^{*\prime} M_{Z_{lt}}^{(h)} W_t^* + \sum_{t=2}^{T} W_t^{\prime} M_{Z_{dt}}^{(1)} W_t\right]^{-1} \times \left[\sum_{t=1}^{T-1} W_t^{*\prime} M_{Z_{lt}}^{(h)} y_t^* + \sum_{t=2}^{T} W_t^{\prime} M_{Z_{dt}}^{(1)} y_t\right]. \quad h = 2, 1 \text{ and}$$

$$\widehat{\delta}_{\text{GMMs}(r)^{(h1)}} = \left[\sum_{t=1}^{T-1} W_t^{*\prime} M_{Z_{lt}}^{(h)} W_t^{*\prime} + \sum_{t=2}^{T} W_t^{+\prime} M_{Z_{dt}}^{(1)} W_t^{+}\right]^{-1} \times \left[\sum_{t=1}^{T-1} W_t^{*\prime} M_{Z_{lt}}^{(h)} y_t^* + \sum_{t=2}^{T} W_t^{+\prime} M_{Z_{dt}}^{(1)} y_t^{+}\right] \quad h = 2, 1,$$

where $M_{Z_{lt}}^{(h)}$, $(h = 2, 1)$ is as defined in BK. Note that GMMs$^{(21)}$ is the system GMM estimator BK consider. The difference among these estimators lies in the number of instruments and whether or not the level model is transformed((1) or (4)).

It should be noted that, among the estimators considered in BK and this paper, only GMMfl$^{(h)}$ is consistent for small T and large N even when effect-stationarity (also called as mean-stationarity) does not hold.

3 Finite sample bias

In this section, we first discuss the relationship between the GMMld$(r)^{(h)}$ and GMMcd$(r)^{(h)}$ estimators, and then derive the leading term of the finite sample bias of the estimators.

The following proposition establishes the equivalence of GMMld$(r)^{(2)}$ and GMMcd$(r)^{(2)}$.

Proposition 1. *GMMld$(r)^{(2)}$ and GMMcd$(r)^{(2)}$ are identical. This is also true if $V_{T_1(r)}$ in (2) and (3) is replaced with its consistent estimate $\widehat{V}_{T_1} = N^{-1}\sum_{i=1}^{N} \widehat{\dot{u}}_i \widehat{\dot{u}}_i^{\prime}$, where $\widehat{\dot{u}}_i$ is a consistent estimate of $\dot{u}_i$.*

This result can be proved in exactly the same way as in Arellano and Bover (1995) and Arellano (2003, pp.152–154). Some remarks are in order.

Remark 1. Arellano and Bover (1995) show that the one-step first-difference GMM estimator using the optimal weighting matrix and all available instruments is identical to GMMfl$^{(2)}$. Proposition 1 indicates that a similar relationship holds for GMMld$(r)^{(2)}$ and GMMcd$(r)^{(2)}$ although they are not necessarily optimal.

Remark 2. Since GMMcd$(r)^{(2)}$ is identical to GMMld$(r)^{(2)}$, the order of magnitude of the finite sample bias of GMMcd$(r)^{(2)}$ derived in the proposition below corresponds to that of GMMld$(r)^{(2)}$.

Remark 3. It should be noted that the equivalence result in Proposition 1 holds only when $h = 2$ and weighting matrix $N^{-1}\sum_{i=1}^{N} Z_{di}^{(2)\prime} \widehat{V}_{T_1} Z_{di}^{(2)}$ is used. If $h = 0, 1$ or a more robust weighting matrix such as $N^{-1}\sum_{i=1}^{N} Z_{di}^{(h)\prime} \widehat{\dot{u}}_i \widehat{\dot{u}}_i^{\prime} Z_{di}^{(h)}$ is used, GMMcd$(r)^{(h)}$ and GMMld$(r)^{(h)}$ are not identical.

The following proposition shows the order of magnitude of the finite sample bias of the estimators defined above.

Proposition 2. *The order of magnitude of the finite sample biases of the various (G)MM estimators are given by*

$$(a) \quad B_{\mathrm{GMMld}^{(2)}} = \sigma_\eta^2(T^2+T-2)\bar{Q}^{-1}_{\mathrm{GMMld}^{(2)}} \begin{bmatrix} \frac{1+\beta\pi}{1-\gamma} \\ \pi \end{bmatrix} = O(TN^{-1}),$$

$$(b) \quad B_{\mathrm{GMMcd}(r_o)^{(2)}} = 2\sigma_\varepsilon^2 \bar{Q}^{-1}_{\mathrm{GMMcd}(r_o)^{(2)}} \begin{bmatrix} \sum_{s=1}^{T-1} tr\left\{\gamma r_o V^{-1}_{s(r_o)}\iota_s\iota_s' + V^{-1}_{s(r_o)}L_s\Gamma_s + \beta\phi V^{-1}_{s(r_o)}L_s\Gamma_s L_s\right\} \\ \sum_{s=1}^{T-1} tr\left\{\pi r_o V^{-1}_{s(r_o)}\iota_s\iota_s' + \phi V^{-1}_{s(r_o)}L_s\right\} \end{bmatrix}$$
$$= O(N^{-1}),$$

$$(c) \quad B_{\mathrm{GMMld}^{(1)}} = 2\sigma_\eta^2(T-1)\bar{Q}^{-1}_{\mathrm{GMMld}^{(1)}} \begin{bmatrix} \frac{1+\beta\pi}{1-\gamma} \\ \pi \end{bmatrix} = O(N^{-1}),$$

$$(d) \quad B_{\mathrm{GMMcd}(r_o)^{(1)}} = 2\sigma_\varepsilon^2 \bar{Q}^{-1}_{\mathrm{GMMcd}(r_o)^{(1)}} \begin{bmatrix} tr\left\{\gamma r_o V^{-1}_{T_1(r_o)}\iota_{T_1}\iota_{T_1}' + V^{-1}_{T_1(r_o)}L_{T_1}\Gamma_{T_1} + \beta\phi V^{-1}_{T_1(r_o)}L_{T_1}\Gamma_{T_1}L_{T_1}\right\} \\ tr\left\{\pi r_o V^{-1}_{T_1(r_o)}\iota_{T_1}\iota_{T_1}' + \phi V^{-1}_{T_1(r_o)}L_{T_1}\right\} \end{bmatrix}$$
$$= O(N^{-1}T^{-1}),$$

$$(e) \quad B_{\mathrm{GMMld}^{(0)}} = O(N^{-1}T^{-1}),$$

$$(f) \quad B_{\mathrm{GMMcd}(r_o)^{(0)}} = O(N^{-1}T^{-1}),$$

$$(g) \quad B_{\mathrm{GMMs}^{(21)}} = \bar{Q}^{-1}_{\mathrm{GMMs}^{(21)}}\left(\bar{Q}_{\mathrm{GMMfl}^{(2)}}B_{\mathrm{GMMfl}^{(2)}} + \bar{Q}_{\mathrm{GMMld}^{(1)}}B_{\mathrm{GMMld}^{(1)}}\right)$$
$$= -2T\times\sigma_\varepsilon^2\bar{Q}^{-1}_{\mathrm{GMMs}^{(21)}} \begin{bmatrix} \frac{1+\beta\phi-(1+\beta\pi)r_o}{1-\gamma} \\ \phi-\pi r_o \end{bmatrix} = O(N^{-1}),$$

$$(h) \quad B_{\mathrm{GMMs}(r_o)^{(21)}} = \bar{Q}^{-1}_{\mathrm{GMMs}(r_o)^{(21)}}\left(\bar{Q}_{\mathrm{GMMfl}^{(2)}}B_{\mathrm{GMMfl}^{(2)}} + \bar{Q}_{\mathrm{GMMcd}(r_o)^{(1)}}B_{\mathrm{GMMcd}(r_o)^{(1)}}\right)$$
$$= O(N^{-1}),$$

$$(i) \quad B_{\mathrm{GMMs}^{(11)}} = \bar{Q}^{-1}_{\mathrm{GMMs}^{(11)}}\left(\bar{Q}_{\mathrm{GMMfl}^{(1)}}B_{\mathrm{GMMfl}^{(1)}} + \bar{Q}_{\mathrm{GMMld}^{(1)}}B_{\mathrm{GMMld}^{(1)}}\right)$$
$$= 2\sigma_\eta^2(T-1)\bar{Q}^{-1}_{\mathrm{GMMs}^{(11)}} \begin{bmatrix} \frac{1+\beta\pi}{1-\gamma} \\ \pi \end{bmatrix} + O(N^{-1}T^{-1}) = O(N^{-1})$$

$$(j) \quad B_{\mathrm{GMMs}(r_o)^{(11)}} = \bar{Q}^{-1}_{\mathrm{GMMs}(r_o)^{(11)}}\left(\bar{Q}_{\mathrm{GMMfl}^{(1)}}B_{\mathrm{GMMfl}^{(1)}} + \bar{Q}_{\mathrm{GMMcd}(r_o)^{(1)}}B_{\mathrm{GMMcd}(r_o)^{(1)}}\right)$$
$$= O(N^{-1}T^{-1}),$$

(k) $B_{\mathrm{GMMs}^{(00)}} = O(N^{-1}T^{-1})$, and

(l) $B_{\mathrm{GMMs}(r_o)^{(00)}} = O(N^{-1}T^{-1})$,

where the various $\bar{Q}$s are defined in the same way as in BK.

Note that (a), (c), (e), and (g) are derived in BK, and others are new. In Table 1, we summarize the orders of magnitude of the finite sample biases of the various estimators, which extends Table 1 in BK[5].

Remark 4. From (a) and (b), we find that the orders of magnitude of the bias of GMMld$^{(2)}$ and GMMcd$(r)^{(2)}$ are $O(T/N)$ and $O(1/N)$, respectively. Since GMMcd$(r)^{(2)}$ is not optimal unless $r = r_o$, it can be concluded that whether an optimal weighting matrix is used or not is inconsequential. Instead, it might be considered that what is important for a reduction in the order of magnitude of bias is a particular GLS transformation that reduces the endogeneity bias from $O(1)$ to $O(1/T)$ because the major difference between the original level model (1) and the transformed model (4) is the degree of endogeneity bias. The degree of endogeneity bias in (1) is $E(\dot{W}_i'\dot{u}_i/T_1) = O(1)$ while that in (4) is $E(W_i^{+'}u_i^+/T_1) = O(1/T)$. This difference effectively reduces the bias of the estimator. Also, notice that a similar relationship holds for the models in first-differences and forward orthogonal deviations.

Remark 5. Comparing (c) with (d), it is found that GMMcd$(r)^{(1)}$ has a smaller order of magnitude of finite sample bias than GMMld$^{(1)}$. Since the only difference between the two estimators is whether or not the model is transformed (both estimators are in 2SLS form and use the same instruments $Z_{di}^{(1)}$), this result illustrates the effectiveness of the pseudo GLS transformation as (6).

Remark 6. Comparing (g) with (h), we observe that both GMMs$^{(21)}$ and GMMs$(r)^{(21)}$ estimators have the same order of magnitude of bias. This is because both contain $B_{\text{GMMfl}^{(2)}}$, which is $O(1/N)$. If we reduce the number of instruments from $O(T^2)$ to $O(T)$ for the GMMfl estimator, a reduction of bias is achieved only for GMMs$(r)^{(11)}$, which can be seen by comparing (i) and (j). If we use $Z_{si}^{(00)}$, the order of magnitude of the bias for both GMMs$^{(00)}$ and GMMs$(r)^{(00)}$ is $O(1/NT)$.

Since we usually do not have an information on r in practice, we suggest to use two-step estimators for r_o, which we call $\widehat{r}$. In the next section, we investigate the effect of a two-step procedure on the finite sample bias by Monte Carlo simulation.

4 Simulation results

In this section, we conduct a Monte Carlo simulation to compare the performance of several estimators and assess the accuracy of the bias approximation. In addition to the estimators reported in BK, we also report the results for GMMcd$(r_o)^{(h)}$ ($h = 2, 1, 0$), GMMs$^{(k)}$ and GMMs$(r_o)^{(k)}$ ($k = 21, 11, 00$), and some of their two-step versions. The simulation design is exactly the same as BK and details are omitted here[6]. To save space, we only report Table 3 (designs 11 and 12) in BK as Table 2. All other results are available from the author upon request. Note that the following simulation summaries are observed from the all simulation results and might not be observed from Table 2 only.

We now summarize the simulation results. We first compare the overall performances of GMMld$^{(h)}$, GMMcd$(r_o, \widehat{r})^{(h)}$, GMMs$^{(k)}$, GMMs$(r_o, \widehat{r})^{(k)}$ ($h = 2, 1, 0;\ k =$

[5]We omitted the restrictions on N, T, and K.

[6]The magnitude of the theoretical bias of GMMld$^{(1)}$ reported in BK is incorrect. Their results should be doubled.

$21, 11, 00$) and then investigate the effects of the changes in some parameters. For two-step estimators, we use $\widehat{r}$ obtained from GMMfl$^{(0)}$[7]. By comparing the results for estimators using r_o and $\widehat{r}$, we can investigate the effect of the two-step procedure. From the results, we find that the difference in bias between the estimators using $\widehat{r}$ and r_o is marginal in many cases. Further, it is found that the two-step estimators have larger dispersion than the infeasible estimators. This seems natural since the first-step estimate generally causes an additional variability in the two-step estimate.

With regard to the performance of GMMld$^{(h)}$ and GMMcd$(\widehat{r})^{(h)}$, we find that the bias of GMMcd$(\widehat{r})^{(h)}$ $(h = 2, 1)$ is significantly smaller than that of GMMld$^{(h)}$ $(h = 2, 1)$ in many cases. This result supports the theoretical implication that the order of magnitude of the bias of GMMcd$(r_o)^{(h)}$ $(h = 2, 1)$ is smaller than that of GMMld$^{(h)}$ $(h = 2, 1)$, and indicates that the pseudo GLS transformation is effective in reducing the bias. However, surprisingly, on observing the standard deviation, we find that GMMcd$(r_o, \widehat{r})^{(h)}$ are more dispersed than GMMld$^{(h)}$ even though GMMcd$(r_o)^{(2)}$ uses the optimal weighting matrix and GMMld$^{(2)}$ does not. This result seems to be design specific since unreported simulation results using other DGPs show that GMMcd$(r_o)^{(2)}$ has smaller standard deviations than GMMld$^{(2)}$. Further, it is observed that GMMld$^{(2),(1)}$ are vulnerable for large $\sigma_\eta^2/\sigma_\varepsilon^2$, while GMMcd$(r_o, \widehat{r})^{(2),(1)}$ are not.

For the GMMs$^{(21)}$ and GMMs$(\widehat{r})^{(21)}$ estimators, because both estimators have a bias of order $O(1/N)$, the results are similar and the superiority depends on the design. However, for GMMs$^{(11)}$ and GMMs$(r_o, \widehat{r})^{(11)}$, although the order of the bias of GMMs$(r)^{(11)}$ is smaller than that of GMMs$^{(11)}$, the magnitude of bias of both estimators are similar when r_o is small. This is partly caused by the structure of GMMs$^{(k)}$ (GMMs$(r_o, \widehat{r})^{(k)}$), which is a matrix weighted average of GMMfl$^{(h)}$ and GMMld$^{(h)}$(GMMcd$(r_o, \widehat{r})^{(h)}$) with opposite signs[8]. With regard to dispersion, although it is observed that GMMs$(\widehat{r})^{(21)}$ is less efficient than GMMcd$(\widehat{r})^{(2)}$ in many cases, this may be the result of using a non-optimal weighting matrix for GMMs$(\widehat{r})^{(21)}$[9]. However, it should be noted that the bias of GMMs$^{(21),(11)}$ becomes substantial if $\sigma_\eta^2/\sigma_\varepsilon^2$ is large, while that of GMMs$(r_o, \widehat{r})^{(21),(11)}$ does not.

Next, we investigate the effect of changes in key parameters ρ, μ, ζ, ϕ, and π. Comparing designs 1 and 2, it is observed that as ρ gets larger, the biases of GMMcd$(\widehat{r})^{(h)}$ and GMMs$(\widehat{r})^{(h)}$ increase slightly, while the dispersion of the estimates of β increases substantially. Next, we investigate the effect of μ by comparing design 3 with 4, 5 with 6, and 11 with 12. From the results, we find that although the biases of GMMld$^{(2),(1)}$ and GMMs$^{(21),(11)}$ increase with μ, those of GMMcd$(\widehat{r})^{(2),(1)}$ and GMMs$(\widehat{r})^{(21),(11)}$ do not. This indicates that in terms of bias, GMMcd$(\widehat{r})^{(2),(1)}$ and GMMs$(\widehat{r})^{(21),(11)}$ are robust to large μ, while GMMld$^{(2),(1)}$ and GMMs$^{(21),(11)}$ are not. To assess the effect of ζ, we compare design 2 with 5 and 4 with 6, and

[7]The reason why we use GMMfl$^{(0)}$ is that GMMfl$^{(0)}$ has the smallest bias among the GMMfl estimators and its consistency is obtained under conditions weaker than those for the other estimators such as GMMs$^{(k)}$. The last column of the table shows the average over all Monte Carlo replications of the estimate of r. From the results, we find that $\widehat{r}$ obtained from GMMfl$^{(0)}$ performs best in many cases.

[8]An analysis for the AR(1) case is given by Hayakawa (2007).

[9]With the sample size considered here, it is not possible to compute GMMs$^{(21)}$ with the optimal weighting matrix.

find that although the magnitude of the biases of GMMcd$(\widehat{r})^{(h)}$ and GMMs$(\widehat{r})^{(k)}$ are hardly affected by ζ, their dispersion decreases as ζ increases. With regard to the effect of ϕ, on comparing designs 2, 7, and 8, we find that a change in ϕ does not have a significant effect on the biases of GMMcd$(\widehat{r})^{(h)}$ and GMMs$(\widehat{r})^{(k)}$. However, the standard deviation for the estimates of β with $\phi = \pm 1$ is much smaller than that for the estimate of β with $\phi = 0$. With regards to the change in π, on comparing design 2 with 9 and 10, it is observed that the effect of a change in π is only marginal for GMMcd$(\widehat{r})^{(h)}$ and GMMs$(\widehat{r})^{(k)}$.

Table 5, which is not reported here, shows the results for $N = 50, T = 5, 10, 20, 50$ with design 11. From the results, we find that as T increases, the biases of all the GMM estimators, except for GMMld$^{(2)}$, decrease. This is consistent with the theoretical results. Comparing the results for scheme 2 with those for scheme 1, we find that although the changes in bias are marginal, those in standard deviation, especially for β, are substantial. These results are similar to those of BK.

Although the simulation setting is somewhat limited, it may be useful to provide a guidance for practical use. Although there is no estimator that dominates over the rest in all cases, under the assumption of effect stationarity, it may be advisable to use GMMcd$(\widehat{r})^{(2)}$ and GMMs$(\widehat{r})^{(k)}$ $(k = 21, 11, 00)$ since, on the whole, these are robust to large $\sigma_\eta^2/\sigma_\varepsilon^2$ and their bias and RMSE are reasonably small.

5 Conclusion

In this paper, we showed that the orders of magnitude of the finite sample biases of GMMld estimators in 2SLS form can be improved if the model is transformed by the upper triangular Cholesky factorization of the inverse of the pseudo variance matrix of error component u_i wherein true values of the variances of individual effects and disturbances may not be used. For instance, we showed that the order of magnitude of the finite sample bias of GMMcd$(r)^{(2)}$ estimator is $O(1/N)$, while that of GMMld$^{(2)}$ is $O(T/N)$ where r may be different from its true value r_o. We also considered some further variants of GMM estimators that are associated with the Cholesky-transformed model, and derived their order of magnitudes of the finite sample biases.

With regard to the relationship between GMMcd(r) and GMMld(r) estimators, we showed that GMMld$(r)^{(2)}$, which uses all available instruments, is identical to GMMcd$(r)^{(2)}$, a GMM estimator for the Cholesky-transformed model that uses all available instruments. This relationship is parallel to that between the first-difference GMM estimator using an optimal weighting matrix and GMMfl$^{(2)}$ estimator.

Simulation results show that transforming the model by Cholesky factorization is effective in reducing the bias and makes the GMM estimators robust to large $\sigma_\eta^2/\sigma_\varepsilon^2$. Hence, although two-step procedure is required, it may be advisable to use a Cholesky-transformed model instead of the original level models when using GMM estimators such as system GMM estimators.

A Proof of Proposition 2

We shall prove Proposition 2. For the proofs of (a), (c), (e), and (g), see BK.
(b), (d): Since $u_{it}^{+} = v_t'\dot{u}_i$, $y_{i,t-1}^{+} = v_t'\dot{y}_{i(-1)}$, and $x_{it}^{+} = v_t'\dot{x}_i$, where v_t is the $(t-1)$th row of $V_{T_1(r)}^{-1/2}$, we have

$$\begin{aligned} E\left(W_t^{+'} M_{Z_{dt}}^{(h)} u_t^{+}\right) &= m_{dt}^{(h)} \begin{bmatrix} E\left(u_{it}^{+} y_{i,t-1}^{+}\right) \\ E\left(u_{it}^{+} x_{it}^{+}\right) \end{bmatrix} = m_{dt}^{(h)} \begin{bmatrix} E\left(v_t'\dot{u}_i v_t'\dot{y}_{i(-1)}\right) \\ E\left(v_t'\dot{u}_i v_t'\dot{x}_i\right) \end{bmatrix} \\ &= m_{dt}^{(h)} \begin{bmatrix} tr\left(E[\dot{u}_i\dot{y}_{i(-1)}']v_t v_t'\right) \\ tr\left(E[\dot{u}_i\dot{x}_i']v_t v_t'\right) \end{bmatrix}. \end{aligned}$$

Hence, for $h = 2$, using $m_{dt}^{(2)} = 2(t-1)$ and

$$\begin{aligned} E(\dot{u}_i\dot{y}_{i(-1)}') &= \sigma_\varepsilon^2 \left[\gamma r \iota_{T_1}\iota_{T_1}' + (I_{T_1} + \beta\phi L_{T_1}')\Gamma_{T_1}' L_{T_1}'\right] \\ E(\dot{u}_i\dot{x}_i') &= \sigma_\varepsilon^2 \left[\pi r \iota_{T_1}\iota_{T_1}' + \phi L_{T_1}'\right], \end{aligned}$$

we have

$$\begin{aligned} \sum_{t=2}^{T} E(W_t^{+'} M_{Z_{dt}} u_t^{+}) &= \begin{bmatrix} tr\left\{E\left(\dot{u}_i\dot{y}_{i(-1)}'\right)\sum_{t=2}^{T} m_{dt}^{(2)} v_t v_t'\right\} \\ tr\left\{E\left(\dot{u}_i\dot{x}_i'\right)\sum_{t=2}^{T} m_{dt}^{(2)} v_t v_t'\right\} \end{bmatrix} \\ &= 2\sigma_\varepsilon^2 \begin{bmatrix} \sum_{s=1}^{T-1} tr\left\{\left(\gamma r\iota_{T_1}\iota_{T_1}' + (I_{T_1} + \beta\phi L_{T_1}')\Gamma_{T_1}' L_{T_1}'\right)\dot{H}_s V_{s(r)}^{-1}\dot{H}_s'\right\} \\ \sum_{s=1}^{T-1} tr\left\{(\pi r\iota_{T_1}\iota_{T_1}' + \phi L_{T_1}')\dot{H}_s V_{s(r)}^{-1}\dot{H}_s'\right\} \end{bmatrix} \\ &= 2\sigma_\varepsilon^2 \begin{bmatrix} \sum_{s=1}^{T-1} tr\left\{\gamma r V_{s(r)}^{-1}\iota_s\iota_s' + V_{s(r)}^{-1} L_s\Gamma_s + \beta\phi V_{s(r)}^{-1} L_s\Gamma_s L_s\right\} \\ \sum_{s=1}^{T-1} tr\left\{\pi r V_{s(r)}^{-1}\iota_s\iota_s' + \phi V_{s(r)}^{-1} L_s\right\} \end{bmatrix} \qquad (11) \\ &= O(T) \end{aligned}$$

where $V_{s(r)} = I_s + r\iota_s\iota_s'$. The second equality is obtained from

$$\sum_{t=2}^{T}(t-1)v_t v_t' = \sum_{s=1}^{T-1} s v_{s+1} v_{s+1}' = \sum_{s=1}^{T-1} \dot{H}_s V_{s(r)}^{-1}\dot{H}_s'$$

with $\dot{H}_s = (O \ : \ I_s)'$ being a $T_1 \times s$ matrix. The last equality holds since, using (5) and results of BK(p.439), and after some algebra, the summand of (11) can be written as

$$\begin{aligned} & tr\left\{A_s L_s\Gamma_s + \beta\phi A_s L_s\Gamma_s L_s + \gamma r V_{s(r)}^{-1}\iota_s\iota_s' + B_{s(r)} L_s\Gamma_s + \beta\phi B_{s(r)} L_s\Gamma_s L_s\right\} \qquad (12) \\ &= O(1) + \gamma r\left(\frac{s}{1+rs}\right) + \frac{1}{(1+rs)s}\frac{1}{1-\gamma}\left[(s-1) - \frac{\gamma-\gamma^s}{1-\gamma}\right] \\ &\qquad + \frac{\beta\phi}{(1+rs)s}\frac{1}{1-\gamma}\left[(s-2) - \frac{\gamma-\gamma^{s-1}}{1-\gamma}\right] \\ &= O(1) \end{aligned}$$

and

$$tr\left\{\pi r V_{s(r)}^{-1}\iota_s\iota_s' + \phi V_{s(r)}^{-1} L_s\right\} = tr\left\{\phi A_s L_s + \pi r V_{s(r)}^{-1}\iota_s\iota_s' + B_{s(r)} L_s\right\} \qquad (13)$$

$$
\begin{aligned}
&= O(1) + \pi r \left(\frac{s}{1+rs} \right) + \frac{s-1}{(1+rs)s} \\
&= O(1).
\end{aligned}
$$

Note that deriving the leading term of (11) is not easy since it is difficult to derive the leading term of $\sum_{s=1}^{T-1} s/(1+rs)$.

For $h = 1$, since $m_{dt}^{(1)} = 2$,

$$
\begin{aligned}
\sum_{t=2}^{T} E(W_t^{+\prime} M_{Z_{dt}} u_t^{+}) &= \begin{bmatrix} tr \left\{ E\left(\dot{u}_i \dot{y}_{i(-1)}' \right) \sum_{t=2}^{T} m_{dt}^{(1)} v_t v_t' \right\} \\ tr \left\{ E\left(\dot{u}_i \dot{x}_i' \right) \sum_{t=2}^{T} m_{dt}^{(1)} v_t v_t' \right\} \end{bmatrix} \\
&= 2\sigma_\varepsilon^2 \begin{bmatrix} tr \left\{ \left(\gamma r \iota_{T_1} \iota_{T_1}' + \Gamma_{T_1}' L_{T_1}' + \beta \phi L_{T_1}' \Gamma_{T_1}' L_{T_1}' \right) V_{T_1(r)}^{-1} \right\} \\ tr \left\{ \left(\pi r \iota_{T_1} \iota_{T_1}' + \phi L_{T_1}' \right) V_{T_1(r)}^{-1} \right\} \end{bmatrix} \\
&= 2\sigma_\varepsilon^2 \begin{bmatrix} tr \left\{ \gamma r V_{T_1(r)}^{-1} \iota_{T_1} \iota_{T_1}' + V_{T_1(r)}^{-1} L_{T_1} \Gamma_{T_1} + \beta \phi V_{T_1(r)}^{-1} L_{T_1} \Gamma_{T_1} L_{T_1} \right\} \\ tr \left\{ \pi r V_{T_1(r)}^{-1} \iota_{T_1} \iota_{T_1}' + \phi V_{T_1(r)}^{-1} L_{T_1} \right\} \end{bmatrix} \\
&= O(1)
\end{aligned}
$$

where in the second equality, we used the following fact:

$$
\sum_{t=2}^{T} v_t v_t' = V_{T_1(r)}^{-1}.
$$

The last equality is proved by replacing s with T_1 in (12) and (13).

(f): The proof is exactly the same as BK, and hence omitted.

(g),(i),(j): Using the results of BK and above, the proofs are straightforward.

(k),(l): The proofs are exactly the same as BK, and hence omitted.

References

[1] Alvarez, J. and M. Arellano (2003) "The Time Series and Cross-Section Asymptotics of Dynamic Panel Data Estimators," *Econometrica*, 71, 4, 1121–1159.

[2] Arellano, M. (2003) *Panel Data Econometrics*, Oxford University Press.

[3] Arellano, M. and O. Bover (1995) "Another Look at the Instrumental Variable Estimation of Error-Components Models," *Journal of Econometrics*, 68, 1, 29–51.

[4] Bun, M. J. G. and J. F. Kiviet (2006) "The Effects of Dynamic Feedbacks on LS and MM Estimator Accuracy in Panel Data Models," *Journal of Econometrics*, 127, 2, 409–444.

[5] Chigira, H. and T. Yamamoto (2006) "A Bias Corrected Estimation for Dynamic Panel Models in Small Samples," Hi-Stat Discussion Paper No. 177, Hitotsubashi University.

[6] Hayakawa, K. (2007) "Small Sample Bias Properties of the System GMM Estimator in Dynamic Panel Data Models," *Economics Letters*, 95, 1, 32–38.

[7] Hayakawa, K. (2008) “The Asymptotic Properties of the System GMM Estimator in Dynamic Panel Data Models When Both N and T are Large.” mimeo.

Table 1: Characteristics and order of magnitude of finite sample bias of LS and MM

estimator	model	instruments	finite sample bias	Bias affected by $\sigma_\eta^2/\sigma_\varepsilon^2$ and π
LSDV	FOD	—-	$O(N^0T^{-1})$	No
GLS	ChLev	—-	$O(N^0T^{-1})$	Heavily
GMMfl$^{(2)}$	FOD	L2	$O(N^{-1}T^0)$	Yes
GMMfl$^{(1)}$	FOD	L1	$O(N^{-1}T^{-1})$	Yes
GMMfl$^{(0)}$	FOD	L0	$O(N^{-1}T^{-1})$	Yes
GMMld$^{(2)}$	Lev	D2	$O(N^{-1}T)$	Heavily
GMMcd$(r)^{(2)}$	ChLev	D2	$O(N^{-1}T^0)$	Yes
GMMld$^{(1)}$	Lev	D1	$O(N^{-1}T^0)$	Heavily
GMMcd$(r)^{(1)}$	ChLev	D1	$O(N^{-1}T^{-1})$	Yes
GMMld$^{(0)}$	Lev	D0	$O(N^{-1}T^{-1})$	Heavily
GMMcd$(r)^{(0)}$	ChLev	D0	$O(N^{-1}T^{-1})$	Yes
GMMs$^{(21)}$	FOD & Lev	L2 & D1	$O(N^{-1}T^0)$	Heavily
GMMs$(r)^{(21)}$	FOD & ChLev	L2 & D1	$O(N^{-1}T^0)$	Yes
GMMs$^{(11)}$	FOD & Lev	L1 & D1	$O(N^{-1}T^0)$	Heavily
GMMs$(r)^{(11)}$	FOD & ChLev	L1 & D1	$O(N^{-1}T^{-1})$	Yes
GMMs$^{(00)}$	FOD & Lev	L0 & D0	$O(N^{-1}T^{-1})$	Heavily
GMMs$(r)^{(00)}$	FOD & ChLev	L0 & D0	$O(N^{-1}T^{-1})$	Yes

FOD: $y_{it}^* = \delta' W_{it}^* + \varepsilon_{it}^*$, ChLev: $y_{it}^+ = \delta' W_{it}^+ + u_{it}^+$, Lev: $y_{it} = \delta' W_{it} + u_{it}$

L2: $Z_{li}^{(2)}$, L1: $Z_{li}^{(1)}$, L0: $Z_{li}^{(0)}$, D2: $Z_{di}^{(2)}$, D1: $Z_{di}^{(1)}$, D0: $Z_{di}^{(0)}$

Table 2: Simulation results for $T = 10$, $N = 20$, scheme 1, disturbances normal

Design	Estimator	$\bar{B}_{\gamma}$	$\bar{B}_{\beta}$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
1	LSDV	-0.14	0.00	-0.14	0.00	0.06	0.04	0.15	0.04	
	GLS	0.03	0.00	0.02	0.00	0.04	0.04	0.05	0.04	
$\gamma = 0.75$	FGLS			0.12	-0.03	0.04	0.04	0.13	0.05	
$\zeta = 3$	GMMfl$^{(2)}$	-0.16	-0.01	-0.14	-0.01	0.08	0.04	0.16	0.04	0.54
$\pi = 0$	GMMfl$^{(1)}$	-0.04	-0.01	-0.05	-0.01	0.10	0.05	0.11	0.05	0.31
$\phi = 0$	GMMfl$^{(0)}$			-0.01	0.00	0.11	0.05	0.11	0.05	0.23
$\mu = 1$	GMMld$^{(2)}$	0.09	-0.01	0.07	-0.01	0.05	0.04	0.08	0.04	
$\rho = 0.5$	GMMcd$(r_o)^{(2)}$	0.04	0.00	0.03	0.00	0.05	0.04	0.06	0.04	
$r_o \approx 0.143$	GMMcd$(\widehat{r})^{(2)}$			0.02	0.00	0.06	0.04	0.06	0.04	
	GMMld$^{(1)}$	0.05	0.00	0.05	0.00	0.08	0.05	0.09	0.05	
	GMMcd$(r_o)^{(1)}$	0.01	0.00	0.01	0.00	0.09	0.06	0.10	0.06	
	GMMcd$(\widehat{r})^{(1)}$			0.00	0.00	0.11	0.06	0.11	0.06	
	GMMld$^{(0)}$			-0.01	0.00	0.09	0.05	0.09	0.05	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.11	0.06	0.11	0.06	
	GMMcd$(\widehat{r})^{(0)}$			-0.02	0.00	0.11	0.06	0.11	0.06	
	GMMs$^{(21)}$	-0.04	0.00	-0.05	0.00	0.06	0.04	0.08	0.04	
	GMMs$(r_o)^{(21)}$	-0.07	0.00	-0.09	-0.01	0.07	0.04	0.11	0.04	
	GMMs$(\widehat{r})^{(21)}$			-0.09	-0.01	0.08	0.04	0.12	0.04	
	GMMs$^{(11)}$	0.02	0.00	0.01	0.00	0.07	0.04	0.07	0.04	
	GMMs$(r_o)^{(11)}$	-0.01	0.00	-0.01	0.00	0.07	0.04	0.07	0.04	
	GMMs$(\widehat{r})^{(11)}$			-0.02	0.00	0.09	0.05	0.09	0.05	
	GMMs$^{(00)}$			-0.01	0.00	0.08	0.04	0.08	0.04	
	GMMs$(r_o)^{(00)}$			-0.01	0.00	0.08	0.05	0.08	0.05	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.00	0.09	0.05	0.09	0.05	

Design	Estimator	$\bar{B}_{\gamma}$	$\bar{B}_{\beta}$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
2	LSDV	-0.20	0.06	-0.20	0.06	0.07	0.15	0.21	0.16	
	GLS	0.03	-0.02	0.02	-0.02	0.05	0.08	0.05	0.08	
$\gamma = 0.75$	FGLS			0.15	-0.13	0.04	0.06	0.16	0.15	
$\zeta = 3$	GMMfl$^{(2)}$	-0.23	0.04	-0.22	0.03	0.09	0.23	0.24	0.23	0.94
$\pi = 0$	GMMfl$^{(1)}$	-0.06	0.00	-0.10	0.00	0.14	0.40	0.18	0.40	0.76
$\phi = 0$	GMMfl$^{(0)}$			-0.02	0.00	0.16	0.60	0.16	0.60	0.70
$\mu = 1$	GMMld$^{(2)}$	0.12	-0.09	0.08	-0.07	0.05	0.09	0.10	0.11	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.05	-0.04	0.03	-0.03	0.06	0.10	0.07	0.11	
$r_o \approx 0.143$	GMMcd$(\widehat{r})^{(2)}$			-0.03	0.01	0.09	0.13	0.10	0.13	
	GMMld$^{(1)}$	0.07	-0.05	0.06	-0.04	0.09	0.18	0.11	0.19	
	GMMcd$(r_o)^{(1)}$	0.02	-0.01	0.01	-0.01	0.11	0.23	0.11	0.23	
	GMMcd$(\widehat{r})^{(1)}$			-0.03	0.00	0.14	0.29	0.15	0.29	
	GMMld$^{(0)}$			-0.02	-0.01	0.13	0.42	0.13	0.42	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.14	0.51	0.14	0.51	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	-0.02	0.21	0.80	0.21	0.80	
	GMMs$^{(21)}$	-0.06	0.09	-0.07	0.03	0.07	0.15	0.10	0.15	
	GMMs$(r_o)^{(21)}$	-0.11	0.11	-0.12	0.04	0.08	0.17	0.15	0.18	
	GMMs$(\widehat{r})^{(21)}$			-0.15	0.03	0.10	0.20	0.18	0.20	
	GMMs$^{(11)}$	0.03	0.01	0.02	-0.01	0.08	0.16	0.08	0.16	
	GMMs$(r_o)^{(11)}$	-0.01	0.03	-0.02	0.01	0.09	0.19	0.09	0.19	
	GMMs$(\widehat{r})^{(11)}$			-0.05	0.01	0.11	0.24	0.12	0.24	
	GMMs$^{(00)}$			-0.01	0.00	0.09	0.23	0.09	0.23	
	GMMs$(r_o)^{(00)}$			-0.01	0.00	0.10	0.26	0.10	0.26	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.00	0.12	0.38	0.12	0.38	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
3	LSDV	-0.20	0.06	-0.20	0.06	0.07	0.15	0.21	0.16	
	GLS	0.00	0.00	-0.01	0.01	0.05	0.07	0.05	0.07	
$\gamma = 0.75$	FGLS			0.05	-0.05	0.05	0.07	0.07	0.08	
$\zeta = 3$	GMMfl$^{(2)}$	-0.20	0.03	-0.18	0.03	0.08	0.23	0.20	0.23	0.31
$\pi = 0$	GMMfl$^{(1)}$	-0.04	0.00	-0.06	0.00	0.10	0.38	0.12	0.38	0.31
$\phi = 0$	GMMfl$^{(0)}$			-0.02	0.00	0.12	0.60	0.12	0.60	0.45
$\mu = 0$	GMMld$^{(2)}$	0.00	0.00	-0.01	0.01	0.06	0.09	0.06	0.09	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.00	0.00	-0.01	0.01	0.06	0.09	0.06	0.09	
$r_o = 0$	GMMcd$(\widehat{r})^{(2)}$			-0.08	0.05	0.09	0.11	0.12	0.12	
	GMMld$^{(1)}$	0.00	0.00	-0.01	0.00	0.10	0.18	0.10	0.18	
	GMMcd$(r_o)^{(1)}$	0.00	0.00	-0.01	0.00	0.10	0.18	0.10	0.18	
	GMMcd$(\widehat{r})^{(1)}$			-0.05	0.02	0.13	0.25	0.14	0.26	
	GMMld$^{(0)}$			-0.02	-0.01	0.12	0.40	0.12	0.40	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.12	0.40	0.12	0.40	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	-0.01	0.16	0.62	0.16	0.62	
	GMMs$^{(21)}$	-0.13	0.14	-0.11	0.05	0.07	0.14	0.13	0.15	
	GMMs$(r_o)^{(21)}$	-0.13	0.14	-0.11	0.05	0.07	0.14	0.13	0.15	
	GMMs$(\widehat{r})^{(21)}$			-0.14	0.04	0.08	0.19	0.16	0.19	
	GMMs$^{(11)}$	-0.02	0.03	-0.03	0.01	0.08	0.15	0.08	0.15	
	GMMs$(r_o)^{(11)}$	-0.02	0.03	-0.03	0.01	0.08	0.15	0.08	0.15	
	GMMs$(\widehat{r})^{(11)}$			-0.05	0.01	0.09	0.23	0.10	0.23	
	GMMs$^{(00)}$			-0.01	0.00	0.08	0.22	0.09	0.22	
	GMMs$(r_o)^{(00)}$			-0.01	0.00	0.08	0.22	0.09	0.22	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.00	0.10	0.37	0.10	0.37	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
4	LSDV	-0.20	0.06	-0.20	0.06	0.07	0.15	0.21	0.16	
	GLS	0.05	-0.02	0.05	-0.02	0.04	0.13	0.07	0.13	
$\gamma = 0.75$	FGLS			0.24	-0.21	0.01	0.05	0.24	0.21	
$\zeta = 3$	GMMfl$^{(2)}$	-0.28	0.04	-0.26	0.04	0.10	0.23	0.28	0.23	16.88
$\pi = 0$	GMMfl$^{(1)}$	-0.17	0.00	-0.27	-0.01	0.22	0.44	0.35	0.44	17.52
$\phi = 0$	GMMfl$^{(0)}$			-0.01	0.02	0.45	0.67	0.45	0.67	9.01
$\mu = 5$	GMMld$^{(2)}$	0.28	-0.22	0.23	-0.18	0.02	0.08	0.23	0.20	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.12	-0.05	0.10	-0.04	0.05	0.18	0.11	0.19	
$r_o \approx 3.57$	GMMcd$(\widehat{r})^{(2)}$			0.06	-0.04	0.13	0.19	0.14	0.20	
	GMMld$^{(1)}$	0.23	-0.15	0.22	-0.14	0.04	0.18	0.23	0.23	
	GMMcd$(r_o)^{(1)}$	0.04	-0.01	0.02	-0.01	0.13	0.43	0.14	0.43	
	GMMcd$(\widehat{r})^{(1)}$			0.02	-0.02	0.17	0.42	0.18	0.42	
	GMMld$^{(0)}$			0.04	0.01	0.31	0.93	0.31	0.93	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.29	1.44	0.29	1.44	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	-0.01	0.31	1.58	0.31	1.58	
	GMMs$^{(21)}$	0.13	-0.10	0.17	-0.07	0.04	0.15	0.17	0.17	
	GMMs$(r_o)^{(21)}$	-0.15	0.13	-0.16	0.02	0.09	0.22	0.18	0.22	
	GMMs$(\widehat{r})^{(21)}$			-0.13	0.02	0.15	0.22	0.20	0.22	
	GMMs$^{(11)}$	0.16	-0.17	0.21	-0.10	0.04	0.16	0.21	0.19	
	GMMs$(r_o)^{(11)}$	-0.03	0.07	-0.04	0.00	0.11	0.28	0.12	0.28	
	GMMs$(\widehat{r})^{(11)}$			-0.04	0.00	0.17	0.28	0.17	0.28	
	GMMs$^{(00)}$			0.07	-0.01	0.12	0.31	0.14	0.31	
	GMMs$(r_o)^{(00)}$			0.01	0.01	0.16	0.41	0.16	0.41	
	GMMs$(\widehat{r})^{(00)}$			-0.01	0.00	0.18	0.41	0.18	0.41	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
5	LSDV	-0.16	0.05	-0.16	0.05	0.06	0.07	0.18	0.08	
	GLS	0.03	-0.02	0.02	-0.02	0.04	0.05	0.05	0.05	
$\gamma = 0.75$	FGLS			0.11	-0.09	0.04	0.04	0.11	0.10	
$\zeta = 9$	GMMfl$^{(2)}$	-0.17	0.03	-0.16	0.02	0.08	0.11	0.18	0.11	0.86
$\pi = 0$	GMMfl$^{(1)}$	-0.04	0.00	-0.06	0.00	0.11	0.18	0.13	0.18	0.71
$\phi = 0$	GMMfl$^{(0)}$			-0.01	0.00	0.12	0.28	0.13	0.28	0.76
$\mu = 1$	GMMld$^{(2)}$	0.09	-0.07	0.06	-0.05	0.05	0.05	0.08	0.07	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.04	-0.03	0.03	-0.02	0.05	0.06	0.06	0.06	
$r_o \approx 0.143$	GMMcd$(\widehat{r})^{(2)}$			-0.02	0.01	0.08	0.07	0.08	0.07	
	GMMld$^{(1)}$	0.05	-0.03	0.04	-0.03	0.08	0.09	0.09	0.10	
	GMMcd$(r_o)^{(1)}$	0.01	-0.01	0.01	-0.01	0.09	0.11	0.09	0.11	
	GMMcd$(\widehat{r})^{(1)}$			-0.02	0.00	0.11	0.14	0.12	0.14	
	GMMld$^{(0)}$			-0.02	0.00	0.12	0.21	0.12	0.21	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.13	0.24	0.13	0.24	
	GMMcd$(\widehat{r})^{(0)}$			-0.02	-0.01	0.16	0.33	0.16	0.33	
	GMMs$^{(21)}$	-0.04	0.05	-0.05	0.02	0.06	0.07	0.08	0.08	
	GMMs$(r_o)^{(21)}$	-0.08	0.06	-0.09	0.03	0.07	0.08	0.11	0.09	
	GMMs$(\widehat{r})^{(21)}$			-0.11	0.02	0.08	0.09	0.14	0.09	
	GMMs$^{(11)}$	0.02	0.01	0.01	-0.01	0.06	0.08	0.07	0.08	
	GMMs$(r_o)^{(11)}$	-0.01	0.01	-0.01	0.00	0.07	0.09	0.07	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.04	0.00	0.09	0.11	0.09	0.11	
	GMMs$^{(00)}$			0.00	0.00	0.08	0.10	0.08	0.10	
	GMMs$(r_o)^{(00)}$			-0.01	0.00	0.08	0.12	0.08	0.12	
	GMMs$(\widehat{r})^{(00)}$			-0.01	0.00	0.10	0.17	0.10	0.17	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
6	LSDV	-0.16	0.05	-0.16	0.05	0.06	0.07	0.18	0.08	
	GLS	0.04	-0.02	0.05	-0.02	0.04	0.06	0.06	0.07	
$\gamma = 0.75$	FGLS			0.24	-0.20	0.01	0.03	0.24	0.20	
$\zeta = 9$	GMMfl$^{(2)}$	-0.21	0.03	-0.20	0.03	0.09	0.11	0.22	0.11	13.26
$\pi = 0$	GMMfl$^{(1)}$	-0.11	0.00	-0.20	-0.01	0.21	0.20	0.29	0.20	13.77
$\phi = 0$	GMMfl$^{(0)}$			0.00	0.01	0.25	0.30	0.25	0.30	6.76
$\mu = 5$	GMMld$^{(2)}$	0.27	-0.21	0.23	-0.17	0.02	0.05	0.23	0.18	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.11	-0.04	0.09	-0.03	0.05	0.09	0.10	0.10	
$r_o \approx 3.57$	GMMcd$(\widehat{r})^{(2)}$			0.07	-0.04	0.10	0.10	0.12	0.11	
	GMMld$^{(1)}$	0.21	-0.13	0.21	-0.12	0.04	0.09	0.21	0.15	
	GMMcd$(r_o)^{(1)}$	0.03	-0.01	0.02	-0.01	0.12	0.20	0.12	0.20	
	GMMcd$(\widehat{r})^{(1)}$			0.02	-0.01	0.15	0.20	0.15	0.20	
	GMMld$^{(0)}$			0.02	0.01	0.26	0.41	0.26	0.41	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.24	0.69	0.24	0.69	
	GMMcd$(\widehat{r})^{(0)}$			-0.02	0.00	0.25	0.67	0.25	0.67	
	GMMs$^{(21)}$	0.11	-0.05	0.15	-0.06	0.05	0.08	0.16	0.10	
	GMMs$(r_o)^{(21)}$	-0.11	0.06	-0.12	0.02	0.08	0.10	0.15	0.10	
	GMMs$(\widehat{r})^{(21)}$			-0.11	0.01	0.12	0.11	0.16	0.11	
	GMMs$^{(11)}$	0.15	-0.08	0.19	-0.09	0.04	0.08	0.20	0.12	
	GMMs$(r_o)^{(11)}$	-0.01	0.03	-0.03	0.00	0.10	0.13	0.10	0.13	
	GMMs$(\widehat{r})^{(11)}$			-0.03	0.00	0.14	0.13	0.14	0.13	
	GMMs$^{(00)}$			0.05	-0.01	0.11	0.15	0.12	0.15	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.13	0.19	0.13	0.19	
	GMMs$(\widehat{r})^{(00)}$			-0.01	0.00	0.14	0.20	0.14	0.20	

Table 3: Simulation results for $T = 10$, $N = 20$, scheme 1, disturbances normal

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
7	LSDV	-0.21	0.06	-0.21	0.05	0.07	0.08	0.22	0.10	
	GLS	0.04	-0.09	0.04	-0.08	0.05	0.07	0.06	0.11	
$\gamma = 0.75$	FGLS			0.16	-0.14	0.04	0.06	0.17	0.15	
$\zeta = 3$	GMMfl$^{(2)}$	-0.25	0.03	-0.23	0.03	0.10	0.09	0.25	0.09	1.15
$\pi = 0$	GMMfl$^{(1)}$	-0.08	0.02	-0.11	0.02	0.15	0.11	0.19	0.11	0.70
$\phi = 1$	GMMfl$^{(0)}$			-0.02	0.01	0.15	0.10	0.15	0.10	0.40
$\mu = 1$	GMMld$^{(2)}$	0.15	-0.12	0.11	-0.09	0.05	0.08	0.13	0.12	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.07	-0.08	0.06	-0.07	0.06	0.08	0.08	0.11	
$r_o \approx 0.21$	GMMcd$(\widehat{r})^{(2)}$			0.03	-0.06	0.08	0.08	0.09	0.10	
	GMMld$^{(1)}$	0.09	-0.05	0.08	-0.05	0.09	0.09	0.12	0.10	
	GMMcd$(r_o)^{(1)}$	0.02	-0.02	0.01	-0.01	0.12	0.09	0.12	0.09	
	GMMcd$(\widehat{r})^{(1)}$			0.00	-0.01	0.14	0.10	0.14	0.10	
	GMMld$^{(0)}$			-0.01	0.00	0.12	0.10	0.12	0.10	
	GMMcd$(r_o)^{(0)}$			-0.02	0.00	0.14	0.10	0.14	0.10	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	0.00	0.15	0.10	0.15	0.10	
	GMMs$^{(21)}$	-0.04	0.00	-0.06	-0.01	0.08	0.08	0.10	0.09	
	GMMs$(r_o)^{(21)}$	-0.11	0.02	-0.13	0.01	0.08	0.08	0.15	0.08	
	GMMs$(\widehat{r})^{(21)}$			-0.13	0.01	0.12	0.09	0.17	0.09	
	GMMs$^{(11)}$	0.04	-0.01	0.03	-0.02	0.08	0.09	0.09	0.09	
	GMMs$(r_o)^{(11)}$	-0.01	0.00	-0.03	0.00	0.09	0.09	0.10	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.04	0.00	0.12	0.09	0.13	0.09	
	GMMs$^{(00)}$			0.00	0.00	0.10	0.09	0.10	0.09	
	GMMs$(r_o)^{(00)}$			-0.01	0.01	0.11	0.09	0.11	0.09	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.01	0.12	0.09	0.12	0.09	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
8	LSDV	-0.20	-0.03	-0.20	-0.03	0.09	0.08	0.22	0.09	
	GLS	0.03	0.02	0.02	0.01	0.04	0.05	0.05	0.05	
$\gamma = 0.75$	FGLS			0.13	-0.08	0.04	0.05	0.13	0.09	
$\zeta = 3$	GMMfl$^{(2)}$	-0.25	-0.07	-0.23	-0.07	0.14	0.11	0.27	0.13	0.90
$\pi = 0$	GMMfl$^{(1)}$	-0.09	-0.05	-0.14	-0.08	0.25	0.21	0.28	0.22	0.92
$\phi = -1$	GMMfl$^{(0)}$			-0.03	-0.02	0.28	0.20	0.28	0.20	0.68
$\mu = 1$	GMMld$^{(2)}$	0.08	-0.01	0.05	-0.01	0.05	0.05	0.07	0.05	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.04	0.01	0.03	0.01	0.05	0.06	0.06	0.06	
$r_o \approx 0.10$	GMMcd$(\widehat{r})^{(2)}$			-0.02	0.02	0.08	0.06	0.08	0.07	
	GMMld$^{(1)}$	0.05	0.01	0.04	0.01	0.10	0.09	0.10	0.09	
	GMMcd$(r_o)^{(1)}$	0.03	0.02	0.02	0.01	0.12	0.10	0.12	0.10	
	GMMcd$(\widehat{r})^{(1)}$			-0.03	-0.01	0.17	0.12	0.18	0.12	
	GMMld$^{(0)}$			-0.01	-0.01	0.22	0.15	0.22	0.15	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.24	0.17	0.25	0.17	
	GMMcd$(\widehat{r})^{(0)}$			-0.04	-0.02	0.31	0.20	0.32	0.20	
	GMMs$^{(21)}$	-0.03	0.04	-0.06	0.01	0.08	0.08	0.10	0.08	
	GMMs$(r_o)^{(21)}$	-0.06	0.03	-0.10	-0.01	0.09	0.08	0.14	0.08	
	GMMs$(\widehat{r})^{(21)}$			-0.15	-0.04	0.13	0.10	0.20	0.11	
	GMMs$^{(11)}$	0.04	0.01	0.01	0.01	0.08	0.08	0.09	0.08	
	GMMs$(r_o)^{(11)}$	0.01	0.01	-0.01	0.00	0.10	0.09	0.10	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.06	-0.02	0.15	0.11	0.16	0.12	
	GMMs$^{(00)}$			-0.01	-0.01	0.13	0.11	0.13	0.11	
	GMMs$(r_o)^{(00)}$			-0.01	-0.01	0.15	0.13	0.15	0.13	
	GMMs$(\widehat{r})^{(00)}$			-0.03	-0.02	0.20	0.15	0.20	0.16	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
9	LSDV	-0.20	0.06	-0.20	0.06	0.07	0.15	0.21	0.16	
	GLS	0.03	0.02	0.02	0.02	0.05	0.08	0.05	0.08	
$\gamma = 0.75$	FGLS			0.13	-0.09	0.04	0.06	0.14	0.11	
$\zeta = 3$	GMMfl$^{(2)}$	-0.22	0.02	-0.21	0.02	0.09	0.23	0.23	0.23	0.80
$\pi = 1$	GMMfl$^{(1)}$	-0.06	-0.01	-0.09	-0.03	0.13	0.40	0.16	0.40	0.67
$\phi = 0$	GMMfl$^{(0)}$			-0.02	0.00	0.15	0.64	0.15	0.64	0.71
$\mu = 1$	GMMld$^{(2)}$	0.09	-0.03	0.06	-0.02	0.05	0.10	0.08	0.10	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.04	0.01	0.02	0.01	0.06	0.10	0.06	0.10	
$r_o \approx 0.09$	GMMcd$(\widehat{r})^{(2)}$			-0.05	0.06	0.09	0.13	0.10	0.14	
	GMMld$^{(1)}$	0.05	-0.01	0.04	0.00	0.09	0.18	0.10	0.18	
	GMMcd$(r_o)^{(1)}$	0.02	0.02	0.01	0.02	0.11	0.22	0.11	0.22	
	GMMcd$(\widehat{r})^{(1)}$			-0.03	0.03	0.14	0.28	0.14	0.29	
	GMMld$^{(0)}$			-0.02	-0.01	0.13	0.42	0.13	0.42	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.14	0.48	0.14	0.48	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	-0.02	0.18	0.74	0.18	0.74	
	GMMs$^{(21)}$	-0.08	0.13	-0.08	0.06	0.07	0.15	0.11	0.16	
	GMMs$(r_o)^{(21)}$	-0.11	0.14	-0.12	0.06	0.07	0.17	0.14	0.18	
	GMMs$(\widehat{r})^{(21)}$			-0.15	0.04	0.10	0.20	0.17	0.20	
	GMMs$^{(11)}$	0.01	0.05	0.00	0.02	0.08	0.16	0.08	0.16	
	GMMs$(r_o)^{(11)}$	-0.01	0.05	-0.02	0.03	0.08	0.18	0.08	0.18	
	GMMs$(\widehat{r})^{(11)}$			-0.05	0.02	0.10	0.24	0.12	0.24	
	GMMs$^{(00)}$			-0.01	0.00	0.09	0.22	0.09	0.22	
	GMMs$(r_o)^{(00)}$			-0.01	0.00	0.10	0.25	0.10	0.25	
	GMMs$(\widehat{r})^{(00)}$			-0.02	-0.01	0.11	0.39	0.11	0.39	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
10	LSDV	-0.20	0.06	-0.20	0.06	0.07	0.15	0.21	0.16	
	GLS	0.02	-0.10	0.02	-0.09	0.05	0.08	0.05	0.12	
$\gamma = 0.75$	FGLS			0.18	-0.19	0.03	0.05	0.18	0.20	
$\zeta = 3$	GMMfl$^{(2)}$	-0.24	0.06	-0.23	0.05	0.09	0.23	0.25	0.24	1.24
$\pi = -1$	GMMfl$^{(1)}$	-0.08	0.03	-0.13	0.05	0.16	0.42	0.20	0.42	1.07
$\phi = 0$	GMMfl$^{(0)}$			-0.02	0.00	0.17	0.60	0.17	0.60	0.88
$\mu = 1$	GMMld$^{(2)}$	0.15	-0.18	0.11	-0.15	0.05	0.08	0.12	0.17	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.06	-0.13	0.04	-0.11	0.06	0.10	0.07	0.15	
$r_o \approx 0.25$	GMMcd$(\widehat{r})^{(2)}$			-0.02	-0.07	0.10	0.12	0.10	0.14	
	GMMld$^{(1)}$	0.09	-0.12	0.08	-0.12	0.09	0.17	0.12	0.21	
	GMMcd$(r_o)^{(1)}$	0.01	-0.08	0.01	-0.07	0.12	0.25	0.12	0.26	
	GMMcd$(\widehat{r})^{(1)}$			-0.03	-0.05	0.15	0.30	0.15	0.31	
	GMMld$^{(0)}$			-0.02	0.01	0.14	0.53	0.15	0.53	
	GMMcd$(r_o)^{(0)}$			-0.02	-0.01	0.16	0.67	0.16	0.67	
	GMMcd$(\widehat{r})^{(0)}$			-0.04	-0.02	0.19	0.81	0.19	0.81	
	GMMs$^{(21)}$	-0.04	-0.01	-0.06	-0.04	0.07	0.15	0.09	0.16	
	GMMs$(r_o)^{(21)}$	-0.12	0.06	-0.14	0.00	0.08	0.18	0.16	0.18	
	GMMs$(\widehat{r})^{(21)}$			-0.16	0.02	0.10	0.20	0.19	0.20	
	GMMs$^{(11)}$	0.05	-0.08	0.04	-0.08	0.08	0.16	0.09	0.18	
	GMMs$(r_o)^{(11)}$	-0.02	-0.03	-0.03	-0.04	0.09	0.20	0.10	0.21	
	GMMs$(\widehat{r})^{(11)}$			-0.06	-0.01	0.12	0.25	0.13	0.25	
	GMMs$^{(00)}$			0.00	-0.01	0.10	0.23	0.10	0.23	
	GMMs$(r_o)^{(00)}$			-0.01	-0.01	0.11	0.28	0.11	0.28	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.01	0.12	0.38	0.12	0.38	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
11	LSDV	-0.21	0.06	-0.21	0.05	0.07	0.08	0.22	0.10	
	GLS	0.04	-0.04	0.04	-0.04	0.05	0.08	0.06	0.08	
$\gamma = 0.75$	FGLS			0.14	-0.08	0.04	0.07	0.14	0.10	
$\zeta = 3$	GMMfl$^{(2)}$	-0.24	0.02	-0.22	0.02	0.10	0.09	0.24	0.09	0.95
$\pi = 1$	GMMfl$^{(1)}$	-0.07	0.00	-0.10	0.00	0.14	0.11	0.17	0.11	0.53
$\phi = 1$	GMMfl$^{(0)}$			-0.02	0.01	0.14	0.10	0.14	0.10	0.29
$\mu = 1$	GMMld$^{(2)}$	0.11	-0.06	0.09	-0.05	0.05	0.08	0.10	0.09	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.06	-0.04	0.05	-0.04	0.06	0.08	0.07	0.09	
$r_o \approx 0.13$	GMMcd$(\widehat{r})^{(2)}$			0.03	-0.03	0.08	0.08	0.08	0.09	
	GMMld$^{(1)}$	0.07	-0.03	0.06	-0.03	0.10	0.09	0.11	0.10	
	GMMcd$(r_o)^{(1)}$	0.02	-0.01	0.02	-0.01	0.11	0.09	0.11	0.09	
	GMMcd$(\widehat{r})^{(1)}$			0.00	-0.01	0.13	0.10	0.13	0.10	
	GMMld$^{(0)}$			-0.01	0.00	0.12	0.09	0.12	0.09	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.13	0.10	0.13	0.10	
	GMMcd$(\widehat{r})^{(0)}$			-0.02	0.00	0.14	0.10	0.14	0.10	
	GMMs$^{(21)}$	-0.06	0.02	-0.07	0.00	0.08	0.08	0.10	0.08	
	GMMs$(r_o)^{(21)}$	-0.11	0.02	-0.12	0.02	0.08	0.08	0.14	0.09	
	GMMs$(\widehat{r})^{(21)}$			-0.13	0.01	0.11	0.09	0.17	0.09	
	GMMs$^{(11)}$	0.02	0.00	0.02	-0.01	0.08	0.09	0.08	0.09	
	GMMs$(r_o)^{(11)}$	-0.01	0.00	-0.02	0.00	0.09	0.09	0.09	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.04	0.00	0.12	0.09	0.12	0.09	
	GMMs$^{(00)}$			-0.01	0.00	0.10	0.09	0.10	0.09	
	GMMs$(r_o)^{(00)}$			-0.01	0.01	0.10	0.09	0.10	0.09	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.01	0.11	0.09	0.11	0.09	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
12	LSDV	-0.21	0.06	-0.21	0.05	0.07	0.08	0.22	0.10	
	GLS	0.06	-0.06	0.07	-0.07	0.05	0.08	0.08	0.11	
$\gamma = 0.75$	FGLS			0.22	-0.13	0.02	0.06	0.22	0.15	
$\zeta = 3$	GMMfl$^{(2)}$	-0.29	0.03	-0.26	0.03	0.10	0.08	0.28	0.09	19.55
$\pi = 1$	GMMfl$^{(1)}$	-0.17	0.01	-0.22	0.01	0.20	0.11	0.29	0.12	17.52
$\phi = 1$	GMMfl$^{(0)}$			-0.01	0.01	0.32	0.11	0.32	0.11	9.43
$\mu = 5$	GMMld$^{(2)}$	0.25	-0.13	0.21	-0.11	0.02	0.07	0.21	0.13	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.13	-0.08	0.11	-0.08	0.04	0.09	0.12	0.12	
$r_o \approx 3.29$	GMMcd$(\widehat{r})^{(2)}$			0.07	-0.06	0.13	0.09	0.14	0.11	
	GMMld$^{(1)}$	0.20	-0.09	0.20	-0.08	0.04	0.08	0.20	0.12	
	GMMcd$(r_o)^{(1)}$	0.05	-0.01	0.02	-0.01	0.12	0.10	0.13	0.10	
	GMMcd$(\widehat{r})^{(1)}$			0.01	-0.01	0.17	0.10	0.17	0.10	
	GMMld$^{(0)}$			0.10	-0.03	0.18	0.11	0.21	0.12	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.27	0.11	0.27	0.11	
	GMMcd$(\widehat{r})^{(0)}$			-0.02	0.00	0.27	0.11	0.27	0.11	
	GMMs$^{(21)}$	0.09	-0.01	0.16	-0.08	0.05	0.08	0.16	0.11	
	GMMs$(r_o)^{(21)}$	-0.14	0.02	-0.14	0.00	0.10	0.09	0.17	0.09	
	GMMs$(\widehat{r})^{(21)}$			-0.12	0.00	0.16	0.09	0.20	0.09	
	GMMs$^{(11)}$	0.11	0.01	0.18	-0.06	0.04	0.08	0.19	0.10	
	GMMs$(r_o)^{(11)}$	-0.01	0.00	-0.03	0.00	0.11	0.09	0.12	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.04	-0.01	0.17	0.09	0.17	0.09	
	GMMs$^{(00)}$			0.08	-0.02	0.12	0.10	0.14	0.10	
	GMMs$(r_o)^{(00)}$			0.00	0.01	0.16	0.09	0.16	0.09	
	GMMs$(\widehat{r})^{(00)}$			-0.02	0.00	0.18	0.09	0.18	0.09	

Table 4: Simulation results for $T = 10$, $N = 20$, scheme 1, disturbances normal

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
13	LSDV	-0.11	0.07	-0.11	0.07	0.07	0.12	0.13	0.14	
	GLS	0.01	-0.01	0.01	-0.01	0.07	0.10	0.07	0.10	
$\gamma = 0.25$	FGLS			0.46	-0.44	0.08	0.10	0.47	0.45	
$\zeta = 3$	GMMfl$^{(2)}$	-0.11	0.06	-0.11	0.05	0.08	0.18	0.13	0.18	0.94
$\pi = 0$	GMMfl$^{(1)}$	-0.02	0.01	-0.04	0.02	0.11	0.30	0.11	0.30	0.91
$\phi = 0$	GMMfl$^{(0)}$			0.00	0.01	0.10	0.47	0.10	0.47	0.97
$\mu = 1$	GMMld$^{(2)}$	0.29	-0.26	0.22	-0.19	0.09	0.13	0.23	0.23	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.03	-0.02	0.02	-0.02	0.08	0.13	0.08	0.13	
$r_o = 0.6$	GMMcd$(\widehat{r})^{(2)}$			0.00	-0.01	0.09	0.13	0.09	0.13	
	GMMld$^{(1)}$	0.10	-0.08	0.10	-0.07	0.10	0.19	0.14	0.20	
	GMMcd$(r_o)^{(1)}$	0.00	0.00	0.01	0.00	0.11	0.26	0.11	0.26	
	GMMcd$(\widehat{r})^{(1)}$			0.00	0.00	0.11	0.27	0.11	0.27	
	GMMld$^{(0)}$			-0.01	0.00	0.10	0.42	0.10	0.42	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.11	0.66	0.11	0.66	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	0.00	0.11	0.74	0.11	0.74	
	GMMs$^{(21)}$	-0.02	0.03	-0.01	0.01	0.08	0.14	0.08	0.14	
	GMMs$(r_o)^{(21)}$	-0.08	0.07	-0.07	0.03	0.08	0.16	0.10	0.17	
	GMMs$(\widehat{r})^{(21)}$			-0.07	0.03	0.08	0.17	0.11	0.17	
	GMMs$^{(11)}$	0.06	-0.03	0.05	-0.03	0.08	0.16	0.10	0.16	
	GMMs$(r_o)^{(11)}$	-0.01	0.01	-0.01	0.00	0.09	0.20	0.09	0.20	
	GMMs$(\widehat{r})^{(11)}$			-0.01	0.01	0.09	0.21	0.09	0.21	
	GMMs$^{(00)}$			0.01	0.00	0.09	0.22	0.09	0.22	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.09	0.27	0.09	0.27	
	GMMs$(\widehat{r})^{(00)}$			0.00	0.00	0.09	0.32	0.09	0.32	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
14	LSDV	-0.11	0.07	-0.11	0.07	0.07	0.12	0.13	0.14	
	GLS	0.01	-0.01	0.01	-0.01	0.07	0.12	0.07	0.12	
$\gamma = 0.25$	FGLS			0.73	-0.70	0.01	0.04	0.73	0.70	
$\zeta = 3$	GMMfl$^{(2)}$	-0.13	0.06	-0.12	0.06	0.08	0.18	0.14	0.19	21.00
$\pi = 0$	GMMfl$^{(1)}$	-0.08	0.04	-0.13	0.05	0.20	0.33	0.24	0.33	21.11
$\phi = 0$	GMMfl$^{(0)}$			0.02	0.00	0.17	0.43	0.17	0.43	14.73
$\mu = 5$	GMMld$^{(2)}$	0.83	-0.74	0.68	-0.60	0.04	0.10	0.68	0.61	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.05	-0.03	0.05	-0.02	0.08	0.19	0.09	0.19	
$r_o = 15$	GMMcd$(\widehat{r})^{(2)}$			0.07	-0.04	0.12	0.20	0.14	0.20	
	GMMld$^{(1)}$	0.60	-0.44	0.57	-0.41	0.09	0.23	0.58	0.47	
	GMMcd$(r_o)^{(1)}$	0.00	0.00	0.01	0.00	0.12	0.39	0.12	0.39	
	GMMcd$(\widehat{r})^{(1)}$			0.02	-0.01	0.13	0.39	0.13	0.39	
	GMMld$^{(0)}$			0.00	0.02	0.25	1.33	0.25	1.33	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.16	1.62	0.16	1.62	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	0.00	0.15	1.52	0.15	1.52	
	GMMs$^{(21)}$	0.36	-0.31	0.42	-0.27	0.10	0.21	0.43	0.34	
	GMMs$(r_o)^{(21)}$	-0.09	0.07	-0.08	0.04	0.08	0.18	0.11	0.18	
	GMMs$(\widehat{r})^{(21)}$			-0.07	0.03	0.09	0.18	0.12	0.18	
	GMMs$^{(11)}$	0.46	-0.43	0.54	-0.35	0.09	0.21	0.55	0.41	
	GMMs$(r_o)^{(11)}$	-0.02	0.02	-0.02	0.01	0.11	0.24	0.11	0.24	
	GMMs$(\widehat{r})^{(11)}$			-0.02	0.01	0.12	0.24	0.12	0.24	
	GMMs$^{(00)}$			0.09	-0.03	0.16	0.49	0.18	0.49	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.11	0.35	0.11	0.35	
	GMMs$(\widehat{r})^{(00)}$			0.00	0.00	0.11	0.35	0.11	0.35	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
15	LSDV	-0.12	0.02	-0.12	0.02	0.07	0.09	0.14	0.09	
	GLS	0.01	-0.08	0.01	-0.08	0.07	0.09	0.07	0.12	
$\gamma = 0.25$	FGLS			0.54	-0.52	0.07	0.09	0.55	0.53	
$\zeta = 3$	GMMfl$^{(2)}$	-0.13	0.00	-0.12	0.00	0.08	0.09	0.14	0.09	1.73
$\pi = 0$	GMMfl$^{(1)}$	-0.04	0.00	-0.05	0.00	0.12	0.13	0.14	0.13	1.46
$\phi = 1$	GMMfl$^{(0)}$			0.00	0.01	0.11	0.10	0.11	0.10	1.24
$\mu = 1$	GMMld$^{(2)}$	0.44	-0.39	0.34	-0.30	0.09	0.11	0.36	0.32	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.04	-0.10	0.03	-0.08	0.08	0.09	0.09	0.12	
$r_o \approx 1.16$	GMMcd$(\widehat{r})^{(2)}$			0.03	-0.08	0.09	0.10	0.10	0.13	
	GMMld$^{(1)}$	0.19	-0.11	0.17	-0.10	0.12	0.11	0.21	0.15	
	GMMcd$(r_o)^{(1)}$	-0.01	-0.01	-0.01	-0.01	0.11	0.10	0.11	0.10	
	GMMcd$(\widehat{r})^{(1)}$			-0.01	-0.01	0.12	0.10	0.12	0.10	
	GMMld$^{(0)}$			0.00	0.00	0.11	0.11	0.11	0.11	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.11	0.10	0.11	0.10	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	0.00	0.11	0.10	0.11	0.10	
	GMMs$^{(21)}$	0.03	-0.05	0.03	-0.06	0.09	0.10	0.09	0.11	
	GMMs$(r_o)^{(21)}$	-0.06	-0.01	-0.08	-0.01	0.08	0.09	0.11	0.09	
	GMMs$(\widehat{r})^{(21)}$			-0.08	-0.01	0.08	0.09	0.12	0.09	
	GMMs$^{(11)}$	0.09	-0.06	0.10	-0.07	0.10	0.10	0.15	0.12	
	GMMs$(r_o)^{(11)}$	-0.02	-0.01	-0.02	0.00	0.09	0.10	0.10	0.10	
	GMMs$(\widehat{r})^{(11)}$			-0.02	0.00	0.10	0.10	0.10	0.10	
	GMMs$^{(00)}$			0.01	0.00	0.09	0.10	0.09	0.10	
	GMMs$(r_o)^{(00)}$			0.00	0.01	0.10	0.10	0.10	0.10	
	GMMs$(\widehat{r})^{(00)}$			-0.01	0.00	0.10	0.10	0.10	0.10	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
16	LSDV	0.01	0.07	0.01	0.08	0.07	0.07	0.07	0.11	
	GLS	0.06	0.07	0.06	0.07	0.05	0.06	0.08	0.09	
$\gamma = 0.25$	FGLS			0.31	-0.22	0.08	0.10	0.32	0.24	
$\zeta = 3$	GMMfl$^{(2)}$	0.03	0.10	0.03	0.10	0.09	0.11	0.09	0.15	0.82
$\pi = 0$	GMMfl$^{(1)}$	0.02	0.03	0.03	0.05	0.15	0.19	0.15	0.20	0.94
$\phi = -1$	GMMfl$^{(0)}$			0.01	0.02	0.16	0.23	0.16	0.23	0.98
$\mu = 1$	GMMld$^{(2)}$	0.17	-0.03	0.14	-0.02	0.07	0.09	0.16	0.09	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.07	0.09	0.06	0.08	0.06	0.08	0.09	0.11	
$r_o \approx 0.71$	GMMcd$(\widehat{r})^{(2)}$			0.06	0.08	0.06	0.08	0.09	0.11	
	GMMld$^{(1)}$	0.08	0.04	0.07	0.04	0.10	0.13	0.12	0.14	
	GMMcd$(r_o)^{(1)}$	0.04	0.07	0.05	0.07	0.12	0.18	0.13	0.20	
	GMMcd$(\widehat{r})^{(1)}$			0.05	0.07	0.12	0.19	0.13	0.20	
	GMMld$^{(0)}$			0.01	0.01	0.16	0.24	0.16	0.24	
	GMMcd$(r_o)^{(0)}$			0.02	0.03	0.20	0.30	0.20	0.30	
	GMMcd$(\widehat{r})^{(0)}$			0.02	0.03	0.20	0.32	0.20	0.32	
	GMMs$^{(21)}$	0.05	0.07	0.05	0.07	0.07	0.09	0.09	0.12	
	GMMs$(r_o)^{(21)}$	0.03	0.09	0.04	0.09	0.08	0.10	0.09	0.14	
	GMMs$(\widehat{r})^{(21)}$			0.04	0.10	0.08	0.10	0.09	0.14	
	GMMs$^{(11)}$	0.07	0.05	0.06	0.05	0.08	0.11	0.10	0.12	
	GMMs$(r_o)^{(11)}$	0.03	0.04	0.04	0.05	0.10	0.13	0.11	0.14	
	GMMs$(\widehat{r})^{(11)}$			0.04	0.06	0.10	0.14	0.11	0.15	
	GMMs$^{(00)}$			0.01	0.01	0.11	0.14	0.11	0.14	
	GMMs$(r_o)^{(00)}$			0.01	0.01	0.13	0.17	0.13	0.17	
	GMMs$(\widehat{r})^{(00)}$			0.01	0.02	0.13	0.18	0.13	0.18	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
17	LSDV	-0.11	0.07	-0.11	0.07	0.07	0.12	0.13	0.14	
	GLS	0.01	0.04	0.01	0.04	0.06	0.09	0.07	0.10	
$\gamma = 0.25$	FGLS			0.29	-0.24	0.09	0.10	0.30	0.26	
$\zeta = 3$	GMMfl$^{(2)}$	-0.11	0.05	-0.10	0.04	0.07	0.18	0.12	0.18	0.42
$\pi = 1$	GMMfl$^{(1)}$	-0.02	0.00	-0.02	0.01	0.09	0.30	0.09	0.30	0.48
$\phi = 0$	GMMfl$^{(0)}$			0.00	0.01	0.09	0.54	0.09	0.54	0.65
$\mu = 1$	GMMld$^{(2)}$	0.12	-0.05	0.09	-0.03	0.08	0.11	0.12	0.11	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.02	0.04	0.01	0.04	0.07	0.11	0.07	0.11	
$r_o \approx 0.20$	GMMcd$(\widehat{r})^{(2)}$			-0.02	0.06	0.08	0.11	0.08	0.13	
	GMMld$^{(1)}$	0.04	0.01	0.04	0.01	0.10	0.16	0.10	0.16	
	GMMcd$(r_o)^{(1)}$	0.01	0.04	0.01	0.03	0.10	0.20	0.10	0.21	
	GMMcd$(\widehat{r})^{(1)}$			0.00	0.03	0.11	0.24	0.11	0.24	
	GMMld$^{(0)}$			0.00	-0.01	0.10	0.37	0.10	0.37	
	GMMcd$(r_o)^{(0)}$			0.00	-0.01	0.10	0.53	0.10	0.53	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	0.00	0.11	0.74	0.11	0.74	
	GMMs$^{(21)}$	-0.05	0.09	-0.04	0.05	0.07	0.13	0.08	0.14	
	GMMs$(r_o)^{(21)}$	-0.07	0.09	-0.06	0.05	0.07	0.15	0.09	0.15	
	GMMs$(\widehat{r})^{(21)}$			-0.06	0.04	0.08	0.16	0.10	0.16	
	GMMs$^{(11)}$	0.02	0.04	0.01	0.03	0.08	0.14	0.08	0.14	
	GMMs$(r_o)^{(11)}$	-0.01	0.04	-0.01	0.03	0.08	0.17	0.08	0.17	
	GMMs$(\widehat{r})^{(11)}$			-0.01	0.02	0.08	0.20	0.08	0.20	
	GMMs$^{(00)}$			0.00	0.00	0.08	0.20	0.08	0.20	
	GMMs$(r_o)^{(00)}$			0.00	0.01	0.08	0.23	0.08	0.23	
	GMMs$(\widehat{r})^{(00)}$			0.00	0.00	0.09	0.32	0.09	0.32	

Design	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
18	LSDV	-0.11	0.07	-0.11	0.07	0.07	0.12	0.13	0.14	
	GLS	-0.04	-0.18	-0.04	-0.18	0.07	0.11	0.08	0.22	
$\gamma = 0.25$	FGLS			0.64	-0.75	0.04	0.02	0.64	0.75	
$\zeta = 3$	GMMfl$^{(2)}$	-0.12	0.08	-0.12	0.07	0.08	0.18	0.14	0.19	12.55
$\pi = -1$	GMMfl$^{(1)}$	-0.05	0.07	-0.09	0.12	0.17	0.39	0.19	0.40	13.66
$\phi = 0$	GMMfl$^{(0)}$			0.01	-0.04	0.17	0.65	0.17	0.66	11.00
$\mu = 1$	GMMld$^{(2)}$	0.60	-0.88	0.47	-0.73	0.08	0.06	0.48	0.74	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	-0.04	-0.39	-0.04	-0.35	0.08	0.15	0.09	0.38	
$r_o = 9.6$	GMMcd$(\widehat{r})^{(2)}$			-0.01	-0.38	0.12	0.21	0.12	0.44	
	GMMld$^{(1)}$	0.30	-0.71	0.27	-0.70	0.12	0.13	0.30	0.71	
	GMMcd$(r_o)^{(1)}$	-0.01	-0.19	-0.01	-0.18	0.12	0.34	0.12	0.39	
	GMMcd$(\widehat{r})^{(1)}$			-0.01	-0.22	0.12	0.36	0.12	0.42	
	GMMld$^{(0)}$			0.01	-0.25	0.22	1.11	0.22	1.13	
	GMMcd$(r_o)^{(0)}$			-0.01	-0.19	0.15	1.34	0.15	1.35	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	-0.18	0.15	1.30	0.15	1.32	
	GMMs$^{(21)}$	0.09	-0.54	0.13	-0.60	0.09	0.13	0.16	0.62	
	GMMs$(r_o)^{(21)}$	-0.09	0.05	-0.08	0.00	0.08	0.18	0.11	0.18	
	GMMs$(\widehat{r})^{(21)}$			-0.07	-0.03	0.09	0.21	0.11	0.21	
	GMMs$^{(11)}$	0.19	-0.60	0.24	-0.67	0.10	0.12	0.26	0.68	
	GMMs$(r_o)^{(11)}$	-0.03	-0.02	-0.02	-0.06	0.10	0.24	0.10	0.25	
	GMMs$(\widehat{r})^{(11)}$			-0.01	-0.10	0.11	0.28	0.11	0.30	
	GMMs$^{(00)}$			0.04	-0.19	0.13	0.38	0.13	0.43	
	GMMs$(r_o)^{(00)}$			0.00	-0.05	0.11	0.36	0.11	0.37	
	GMMs$(\widehat{r})^{(00)}$			0.00	-0.03	0.11	0.37	0.11	0.37	

Table 5: Simulation results design 11, scheme 1, disturbances normal

T	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
5	LSDV	-0.41	0.05	-0.41	0.05	0.08	0.08	0.42	0.09	
	GLS	0.10	-0.10	0.09	-0.08	0.04	0.07	0.09	0.11	
	FGLS			0.11	-0.07	0.03	0.06	0.12	0.09	
	GMMfl$^{(2)}$	-0.19	-0.02	-0.20	-0.02	0.16	0.09	0.26	0.09	1.24
	GMMfl$^{(1)}$	-0.09	-0.01	-0.10	-0.02	0.20	0.11	0.22	0.11	0.80
	GMMfl$^{(0)}$			-0.03	0.00	0.21	0.11	0.21	0.11	0.55
	GMMld$^{(2)}$	0.07	-0.03	0.04	-0.02	0.08	0.08	0.09	0.09	
	GMMcd$(r_o)^{(2)}$	0.06	-0.03	0.03	-0.02	0.08	0.08	0.09	0.09	
	GMMcd$(\widehat{r})^{(2)}$			0.01	-0.01	0.10	0.09	0.10	0.09	
	GMMld$^{(1)}$	0.05	-0.02	0.03	-0.02	0.11	0.09	0.12	0.09	
	GMMcd$(r_o)^{(1)}$	0.03	-0.01	0.02	-0.01	0.13	0.09	0.13	0.09	
	GMMcd$(\widehat{r})^{(1)}$			-0.01	-0.01	0.15	0.09	0.15	0.09	
	GMMld$^{(0)}$			-0.01	0.00	0.12	0.09	0.12	0.09	
	GMMcd$(r_o)^{(0)}$			-0.01	0.00	0.13	0.09	0.13	0.09	
	GMMcd$(\widehat{r})^{(0)}$			-0.03	0.00	0.15	0.09	0.15	0.09	
	GMMs$^{(21)}$	-0.03	0.00	-0.03	-0.01	0.10	0.08	0.10	0.08	
	GMMs$(r_o)^{(21)}$	-0.05	0.00	-0.05	0.00	0.11	0.08	0.12	0.08	
	GMMs$(\widehat{r})^{(21)}$			-0.08	0.00	0.14	0.08	0.16	0.08	
	GMMs$^{(11)}$	0.01	0.00	0.00	-0.01	0.10	0.09	0.10	0.09	
	GMMs$(r_o)^{(11)}$	0.00	0.00	-0.01	0.00	0.11	0.09	0.11	0.09	
	GMMs$(\widehat{r})^{(11)}$			-0.04	0.00	0.13	0.09	0.14	0.09	
	GMMs$^{(00)}$			-0.01	0.00	0.11	0.09	0.11	0.09	
	GMMs$(r_o)^{(00)}$			-0.02	0.00	0.11	0.09	0.11	0.09	
	GMMs$(\widehat{r})^{(00)}$			-0.04	0.00	0.13	0.09	0.14	0.09	

T	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
10	LSDV	-0.21	0.06	-0.20	0.05	0.04	0.05	0.21	0.07	
	GLS	0.04	-0.04	0.04	-0.04	0.03	0.05	0.05	0.06	
	FGLS			0.14	-0.08	0.02	0.04	0.14	0.09	
	GMMfl$^{(2)}$	-0.13	0.01	-0.13	0.01	0.07	0.06	0.15	0.06	0.55
	GMMfl$^{(1)}$	-0.03	0.00	-0.04	0.00	0.09	0.07	0.10	0.07	0.28
	GMMfl$^{(0)}$			-0.01	0.00	0.09	0.06	0.09	0.06	0.19
	GMMld$^{(2)}$	0.08	-0.04	0.06	-0.03	0.04	0.05	0.07	0.06	
	GMMcd$(r_o)^{(2)}$	0.04	-0.02	0.03	-0.02	0.04	0.05	0.05	0.06	
	GMMcd$(\widehat{r})^{(2)}$			0.02	-0.02	0.05	0.06	0.06	0.06	
	GMMld$^{(1)}$	0.05	-0.02	0.04	-0.02	0.07	0.06	0.08	0.06	
	GMMcd$(r_o)^{(1)}$	0.01	-0.01	0.01	-0.01	0.08	0.06	0.09	0.06	
	GMMcd$(\widehat{r})^{(1)}$			0.01	-0.01	0.09	0.06	0.09	0.06	
	GMMld$^{(0)}$			0.00	0.00	0.07	0.06	0.07	0.06	
	GMMcd$(r_o)^{(0)}$			0.00	0.00	0.08	0.06	0.08	0.06	
	GMMcd$(\widehat{r})^{(0)}$			-0.01	0.00	0.08	0.06	0.08	0.06	
	GMMs$^{(21)}$	-0.04	0.01	-0.04	0.00	0.06	0.05	0.07	0.05	
	GMMs$(r_o)^{(21)}$	-0.08	0.01	-0.07	0.01	0.06	0.05	0.09	0.06	
	GMMs$(\widehat{r})^{(21)}$			-0.07	0.01	0.07	0.06	0.10	0.06	
	GMMs$^{(11)}$	0.02	0.00	0.01	-0.01	0.06	0.06	0.06	0.06	
	GMMs$(r_o)^{(11)}$	-0.01	0.00	-0.01	0.00	0.06	0.06	0.06	0.06	
	GMMs$(\widehat{r})^{(11)}$			-0.02	0.00	0.07	0.06	0.08	0.06	
	GMMs$^{(00)}$			0.00	0.00	0.06	0.06	0.06	0.06	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.07	0.06	0.07	0.06	
	GMMs$(\widehat{r})^{(00)}$			-0.01	0.00	0.07	0.06	0.07	0.06	

T	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
20	LSDV	-0.10	0.03	-0.10	0.03	0.03	0.03	0.10	0.05	
	GLS	0.01	-0.01	0.01	-0.01	0.02	0.03	0.03	0.04	
	FGLS			0.17	-0.11	0.02	0.03	0.17	0.12	
	GMMfl$^{(2)}$	-0.09	0.02	-0.09	0.02	0.03	0.04	0.10	0.04	0.33
	GMMfl$^{(1)}$	-0.01	0.00	-0.02	0.00	0.04	0.04	0.05	0.04	0.18
	GMMfl$^{(0)}$			0.00	0.00	0.04	0.04	0.04	0.04	0.14
	GMMld$^{(2)}$	0.09	-0.05	0.08	-0.04	0.02	0.04	0.08	0.05	
	GMMcd$(r_o)^{(2)}$	0.03	-0.02	0.02	-0.02	0.03	0.04	0.03	0.04	
	GMMcd$(\widehat{r})^{(2)}$	0.00	0.00	0.02	-0.02	0.03	0.04	0.04	0.04	
	GMMld$^{(1)}$	0.05	-0.02	0.04	-0.02	0.05	0.04	0.07	0.05	
	GMMcd$(r_o)^{(1)}$	0.00	0.00	0.00	0.00	0.06	0.04	0.06	0.04	
	GMMcd$(\widehat{r})^{(1)}$			0.00	0.00	0.06	0.04	0.06	0.04	
	GMMld$^{(0)}$			0.00	0.00	0.05	0.04	0.05	0.04	
	GMMcd$(r_o)^{(0)}$			0.00	0.00	0.05	0.04	0.05	0.04	
	GMMcd$(\widehat{r})^{(0)}$			0.00	0.00	0.05	0.04	0.05	0.04	
	GMMs$^{(21)}$	-0.04	0.01	-0.05	0.01	0.03	0.04	0.06	0.04	
	GMMs$(r_o)^{(21)}$	-0.06	0.01	-0.07	0.01	0.03	0.04	0.08	0.04	
	GMMs$(\widehat{r})^{(21)}$			-0.07	0.01	0.04	0.04	0.08	0.04	
	GMMs$^{(11)}$	0.02	0.00	0.01	0.00	0.04	0.04	0.04	0.04	
	GMMs$(r_o)^{(11)}$	0.00	0.00	-0.01	0.00	0.04	0.04	0.04	0.04	
	GMMs$(\widehat{r})^{(11)}$			-0.01	0.00	0.04	0.04	0.04	0.04	
	GMMs$^{(00)}$			0.00	0.00	0.04	0.04	0.04	0.04	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.04	0.04	0.04	0.04	
	GMMs$(\widehat{r})^{(00)}$			0.00	0.00	0.04	0.04	0.04	0.04	

T	Estimator	$\bar{B}_\gamma$	$\bar{B}_\beta$	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
50	LSDV	-0.04	0.02	-0.04	0.02	0.01	0.02	0.04	0.03	
	GLS	0.00	0.00	0.00	0.00	0.01	0.02	0.01	0.02	
	FGLS			0.20	-0.16	0.01	0.02	0.20	0.16	
	GMMfl$^{(1)}$	0.00	0.00	-0.01	0.00	0.02	0.02	0.02	0.02	0.14
	GMMfl$^{(0)}$			0.00	0.00	0.02	0.02	0.02	0.02	0.13
	GMMld$^{(1)}$	0.05	-0.02	0.05	-0.02	0.03	0.03	0.05	0.03	
	GMMcd$(r_o)^{(1)}$	0.00	0.00	0.00	0.00	0.03	0.03	0.03	0.03	
	GMMcd$(\widehat{r})^{(1)}$			0.00	0.00	0.04	0.03	0.04	0.03	
	GMMld$^{(0)}$			0.00	0.00	0.03	0.03	0.03	0.03	
	GMMcd$(r_o)^{(0)}$			0.00	0.00	0.03	0.03	0.03	0.03	
	GMMcd$(\widehat{r})^{(0)}$			0.00	0.00	0.03	0.03	0.03	0.03	
	GMMs$^{(11)}$	0.01	-0.01	0.01	-0.01	0.02	0.02	0.02	0.02	
	GMMs$(r_o)^{(11)}$	0.00	0.00	0.00	0.00	0.02	0.02	0.02	0.02	
	GMMs$(\widehat{r})^{(11)}$	0.00	0.00	0.00	0.00	0.02	0.02	0.02	0.02	
	GMMs$^{(00)}$			0.00	0.00	0.02	0.02	0.02	0.02	
	GMMs$(r_o)^{(00)}$			0.00	0.00	0.02	0.02	0.02	0.02	
	GMMs$(\widehat{r})^{(00)}$			0.00	0.00	0.02	0.02	0.02	0.02	

Table 6: Simulation results for $T = 10$, $N = 20$, scheme 2, disturbances normal

Design	Estimator	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
11	LSDV	-0.21	0.03	0.07	0.17	0.22	0.18	
	GLS	0.02	0.01	0.05	0.08	0.05	0.08	
$\gamma = 0.75$	FGLS	0.14	-0.11	0.04	0.07	0.14	0.13	
$\zeta = 3$	GMMfl$^{(2)}$	-0.22	-0.02	0.09	0.27	0.24	0.27	0.93
$\pi = 1$	GMMfl$^{(1)}$	-0.10	-0.07	0.14	0.48	0.17	0.49	0.84
$\phi = 1$	GMMfl$^{(0)}$	-0.02	-0.01	0.17	0.84	0.17	0.84	0.95
$\mu = 1$	GMMld$^{(2)}$	0.06	-0.03	0.06	0.10	0.08	0.10	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.03	0.01	0.06	0.11	0.06	0.11	
$r_o \approx 0.13$	GMMcd$(\widehat{r})^{(2)}$	-0.06	0.06	0.10	0.14	0.12	0.16	
	GMMld$^{(1)}$	0.04	-0.01	0.10	0.20	0.11	0.20	
	GMMcd$(r_o)^{(1)}$	0.01	0.02	0.11	0.24	0.11	0.24	
	GMMcd$(\widehat{r})^{(1)}$	-0.04	0.02	0.15	0.33	0.15	0.33	
	GMMld$^{(0)}$	-0.02	0.00	0.14	0.54	0.14	0.54	
	GMMcd$(r_o)^{(0)}$	-0.02	0.00	0.16	0.71	0.16	0.71	
	GMMcd$(\widehat{r})^{(0)}$	-0.04	-0.03	0.20	0.96	0.20	0.96	
	GMMs$^{(21)}$	-0.09	0.06	0.07	0.17	0.11	0.18	
	GMMs$(r_o)^{(21)}$	-0.12	0.05	0.08	0.19	0.14	0.19	
	GMMs$(\widehat{r})^{(21)}$	-0.16	0.02	0.10	0.23	0.19	0.23	
	GMMs$^{(11)}$	0.00	0.02	0.08	0.17	0.08	0.17	
	GMMs$(r_o)^{(11)}$	-0.02	0.03	0.09	0.20	0.09	0.20	
	GMMs$(\widehat{r})^{(11)}$	-0.06	0.00	0.11	0.28	0.12	0.28	
	GMMs$^{(00)}$	-0.01	0.00	0.09	0.26	0.09	0.26	
	GMMs$(r_o)^{(00)}$	-0.01	0.00	0.10	0.29	0.10	0.29	
	GMMs$(\widehat{r})^{(00)}$	-0.02	-0.02	0.12	0.48	0.12	0.49	

Design	Estimator	bias γ	bias β	ster γ	ster β	rmse γ	rmse β	$\widehat{r}$
12	LSDV	-0.21	0.03	0.07	0.17	0.22	0.18	
	GLS	0.04	0.16	0.04	0.14	0.06	0.22	
$\gamma = 0.75$	FGLS	0.23	-0.17	0.01	0.06	0.23	0.18	
$\zeta = 3$	GMMfl$^{(2)}$	-0.27	-0.03	0.10	0.27	0.29	0.27	14.75
$\pi = 1$	GMMfl$^{(1)}$	-0.25	-0.15	0.22	0.52	0.34	0.54	15.96
$\phi = 1$	GMMfl$^{(0)}$	-0.01	0.02	0.37	0.96	0.37	0.96	9.51
$\mu = 5$	GMMld$^{(2)}$	0.21	-0.09	0.03	0.10	0.21	0.13	
$\rho = 0.95$	GMMcd$(r_o)^{(2)}$	0.07	0.20	0.05	0.19	0.09	0.27	
$r_o \approx 3.29$	GMMcd$(\widehat{r})^{(2)}$	0.01	0.16	0.13	0.23	0.13	0.28	
	GMMld$^{(1)}$	0.18	-0.02	0.05	0.20	0.19	0.21	
	GMMcd$(r_o)^{(1)}$	0.02	0.16	0.13	0.45	0.13	0.47	
	GMMcd$(\widehat{r})^{(1)}$	-0.01	0.10	0.17	0.46	0.17	0.47	
	GMMld$^{(0)}$	0.02	0.19	0.26	0.80	0.26	0.82	
	GMMcd$(r_o)^{(0)}$	-0.01	0.16	0.27	1.34	0.27	1.34	
	GMMcd$(\widehat{r})^{(0)}$	-0.03	0.08	0.30	1.65	0.30	1.65	
	GMMs$^{(21)}$	0.11	0.13	0.05	0.17	0.12	0.22	
	GMMs$(r_o)^{(21)}$	-0.15	0.08	0.09	0.25	0.17	0.26	
	GMMs$(\widehat{r})^{(21)}$	-0.15	0.05	0.14	0.26	0.21	0.27	
	GMMs$^{(11)}$	0.16	0.04	0.05	0.18	0.17	0.18	
	GMMs$(r_o)^{(11)}$	-0.03	0.09	0.11	0.31	0.11	0.32	
	GMMs$(\widehat{r})^{(11)}$	-0.06	0.03	0.15	0.33	0.17	0.33	
	GMMs$^{(00)}$	0.05	0.10	0.11	0.31	0.12	0.33	
	GMMs$(r_o)^{(00)}$	0.01	0.05	0.15	0.46	0.15	0.46	
	GMMs$(\widehat{r})^{(00)}$	-0.02	0.00	0.18	0.52	0.18	0.52	