The Development of Mathematics Education Based on Ethnomathematics (2) —Analysis of Universal Activities in terms of Verbs —

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In the development of a new curriculum whose focus is mathematical activity as signified, the verb as its signifier should be a center of consideration. This is why the proposed curriculum is named verb-based. In mathematics education, an activity deepens itself in a recursive manner through symbolization of activity. Bishop (1991) has widened this concept of activity and claimed that each culture has developed its own mathematics through six universal activities. The verb-based curriculum intends to make use of these cultural activities.

The overall theme of the series of researches concerns the international cooperation in mathematics education especially with Kenya as a leading country on the African continent. In the development of the verb-based curriculum, the authors have this concern, with resonance of mathematics educators such as D'Ambrosio (1985), Gerdes (1990), Nebres (1988) in the developing countries, who have proposed cultural perspectives of mathematics education. And the target of this paper was the analysis of the Japanese course of study at the primary level, and the verbs in it were collected and analyzed. As a result, it has shown the structure of mathematical activities under the theory of internalization.

The six universal activities and the verb-based curriculum

In the process of teaching and learning mathematics in a classroom, an interaction develops among three components, children, teacher and mathematics. Among these components, children play an important role in the constructivism, which is based on the Piaget's epistemology and has developed its theory by focusing on their activity. According to its theory, the operation is interpreted as an activity to be internalized as a result of action towards the environment, and furthermore mathematics as a system of such operations.

When we regard children's activity as acting upon their own environment, the environment should include not only mathematical objects but also other objects, which are highly related to their own culture. Acting upon the objects does not occur in the vacuum of abstractness. Even mathematical objects of today have been created through such activities for many years by abstracting some properties of objects in the environment. In this way, focusing on activities in this paper means also to seek for the possibility of widening up the range of objects.

From a different perspective, Bishop (1991) argues that each culture has developed

mathematics and there exist *six universal activities,* that are 'to count', 'to measure', 'to locate', 'to design', 'to explain' and 'to play', at the basement of these mathematics. For example, there may be different counting systems and expressions of numbers from culture to culture, but there is no culture that does not have a counting activity. In this sense the six activities possess a kind of universality and at the same time their various appearances in different cultures indicate the existence of sub-activities, depending on their environment. It is there that the possibility lies for the analysis of the inter-structure of various cultural activities and sub-activities.

A course of study usually has an arrangement and a structure of knowledge as a target to achieve. More weight used to be given to the knowledge acquired at the end of a class, and so naturally the main concern of mathematics education was the formula and technique to enable the speed and accuracy of solution. However, recently this tendency has been under criticism, and instead the affective aspects of the process such as interests, willingness and attitudes, have been put more emphasis upon. If the curriculum is based upon the knowledge, which is expressed in noun form, then it is called the noun-based curriculum. On the other hand, the curriculum to be considered in this paper is called the verb-based curriculum whose concern is to arrange mathematical activities in the sense of the constructivism, following their development.

Naturally, it is important to have actual activities in mind in accordance with verbs listed in this curriculum. In the teaching and learning process, there are such activities as interaction between teacher and children, mutual interaction among children, and an individual activity by children, and their details may differ from one another, depending upon time, place and after all culture. The authors, however, have assumed that the essence of activities is expressed in terms of verbs. This paper is a first step towards the development of this new curriculum principle.

For example, one of the universal activities, to measure, develops from the activity 'to compare' through the activities, 'to measure' and 'to know' to the activities, 'to estimate' and 'to understand' relations of different kinds of units. All these activities, signified by verbs, seem to manifest a development of measuring activity. And the verb-based curriculum focuses on these activities and expresses rigorously this development in terms of verbs.

From the above consideration, the objectives of this paper are as follows;

(1) To analyze general characteristics of verbs,

(2) To categorize the verbs in the course of study and to analyze their inter-structure,

(3) To consider a theoretical foundation for the verb-based curriculum.

Characteristics of verbs

The general characteristics of verbs are analyzed before the analysis of the verbs in the course of study. Like shown in the table 1 and figure 1, nouns occupy more percentage of the vocabulary than verbs, and the percentage goes even higher as the number of vocabulary, which are most frequently used, increases in the Japanese language (National Language Institute, 1984). The first characteristic of verbs, in other words, indicates that one verb can take many nouns as an object, a place, means and so on. Of course a noun can take a certain number of verbs as well, but the verb has a large versatility considering the number of objects. For example the verb 'to count', which is typical in mathematics education, can take such objects as cows, papers

and many others. The counting action for cows and the one for papers differ in appearance, but both the activity 'to count' and its corresponding word are abstracted as a common property from these activities.

Table 1. Vocabulary and percentage of noun and verb: National Language Institute

Vocabulary	Noun	Verb
100	44.5	27.0
200	49.8	23.5
300	48.5	20.5
400	51.4	18.4
500	52.0	18.5
600	52.9	18.4
700	52.4	18.8
800	53.8	18.4
900	54.2	18.3
1000	54.5	18.2
1100	55.3	18.2
1200	55.5	18.7

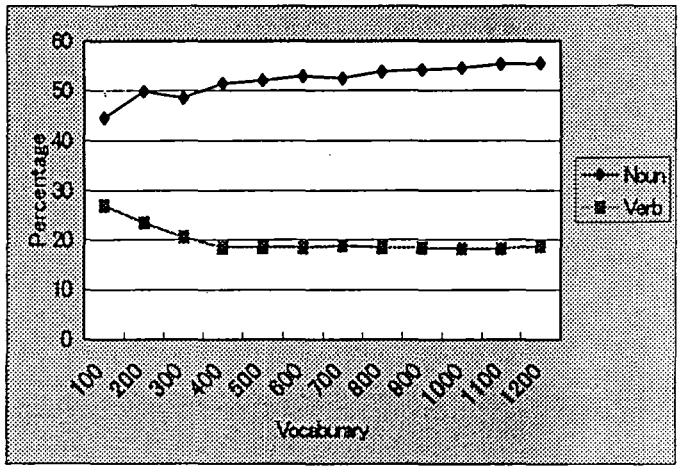

Fig. 1. The graph of percentage of noun and verb

The reason why verbs as signifier are fewer than nouns, lies also in the property of activity as signified that is instantaneous and does not retain its locus of movement very long. For example, the activity 'to hit' can be perceived by means of eyes and ears, and it remains as an afterimage for only a while. Thus verbs have the second characteristic that the activity as its signified is instantaneous.

The last property of verbs is flexibility towards the object to take. The way of hitting a door and the way of hitting a shoulder are different in terms of an object of activity, movement of hand and even the part of hand to use. And the intention of activity also differs from each other. While the former is to call the attention of somebody inside the room, the latter is to alleviate stiffness in the shoulder or to inform somebody about demotion to the lower post in a certain context. These two activities are differentiated not in terms of verbs but in terms of nouns. So when a new object emerges, the activity towards it can be expressed in analogy with the existing activities. For example, the computer-related facilities are fast permeating our daily life and the emergence of new objects such as keyboard and mouse urges an extensive interpretation of the existing activities and a widened usage of the corresponding verbs in order to cover a new entry.

Three characteristics regarding verbs have been discussed so far. They are in short 'the comparatively small number of verbs', 'the activity as signified of verbs is instantaneous' and 'the flexibility of verbs towards its objects'. This last one can integrate all the characteristics of verbs and represent the manifoldness in the verbs' association with other words, especially nouns. Then this characteristic is to be named an 'elasticity' of verbs to accommodate newly created objects in analogy with elasticity of rubber that absorbs a shock coming from outside.

Analysis of verbs in the Japanese course of study

The analysis has been done against verbs in the teaching content of Mathematics, the course of study (Ministry of Education, 1989) at the primary school level in Japan. The reason for this is that Kenyan latest syllabus for primary school (Republic of Kenya) was published in 1992 and we have a plan to apply the result here to the analysis of this Kenyan syllabus.

The table 2 shows the verbs at the grade 1 level as a part of result. A, B and C in the table show domains in the course of study. They are namely Numbers and Calculations, Quantities and Measurements, and Geometric Figures respectively.

The methodology of collection of verbs is as follows. Originally verbs are collected per domain and per grade from the course of study in Japanese. Even if they appear more than twice, the repetition is avoided. They are listed here in the dictionary form for the purpose of uniformity and are placed inside the brackets. Then, they are given an English translation with reference to the Japan Society of Mathematics Education (1990), which is placed in front of the brackets. Some of the Japanese verbs are not verbs any longer when they are translated into English (e.g. through (通す)), nor they have one-to-one correspondence with each other (e.g. to understand (理解する), to know (理解する)). Because of this reason, both Japanese and English are provided for reference.

N.A. (Not Available) in the table 2 means that there are no English words that correspond with these Japanese verbs according to Japan Society of Mathematics Education (1990). They don't have a very significant meaning and their omission does not give much impact on further analysis.

Table 2. Verbs for grade 1 in the course of study

	Verb		Quasi-verb
A	to represent (表す) to compare (比べる) to count (数える) to know (知る) regarding (みる) N.A. (する) to consider... in relation (関係付ける) to divide... into equal parts (等分する) in a neatly arranged way (整理する)	to understand, to know (理解する) to interpret, to read (よむ) to group (まとめる) to make (作る) to be able to, to be applied (できる) to use, to be applied (用いる)	understand (理解) correspondence (対応) manipulation (操作) carry out (計算) meaning (意味)
B	through (通す) to read (よむ) N.A. (できる) N.A. (なる)	to compare (比べる) around (ある) N.A. (する)	comparing (比較) experiences (経験) measurement (測定) manipulation (操作) understand (理解)
C	through (通す) to recognize (認める) to grasp (とらえる) to construct (作る) N.A. (なる)	to decompose (分解する) to use (用いる) to represent (言い表す) N.A. (する)	observing (観察) experiences (経験) constructing (構成) manipulative activities (操作) understand (理解)

(NOTE: Domain D, Quantitative Relations, starts only at grade 3.)

The quasi-verb in this table is categorized as a noun in Japanese that becomes easily a verb when it is suffixed by 'suru', which means 'to do'. Its strong association with the verb can be known also from the English translation. It is a combination of

two verbs or a combination of a verb and a noun in Chinese characters, but as a whole it has a verb-like meaning.

There are four points noted from the analysis of verbs, as shown below.

(1) There are more transitive verbs than intransitive ones.

Table 3. Number of verbs per domain

Domain	A	B	C	D
Number of verbs in each domain	43	17	22	34

The table 3 shows the number of words when they are counted only once per domain from grade 1 to grade 6. In totality, there are 68 verbs that appear in the course of study, avoiding the repetition. Only 8 verbs out of them are intransitive verbs. While intransitive verbs express a certain *state* of things, transitive verbs represent *'activity towards objects'* which is fundamental in mathematics education. And the large number of transitive verbs demonstrates that mathematics education focuses on these activities.

(2) Making activity an object of another activity

* Usage of 'masu' form of verb as a noun

(Example) duplications (重なり), dispersion (散らばり) etc.

* How-to

(Example) how to represent (表し方) etc.

For this same Japanese, there are different English translations, which are the way of representing, notation, how...are set, representation.

* Quasi-verbs

(Example) manipulation, measurement, etc.

They are related to *objectification of activity.* This means that the primary activity itself becomes an object of another activity. After having the primary activity, say to measure, we now consider the meaning or the method of the activity. Converting the verb to a noun (e.g. 'to measure' to 'measurement' or 'the way of measuring') can play an important role in this objectification process from just 'doing' to 'considering the meaning or the method of doing'

(3) The role of verb 'to deepen'

Among 6 verbs, 'to use', 'to interpret', 'to understand', 'to represent', 'to know' and 'to deepen', which appear in all four domains, the last verb has a special function in such a sense that the signified activity develops based on the knowledge that has been learned before. Its first appearances for domains A, B, C and D are at 2nd grade, 3rd grade, 3rd grade and 5th grade respectively. They can be regarded as boundary between basic and advanced topics.

(4) There are 20 verbs that belong to both domains A and D.

This number constitutes approximately 60% of verbs in domain D, which starts only at grade 3. It is known from this fact that activities in domain D develop on the basis of the activities in domain A.

The first point shows that activities towards objects are given more focus on in mathematics education, and the other three points show that the secondary activities develop through the *objectification* and *deepening* of the primary activities. In totality they briefly describe that due attention should be given to development of activities in mathematics class. Now it is a next step to consider structural relation of activities and thus that of verbs to attain our goals.

From the structure of activities to the one of verbs

In the previous section the 68 verbs were extracted from the course of study and their general properties were considered preliminarily. Further analysis should be done to explore the structure of verbs in this section.

They consist of 60 transitive verbs and 8 intransitive ones. Furthermore, we excluded from the former category both 4 verbs that do not take human being as a subject, and 5 verbs that take only specific objects and are not related very much to mathematical activity. It is because the purpose of this research is to consider children's activities towards objects in mathematics education. In short, the 51 verbs are finally selected and analyzed as shown in the table 4.

Considering the nature of activities, they are categorized into personal ones and inter-personal ones, and the former is further subdivided into the ones that involve external action and the ones that do not. Naturally personal external and personal internal activities go hand in hand, but also all three groups of activities are related to each other. However, if more than two categories are involved in a certain activity, then preference is given to the category of personal external activities, because this category represents most explicitly children's activities towards objects. The categorization of verbs in the table 4 is a reflection of three categories of activities.

Table 4. Categories of 51 verbs

Domains	Personal external verbs	Personal internal verbs	Inter-personal verbs
A	to consider... in relation to, to count, to divide into equal parts, to be multiplied, to estimate, to be divided, to apply, to be classified, to move, to transform, to be multiplied or divided, to be determined, to check the results		
B	to measure (測る), to measure (測定する), to decompose (分ける)		
C	to decompose (分解する), to observe, to recognize, to find		to represent (言い表す)
D	to substitute (to be filled in), to gather, to check up, to interpret (よみとる),to devise		to represent (表現する)
AB	to compare		
AC	to construct (to compose)		
AD	to classify, to arrange, to classify and arrange, to calculate, to regard	to consider (考える)	
BC	to grasp (to be represented)		
BD	to choose, to find (to get, to measure)		
CD	to draw	to consider (考察する)	
ABC	through (通して)		
ABD			
ACD	to pay attention, to investigate, to summarize, to make		
BCD			
ABCD	to use, to interpret (to read), to deepen	to understand, to know	to represent (表す)

(NOTE: Japanese verbs are also indicated in the brackets where the English translations are the same. The English verbs in the brackets show an alternative for the same Japanese verbs. For example, both 'to interpret' and 'to read' in the last row correspond with '読む'.)

Now the six universal activities are revisited for further analysis. The two activities 'to explain' and 'to play' belong to the category of inter-personal activities and the remaining four, 'to count', 'to measure', 'to locate' and 'to design' belong to the category of personal external activities. Of course the activity to count may be exercised among several persons in a certain context, but it is regarded as a personal activity because of the same reason as in the above case.

Bishop (1991) analyzed that the activity 'to explain' belongs to the meta-level of the other activities. This inter-personal activity is related closely with the personal internal activity 'to understand'. The latter activity develops within oneself but the former gives a boost to its development. In other words the activities 'to understand' and 'to explain' represent dual process in which they stimulate each other, and the clog in this process of interaction causes a reflection on the activity.

'To play' seems to have a lot of interesting implications for the teaching and learning process. It has two aspects of personal and social activities but in any case it does not put much importance on the outcome of that activity. Enjoyment is its main proponent, and in this sense it is uniquely related to the affective aspect of the learning activity. The clarification of the activity to play, is a future task in mathematics education.

Although all the six activities may possess a part of creation, the activity 'to design' has an active role in this.

> "Designing involves imagining nature without the 'unnecessary' parts, and perhaps even emphasizing some aspects more than others. To great extent, then, designing concerns abstracting a shape from the natural environment." (Bishop, 1991, p.39)

This is very important not only from a mathematical point of view, but also a cultural point of view. While most of our activities are directed towards the environment that already exists, the creative aspect of activities adds a new entry into the list of objects in the environment.

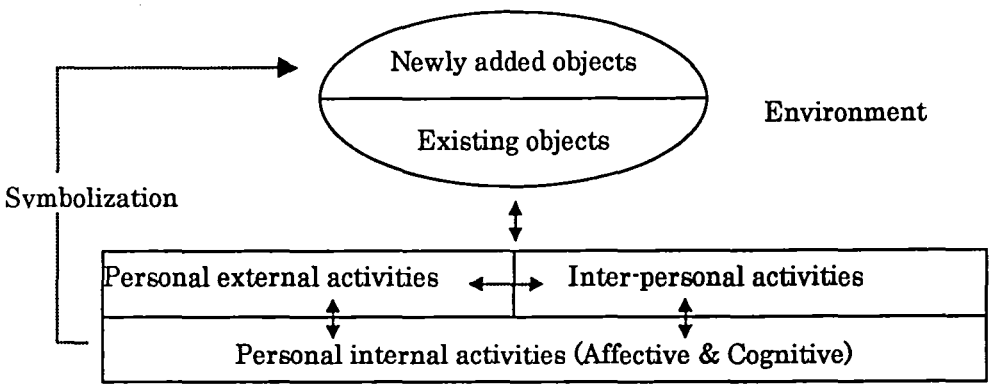

Fig. 2. Structure of categories of activities

From this analysis of universal activities, the figure 2 is drawn. Through personal external activities and inter-personal activities, we act on objects in the environment, and simultaneously we have personal internal activities within ourselves. With this interaction, mere action is changed to the activity with the intention, the activity upon objects is then internalized as an operation, and this internalized operation, by being given a symbol, becomes a new entry into the environment. For example, the activity

to count gives impetus to the formation of internalized operations and symbols such as '1', '+', and '='. Once they are added as new objects into the environment, we can act upon them. As a result, it may lead to realization of the calculation properties or the number system.

Now we would like to take a step further towards development of verb-based curriculum. Verbs are picked from the measurement domain and are arranged according to the grade and the quantities of measurement, which are length, area, capacity and volume (Fig. 3). Because of space limitation, 'to' is removed from all verbs. In the figure, we have observed the following two points.

Firstly the process of measuring activity is more or less similar for various quantities to be dealt with. It is considered to indicate that the universal activity 'to measure' develops itself in stages, taking somewhat the same route despite objects and quantities to measure.

Secondly both personal internal verbs and personal external verbs are found almost in each stage of the process. For example 'to compare' and 'to measure' show external activities, which are practiced as primary activities in the beginning, and they stimulate children 'to understand' as an internal activity important aspects of measuring. The inter-personal activity 'to represent' is indicated only once in this part of the curriculum. It may be assumed that the activity occurs naturally in the course of discussion, but in the other domains 'to represent' plays a more dynamic role in deepening the understanding. The role is recognized more intensively and is being rigorously sought as in communication abilities (e.g. Kanemoto, 1998). While these three categories interact each other as shown in the figure 2, children objectify the primary activities in the following stages.

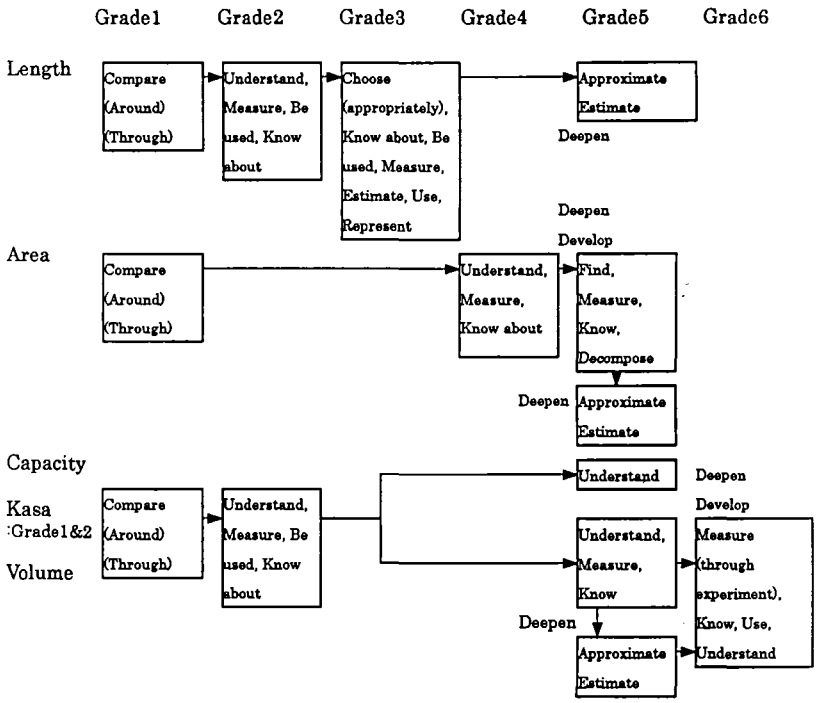

Fig. 3. Verbs in measurement domain

From the figure 3, we can summarize the development of the measuring activity as follows.

(1) Pre-measuring activity

Children grasp rough idea by the activity to 'compare'.

(2) Basic measuring activity 1

Children 'understand' and 'know about' concept of units and meaning of measurement through the activity to 'measure'.

(3) Basic measuring activity 2

Children 'choose appropriately' the measuring instrument, 'know about' the units of measuring, and 'represent' quantity 'using' the appropriate unit.

(4) Estimating activity

Children 'deepen' the understanding of measurement, 'approximate' a figure by the fundamental ones, and 'estimate' the quantity.

(5) Integrative activity

Children 'measure through experiment' various quantities, 'know' how to measure more complicated figures, and 'understand' relations of different kinds of units.

Towards a theoretical foundation of the verb-based curriculum

In Kenya the importance of mathematics and science education at secondary level has been stressed in the seventh development plan for the purpose of industrialization and sustainable development of the society. On the other hand, the Japanese government has expressed concerns over the international cooperation, especially in African education, during the general meeting of UNCTAD (United Nations Conference on Trade and Development) in April 1996, and the basic education is one of the target areas in the educational cooperation. Under this circumstance, the Kenyan government has made a request to the Japanese government to assist her on the efforts of improving the quality of education through establishment of the in-service course for teachers.

This is just one example of the international cooperation, but a very crucial one. It is because Kenya has been one of the biggest recipient countries of Japanese aids and this is a pioneer project in the area of basic education, although Japan has been requested to make similar contribution also by some other countries. Considering the fact that the education has played a very crucial role in the modernization and industrialization process of Japanese society, we should re-examine our role as a member of international community toward the development of education not only financially but also intellectually.

Education is a very complicated enterprise, which involves so many factors of society. In the center of modern education, however, there is a school and the curriculum contains the essence of what is to be taught or to be learned there. Thus consideration of curriculum development constitutes a very first step of the researches on the educational cooperation.

In this paper we have concentrated our efforts on development of curriculum, bearing in mind cultural activities in resonance with proposals of ethnomathematics by many mathematics educators in the developing countries (D'Ambrosio, 1985; Nebres, 1988; Gerdes, 1990).

More clearly, ethnomathematics provides substance to the verb-based curriculum

in terms objects and activities existing within culture. For example, Sona, a traditional drawing by Tchokwe in Angola, was presented and analyzed for mathematics education (Gerdes,1990, 1999). This is one example of cultural activity for the curriculum. Of course in school mathematics, just to be able to draw many patterns of Sona cannot be an educational objective. There are, however, many indicative hints in this example not only for children in the Angola where they were originally drawn but also for 'demathematized members in the highly mathematized society' (Keitel, 1998). The children within the Angolan community of course feel mathematics closer to them and thus cultural tie. It boosts their learning activity cognitively and affectively. On the other hand, the children in a developed country may be able to appreciate other culture and so it enables them to recognize possibilities of alternative design, to grow interest in other patterns and to create new shapes.

Besides the substance of cultural activities, ethnomathematics has posed critical arguments about mathematics especially in the mathematics education (D'Ambrosio, 1985). In Baba, Iwasaki (1998), we discussed that it is important both to view mathematics and mathematics education critically with ethnomathematics and to view ethnomathematics itself critically by application of the three elements of critical mathematics education, that are critical competence, critical distance, and critical engagement.

In summary, it is important in the verb-based curriculum to base oneself within a culture, to view the object critically, to create a new object through objectification and symbolization of cultural activities and to act again on the newly created objects. This recursive process is the same as the structure of activities discussed in the previous section.

As future directions of this research, we have plans;

(1) To analyze the verbs in the Kenyan syllabus and thus activities and to make a comparative study between Japanese and Kenyan curriculum,

(2) To design and to analyze classroom practice using the verb-based curriculum.

In this paper it is assumed that mathematical activity is internalized into the individual learner's schema, but this assumption itself should be examined as a part of future research.

(NOTE: This is an expanded edition of the research paper, which was presented at 24th, Conference of International Group for the Psychology of Mathematics Education at Hiroshima, Japan in July 23rd to 27th, 2000.)

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