Bound fluxon pair in one-dimensional SQUID array

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Abstract

We propose a one-dimensional array of superconducting quantum interference devices (SQUIDs) composed of three asymmetrically positioned Josephson junctions to realize a discrete double sine-Gordon (DSG) model. Two fluxons in this SQUID array attract each other and form bound states with internal oscillation modes. We conduct numerical simulations of a discrete DSG equation, and show that the period of the internal oscillation of a moving fluxon pair exhibits relativistic time dilation except near the speed of light. We also show that driving with a pure alternating current causes progressive motion of the bound fluxon pair even in the presence of dissipation.

Key words: SQUID, fluxon, double sine-Gordon equation
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1. Introduction

Fluxons in Josephson junction systems can be considered topological excitations, which have excellent stability against various perturbations. This atomic nature of fluxons makes it possible to utilize them as building blocks for information technology such as rapid single-flux-quantum (RSFQ) logic circuits [1]. If two fluxons are bound together and form a kind of artificial molecule, the dynamics becomes rich due to the internal degrees of freedom as regards the relative position of the fluxons.

In a recent paper [2], we showed that the phase difference of a long Josephson junction with a ferromagnetic insulating layer obeys a double sine-Gordon (DSG) equation involving bound half-fluxon pairs. This fluxon pair could be a candidate for use in testing the relativistic time dilation where moving clocks run slowly, since it has an internal oscillation acting as its own clock. Moreover, the quantized levels of the nonlinear internal oscillation of a fluxon pair can store quantum information, which means that a fluxon pair could become a mobile qubit [3].

Here, we propose a one-dimensional superconducting quantum interference device (SQUID) array as another candidate for Josephson junction systems that possess bound fluxon pairs. In our proposed system, each SQUID is composed of three asymmetrically positioned Josephson junctions. The asymmetry in the SQUID yields a double sine term in the current-phase relation for each SQUID. Thus, the phase differences of the SQUIDs obey a DSG equation in a dense array limit, which involves bound fluxon pairs.

We numerically analyze the dynamics of a fluxon pair in this system, and show that the pair actually forms a bound state and possesses internal oscillation modes whose period has a velocity dependence coincident with the relation of the relativistic time dilation except near the speed of light. We also show that driving with a pure alternating current (AC) causes the progressive motion of the bound fluxon pair even in the presence of dissipation.

2. 1D SQUID array

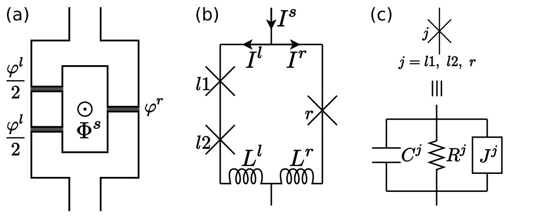

Fig. 1. (a) Schematic diagram of an asymmetric SQUID with three Josephson junctions. (b) Representation of the equivalent circuit, where the cross denotes a Josephson junction element that is described by the resistively shunted junction model as shown in (c).

Figure 1(a) is a schematic diagram of an asymmetric SQUID with three Josephson junctions. This type of SQUID was previously discussed by Zapata et al., regarding the realization of a rocking ratchet mechanism [4]. Using a series of two identical Josephson junctions, a $\sin(\varphi/2)$ term appears in the total current through the SQUID. Here, we utilize this feature to realize a discrete double sine-Gordon model.

As in Ref. [4], we assume that the SQUID is formed of conventional junctions whose gauge invariant phase differences φ^j $(j = l1, l2, r)$ can be adequately described by the resistively and capacitively shunted junction (RCSJ) model [5] as shown in Fig. 1(c). We also assume that the two junctions in the left arm are identical and take $C^{l1} = C^{l2} \equiv 2C^l$, $R^{l1} = R^{l2} \equiv R^l/2$, and $J^{l1} = J^{l2} \equiv J^l$, where R^j, C^j, and J^j are the resistance, capacitance, and critical current of junction j. In this case, $\varphi^{l1} = \varphi^{l2}$ could be expected, and the current through the left arm, I^l, is expressed by the total phase difference $\varphi^l = \varphi^{l1} + \varphi^{l2}$ of the left arm as

$$I^l = J^l \sin\left(\frac{\varphi^l}{2}\right) + \frac{\hbar}{2eR^l}\dot{\varphi}^l + \frac{\hbar C^l}{2e}\ddot{\varphi}^l. \qquad (1)$$

On the other hand, the current through the right arm is expressed as

$$I^r = J^r \sin\left(\varphi^r\right) + \frac{\hbar}{2eR^r}\dot{\varphi}^r + \frac{\hbar C^r}{2e}\ddot{\varphi}^r. \qquad (2)$$

Here, the dot denotes the time differentiation.

Figure 1(b) shows the equivalent circuit of the SQUID. In the limit where the contributions L^l and L^r to the total loop inductance $L^s = L^l + L^r$ are such that $|L^s I^s| \ll \Phi_0$ with $\Phi_0 = h/2e$ being the flux quantum, the total flux Φ^s is approximately the external flux $\Phi^s_{\rm ext}$. Then, the fluxoid quantization condition for the loop within a SQUID is

$$\varphi^l - \varphi^r = -\varphi^s_{\rm ext} - 2\pi n^s, \qquad (3)$$

where $\varphi^s_{\rm ext} = 2\pi\Phi^s_{\rm ext}/\Phi_0$ and n^s denotes the vorticity of the SQUID. In the following, we restrict ourselves to the limit of a small loop size and neglect $\varphi^s_{\rm ext}$ and n^s for simplicity. Then, the SQUID can be described by a single phase difference $\varphi = \varphi^l = \varphi^r$.

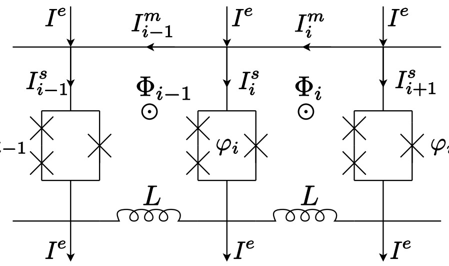

Fig. 2. Equivalent circuit of a SQUID array.

Figure 2 shows the equivalent circuit of our proposed SQUID array. The fluxoid quantization condition for the ith mesh between SQUIDs is

$$\varphi_i - \varphi_{i+1} = -2\pi\frac{\Phi_i}{\Phi_0} = -\varphi_{\rm ext} - 2\pi\frac{L}{\Phi_0}I^m_i, \qquad (4)$$

where Φ_i is the magnetic flux through the mesh, L is the loop inductance of the mesh, I^m_i is the mesh current circulating in each loop, and $\varphi_{\rm ext} = 2\pi\Phi_{\rm ext}/\Phi_0$ with $\Phi_{\rm ext}$ being the external flux through the mesh. The vorticity of the mesh is absorbed in the definition of φ_i, which can be considered in the boundary condition. The current conservation reads

$$I^e + I^m_i = I^s_i + I^m_{i-1}, \quad I^s_i = I^l_i + I^r_i, \qquad (5)$$

where I^e is the external current. Substituting Eqs. (1), (2), and (4) into Eq. (5), and after appropriate normalization we obtain

$$\frac{\partial^2\varphi_i}{\partial t^2} + \alpha\frac{\partial\varphi_i}{\partial t} + \frac{2}{1+2\zeta}\left\{\sin\left(\frac{\varphi_i}{2}\right) + \zeta\sin(\varphi_i)\right\} - \left(\varphi_{i+1} + \varphi_{i-1} - 2\varphi_i\right)/d^2 = \gamma(t), \qquad (6)$$

where $\alpha = t_0/RC$, $\zeta = J^r/J^l$, $\gamma(t) = I^e/I_0$, and $d = \sqrt{2\pi L I_0/\Phi_0}$ with $t_0 = \sqrt{\Phi_0 C/2\pi I_0}$ being the unit of time t, $I_0 = J^r + J^l/2$ being the unit of current, $R = (1/2R^l + 1/R^r)^{-1}$, and $C = C^l/2 + C^r$. Thus, the SQUID array obeys a discrete DSG equation with friction and external driving.

The parameter d can be considered the effective mesh size of the SQUID array normalized by the typical width of a single fluxon. In the limit of a small d, i.e., the dense array limit, and when $\alpha = \gamma = 0$, Eq. (6) becomes a continuous DSG equation:

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{2}{1+2\zeta}\left\{\sin(\varphi/2) + \zeta\sin(\varphi)\right\} = 0. \qquad (7)$$

Here, the normalization unit of velocity is the speed of light in this system $c = d_0/dt_0 = d_0/\sqrt{LC}$ with d_0 being the actual width of the mesh. The DSG equation is known to have a static kink-pair solution:

$$\varphi_{\rm kp}(x) = \sigma_{\rm SG}(x + r_0/2) + \sigma_{\rm SG}(x - r_0/2), \qquad (8)$$
$$\sigma_{\rm SG}(x) = 4\arctan(\exp(x)),$$

where $r_0 = 2\text{arcsinh}(\sqrt{2\zeta})$ and $\sigma_{\rm SG}(x)$ is the kink-soliton solution of the sine-Gordon equation [6].

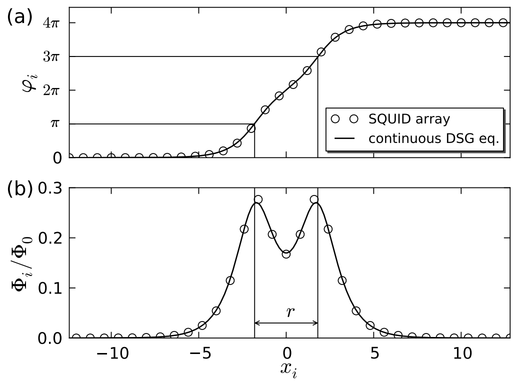

Fig. 3. (a) Phase profile for a fluxon pair when $d = 0.8$, $\zeta = 4$, and $N = 32$. (b) Magnetic flux through the mesh.

Figure 3(a) shows the plot of the lowest energy configurations of φ_i for $d = 0.8$, $\zeta = 4$, and $N = 32$ (N is the number of SQUIDS) as a function of $x_i = d \cdot i$ under the boundary condition; $\varphi_N = \varphi_0 + 4\pi$. The solid curve denotes the kink-pair solution [Eq. 8]. Figure 3(b) shows the normalized magnetic flux through the mesh, $\Phi_i/\Phi_0 = (\varphi_{i+1} - \varphi_i)/2\pi$, together with the curve of $\varphi'_{\rm kp}(x) \cdot d/2\pi$ where the prime denotes the differentiation with respect to x. These figures show that the kink-pair solution well describes the phase differences of the SQUID array system even when the discreteness is not weak.

Therefore, we expect that the SQUID array system also has a bound fluxon pair with internal oscillation modes [6]. In the following section, we show the results of numerical simulations of the internal oscillations of the bound fluxon pair. In these simulations, we adopt the velocity Verlet integration scheme [7] and estimate the separation between the two fluxons $r(t)$ by numerically searching for the positions where $\varphi = \pi$ and 3π as shown in Fig. 3(b). The center of mass position of the pair $x_c(t)$ is determined in the same way, and the average velocity v is estimated from $x_c(t)$.

3. Relativistic time dilation

Since the continuous DSG equation [Eq. 7] is Lorentz invariant, the period of the kink-pair oscillation T should increase with v as $T = T_0/\sqrt{1-v^2}$, where T_0 is the period at $v = 0$ [2]. Also in a SQUID array system that obeys a discrete DSG equation, it would be possible to observe the relativistic time dilation by using the resonance effect [2], if the discreteness is not strong. We performed numerical simulations based on Eq. (6) with

$$\gamma(t) = \gamma_{\rm dc} + \gamma_{\rm ac}\sin(2\pi\nu_e t). \qquad (9)$$

The balance between α and $\gamma_{\rm dc}$ determines the terminal velocity of the fluxon pair. Resonance occurs when the frequency ν_e of the external AC current matches the frequency of the internal oscillation, and the amplitude of the oscillation becomes large. Thus, we can measure the frequency of the internal oscillation by detecting the variance of the total width of a fluxon pair as a function of the frequency of the applied alternating current.

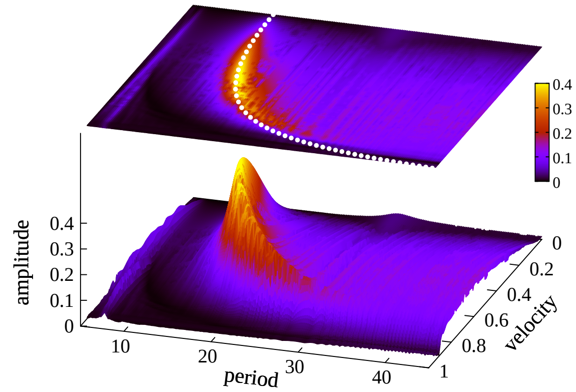

Fig. 4. (Color online). Amplitude of the internal oscillation as a function of the period of external alternating current and the velocity of the bound fluxon pair. The (white) dotted line denotes the relation of the relativistic time dilation.

Figure 4 shows the amplitude of the oscillation of r as a function of the period of the external alternating current, $1/\nu_e$, and the average velocity v of a pair after convergence to the terminal velocity. Here, we choose parameters of $d = 0.25$, $N = 100$, $\zeta = 4$, $\alpha = 0.05$, and $\gamma_{\rm ac} = 0.02$. The amplitude is estimated from the Fourier spectra of $r(t)$. We can easily find the resonance peaks of the internal oscillation, which coincide with the (white) dotted curve, namely, the relation of the relativistic time dilation for $T_0 = 14.18$ except near the speed of light. When the velocity of the fluxon pair approaches to the speed of light, the width of the fluxon becomes comparable to the mesh size d due to the Lorentz contraction, and the discreteness becomes effectively strong, which results in this discrepancy.

4. Progressive motion under AC drive

Although discreteness is an obstacle to exploring the relativistic effect near the speed of light, there are interesting phenomena that only occur in discrete systems. One such phenomenon is the stable progressive motion of kinks with homogeneous pure AC driving [8–11]. In a discrete system, kinks experience a periodic potential, namely the Peierls-Nabarro (PN) potential, whose period is the lattice constant of the system, which here is d. Therefore, stable propagation can be maintained only when a kind of mode locking occurs between the external driving and the oscillation of the propagating kinks in the PN potential. Thus, the mean velocity is predicted to be $v = \nu_e d \cdot m/M$, where m and M are the super- and subharmonic resonance orders [11].

Figure 5 shows the d dependence of the average velocity v of the fluxon pair calculated by the numerical simulation of Eq. (6) for $N = 32$, $\zeta = 4$, $\alpha = 0.01$, $\gamma_{\rm dc} = 0$, $\gamma_{\rm ac} = 0.03$, and $\nu_e = 0.04$. We find that the velocity v of the stable progressive motion coincides with the dotted line denoting the predicted velocity for the harmonic resonance, $v = \nu_e d$.

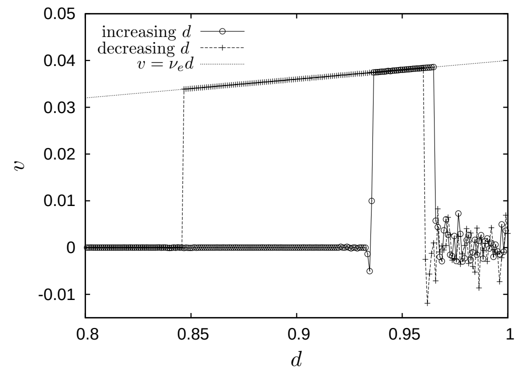

Fig. 5. Mesh size dependence of the average velocity of the fluxon pair.

From results obtained under various conditions, we see that both propagation directions are equally possible and are sensitive to small variations in the conditions. However, if the effect of the external magnetic flux through the SQUID, namely, $\varphi^s_{\rm ext}$ in Eq. (3) is considered, the fluxon pair would experience asymmetric PN potential, and the direction of progress would be fixed. We will discuss this effect in a future study.

5. Conclusions

We have proposed a 1D array of asymmetrical SQUIDs to realize a discrete DSG model, which has bound fluxon states with internal oscillation modes. We have numerically shown that the internal oscillation of a moving fluxon pair exhibits a relativistic time dilation and that pure AC driving results in stable propagation of the fluxon pair.

It will be possible to create our proposed system using current technology [12]. The parameter d for the mesh loop should be no larger than 1, and yet it should be sufficiently larger than that for the SQUID component. In particular, in order to observe relativistic effects, it is necessary to create smaller underdamped SQUID components, e.g., a quarter of the length of that used in [12]. It will also be possible to observe these phenomena using, e.g., low-temperature scanning electron microscopy [13].

Acknowledgements

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