

Number and Calculations, Number and Algebraic Expressions

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Abstract : The purpose of this paper is to summarize the changes of teaching and learning contents in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in the Course of Studies for elementary and secondary school mathematics after 1945 in Japan. As a result, the followings are found out as three main characteristics. (a) There is a remarkable difference between the period of “Life-Unite Learning” (1945 – 1957) and the period of “Systematic Learning” (1958 – 1967) regarding to the teaching and learning contents and their sequences in these areas shown in the Course of Studies. (b) The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). In the new one (1998), however, we notice that calculations are rather lightened; decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. (c) The dealing with equations and inequalities in secondary schools is gradually lightened in recent years.

1. Introduction

The purpose of this paper is to summarize the changes of teaching and learning contents in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in elementary and secondary school mathematics after 1945 in Japan. In doing so, we refer to the Course of Studies that have been revised every about ten years by the Ministry of Education.

2. Changes in “Number and Calculations”

The tables 1, 2, 3, and 4 show the summary of changes of the teaching and learning contents regarding to the number and calculations shown in the Course of Studies (1947, 1951, 1958, 1968, 1977, 1989, and 1998).

In the tentative Course of Study (1947), the four operations with fractions were dealt with in the sixth grade. In the tentative one (1951), the timing of dealing with teaching and learning contents in this area was generally one- or two-year later than the before. For example, multiplication facts was in third grade; division by two-digit numbers was in fifth grade; multiplication and division with fractions were in the first grade at lower secondary level; and four operations with positive and negative numbers were in the second at lower secondary level (Table 1).

In the period of “Systematic Learning” (1958 – 1967), the timing was modified to be one-year earlier than the before.

For example, the simple addition and subtraction with two-digit and one-digit numbers were in the first grade; multiplication facts were in second grade; division by two-digit numbers was in fourth grade; multiplication and division with fractions were in the sixth grade; and four operations with positive and negative numbers were in the first grade at lower secondary school level (Table 2). The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989).

In the period of “Modernization” (1968 – 1976), at elementary school level no remarkable changes were made except completing the four operations with fractions in elementary school. On the other hand, at lower secondary level, the idea and structure of number sets was introduced as new teaching and learning content in the second grade (Table 2).

In the period of “Back to Basics” (1977 – 1988), however, the idea and structure of number sets was eliminated (Table 3). In this period, the need for careful selection of the teaching and learning contents in mathematics was emphasized. But we cannot find out the remarkable changes in this area of number and calculations. It means that the area seems to be basic and important in school mathematics.

In the Course of Study (1989), there were no changes in this area at elementary level. At secondary level, some

Table 1. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 1)

		Tentative Course of Study (1947)			Tentative Course of Study (1951)		
		Number	Addition & Subtraction	Multiplication & Division	Number	Addition & Subtraction	Multiplication & Division
PRIMARY	1st Grade	Counting, reading, and writing numbers up to 100	Meanings and mutual relations		Numbers up to 50 Principle of place unit half & half of half	Knowing the meanings of Addition and Subtraction	Having a rich background of multiplication and division
	2nd Grade	Numbers up to 1000	Addition & Subtraction with 2-digit numbers (sum up to 100) Rough estimation of Addition & Subtraction with bridging	Multiplication facts Understanding of meanings of equal division and inclusive division	Numbers up to 500 Principle of place unit Estimation of numbers 1/2, 1/3, 1/4	Facts Written calculation Addition & Subtraction with 2-digit numbers Mutual relations of addition & subtraction	Having a rich background by using the ideas of multiple and fraction
	3rd Grade	Unit fractions	Addition & Subtraction with abacus Written calculation Addition & Subtraction with same denominator fractions	Mental computation 2-digit × 1-digit 1-digit + 1-digit 3-digit ÷ 2-digit	Numbers up to 5000 Extension of Principle of place unit Simple fractions (units) Using decimal numbers	Addition & Subtraction with 3-digit numbers Mixed calculations	Meanings & multiplication facts Mutual relations Connecting the meanings of unit fractions to multiplication & division
	4th Grade	Numbers less than one hundred million Meanings of decimal numbers and decimal point	Addition & Subtraction with compound numbers	Written calculation Multiplication with 3-digit numbers Division with 3-digit numbers	Numbers up to 50000 Decimal positional notation Size of fractions & decimal numbers Relative size & Round numbers	Addition & Subtraction with 4-digit numbers With abacus Mutual relations among four computations	Written calculation with 4-digit numbers 2-digit × 2-digit Division by 1-digit
	5th Grade	Multiple relations of natural numbers and fractions Rate and percentage Approximate numbers		Multiplication & Division with decimal number Multiplication & Division with abacus	Numbers up to 1000000 Equality of fractions Mutual relations of decimals and fractions	With abacus Addition & Subtraction with same denominator fractions With decimal numbers	3-digit × 3-digit Division by 2-digit Multiplication & Division with decimal numbers
	6th Grade	Fractions	Four computations with fractions		Approximate numbers Size of fractions Ratio Rate & percentage	Abacus Four computations with fractions	Division by 3-digit Dealing with 0
LOWER S.	1st Grade	Positive numbers & negative numbers	Four computations with positive and negative numbers Understanding and applying the rules of computations		Numbers less than one trillion Fraction & decimals Ratio, rate, percentage	Four computations with whole numbers, decimal numbers, and fractions	
	2nd Grade	Square numbers & Square root numbers	Using the table of square and square root numbers		Positive numbers & negative numbers	Four computations with positive and negative numbers	
	3rd Grade	History of numbers and computations, and its relation to our daily life			Square numbers & Square root numbers	Computations with square numbers & Using the table of square root numbers	
UPPER S.	Complex numbers	Four computations with complex numbers		Complex numbers	Four computations with complex numbers		

Table 2. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 2)

		Course of Study (1958)			Course of Study (1968)		
		Number	Addition & Subtraction	Multiplication & Division	Number	Addition & Subtraction	Multiplication & Division
PRIMARY	1st Grade	Counting, reading, and writing numbers up to 100 Constructing numbers up to 10 Meanings of place unit of 2-digit numbers Sequence, size and order of numbers	Situations and signs of Addition & Subtraction 2-digit + 1-digit 2-digit - 1-digit	Counting collectively and dividing equally (background experiences)	Counting things and representing with numbers (tens-unit) Sequence, size and order of numbers Relating one number to the other number Representing 2-digit numbers	Situations of Addition & Subtraction (meanings) Addition & Subtraction with simple 2-digit numbers	Counting collectively and dividing equally (Understanding the basics)
	2nd Grade	Reading and writing of numbers up to 1000 Understanding of the idea of place unit Meaning of 0 Basic idea of rate	Addition & Subtraction with 3-digit numbers Written calculations Understanding and using the basic relations of Add. & Subtraction	Situations and sign of multiplication Multiplication facts	Thousands-unit Decimal positional notation, size, order of numbers up to 4-digit numbers Simple fractions such as 1/2, 1/3 etc.	Mutual relations of Addition & Subtraction with 3-digit numbers Written calculation Using the properties of Addition & Subtraction	Situations of multiplication Multiplication facts
	3rd Grade	Reading and writing of numbers up to 10000 Principle of place unit Writing the numbers of 10, 100, 1/10 times Simple fractions Simple decimals	Addition & Subtraction with 4-digit numbers Devising calculation of Add. & Subtraction	Written calculation of multiplication 3-digit × 1-digit Using the relations of multiplication Situations and sign of division Division by 1-digit	Ten Thousands-unit Writing the numbers of 10, 100, 1/10 times How to write decimals numbers and fractions	Using the properties of Addition & Subtraction Addition & Subtraction with decimal numbers and fractions With abacus	Using the properties of multiplication Written calculation of multiplication 3-digit × 2-digit Written calculation of division Division by 1-digit
	4th Grade	Decimal positional notation Approximate numbers Mutual relations of decimals and fractions with natural numbers Meanings of unit fractions Equality and size of fractions	Meanings and relations of four operations Addition & Subtraction with decimal numbers Addition & Subtraction with same denominator fractions With abacus	Multiplication by 3-digit Written calculation of division Division by 2-digit Multiplication & Division of decimal numbers by natural numbers	Units of million and trillion Decimal positional notation Approximate numbers Meanings and writing of decimal numbers How to write fractions and meanings of numerator & denominator	Meanings and properties of four operations Addition & Subtraction with decimal numbers Addition & Subtraction with same denominator With abacus	Division by 2-digit Multiplication & Division of decimal numbers by natural numbers
	5th Grade	Units of million and trillion Understanding of decimals Mutual relation among natural numbers, decimal numbers and fractions Percentage and rate	Addition & Subtraction with decimal numbers Addition & Subtraction with different denominator fractions With abacus	Multiplication & Division with decimal numbers Multiplication & Division of fractions by natural numbers	Classification of natural numbers (odd and even number) Multiples and divisors Understanding the decimal numbers deeply Equality and size of fractions Percentage and rate	Addition & Subtraction with different denominator fractions	Division with decimal numbers Multiplication & Division of fractions by natural numbers
	6th Grade	Approximate numbers Comparing sizes of numbers	Meanings of four operations Addition & Subtraction with fractions		Mutual relation among natural numbers, decimal numbers and fractions	Possibility of four computations Properties of addition and multiplication With fractions	
LOWER S.	1st Grade	Properties of natural numbers (multiples and divisors) Positive numbers & negative numbers Approximate values (error and significant figure)	Mutual relations and laws of four operations		[Course of Study for Lower Secondary (1970)] Positional notation Properties of natural numbers (primes, multiples, divisors) Positive numbers & negative numbers Approximate values (error and significant figure)		Four computations with positive and negative numbers With slide rule
	2nd Grade	Positive numbers & negative numbers	Four computations with positive and negative numbers		Structure of number sets		
	3rd Grade	Square root (rational and irrational numbers)	Using the table of square root & application of the laws of calculation		Square root (rational and irrational numbers)		Using the table of square root and slide rule & computation of simple algebraic expressions
UPPER SECOND.	[Course of Study for Upper Secondary (1960): Mathematics I] Complex numbers			Four computations with complex numbers		[Course of Study for Upper Secondary (1970): Mathematics I] Real and complex numbers Operations with real numbers	

Table 3. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 3)

	Course of Study (1977)			Course of Study (1989)			
	Number	Addition & Subtraction	Multiplication & Division	Number	Addition & Subtraction	Multiplication & Division	
PRIMARY	1st Grade	<ul style="list-style-type: none"> Representing the number and order of things with numbers (tens-unit) Size, order, and sequence of numbers Relating one number to the other number Representing 2-digit numbers 	<ul style="list-style-type: none"> Situations of Addition & Subtraction Addition & Subtraction with simple 2-digit numbers 	<ul style="list-style-type: none"> Counting collectively and dividing equally 	<ul style="list-style-type: none"> Representing the number and order of things with numbers (tens-unit) Size, order, and sequence of numbers Relating one number to the other number Representing 2-digit number 	<ul style="list-style-type: none"> Situations of Addition & Subtraction (meanings) Addition & Subtraction with simple 2-digit numbers 	<ul style="list-style-type: none"> Counting collectively and dividing equally
	2nd Grade	<ul style="list-style-type: none"> Thousands-unit Decimal positional notation, size, order of numbers up to 4-digit numbers 	<ul style="list-style-type: none"> Mutual relations of Addition & Subtraction with 3-digit numbers Written calculations Using the properties of Addition & Subtraction 	<ul style="list-style-type: none"> Situations of multiplication Using the properties of multiplication Multiplication facts 	<ul style="list-style-type: none"> Thousands-unit Decimal positional notation, size, order of numbers up to 4-digit numbers Relative size of number 	<ul style="list-style-type: none"> Mutual relations of Addition & Subtraction with 3-digit numbers Written calculations Using the properties of Addition & Subtraction 	<ul style="list-style-type: none"> Situations of multiplication Using the properties of multiplication Multiplication facts
	3rd Grade	<ul style="list-style-type: none"> Ten Thousands-unit Writing the numbers of 10, 100, 1/10 times How to write decimal numbers and fractions 	<ul style="list-style-type: none"> Using the properties of Addition & Subtraction Addition & Subtraction with decimal numbers and fractions With abacus 	<ul style="list-style-type: none"> Using the properties of multiplication Written calculation 3-digit \times 2-digit Meanings of division Written calculation of division Division by 1-digit 	<ul style="list-style-type: none"> Ten Thousands-unit Writing the numbers of 10, 100, 1/10 times Relative size of number How to write decimal numbers and fractions 	<ul style="list-style-type: none"> Using the properties of Addition & Subtraction Addition & Subtraction with decimal numbers and fractions With abacus 	<ul style="list-style-type: none"> Using the properties of multiplication Written calculation 3-digit \times 2-digit Meanings of division Written calculation of division Division by 1-digit
	4th Grade	<ul style="list-style-type: none"> Units of million and trillion Decimal positional notation Approximate numbers Meanings and writing of decimal numbers How to write fractions and its meanings 	<ul style="list-style-type: none"> Meanings and properties of Addition & Subtraction with decimal numbers Addition & Subtraction with same denominator fractions 	<ul style="list-style-type: none"> Meanings and properties of four operations Division by 2-digit Multiplication and division of decimal numbers by natural number 	<ul style="list-style-type: none"> Units of million and trillion Decimal positional notation Approximate numbers and its usefulness Meanings and writing of decimal numbers How to write fractions and its meanings 	<ul style="list-style-type: none"> Meanings and properties of Addition & Subtraction with decimal numbers Addition & Subtraction with same denominator fractions With abacus Estimation of sum and difference 	<ul style="list-style-type: none"> Meanings and properties of four operations Division by 2-digit Using the properties of division Multiplication and division of decimal numbers by natural numbers
	5th Grade	<ul style="list-style-type: none"> Classification of natural numbers (odd and even number) Multiplies and divisors Understanding decimal numbers deeply Equality and size of fractions Percentage (rate) 	<ul style="list-style-type: none"> Addition & Subtraction with different denominator fractions 	<ul style="list-style-type: none"> Division with decimal numbers Multiplication & division of fractions by natural numbers 	<ul style="list-style-type: none"> Classification of natural numbers (odd and even number) Multiplies and divisors Understanding decimal numbers deeply Equality and size of fraction Percentage and rate 	<ul style="list-style-type: none"> Addition & Subtraction with different denominator fractions 	<ul style="list-style-type: none"> Division with decimal numbers Estimation of product and quotient
	6th Grade	<ul style="list-style-type: none"> Mutual relation among natural numbers, decimal numbers and fractions Using numbers to represent possibility of uncertain events 	<ul style="list-style-type: none"> Possibility of four operations Multiplication & division with fractions 				<ul style="list-style-type: none"> Multiplication & division with fractions
LOWER S.	1st Grade	<ul style="list-style-type: none"> Properties of natural numbers (primes, multiples, divisors) Positive numbers & negative numbers Approximate values 	<ul style="list-style-type: none"> Four operations with positive and negative numbers 		<ul style="list-style-type: none"> Positive numbers & negative numbers 	<ul style="list-style-type: none"> Four operations with positive and negative numbers 	
	2nd Grade				<ul style="list-style-type: none"> Representation of numbers (binary system approximate value) 		
	3rd Grade	<ul style="list-style-type: none"> Square root (rational and irrational numbers) 	<ul style="list-style-type: none"> Calculation of basic expressions including square root 		<ul style="list-style-type: none"> Square root Prime numbers 	<ul style="list-style-type: none"> Calculation of basic expressions including square root 	
UPPER SECOND.	<ul style="list-style-type: none"> Course of Study for Upper Secondary (1978): Mathematics I Real and complex num. 			<ul style="list-style-type: none"> Course of Study for Upper Secondary (1989): Mathematics A, Mathematics B Real and Complex num. Calculations with real and complex numbers 			

Table 4. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 4)

	Course of Study (1998)			
	Number	Addition & Subtraction	Multiplication & Division	
PRIMARY	1st Grade	<ul style="list-style-type: none"> Representing the number and order of things with numbers (tens-unit) Size, order, and sequence of numbers Relating one number to the other number Representing numbers up to 100 	<ul style="list-style-type: none"> Situations of Addition & Subtraction (meanings) Addition & Subtraction with 1-digit numbers 	<ul style="list-style-type: none"> Counting collectively and dividing equally
	2nd Grade	<ul style="list-style-type: none"> Thousands-unit Decimal positional notation, size, order of numbers up to 4-digit numbers Relative size of number 	<ul style="list-style-type: none"> Mutual relations of Addition & Subtraction with 2-digit numbers Written calculations Using the properties of Addition & Subtraction 	<ul style="list-style-type: none"> Situations of multiplication and division with decimal numbers Multiplication facts
	3rd Grade	<ul style="list-style-type: none"> Ten Thousands-unit Writing the numbers of 10, 100, 1/10 times Relative size of number 	<ul style="list-style-type: none"> Addition & Subtraction with 3-digit numbers Using the properties of Addition & Subtraction With abacus 	<ul style="list-style-type: none"> Using the properties of multiplication Written calculations of multiplication 3-digit \times 2-digit Meanings of division Division by 1-digit
	4th Grade	<ul style="list-style-type: none"> Units of million and trillion Decimal positional notation Approximate numbers Meanings and writing of decimal numbers and fractions 	<ul style="list-style-type: none"> Meanings of Addition & Subtraction with decimal numbers Addition & Subtraction with simple decimal numbers (unit of 1/10) 	<ul style="list-style-type: none"> Using the properties of division Written calculations of division Division by 2-digit
	5th Grade	<ul style="list-style-type: none"> Classification of natural numbers (odd and even number) Understanding decimal numbers deeply Meanings and writing of decimal numbers and fractions Equality of fractions Percentage (rate) 	<ul style="list-style-type: none"> Meanings of Addition & Subtraction with fractions Addition & Subtraction with same denominator fractions (proper fractions) Estimation of sum and difference 	<ul style="list-style-type: none"> Meanings of multiplication and division with decimal numbers Multiplication & Division with simple decimal numbers (unit of 1/10)
	6th Grade	<ul style="list-style-type: none"> Divisors and multiples Equality and size of fraction 	<ul style="list-style-type: none"> Addition & Subtraction with different denominator fractions (proper fractions) 	<ul style="list-style-type: none"> Meanings of multiplication and division with fractions Multiplication & Division with fractions (except mixed fractions) Estimation of product and quotient
LOWER S.	1st Grade	<ul style="list-style-type: none"> Positive numbers & negative numbers 	<ul style="list-style-type: none"> Four operations with positive and negative numbers 	
	2nd Grade			
	3rd Grade	<ul style="list-style-type: none"> Square root (except square root table) Prime numbers 	<ul style="list-style-type: none"> Calculation of basic expressions including square root 	
UPPER SECOND.	<ul style="list-style-type: none"> Course of Study for Upper Secondary (1999): Basic Mathematics, Math I, Math II Number & Human Real numbers (rational and irrational numbers) Complex numbers Calculations with real numbers 			

changes were made. For example, the teaching and learning of real numbers and complex numbers was set in an optional mathematics subject at upper secondary school level.

Then in the new Course of Study (1998), the total number of school hours for mathematics is inevitably decreased by the implementation of the five-day schooling system. As a result, calculations are rather lightened; decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. In contrast, the meanings of number and four operations, the making use of properties of four operations to construct and learn algorithms of complicate calculations (Table 4).

3. Changes in “Algebraic Expression and Operations”

The table 5 shows the summary of changes of the teaching and learning contents regarding to the algebraic expression and operations. In this table, not only the content of algebraic expressions but also the content of symbolic expressions at elementary level and the content of equation and inequalities at secondary levels are included. In addition, there were no explicit mentions about algebraic expressions in the tentative Course of Studies (1947 and 1948). So the contents in this area in the Course of Studies (1951, 1958, 1968, 1977, 1989, and 1998) are summarized in the table 5.

As you can see in the table, regarding to the algebraic expression and operations, the contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). According to the emphasis of mathematical thinking, even at elementary school level, the following contents were gradually introduced; presenting the quantitative relations in symbolic expressions and interpreting these expressions in some real contexts; presenting and using mathematical formulas; using alphabetic letters and symbols such as x and \square . At the lower secondary level, basing on the experiences in elementary school, the rule of writing algebraic expressions, the multiplication and division with algebraic expressions, and the linear equations and quadratic equations were set as the teaching and learning contents. In the Course of Study (1960) for upper secondary mathematics, the fractional and irrational expressions, the higher-order equations, the quadratic inequalities were set as the teaching and learning

contents.

In the Course of Study (1968), at elementary school, using symbols such as \square and \triangle to represent numbers and quantities was set in the third grade; and the using alphabetic letters such as a and x instead of words and symbols of \square and \triangle was set in the fifth grade. At the lower secondary level, the addition and subtraction with linear expressions and the properties of equality and linear equations were set in the first grade; the properties of inequality and linear inequalities were set in the second grade; the multiplication and division with linear expressions and the quadratic equations were set in the third grade as main contents. The one prominent characteristic in this period is emphasizing the integrated view on equations and inequalities.

In the Course of Study (1977), the more careful attention was paid to introducing of symbols and alphabetic letters gradually in order to make a connection with algebraic expressions in elementary school as follows; using the symbol of \square in the third grade; using the symbols of \square and \triangle was in the fourth grade; and using the alphabetic letters such as a and x in the fifth grade. On the other hand, at lower secondary level, linear inequalities with two unknowns was eliminated.

In the Course of Study (1989), at elementary level, the activity of representing and interpreting symbolic expressions of four operations with whole numbers was put more emphasized. At lower secondary school, the simultaneous system of linear equation and inequality was eliminated. Moreover, at upper secondary level, the dealing with equations and inequalities was lightened.

Then in the new Course of Study (1998), as the total number of school hours for mathematics is decreased, some remarkable changes are made. At elementary school, using the sign of inequality is eliminated; using the symbols of \square and \triangle is introduced one year later than the before (1989). On the other hand, the activity of representing and interpreting symbolic expressions is put more emphasized. The using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. Moreover, at lower secondary level, the properties of inequality and linear inequalities are eliminated; the quadratic equations are limited to the simple and the solution formula of quadratic equations is not dealt in lower secondary school. These contents eliminated in the

Table 5. Changes of the Teaching and Learning Contents in "Algebraic Expression and Operations"

	C. S. (1951)	C. S. (1958)	C. S. (1968)	C. S. (1977)	C. S. (1989)	C. S. (1998)	
P R I M A R Y	1st Grade	·Writing and reading symbolic expressions by using signs of +, -			·Representing and interpreting situations of addition and subtraction with symbolic expressions	·Representing and interpreting situations of addition and subtraction with symbolic expressions	
	2nd Grade	·Writing and reading the relations among concrete things by using signs of +, -, ×	·Representing and interpreting quantitative relations (=, <)	·Representing and interpreting the things and relations (=, <)	·Representing and interpreting situations of multiplication ·Representing and interpreting the things and relations (=, <)	·Representing and interpreting situations of multiplication	
	3rd Grade	·Using symbolic expressions	·Putting the quantitative relations into formulas with words ·Making symbolic expressions by using a sign □ as unknown	·Putting the quantitative relations into formulas ·Making symbolic expressions by using signs □, Δ and inquiring numbers fit the signs	·Putting the quantitative relations into formulas ·Making symbolic expressions by using a sign □ and inquiring numbers fit the sign	·Representing and interpreting situations of division ·Putting the quantitative relations into formulas and interpreting them ·Making symbolic expressions by using a sign □ and inquiring numbers fit the sign	·Representing and interpreting situations of division
	4th Grade	·Putting symbolic expressions into one ·Writing expressions by using ()	·Order of Calculation ·Making symbolic expressions with () ·Meaning of sign = ·Putting the quantitative relations into formulas and using them	·Meanings and calculations expressions with () ·Properties of sign = ·Understanding the ideas of formula and using them	·Meanings and calculations expressions with () ·Ideas of formula ·Making symbolic expressions by using signs □, Δ and inquiring numbers fit the signs	·Meanings and calculations expressions with () ·Ideas of formula ·Making symbolic expressions by using signs □, Δ and inquiring numbers fit the signs	·Putting the relation regarding to division into formula ·Meanings and calculations expressions with () ·Understanding the ideas of formula and using them
	5th Grade	·Knowing the meaning of sign =	·Using expressions to solve quantitative problems ·Making expressions by using a symbol x as unknown and finding out the value of x ·Using formulas	·Understanding the applicability of formulas widely ·Using alphabetic letters a, x to represent quantity and finding out the values	·Understanding the applicability of formulas widely ·Using alphabetic letters a, x to represent quantity and finding out the values	·Understanding the applicability of formulas widely ·Using alphabetic letters a, x to represent quantity and finding out the values	·Summarizing the properties regarding to four operations by using signs □, Δ ·Understanding the applicability of formulas widely
	6th Grade		·Applying formulas in case of fractions and interpreting the relations expressed more generally	·Applying formulas in case of fractions and interpreting the relations expressed more generally ·Interpreting expressions in the form by noticing addition and multiplication structures			
L O W E R S E C O N D A R Y	1st Grade	·Representing quantities and the relations and rules by using alphabetic letters ·Alphabetic letters as unknowns ·Substituting value for x ·Rules for writing algebraic expressions ·Finding out a value of x by inverse operation	[C. S. (1969)] ·Representing algebraic expressions by using alphabetic letters ·Rules for writing algebraic expressions ·Addition & Subtraction with linear expressions ·Meanings of equation and inequality ·Properties of equality and linear equations	·Rules for writing algebraic expressions ·Addition & Subtraction with linear expressions ·Meanings of equation and linear equations with one unknown	·Using alphabetic letters ·Rules for writing algebraic expressions ·Addition & Subtraction with linear expressions ·Meanings of equation ·Properties of equality and linear equations with one unknown	·Using alphabetic letters ·Rules for writing algebraic expressions ·Addition & Subtraction with linear expressions ·Meanings of equation ·Properties of equality and linear equations with one unknown	
	2nd Grade	·Representing the quantities and relation by using letters and expressions (=, <) ·Rules for writing algebraic expressions ·Using letter x ·Properties of equality ·Simple equations	·Meanings of alphabetic letters and algebraic expressions ·Calculations with algebraic expressions (Add & Sub; polynomials) ·Properties of equality ·Linear equations and simultaneous equations	·Calculations with algebraic expressions ·Properties of inequality ·Linear inequalities with one unknown and with two unknowns ·Linear equations with two unknowns ·Simultaneous system of linear equation and inequality	·Calculations with algebraic expressions ·Using algebraic expression ·Transforming simple equalities ·Properties of inequality & linear inequalities with one unknown ·Linear equations with two unknowns ·Simultaneous system of linear equation and inequality	·Calculations with algebraic expressions (Add & Sub; polynomials) (Mul & Div; monomials) ·Using algebraic expression ·Transforming simple equalities ·Properties of inequality & linear inequalities with one unknown ·Simultaneous system of linear equations with two unknowns	·Calculations with algebraic expressions (Add & Sub; polynomials) (Mul & Div; monomials) ·Using algebraic expression ·Transforming simple equalities ·Simultaneous system of linear equations with two unknowns
	3rd Grade	·Linear equations ·Simultaneous system of linear equations ·Transforming simple equalities ·Calculations with algebraic expressions ·Expanding and factorizing algebraic expressions	·Calculation with algebraic expressions (expansion & factorizing by formulas) ·Replacing an expression by one alphabetic letter ·Quadratic equations ·Simultaneous system of linear equations	·Multiplication with linear expressions ·Expanding and factorizing algebraic expressions by formulas ·Quadratic equations ·Linear inequalities with two unknowns	·Multiplication with linear expressions ·Expanding and factorizing algebraic expressions by formulas ·Quadratic equations ·Quadratic equations (by factorizing, solution formula etc.)	·Multiplication with linear expressions ·Expanding and factorizing algebraic expressions by formulas ·Quadratic equations (by factorizing, solution formula etc.)	·Multiplication with linear expressions ·Expanding and factorizing algebraic expressions by formulas ·Simple Quadratic equations (except solution formula)
U P P E R S E C O N D A R Y	[Analysis I] ·Four operations with integral expressions and fractional expressions ·Simultaneous system of linear equations ·Quadratic equations (relations between roots and coefficients; discriminant) ·Properties of equality and inequality ·Fractional equation & irrational equation ·Properties of proportional expressions	[C. S. (1960)] [Mathematics I] ·Four operations with integral expressions and fractional expressions ·Irrational expressions ·Quadratic equations (discriminant) ·Fractional equation & irrational equation ·Higher-order equations ·Simultaneous system of quadratic equations with two unknowns ·Linear inequalities & quadratic inequalities	[C. S. (1970)] [Mathematics I] ·Four operations with integral expressions and rational expressions ·Quadratic equations ·Equations & factor theorem ·Simple absolute inequalities ·Quadratic inequalities	[C. S. (1978)] [Mathematics I] ·Integral expressions and rational expressions ·Quadratic equations ·Simple higher-order equations ·Simultaneous system of equations ·Quadratic inequalities ·Algebraic expressions and proof	[Mathematics I] ·Quadratic equations & Quadratic inequalities (only real roots) [Mathematics A] ·Integral expressions ·Equalities and inequalities [Mathematics B] ·Complex numbers & Quadratic equations	[C. S. (1999)] [Mathematics I] ·Linear inequalities ·Quadratic equations & Quadratic inequalities (only real roots) [Mathematics II] ·Division with integral expressions & fractional expressions ·Complex numbers & Quadratic equations	

lower secondary are shifted to and set in the mathematic subject at upper secondary school level.

4. Conclusion

The purpose of this paper was to summarize the changes of teaching and learning contents in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in the Course of Studies for elementary and secondary school mathematics after 1945 in Japan. To do so, the tables of 1 to 5 were shown.

As a result of overview and some considerations, the followings are found out as three main characteristics. (a) There is a remarkable difference between the period of “Life-Unite Learning” (1945 – 1957) and the period of “Systematic Learning” (1958 – 1967) regarding to the teaching and learning contents and their sequences in these areas shown in the Course of Studies. (b) The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). In the new one (1998), however, we notice that calculations are rather lightened; decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. (c) The dealing with equations and inequalities in secondary schools is gradually lightened in recent years.

Generally speaking, education is the dynamic system with purposes and designs to meet and supply the needs of society and students through long generations, not excepting mathematics education. In this sense, it is important for us as mathematics researchers/educators to rethink the teaching and learning contents and their sequences in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in one whole vision including their aims, teaching methods, and the ways of evaluation in light of historical lessons.

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